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The Application of Non-Dimensional Methods to the Planning of Helicopter Performance Flight Trials and the Analysis of Results

By

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P. A. Knowles

Summary

The advantages of using non-dimensional methods in helicopter performance trials are outlined and the non-dimensional groups of parameters used to define level flight, partial climb, vertical climb, autorotation and hovering are stated. Each of these flight modes is examined in turn and the methods of planning trials, preparing flight data and analysing results are discussed. Some examples of trial planning are included as an Appendix.

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1. Introduction

As experience has been gained at A. and A.E.E. on the testing of helicopters fitted with turbine engines, methods of testing and analysis have been modified and, in particular, more use has been made of non-dimensional methods. Earlier Reports (Ref. 1, 2, 3, 4) have dealt with the basis of the methods and with particular techniques; the present Report extends the work to describe the detailed planning of flight trials and, for convenience and because of the limited circulation of some of the previous work, contains sufficient of the earlier material to form a coherent account of both the planning of trials and the analysis of the results.

The use of non-dimensional methods in helicopter performance flight testing, can eliminate a large part of the labour of reducing test results to standard conditions, which occurs with conventional flight test techniques, while at the same time increasing the probable accuracy of the results and, in some cases, decreasing the flying time necessary to obtain the required information from a trial. It will be shown, however, that to obtain the maximum information from this method, it is essential that trials should be carefully planned in advance. The planning process will be examined for various types of test, but the methods shown here are not unique and may be adapted or replaced according to the needs of the test programme under consideration and the preferences of the Trials' Officer, provided that certain essential conditions are fulfilled. These may be summarised by four questions which will have been answered when a trial has been planned:-

1. What results are required?

2. (a) Can these results be obtained at the trial site?

or (b) What characteristics (temperature and altitude) should the trial site have?

3. What flights are necessary to obtain the required results?

4. What information is necessary to allow these flights to be made?

The importance of pre-trial planning can be seen by considering a test for which results are required for the helicopter operating in both temperate and tropical conditions. By careful planning it may sometimes be possible to obtain all the results by operating from one site but even if this is not feasible it will usually be possible to obtain, from flights made in, say, temperate conditions, sufficient results applicable to tropical conditions to enable the tropical trial to be planned economically and to spot at an early stage any inconsistency between the temperate and tropical results.

2. Parameters used in the Non-Dimensional Method

The non-dimensional parameters which define a particular mode of flight can be conveniently and most accurately obtained by dimensional analysis and the results of doing this in relation to helicopter performance are stated in Ref. 1. These results will form the basis of the methods discussed in this Report and for completeness the assumptions made and the results obtained will be stated briefly.

/It

It is assumed that the performance of a member of a family of geometrically similar helicopters depends only on its size, power, steady state motion, weight and rotor speed and on air pressure and temperature. It will be shown that it is sometimes necessary to consider the case where pressure and temperature are replaced by density. This simplifies the application of the method in some ways but some loss of accuracy may be incurred since the results obtained will not be at a standard, constant Mach No.. The viscosity and the humidity of the air and the mass distribution of the ro or system are neglected.

The most general case that will be considered is that of a machine in climbing and translational flight with the state of the air defined by its pressure and temperature. Then, using symbols defined on pages 29 and 30

$$P = f(R,\ V,\ V_c,\ W,\ \Omega,\ p,\ T)$$

and dimensional analysis gives

$$\frac{P}{pR^2\sqrt{T}} = f\left(\frac{V}{\Omega R}\right)\left(\frac{\Omega R}{\sqrt{T}}\right)\left(\frac{V_c}{\Omega R}\right)\left(\frac{W}{pR^2}\right) \qquad \text{—— (1)}$$

If instead, air density is used

$$\frac{P}{\rho\Omega^3R^5} = f\left(\frac{V}{\Omega R}\right)\left(\frac{V_c}{\Omega R}\right)\left(\frac{W}{\rho\Omega^2R^4}\right) \qquad \text{—— (2)}$$

It is, of course, possible to define the state of the air in terms of its density and temperature in which case a form similar to (2) is obtained, except that there is the additional term $\left(\frac{\Omega R}{\sqrt{T}}\right)$ and results using this form would be at a standard Mach No. In practice this form is not used because air density cannot be easily measured and pressure and temperature can: the only advantage of bringing density into the picture is to obtain the form given by (2) in which the power parameter is a function of three variables - one less than that found in (1).

It is convenient to replace the atmospheric pressure, density and temperature by their ratios to sea level standard values and also to refer the rotor speed to a standard value. Then for a particular helicopter where R is constant, (1) and (2) may be written

$$\frac{P}{\delta\sqrt{\theta}} = f\left(\frac{V}{\omega}\right)\left(\frac{\omega}{\sqrt{\theta}}\right)\left(\frac{V_c}{\omega}\right)\left(\frac{W}{\delta}\right) \qquad \text{—— (3)}^*$$

$$\text{and } \frac{P}{\sigma\omega^3} = f\left(\frac{V}{\omega}\right)\left(\frac{V_c}{\omega}\right)\left(\frac{W}{\sigma\omega^2}\right) \qquad \text{—— (4)}^*$$

/If

* These quantities are no longer, of course, non-dimensional but give convenient numerical values. They are often, loosely, referred to as "non-dimensional parameters".

If a test has to be made with constant rotor speed it is impossible to maintain $^{\omega}/\sqrt{\theta}$ constant and results will be at a non-standard Mach No. unless the ambient temperature during the tests is that of the standard condition for which results are required. The form obtained from (4), with ω = constant, must then be used. Thus

$$\frac{P}{\sigma} = f\left(\frac{W}{\sigma}\right)\left(V\right)\left(V_c\right) \qquad (5)^{*}$$

Equations (3) - (5) are the basic forms used in performance measurement but it will be found convenient to rearrange them for certain tests: this will be noted in the discussion of the test concerned. As has already been mentioned results obtained by the use of the forms given in equations (4) and (5) will not be at a standard Mach No. This is an important qualification and its implications will be discussed in Section 3. If equation (3) is used all results will be at a standard Mach No. and both the Mach No. based on the forward speed, V, and the Mach No. based on the rotor speed, Ω, will be correct if $^{\omega}/\sqrt{\theta}$ and $^{V}/\omega$ are kept constant. It is possible to write equation (3) as

$$\frac{P}{\delta\sqrt{\theta}} = f\left(\frac{V}{\sqrt{\theta}}\right)\left(\frac{\omega}{\sqrt{\theta}}\right)\left(\frac{V_c}{\sqrt{\theta}}\right)\left(\frac{W}{\delta}\right)$$

and it is seen that in this form the various parameters proportional to Mach No. are explicity stated.

In obtaining (3) - (5) the viscosity of air has been neglected and thus results will not be at a standard Reynolds No. It is not possible to maintain Reynolds No. constant if $^{V}/\omega$ and $^{\omega}/\sqrt{\theta}$ are to be constant but it is possible to design special tests to check the effect of changing Reynolds No. These tests will be discussed in Section 4 but for current helicopters it has been found (from a limited number of tests) that the error in neglecting Reynolds No. is small.

The non-dimensional groups used to define performance have been obtained by dimensional analysis and this is the recommended method. Dimensional analysis, however, does not help in deciding the relative importance of the parameters obtained and it is sometimes necessary to examine the power equation to decide the best method of grouping the basic quantities during the dimensional analysis. A good example of this technique will be used in Section 6 but since it is of general interest the equations will be examined now.

The power required at the rotor in steady flight may be stated as

Power required = Induced power + Climb power + Parasitic power + Profile power

and for performance calculations a commonly used form of this equation is

/P =

* These quantities are no longer, of course, non-dimensional but give convenient numerical values. They are often, loosely, referred to as "non-dimensional" parameters".

$$P = Wv + WV_c + \frac{D_{100}\rho V^3}{10^4 \rho_o} + \frac{\rho}{8} As\Omega^3 R^3 \; C_D \; (1 + n\mu^2) \qquad \text{——— (6)}$$

where $C_D = f\left(\frac{W}{\rho\Omega^2}\right)$ and $v = W/\left(2\Pi R^2 \rho \sqrt{V^2 + (V_c + v)^2}\right)$

rearranging with all constants denoted by C where the definition and numerical value of C varies on each occasion it is used, gives

$$\frac{P}{\sigma\omega^3} = \left(\frac{W}{\sigma\omega^2}\right)\left(\frac{v}{\omega}\right) + \left(\frac{W}{\sigma\omega^2}\right)\left(\frac{V_c}{\omega}\right) + C\left(\frac{V}{\omega}\right)^3 + CC_D \; [1 + C\left(\frac{V}{\omega}\right)^2]$$

where $C_D = f\left(\frac{W}{\sigma\omega^2}\right)$ and $\frac{v}{\omega} = C \frac{W/\sigma\omega^2}{\sqrt{(V/\omega)^2 + [(^Vc/\omega) + (^v/\omega)]^2}}$

Thus $\frac{P}{\sigma\omega^3} = f\left(\frac{W}{\sigma\omega^2}\right)\left(\frac{V_c}{\omega}\right)\left(\frac{V}{\omega}\right)$

It would be expected that the power equation, being dimensionally correct, would yield non-dimensional groups and in this case the result agrees with equation (4) but it is seen that the use of a simple form of the power equation has resulted in the power parameter being independent of Mach No. In fact, of course, Mach No. effects could have been included by writing the blade drag coefficient as a function of $^\omega/\sqrt{\theta}$ as well as $^W/_{\sigma\omega}2$.

If a test has to be made with constant rotor speed then

$\frac{P}{\sigma} = f\left(\frac{W}{\sigma}\right) \; (V_c) \; (V)$

This form is convenient for level flight tests (with $V_c = 0$) but for climbing flight it is more convenient to change the form to

$\frac{P}{W} = f\left(\frac{W}{\sigma}\right)(V_c)(V)$

so that power and density are both functions of weight alone.

When operating close to the ground to measure vertical performance it is not possible to keep $^W/\sigma$ constant without ballasting between tests. To see the effect of changes in $^W/\sigma$ consider equation (6) which may be written

$$^P/W = v + V_c + \frac{CV^3}{^W/\sigma} + \frac{CC_D}{^W/\sigma} \; [1 + CV^2] \qquad \text{——— (7)}$$

where $C_D = f\;(^W/\sigma)$ and $v = C \frac{^W/\sigma}{\sqrt{V^2 + [V_c + v]^2}}$

/and

and it is seen that since both the induced power and profile power terms depend on W/σ neither will be correct if W/σ varies. To use either of these forms would lead to serious inaccuracies but the problem can be, at least, partially solved. From equation (7), multiplying through by $\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}$

$$\frac{P\sigma^{\frac{1}{2}}}{W^{3/2}} = \frac{v\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} + \frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} + C\left(\frac{V\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)^3 + CC_D\left(\frac{W}{\sigma}\right)^{-3/2}\left[1 + CV^2\right]$$

where $C_D = f\left(W/\sigma\right)$ and $\dfrac{v\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} = \dfrac{1}{C\sqrt{\left(\dfrac{V\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)^2 + \left[\left(\dfrac{v\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right) + \left(\dfrac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)\right]^2}}$

and since $v\sigma^{\frac{1}{2}}/W^{\frac{1}{2}} = f\left(V\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}\right)\left(V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}\right)$,

$$V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}} = f\left(\frac{P\sigma^{\frac{1}{2}}}{W^{3/2}}\right)\left(V\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}\right)\left(\frac{W}{\sigma}\right)\left(V\right) \qquad \text{——— (8)}$$

Only the profile power term contains W/σ and V and thus, if when measuring rate of climb near the ground, the terms $P\sigma^{\frac{1}{2}}/W^{3/2}$ and $V\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}$ are kept constant, the results will be correct except for profile drag. This error will be significant but for any test it would be worthwhile to estimate the error for the probable change in weight to see if, or how often, reballasting would be necessary.

In this section power, P, has been taken to mean power required by the rotor and it should be emphasised that this is not the same thing as the power given by the engine. Nor, in general, do the two have a simple relationship: The power absorbed by the tail rotor and in mechanical losses in the engine and transmission will vary not only with air density, temperature etc. but also with factors like the grade of oil used in the engine and the variation may not be a function of the parameters used to define the power required at the rotor. Results predicted for standard conditions from tests made in non-standard conditions will only be true in terms of the power required at the rotor. The error in speaking of engine power is likely to be small and since eventually most results have to be related to engine power to obtain, for instance, maximum rate of climb or hover ceiling it is better in practice to measure the engine power rather than rotor power if a torquemeter is being used. Exceptionally, if an accurate engine calibration for the engine to be used in the trial is available rotor power may be measured.

3. General Principles

If a non-dimensional approach is not employed the most common method of testing helicopters is to fly in conditions as near to the standard conditions (at which results are required) as possible and to make allowance for variations from the standard conditions when reducing the results. The variations are of two sorts; first those which are inherent in the non-standard condition but which remain constant during a flight, e.g. ambient temperature, and, secondly, those which, although they may be correct at some point in time, vary during flight, e.g. weight. Now, if a non-dimensional approach is used the standard conditions will imply certain values of the non-dimensional parameters which

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define the particular test under consideration and provided that these are numerically correct during the test the results obtained (in non-dimensional form) will be valid, within the assumptions made to obtain the non-dimensional parameters, and no reduction will be necessary except to obtain the dimensional results required from the non-dimensional test results: similarly when a non-dimensional result has been obtained for the particular value of the non-dimensional parameter it is also valid for any combination of the dimensional variables provided that they yield the correct value of the parameter, and provided that, for any given set of the dimensional variables the value of the dependent variable inherent in the value of the non-dimensional result is feasible. For example, it is possible to predict a power for level flight in certain conditions where, in those conditions, the engine would be incapable of giving that power. Engine and airframe limitations must also be taken into account when predicting such results. Once the value of a non-dimensional parameter appropriate to the standard conditions has been decided it is immaterial what values the individual variables take for the test provided the value of the parameter as a whole is correct. If, as is usual during a flight, more than one test is made it is necessary to ensure that the value of the non-dimensional parameter is correct at the start of each test. Thus if the parameter $(^{W}/\delta)$ is being maintained constant the altitude for successive tests has to be increased as fuel is used and the A.U.W. decreases.

If a trial is to be made using non-dimensional methods five aspects should be considered when planning the trial.

1. The choice of non-dimensional parameters for the particular test to be made. In all cases discussed in this Report these will be one of the sets of parameters contained in equations (3), (4) or (5) or a form derived from one of these equations. If it is possible to vary the rotor speed either equation (3) or equation (4) will be used but as explained in Section 2 results obtained using equation (3) will be at a standard Mach No. and will, therefore, be more accurate. Besides this advantage it will be shown that this method is also easier to use in the air but against these advantages is the fact that equation (4) contains one less parameter than equation (3) and the use of equation (4) allows all the results from level flight tests to be presented in one diagram.

Up to now most level flight testing has been carried out using the $^{W}/\delta$ method of equation (3) but recently an attempt has been made to assess the probable loss in accuracy if the $^{W}/_{\sigma\omega}2$ method of equation (4) is used. The method of doing this is discussed in Section (4) and, so far, from a limited number of tests, the results have been inconclusive. When considering this question it should be borne in mind that the loss of accuracy will depend to a great extend on the differences between the standard conditions for which results are required and the test conditions.

It is recommended that, at present, the $^{W}/\delta$ method should be used unless there is experimental evidence that the loss in accuracy inherent in the $^{W}/_{\sigma\omega}2$ method will be tolerable.

Turning now to helicopters which have a fixed rotor speed. First, it should be pointed out that the variation in rotor speed needed in many tests is small and that the rotor speed of machines, which are normally flown at constant rotor speed can often be changed sufficiently by altering the governor datum etc. If, however, the rotor speed cannot be varied the results will be at a non-standard Mach No. and the loss in accuracy which will result from differences between the standard and test conditions must be considered.

/Until

Until more tests are made it is not possible to state what variation in, say, temperature is tolerable but two criteria may be considered. First, what variation of rotor speed would be needed to obtain the correct Mach No.? If this is of the same order as the variation a pilot would normally accept as usual then it is probably fair to assume that the corresponding Mach No. effects will be small since, in any case, it must be accepted that such variation will occur during normal operations. A 1% change in rotor speed is approximately equivalent to a 6°C change in temperature at constant altitude. Secondly, what alternative method of testing is available? The obvious alternative is to use one of the conventional reduction methods. These vary in detail but do not, usually, account for Mach No. effects and therefore for a given difference between the standard and test temperature there is no advantage in not using the non-dimensional method on this account. Additionally the other advantages of the non-dimensional approach remain and in particular the need to assume a constant mean blade drag coefficient, inherent in many reduction methods, is not necessary.

2. Expressing the ranges of parameters (altitude, weight, temperature etc.) for which results are required in terms of the non-dimensional parameters, and predicting the amount of information which it will be possible to obtain from the ambient conditions at a given test site. Once the choice of non-dimensional parameters has been made it is quite straight forward to obtain the extremes of the ranges required: for example if information is required at speeds up to a value, V, and at several rotor speeds the lowest of which is Ω, then the highest value of the non-dimensional parameter, μ, is $^{V}/_{\Omega R}$. Then, if only one non-dimensional parameter is to be varied in the test and the range of interest in that parameter has been found, it is then possible to decide at how many values of the parameter tests should be made to insure satisfactory results. One difficulty that may sometimes arise is the case of varying a parameter which will give a discontinuity in the results. An example is when results are being obtained by varying the test altitude using a gas turbine engine giving maximum power. Then, if the type of engine limitation changes with altitude, the plot of the results may change slope abruptly at the point where the limitation changed and this may mean that rather more test points than usual would be needed to define adequately the test plot on either side of the point at which the discontinuity occurs.

If two or more non-dimensional parameters are to be varied during the trial then it is necessary to calculate the range of each over which tests must be made. It does not follow, however, that it is necessary to consider every possible combination of, say, two non-dimensional parameters to obtain results over the full range of the basic (dimensional) parameters. Only those combinations which can arise from the basic parameters need be considered. To make this clear consider a test to find the power required for level flight at A.U.W., W, and rotor speed, ω, over a range of altitudes, in two standard atmospheres. The non-dimensional parameters are $^{W}/_{\delta}$ and $^{\omega}/_{\sqrt{\theta}}$ and it is clear that the highest value of $^{W}/_{\delta}$, obtained from the lowest value of δ, i.e. the highest altitude, need not be considered with the lowest value of $^{\omega}/_{\sqrt{\theta}}$ obtained by using the highest temperature appropriate to the hotter of the two standard atmospheres specified, i.e. at sea level. This is, of course, obvious in this instance but it is always necessary to ensure that anomalies do not occur. If there is any doubt it is usually possible to construct a simple diagram to show the areas of interest and some examples will be described in this Report. Such a diagram is also useful in the next stage of planning a trial, deciding the amount of information which it will be possible to obtain by flying from the trial site (if this has been fixed) or alternatively deciding the necessary characteristics of the trial site(s). For a particular site and for a particular helicopter the envelope of possible performance in terms of the non-dimensional parameters involved can be drawn, and superimposition of the diagram showing the required envelope will allow the areas (in terms of the non-dimensional quantities) which cannot be covered to be seen: this information can then, if required, be transformed to show the deficiency in terms of the dimensional qualities.

/3.

3. Provision of data to be used in flight. During each flight it is necessary to maintain the values of the non-dimensional parameter(s) at some predetermined value or values and it is therefore necessary to provide means to do this. There is often more than one way in which information can be presented and it is worthwhile to take care to choose the simplest way for use in the air. Basically there are two ways of presenting data: graphically or in tabular form. Thus for the parameter $^W/\delta$, which, if kept constant, means that the operating altitude must be increased as the A.U.W. decreases, a list of altitudes corresponding to a series of weights might be drawn up or alternatively a line of $^W/\delta$ = constant, drawn on a plot of A.U.W. versus altitude (see Fig. 1). The choice between these methods may sometimes depend on the preferences of the man who is to use the data but, in general, the graphical method is to be preferred because it gives a continuous set of values and allows a lot of information to be given on one piece of paper. One further advantage is that there is usually less computational work required in constructing a graph: when tables have been used, it has been found that the use of a digital computer to compile them is often worthwhile.

4. Consideration of priorities during the trial. It is often the case that when a trial has been planned to take place from a particular location some of the tests will only be possible when weather conditions are favourable. One obvious example is tethered hovering which can only be done in very light winds. The other factor which is usually important is temperature and whilst it may be obvious that advantage should be taken of extremes which may occur infrequently, doing so may involve a different sequence of tests from that which would otherwise be chosen. This point will be exemplified in the Sections of this Report dealing with individual tests but, in general, it is worthwhile to consider the matter before the trial starts, during the planning stage.

5. Presentation of results. The results obtained from tests should be plotted against the parameters used to define the test and the dimensional results should be obtained by reduction from values taken from faired lines through the test points. The reduction of test data is usually extremely simple and little or no preparation for reduction is required when planning a trial; there are however two aspects worth considering and these have both been briefly mentioned in the last Section. First, the choice of non-dimensional parameters to define a particular test can affect the presentation of results since if the number of variables can be reduced to three all the results can be shown on a single graph. This is a valuable simplification but must not be an overriding factor in the choice of parameters. Secondly, it is sometimes impossible to plot the results satisfactorily against any other parameters than those used during the flight. In particular a level flight test made at constant $^W/_{\sigma\omega^2}$ will not usually yield a set of results at constant $^W/\delta$.

Although the results obtained from a trial, where the non-dimensional method was used would normally be presented as a plot of non-dimensional quantities and the required results obtained from this plot, it is still possible to apply a conventional reduction method to the data. This would not normally be necessary since this method is much more complicated and the accuracy will depend on the assumptions made in the reduction process, but is mentioned to allay any feeling that if non-dimensional tests are made and something goes wrong, all is lost.

/Finally

Finally it should be pointed out that although using the non-dimensional methods calls for a rather different flying technique to that hitherto employed, the effect of deviations from the required operating conditions will, in general, be of the same order in both cases.

4. Level Flight

The most usual reason for this test is to find the power required to maintain level flight for a range of forward speeds at specified weights, rotor speeds, heights and ambient temperatures. The results are presented as curves of power required against forward speed.

Dealing first with the construction of the programme, it is necessary to decide at the outset which set of parameters is to be used to define level flight. The choice is

$$P/\delta\sqrt{\theta} = f\left(W/\delta\right)\left(\omega/\sqrt{\theta}\right)\left(V/\omega\right) \quad \text{from equation (3)}$$

$$\text{or } P/\sigma\omega^3 = f\left(W/\sigma\omega^2\right)\left(V/\omega\right) \quad \text{from equation (4)}$$

As explained in Section 3 the former expression is preferable for greater accuracy and simplicity of use in the air but the second method will often allow a wider range of results to be obtained in a given situation and the results are more easily presented.

If the rotor speed is fixed there is no choice and the parameters are

$$P/\sigma = f\left(W/\sigma\right)\left(V\right) \quad \text{from equation (5)}$$

The standard conditions, for which results are required, may include a range of weight, height, temperature, rotor speed and forward speed and to design a programme it is necessary to find the extreme limits of the ranges of the parameters to be used. If, for example, the parameters are W/δ, $\omega/\sqrt{\theta}$ and V/ω the upper limit of W/δ would be given by the maximum weight and maximum altitude. The programme will then consist of tests at specified values of W/δ, $\omega/\sqrt{\theta}$ and V/ω, and will automatically include the range of W, h, T, Ω, V provided the appropriate range of W/δ, $\omega/\sqrt{\theta}$ and V/ω are covered. It can be seen that the number of parameters has been reduced. It may be found that it is physically impossible to obtain the required values of the parameters at a particular trial site, but this limitation would have occurred whatever test method had been used. Obviously the same care is needed here, as in any other system, to ensure, in the planning stage, that it is possible to obtain the required results at the trial location. If the parameters chosen are $P/\sigma\omega^3$, $W/\sigma\omega^2$ and V/ω or if the rotor speed is fixed, a similar process is adopted; planning in terms of $W/\sigma\omega^2$ and V/ω or W/σ and V.

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If more than one non-dimensional parameter is to be altered during the tests the exact combinations of the parameters which need to be flown must be found. The best way to explain this is by a worked example: one is shown in Appendix I. This example also illustrates a method of depicting graphically the results which may be obtained by flying in any given set of ambient conditions.

It is also necessary when planning a trial to ensure that the tests can be made without reaching a limitation which occurs in the test conditions but not in the standard conditions. The limitation might be the maximum permitted speed, engine power available or maximum available collective pitch. In general, this difficulty is not usually serious in level flight tests but it may occur especially if results are required at or near maximum forward speed.

Two other results are sometimes required from level flight trials: collective pitch and fuel flow measurements. Dealing first with collective pitch, the non-dimensional parameter here is ϑ, the collective pitch angle measured from an arbitrary datum, and this may replace the power parameter in any of the expressions given. Thus

$$\vartheta = f\left({}^{W}/\delta\right)\left({}^{\omega}/\sqrt{\theta}\right)\left({}^{V}/\omega\right) \text{ etc.}$$

Fuel flow is less simple. For a turbine engine (from Ref. 2) the non-dimensional fuel flow parameter ${}^{G}/\delta\sqrt{\theta}$ is a function of ${}^{N_c}/\sqrt{\theta}$, ${}^{\omega}/\sqrt{\theta}$ and Δ assuming that the effect of airspeed can be neglected and that the efficiency of the engine is constant. ${}^{P}/\delta\sqrt{\theta}$ is also a function of ${}^{N_c}/\sqrt{\theta}$, ${}^{\omega}/\sqrt{\theta}$ and Δ and thus ${}^{G}/\delta\sqrt{\theta}$ can replace ${}^{P}/\delta\sqrt{\theta}$ in eqn. (3) provided that the transmission and tail rotor losses are assumed constant, giving

$${}^{G}/\delta\sqrt{\theta} = f\left({}^{W}/\delta\right)\left({}^{\omega}/\sqrt{\theta}\right)\left({}^{V}/\omega\right)\left({}^{V_c}/\omega\right)$$

Alternatively if the engine is to be calibrated by air tests giving plots of ${}^{P}/\delta\sqrt{\theta}$ against ${}^{N_c}/\sqrt{\theta}$, ${}^{\omega}/\sqrt{\theta}$ and Δ, the fuel flow can be measured at the same time and curves of ${}^{G}/\delta\sqrt{\theta}$ against ${}^{N_c}/\sqrt{\theta}$, ${}^{\omega}/\sqrt{\theta}$ and Δ drawn. Thereafter the fuel flow can be found from the engine setting in any condition of flight. In practice it may be difficult to obtain the variation of ${}^{G}/\delta\sqrt{\theta}$ with Δ if the latter cannot be adjusted independently of the other engine controls but in many cases it will be found that in the regions of interest the guide vanes are fully open and therefore Δ = constant. The variations of ${}^{G}/\delta\sqrt{\theta}$ with ${}^{\omega}/\sqrt{\theta}$ and Δ at constant ${}^{N_c}/\sqrt{\theta}$ are often small but should be checked for each type of engine.

Similar methods may be developed for a piston engine but may not be satisfactory in practice. Thus for an unsupercharged engine ${}^{G}/\delta\sqrt{\theta}$ may be written as a function of ${}^{W}/\delta$, ${}^{\omega}/\sqrt{\theta}$, ${}^{V}/\omega$ and ${}^{V_c}/\omega$ as before but one difficulty is to decide what temperature is appropriate for θ. The power demanded from

/the

the engine depends on the ambient air temperature but the power produced by the engine and hence the fuel flow depends on the carburettor air temperature. The temperatures may vary widely especially if carburettor heat is used. A further objection is that the expression for $^G/\delta\sqrt{\theta}$ is only true provided that the engine efficiency remains constant. The uncertainty about temperature may be removed by working in terms of engine parameters when, with manifold pressure, B, and relative carburettor air temperature θ_c, $^G/\delta\sqrt{\theta_c} = f\left(^B/\delta\right)\left(^\omega/\sqrt{\theta_c}\right)$. The doubt about engine efficiency remains, however, and this method should be used with caution.

Consider now the operating conditions for the case of a helicopter with variable rotor speed, and assuming that for a particular test $^W/\delta$ and $^\omega/\sqrt{\theta}$ are known. It is then necessary to decide the take-off weight, the initial height and the initial rotor speed in relation to the ambient temperature on the day of the test. If $^\omega/\sqrt{\theta}$ is specified, the temperature range for the available rotor speed range is fixed and since during the course of the flight the machine will have to gain height it is desirable to start the test with a high rotor speed. Thus the initial operating temperature can be found and assuming a lapse rate the approximate initial height may be estimated from the temperature on the ground. It then remains to select a suitable take-off weight to allow the helicopter to operate at this height. It may prove impracticable to start flying near the highest rotor speed available and this may mean that the flight may have to be curtailed when the rotor speed drops below the minimum permitted value. It would be possible to decide in the planning stage what ranges of ground temperatures would permit flying at any given combination of $^W/\delta$ and $^\omega/\sqrt{\theta}$ and hence utilise variations in temperature on the test site as effectively as possible. Such an exercise may or may not appeal to any individual but in any case it would be worthwhile considering the problem in the planning stage and bearing it in mind during the actual trial. The question of engine limitations must also be considered since these may be reached at certain combinations of temperature and weight at the test site and prevent the completion of all the tests necessary to obtain results for the standard conditions.

If it is impossible to meet the rotor speed requirement in the prevailing temperatures it may sometimes be best to fix the rotor speed at its value for the standard conditions and fly with $^W/\sigma$ constant. $^W/\sigma$ constant is the only condition for a fixed rotor speed machine and at a given ground temperature the take-off weight simply specifies the height at which the tests will be flown.

The flight plan will vary depending on whether a fixed or variable rotor speed machine is used. With a variable rotor speed it has been shown that $^W/\delta$ and $^\omega/\sqrt{\theta}$ will have been decided for the flight in question and must then be maintained constant throughout the required range of forward speeds in order that $^P/\delta\sqrt{\theta}$ shall yield a true value of P at any given $^V/\omega$. In order to maintain $^W/\delta$ constant as fuel is used it will be necessary to increase the height at which tests are made as the flight proceeds. At the same time any temperature variations, whether due to atmospheric changes or to a change of operating height, must be compensated by a change of rotor speed. Obviously during any one run, the height and rotor speed must remain constant. The

/height

height appropriate to any fuel state and the rotor speed at any temperature will be presented in a suitable form for use in the air. A machine with a fixed rotor speed is rather less straightforward. Only one parameter, W/σ, must be maintained constant, but since air density cannot be displayed directly in the aircraft a method of showing the correct altitude for any fuel state and any temperature is necessary. Thus for a given fuel state an altitude must be found where the ambient temperature is such as to give the required air density. Some anticipation will be required to find the initial operating height but thereafter changes will be small.

It is possible to present the necessary information in tabular form. In all cases weight variation must be taken into account and there are two methods of doing this: either by forming the tables in terms of the aircraft weight or in terms of the fuel state. Usually it will be simpler to consider aircraft weight since, although it may sometimes be possible to read the fuel state directly off a gauge, there is little difficulty converting this reading to aircraft weight and if aircraft weight is used, one table only is necessary for a given value of W/δ whereas if fuel state is used a separate table is necessary if the payload (equipment, personnel or ballast) is varied.

A typical table for a helicopter with variable rotor speed is shown below

Aircraft Weight	Altitude	O.A.T.	Rotor Speed
Max.	Increasing	High	High
↓		↓	↓
Min.		Low	Low

W/δ = constant

$\omega/\sqrt{\theta}$ = constant

One such table is required for each value W/δ and $\omega/\sqrt{\theta}$ to be tested. An alternative method is to construct two diagrams showing altitude against weight for the values of W/δ required and rotor speed against temperature for the values of $\omega/\sqrt{\theta}$ to be tested. This method (devised by G.M.J. Davis, A. & A.E.E.) is obviously more compact than using tables; it is simpler in use and more quickly constructed. An example is shown in Fig. 1 for a small piston engined helicopter. Lines showing maximum permitted airspeed with altitude are superimposed. The horizontal scale of A.U.W. may be replaced by an equivalent quantity if convenient: thus if the change of weight is to be found from a flow meter counter the initial reading of the counter could be marked at the corresponding A.U.W. and a scale in terms of the counter reading superimposed on the weight scale. Then during the flight the altitude is found directly from the line of W/δ constant and the counter reading. This method is very easy to use but involves marking out the counter scale immediately before each flight when the A.U.W. for the flight and the counter reading are known: a counter which could be reset to zero would be convenient.

If the helicopter is to be operated at constant rotor speed the table may be formed as follows

/Table

AMBIENT TEMPERATURE	MIN. ——— MAX.
Aircraft Weight	Altitude
Max.	
↓	↓ ↓ ↓ ↓ ↓
Min.	

$^{W}/\sigma$ = constant

ω = constant

It will be appreciated that for any given fuel state, and therefore aircraft weight, the helicopter must fly at an altitude where the temperature, at that altitude, corresponds to the temperature shown for the same altitude in the table. This is a unique point. Again a graphical presentation may be preferred and an example is shown in Fig. 2. For a given A.U.W., σ is found and then combinations of the altitude at which this value of σ occurs may be found for a given ambient temperature. It may be noted that the plot of σ against height and temperature will be the same whatever helicopter is being tested, although the ranges of these values which are of interest may vary.

The problem of anticipating what the temperature at a given altitude will be, remains, of course. If any difficulty is found a standard method is as follows: from the starting point (altitude and temperature) follow a constant temperature line from that point to the density required. Alter height to that appropriate to the new point and obtain the ambient temperature at that height. Plot on the chart this new temperature and the altitude: then follow the constant temperature line as before and repeat the process until the correct altitude/temperature combination is found. This method sounds complicated when written down but is, in fact, quite easy to use in practice. Two or three approximations are usually needed to find the initial operating altitude but these are made during the climb and do not waste any time. Once the operating height has been reached all that is usually necessary is to alter the height by 100–200 ft. between each run. The accuracy required is to read the graphs to the nearest 50 ft. and one degree (C) of temperature.

The other case which may occur is when the rotor speed is variable and the parameter $^{W}/\sigma\omega^2$ is to be used. To use tables showing altitude against temperature for a range of rotor speeds would be unwieldy: a diagram is simpler although it is still undesirably complicated for use in the air. One such diagram is shown in Fig. 3 where for a given weight, altitude and temperature the correct engine speed may be found. Alternatively for a given weight and engine speed the density and therefore combinations of altitude and temperature can be found, but this method of use is more complicated since anticipation of the temperature change with altitude is required. Actual rotor speed (or if more convenient engine speed, depending on the pattern of tachometer fitted) is needed on the diagram used in flight but if several trials are contemplated it might be worthwhile constructing a master table in terms of the relative rotor speed.

The results of tests may be plotted and faired to give curves of $^{P}/\delta\sqrt{\theta}$ against $^{V}/\omega$ (or $^{P}/\sigma\omega^3$ against $^{W}/\sigma\omega^2$) for the various combinations of $^{W}/\delta$ and $^{\omega}/\sqrt{\theta}$ tested (or $^{P}/\sigma$ - V for various $^{W}/\sigma$ if constant rotor speed is used).

/From

From these plots, curves of P - V for the required standard conditions may be obtained by inserting the appropriate values of W, ω, δ, θ, or W, σ, ω. Care must be taken not to present results which imply the use of more power than the engine can give due to an engine limitation which would be apparent under the standard conditions but which may not have applied in the test condition.

Collective pitch measurements provide a valuable guide for several aspects of level flight performance and should be obtained whenever possible. For example, confirmation that two tests made under nominally the same conditions but yielding different results were in fact flown in similar conditions: vertical movement of air for instance would alter the collective pitch setting required to maintain level flight.

Finally two tests which may be made to check the effects of Mach No. and Reynolds No. variations will be discussed. First, Mach No.: this test might be made to decide whether it is permissible to use the $W/\sigma\omega^2$ rather than the more exact W/δ, $\omega/\sqrt{\theta}$ method. $W/\sigma\omega^2$ can be written $\left(W/\delta\right) \times \left(\omega/\sqrt{\theta}\right)^{-2}$ and it is therefore possible to make tests at a constant value of $W/\sigma\omega^2$ while varying $\omega/\sqrt{\theta}$ and W/δ. The results, which should be a plot of $P/\sigma\omega^3$ against V/ω, will show whether there is any difference in the power required at a given value of V/ω, over the range of $\omega/\sqrt{\theta}$ used in the tests, at the value of $W/\sigma\omega^2$ used. The results obtained are only applicable at the value of $W/\sigma\omega^2$ at which the tests were made since the effect of Mach No. will depend on the incidence of the blades as well as $\omega/\sqrt{\theta}$, and the collective pitch angle depends on $W/\sigma\omega^2$. It is, however, permissible to assume that if no Mach No. effect can be found at some value of $W/\sigma\omega^2$ this result will hold for lower values of $W/\sigma\omega^2$. It may not be possible to simulate the lower values of $\omega/\sqrt{\theta}$ which may occur in tropical conditions by tests made in temperate conditions, but if tests can be made over a range of $\omega/\sqrt{\theta}$ it may be assumed that, at constant $W/\sigma\omega^2$, the power required will not change due to Mach No. effects at values of $\omega/\sqrt{\theta}$ lower than those tested if no change can be found over the range tested and provided that the range tested is at least as great as the difference between the lowest value tested and the lowest value which it is proposed to simulate.

As an example consider a case where results are required for I.S.A. +30°C conditions, for altitudes between 0 and 7000 ft., with $\omega = 1$ and $W = 5000$ lb. Tests are to be made in I.S.A. conditions, at an A.U.W. of 5000 lb. and a rotor speed range of from $\omega = 1.025$ to 0.95 is available. The tests must be made at altitudes of less than 10000 ft. The maximum value of $W/\sigma\omega^2$ in the hot climate is 6850 lb. and the range of $\omega/\sqrt{\theta}$ which will be experienced in these conditions is from 0.952 to 0.975. Now with $W/\sigma\omega^2 = 6850$ lb. in the temperate tests the rotor speed range that may be used if the test altitude is not to exceed 10000 ft. is from $\omega = 0.95$ at 7000 ft. to $\omega = 0.995$ at 10000 ft. The corresponding values of $\omega/\sqrt{\theta}$ are 0.975 to 1.032. Thus although the actual lowest value required ($\omega/\sqrt{\theta} = 0.952$) cannot be obtained an adequate range can be covered and if no Mach No. effects are found, up to the highest V/ω needed, it could be assumed that results predicted for $\omega/\sqrt{\theta}$ in the range 0.952 to 0.975 would be valid.

Next the question of Reynolds No., R_N. For a helicopter there are two values of R_N to be considered: one based on the forward speed, V, and one on the rotor speed, ω, and these are proportional to ${}^{V\sigma}/\mu_A$ and ${}^{\omega\sigma}/\mu_A$ respectively, where μ_A, the coefficient of viscosity of air is independent of pressure but increases with increasing temperature. If a test has been made at certain values of ${}^{W}/\delta$, ${}^{\omega}/\sqrt{\theta}$ and ${}^{V}/\omega$ the result, say, ${}^{P}/\delta\sqrt{\theta}$, will, strictly, only be true for the values of ω, V, δ, θ at which the flight was made since at any other values R_N will, in general, be different even though ${}^{W}/\delta$, ${}^{\omega}/\sqrt{\theta}$ and ${}^{V}/\omega$ are the same.

If there is doubt about the effect of R_N when, say, results from a trial are to be used to predict performance in temperatures different from that in which the tests were made it is possible to make special tests to find the variation of performance with R_N. As an example the power could be measured for a range of ${}^{V}/\omega$ at constant ${}^{W}/\delta$, ${}^{\omega}/\sqrt{\theta}$ at two distinctly different altitudes thus giving the power required at two values of R_N. In order to maintain ${}^{W}/\delta$ and ${}^{\omega}/\sqrt{\theta}$ constant the weight and the rotor speed would be varied between the altitudes chosen for the tests. Then rewriting the expressions for R_N $\left({}^{V\sigma}/\mu_A \text{ and } {}^{\omega\sigma}/\mu_A\right)$ as $\left({}^{V}/\omega\right)\left({}^{\omega}/\sqrt{\theta}\right)\left({}^{\delta}/\mu_A\sqrt{\theta}\right)$ and $\left({}^{\omega}/\sqrt{\theta}\right)\left({}^{\delta}/\mu_A\sqrt{\theta}\right)$ it is seen that at constant ${}^{V}/\omega$ and ${}^{\omega}/\sqrt{\theta}$, R_N varies with ${}^{\delta}/\mu_A\sqrt{\theta}$, and decreases with increasing altitude. The limit on the reduction of R_N by increasing altitude will occur when it is no longer possible to keep ${}^{W}/\delta$ constant by decreasing weight. In a typical case a reduction in R_N of 30% could be obtained between flying at S.L. and 12000 ft. in I.S.A. conditions and this would be equivalent to the change in R_N which would be experienced when predicting the performance in I.S.A. +30°C, 7000 ft. conditions from results obtained from tests made in I.S.A., sea level conditions.

5. Partial Climbs

The object of these tests may be to find (1) the best climbing speed at some specified conditions or to find the variation of the best climbing speed with altitude etc. The best climbing speed may be known from level flight tests but some partial climbs are still necessary to find (2) the position error in climbing flight which is usually significantly different from that in level flight: (3) the variation of rate of climb, at the best climbing speed, with particular parameters such as altitude or weight.

Before considering how these tests can best be made using a non-dimensional approach the basic non-dimensional technique for partial climbs will be discussed. It will be seen that the tests are sometimes difficult to carry out especially if a torquemeter is not fitted to the helicopter and the economy in flying time usually associated with non-dimensional tests may not be obtained unless results are required for a large number of standard conditions. Mainly for this reason extensive use of non-dimensional methods has not, so far, been made but it is, nevertheless, worthwhile to consider methods for flying partial climbs non-dimensionally because the essential accuracy and ease of reducing results are still important.

/The

The parameters which define partial climbs are given in equations (3) and (5); with variable rotor speed, $V_c/\omega = f\left(P/\delta\sqrt{\theta}\right)\left(W/\delta\right)\left(\omega/\sqrt{\theta}\right)\left(V/\omega\right)$ and with fixed rotor speed a modified form of equation (3), $V_c = f\left(P/W\right)\left(1/\sigma\right)\left(V\right)$.

An alternative if the rotor speed can be varied is $V_c/\omega = f\left(P/W\omega\right)\left(W/\sigma\omega^2\right)\left(V/\omega\right)$

The best form to use will depend on various factors but the problem is largely that the power parameter must be maintained constant, and the method of achieving this in the air depends on whether or not the machine is fitted with a torquemeter and whether it has a piston engine or a gas turbine. Consider, first, a machine with fixed rotor speed: if a torquemeter is available, P/W constant is the same as Q/W constant and hence maintaining P/W constant is straightforward: if no torquemeter is fitted, it is necessary to use some form of engine curves in the air to obtain manifold pressure, compressor speed etc. appropriate to some new power requirement - this is undesirable and may not always be possible but appears unavoidable.

Next consider a helicopter with a variable rotor speed: apart from the question of maintaining the power term constant the results from the form involving $P/\delta\sqrt{\theta}$ will be at the standard Mach No. but on the other hand the $P/W\omega$ form allows less complication in presenting results because there is one less variable. For a gas turbine it will be shown that keeping $P/\delta\sqrt{\theta}$ constant involves either flying at constant $N_c/\sqrt{\theta}$ or at constant Q/W. If $N_c/\sqrt{\theta}$ constant is indicated, the $P/\delta\sqrt{\theta}$ form of the power parameter is preferable since any engine limitation will be directly apparent. If Q/W is to be constant there is no difference in the two methods since $P/W\omega \equiv Q/W$. With piston engined machines fitted with a torquemeter Q/W can be kept constant and this is equivalent to maintaining $P/W\omega$ or $P/\delta\sqrt{\theta}$ constant: without a torquemeter power curves are again necessary and the flight charts are somewhat simpler if $P/W\omega$ is used rather than $P/\delta\sqrt{\theta}$.

These are the possibilities but before going further it is worthwhile to examine more closely the difficulties if a torquemeter is not used. First it is necessary to use some form of engine power curves and unless these are known to be accurate for the particular engine(s) being used very little reliance can be placed on the results obtained. Secondly, even if accurate power curves can be obtained there remains the problem of presenting information in a suitable form for use in flight. This is considered impractical if power is a function of several variables as in the case of a piston engine. ith a gas turbine it may be feasible to fly at constant $N_c/\sqrt{\theta}$ but this has not been done and tests would be desirable using this method before it was used for a trial.

Consider now the question of trial planning and engine limitations.

First, turbine engined helicopters with variable rotor speed. If results are required for a specific set of standard conditions the values of $P^\circ/\delta\sqrt{\theta}$, W/δ, V/ω and $\omega/\sqrt{\theta}$ (or $P/W\omega$, $W/\sigma\omega^2$, V/ω) may be found for those conditions and

/it

it is then necessary to ensure that these values are maintained in the conditions prevailing when the test is performed. Thus the weight and rotor speed may be varied to allow for a non-standard altitude or temperature. The power parameter, however, needs some care since an engine limitation other than that relevant to the standard condition may operate during the test. The result normally wanted will be the rate of climb for a given engine rating in the standard conditions and, in these conditions, the engine limitation may be compressor speed, jet pipe temperature, fuel flow or torque depending on the altitude and ambient temperature. The limitation in the standard conditions may be found from the makers curves or, preferably, from level flight tests using the particular engine which will be used in the partial climbs. Having found the limitation in the standard conditions it is useful to examine the engine parameters to see how $P/\delta\sqrt{\theta}$ may be kept constant. Quoting Ref. 2,

$$\frac{P}{\delta\sqrt{\theta}} = f_1 \left(\frac{N_c}{\sqrt{\theta}}\right) \left(\frac{\Omega}{\sqrt{\theta}}\right) \left(\Delta\right)$$

$$\frac{G}{\delta\sqrt{\theta}} = f_2 \left(\frac{N_c}{\sqrt{\theta}}\right) \left(\frac{\Omega}{\sqrt{\theta}}\right) \left(\Delta\right)$$

$$\frac{T_j}{\theta} = f_3 \left(\frac{N_c}{\sqrt{\theta}}\right) \left(\frac{\Omega}{\sqrt{\theta}}\right) \left(\Delta\right)$$

and thus, the non-dimensional power, fuel flow and jet pipe temperature are all functions of the non-dimensional compressor speed, rotor speed and guide vane position. It may be assumed that the guide vanes are fully open during partial climbs (i.e. Δ constant), and that $\Omega/\sqrt{\theta}$ will be kept constant. Then $P/\delta\sqrt{\theta}$ will vary with $N_c/\sqrt{\theta}$ only and if the engine limitation in the standard conditions was compressor speed, the value of $N_c/\sqrt{\theta}$ appropriate to the limitation in the standard conditions should be maintained in the tests. It will not always be possible to test in a hotter climate than that for which results are required since although the ambient temperature may be reduced by flying higher, the weight must be reduced to keep W/δ constant. If the tests are carried out at temperatures other than that specified some other engine limitation may appear e.g. a torque limit when operating at low temperatures.

If the engine limitation in standard conditions is either the fuel flow or the JPT it follows that it is necessary to fly at the values of $G/\delta\sqrt{\theta}$ or T_j/θ appropriate to the standard conditions. Since Δ and $\Omega/\sqrt{\theta}$ will be constant, then $N_c/\sqrt{\theta}$ is constant if $G/\delta\sqrt{\theta}$ or T_j/θ is constant, and hence $P/\delta\sqrt{\theta}$ must be constant. This implies, however, that it will also be correct to fly with $N_c/\sqrt{\theta}$ constant at a value corresponding to that appropriate to the engine limitation. The remarks made about the effect of ambient temperature when considering a compressor speed limitation also apply here.

/A

A torque limitation in standard conditions may be shown to call for Q/W constant to maintain $P/\delta\sqrt{\theta}$ constant. Thus, if suffix o denotes standard conditions, and suffix 1 test conditions, the non-dimensional parameters are, with $P = \Omega Q$

$$\frac{\Omega_o Q_o}{\delta_o\sqrt{\theta_o}} = \frac{\Omega_1 Q_1}{\delta_1\sqrt{\theta_1}}, \frac{W_o}{\delta_o} = \frac{W_1}{\delta_1}, \frac{\Omega_o}{\sqrt{\theta_o}} = \frac{\Omega_1}{\sqrt{\theta_1}}$$

and rewriting, $\frac{Q_o}{W_o} = \frac{Q_1}{W_1}$ i.e. Q/W constant.

Again on a hot day a lighter weight would help, if, say, a JPT limitation was encountered, since the helicopter could fly higher (from W/δ) and less torque would be required (from Q/W).

So far only the case of testing to obtain results at one set of standard conditions has been considered. If results are required for a large number of conditions it may sometimes be possible to reduce the amount of testing by working within the range of non-dimensional parameters involved. The engine limitations will govern whether this is possible. If, for instance, it is found that the engine will be torque limited for a number of standard conditions it may be worthwhile. On the other hand if some other engine limitation is indicated the power parameter is $N_c/\sqrt{\theta}$ and each standard condition will require a different value depending on the temperature and type of limitation. Generally this case does not lend itself to planning a short series of tests in terms of non-dimensional parameters. It will, of course, still be useful to conduct each individual test on the basis of the non-dimensional parameters appropriate to each set of standard conditions. It is worth mentioning here that if a series of tests have been made over a range of values of $N_c/\sqrt{\theta}$ and the engine limitation in terms of a parameter which can be expressed as a value of $N_c/\sqrt{\theta}$, is subsequently changed, results will be available for the new limitation provided, of course, that it is within the range of $N_c/\sqrt{\theta}$ tested. One further difficulty may be noted: if a series of tests is planned using non-dimensional parameters and if these include more than one type of engine limitation the results will include discontinuities where the type of limitation changed. If, then, a plot contains too few points to show these discontinuities accurately, results obtained by interpolation may be relatively inaccurate.

If the rotor speed is fixed, the parameters P/W, W/σ have to be used and since the rotor speed is constant, the power is directly related to the torque. As in the case of a variable rotor speed machine the engine limitation in the standard conditions must be found but here it should be expressed as a torque since the engine parameter $\Omega/\sqrt{\theta}$ cannot be maintained constant. The error if the engine parameter $N_c/\sqrt{\theta}$ were used instead of Q/W would be more serious with a fixed shaft turbine engine.

/With

With piston engined helicopters the principles underlying the remarks made for turbine engined machines still apply. It is most important that the power available in the standard condition for which results are required and in the test conditions should be accurately known but given this and also a knowledge of such limitations as cylinder head and oil temperatures for the standard conditions, trial planning is possible.

Turning now to the preparation of information to be used in flight. This will depend on the set of parameters chosen. First, consider V_c/ω as a function of $P/\delta\sqrt{\theta}$, W/δ, $\omega/\sqrt{\theta}$, V/ω. The test will consist of measuring V_c for a number of values of V (the small variation in ω usually makes it possible and more convenient to work in terms of V rather than V/ω in the air) at specified values of $P/\delta\sqrt{\theta}$, W/δ, $\omega/\sqrt{\theta}$. Then as the weight changes the altitude (at the start of the climb) must be increased to maintain W/δ constant, the rotor speed adjusted for temperature changes, and the power setting varied to keep $P/\delta\sqrt{\theta}$ at the correct value. The first two of these conditions are the same as those used in the level flight test and a chart similar to that shown in Fig. 1 will be the best was of presenting the necessary information. It has been shown that there is no single way of presenting information for the power parameter. There are two cases, first, if the engine is torque limited, or more generally to maintain $P/\delta\sqrt{\theta}$ constant in the absence of any engine limitation, Q/W must be kept constant. Secondly, for a compressor speed, fuel flow or jet pipe temperature limitation N_c/Ω (or T_j/θ, $G/\delta\sqrt{\theta}$) must be maintained constant.

If Q/W is required constant a table or graph may be used to show the torque required at any weight for a given value of Q/W. For a compressor speed, fuel flow or jet pipe temperature limitation it has been shown that the tests can be made at constant N_c/Ω and this is usually the simplest method since in addition to the charts shown in Fig. 1 the only additional information required is a table or curve of N_c against Ω for the required value of N_c/Ω.

Alternatively charts showing $G/\delta\sqrt{\theta}$ or T_j/θ could be used but the first form is rather more complicated and the gauge showing jet pipe temperature is not usually suitable for accurate interpretation. It should be noted, however, that although experience has shown that the relationship between G, T_j and N_c is valid under the assumptions stated, if this is shown not to be the case for a particular engine the test must be made in terms of the limiting parameter.

If the parameters chosen are V_c/ω as a function of $P/W\omega$, $W/\sigma\omega^2$, V/ω or with a fixed rotor speed, V_c as a function of P/W, W/σ, V, the power term has been shown to be the equivalent to keeping Q/W constant and the information required and its presentation is similar to the case discussed above. The other parameters are similar to the equivalent level flight cases and the appropriate information is shown in Figs. 2 and 3.

/Now

Now to consider the best way of achieving the objects of the partial climb tests, set out at the beginning of the Section, using the non-dimensional approach. It will be assumed that results are required for a set of standard conditions which are not those at the test site. First, suppose that the best climb speed is known from level flight tests. It is then necessary to find the position error for climbing flight and this may be done by towing an airlog during climbing flight at two or three speeds near to those indicated by the level flight results. These tests may be made over the full range of the non-dimensional parameters found from the standard conditions, if necessary, but it is often found that only the extremes of the ranges need to be checked since the position error usually remains sensibly constant for all conditions of climbing flight that are of interest. If these flights are made at values of the non-dimensional parameters which need to be flown to obtain climbing performance some saving of flying time may be achieved. Once the best indicated climbing speed has been found the remaining task of measuring the rate of climb for the simulated standard conditions may be accomplished by flying at the correct value of $^V/\omega$ or V.

If no level results are available the best indicated climbing speed and its variation with altitude and weight, which is usually large enough to be detected, and with rotor speed and air temperature, which may be negligible, must first be found. In any case the overall variation is normally small enough, over the ranges of the parameters which are usually of interest, for it to be sufficient to make tests (over a suitable range of $^V/\omega$ or V) at the extremes of the values of the non-dimensional quantities. These tests will show the best indicated climbing speed and from the point of view of obtaining climb performance it is unnecessary to know the position error although this can, of course, be found, if required, by using an airlog during these tests. Once the best indicated climbing speed is known the climb performance can be measured using the method indicated above.

6. Vertical Climb

The object of this test will be, first, to produce curves of vertical rate of climb against one or more of the variables, altitude, temperature, weight and rotor speed, with the remaining variables constant, at a specified engine rating, for zero airspeed. Secondly, to establish the effect of windspeed on the climb performance. Ideally the zero airspeed results should be obtained in still air but since this condition is rarely available when required, the tests are made by climbing vertically relative to the ground in low winds and correcting to zero airspeed from the results of tests flown over a range of low airspeeds, during which a ground runner is used to define the airspeed for each climb. The rate of climb in any particular wind speed can then be obtained by modifying the zero airspeed results using correction factors found from the results of the tests made at low airspeeds. These techniques and the method of handling results and making corrections are contained in Ref. 3. For ease of reference the first type of test, that is flying vertically relative to the ground, will be called 'reduced power verticals' and the second, when the climb is made following a ground runner, 'low airspeed verticals'. The results of both these tests may be expressed in non-dimensional terms but, whereas, the advantage of using a non-dimensional approach to low airspeed verticals is debatable, this approach is inherent in reduced power verticals: this test will be discussed first.

/As

As stated above climbs are made vertically relative to the ground in low windspeeds and the results are corrected to zero windspeed. It is, therefore, legitimate to consider this test as a vertical climb at zero forward speed. If the rotor speed is variable the relevant parameters are given in eqn. (4) and are $V_c/\omega = f\left(W/\sigma\omega^2\right)\left(P/\sigma\omega^3\right)$. The density is not an effective variable since the tests are made in a narrow altitude range near the ground and hence the rotor speed must be varied to compensate for weight changes to keep $W/\sigma\omega^2$ constant. Then to maintain $P/\sigma\omega^3$ constant, the power must be changed when the rotor speed is varied. This form has the advantage of using the same parameters, $W/\sigma\omega^2$ and $P/\sigma\omega^3$, that are used for tethered hovering tests and sometimes for level flight tests, thus making a comparison of the results easy but it is easier to conduct the tests using a modified form, $V_c/\omega = f\left(W/\sigma\omega^2\right)\left(P\sigma^{\frac{1}{2}}/W^{3/2}\right)$. Here, with air density constant, the rotor speed and power are both simply functions of weight and this allows the flight data necessary to keep the parameters constant to be presented in a simpler form.

If the rotor speed is fixed the parameters shown in equation (8), $V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}} = f\left(W/\sigma\right)\left(P\sigma^{\frac{1}{2}}/W^{3/2}\right)$ should be used, since, as explained in Section 2, these make the induced power loss independent of W/σ and provided $P\sigma^{\frac{1}{2}}/W^{3/2}$ is maintained constant as fuel is used, the rate of climb parameter, $V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}$, will only be in error by an amount dependent on the change of profile power with W/σ. Since it is impossible to maintain W/σ constant except by ballasting it must be decided in advance what change of weight can be tolerated before reballasting is necessary. A method of doing this is shown in Appendix 2: the result of applying this method to the Scout shows that, for a particular case, a decrease in W/σ of 5%, keeping $P\sigma^{\frac{1}{2}}/W^{3/2}$ constant, would cause the rate of climb to decrease from 180 ft./min. to 90 ft./min. or from 900 ft./min. to 820 ft./min. It is seen that in the 180 ft./min. case (often a Specification requirement) the error for a 5% change of W/σ is excessive and, in fact, to keep the error within 10% (of 180 ft./min.) the change in W/σ must be not greater than 1%: in this case a weight change of 50 lb.

This calculation illustrates, incidently, the importance of knowing the weight of fuel, equipment and aircrew in this type of work.

In general the approach to planning a trial will be similar to that for the level flight case. Once the range of parameters to be covered is known the corresponding range of the non-dimensional parameters may be found and the trial planned to cover these. The power available from the engine to be used in the tests, at any given rating, may be known in advance from previous tests and these figures, rather than the maker's curves, should be used if available. Engine limitations may cause difficulties in planning and each case has to be considered individually. Since the ambient conditions at the test site may be markedly different from those for which results are required a full knowledge

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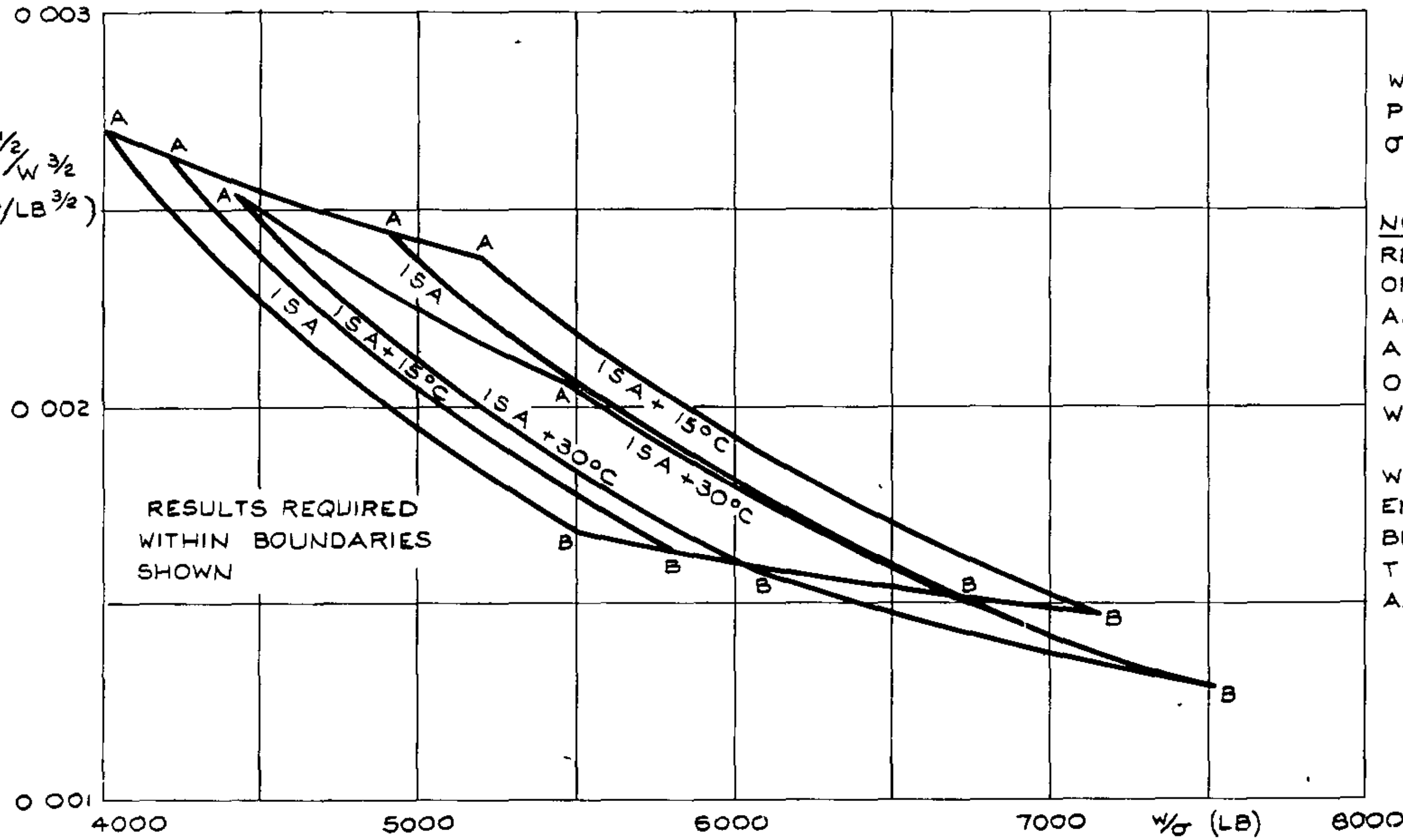


W = A U W (LB)
P = POWER REQUIRED (HP)
σ = RELATIVE DENSITY

NOTE LINES A-B REPRESENT AN INCREASE OF WEIGHT AT CONSTANT ALTITUDE AND LINES A-A, B-B AN INCREASE OF ALTITUDE AT CONSTANT WEIGHT.

POWER IS CONSTANT WITH ALTITUDE IF THE ENGINE IS TORQUE LIMITED, BUT DECREASES IF A TEMPERATURE LIMITATION APPLIES

TEST: REDUCED POWER VERTICALS WITH CONSTANT ROTOR SPEED

DIAGRAM SHOWING COMBINATIONS OF W/σ & $P\sigma^{1/2}/W^{3/2}$ FOR WHICH TEST RESULTS ARE REQUIRED IN ORDER TO BE ABLE TO PREDICT RESULTS FROM SEA LEVEL TO 7000 FT UNDER THE STATED AMBIENT CONDITIONS.

FIG. A.8.

sometimes been found that the best line is a straight one i.e. $^{\partial V_c}/\partial V$ = constant. This is surprising since, for example, in the case of the Scout the theoretical value of $^{\partial V_c}/\partial V$ varies from 0 to 30 ft./min./kt. in the speed range 0-10 kt. for typical values of $^{P\sigma^{\frac{1}{2}}}/W^{3/2}$ and $^{W}/\sigma\omega^2$. It is hoped to make some special tests to elucidate this point in the future.

Meanwhile, the following precautions are worth considering. First, the value of V_c at V=0 is very important and should be well established: the line or curve which is finally used must pass through (or near) this point. The theoretical curve is often very nearly a straightline at speeds above about 5 kt. and since the correction for a change of speed ΔV is $\int_0^{\Delta V} \frac{\partial V_c}{\partial V}.dV$ and not $\frac{\partial V_c}{\partial V}.\Delta V$ unless $^{\partial V_c}/\partial V$ = constant, a serious error could be introduced unless the whole speed range from zero upwards is considered when drawing a line on the plot of V_c versus V. Secondly, a straight line (passing through the point at V=0) should only be used if it is clearly a better fit than a curve with zero slope at the point at V=0.

Calculations for the Scout indicate an appreciable variation of $^{\partial V_c}/\partial V$ with the power and weight parameters at constant speed. As an example, at 10 kts. the variation between two extreme combinations of $^{P\sigma^{\frac{1}{2}}}/W^{3/2}$ and $^{W}/\sigma\omega^2$ was 20 ft./min./kt. Variations of this order might be of importance if the conditions for say, 180 ft./min. vertical performance were being established and until more experimental evidence has been accumulated it would seem advisable to obtain corrections for at least two distinctly different combinations of the weight and power parameters if there is any doubt as to the variation of the correction with these quantities. Again, in the absence of any better guide, it is suggested that the variation of the correction should be checked using simple theory to establish the worst cases for any particular trial.

If tests are made at some combination of the power and weight parameters, these parameters should normally be those used for the reduced power verticals plus the appropriate term for forward speed. That is $^{V}/\omega$ if the climb parameter is $^{V_c}/\omega$ or $^{V\sigma^{\frac{1}{2}}}/W^{\frac{1}{2}}$ if the climb parameter is $^{V_c\sigma^{\frac{1}{2}}}/W^{\frac{1}{2}}$. As in partial climbs it is necessary to be able to change the power setting during the test and the remarks made on this subject in Section 5 are applicable to this case.

7. Autorotation

This test is analogous to Partial Climbs (Section 5) except that the power is zero. The same parameters apply, omitting power, so that, if as is usually the case the rotor speed can be varied, it is necessary to maintain $^{W}/\delta$ and $^{\omega}/\sqrt{\theta}$ or $^{W}/\sigma\omega^2$ constant in flight and, exceptionally, if a fixed rotor speed is used, $^{W}/\sigma$. The choice between $^{W}/\delta$ and $^{\omega}/\sqrt{\theta}$ or $^{W}/\sigma\omega^2$ is as before: the first form gives results at a standard Mach No. while the second allows a simplified presentation of results but is more complicated to use in the air. If the

/tests

tests are to be made at only one value of forward speed, the parameters $^W/\delta$, $^\omega/\sqrt{\theta}$ and $^{V}c/\omega$ could be used to define the results provided that the tests were made at a standard value of $^V/\omega$ by changing V as the rotor speed was altered.

Since there are no engine limitations to consider, the trial can usually be planned in terms of the non-dimensional parameters. The most general type of test is to find the variation in rate of descent with rotor speed and forward speed in one or more standard conditions. Consider the parameter $^{V_c}/\omega$, $^V/\omega$, $^W/\delta$, $^\omega/\sqrt{\theta}$. $^W/\delta$ will be known from the standard conditions and also the values or ranges of values of $^V/\omega$ and $^\omega/\sqrt{\theta}$. Tests can then be planned to cover these parameters and since the rotor speed can usually be varied over quite a large band there is usually no difficulty in simulating temperature conditions far removed from those in which the tests are to take place. The rotor speed for the test will depend on the value of $^\omega/\sqrt{\theta}$ and the ambient temperature and then the forward speed is found from the rotor speed and the value of $^V/\omega$. Suitable graphs showing altitude and A.U.W. for constant $^W/\delta$, rotor speed and temperature for constant $^\omega/\sqrt{\theta}$ and rotor speed and forward speed for constant $^V/\omega$ are required. Similar methods can be used if the parameters $^W/\sigma\omega^2$, $^V/\omega$, $^{V_c}/\omega$ are used.

8. Hovering

The hovering performance of a helicopter is usually defined in terms of the power required to maintain height for given ambient conditions, weight, rotor speed and, if subject to ground effect, wheel clearance. The recommended method of obtaining the hover performance by tethering the machine is fully described in Ref. 4 and forms an interesting application of the non-dimensional approach. For completeness a brief account of the basis of this method will be included here.

The power required to hover with no ground effect can be expressed as $^P/\sigma\omega^3$ and is a function of $^W/\sigma\omega^2$ only.

If ground effect is present a further parameter, l, can be introduced to define the wheel clearance. In tethered hovering l is fixed by the length of the cable used and by varying the cable tension and rotor speed a wide range of $^W/\sigma\omega^2$ may be obtained during each flight. The tests can, of course, be repeated with various lengths of cable, depending on the specification for the trials.

Planning is relatively simple: the range of $^W/\sigma\omega^2$ for which results are required can be calculated and the range which is likely to be available at the trial site estimated, bearing in mind that the range of $^W/\sigma\omega^2$ may be limited by the power available in the test conditions. As usual, engine limitations should be examined in advance to see if they will restrict the tests in any way.

The actual tests are normally made by increasing the power in stages from the point at which the cable is just taut until either maximum power is being used or until the maximum permissible tension in the cable is achieved: no data for use in flight are required.

/The

The method, outlined above, gives results at a non-standard Mach No. and with current helicopters this method seems satisfactory: no difference has been found between results obtained in temperate and tropical conditions. It is possible to take Mach No. into account by using the parameters $^P/\delta\sqrt{\theta}$, $^W/\delta$ and $^\omega/\sqrt{\theta}$, and apart from the slight increase in complexity involved in having to calculate the rotor speed to obtain the correct value of $^\omega/\sqrt{\theta}$, the only disadvantage is that if it is possible to use a rotor speed lower than that called for in the standard conditions during the test, the range of $^W/\delta$ available may be appreciably less than that of $^W/\sigma\omega^2$.

9. Discussion

Although the advantages of the non-dimensional approach are clearly apparent, only its use in actual trials over a period of time will show all the difficulties and limitations. The application of the method falls into two parts which are quite distinct. First, for any given set of standard conditions the appropriate non-dimensional parameters may be found, and, by maintaining these constant during the test, the reduction of results becomes merely simple arithmetic. The results will usually be more accurate than those found using the standard reduction method since fewer assumptions have to be made. Secondly, an extensive trial can be planned in terms of non-dimensional parameters and, although this may not always be practical for all flight modes, when used, a reduction in the number of tests needed and/or an increase in the amount of information which can be obtained at any one test site may be obtained.

The first application is relatively straightforward and easy to use, whereas the second is more complicated because of engine limitations etc. which make advance planning more difficult. Nevertheless, in practice, complete trials are being planned in this manner, as a matter of course, and it is suggested that, even when it is not feasible to plan a trial non-dimensionally, the exercise of setting out all the relevant facts systematically to see if it is possible, has a value of its own in giving an insight into the problems which are likely to arise no matter how the trial is conducted.

It has already been pointed out that each trial needs a separate approach in planning and it is not practicable to lay down rules which would enable the planning to be done by rote. Undoubtedly as non-dimensional testing becomes widespread the planning of certain types of test will become almost standardised but meanwhile some examples of actual trial planning are included in an Appendix to this Report in the hope that they will help to show some of the methods of approach available, and some of the difficulties which may arise.

REFERENCES

Ref. No.	Author	Title, etc.
1	G.F. Langdon	Experimental Methods for Helicopter Performance Reduction. A.H.D. Note 54. June 1961.
2	G.F. Langdon	A Note on Performance Reduction for Helicopters fitted with Gas Turbine Engines with Free Power Turbines and Torquemeters. A. & H. Note 53. March 1961.

/3

Ref. No.	Author	Title, etc.
3	I.A. Fisher	The Measurement and Prediction from Flight Test Results of a Helicopter's Vertical Climb Performance, using a non-Dimensional Method. AAEE/Res/310. Jan. 1964.
4	G.F. Langdon	Measurements of the Hovering Performance of a Helicopter by Tethering. AAEE/Tech/217. May 1962.

/Notation

Notation

A rotor disc area = ΠR^2

B manifold pressure

C constant defined in Section 2

C_D drag coefficient

D_{100} drag at speed of 100 ft./sec.

G fuel flow

h altitude

l rotor clearance above ground

n constant

N_c compressor speed

P power

p atmospheric pressure

p_o atmospheric pressure at sea level

Q torque

R rotor radius

R_N Reynolds No.

s rotor solidity = $\frac{\text{total blade area}}{A}$

T ambient air temperature

T_c carburettor air temperature

T_j jet pipe temperature (JPT)

T_o ambient air temperature at I.S.A., sea level = 288°K.

V forward speed

V_c rate of climb

v rotor induced velocity

W aircraft weight

δ relative atmospheric pressure = $^p/p_o$

/Δ

Δ inlet guide vane position

θ relative air temperature = $^{T}/T_o$

θ_c relative carburettor air temperature = $^{T_c}/T_o$

T, T_o, T_c, must be in degrees absolute

ϑ collective pitch

μ rotor tip speed ratio = $^{V}/\Omega R$

μ_A coefficient of viscosity of air

ρ air density

ρ_o air density at I.S.A., sea level

σ relative air density = $^{\rho}/\rho_o$

ω relative rotor speed = $^{\Omega}/\Omega_o$

Ω rotor speed

Ω_o standard rotor speed

Examples of Planning

1.1 Tethered Hovering

This example is based on a trial which was made on a small piston engined helicopter. The trial sites were at Boscombe Down and at Idris. Results were required for the following conditions

Weight	1250-1600 lb.
Engine speed	2900 R.P.M. = Ω_o
Altitude	0-5000 ft.
Temperature	I.S.A. - I.S.A. +25°C.

The operating conditions relevant to the test were

Minimum take-off weight with 30 minutes fuel, 1420 lb.
Maximum permitted tension in cable, 330 lb.
(The tension in the cable was restricted to a figure such that the helicopter weight plus the tension did not exceed the maximum overload A.U.W. of 1750 lb. Thus under no circumstances was there any advantage in flying at anything above the minimum take-off weight.)
Engine speed range, 2500-2900 R.P.M. = Ω
Altitude, approximately sea level

The parameters for this test are $W/\sigma\omega^2$ and $P/\sigma\omega^3$, $(\omega = \Omega/\Omega_o)$. The object is to test over the necessary range of $W/\sigma\omega^2$. The extremes of this range are

Maximum $W/\sigma\omega^2$, at maximum weight and 5000 ft., I.S.A. +25°C, $= \dfrac{1600}{0.767 \times 1^2} = 2090$ lb.

Minimum $W/\sigma\omega^2$, at minimum weight and I.S.A., sea level, $= \dfrac{1250}{1 \times 1^2} = 1250$ lb.

Now, consider the values of $W/\sigma\omega^2$ which are likely to be available. It is necessary to assume values of σ for the test sites and these were chosen as $\sigma = 1$ for Boscombe Down (I.S.A.) and $\sigma = 0.936$ for Idris (I.S.A. +20°C). These values are well below the maximum temperatures found at Boscombe and Idris in August and September when the trials took place but were chosen to allow for the fact that the tests could only be made in zero wind conditions and these periods were most likely to occur early in the morning when the temperatures were comparatively low. The values of $W/\sigma\omega^2$ available are then, using the full rotor speed range, $\omega = 0.863 - 1$ (engine speed 2500-2900 R.P.M.)

	Boscombe	Idris
Maximum	$\dfrac{1750}{1 \times 0.863^2} = 2730$	$\dfrac{1750}{0.936 \times 0.863^2} = 2510$
Minimum	$\dfrac{1420}{1 \times 1^2} = 1420$	$\dfrac{1420}{0.936 \times 1^2} = 1517$.

/Two

Two points may be noted. First, the minimum values available are well above that required: this is because the value of $^W/\sigma\omega^2$ = 1250 represents a case with only a pilot on board whereas for the test an observer must be carried as well. Secondly, the maximum values obtained assume that there will be enough power available at the low engine speed necessary to sustain flight at high values of $^W/\sigma\omega^2$: this may not be the case, depending on the engine characteristics, but in this trial there was a net gain in using the lower speeds. In practice if a sufficiently large range of rotor speeds lower than that at which results are required is available this point causes no difficulty provided that the tests are made in conditions where the maximum power output in the area where results are required is available. In this case, this condition was met at Boscombe Down (I.S.A., sea level) and all the results required could be obtained from these tests: the trial at Idris provided only confirmatory results.

If the tests were to have been made solely at Idris there would be doubt as to what range of values could be obtained with the engine power available. It would be possible to predict this in advance only if the power output of the engine and the power required by the helicopter in the test condition was known. The accuracy of the prediction would then depend on the accuracy of this information. In this case, where the maximum power output occurs at sea level in given ambient conditions results would have certainly been available for temperatures greater or equal to the test condition at all altitudes.

In general, if any doubt exists as to whether the required results will be obtained, a site at which the engine gives maximum power will provide the highest value of $^W/\sigma\omega^2$, if lack of power is likely to be a limitation: otherwise, of course, a site with a low air density will give high values of $^W/\sigma\omega^2$ and vice-versa.

1.2 Level Flight

This example is based on the same machine referred to in Section 1.1, that is, a small piston engined helicopter. Again trials were to be made at Boscombe Down and at Idris. The power required in level flight was to be found for the conditions

Weight	1250-1600 lb.
Engine speed	2700-2900 R.P.M. (Ω_o = 2900 R.P.M.)
Altitude	0-5000 ft.
Temperature	I.S.A. - I.S.A. +25°C.

All the tests were made over as large a speed range as possible and results were required up to at least 70 kts. Thus the possible altitude for the tests might have been limited by the forward speed available but, in fact, this was not so at the maximum altitude at which tests were made of 5000-6000 ft. The rotor speed range available corresponded to engine speeds of 2500 to 2900 R.P.M. and, for planning purposes, it was assumed that temperatures at Boscombe Down and Idris would correspond to I.S.A. +5°C. and I.S.A. +25°C. respectively. As will be shown the minimum weight requirement of 1250 lb. could not be met because this case arises with only the pilot on board and for test purposes an observer had to be carried. The minimum practical weight for test purposes (with 30 minutes fuel) was 1420 lb.

/The

The parameters chosen for the test were $^P/\delta\sqrt{\theta}$, $^W/\delta$, $^\omega/\sqrt{\theta}$ and $^V/\omega$. The envelope containing all required combinations of $^W/\delta$ and $^\omega/\sqrt{\theta}$ is shown in Fig. A.1, and is formed from a series of rectangular envelopes. Each of these shows the combinations for the weight and rotor speed ranges, at one temperature condition and altitude; four extreme cases, I.S.A., sea level; I.S.A. +25°C, sea level; I.S.A., 5000 ft. and I.S.A. +25°C., 5000 ft. are shown.

In order to show what is likely to be possible another diagram (Fig. A.2) was constructed showing the combinations of $^W/\delta$ and $^\omega/\sqrt{\theta}$ which could be obtained under I.S.A. +5°C. and I.S.A. +25°C. conditions. At any given altitude, 1000 and 5000 ft. are shown (1000 ft. was considered the minimum practical pressure altitude for this trial), the combinations available are contained in a rectangle which is formed from the limiting values of weight and rotor speed. The complete envelope is then formed by all possible rectangles. It should be pointed out that in this example the extreme limit of maximum weight has been assumed available regardless of the operating altitude and, in fact, the fuel used to reach 5000 ft. was small enough to be neglected. If, however, tests had been made at much higher altitudes an allowance for this factor would have been made when constructing the diagram. Also shown in Fig. A.2 is the envelope showing the combinations of $^W/\delta$ and $^\omega/\sqrt{\theta}$ required. It can be seen that there is no possibility of obtaining results for the lower values of $^W/\delta$. In the practical range of $^W/\delta$ there is no difficulty in obtaining results at the lower values of $^\omega/\sqrt{\theta}$: in fact all the necessary flights could be made in I.S.A. +5°C. conditions. This arises from the availability of rotor speeds lower than those specified for the results. At the higher values of $^\omega/\sqrt{\theta}$ there is a gap which can only be filled by flying in cooler conditions than I.S.A. +5°C. Thus if a cool day occurred during the tests it would be worth taking advantage of it to fill this corner of the envelope. At the lower values of $^\omega/\sqrt{\theta}$ there is a large area available which is not required to obtain the desired results and there is normally no point in flying tests within this area.

In order to show precisely what results will be obtained it is useful to draw one other diagram (Fig. A.4). This is obtained from Figs. A.1 and A.2 and shows in terms of weight and altitude what results will be available for I.S.A. and I.S.A. +25°C. conditions at 2700 and 2900 engine R.P.M. In order to show the method of obtaining Fig. A.4 parts of Figs. A.1 and A.2 have been redrawn in Fig. A.3. To obtain Fig. A.4(a) consider Figs. A.1 and A.3. At sea level results, for an engine speed of 2900 R.P.M. are required for those combinations of $^W/\delta$ and $^\omega/\sqrt{\theta}$ which lie on the line A, X2. Only those from X1 to X2 are available. Hence with $\delta = 1$ (sea level) the weight range for which results can be obtained is from 1550 to 1600 lb. Now at 5000 ft. the line on which results are required is E, X3 and it is seen that tests can only be made to obtain the portion X4, X3: that is weights of 1550 to 1600 lb. at I.S.A., 5000 ft. The lines corresponding to A, X2 and E, X3 if results are required for an engine speed of 2700 R.P.M. are D, Y2 and Y4, Y3. At sea level results between Y1 and Y2 can be obtained while at 5000 ft. the whole of Y4, Y3 is available. The shape of the line joining Y1 and Y4 (Fig. A.4) must then be found and it may be obtained by joining the points D and Y4 (Fig. A.3) by the appropriate curve and considering the intersection Y5. At this point $^W/\delta = 1472$ and, therefore, at an A.U.W. of 1250 lb., $\delta = 0.848$: hence on Fig. A.4(a) Y5 is at 4500 ft. The remaining cases shown in Fig. A.4 are obtained in a similar manner: the appropriate points are shown in Fig. A.3.

/It

It is seen (from Fig. A.4) that very little information will be obtained for flight in I.S.A. conditions at 2900 R.P.M. with tests being made in conditions no cooler than I.S.A. +5°C. and this emphasises the need to take advantage of any cool weather during the trial. For interest the line showing how the results would be extended by testing in I.S.A. conditions is shown in Fig. A.4(a).

Finally it is necessary to decide at what values of $^W/\delta$ and $^\omega/\sqrt{\theta}$ tests should be flown. Referring to Fig. A.2 it is desirable to cover a range of $^W/\delta$ from the minimum obtainable to the maximum required. The values chosen were 1500, 1700 and 1900: three values being considered the minimum number to allow interpolation. At each value of $^W/\delta$ flights were made at two values of $^\omega/\sqrt{\theta}$ and in order to plot the results the values of $^\omega/\sqrt{\theta}$ must be the same for each value of $^W/\delta$. Thus the maximum value was chosen as $^\omega/\sqrt{\theta} = 1$ since although it is possible to fly at a higher value at the highest value of $^W/\delta$, at $^W/\delta = 1500$ even $^\omega/\sqrt{\theta} = 1$ implies choosing a day slightly cooler than I.S.A. +5°C. for this particular test. The lower value of $^\omega/\sqrt{\theta}$ was chosen as 0.89. These values were appropriate to temperate flying but, obviously it was not possible to fly at $^\omega/\sqrt{\theta} = 1$ in Idris. Thus for the tropical flying $^\omega/\sqrt{\theta} = 0.96$ and 0.89 were used. Since the value of 0.89 was used in both temperate and tropical conditions the results would be expected to agree exactly and thus provide a check, and assuming that this was the case the results at 0.96 could be used with the temperate results to provide data at three values of $^\omega/\sqrt{\theta}$ for cross plotting.

1.3 Vertical Climb

The trial will be planned for a hypothetical helicopter bearing some resemblance to the 'Scout'. Vertical rates of climb are required at maximum available power for the following conditions

1. Altitude, 0-7000 ft.

2. Weight, 4000-5500 lb.

3. Temperature, I.S.A. - I.S.A. +30°C.

4. Rotor speed, 400 R.P.M.

All up weights of 3700-5500 lb. and a rotor speed range of 380-400 R.P.M. will be assumed available. The tests will take the form of reduced power verticals and it will be assumed that the tests will be made in still air or, alternatively, corrections are available to reduce results obtained in light winds to still air values.

For convenience the performance at three conditions of temperature, corresponding to I.S.A., I.S.A. +15°C. and I.S.A. +30°C. will be investigated, but any number of intermediate points could easily be included.

/First

First it is necessary to consider the power available in the various climatic conditions for which results are required. From manufacturers curves or from the results of prior engine calibration tests a table, shown below, may be drawn up showing the power available at a convenient number of altitudes and temperatures, together with the type of engine limitation in each case. The relative density, σ, is also noted for each point.

Temperature	I.S.A.		I.S.A. +15°C.		I.S.A. +30°C.	
Altitude (ft.)	Power	σ	Power	σ	Power	σ
0	685	1.000	685	0.950	676	0.905
3000	685	0.914	685	0.869	648	0.827
5000	685	0.860	685	0.817	629	0.778
7000	685	0.819	685	0.769	610	0.731
Type of Engine Limitation	Torque		Torque		Temperature	

Table 1. Power Available and Relative Density

Thus for I.S.A. and I.S.A. +15°C. the engine is torque limited and the powers quoted correspond to 400 R.R.P.M. A temperature limitation applies at I.S.A. +30°C.

The parameters $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ will be used to define the flight conditions for the vertical climb tests and the next step is to obtain an envelope which will contain all the combinations of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ for which test results must be obtained to cover the ranges of the dimensional parameters for which rates of climb are required.

A diagram showing the envelope of the non-dimensional parameters for three different temperatures is shown in Fig. A.5. If the form of this diagram is not obvious for any particular example, it may be obtained from the type of Table shown below. For each temperature the weight and power parameters corresponding to a range of altitudes and weights are calculated and set down in a systematic fashion:

/Table 2

Altitude (ft)	Temperature	I.S.A.				I.S.A. +15°C.				I.S.A. +30°C.			
	Weight (lb)	4000	4500	5000	5500	4000	4500	5000	5500	4000	4500	5000	5500
7000	$^W/\sigma\omega^2$	4950	5560	6180	6800	5210	5850	6500	7150	5470	6160	6840	7530
	$^P/\sigma\omega^3$.	847	847	847	847	891	891	891	891	835	835	835	835
5000	$^W/\sigma\omega^2$	4650	5230	5820	6400	4890	5510	6120	6730	5140	5790	6430	7070
	$^P/\sigma\omega^3$	796	796	796	796	838	838	838	838	809	809	809	809
3000	$^W/\sigma\omega^2$	4380	4920	5470	6020	4610	5180	5760	6330	4840	5440	6040	6650
	$^P/\sigma\omega^3$	749	749	749	749	788	788	788	788	784	784	784	784
0	$^W/\sigma\omega^2$	4000	4500	5000	5500	4220	4730	5260	5790	4420	4970	5520	6080
	$^P/\sigma\omega^3$	685	685	685	685	721	721	721	721	747	747	747	747

Table 2. Non-dimensional Parameters

The limits of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ are seen to be 4000, 7530 and 685, 891 respectively. These results may now be used to construct Fig. A.5. Note that the 'sides' of the I.S.A. +30°C. envelope are curves due to the engine characteristics.

Fig. A.5 shows the combinations of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ for which it is necessary to obtain results in order to predict performance in I.S.A., I.S.A. +15°C. or I.S.A. +30°C. conditions. It is now necessary to find out either (a) the characteristics of a site or sites which will allow these tests to be made or (b) what results will be available if tests are made in given ambient conditions.

Starting with (a), assume for the moment that the characteristics of a site to obtain results for I.S.A. conditions are required. As a starting point the maximum value of σ for which the highest values of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ can be obtained will be found. A lower value of σ would allow the highest values of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ to be obtained but would reduce the possibility of covering the lower values of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$. From Fig. A.5 the highest value of $^W/\sigma\omega^2$ is 6800 and therefore $\sigma = {}^W/6800\omega^2$. The highest value of σ is with the lowest rotor speed ($\omega = 0.95$) and the highest weight, 5500 lb. Then $\sigma = 0.897$. The highest value of $^P/\sigma\omega^3$ is 847 and hence $\sigma = {}^P/847\omega^3$. If the engine is torque limited $P = 685\omega$ and $\sigma = {}^{685}/847\omega^2$. At the lowest rotor speed σ is again 0.897: the fact that the relative density is the same for the highest values of both $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ arises from the engine being torque limited in both the standard condition and in the test condition. If the engine is not torque limited the power available will depend on altitude and this case will be considered separately. The limits of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ may now be found for $\sigma = 0.897$.

Thus Maximum $^W/\sigma\omega^2 = {}^{5500}/0.897 \times 0.95^2 = 6800$, Minimum $^W/\sigma\omega^2 = {}^{3700}/0.897 \times 1^2 = 4120$

Maximum $^P/\sigma\omega^3 = {}^{685 \times 0.95}/0.897 \times 0.95^3 = 847$, Minimum $^P/\sigma\omega^3 = 0$

It is now necessary to establish which combinations of these parameters can be flown within the permissible weight and rotor speed ranges. Results are shown (Fig. A.6) superimposed on the envelopes shown in Fig. A.5. Fig. A.6 may be constructed as follows. Consider the line on which the maximum value of $^W/\sigma\omega^2$ is constant (i.e. $^W/\sigma\omega^2 = 6800$). On this line $\omega = 0.95$ and the maximum value of $^P/\sigma\omega^3$ is 847. Now consider the line of constant $^P/\sigma\omega^3 = 847$ on which $\omega = 0.95$. The minimum value of $^W/\sigma\omega^2 = {}^{3700}/0.897 \times 0.95^2 = 4570$. Next consider the line of constant $^P/\sigma\omega^3 = 685$ (this being the minimum value of $^P/\sigma\omega^3$ required). The rotor speed may take any value within the permitted range and therefore the minimum value of $^W/\sigma\omega^2 = {}^{3700}/0.897 \times 1^2 = 4120$. Finally consider the line of constant $^W/\sigma\omega^2 = 4120$: on this line $\omega = 1$ and therefore the maximum value of $^P/\sigma\omega^3 = {}^{685}/0.897 \times 1^3 = 764$. These points then define the envelope of available combinations of parameters at $\sigma = 0.897$. It can be seen from Fig. A.6 that the possible combinations will allow the prediction of results for most I.S.A. condition: only those I.S.A. cases in the shaded area to the left of the line A-B are excluded. To see exactly what combinations of weight and altitude are not covered it is useful to construct another diagram (Fig. A.7(a)) showing the boundary of results obtainable in the I.S.A. condition. This boundary corresponds to the relevant boundary on Fig. A.6, that is the intersections of the boundary lines showing what values of $^W/\sigma\omega^2$ and $^P/\sigma\omega^3$ are available with the envelope containing the required combinations of these parameters. It shows for exactly what combinations of altitude and weight, results can be obtained for I.S.A. conditions by flying at $\sigma = 0.897$ with the engine torque limited. Fig. A.7 indicates that it will also be possible to obtain some results applicable to I.S.A. +15°C. and I.S.A. +30°C.; their extent is shown in Figs. A.7(b) and A.7(c).

Fig. A.6 also shows that it is not necessary to test at all combinations of the weight and power parameters: only those which lie within the I.S.A. boundary if only I.S.A. results are required, etc.

As yet only the relative density of the trial site has been defined. Translating this into combinations of altitude and temperature a site at 3600 ft., I.S.A. or 1900 ft., I.S.A. +15°C. etc. would be suitable. Obviously it will not usually be possible to find a site which meets these requirements exactly but having selected an optimum it is a simple matter to predict the range of results obtainable at any site. In general a hotter

/and/or

and/or higher site (lower relative density) will yield results at high weights/altitudes and vice-versa. These remarks only, so far, apply to sites where the engine is torque limited. To see the effect if some other limitation applies at the trial site consider a site at 300 ft., I.S.A. +30°C. The relative density is 0.897 as before but the power available is reduced to 673 H.P. Since, however, the engine will still be torque limited at low rotor speeds (380 r.r.p.m. ≡ 651 H.P.) the existing envelope on Fig. A.5 will be valid at rotor speeds less than 393 r.p.m. Changes for rotor speeds above this value are shown on Fig. A.6 and it is seen that there will be no change in the results obtained. This is, however, not a general result and each case must be examined separately.

If it is considered necessary to obtain results for the I.S.A. conditions not already covered i.e. low weights combined with low altitudes, a second site must be found to enable $^W/\sigma\omega^2$ = 4000 to be obtained. Suitable sites will be discussed at a later stage.

A site condition to allow a complete set of results to be obtained at I.S.A. +15°C. can be found in the same way as that for the I.S.A. case. As for the I.S.A. condition, the engine is torque limited and as no new points emerge this case will not be examined in detail.

To obtain results for the high weight, high altitude region at I.S.A. +30°C. an even smaller relative density of 0.81 is needed and in this case is defined by the maximum value of $^W/\sigma\omega^2$ (7530) required. Results are shown for this case assuming full (torque limited) power available, and also for a site at 3650 ft., I.S.A. +30°C. (i.e. σ = 0.81) where the engine is temperature limited. In this case the power available is 642 H.P. and the engine is temperature limited over the entire rotor speed range. Hence the maximum value of the power parameter is slightly reduced but since the relative density was defined by the required value of $^W/\sigma\omega^2$ there is a large reserve margin for $^P/\sigma\omega^3$ and no results are effected.

Two specific cases will now be considered. First, testing at Idris (assumed pressure altitude, 400 ft.) where a temperature range corresponding to I.S.A. +10°C. to I.S.A. +25°C. is available during the day. I.S.A. +25°C. at 400 ft. corresponds to σ = 0.908 and the engine is torque limited. In Fig. A.7 the lines drawn for σ = 0.897 will give a good indication of the results available and it is seen that full altitude coverage cannot be obtained for any of the ambient temperatures shown. No tests, apart from checking, are needed at σ< 0.908 but, in fact, at I.S.A. +10°C. (σ = 0.957) a large part of the range of non-dimensional parameters can be tested. The lines appropriate to σ = 0.957 are shown in Fig. A.7.

If a trial were being planned the next step, having obtained and accepted an overall view of what results can be obtained, might be to select the combinations of parameters at which it is desirable to obtain results on the trial. The following list might be appropriate in this case and has been drawn up using Fig. A.7. Where it is possible, the test values exceed the required values slightly.

$^{P}/\sigma\omega^{3}$	680	720	760	795	820	840
$^{W}/\sigma\omega^{2}$	3900	4100	4300	4500	4700	4800
	4325	4550	4825	5075	5225	5300
	4750	5000	5350	5650	5750	5800
	5175	5450	5875	6225	6275	6300
	5600	5900	6400	6800	6800	6800

Of these all to the left of the solid line can be flown at I.S.A. +10°C. The combinations shown represent a minimum, and more may be necessary especially in regions where the performance is marginal. Obviously results would be plotted at the trial site and tests flown as necessary. It would be useful to prepare in advance a table showing what combinations of parameters could be flown at any given ambient temperature.

In addition, flight data, showing the variation of rotor speed and power with weight, must be available for each combination of the non-dimensional parameters to be tested.

The second example will be testing in France where a range of airfields are available at various altitudes up to 10000 ft. Two site temperatures corresponding to I.S.A. and I.S.A. +15°C. will be considered. Taking I.S.A. first; to cover the high weight, altitude 'corner' of the I.S.A. +30°C. requirement σ = 0.810 is needed, which corresponds to a height of 7000 ft. In order to allow for the low weight, altitude condition 0.975 $\geq\sigma\geq$ 0.905, and the corresponding altitudes are 900 ft. to 3300 ft. Thus two test sites are needed, one at or near 7000 ft. and one between 900 ft. and 3300 ft. The high altitude site could be higher than 7000 ft. but if it is the requirement for the second site will be altered. In I.S.A. +15°C. conditions the corresponding altitudes are 5200 ft. for one site and from sea level to 1600 ft. for the second.

If it is impossible to vary the rotor speed of a particular helicopter the trial must be planned using different parameters, $^{W}/\sigma$ and $P\sigma^{\frac{1}{2}}/W^{3/2}$. The basic method will be similar to that described for the variable speed rotor machine and results for the same helicopter as used hitherto are shown in Figs. A.8, A.9, A.10 and A.11. These are for the same values of σ used in previous cases to emphasise the differences in the ranges of results obtainable. It is seen that far fewer results are available for a given value of σ and in no case can results for all the I.S.A. +15°C. altitudes or I.S.A. +30°C. conditions be obtained, at the same relative density as that when the rotor speed was variable. Another point of interest is that once the engine cannot supply the maximum permitted torque at the nominal rotor speed the results are effected immediately since the rotor speed cannot be reduced. Results of this are shown in Fig. A.11 for the I.S.A. +30°C. test sites considered for the variable rotor speed case; the appropriate lines have been omitted from Fig. A.9 for clarity, but are of similar form to those shown.

A Method of Calculating the Effect of a Change in A.U.W. on Vertical Climb Performance

This calculation is necessary when a test consisting of reduced power verticals' is planned for a helicopter with a fixed rotor speed, to assess what change of weight can be permitted before ballasting (to restore the A.U.W. to its nominal value) is necessary. The non-dimensional parameters for this test are

$$\frac{V_c\sigma^{\frac{1}{2}}}{W^{1/2}} = f\left(\frac{P\sigma^{\frac{1}{2}}}{W^{3/2}}\right)\left(\frac{W}{\sigma}\right)$$

and rewriting equation (6) with V = 0

$$\frac{P\sigma^{\frac{1}{2}}}{W^{3/2}} = \frac{v\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} + \frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} + \frac{1}{8}\rho_o\ \Omega^3R^3\ As\left(\frac{W}{\sigma}\right)^{-3/2} C_D \qquad \text{A.1}$$

where $\frac{v\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} = \frac{k}{2A\rho_o}\ \frac{1}{\left(v\sigma^{\frac{1}{2}}/W^{\frac{1}{2}} + V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}\right)}$ and $C_D = f\left(\frac{W}{\sigma}\right)$

Solving the induced velocity equation and substituting into A.1

$$\frac{P\sigma^{\frac{1}{2}}}{W^{3/2}} = \frac{1}{2}\cdot\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}} + \sqrt{\left(\frac{1}{2}\cdot\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)^2 + \frac{k}{2A\rho_o}} + \frac{1}{8}\ \rho_o\ \Omega^3R^3\ As\left(\frac{W}{\sigma}\right)^{-3/2} C_D \qquad \text{A.2}$$

Two cases will now be considered

(1) If $\frac{k}{2A\rho_o} >> \left(\frac{1}{2}\cdot\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)^2$. This is usually true when the rate of climb is less than 500 ft./min. Then for two conditions, suffix 1, 2 the change of V_c at constant $P\sigma^{\frac{1}{2}}/W^{3/2}$ is

$$\left(\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)_1 - \left(\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)_2 = \frac{1}{4}\ \rho_o\ \Omega^3R^3\ As\left[\frac{C_{D2}}{(W/\sigma)_2^{\ 3/2}} - \frac{C_{D1}}{(W/\sigma)_1^{\ 3/2}}\right] \qquad \text{A.3}$$

assuming that k remains constant. C_D is a function of W/σ and may be known. If it is not known there are two alternatives. First, C_D may be assumed constant: this results in a larger change of $\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}$ than actually occurs since C_D falls as W/σ decreases, and this is, therefore, a safe course. Secondly, an empirical formula may be used. These are usually of the form C_D = constant

/+ f

$+ f\left(\frac{W}{\sigma}\right)$ and one typical of a 12% thick symmetrical section is $C_D = 0.00883 + 0.0126\left(\frac{7}{\rho_o\Omega^2R^2As}\cdot\frac{W}{\sigma}\right)^2$. A safe course is to assume a drag 10 or 20% larger than that given by any such formula since this simply increases by the same amount the change of $V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}}$ for a given change of W/σ.

(2) If $\frac{k}{2A\rho_o}$ is not large compared with $\left(\frac{1}{2}\cdot\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)^2$. It is then difficult to treat equation A.2 in the way shown above and the best method is to differentiate A.2 to obtain $\frac{\partial(V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}})}{\partial(W/\sigma)}$. Then for changes of W/σ not exceeding 5% it may be assumed that

$$\left(\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)_1 - \left(\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)_2 = \frac{\partial(V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}})}{\partial(W/\sigma)}\cdot\left[\left(\frac{W}{\sigma}\right)_1 - \left(\frac{W}{\sigma}\right)_2\right] \qquad \text{A.4}$$

From A.2
$$\frac{\partial(V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}})}{\partial(W/\sigma)} = \frac{\frac{1}{8}\rho_o\Omega^3R^3As\left[\frac{3C_D}{(W/\sigma)^2} - \frac{2\partial C_D/\partial(W/\sigma)}{(W/\sigma)}\right]}{(W/\sigma)^{\frac{1}{2}}\left[1 + \frac{V_c\sigma^{\frac{1}{2}}}{2W^{\frac{1}{2}}}\Big/\sqrt{\left(V_c\sigma^{\frac{1}{2}}/2W^{\frac{1}{2}}\right)^2 + \frac{k}{2A\rho_o}}\right]}$$

Again some knowledge of the blade drag is necessary and the suggestions made above still apply. If the drag is assumed constant $\frac{\partial C_D}{\partial(W/\sigma)} = 0$.

For $C_D = 0.00883 + 0.0126\left(\frac{7}{\rho_o\Omega^2R^2As}\cdot\frac{W}{\sigma}\right)^2$

$$\frac{\partial C_D}{\partial(W/\sigma)} = 0.0252\left(\frac{7}{\rho_o\Omega^2R^2As}\right)^2\cdot\frac{W}{\sigma}.$$

If changes of W/σ in excess of 5% are considered equation A.4 is no longer valid and the form

$$\left(\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)_1 - \left(\frac{V_c\sigma^{\frac{1}{2}}}{W^{\frac{1}{2}}}\right)_2 = \frac{\partial(V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}})}{\partial(W/\sigma)}\cdot\left[\left(\frac{W}{\sigma}\right)_1 - \left(\frac{W}{\sigma}\right)_2\right] + \frac{1}{2}\cdot\frac{\partial^2(V_c\sigma^{\frac{1}{2}}/W^{\frac{1}{2}})}{\partial(W/\sigma)^2}\times$$
$$\times\left[\left(\frac{W}{\sigma}\right)_1 - \left(\frac{W}{\sigma}\right)_2\right]^2 \qquad \text{A.5}$$

should be used. From A.2

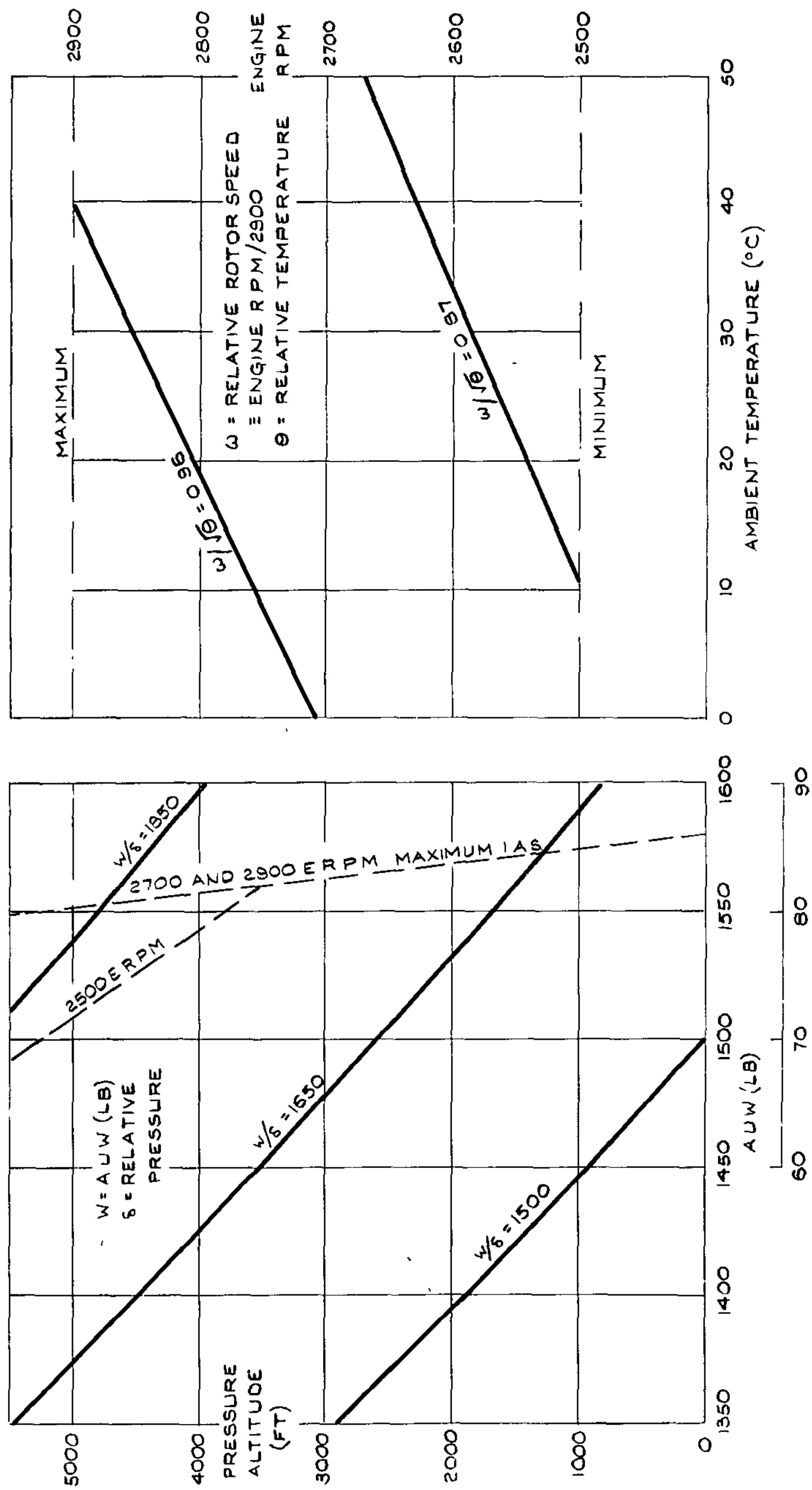
$$\frac{\partial^2 (V_c \sigma^{\frac{1}{2}} / W^{\frac{1}{2}})}{\partial (W/\sigma)^2} = \frac{\frac{1}{8} \rho_o \Omega^3 R^3 As \left[- \frac{15}{2} \frac{C_D}{(W/\sigma)^3} + \frac{3 \partial C_D / \partial (W/\sigma)}{(W/\sigma)^2} - \frac{2 \partial^2 C_D / \partial (W/\sigma)^2}{(W/\sigma)} \right]}{(W/\sigma)^{\frac{1}{2}} \left[1 + \frac{V_c \sigma^{\frac{1}{2}}}{2 W^{\frac{1}{2}}} \Bigg/ \sqrt{\left(V_o \sigma^{\frac{1}{2}} / 2 W^{\frac{1}{2}} \right)^2 + \frac{k}{2 A \rho_o}} \right]}$$

and from the equation for blade drag quoted above

$$\frac{\partial^2 C_D}{\partial (W/\sigma)^2} = 0.0252 \left(\frac{1}{\rho_o \Omega^2 R^2 As} \right)^2$$

Since there is little extra work involved in evaluating the second differential coefficient once the first has been found it is probably worthwhile to check its value even if the change in W/σ is less than 5%.

FIG. 1.



DIAGRAMS SHOWING FLIGHT INFORMATION REQUIRED IN LEVEL FLIGHT TRIALS FOR A SMALL PISTON ENGINED HELICOPTER WITH VARIABLE ROTOR SPEED, & W/δ, $\omega/\sqrt{\theta}$ CONSTANT.

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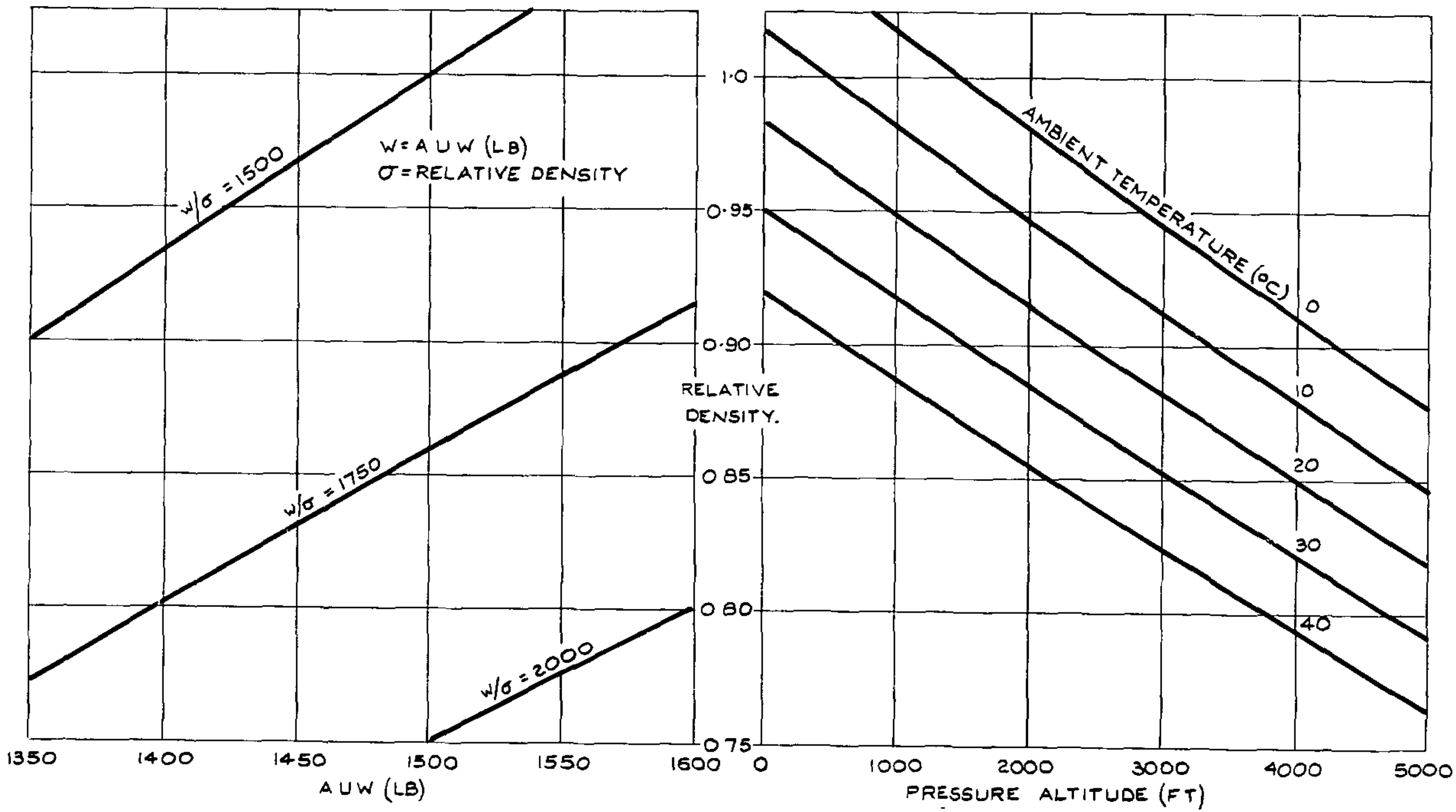


DIAGRAMS SHOWING FLIGHT INFORMATION REQUIRED IN LEVEL FLIGHT TRIALS FOR A SMALL PISTON ENGINED HELICOPTER WITH FIXED ROTOR SPEED.

FIG. 2.

DIAGRAM SHOWING FLIGHT INFORMATION REQUIRED IN LEVEL FLIGHT TRIALS FOR A SMALL PISTON ENGINED HELICOPTER WITH VARIABLE ROTOR SPEED, & $W/\sigma\omega^2$ CONSTANT.

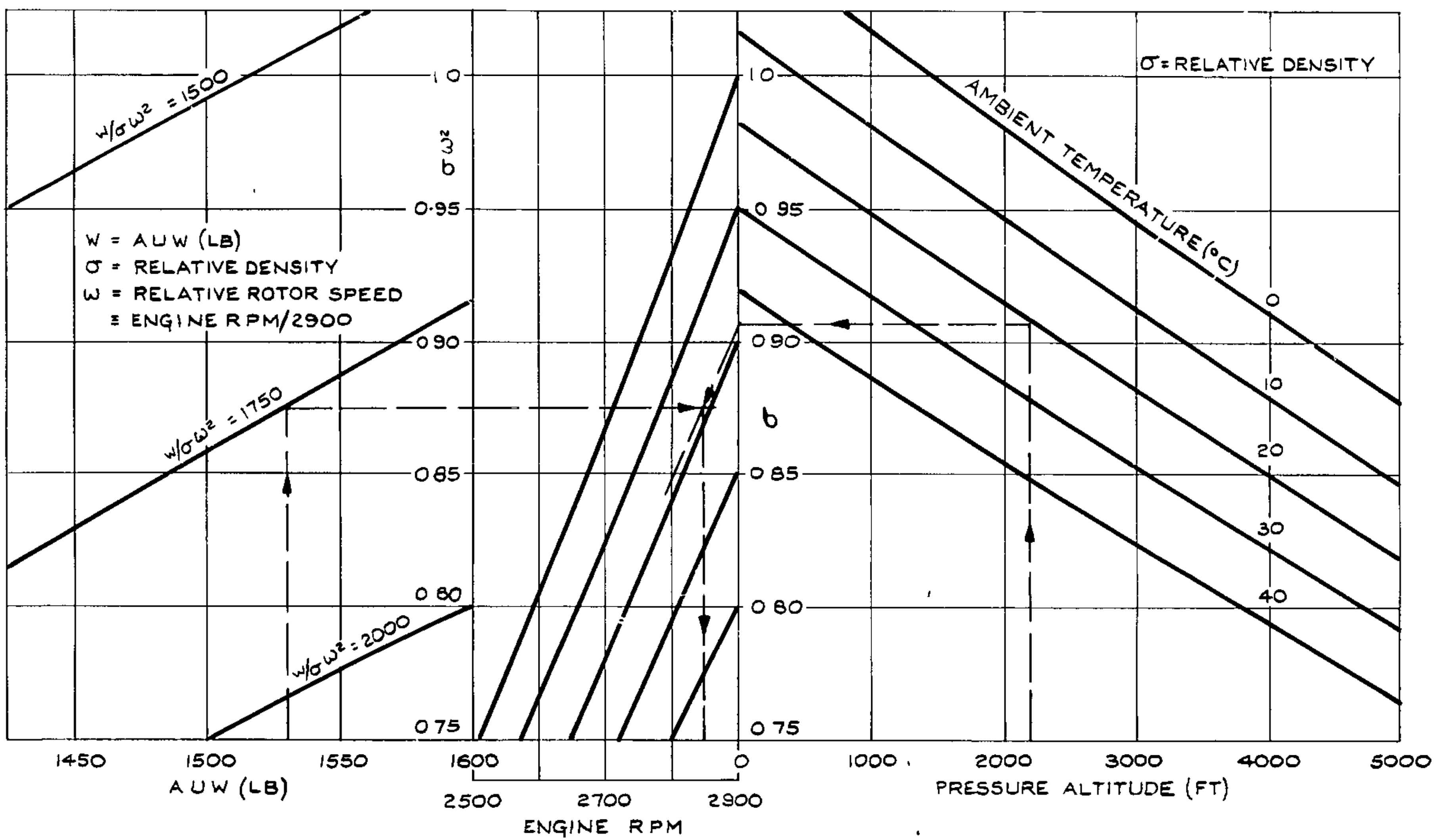
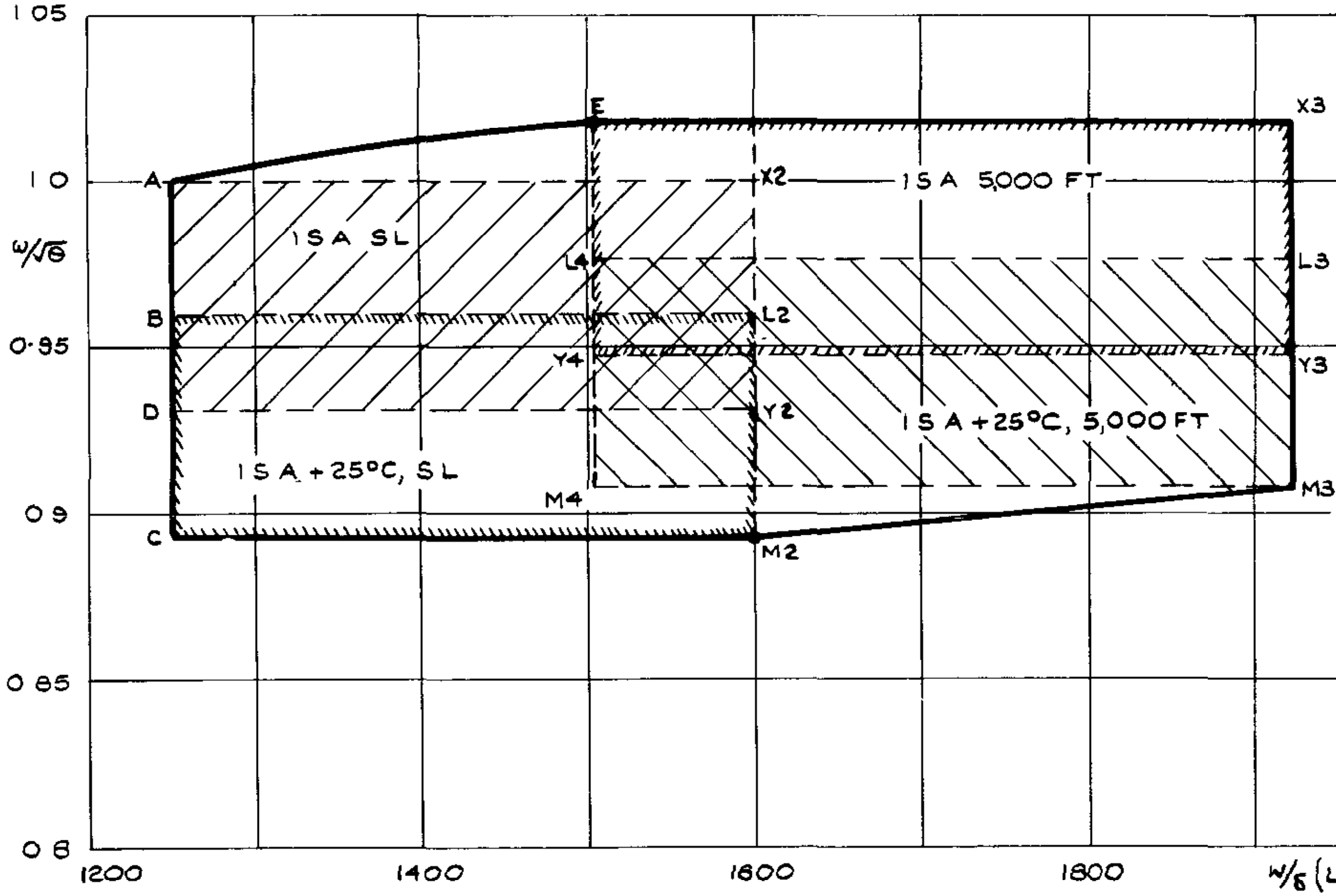


FIG. 3.



NOTE THE ENVELOPE SHOWN IS MADE UP OF RECTANGLES, EACH OF WHICH CONTAINS THE COMBINATIONS OF W/δ & $\omega/\sqrt{\theta}$ NECESSARY TO PREDICT RESULTS FOR THE STATED CONDITIONS THE FOUR EXTREME RECTANGLES ARE SHOWN TYPICALLY, FOR I S A + 25°, 5,000 FT AND I S A S L CASES, LINES A, X2 & L4, L3 REPRESENT AN INCREASE OF A U W AT CONSTANT, MAXIMUM ROTOR SPEED; LINES X2, Y2 & L3, M3 A DECREASE OF ROTOR-SPEED AT CONSTANT A U W, LINES Y2, D & M3, M4 A DECREASE OF A U W AT CONSTANT, MINIMUM ROTOR SPEED & LINES D, A & M 4, L4 AN INCREASE OF ROTOR SPEED AT CONSTANT A U W.

W = A U W (LB)
δ = RELATIVE PRESSURE
θ = RELATIVE TEMPERATURE
ω = RELATIVE ROTOR SPEED
$= \dfrac{\text{ENGINE SPEED (R P M)}}{2900}$

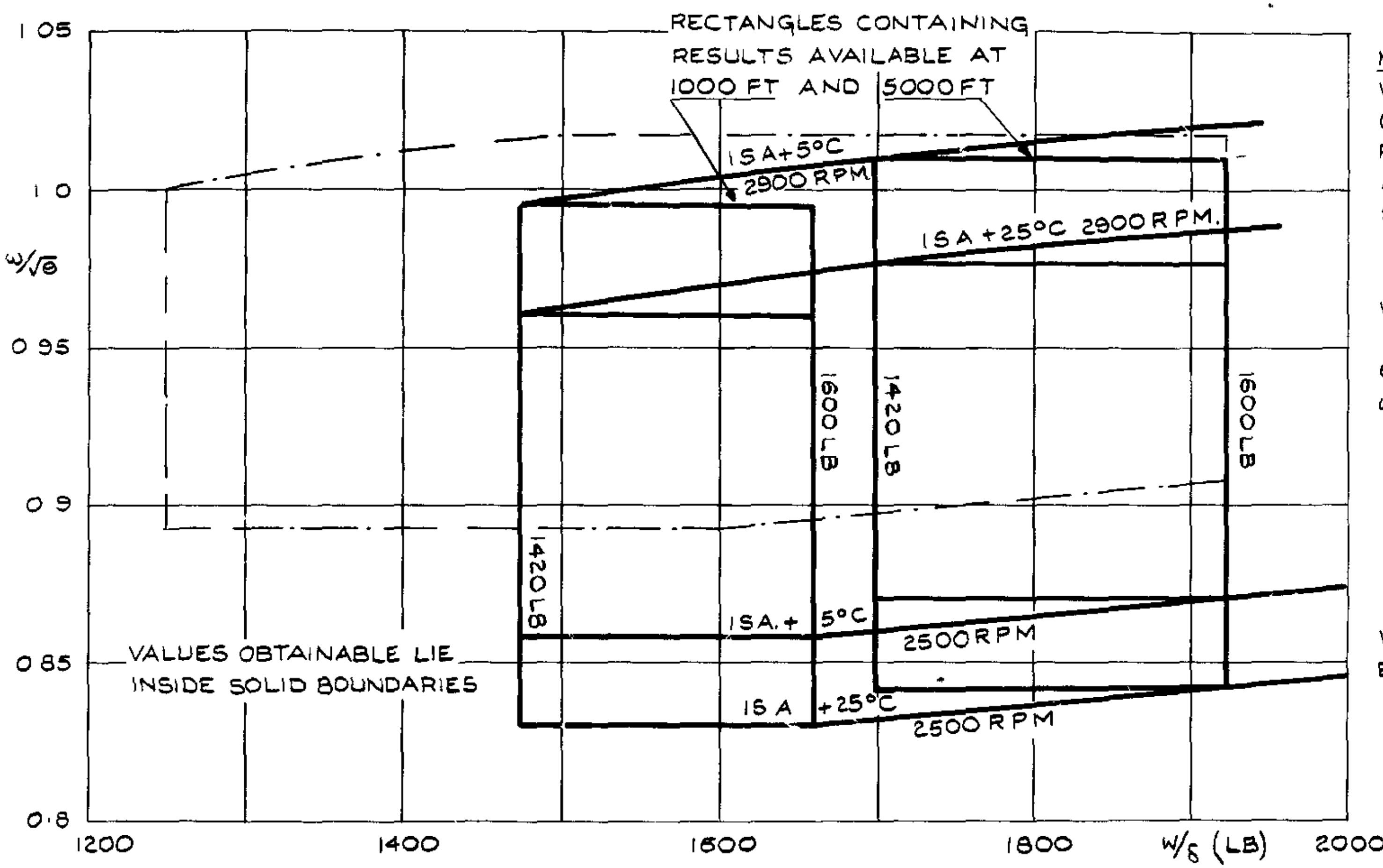
RANGE OF W 1250-1600 LB
RANGE OF ENGINE R P M 2700-2900

TEST LEVEL FLIGHT WITH VARIABLE ROTOR SPEED

DIAGRAM SHOWING COMBINATIONS OF W/δ & $\omega/\sqrt{\theta}$ FOR WHICH TEST RESULTS ARE REQUIRED TO BE ABLE TO PREDICT RESULTS OVER THE REQUIRED RANGES OF A.U.W, ALTITUDE, TEMPERATURE AND ROTOR SPEED.

FIG. A.I.

DIAGRAM SHOWING COMBINATIONS OF W/δ & $\omega/\sqrt{\theta}$ OBTAINABLE WHEN TESTING IN I.S.A.+5°C AND I.S.A.+25°C CONDITIONS.



NOTE THE AREAS LYING WITHIN DASHED LINES CONTAIN COMBINATIONS OF W/δ & $\omega/\sqrt{\theta}$ FOR WHICH TESTS RESULTS ARE REQUIRED FOR DETAILS SEE FIG A I

W = A U W (LB)

δ = RELATIVE DENSITY

θ = RELATIVE TEMPERATURE

ω = RELATIVE ROTOR SPEED

$= \dfrac{\text{ENGINE SPEED(RPM)}}{2900}$

RANGE OF VALUES AVAILABLE

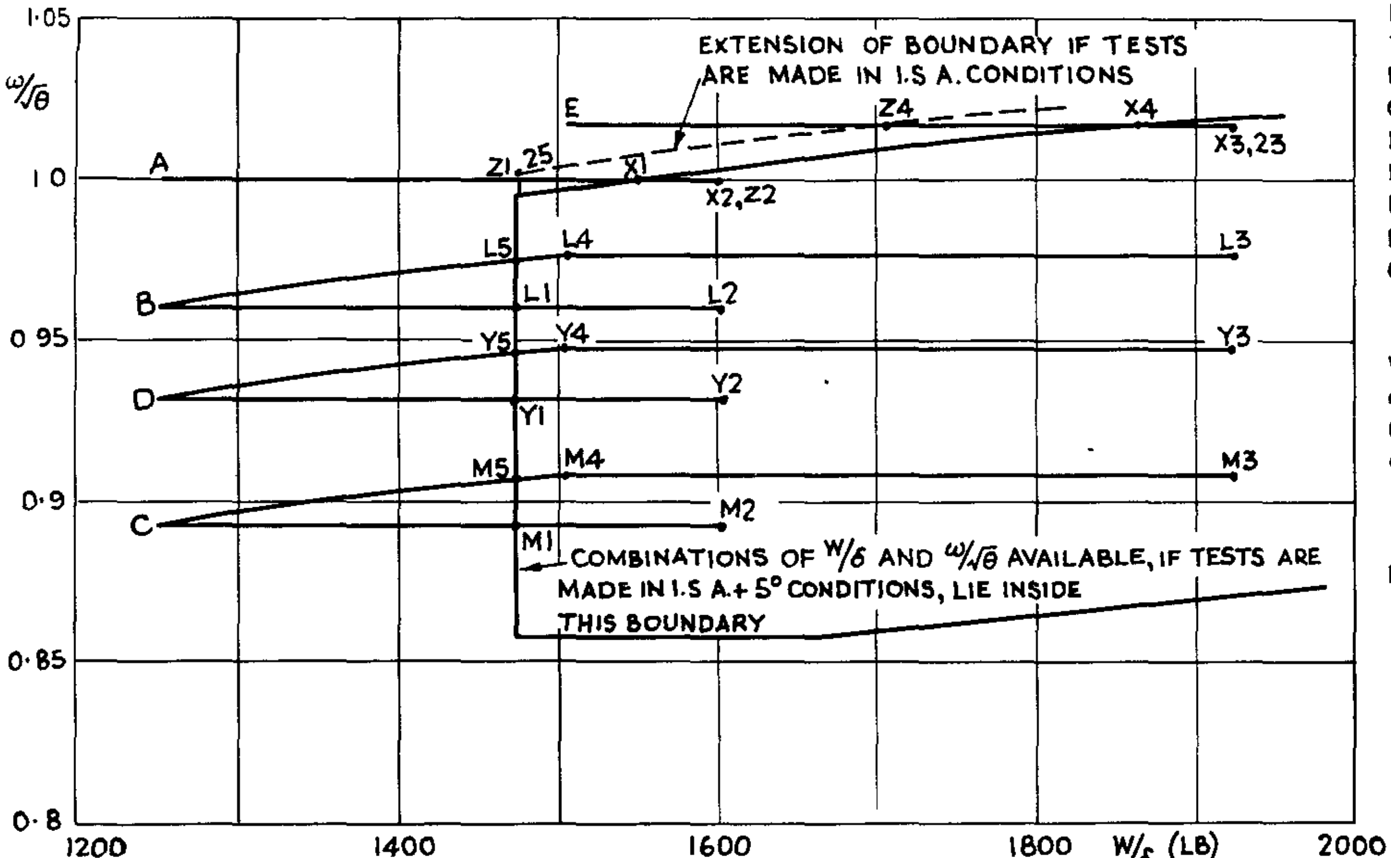
W: 1420 - 1600 LB

ENGINE RPM 2500-2900

TEST LEVEL FLIGHT WITH VARIABLE ROTOR SPEED

FIG. A.2.

PARTS OF FIGS. A.1. AND A.2. REDRAWN TO SHOW THE METHOD OF OBTAINING FIG. A.4.



TEST: LEVEL FLIGHT WITH VARIABLE ROTOR SPEED.

LINE	SHOWS RESULTS REQUIRED AT
A, X2	I.S.A., S.L. 2900 R.P.M.
D, Y2	I.S.A., S.L. 2700 R.P.M.
E, X3	I.S.A., 5000 FT 2900 R.P.M.
Y4, Y3	I.S.A., 5000 FT 2700 R.P.M.
B, L2	I.S.A., +25°C, S.L. 2900 R.P.M.
C, M2	I.S.A. + 25°C, S.L. 2700 R.P.M.
L4, L3	I.S.A., +25°C, 5000 FT 2900 R.P.M.
M4, M3	I.S.A. + 25°C, 5000 FT 2700 R.P.M.
D, Y4	I.S.A. 2700 R.P.M. W = 1250 LB
B, L4	I.S.A. + 25°C, 2900 R.P.M. W = 1250 LB.
C, M4	I.S.A. + 25°C, 2700 R.P.M. W = 1250 LB

W = A.U.W. (LB)

δ = RELATIVE PRESSURE

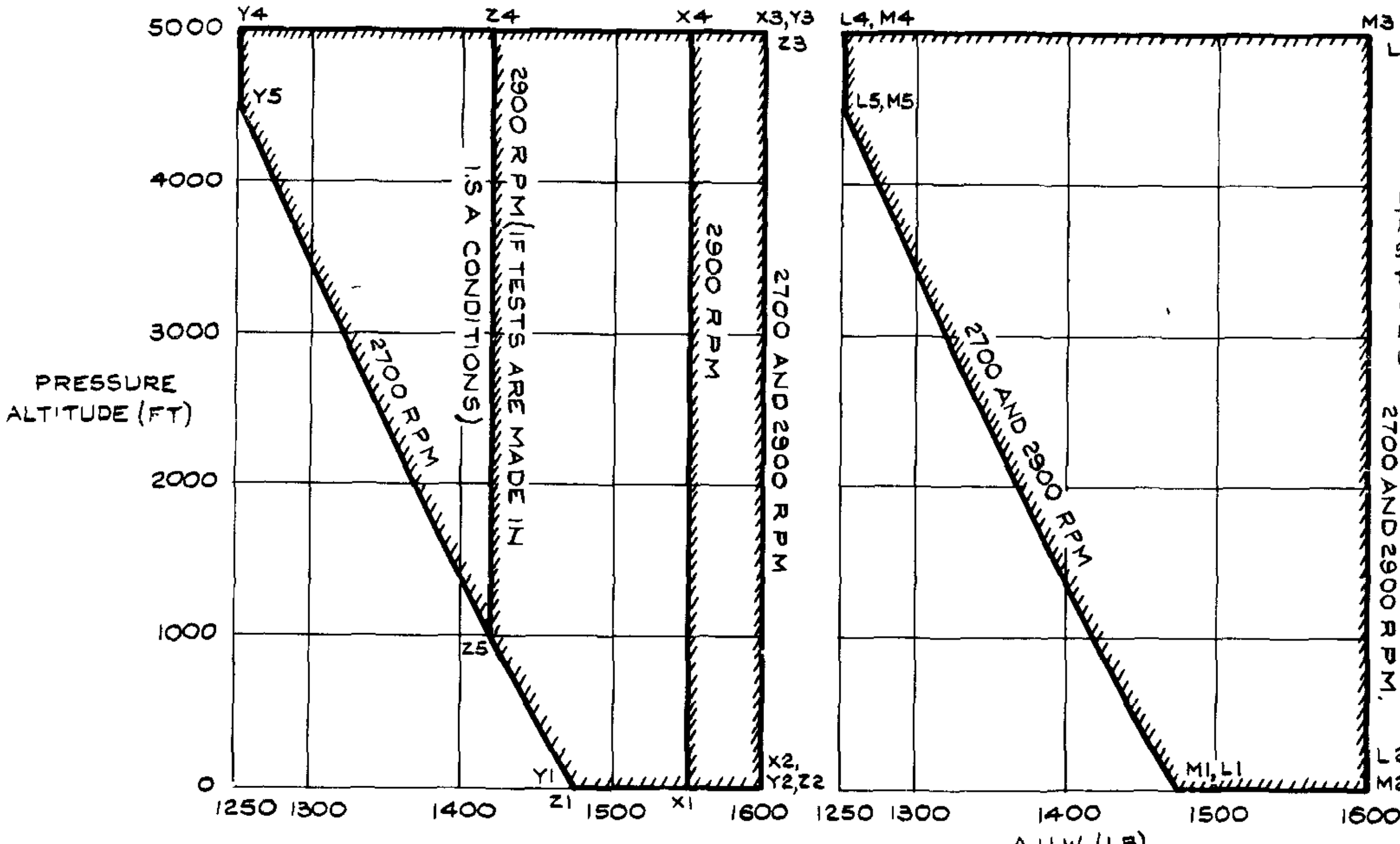
θ = RELATIVE TEMPERATURE

ω = RELATIVE ROTOR SPEED $= \dfrac{\text{ENGINE SPEED (RPM)}}{2900}$

NOTE: THE PURPOSE OF THIS FIGURE IS DESCRIBED IN SECTION A.12

FIG. A.3.

DIAGRAMS SHOWING COMBINATIONS OF WEIGHT AND ALTITUDE FOR WHICH RESULTS MAY BE OBTAINED, AT THE STATED ENGINE (ROTOR) SPEEDS BY TESTING IN I.S.A+5°C CONDITIONS



(a) I S A RESULTS

(b) I S A + 25°C RESULTS

NOTE
POINTS LETTERED ARE SHOWN IN FIGS A.1 AND A.3
THE METHOD OF CONSTRUCTING THIS FIGURE IS DESCRIBED IN SECTION A 1 2

TEST LEVEL FLIGHT WITH VARIABLE ROTOR SPEED.

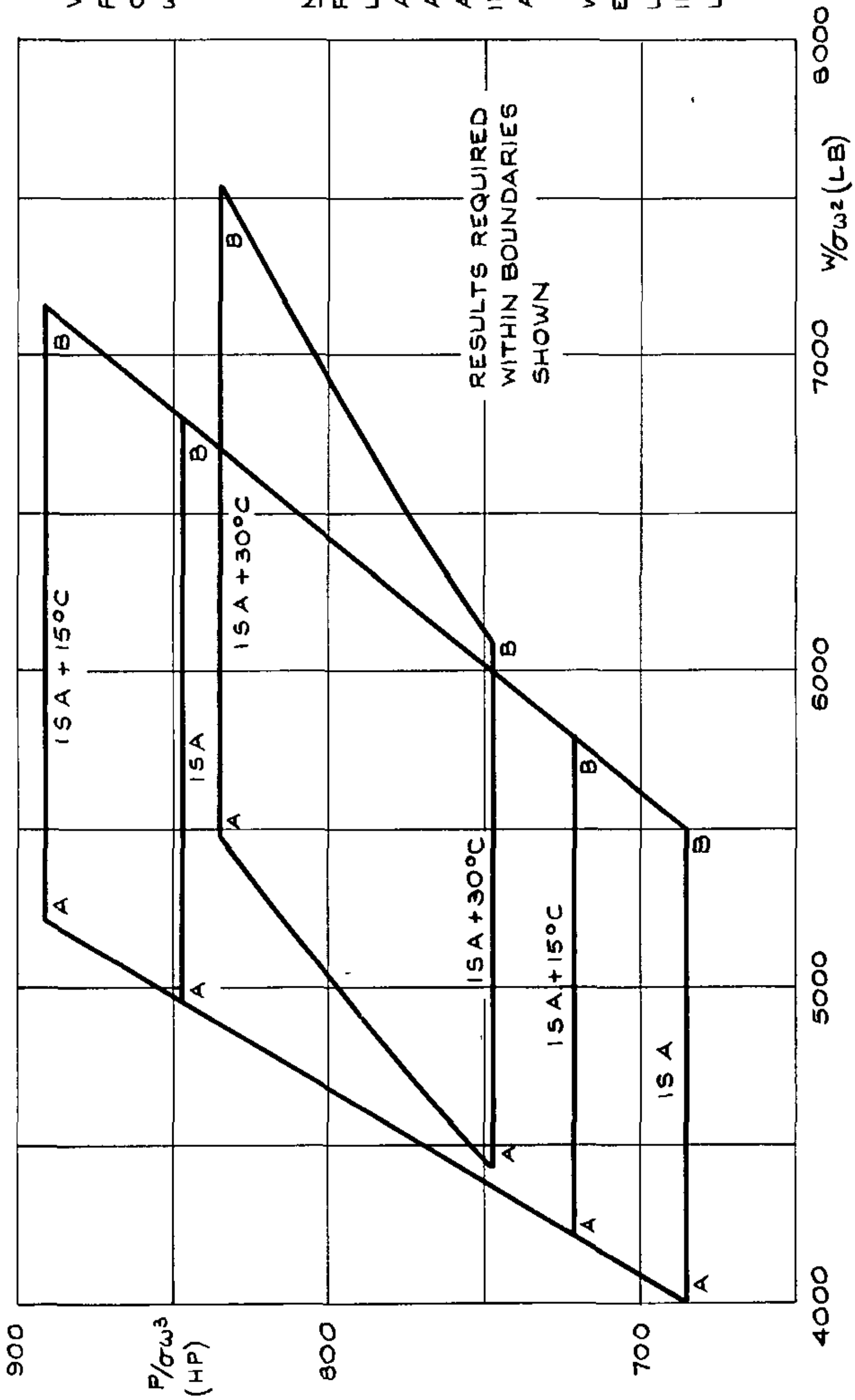
FIG. A.4.

FIG. A.5.

W = A U W (LB)
P = POWER REQUIRED (HP)
σ = RELATIVE DENSITY
ω = RELATIVE ROTOR SPEED
= ROTOR SPEED (R P M) / 400

NOTE ALL RESULTS ARE REQUIRED AT ω=1 THEN LINES 'A'-'B' REPRESENT AN INCREASE OF WEIGHT AT CONSTANT ALTITUDE, AND LINES 'A'-'A', 'B'-'B' AN INCREASE OF ALTITUDE AT CONSTANT WEIGHT.

POWER IS CONSTANT WITH ALTITUDE IF THE ENGINE IS TORQUE LIMITED, BUT DECREASES IF A TEMPERATURE LIMITATION APPLIES



TEST REDUCED POWER VERTICALS WITH VARIABLE ROTOR SPEED

DIAGRAM SHOWING COMBINATIONS OF $W/\sigma\omega^2$ & $P/\sigma\omega^3$ FOR WHICH TEST RESULTS ARE REQUIRED IN ORDER TO BE ABLE TO PREDICT RESULTS FROM SEA LEVEL TO 7000 FT. UNDER THE STATED AMBIENT CONDITIONS.

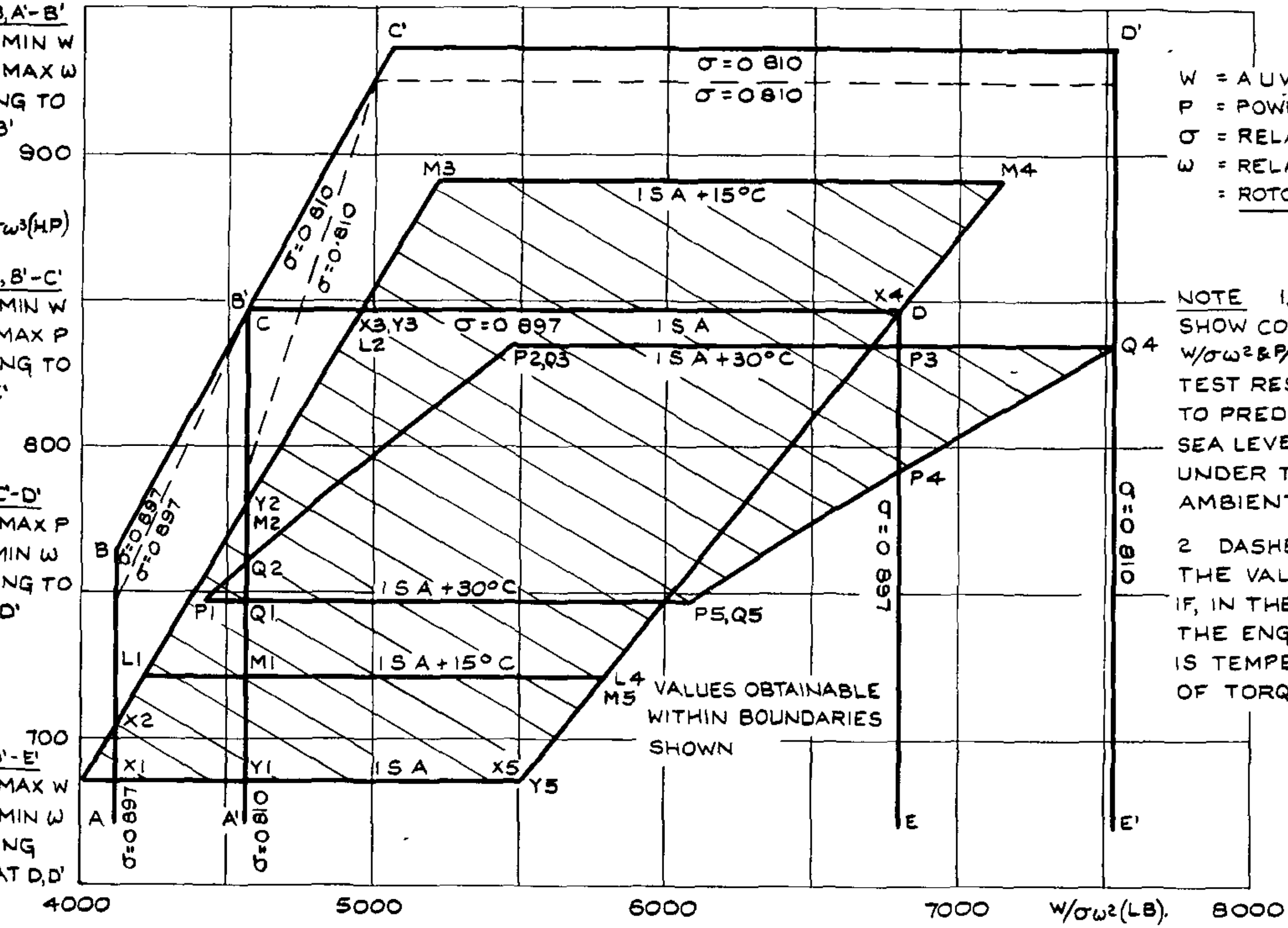
DIAGRAM SHOWING COMBINATIONS OF $W/\sigma\omega^2$ & $P/\sigma\omega^3$ OBTAINABLE AT VARIOUS VALUES OF RELATIVE DENSITY

LINES A-B, A'-B'
CONSTANT MIN W
CONSTANT MAX ω
P INCREASING TO MAX AT B, B'

LINES B-C, B'-C'
CONSTANT MIN W
CONSTANT MAX P
ω DECREASING TO MIN AT C, C'

LINES C-D, C'-D'
CONSTANT MAX P
CONSTANT MIN ω
W INCREASING TO MAX AT D, D'

LINES D-E, D'-E'
CONSTANT MAX W
CONSTANT MIN ω
P DECREASING FROM MAX AT D, D'



W = A U W (LB).
P = POWER REQUIRED (H P)
σ = RELATIVE DENSITY
ω = RELATIVE ROTOR SPEED = $\frac{\text{ROTOR SPEED (R P M)}}{400}$

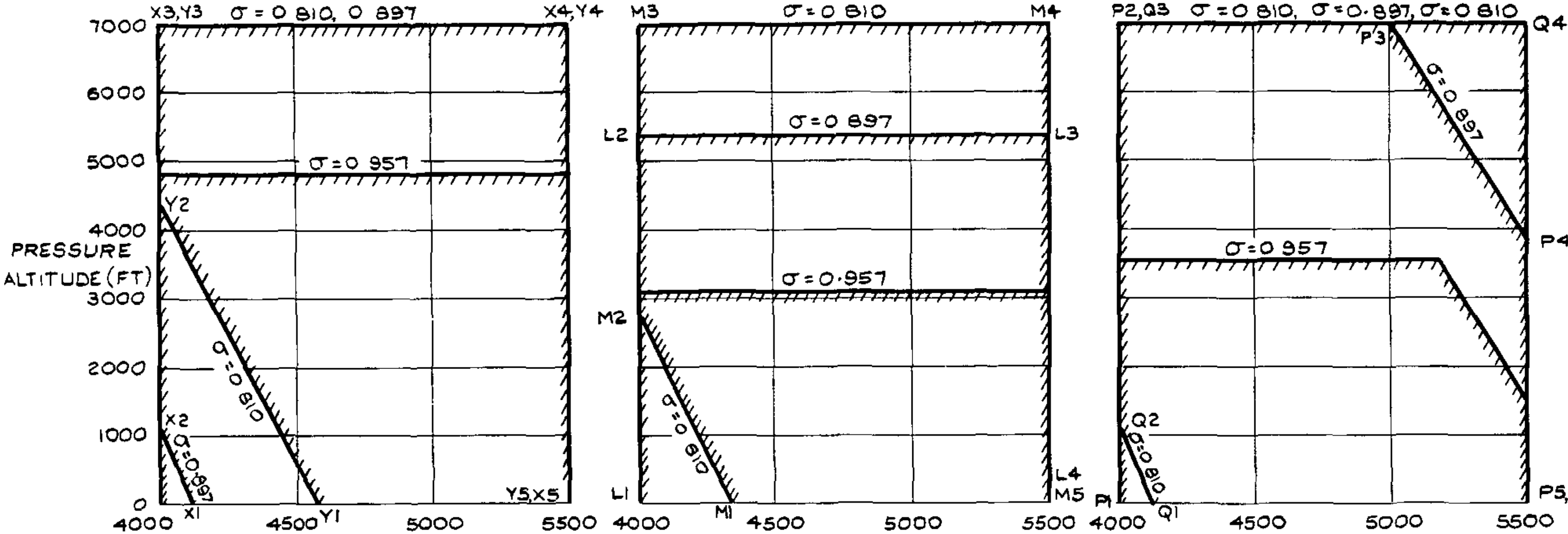
NOTE 1. SHADED AREAS SHOW COMBINATIONS OF $W/\sigma\omega^2$ & $P/\sigma\omega^3$ FOR WHICH TEST RESULTS ARE REQUIRED TO PREDICT RESULTS FROM SEA LEVEL TO 7000 FT UNDER THE STATED AMBIENT CONDITIONS

2 DASHED LINES SHOW THE VALUES AVAILABLE IF, IN THE TEST CONDITION, THE ENGINE LIMITATION IS TEMPERATURE INSTEAD OF TORQUE

TEST REDUCED POWER VERTICALS WITH VARIABLE ROTOR SPEED

FIG. A.6.

DIAGRAMS SHOWING COMBINATIONS OF WEIGHT AND ALTITUDE FOR WHICH RESULTS MAY BE OBTAINED BY TESTING AT THE STATED VALUES OF RELATIVE DENSITY



(a) ISA RESULTS

(b) ISA + 15°C RESULTS

(c) ISA + 30°C RESULTS

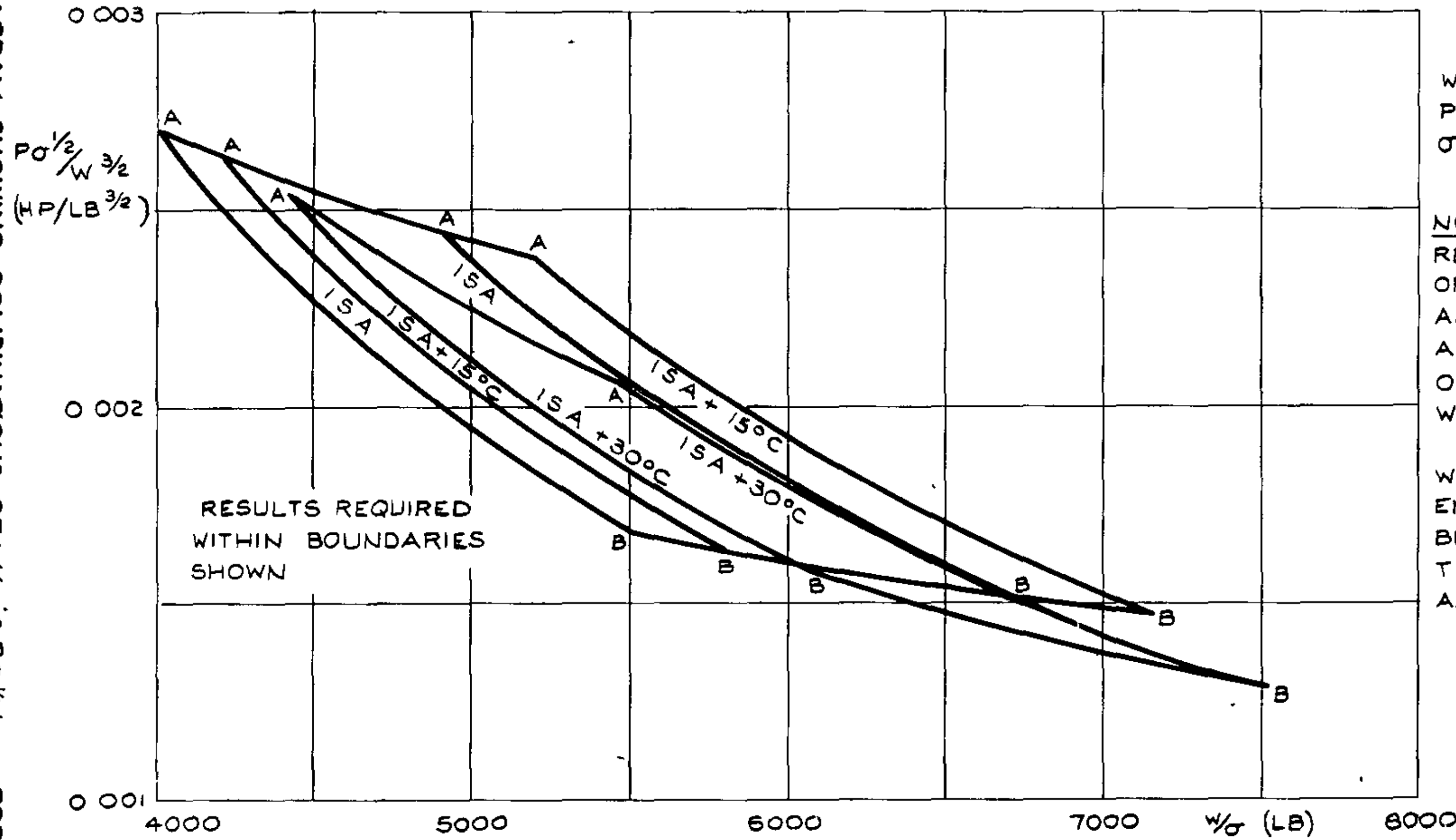
TEST REDUCED POWER VERTICALS WITH VARIABLE ROTOR SPEED

σ = RELATIVE DENSITY X, Y, L, M, P, Q DEFINE POINTS WHOSE EQUIVALENTS ARE SHOWN IN FIG A6.

FIG. A.7.

SK.B 3636 TECH/322/PERF

TRAEN CH P KNOWLES APP for S of P 10 9 65



W = AUW (LB)
P = POWER REQUIRED (HP)
σ = RELATIVE DENSITY

NOTE LINES A-B REPRESENT AN INCREASE OF WEIGHT AT CONSTANT ALTITUDE AND LINES A-A, B-B AN INCREASE OF ALTITUDE AT CONSTANT WEIGHT.

POWER IS CONSTANT WITH ALTITUDE IF THE ENGINE IS TORQUE LIMITED, BUT DECREASES IF A TEMPERATURE LIMITATION APPLIES

DIAGRAM SHOWING COMBINATIONS OF W/σ & $P\sigma^{1/2}/W^{3/2}$ FOR WHICH TEST RESULTS ARE REQUIRED IN ORDER TO BE ABLE TO PREDICT RESULTS FROM SEA LEVEL TO 7000 FT UNDER THE STATED AMBIENT CONDITIONS.

TEST: REDUCED POWER VERTICALS WITH CONSTANT ROTOR SPEED

FIG. A.8.

10.9.65 SK B 3637

FIG. A.9.

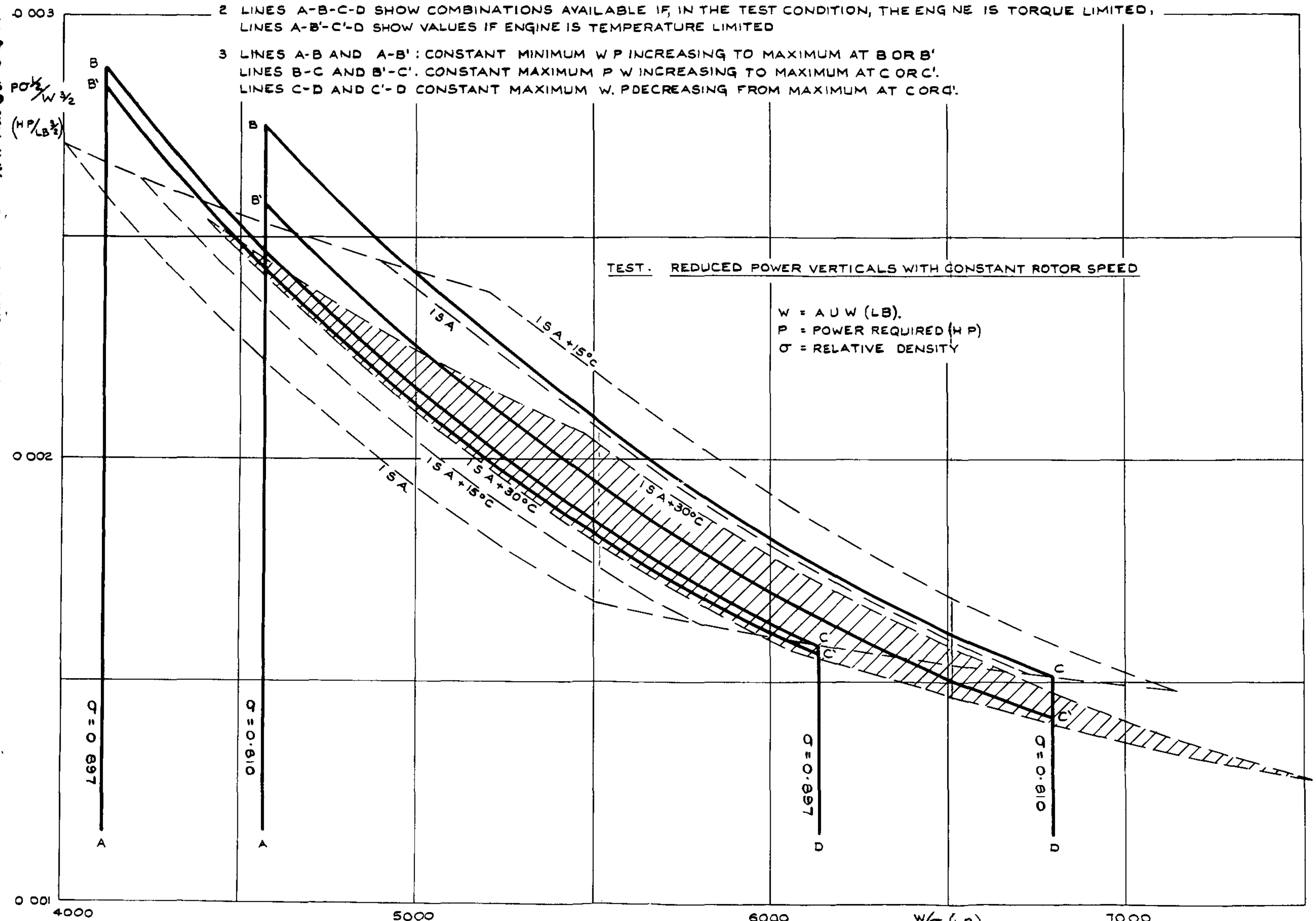


DIAGRAM SHOWING COMBINATIONS OF W/σ & $P\sigma^{1/2}/W^{3/2}$ OBTAINABLE AT TWO VALUES OF RELATIVE DENSITY.

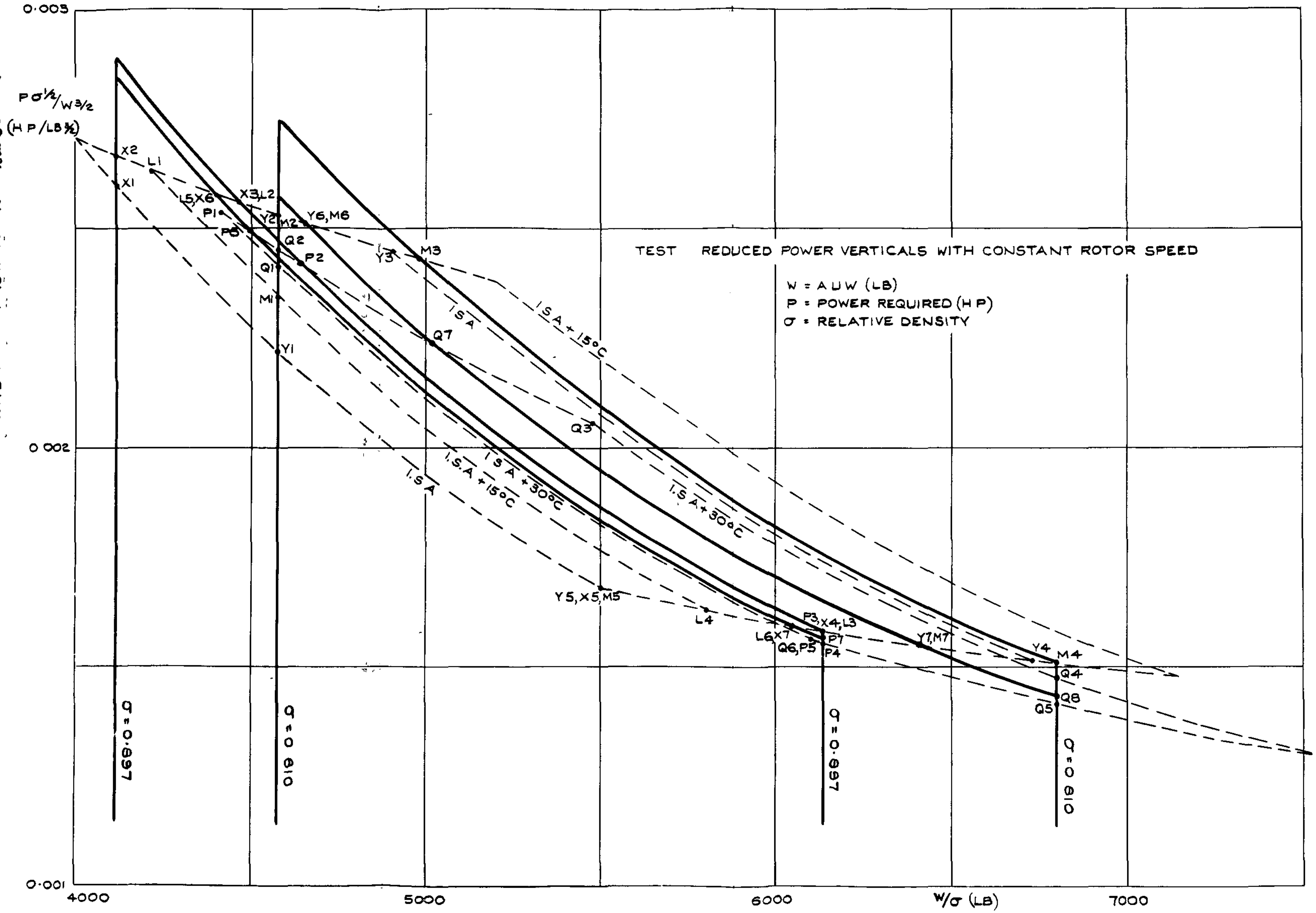
TR.AEN CH. P KNOWLES APP. [signature] for S of P

TECH/322/PERF

SK.B 3637

FIG. A.9.

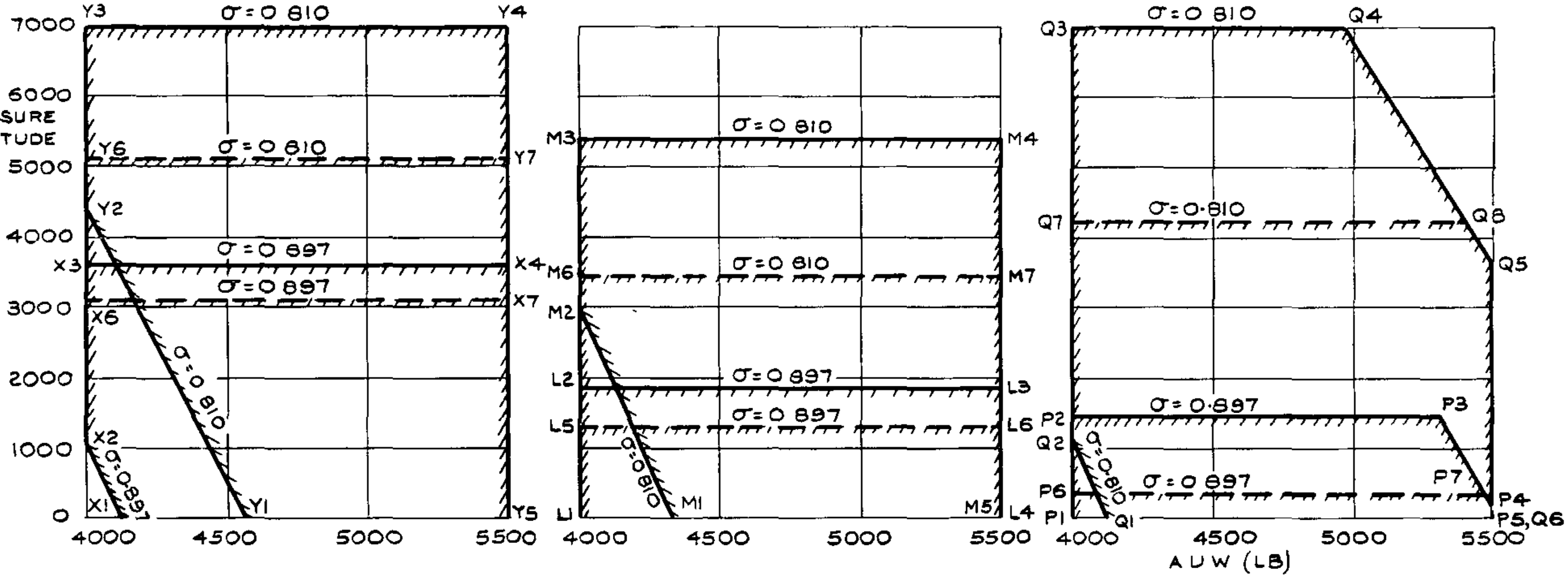
TR.A E N CH. P KNOWLES. APP 10 9 65



TECH/322/PERF

FIG.A.9. MARKED TO SHOW THE POINTS USED TO CONSTRUCT FIG.A.11.

SK B 3639   TECH / 322/PERF   TRAEN CH P KNOWLES   APP KW for S of P 10 9 65



(a) I S A RESULTS   (b) I S A + 15° C RESULTS   (c) I S A + 30°C RESULTS

TEST: REDUCED POWER VERTICALS WITH CONSTANT ROTOR SPEED

X, Y, L, M, P, Q DEFINE POINTS WHOSE EQUIVALENTS ARE SHOWN IN FIG A 10.

σ = RELATIVE DENSITY

DASHED LINES SHOW THE VALUES AVAILABLE IF, IN THE TEST CONDITION, THE ENGINE LIMITATION IS TEMPERATURE INSTEAD OF TORQUE

DIAGRAMS SHOWING COMBINATIONS OF WEIGHT AND ALTITUDE FOR WHICH RESULTS MAY BE OBTAINED BY TESTING AT THE STATED VALUES OF RELATIVE DENSITY

FIG. A.11.

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