C.P. No. 905

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by

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LONDON· HER MAJESTY'S STATIONERY OFFICE

1967

PRICE 7*s* 6*d* NET

U.D.C. No. 678.664 : 621-496 : 539.531 : 534.372 : 539.434

C.P. No.905*
August 1965

STIFFNESS, DAMPING AND CREEP PROPERTIES OF A POLYURETHANE FOAM INCLUDING THE EFFECTS OF TEMPERATURE AND HUMIDITY

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D. B. Payen, H.N.D.

SUMMARY

Experiments have been made to establish the physical properties of a flexible polyurethane foam plastic used in the construction of aeroelastic models for use in low-speed wind tunnels. Simple static and dynamic bending tests were made on a composite cantilever of plastic foam and a light alloy bar. It is shown that dynamic stiffness and damping capacity measurements must take account of temperature and absolute humidity, and that the dynamic stiffness was approximately double the static value over the very limited frequency range considered. Other phenomena such as creep and the effect of 'working' are discussed.

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* Replaces R.A.E. Technical Report No.65198 - A.R.C.27597

CONTENTS

1 INTRODUCTION

The advent of high speed aircraft has created a need for fresh techniques in aeroelastic modelling. In particular the chordwise flexibility of low-aspect-ratio wings cannot be represented adequately by the single-spar or torsion-box type of construction used extensively in the past.

A form of flexible construction for low-speed wind-tunnel models is to make a plane network of interconnected metal beams with weights attached, which represents the required stiffness and mass distribution. Around this skeleton, a light expanded polyurethane plastic, hereafter called foam, is cast to provide the desired aerodynamic profile. Ideally the foam should make a negligible contribution to the elastic properties of the completed model, but in practical applications it is found to be significant.

No relevant information regarding the stiffness properties of foam is available, but Redshaw[1] has shown that the static and dynamic stiffnesses of solid xylonite plastic differ, and that the static value is affected by humidity. These peculiarities of a related material indicated the need to examine the stiffness characteristics of the foam.

The investigation comprised static stiffness, dynamic stiffness and damping tests, made on a composite cantilever of low density foam reinforced with a central duralumin bar. The static tests were simple bending and were extended to examine qualitatively the creep properties of the foam and the effect of systematic squeezing or 'working'. The dynamic experiments were more involved and provided a convenient means to examine the effect of humidity and temperature on the stiffness and damping in the foam. An oscillatory technique was used where the cantilever was vibrated and its response monitored with a proximity transducer. For the stiffness tests the oscillation was forced and the resonant condition established. Damping was measured by the direct comparison of the response in free vibration with a known electrical signal of controllable exponential decay. The results were analysed on the assumption that the cantilever could be treated by engineering beam theory and its stiffness is used to determine an effective Young's modulus of the foam.

In Section 5 the detailed observations of the experiments are presented. It is shown that there is a large variation of dynamic stiffness and damping with normal day to day changes in temperature and humidity. The dynamic stiffness is almost double the static stiffness in states of comparable humidity, and the initial static stiffness may be reduced considerably by 'working' the foam without causing appreciable permanent deformation. The damping in the foam is found to be hysterotic. An explanation of these findings in terms of the chemical composition and structure of the foam, is given in Section 6.

2 SYNOPSIS OF TESTS

A broad outline of the investigations is presented in the table below followed by summaries of test procedures which show the state and environment of the specimen at each stage of the experiment.

Table of tests

Type	Breakdown		Condition of foam	Environment of specimen	Remarks
Static	1 Creep		W	Open laboratory	Numerous checks on H & T
	2 Stiffness	a Effect of working	U → W	Open laboratory	-
		b Values measured	W	Open laboratory	H & T measured
Dynamic	3 Stiffness	a Effect of H & T	W	Humidity chamber	Controlled H & T
		b Effect of frequency	W	Open laboratory	Test completed in under one hour
	4 Damping	a Type of	W	Open laboratory	H & T measured
		b Effect of H & T	W	Humidity box	Limited H & T control

Key:- W = worked; U = unworked; H = humidity; T = temperature

2.1 Creep test

The creep* investigation was qualitative and simply entailed loading the composite cantilever specimen and noting its deflection at set intervals thereafter until all motion had virtually ceased. The load was then removed and deflections measured as before.

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* The creep property of the foam is probably anelastic[2], i.e. response to stress is diffusion-controlled and molecular orientation is induced by shear. The configurational entropy, or molecular disorder, is thus decreased so that release of the stress, results in an attempt by the molecules to restore maximum entropy and hence produces a definite, though low, restoring force, with no permanent set.

2.2 Static stiffness tests

(a) Effect of 'working'

Simple experiments indicated that the stiffness of the foam could be reduced by squeezing or manipulating it between the fingers and thumb. This process covering the whole of its surface is herein referred to as 'working'.

A stiffness value for the cantilever was found by evaluating the average slope of a load against deflection graph. This plot was obtained by loading the beam in equal increments and measuring the deflection 30 seconds after each increment. In this way, the stiffness of 'unworked' foam and foam after a number of successive 'workings' were compared.

Although allowances for humidity and temperature were not included at this stage, it is clear that the effect of working is substantial.

(b) Measurement of stiffness

It was difficult to assess a true value for the static stiffness because of the rapid creep of the foam. However, the following procedure was adopted. A single load was applied to the beam and the deflection was measured after thirty minutes - a period long enough to allow most of the creep to take place. The response was then assumed to be linear* with the load applied.

2.3 Dynamic stiffness tests

In experiments to determine the dynamic stiffness of the foam the beam was mounted vertically with the mass above the support and harmonically forced by an electromagnetic exciter. By adjusting the excitation frequency until the displacement response under constant input power was a maximum, the resonant frequency of the beam was established and its rigidity deduced.

(a) Effect of humidity and temperature

The composite cantilever was mounted in a closed chamber described in Section 3.4(i), and its stiffness variation with temperature examined while maintaining a constant relative humidity. This was repeated for a number of humidity values covering a wide range of conditions.

(b) Frequency effect

An attempt was made to examine the variation of stiffness with frequency by varying the concentrated mass attached to the end of the beam.

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* Plastics and polymers are materials that obey a generalised Hooke's Law[2] i.e. the total effect on a strain-time curve of doubling the stress throughout the test is to double the strain throughout without changing the time scale.

The range of frequency was however, limited to 2·54 c/s to 6·07 c/s because of the so called pendulum stiffness incurred by the mass.

2.4 Damping tests

Damping measurement was made by comparing the response 'envelope' of the beam in free vibration with the curve produced by the exponential decay of potential across a capacitor discharging through a resistance. The procedure used is detailed in Section 4.3.

(a) Nature of damping

The form of damping present in the foam was found by examination of the variation of damping force with frequency in conditions of constant temperature and humidity.

(b) Effect of temperature and humidity

For this test the cantilever was mounted in the humidity box described in Section 3.4(ii), and the enclosed air dried as much as possible with sulphuric acid. Damping and humidity measurements were then made at regular intervals as the humidity slowly varied with ambient temperature and gradual dilution of the acid. The moisture content in the air slowly rose over several days since water was always being introduced via the wet bulb of a psychrometer. A small number of tests were made at higher temperatures with the aid of an electrically heated wire.

3 TEST EQUIPMENT

3.1 Specimen

Two main considerations influenced the design of the specimen cantilever. Some form of reinforcement was necessary to minimise static droop, since the strength-weight ratio of the foam is very low and the length of the beam had to be such that deflection due to shear and end fixing effects were negligible. Secondly the stiffness contribution of the foam had to be sufficiently large compared to that of the reinforcement to ensure maximum accuracy of results. A stiffness ratio of foam to reinforcement of about 3-1 was achieved.

The composite cantilever is illustrated in Fig.1, and was made by glueing two strips of foam of 9 lb/ft^3 density, on either side of a flat duralumin bar. These strips were cut from a block about 7/8 inch thick and stuck to the metal by roughening the surfaces to be glued with glass cloth and applying black Bostik compound. A good bond was produced which lasted throughout the tests. The metal bar was clamped securely at one end between two steel blocks, and one end of each foam strip glued to its respective block whilst the beam was unloaded. Provision for loading and adding mass to the bar was made at the free end.

3.2 Static deflection measurement apparatus

The end deflection of the cantilever was measured using an optical system that enabled a reading to be taken by simply viewing through a telescope the reflection in a small mirror of a concave illuminated scale. The mirror was mounted on a tripod, one leg being located in a dimple drilled into the head of a small bolt attached to the loading point on the duralumin bar, and the other two legs on a flat horizontal platform held nearby. Any deflection of the bar thereby caused the mirror to rotate.

3.3 Dynamic test apparatus

To test the plastic under dynamic loading the composite cantilever was oscillated by an electromagnetic exciter and its response measured with a Wayne Kerr vibration meter employing a capacity pick-up. The light weight coil of the exciter was attached to the free end of the beam and a small disc of tin foil glued to the plastic surface provided an earth plate for the pick-up. The arrangement is shown in Fig.2.

3.4 Humidity chambers

(i) Humidity chamber for stiffness experiments

Stiffness experiments in humidity and temperature controlled conditions were made with the specimen mounted in a closed chamber, which was of massive construction and heavily lagged. A fan heater and refrigeration unit were incorporated together with a number of thermocouples sited to provide a check on the uniformity of temperature in the working section. In order to measure relative humidity a hair hygrometer was used which was later calibrated against an aspirated wet and dry bulb psychrometer. No direct humidity, control was available and the use of saturated salt solutions proved to be too slow for convenience. However, by careful manipulation of the heater fan and refrigerator the required conditions were obtained and since the insulation was efficient the conditions lasted long enough for the specimen to 'soak' and a test to be conducted. For control at extremes of humidity, the introduction of steam in one case, or the use of a drying agent at the other, was found to be helpful.

(ii) Humidity box for damping experiments

A wooden box lined with heavy gauge polythene and measuring approximately 20 inch × 14 inch × 14 inch was used to obtain the required atmospheric condition. A small fan was incorporated, and the moisture content of the enclosed space was controlled using concentrated sulphuric acid in a glass container, the lid of which could be raised at will.

Humidity measurement was made with an aspirated wet and dry bulb psychrometer that employed thermocouples. This instrument, which was found to be sensitive and simple to operate, had been developed by A.O.R.E., Farnborough.

4 EXPERIMENTAL PROCEDURE AND METHODS OF ANALYSIS

The aim in all the stiffness tests was to establish the rigidity of the composite beam $(EI)_c$ and hence an effective Young's modulus for the foam in a known condition:-

$$E_f = \frac{(EI)_c - (EI)_d}{I_f}$$

The suffixes 'f' and 'd' refer to foam and duralumin respectively. The rigidity of the duralumin bar was determined by static and dynamic measurements before it was built into the specimen.

The experimental procedures employed to obtain the stiffness of the specimen are described in Section 2.

4.1 Static stiffness analysis

From the measured stiffness of the cantilever the rigidity was calculated using simple beam theory, viz:-

$$(EI)_c = \frac{S\ell^3}{3}$$

where S is the direct cantilever stiffness for tip loading.

It can be shown that, for small deflections, the effect of anticlastic curvature restraint in the duralumin bar can be neglected.

4.2 Dynamic stiffness analysis

The dynamic rigidity of the specimen was found from the following relationship which is derived in Appendix A for a cantilever held vertically with its mass below the support.

$$(EI)_c = \frac{\omega^2 M\ell^3}{3}\left\{1 + \frac{0\cdot2357}{\lambda} - \frac{1\cdot2g}{\ell\omega^2} - \frac{0\cdot375g}{\ell\omega^2\lambda}\right\} + 0\cdot75\,\omega^2 M_T \ell K_T^2 \;.$$

ω is assumed equal to the measured resonant frequency of the beam, and the sign of the terms in g is reversed when it is held with its mass above the support.

This formula was derived by assuming that the deflection mode of the vibrating beam is the same as the static deflection curve of a weightless uniform cantilever with a concentrated load at its end. The term $0\cdot75\omega^2 M_T \ell K_T^2$ is an allowance made for the rotary inertia of the exciter coil. An exact solution for the fundamental frequency of a horizontal cantilever of uniformly distributed mass with a load at its end has been given by Timoshenko[3]. The frequency given by the above formula in the absence of gravity (beam in horizontal position) is only 1·5% higher than the exact solution for the extreme case of the end load equal to zero. As a check on the method, the above procedure was performed on a duralumin bar, and the results, shown in Tables 1(a) and 1(b), were very consistent for the beam held in all attitudes and at all tested frequencies. Furthermore the average EI given by this technique was only 1·4% lower than that obtained by static measurement.

4.3 Damping tests

Measurements of damping of the composite cantilever vibrating freely in the fundamental mode were accomplished by the procedure outlined in Section 2.4. The beam was initially displaced onto a stop by passing direct current through the coil of the electromagnetic exciter attached to its free end, and its deflection and subsequent oscillation monitored with the vibration meter. (See Section 3.3.) Air damping in the exciter was minimised by moving the magnetic core just out of the coil. (See Fig.2.)

The circuit used to set the beam in motion and enable a measurement to be made is shown in Fig.3(a). On closing the hand-operated micro-switch the capacitor charged up, the coil was energised and the beam displaced. On opening it the capacitor discharged through the resistance R and the beam was released and performed damped oscillations about the datum position; both processes beginning at the same time. The voltage appearing across the capacitor and resistance R was applied to one Y amplifier of a high input impedance double beam oscilloscope, while the attenuated signal from the vibration meter was applied to the other. By adjusting the controls on the oscilloscope and potentiometer P the two traces were made coincident, both when the beam was at rest in the datum position and the capacitor discharged, and when the beam was fully displaced and the capacitor charged. Thus while the beam was oscillating the pattern produced on the face of the tube was similar to those of Fig.3(b). Damping measurement was made by varying the resistance R and shaping the Y_2 curve until it touched the wavy Y_1 curve at points m_1, m_2, m_3, ..., and reading the final value of R off a calibrated scale.

The damping coefficients were evaluated by identifying the expression for the decay envelope of a single-degree-of-freedom system with that governing the decay of potential of a capacitor discharging through a resistance on the assumption that damping may be represented by velocity dependent forces, i.e.

$$A_0 \, e^{-b\omega t} = E_0 \, e^{-\frac{1}{T} t} .$$

T denotes the time constant of the capacitance resistance circuit and is equal to the product of R and the capacitance, and b is the ratio of the actual damping force coefficient C and the critical value for the system given by,

$$C_{crit} = 2 \sqrt{a\,k}$$

where a and k are the generalised mass and stiffness respectively. A_0 and E_0 are arbitrary constants indicating initial quantities and must be equal. Hence

$$b = \frac{1}{T\omega} . \tag{1}$$

Because in the cases considered the difference between damped and undamped resonant frequencies is negligible, we have, by definition,

$$b \simeq \frac{C}{2\,a\,\omega} \tag{2}$$

so that,

$$C = \frac{2a}{T} . \tag{3}$$

If it is found that the damping is hysteretic rather than viscous, the concept of equivalent viscous damping may be used. The energy dissipated per cycle of hysteresis for most engineering materials is proportional to the square of the amplitude;

so that,

$$(E.D.)_h = K\theta^2 . \qquad (4)$$

The energy dissipated per cycle by viscous damping at resonance is,

$$(E.D.)_v = \pi\omega C\theta^2 . \qquad (5)$$

Equating (4) and (5) the equivalent viscous damping coefficient is deduced as,

$$C_{equiv} = \frac{K}{\pi\omega} = \frac{h}{\omega} \qquad (6)$$

where h is the hysteretic constant for the material. By substitution for C from (3) in (6) we get:-

$$h = \frac{2a\omega}{T} . \qquad (7)$$

In tests where the concentrated mass at the end of the beam was varied while the stiffness was kept constant it was convenient to express equations (3) and (7) in the following forms.

$$C = \frac{2k}{T\omega^2} \qquad (8)$$

$$h = \frac{2k}{T\omega} . \qquad (9)$$

5 PRESENTATION OF RESULTS

5.1 Creep

A typical delayed strain curve is shown in Fig.4, which demonstrates that even though a state of quasi-steady equilibrium is reached after thirty minutes, movement continues slowly. Growth of deflection was never observed to have ceased despite waiting for several hours on one occasion. There is evidence that the creep property is anelastic in character since a high degree of symmetry exists between the load on and load off cases and there is ultimately very little permanent set. (See footnote of Section 2.1.)

5.2 The effect of 'working' the foam

The procedure described briefly in Section 2.2 yielded an interesting result, namely that almost immediately after the foam was 'worked' it recovered its overall dimensions, but took several hours to reach a steady stiffness. This is shown in Table 2. There is however, a permanent effect of successive 'workings' indicated by Table 3 where the foam is gradually weakened until a minimum stiffness is reached that is approximately 65% of the 'unworked' state.

No obvious deterioration of the foam was observed as a consequence of this harsh treatment.

In Fig.5 a typical load-deflection curve for the beam is drawn which exhibits considerable hysteresis. Nevertheless the curves are sufficiently linear over their major portions to give a reliable average stiffness value.

5.3 Dynamic stiffness and the effect of humidity

The variation of the dynamic stiffness of the foam with temperature and relative humidity is plotted as a family of curves in Fig.6. For a given relative humidity there is a small amount of scatter, but the variation of stiffness with humidity and temperature is so great that this becomes insignificant. The possibility of the actual amount of water vapour in the air affecting the stiffness was investigated by replotting Fig.6 in terms of absolute humidity in Fig.7*. The coalescence of the several curves of Fig.6 into a single characteristic indicates that the prevailing absolute humidity is a major factor in determining the stiffness of the foam. In Gt. Britain the normal variation of absolute humidity is from $2{\cdot}5 \times 10^{-4}$ lb/ft^3 to $8{\cdot}0 \times 10^{-4}$ lb/ft^3, so that a 30% change in stiffness could easily occur. Fig.8 is also of interest since it shows that the variation of stiffness with the reciprocal of absolute humidity is fairly linear. When large changes in the air condition were made during the tests, the beam assumed a constant resonant frequency in 15 to 30 minutes, thus illustrating the rapid response of the foam to changing climate.

Static and dynamic stiffness measurements are compared in Figs.8 and 9. In spite of some scatter the curves show that the effective Young's modulus of foam under dynamic loading is nearly double that obtained by static measurement. The range of humidity covered was limited by laboratory conditions, but the indication is that moisture has less effect on the static than on the dynamic stiffness.

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*Absolute humidity is defined as[4], 'the ratio of the mass of water vapour to the volume occupied by the moist air with which it is associated', and can be computed from a knowledge of the relative humidity and air temperature.

Table 4 shows that the variation of dynamic stiffness with frequency is insignificant for the limited range considered.

5.4 Damping characteristics of the foam and the effects of humidity and temperature

The damping capacity of the composite beam and hence the foam was found to be predominantly hysteretic. The results of three separate tests are shown in Table 5, where the variation in hysteretic constant with frequency is negligible in steady conditions of humidity and temperature. In using equation (9) the generalised stiffness was assumed to remain constant and assigned an arbitrary value.

The effects of absolute humidity and temperature on the equivalent viscous damping ratio and hysteretic constant are illustrated in Figs.10 and 11, where each symbol represents measurements made within the temperature range indicated. Although there is some overlap of symbols the influence of temperature on the damping is evident especially at the drier end of the scale. The points marked by asterisks were obtained in an additional test where the temperature was maintained at 31·5°C. These have far less scatter and confirm the trend. The curves are limited at the higher humidities by the onset of condensation in the foam.

The essential difference in character of Figs.10 and 11 is due to the variation in stiffness of the foam with humidity and the accompanying change in resonant frequency of the beam. From Fig.11 the hysteretic constant could change by a factor of two in normal laboratory conditions, and because the air tends to be driest when it is coldest a large proportion of this change could easily occur in a day. This figure has an arbitrary scale since the generalised mass in equation (7) was assumed to remain constant and given a convenient value.

6 AN EXPLANATION OF THE MECHANICAL BEHAVIOUR OF THE FOAM IN TERMS OF ITS COMPOSITION AND STRUCTURE

A simplified description of the chemistry of flexible polyurethane[6] will facilitate further discussion. Basically the material is made by the reaction of an alkyd resin with a liquid di<u>iso</u>cyanate, in which the evolution of carbon dioxide and cross-linking of polymer chains occur simultaneously, generating innumerable bubbles of gas within a gelling mass, to produce a foamed structure. Normally this structure is amorphous, but where a crystalline alkyd is used as one of the ingredients a tendency toward crystallization may appear in the final polyurethane. The molecular composition is one of long polymer chains having a carbon-carbon backbone with considerable freedom of movement at each individual carbon-carbon bond, these chains being tied to each other at infrequent intervals by primary valence bonds or cross links.

The microscopic structure of the foam is shown by a photograph in Fig.12. It consists of small flat-sided cells approximately one millimetre across. Most of the cell walls, however, are punctured so that cell cavities intercommunicate; this being typical of low density flexible polyurethanes. Clearly moisture vapour can penetrate to all parts of the foam, where it will exercise a plasticizing action, that is it reduces the Van der Waals molecular forces between adjacent polymer chains, with the net result that some stiffness is lost. The effect of plasticization has been demonstrated[5] in a study of the widely used 66 nylon which is chemically similar to the foam. A diagram taken from this work is shown in Fig.13 and indicates that a 2% water addition reduces the stiffness by about a quarter.

The load carrying members in polyurethane foam are essentially the edges of each cell, and these - normally termed fibril columns - can buckle and yield in much the same way as conventional struts and ties. During the investigation into the creep characteristics of the foam it was observed that the growth of deflection with time was not continuous, but occurred in sudden jerks which in some cases caused the beam to oscillate momentarily. This implies that there was rapid yielding of parts of the structure at different time intervals. Now if we suppose, that even under small loads there are many compressive elements in the plastic at or near the point of buckling, then a small reduction in their rigidity would lead to a far greater reduction in stiffness of the overall structure. Furthermore this stiffness reduction would depend on the degree of plasticization in the plastic, which in steady conditions will be proportional to the vapour concentration of the air passing through it. The stiffness under prolonged loading however, may not be greatly affected by humidity because of the slow inevitable buckling and yielding of fibril columns as stress relaxation occurs in the polymer; a process that will happen in a random fashion depending on the geometry of the columns within the cellular structure.

Tests showed that when the plastic was 'worked' there was recovery of stiffness overnight, and the initial stiffness was never fully regained. Moreover after several treatments a steady stiffness value was achieved. One may conclude that successive 'workings' lead to permanent damage of the cell walls or very weak fibrils, with consequential loss of efficiency. The recovery effect however, is probably due to the temporary destruction of the stiffer crystalline regions within the polymer, which are restored over a rest period of several hours.

Damping or the dissipation of energy on displacement is associated with molecular activity in the foam; the greater the activity the less the energy loss. Above a so called glass transition temperature Tg individual atoms in the polymer chains rotate and translate continuously, this motion increasing with rising temperature. At Tg molecular motion is very small, and rapidly decreases further as the temperature is lowered and the Van der Waals forces between adjacent chains become dominant.

In the case of polyurethane foams at temperatures between 15° and 30°C the plastic is above its Tg of about -15°C, but well below the temperature at which rapid and complete elastic behaviour is obtained, this normally being 80°C or more. Thus slow elastic response with considerable hysteresis can be expected.

Water vapour, acting as a plasticizer has the effect of lowering Tg, or conversely increasing molecular mobility at a given temperature.

7 CONCLUDING REMARKS

The dynamic stiffness of the foam is greatly affected by variation of absolute humidity, and is approximately double the static value. The stiffness may be considerably reduced in the first instance by 'working' or squeezing it without causing obvious damage.

Temperature and humidity markedly influence the damping capacity, which is hysteretic.

In view of the above properties it is desirable that, in aeroelastic models of the metal frame - foam type of construction, the foam's contribution to the overall stiffness and damping should be kept small. This may be difficult to achieve however, because large stresses can arise in the foam due to its interaction with support members and other components in the model, especially where higher modes of deformation occur. An alternative would be to seal the foam from the atmosphere by coating it with a durable film or skin. No organic skin of this nature could stop the flow of air entirely, but it may slow it down to such an extent that the variation in stiffness of the model during a test would be acceptable.

The qualitative properties of the foam described in this Report are broadly applicable to foams of various densities, but precise values relate to foam of 9 lb/ft^3 density.

8 ACKNOWLEDGEMENTS

The author would like to acknowledge the help given by Mr. L.N. Phillips of CPM Department, R.A.E. with the chemical aspects of the foam's behaviour.

Appendix A

DERIVATION OF FORMULA USED TO CALCULATE THE DYNAMIC STIFFNESS OF THE COMPOSITE CANTILEVER

Taking axes as shown in Fig.14, and assuming that the deflection mode of the vibrating beam is the same as the static deflection curve of a weightless cantilever with a concentrated load at its end, we have

$$E\,I\,x = \frac{M\,\ell\,z^2}{2} - \frac{M\,z^3}{6}$$

or

$$\text{mode } x = (3\eta^2 - \eta^3)\,\ell\,q\,,$$

where $\eta = \frac{z}{\ell}$.

Potential energy

Length of bar from 0 to $\eta\,\ell$ at max displacement is

$$\ell_\eta = \int_0^{\eta\ell} \sqrt{1 + \left(\frac{dx}{dz}\right)^2}\,dz = \ell \int_0^{\eta} \left[1 + (6\eta - 3\eta^2)^2\,q^2\right]^{\frac{1}{2}} d\eta\,.$$

Assuming small oscillations

$$\ell_\eta = \eta\,\ell + (6\eta^3 - 4{\cdot}5\eta^4 + 0{\cdot}9\eta^5)\,q^2\,\ell\,.$$

Length of bar from 0 to $\eta\ell$ at zero displacement is $\eta\ell$ so that height that element of mass $\ell\,d\eta\,\mu$ is raised is

$$h_\eta = \ell(6\eta^3 - 4{\cdot}5\eta^4 + 0{\cdot}9\eta^5)\,q^2\,.$$

Let $\ell\eta_0$ be distance of 0 to M along z axis. Then potential energy of distributed mass is

$$(\text{P.E.})_\mu = \mu\,\ell^2\,g\,q^2 \int_0^{\eta_0} (6\eta^3 - 4{\cdot}5\eta^4 + 0{\cdot}9\eta^5)\,d\eta$$

$$= \ell^2\,\mu\,g\,q^2\,(1{\cdot}5\eta_0^4 - 0{\cdot}9\eta_0^5 + 0{\cdot}15\eta_0^6).$$

Strain energy in mode is

$$(\text{P.E.})_s = \frac{1}{2}\int_0^{\eta_0 \ell} EI\left(\frac{d^2x}{dz^2}\right)^2 dz = \frac{18EI}{\ell}\, q^2 \int_0^{\eta_0} (1-\eta)^2\, d\eta$$

$$= \frac{6\,EI\,q^2}{\ell}\left(1-(1-\eta_0)^3\right) .$$

Work done by concentrated mass is

$$(\text{P.E.})_M = Mg\ell\,(6\eta_0^3 - 4{\cdot}5\eta_0^4 + 0{\cdot}9\eta_0^5)\, q^2 .$$

For small q, $\eta_0 \simeq 1$, and then

$$\text{Total P.E.} = \left[\frac{6EI}{\ell} + 2{\cdot}4\,Mg\ell + 0{\cdot}75\mu\ell^2 g\right] q^2 .$$

<u>Kinetic energy</u>

$$\text{K.E.} = \frac{1}{2}\int_0^{\eta_0\ell} \mu\dot{x}^2\, dz + \frac{M}{2}\,[3\eta_0^2 - \eta_0^3]^2\, \ell^2\, \dot{q}^2$$

$$= \frac{\mu\ell^3}{2}\,\dot{q}^2 \int_0^{\eta_0} (3\eta^2 - \eta^3)^2\, d\eta + 2M\ell^2\dot{q}^2$$

$$= [0{\cdot}4715\,\mu\ell^3 + 2M\,\ell^2]\,\dot{q}^2 .$$

Hence applying Lagrange's equation we have

$$(0{\cdot}4715\,\mu\ell^3 + 2M\,\ell^2)\,\ddot{q} + \left[\frac{6EI}{\ell} + 2{\cdot}4Mg\ell + 0{\cdot}75\mu\ell^2 g\right] q = 0$$

so that the undamped natural frequency is given by

$$\omega^2 = -\frac{\ddot{q}}{q} = \frac{6EI + 2{\cdot}4M\,g\ell^2 + 0{\cdot}75\mu\,\ell^3 g}{0{\cdot}4715\mu\,\ell^4 + 2M\,\ell^3}$$

putting $\lambda = \frac{M}{\mu\ell}$ we get

$$EI = \frac{M\ell^3\omega^2}{3}\left\{1 + \frac{0{\cdot}2357}{\lambda} - \frac{1{\cdot}2g}{\ell\,\omega^2} - \frac{0{\cdot}375g}{\ell\,\omega^2\,\lambda}\right\}$$

Table 1A

DYNAMIC STIFFNESS TESTS ON A DURALUMIN BAR

Mass below support		
Mass attached to beam grams	Resonant frequency cycles/sec	Rigidity of beam $E_D I_D$ lb in^2
1·925	6·43	10·25
3·673	5·48	10·28
4·244	5·26	10·31
5·129	4·95	10·29
8·747	4·13	10·27
14·578	3·41	10·26
24·416	2·80	10·26
44·085	2·25	10·25

Table 1B

Mass above support		
Mass attached to beam grams	Resonant frequency cycles/sec	Rigidity of beam $E_D I_D$ lb in^2
1·925	6·18	10·21
3·670	5·21	10·27
4·265	4·97	10·30
5·516	4·54	10·28
8·894	3·76	10·30
15·284	2·93	10·27
24·699	2·27	10·27
44·178	1·57	10·26

Measured static rigidity $E_D I_D$ = 10·42 lb in^2

Table 2

RECOVERY EFFECT

Condition of foam	E_f lb/in^2
Unworked	64·0
Worked and tested after:-	-
10 minutes	37·6
60 minutes	42·4
One night	50·5

Table 3

EFFECT OF 'WORKING'

Successive workings	E_f lb/in^2
Unworked	64·0
Initial working	50·5
2nd "	49·8
3rd "	43·8
4th "	44·4
5th "	42·3

Table 4

DYNAMIC STIFFNESS TESTS ON COMPOSITE BEAM

A SET OF TYPICAL RESULTS

FOAM IN THOROUGHLY WORKED CONDITION

Mass below support		
Mass attached to beam grams	Resonant frequency cycles/sec	Young's modulus E_f lb/in^2
5·241	6·07	63·5
8·617	5·58	64·1
18·181	4·69	66·0
28·094	4·12	66·7
37·846	3·72	66·5
47·573	3·43	66·4
57·206	3·21	66·5
66·761	3·02	65·5
	Mass above support	
5·241	5·85	64·9
8·617	5·33	65·0
18·181	4·38	66·2
28·094	3·77	67·1
37·846	3·33	66·4
47·573	3·01	66·3
57·206	2·75	65·9
66·761	2·54	65·2

Table 5

FORM OF DAMPING IN THE COMPOSITE BEAM

Absolute humidity lb/ft^3 $\times 10^{-4}$	Temperature °C	Concentrated mass grams	Measured frequency c/s	Hysteretic constant (From arbitrary stiffness)
7·44	21·8	42·13	3·13	0·584
7·54	21·9	22·50	3·86	0·559
7·50	21·9	12·71	4·41	0·580
7·50	21·9	2·35	5·33	0·601
7·50	21·9	0	5·60	0·605
6·05	21·2	42·13	3·21	0·604
6·31	21·5	22·50	3·96	0·612
6·63	21·8	12·71	4·52	0·620
6·54	21·6	2·35	5·45	0·643
6·54	21·6	0	5·80	0·632
5·94	19·0	0	6·00	0·636
5·86	19·0	2·35	5·60	0·631
5·86	19·0	12·71	4·61	0·641
5·90	19·2	22·50	4·02	0·658
5·87	19·2	42·13	3·27	0·637

SYMBOLS

a	generalised inertia
b	ratio of damping to critical damping
e	Napierian log base
g	acceleration due to gravity
h	hysteretic damping constant
k	generalised stiffness
ℓ	length of beam
m	$\mu\ell$ (total mass of beam)
q	generalised co-ordinate
t	time
C	viscous damping coefficient
E_f	Young's modulus for the plastic foam
$(EI)_d$	flexural rigidity of duralumin bar
$(EI)_c$	flexural rigidity of composite beam
I_f	second moment of area of foam about neutral axis of beam
K	constant of proportionality (equation (4))
K_T	radius of gyration of exciter coil
M	concentrated mass at free end of beam
M_T	mass of exciter coil
T	time constant of capacitance resistance circuit
λ	ratio of M to m
θ	amplitude of vibration
μ	mass per unit length of beam
ω	fundamental resonant frequency

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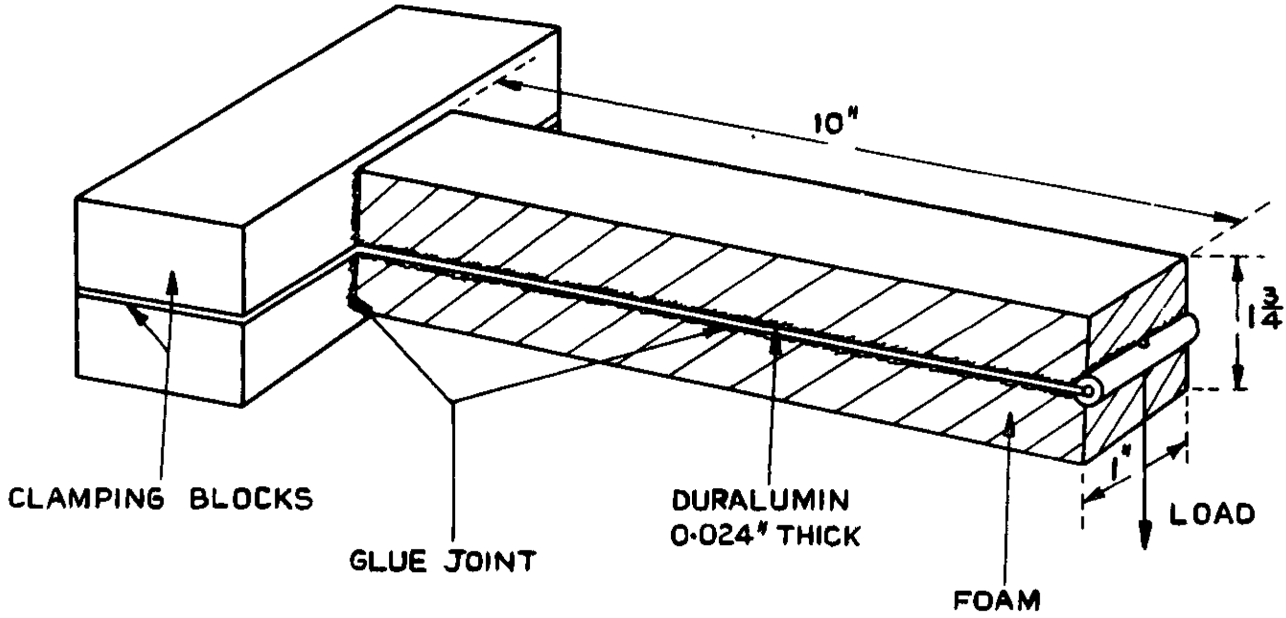

FIG 1 GENERAL VIEW OF COMPOSITE BEAM

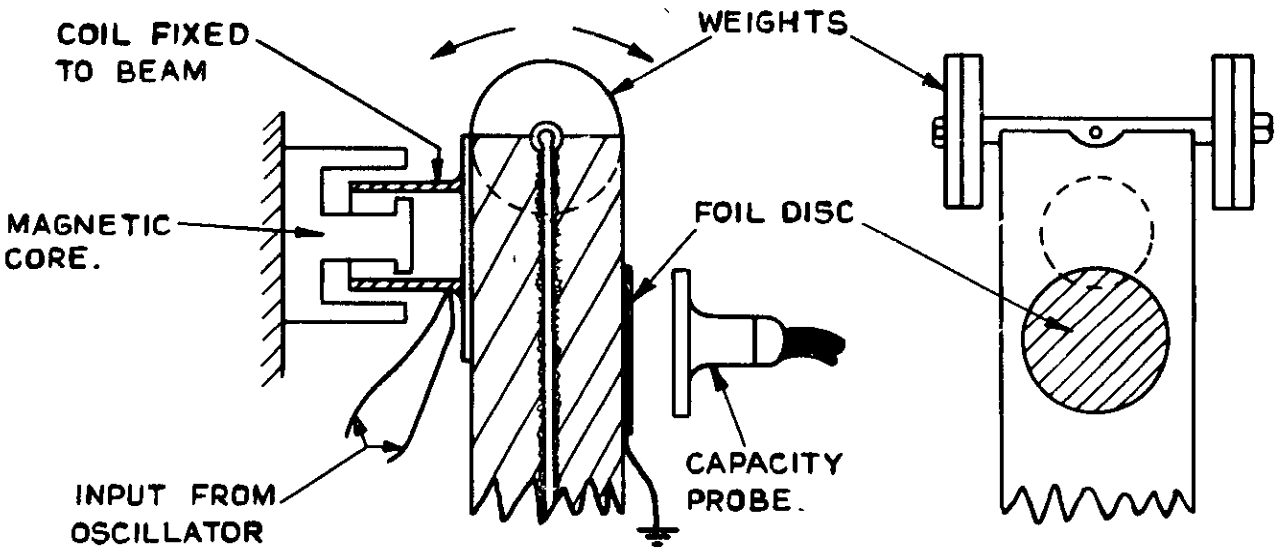

FIG 2 FREE END OF BEAM SHOWING ARRANGEMENT FOR DYNAMIC TESTS

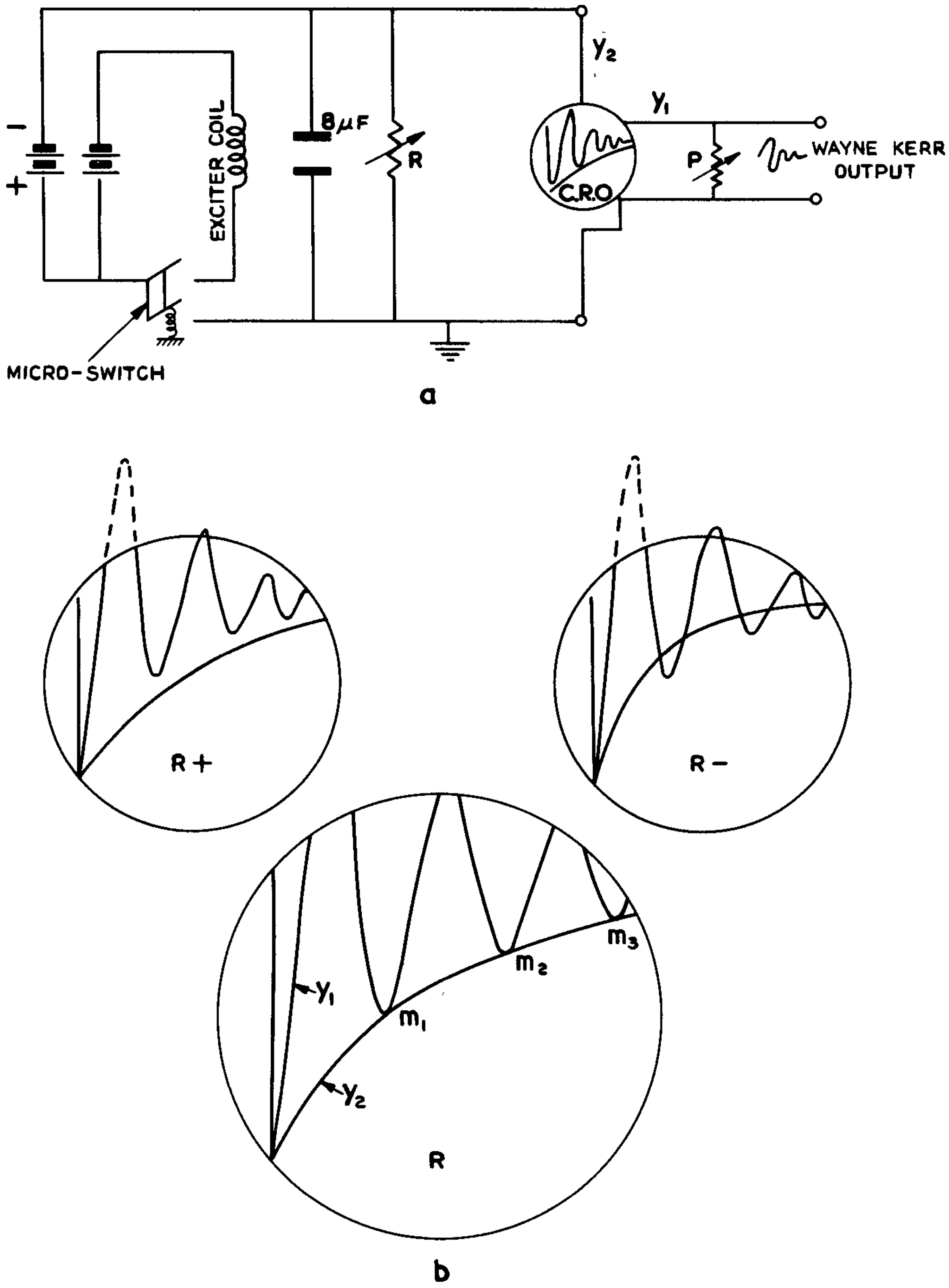

FIG 3 a & b DAMPING MEASUREMENT

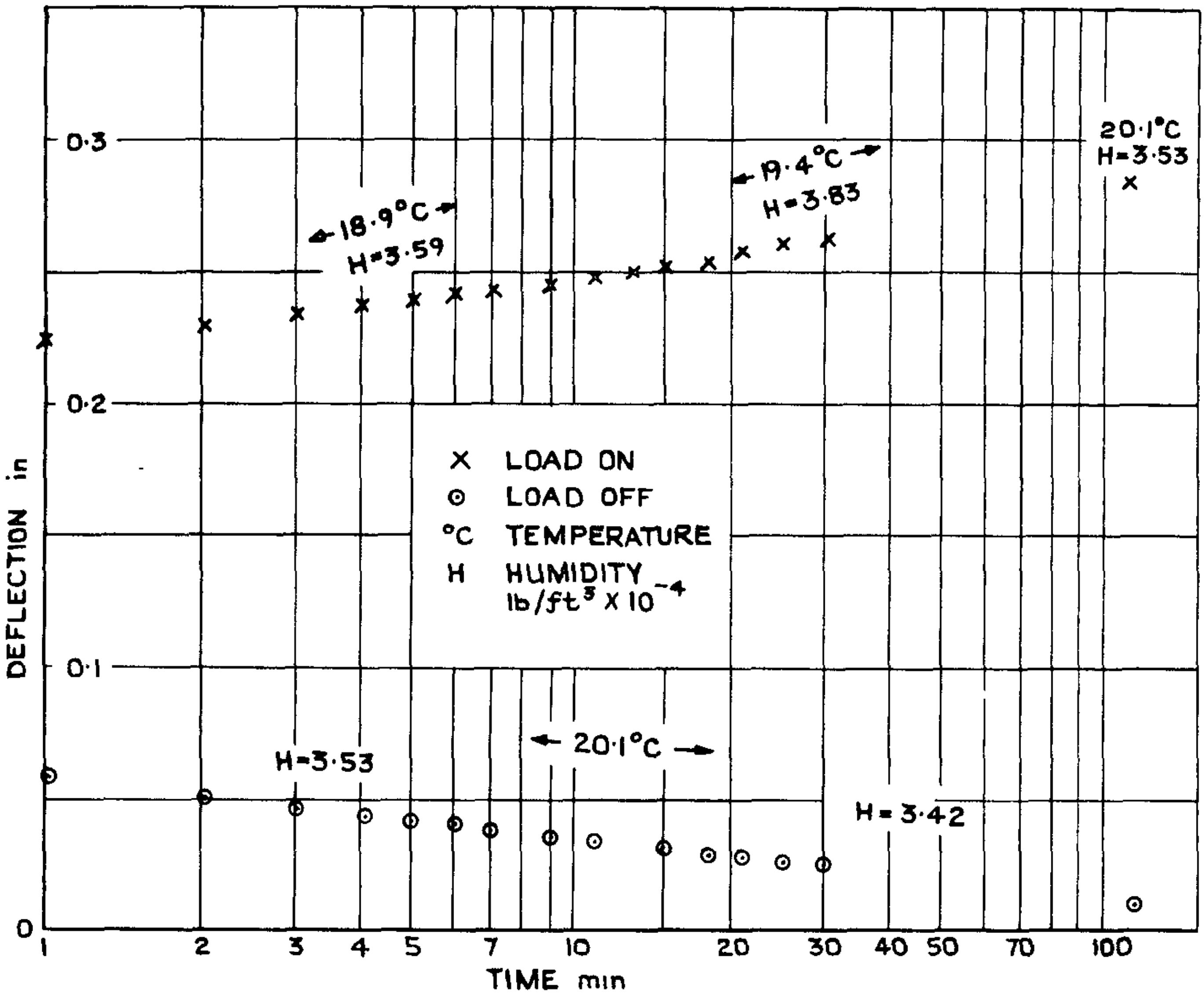

FIG. 4 CREEP CURVE FOR LOAD OF 12 GRAMS

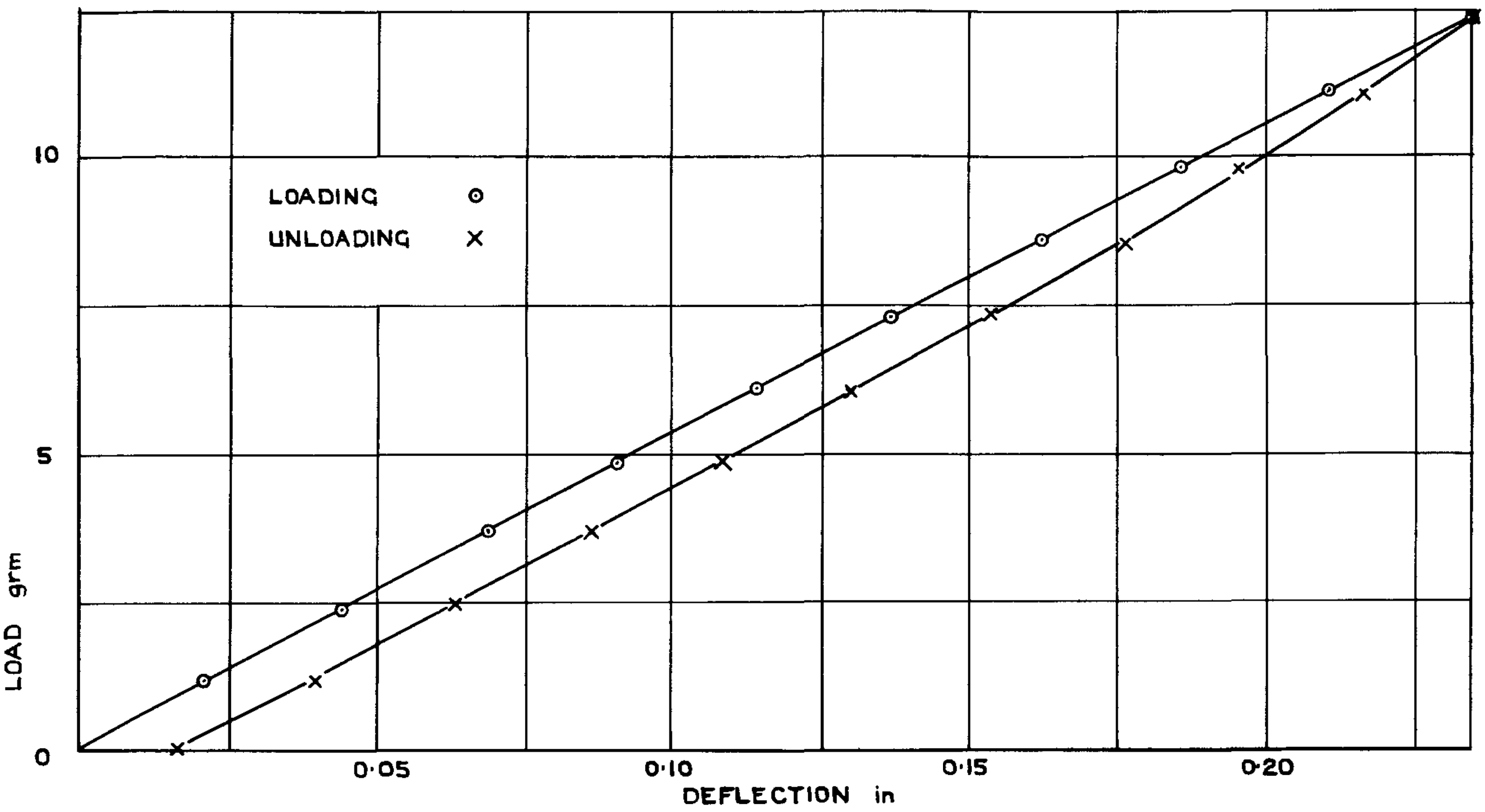

FIG. 5 TYPICAL DEFLECTION CURVE FOR COMPOSITE BEAM

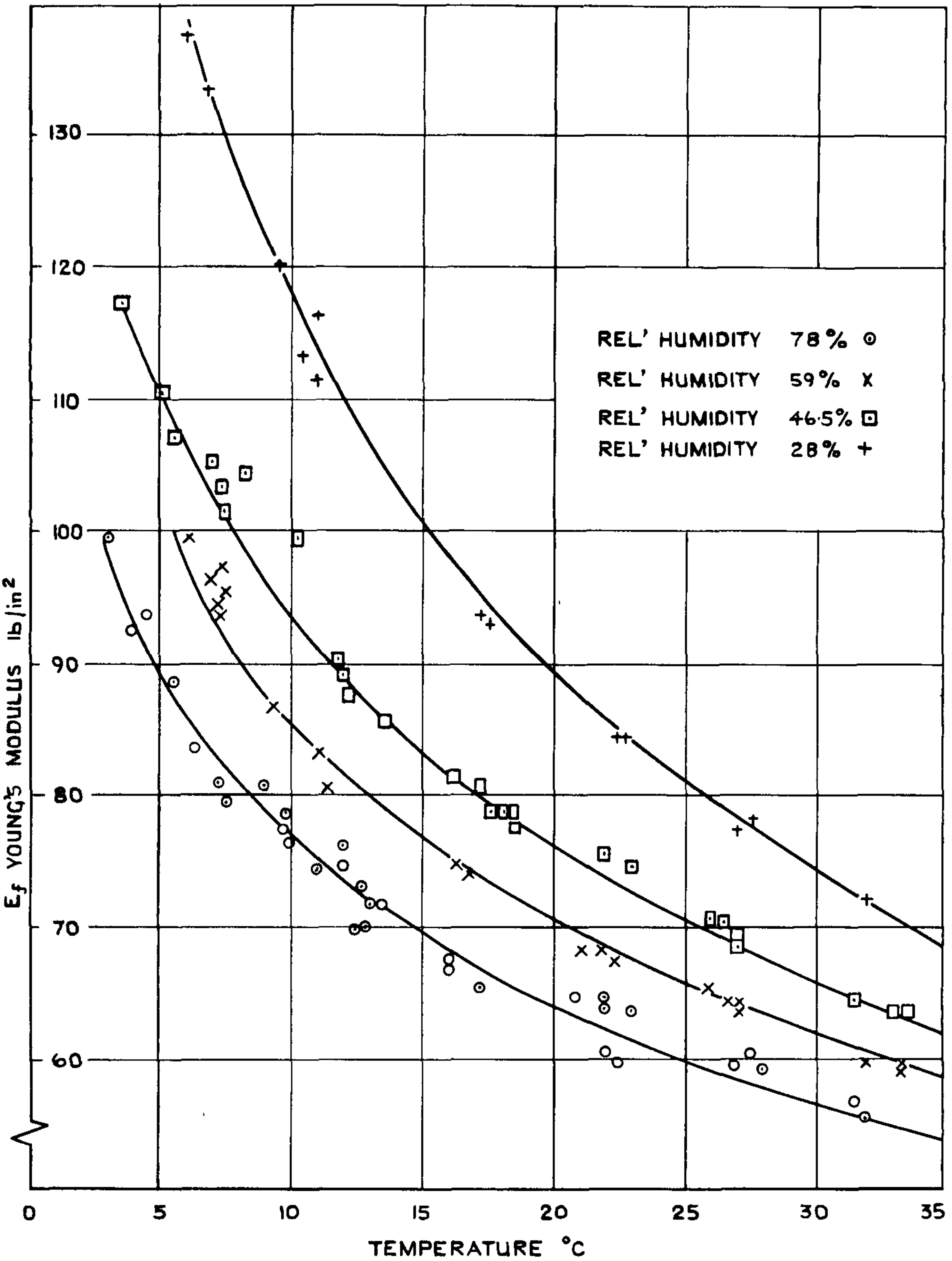

FIG.6 EFFECT OF TEMPERATURE AND RELATIVE HUMIDITY ON THE STIFFNESS OF THE FOAM

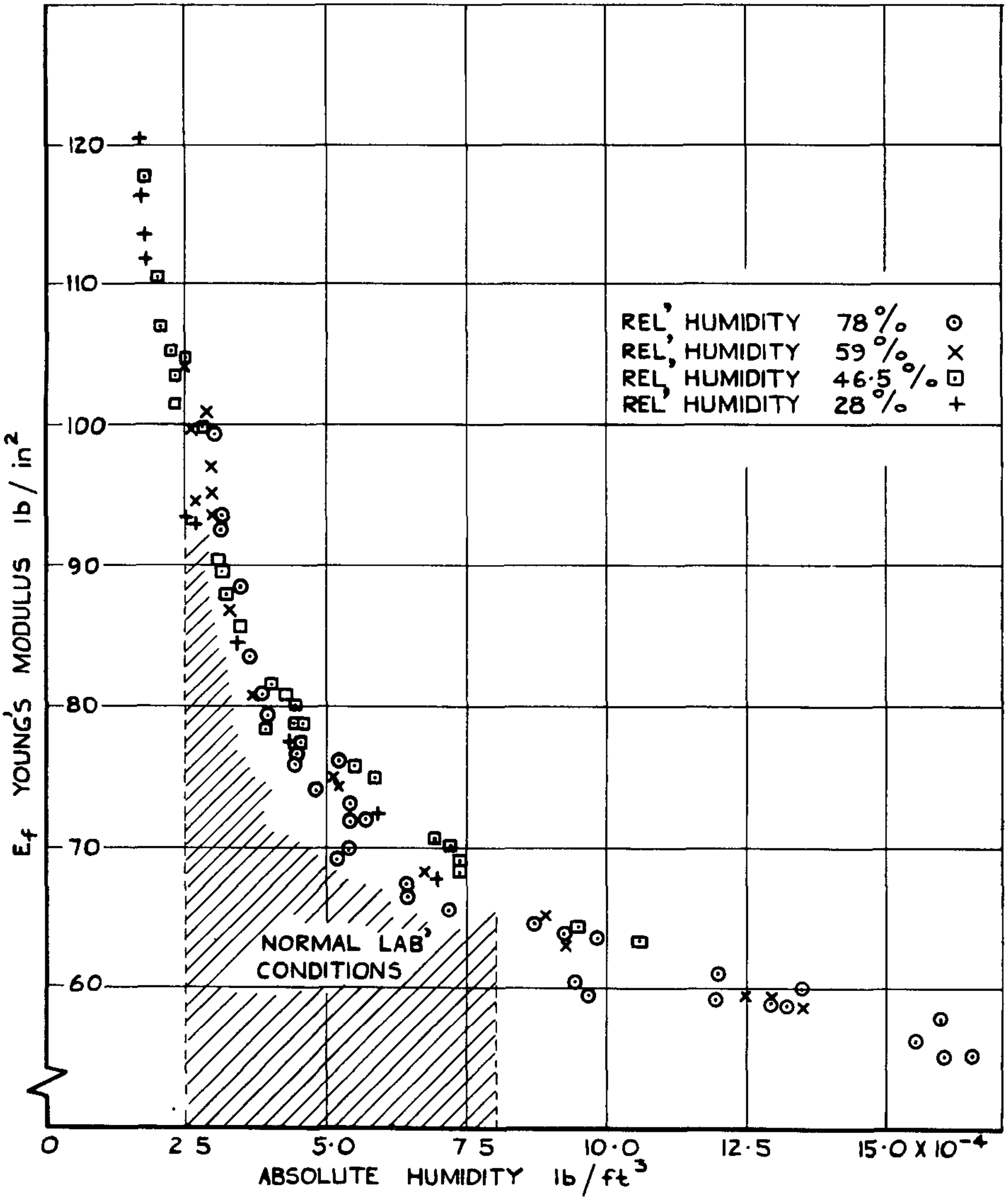

FIG 7 THE EFFECT OF TEMPERATURE AND RELATIVE HUMIDITY RESOLVED TO ONE OF ABSOLUTE HUMIDITY

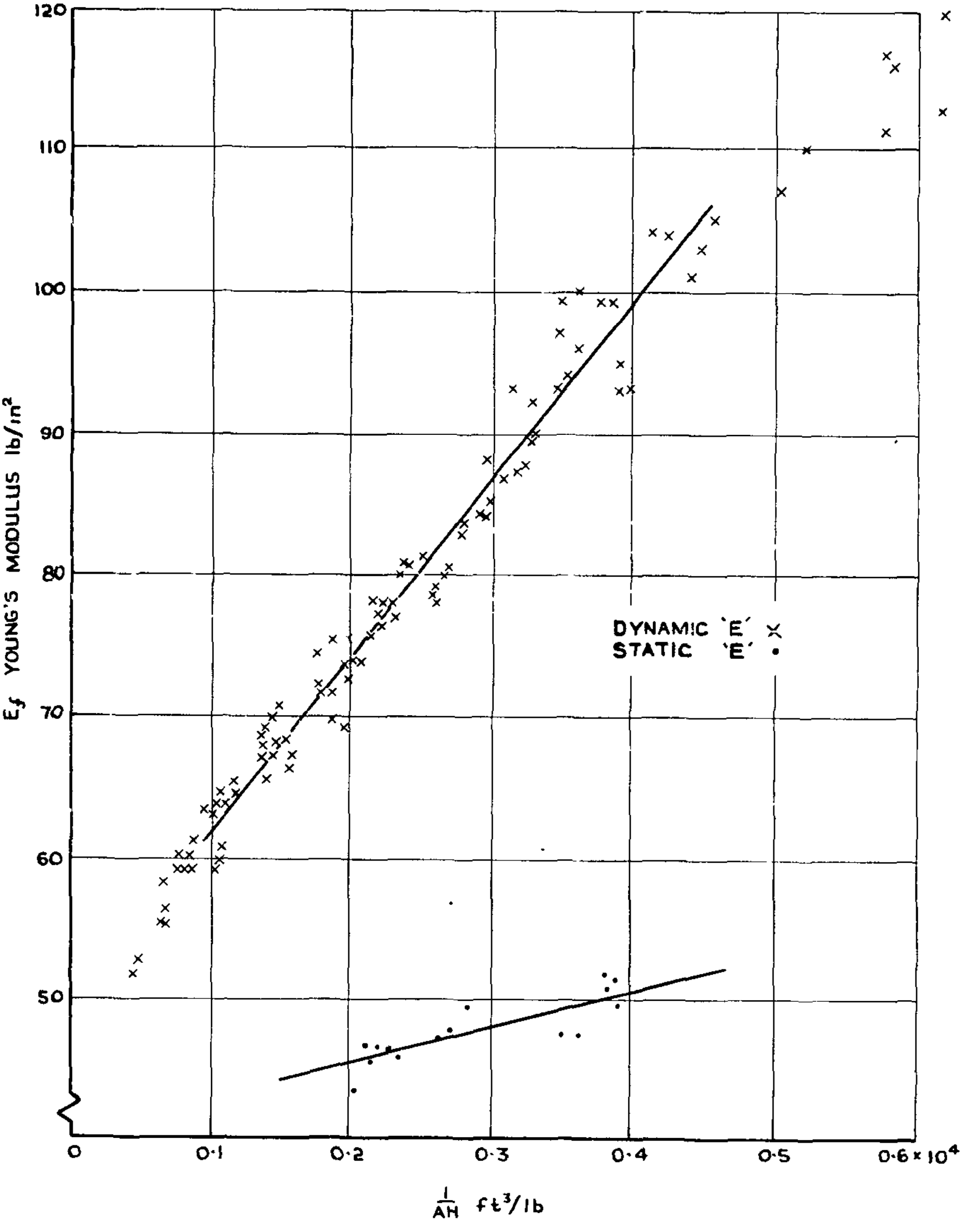

FIG. 8 VARIATION OF THE STIFFNESS OF THE FOAM WITH THE RECIPROCAL OF ABSOLUTE HUMIDITY

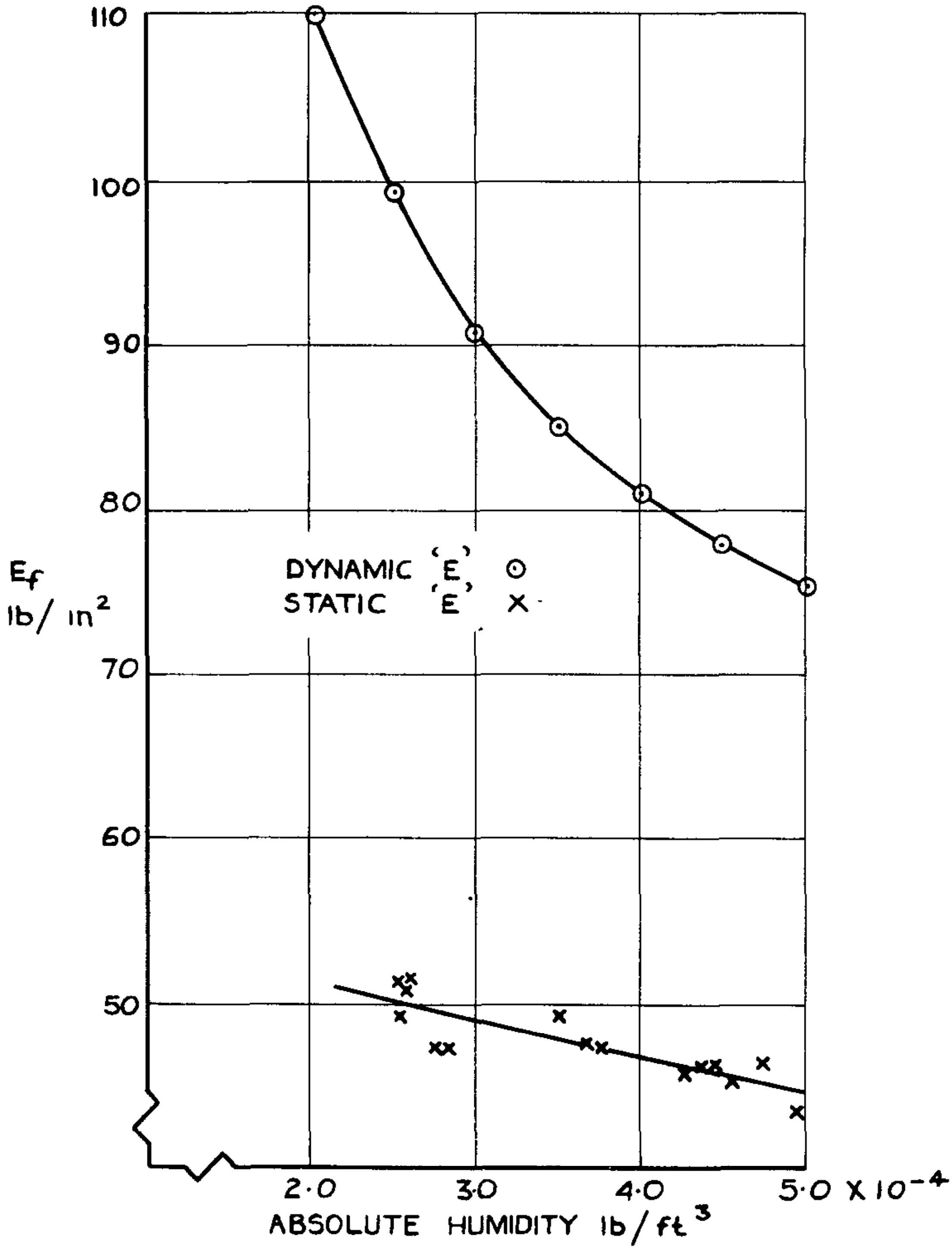

FIG 9 A COMPARISON OF DYNAMIC AND STATIC STIFFNESS

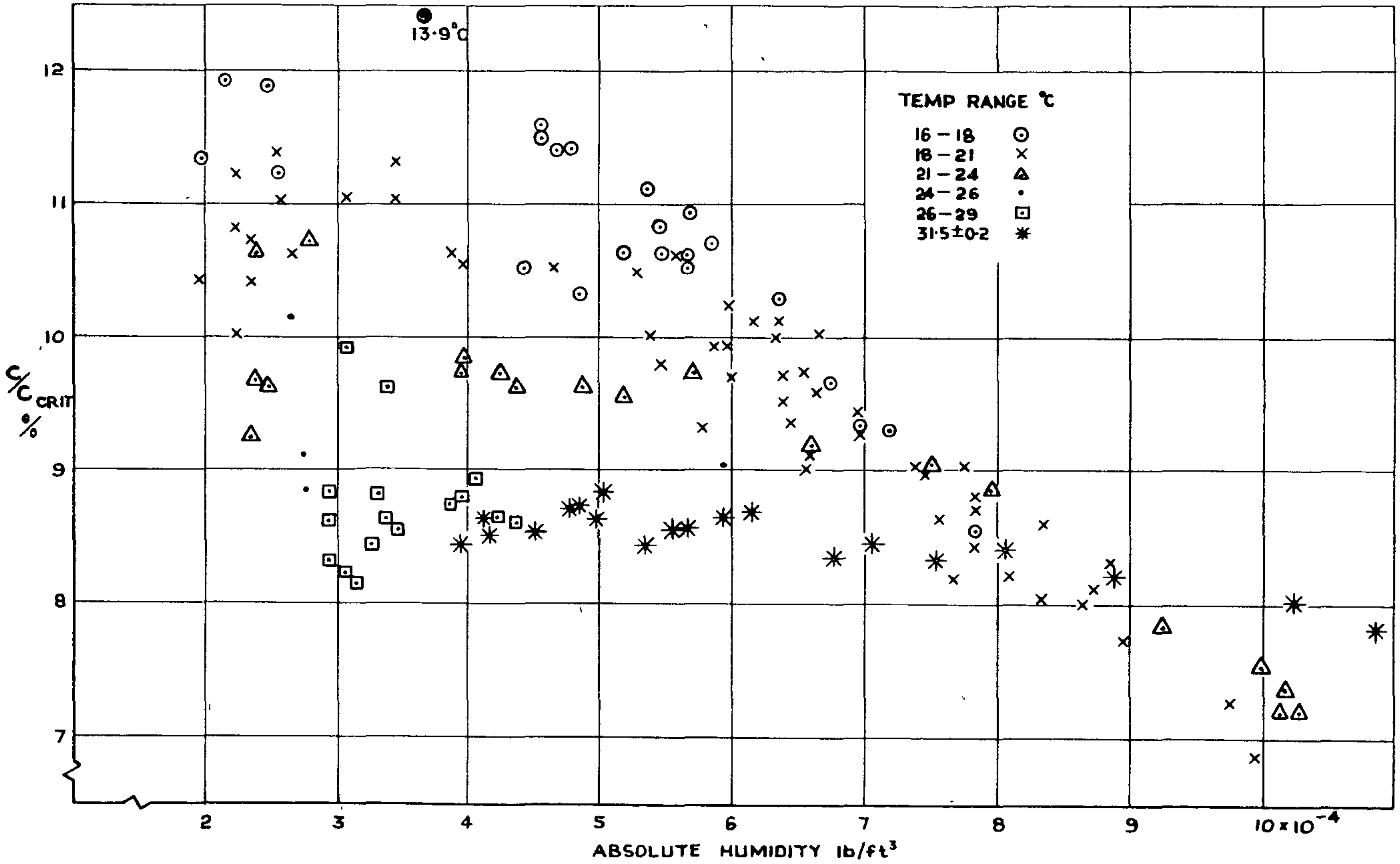

FIG. 10 VARIATION OF DAMPING RATIO WITH HUMIDITY AND TEMPERATURE

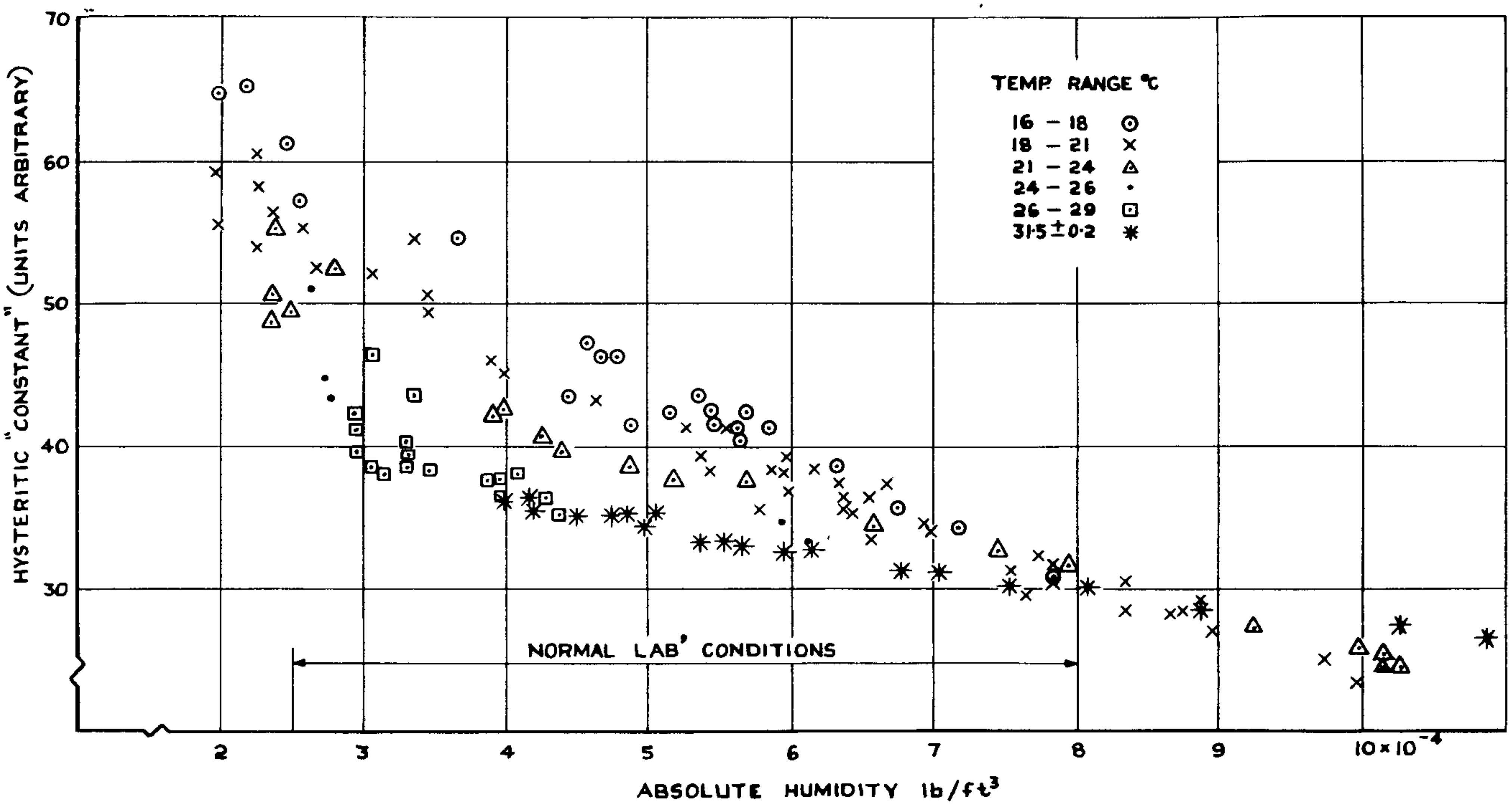

FIG. 11 THE EFFECT OF ABSOLUTE HUMIDITY ON THE HYSTERETIC CONSTANT OF THE FOAM

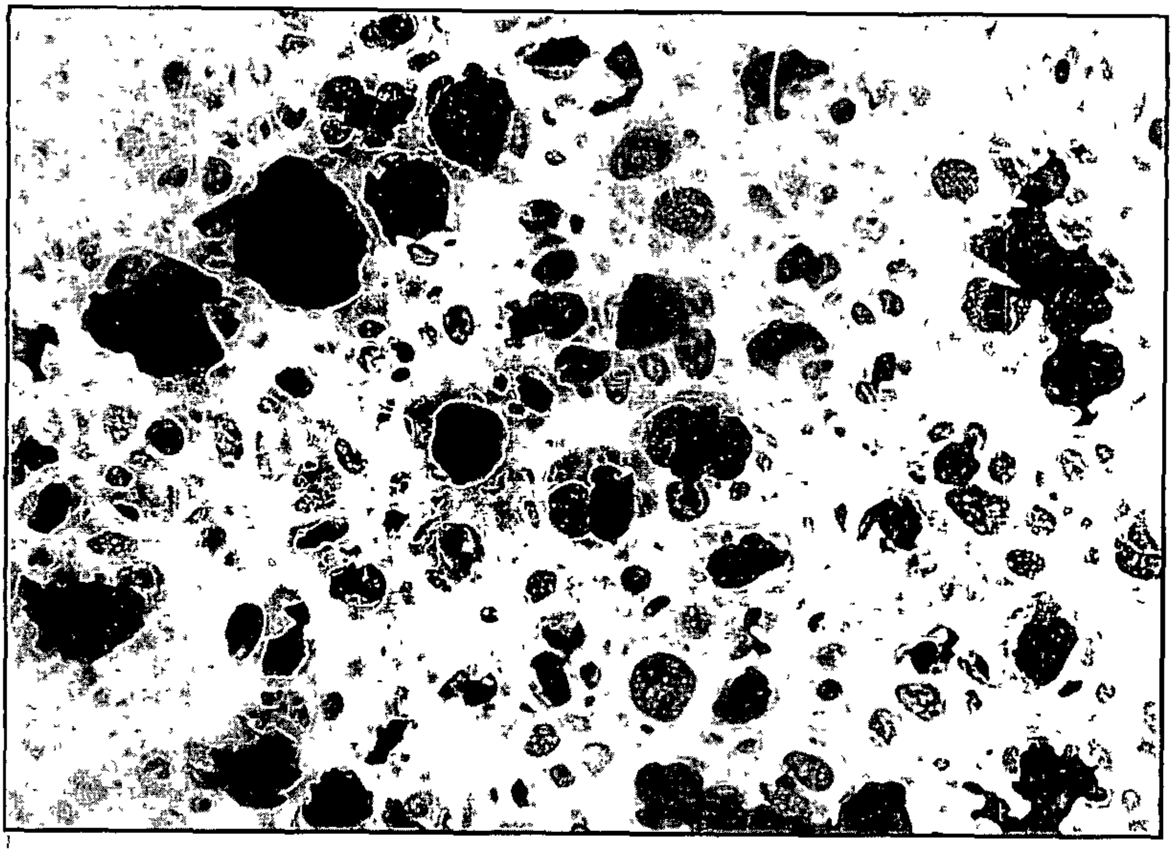

Fig.12. The microscopic structure of polyurethane foam magnified 20 times

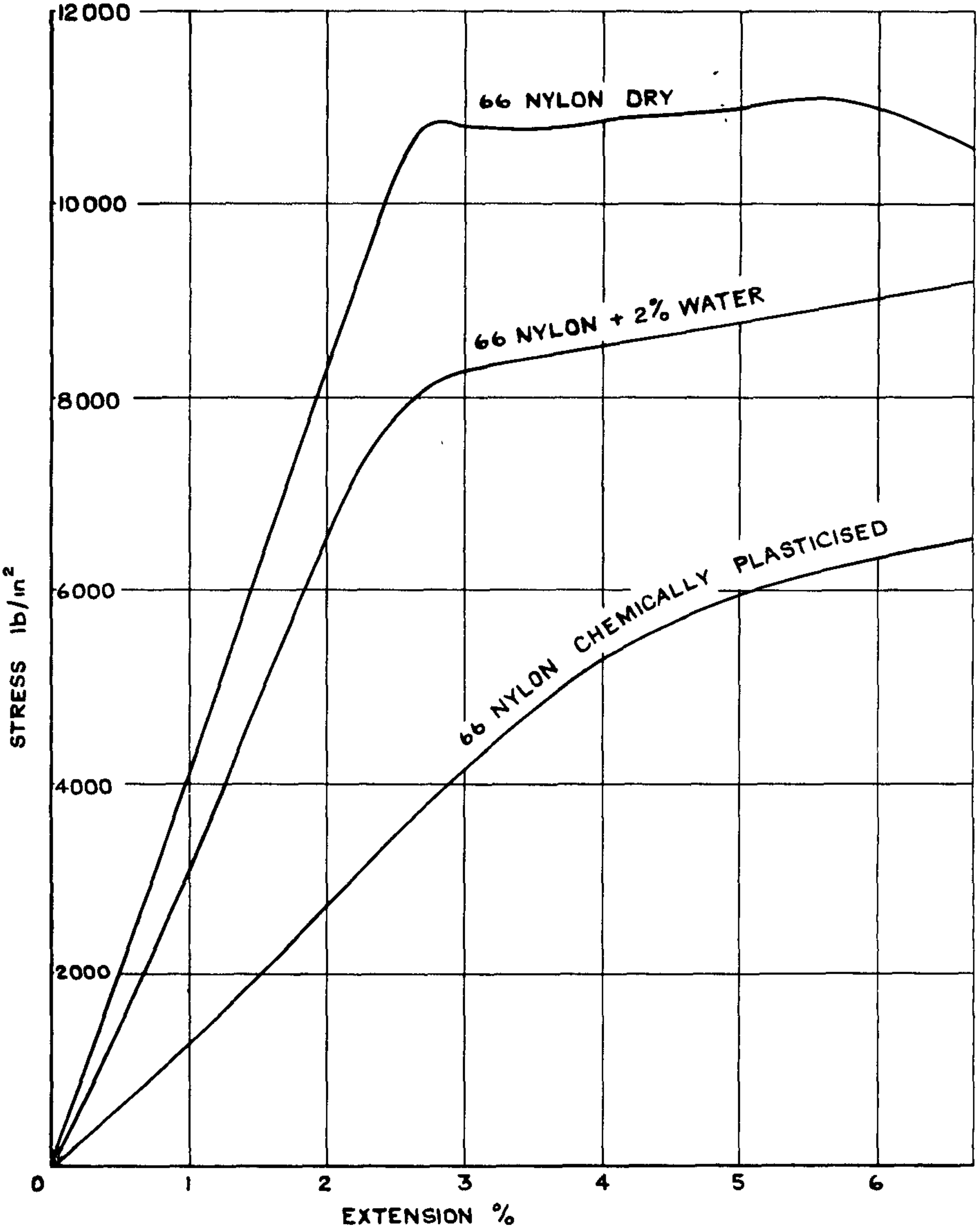

FIG.13 THE EFFECT OF WATER AND PLASTICISATION ON THE STIFFNESS OF 66 NYLON

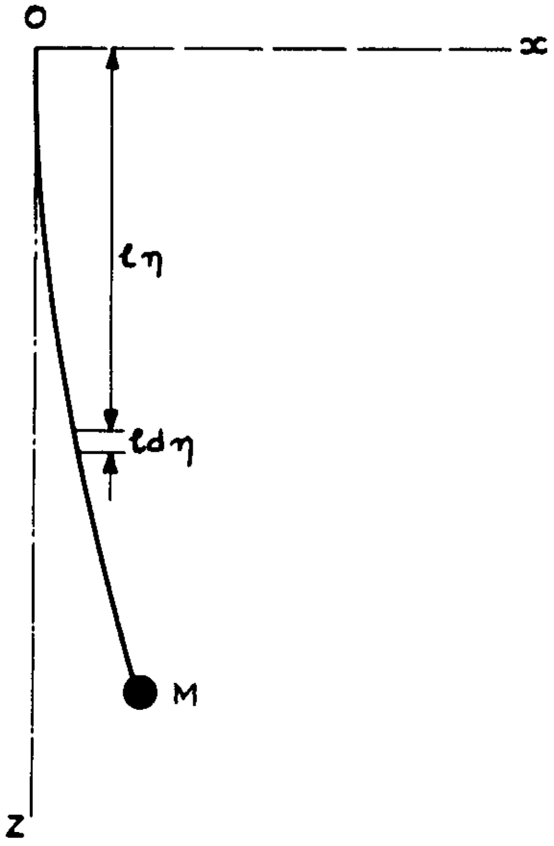

FIG 14 AXES OF CO-ORDINATES FOR VERTICAL BEAM

*Printed in England for Her Majesty's Stationery Office by
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August 1965 678.664:
Payen, D. B. 621-496:
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INCLUDING THE EFFECTS OF TEMPERATURE AND HUMIDITY 534.372:
539.434

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