C.P. No. 895

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MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Notes on Ducted Fan Design

By

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1966

Price 7s 6d. net

U.D.C. No. 629.13.038.23

C.P. No.895*

August, 1964

NATIONAL GAS TURBINE ESTABLISHMENT

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SUMMARY

In general, conventional compressor stages are designed by the cascade method, while high stagger low solidity ducted fans are designed on modified isolated aerofoil theory. The purpose of these notes is to provide a basis for discussion on the relative merits of the two methods and on the desirability of extending one method to cover the whole range of blading likely to be required in compressors and fans. Attention has been mainly confined to low speed two-dimensional considerations.

It is suggested that the cascade approach could provide a basis for the formulation of a unified design method.

A project of this nature would necessitate a programme of testing and performance analysis of typical fans; high stagger cascade tests might also provide supporting data, although there could be doubts as to their significance.

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*Replaces N.G.T.E. M.386 - A.R.C.26 603

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1.0 Introduction

There is no generally accepted definition which distinguishes a "ducted fan" from a single compressor stage. For the purposes of these notes, the term will be taken to refer to a stage containing a row of retarding aerofoils of considerably higher stagger and/or pitch/chord ratio than are generally used in conventional multi-stage compressors. Stagger is defined as the angle between the blade chord line and the axial direction; pitch/chord ratio is the inverse of solidity. These conditions are usually associated with low design velocity ratios, low design temperature rises, high degrees of reaction (in the case of rotor blade rows) or various combinations of the foregoing.

A current application where such requirements may apply is in the hovercraft lifting fan. The jet curtain velocity is low, and it is desirable to keep the velocities right through the system to low values also; otherwise the system losses apart from the outlet loss (inlet, fan, fan outlet) will be comparable to the outlet loss, leading to a relatively large increase in the fan power requirements. A low fan axial velocity will give low values of the axial/peripheral velocity ratio V_a/U, and correspondingly high blading staggers.

The stagger of rotor blades may be further increased if it is desired to use two blade rows only, i.e., inlet guides and rotors or rotors and outlet guides, with axial flow at entry and exit to the stage. In either case, but particularly the former, the reaction will be increased, with a consequent increase of rotor blade stagger.

If the required pressure rise (determined by the total system losses) and hence temperature rise is also low, the air deflection required in the rotor row will be small. If the fan is to work at high efficiency, the pitch/chord ratio will have to be correspondingly large to ensure that the lift/drag ratio is near the maximum.

Broadly similar considerations can apply to circulating fans for nuclear reactors. Here, however, although the axial velocity is generally low, the circuit losses may necessitate a relatively high stage temperature rise, and hence a relatively low blading pitch/chord ratio.

Aircraft lifting and control fans, ventilating fans, fans for low speed wind tunnels, and high reaction multi-stage compressors are other applications where blade rows with high staggers and/or high pitch/chord ratios may be required in varying degrees. Fans in which reversal of stagger is used for control purposes must have a pitch/chord ratio above unity along the whole blade height to avoid mechanical interference.

There is a large amount of experience and published information available on the cascade approach to the design of blade rows at the lower staggers and lower pitch/chord ratios, i.e., in the conventional compressor range of parameters. At much higher staggers and pitch/chord ratios, i.e. in the ducted fan range, modified isolated aerofoil theory is commonly applied[1 to 5]. There is of course a large amount of data available on isolated aerofoil performance, but published information on tests and performance analysis of fans designed by this method is scarce, and accurate prediction of performance, especially at off-design conditions, may be difficult. Another difficulty which may confront the designer is in the selection of the method of design when the blading parameters tend to lie intermediately between the compressor and fan values.

The purpose of these notes is to provide a basis for discussion on the desirability of formulating a unified design method which could be applied to the whole range of stagger and pitch/chord ratio likely to be encountered in fans or compressors. Attention is mainly confined to the two-dimensional low speed design of a single blade row, and no attempt is made to deal in detail with three-dimensional effects, high Mach numbers, etc., although in a practical case such factors may have an important bearing on the two-dimensional design.

2.0 The cascade method

2.1 Basic equations

The blade row is considered primarily as a device for changing the direction of the airflow in a cylindrical surface about the fan axis.

The total temperature rise through the rotor row is given by:-

$$T = \frac{\Omega U V_a}{g J K_p} (\tan \alpha_1 - \tan \alpha_2)$$

assuming that the axial velocity V_a is unchanged through the blade row and that no radial shift of the flow occurs, and where:

ΔT = stage temperature rise

Ω = work done factor[6] (usually 1.0 in a single-stage fan)

U = blade speed at the blade height considered

V_a = air axial velocity

α_1 = air inlet angle relative to the rotor*

α_2 = air outlet angle relative to the rotor*

g = acceleration due to gravity

J = mechanical equivalent of heat

K_p = specific heat of air at constant pressure

Thus for a given U and V_a the temperature rise depends on ($\tan \alpha_1 - \tan \alpha_2$). Now the blades are designed to give the particular value of α_2 (provisionally assumed constant) required by the velocity triangles, which themselves depend on the flow coefficient V_a/U and on the degree of reaction which the designer has specified. The value of ($\tan \alpha_1 - \tan \alpha_2$) for stable and efficient operation can be regarded as mainly a

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*Measured from the axial direction

function of α_2 and of the pitch/chord ratio, and is specified by the use of a loading parameter, several of which are in current use; these are briefly described in the next section.

2.2 Loading parameters

Perhaps the "nominal" deflection[6,7] is the most commonly used loading parameter. It is generally presented as deflection $\mathcal{E}$ (i.e., $\alpha_1 - \alpha_2$) as a function of air outlet angle α_2 with the pitch/chord ratio s/c as a secondary parameter. Figure 1 presents typical curves for three pitch/chord ratios. The basic equation for the curves is generally taken as

$$C_L = 2 \left[\frac{\cos \alpha_1}{\cos \alpha_2} \right]^{2.75}$$

where C_L is the theoretical lift coefficient (neglecting the drag term "$-C_D \tan \alpha_m$") where C_D is the drag coefficient given by

$$C_L = 2 \frac{s}{c} (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m$$

and the vector mean air angle α_m is defined by

$$\tan \alpha_m = 0.5 (\tan \alpha_1 + \tan \alpha_2)$$

and s = blade pitch

c = blade chord

The usual range of application is for α_2 from 0 to 40° and s/c from 0.5 to 1.5.

Another criterion[8] is defined by

$$C_{L_{V_2}} = \text{constant}$$

where $C_{L_{V_2}}$ is the theoretical lift coefficient based on the velocity leaving the blade row and is defined by the equation

$$C_{L_{V_2}} = 2 \frac{s}{c} (\tan \alpha_1 - \tan \alpha_2) \frac{\cos^2 \alpha_2}{\cos \alpha_m}$$

The constant in the previous equation is given in Reference 8 as a function of the pitch/chord ratio, and the recommended value for normal compressor purposes is

$$C_{L_{V_2}} = 1.35 \times \frac{\left(6\frac{s}{c} - 1\right)}{6\frac{s}{c}}$$

Curves are given in Reference 8 for s/c values of 0.5 to 1.5 and for α_2 values up to 50°. For reference purposes, values of δ are tabulated in Appendix I of the present Memorandum for $\alpha_2 = 0$ to 80°, s/c = 1.0, 2.5 and 4.0, and $C_{L_{V_2}} = 1.0$.

The American "diffusion factor"[9] is another loading parameter, and is defined by

$$D = \left(1 - \frac{\cos\alpha_1}{\cos\alpha_2}\right) + \frac{\cos\alpha_1}{2}\,\frac{s}{c}\,(\tan\alpha_1 - \tan\alpha_2)$$

Reference 10 suggests values for the constant "D" of 0.6 for rotor blade roots and 0.5 for rotor blade tips. It is not clear however, over what range of s/c and α_2 this parameter has been checked.

There is also an early American rule[11] given by $\frac{C_L}{s/c}$ = constant, where the constant may have values between 0.8 and 1.1. It has presumably been superseded by the diffusion factor in later American designs. The ranges of pitch/chord ratio and air outlet angle over which it has been applied are not known to the author.

Any of these loading parameters could of course be used outside their normal range of application as a basis for extending cascade methods to the high stagger and/or high pitch/chord range; and individual designers may on occasion have used them in this manner, though probably without any firm experimental backing.

2.3 Deviation and incidence rules

2.3.1 British rules

A commonly used deviation rule is

$$\delta = m\theta\sqrt{\frac{s}{c}}$$

where $\delta = \alpha_2 - \beta_2$ = deviation angle

$\theta = \beta_1 - \beta_2$ = blade camber angle

β_1 = blade inlet angle*, i.e., of tangent to camber line at leading edge

β_2 = blade outlet angle*, i.e., of tangent to camber line at trailing edge

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*Measured from the axial direction

and where the coefficient m is presented as a function of the blade stagger for circular arc and parabolic arc (P40) camber lines. It is a modification of earlier rules[7] and summarises the results of potential flow investigations and cascade tests at N.G.T.E.[12]. Reference 6, which gives a formula for m, states that it is reasonably accurate up to an air outlet angle of 50°, the implied pitch/chord ratio range being 0.5 to 1.5. Individual designers may of course prefer to use modifications of this rule. Reference 12, which gives curves of m (reproduced here in Figure 2 for reference purposes) emphasises that the rule holds only for the cambers and staggers commonly associated with each other, i.e., for normal values of blade loading parameters.

Incidence rules are more open to the individual preference of the designer, and within limits are of less importance in determining the performance of the stage, since although the deviation directly determines α_2, the choice of incidence angle (considering a fixed flow and variable blade geometry) does not affect α_1 or α_2.

An early rule in use at N.G.T.E.[13], is

$$i^\circ = 10\left(2\frac{a}{c} - \frac{s}{c}\right)$$

where i = incidence angle ($= \alpha_1 - \beta_1$)

a = position of maximum camber from the blade leading edge

This rule has a partly empirical and partly theoretical basis, taking into consideration the necessity of avoiding choking at high speeds and the desirability of having the stagnation point near the leading edge. It is intended to apply in the range s/c = 0.5 to 1.5. For circular arc (a/c = 0.5) and parabolic P40 (a/c = 0.4) camber lines, the recommended incidences are then as follows:-

s/c	0.5	0.8	1.0	1.5
i for C50	+5°	+2°	0°	-5°
i for P40	+3°	0°	-2°	-7°

Another criterion for incidence is given in Reference 14 and its derivatives[15,16]. The basis is again the position of the front stagnation point. Curves of design incidence against camber angle can be derived, with pitch/chord ratio as a secondary parameter, and are presented for instance in Reference 17, for camber angles from 0 to 50°, pitch/chord ratios from 0.5 to 1.5, and with the limits of α_2 stated as 0 to 40°. These curves are reproduced in Figure 3 of the present Memorandum.

2.3.2 N.A.C.A. rules

Systematic generalisations of cascade incidence and deviation data are given in Chapter VI of Reference 10, for N.A.C.A. 65-series blades, with modifications for application to circular arc camber line blades.

They are largely based on the extensive cascade tests of Reference 18. The correlations are based on "reference" values of incidence and deviation, i.e., those occurring at the minimum loss condition for the cascade. In the analysis, both deviation and incidence are taken to be linear functions of camber, so that

$$\delta_{ref} = \delta_0 + m\theta$$

and

$$i_{ref} = i_0 + n\theta$$

where

δ_{ref} is the reference deviation angle

δ_0 is the reference deviation for zero camber

θ is the camber angle

i_{ref} is the reference incidence angle

i_0 is the reference incidence angle for zero camber

m and n are functions of α_1 and s/c

Data is given for finding i_0 and δ_0, and curves are given presenting m and n as functions of α_1 (for $\alpha_1 = 0$ to 70°) at solidities of 0.4, 0.6... 2.0, i.e., at pitch/chord ratios of 2.5, 1.667... 0.5.

Additionally, the function $1 - m + n$ is plotted to α_1 for the same values of pitch/chord ratio. This is useful in calculating the blade camber, since

$$\theta = \mathcal{E} - i_{ref} + \delta_{ref}$$

i.e.,

$$\theta = \frac{\mathcal{E} - i_0 + \delta_0}{1 - m + n}$$

where $\mathcal{E}$ is the required deflection derived from the loading parameter. Examination of Figure 57 of Chapter VI of Reference 10 shows that $(1 - m + n)$ decreases as α_1 increases and as the pitch/chord ratio increases; the curve for $s/c = 2.5$ passes through zero at $\alpha_1 = 66.3$. Thus the above equation would give infinite camber, whatever the value of the numerator $\mathcal{E} - i_0 + \delta_0$, and it is found in general that cambers tend to become ridiculously large at high pitch/chord ratios and inlet air angles.

It is interesting therefore to examine the scope of the test results on which this data is based. The major variables of the tests have been deduced from the test points shown in the supporting figures. They are listed in Appendix II, where it is seen that the highest pitch/chord ratio of the tests is 2.0 (at $\alpha_1 = 45$ and 60°), while at $\alpha_1 = 70^\circ$ the highest value is 1.0.

It is thus apparent that the N.A.C.A. data is inadequate as it stands for design work outside these limits. If the incidence rule were neglected, and some arbitrary incidence taken, it would of course be possible to use the deviation rule outside the test limits, although the significance of such a step would have to be examined closely.

Finally, it is well known (and in fact is mentioned in Reference 10) that the N.A.C.A. rules give higher deviations than the N.G.T.E. rule; this need not however prevent the use of both sets of data as guides to design, although it may render difficult the recommendation of a preferred system.

2.4 Lower limits of pitch/chord ratio

In normal multi-stage compressor practice, it is rare for pitch/chord ratios to fall below about 0.5 at any position on the blade height (except perhaps for some low diameter ratio first stages), and in any case the lower values are generally associated with low staggers. Inspection of blade passage geometry suggests that the limiting pitch/chord ratio for efficient operation would increase with increase of stagger. This is quite apart from high speed effects which are not considered in these notes. This subject can be of importance in high reaction multi-stage compressors[19], where the rotor blades are set at high stagger, and where low pitch/chord ratios may be desirable in order to ensure high stage temperature rises. Some tests on medium stagger blading in a water compressor[20] have suggested a serious loss of performance (compared with simple predictions) at a mean diameter pitch/chord ratio of 0.5, while tests on two stages of lower stagger blading[21] have shown good performance at the same value. Reference 22 describes a test of six stages of high reaction blading with a mean diameter rotor pitch/chord ratio of 0.5 and an air outlet angle of 52.3°, i.e., in the fairly high stagger range. The performance was very poor, and the velocity profiles suggested that the biggest losses occurred at the inner diameter, where the pitch/chord ratio was lowest. Apart from these examples, Reference 18 provides a useful guide to cascade performance over a wide range of stagger at medium pitch/chord ratios but it does not indicate the limits of safe design.

It is evident that a comprehensive design method would have to include (as a secondary but important feature) a knowledge of the lower limits of usable pitch/chord ratio over the whole stagger range.

3.0 Isolated aerofoil approach

3.1 Basic equations

As in the cascade method, conditions are examined in a developed cylindrical surface about the axis of rotation, but attention is initially directed to the forces acting on the blades. The basic aerodynamics are of course the same in the two methods. The strip theory equations[23 to 27] relate to the thrust and torque on an element of blade of radial width dr, and are given typically in the form

$$\frac{dT}{dr} = \frac{1}{2}\rho z c V_a^2 G \cot\phi$$

$$\frac{dQ}{dr} = \frac{1}{2}\rho z c V_a^2 H r \cot\phi$$

$$G = \frac{C_L}{\sin\phi} - \frac{C_D}{\cos\phi} = \frac{C_L \cos(\phi + \sigma)}{\sin\phi\cos\phi\cos\sigma}$$

$$H = \frac{C_L}{\cos\phi} + \frac{C_D}{\sin\phi} = \frac{C_L \sin(\phi + \sigma)}{\sin\phi\cos\phi\cos\sigma}$$

$$\tan\sigma = \frac{C_D}{C_L}$$

where

C_L = lift coefficient based on vector mean velocity

C_D = drag coefficient based on vector mean velocity

Q = torque

T = thrust

V_a = axial velocity

c = blade chord

r = radius

z = number of blades

ρ = air density

ϕ = angle of vector mean velocity of air relative to rotor blade, measured from the tangential direction

= $90^\circ - \alpha_m$ in conventional cascade symbols

Reference 24 additionally gives

$$\tan\phi = \frac{V_a}{U}\left(\frac{1}{1 - a}\right)$$

and

$$\frac{1}{1 - a} = 1 + \frac{zcH}{8\pi r}$$

where

U = blade peripheral velocity

a = "rotational inflow factor"

These equations merely express the relationship between ϕ and V_a/U for the case when there are no inlet guides.

The pressure rise and temperature rise may be deduced at any radius from the thrust and torque equations.

As in the design of a fan on cascade principles, the blade stagger, represented roughly by ϕ, will depend largely on the velocity ratio and on the degree of reaction. The designer has to find a blade section which when operating in a flow represented by ϕ, will give the required lift coefficient, when the other factors have been settled. Three main inter-related decisions have in fact to be made. The first is the choice of aerofoil section to be used; the second is the lift coefficient at which it is desirable to operate the section; and the third is the correction (if any) to be applied to allow for cascade interference. These decisions correspond broadly to the choice of aerofoil section, loading parameter, incidence, and deviation in a cascade.

3.2 Choice of aerofoil and lift coefficient

Various profiles have been used in fan designs, the emphasis being on those with a flat under-surface.

The following are taken from some of the references:-

Reference 1	Gottingen 436
	Gottingen 436 with increased camber "Symmetrical aerofoil" on circular arc camber line
	Gottingen 385
	Gottingen 398
References 2, 25	Clark Y
	R.A.F. 6E
References 23, 24, 28	N.P.L. series (flat under-surface)

Details of some aerofoil sections and performances are given in References 29 and 30.

Various empirical rules for allowable lift coefficient are given in the literature, the chief considerations being that too high a value might result in stalling, while too low a value will result in a low lift/drag ratio and hence in a low efficiency. The choice will obviously depend to some extent on the properties of the aerofoil section used. A selection of published rules is given below.

Reference 1	Stalling avoided if the hub s/c is not smaller than 0.9 and if the hub C_L is less than 1.0
Reference 24	$C_L = 0.6$ at the rotor tip is usually chosen, a value in excess of 0.7 not being usually attainable
Reference 25	C_L should not exceed 1.0 at the rotor hub; 0.9 is preferable
Reference 27	$C_L \simeq 1.0$ at the rotor hub $C_L \simeq 0.7$ at the rotor tip.

3.3 Corrections for cascade interference effects

Cascade interference calculations appear to be generally based on the well-known results of Weinig[3,5,31,32] for potential flow through cascades of flat plates.

The lift coefficient of an isolated flat plate in ideal flow is given by

$$C_L = 2\pi \sin(\phi - \sigma)$$

where σ is the angle of the plate relative to some datum and ϕ is the angle of the airflow relative to the same datum, the sign of C_L depending on the sign convention used for the angles.

If the flat plate is now put in cascade with others, ϕ becomes the vector mean air angle, usually taken relative to the tangential direction, and $\sigma + \gamma = 90^\circ$ where γ is the usual cascade stagger angle, taken relative to the axial direction. The equation is then

$$C_L = 2\pi f \sin(\phi - \sigma)$$

where the lift factor f is a function of the pitch/chord ratio s/c and the plate angle σ or γ.

Weinig's curves for f were later deduced independently by Collar[31]. They are given in Figure 4 of the present Note, and show changes in the lift coefficient at a given plate setting angle and vector mean air angle when the plate is subjected to interference by other similar plates at various pitch/chord ratios. It seems reasonable to use these curves as guides when considering thin blades of low camber operating at incidences away from the stall; in such cases, σ becomes the incidence angle relative to the no-lift line, the position of which for a given blade will depend on the pitch/chord ratio and stagger.

What is perhaps of more importance however is the effect of interference on the allowable lift coefficient or alternatively on the stalling value, the two being interdependent. This obviously cannot be predicted by simple potential flow theory, as it is a function of boundary layer behaviour. Any systematic relationship between pitch/chord ratio and allowable lift coefficient would of course correspond to a comprehensive cascade loading parameter.

Reference 25 includes a summary of some of the results of the high stagger experimental and theoretical cascade investigations described in Reference 33. In particular, curves are given presenting the no-lift angles for cambered aerofoils of finite thickness, as functions of the thickness, camber stagger, and pitch/chord ratio. These curves were derived theoretically, but were given some support by the experimental work.

Figure 5 of the present Memorandum reproduces some of the test results of Reference 33; lift and drag coefficients, and the lift/drag ratio are shown for an isolated aerofoil and for the same aerofoil in cascade at pitch/chord ratios of 1.5 and 1.0. The reduction of stalling lift coefficient, the increase of the no-lift incidence angle, and the change of slope as the pitch/chord ratio is decreased are clearly seen. Figure 6 presents the lift coefficients for the cascade tests recalculated on the basis of the outlet velocity; the "theoretical" values which neglect the drag term are also shown for comparison.

The cascade tunnel was of a simple open ended type exhausting to atmosphere and no precautions were taken against contraction and other effects; for many of the tests only five blades were used. The details of the results must therefore be treated with some reserve.

In some intermediate cases it may be possible to check a design based on isolated aerofoil theory against existing cascade data or minor extrapolations of existing cascade data. It should also be noted that in some fans of high pitch/chord ratio the interference correction may be negligible, except perhaps near the hub.

4.0 Effect of blade setting and air angle errors

It is of interest to examine the effect of blade setting errors on the performance of a fan or compressor stage; the subject has a direct bearing on the desirable accuracy of the design method and calculations and of the machine construction as well as on the analysis of the test performance.

The total temperature rise across a rotor blade row is given by

$$\Delta T = \frac{\Omega U V_a}{g J K_p} (\tan \alpha_1 - \tan \alpha_2)$$

using the cascade notation and assumptions of Section 2.1. Hence for a given axial velocity and blade speed the temperature rise depends directly on $(\tan \alpha_1 - \tan \alpha_2)$.

Now simple potential flow considerations[12,31,34] for cascades give

$$\tan \alpha_2 = A + B \tan \alpha_1$$

where A and B are constants depending on the cascade geometry. For conventional low pitch/chord ratio compressor type cascades, the term B tan α_1 is small compared with A, i.e., α_2 is practically constant. This is progressively less true however as the pitch/chord ratio and the stagger increase. Appendix III lists values of A and B for $\alpha_2 = 0^\circ$ to 80° at $C_{L_{V_2}} = 1.0$ and pitch/chord ratios of 1.0, 2.5 and 4.0. These values are based on Weinig's curves for flat plates in potential flow. For comparison, some unpublished test results from a single stage fan are also quoted. The dependence of α_2 on α_1 is presented in a somewhat different manner in Figure 55 of Chapter VI of Reference 10, where the rate of change of deviation angle with incidence angle at the N.A.C.A. "reference" condition is plotted to solidity $\left(= \frac{1}{s/c} \right)$ for various values of the air inlet angle α_1. These curves are said to be based on the cascade tests of Reference 18, with the use of Weinig's investigations as a guide. In a real cascade, the change of the width of the blade wakes may also contribute to the change of α_2 with α_1.

We may thus write

$$\tan \alpha_1 - \tan \alpha_2 = \tan \alpha_1 (1 - B) - A$$

and the effect of a small change (say 0.1°) in α_1 on the temperature rise at constant U and V_a can be readily calculated, if A and B are known. Such a change could arise for instance if the inlet guide blades were incorrectly set or if their deviation angle was incorrectly estimated.

What is perhaps of more practical interest is the effect of a small change in the blade stagger, resulting in changes in A and B. A theoretical treatment covering all cascades is obviously impossible, but Weinig's curves can be used to investigate these changes in cascades of flat plates in ideal flow, and thus provide at least an indication of the trends to be expected.

The effect of a small change in α_2 is also of interest, since the multi-stage compressor designer tends to assume that a small change in the stagger setting will cause an approximately equal change in α_2; this is of course true only at low staggers and low pitch/chord ratios.

Calculations were carried out at pitch/chord ratios of 1.0, 2.5 and 4.0 for $\alpha_2 = 10^\circ, 20^\circ \ldots 80^\circ$, at an initial $C_{L_{V_2}}$ of 1.0 in each case. The percentage changes in $(\tan \alpha_1 - \tan \alpha_2)$ were estimated for changes of 0.1° in the air inlet and outlet angles α_1 and α_2 and in the stagger angle γ; they are plotted in Figures 7, 8 and 9. These curves probably give a reasonable indication of the behaviour of conventional cascades operating away from the stalling condition. Perhaps the most significant feature is the relatively small effect of the change in stagger as compared with that of the change in air inlet or outlet angle, for the two higher values of pitch/chord ratio at the higher values of α_2. Thus for a pitch/chord ratio of 4.0 the percentage error in the temperature rise is -0.75 at $\alpha_2 = 0^\circ$ and -1.12 at $\alpha_2 = 80$ for 0.1° error in γ, while for a similar error in α_2, the corresponding percentage values are -1.4 and -8.6.

The question of blade setting errors is probably of greater interest in the design of multi-stage compressors, rather than in single-stage fans. Errors in the stage performance are not easily corrected in conventional multi-stage machines, since it is not generally practicable to allow for adjustment of the blade stagger in every stage. In a single-stage fan, however, it is probably less inconvenient to make provision for small changes in the stagger of a single row of inlet guide or rotor blades.

5.0 Effect of varying air outlet angle on fan performance

The variation of α_2 with α_1 given by the equation

$$\tan \alpha_2 = A + B \tan \alpha_1$$

will reduce (in magnitude) the slope of the estimated fan temperature rise characteristic, in comparison with the case where α_2 is assumed to be constant. Figure 10 illustrates this for a hypothetical fan design for axial discharge from the rotor, with $\alpha_2 = 70^\circ$ at $C_{L_{V_2}} = 1.0$ and a pitch/chord ratio of 4.0. The values of A and B are again those derived for flat plate cascades in potential flow. The curves show that the assumption of a constant value of α_2 is quite unjustified for even approximate estimates of fan performance at high values of α_2 (or stagger) and high pitch/chord ratios.

6.0 Comparison of some fan designs

It is of interest to compare the leading features of some existing fan designs in terms of the usual cascade parameters at the mean diameter.

Design A is described in Reference 24. It has a two-bladed rotor of 8.5 ft tip diameter, with a hub/tip diameter ratio of 0.35, operating at 1000 rev/min. The blade profiles consist of N.P.L. sections which are flat on the underside. There are no inlet or outlet guides.

Design B is described in Reference 1. It has a four-bladed rotor of 23.6 in. tip diameter, with a diameter ratio of 0.33, operating at about 3000 rev/min. The blade profiles are of symmetrical aerofoil sections on circular arc camber lines. Inlet swirl is provided by guide vanes in a radial inlet, the rotor being designed for axial discharge.

Design C is described in Reference 1. It has a six-bladed rotor of 23.6 in. tip diameter, with a diameter ratio of 0.33, operating at about 3000 rev/min. The blade profiles are Gottingen 398, 436 and 385, depending on the radial position. Inlet swirl is provided by guide vanes in a radial inlet, the rotor being designed for axial discharge.

Design D is described in Reference 1. It has a ten-bladed rotor of 23.6 in. tip diameter, with a diameter ratio of 0.33 operating at about 3000 rev/min. The blade profile is Gottingen 436. Inlet swirl is provided by guide vanes in a radial inlet, the rotor being designed for axial discharge.

Design E is described in Reference 1. It has a twenty-bladed rotor of 23.6 in. tip diameter, with a diameter ratio of 0.7, operating at about 1500 rev/min. The blade profile is Gottingen 436 with increased cambers.

Inlet swirl is provided by guide vanes in a radial inlet, the rotor being designed for axial discharge.

Design F is a purely hypothetical design, which might however be taken as suitable for the circulating fan of a nuclear reactor. It has a twelve-bladed rotor of 48 in. tip diameter, with a diameter ratio of 0.63, operating at about 3000 rev/min. The blade profile is Gottingen 436. Inlet guides are fitted, and the rotor is designed for axial discharge.

In Appendix IV, which tabulates the leading parameters, the usual cascade symbols are used. In general, the figures should be regarded as approximate only, and the last decimal place should be regarded with reserve. The cambers of the blade profiles were found by drawing a mean line through the section; the blade inlet and outlet angles β_1 and β_2 refer to this line, and the incidence i and deviation δ are also based on β_1 and β_2 respectively. The values of the lift coefficients C_L and C_{LV_2} are the theoretical values, the drag coefficient terms being neglected.[2]

The table shows that from the cascade viewpoint, the cambers are of medium value, the incidences are highly negative, and the deviations are fairly conventional. All except one of the values of α_2 are above 70°. The lift coefficients vary from 0.72 to 1.0, with corresponding values based on the outlet velocity of 0.84 to 1.53. The latter value however refers to Fan E which gave a poor test performance.

7.0 General observations

Summarising the foregoing, existing British cascade data is generally adequate for pitch/chord ratios of 0.5 to 1.5 or possibly 2.0 and for air outlet angle of 0 to 40°; there is room however for further refinement of the data. The systematic cascade tests published by the N.A.C.A.[18] and generalised in Reference 10, can serve as a guide over a pitch/chord ratio range of about 0.7 to 2.0 for air outlet angles of about 0 to 60°; but caution should be used in the extrapolation of this data outside the limits of the original tests.

For the pitch/chord ratios above 2.0, in association with air outlet angles of 65 to 75° or higher, the isolated aerofoil approach would generally be used at present. The major problems facing the designer here are the choice of lift coefficient, cascade lift factor and aerofoil section. Attainment of exact design performance is probably of less practical importance in a single-stage fan than in a multi-stage compressor, since sufficient mechanical adjustment is in general more easily accomplished; and in any case, errors in blade setting angle do not appear to be of greatly increased importance at high air outlet angles if the pitch/chord ratio is also reasonably high.

Assuming it to be desirable to have a single method covering the whole practical range of stagger and pitch/chord ratio, these are two obvious lines of attack. The first is to select and extend present cascade rules, with modification where necessary, into the high pitch/chord ratio region, where interference effects become progressively less. The second is to take isolated aerofoil data and to extend it to the low pitch/chord ratio region, where interference and therefore correction factors become progressively larger. This is much less attractive than the first approach, especially as the only available systematic information on interference effects appears to be that of Weinig, with some support from the work of Shimoyama[33]. According to Reference 3, this type of approach has been used in Germany; as described, it appears to be very laborious in application.

For an extended cascade loading parameter, the use of the lift coefficient $C_{L_{V_2}}$, modified as appropriate, appears to be a possible choice, since it is shown in Reference 8 that this offers a measure of theoretical correlation between compressor and turbine cascades and isolated aerofoils. The choice of a comprehensive deviation rule might of course be rendered somewhat difficult by the disparity between the British and N.A.C.A. data, but a satisfactory solution should be attainable. Incidence rules and the setting of lower limits to the pitch/chord ratio are of secondary importance. Any extension of cascade data would of course necessitate further experimental investigations. These could be carried out on actual fans and compressors or possibly on cascade tunnels.

Very little work has been carried out on high stagger cascades in this country. Among the objections to their use are the difficulty of measuring small air deflections sufficiently accurately, and the possibility of excessive wall interference effects. Nevertheless, they could possibly provide useful data in support of that derived from actual fan tests.

Tests on fans go some way towards avoiding the difficulty of accurate estimation of air angles, if torque or temperature rise can be measured, and in general they are probably less laborious than systematic cascade experiments. Analysis of fan tests necessitates the separation of three-dimensional effects from the purely two-dimensional performance of the blade sections. In this respect the influence of blade aspect ratio and tip clearance on efficiency and on stalling incidence might well be important. Mach number effects would also need separate consideration in a high speed fan; this is another field where cascade tests might be of value in the provision of supporting data.

There appears to be relatively little published work on the detailed analysis of high stagger fan tests. References 35, 36 and 37 are useful recent additions to the literature.

8.0 Conclusions

Fans with blade rows of high stagger (e.g., $\alpha_2 = 65^\circ$ or above) and high pitch/chord ratio (e.g., 2.0 or above) are usually designed on isolated aerofoil theory; corrections may be made for cascade interference effects where the designer considers this to be necessary. There is little published information relating performance to design in such fans. Stages of multi-stage compressors, generally in the range $\alpha_2 = 0$ to 40°, and $s/c = 0.5$ to 1.5, are usually designed by the cascade approach. In this case, there is a considerable amount of information relating performance to design.

Designers who work exclusively in the fan or compressor fields are probably reasonably satisfied with the particular methods they employ. There appears to be a case however, for the development of a unified design method which would cover the whole range of geometry likely to be met with in compressors or fans. Most importantly, this might provide a firm basis for designs which fall between normal and fan and compressor practice (i.e., s/c in the range 1.5 to 2.5 approximately) and in addition should result in improved performance prediction in the fan region. The N.A.C.A. cascade data, together with the existing British information, would probably form a useful starting point, although care would be necessary in extrapolating it beyond the limits of the original tests.

A project of this kind would necessitate a supporting programme of tests and detailed performance analyses of typical fans; this could be supported by cascade tests, although the value of the latter is somewhat doubtful at the higher staggers and pitch/chord ratios.

The foregoing relates to purely two-dimensional low speed design considerations; three-dimensional and high speed effects would have to be considered separately, though probably in parallel with the primary investigations.

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APPENDIX I

Deflections and theoretical lift coefficients for $C_{LV_2} = 1.0$

a_2^o	s/c = 1.0		s/c = 2.5		s/c = 4.0	
	ε^o	C_L	ε^o	C_L	ε^o	C_L
0	25.91	0.944	11.25	0.990	7.11	0.997
10	23.13	0.880	10.57	0.959	6.79	0.976
20	20.30	0.828	9.65	0.930	6.29	0.956
30	17.45	0.787	8.56	0.906	5.65	0.940
40	14.58	0.754	7.33	0.886	4.89	0.926
50	11.68	0.729	5.98	0.868	4.02	0.912
60	8.77	0.709	4.56	0.856	3.08	0.903
70	5.85	0.696	3.07	0.846	2.09	0.899
80	2.93	0.690	1.54	0.839	1.05	0.891

APPENDIX II

Range of tests on which generalised cascade data sheets of Reference 10 are based

At each pitch/chord ratio s/c and camber θ, the air inlet angle α_1 was kept constant at a selected value, and measurements were made at various blade staggers. The tests were then repeated at other values of α_1. The data sheets of Reference 10 are based on generalisations of the "reference" or minimum loss conditions of these tests. The following table gives the leading parameters of each of the tests, as judged from the figures of Reference 10.

α_1°	s/c	θ°
30	1.0	10, 20, 30, 37.5, 45
	0.8	10, 30, 45
	0.67	10, 20, 30, 37.5, 45
45	2.0	30, 45
	1.33	30, 45
	1.00	30, 37.5, 45, 52, 59
	0.80	30, 45
	0.67	10, 20, 30, 37.5, 45, 52, 59
60	2.00	30, 45
	1.33	30, 45
	1.00	30, 37.5, 45, 52
	0.80	10, 30, 45
	0.67	10, 20, 30, 37.5, 45
70	1.00	10, 20, 30, 37.5
	0.80	20, 30, 37.5
	0.67	10, 20, 30, 37.5

APPENDIX III

Values of A and B in equation $\tan \alpha_2 = A + B \tan \alpha_1$ for flat plate cascades in potential flow

	s/c = 1.0		s/c = 2.5		s/c = 4.0	
α_2	A	B	A	B	A	B
0	-0.021	0.041	-0.056	0.284	-0.057	0.459
10	0.148	0.043	0.070	0.284	0.038	0.458
20	0.324	0.046	0.197	0.293	0.134	0.466
30	0.524	0.048	0.329	0.311	0.229	0.486
40	0.769	0.050	0.467	0.343	0.325	0.516
50	1.094	0.053	0.617	0.388	0.422	0.559
60	1.577	0.057	0.776	0.455	0.519	0.616
70	2.506	0.061	0.950	0.547	0.611	0.691
80	4.203	0.182	1.137	0.674	0.696	0.784

The stagger setting is that which gives the appropriate value of α_2 at $C_{LV_2} = 1.0$.

Note: Some unpublished test results on a single-stage fan gave $A = 2.10$ and $B = 0.27$, for Gottingen 436 blade sections at a pitch/chord ratio of 1.5 and $\alpha_2 = 72.9^\circ$ at $C_{LV_2} = 1.0$.

APPENDIX IV

Leading mean diameter parameters of some existing fan designs

Design	A	B	C	D	E	F
V_a/U	0.280	0.279	0.270	0.314	0.471	0.300
$K_p\Delta T/\frac{1}{2}U^2$	0.126	0.127	0.232	0.377	1.160	0.570
No. blades	2	4	6	10	20	12
s/c	7.22	7.36	3.87	2.19	1.18	1.50
t/c	0.12	0.08	0.11	0.11	0.11	0.11
h/c	2.20	4.20	3.71	2.52	1.32	1.32
θ	37.7	18.0	25.0	25.0	37.1	25.0
β_1	99.9	82.7	91.2	90.1	92.8	91.9
β_2	62.2	64.7	66.2	65.1	55.7	66.9
α_1	74.4	75.3	76.4	75.2	73.4	76.1
α_2	73.4	74.4	74.9	72.6	64.8	72.1
ε	1.0	0.9	1.5	2.6	8.6	4.0
i	-25.5	-7.4	-14.8	-14.9	-19.4	-20.8
δ	11.2	9.7	8.7	7.5	9.1	5.2
C_L	0.91	0.88	0.82	0.72	1.00	0.770
C_{LV_3}	0.96	0.94	0.91	0.84	1.53	0.99
$\tan\alpha_1 - \tan\alpha_2$	0.23	0.23	0.43	0.59	1.23	0.950

Note: t = blade maximum thickness

h = blade height

$\varepsilon = \alpha_1 - \alpha_2$

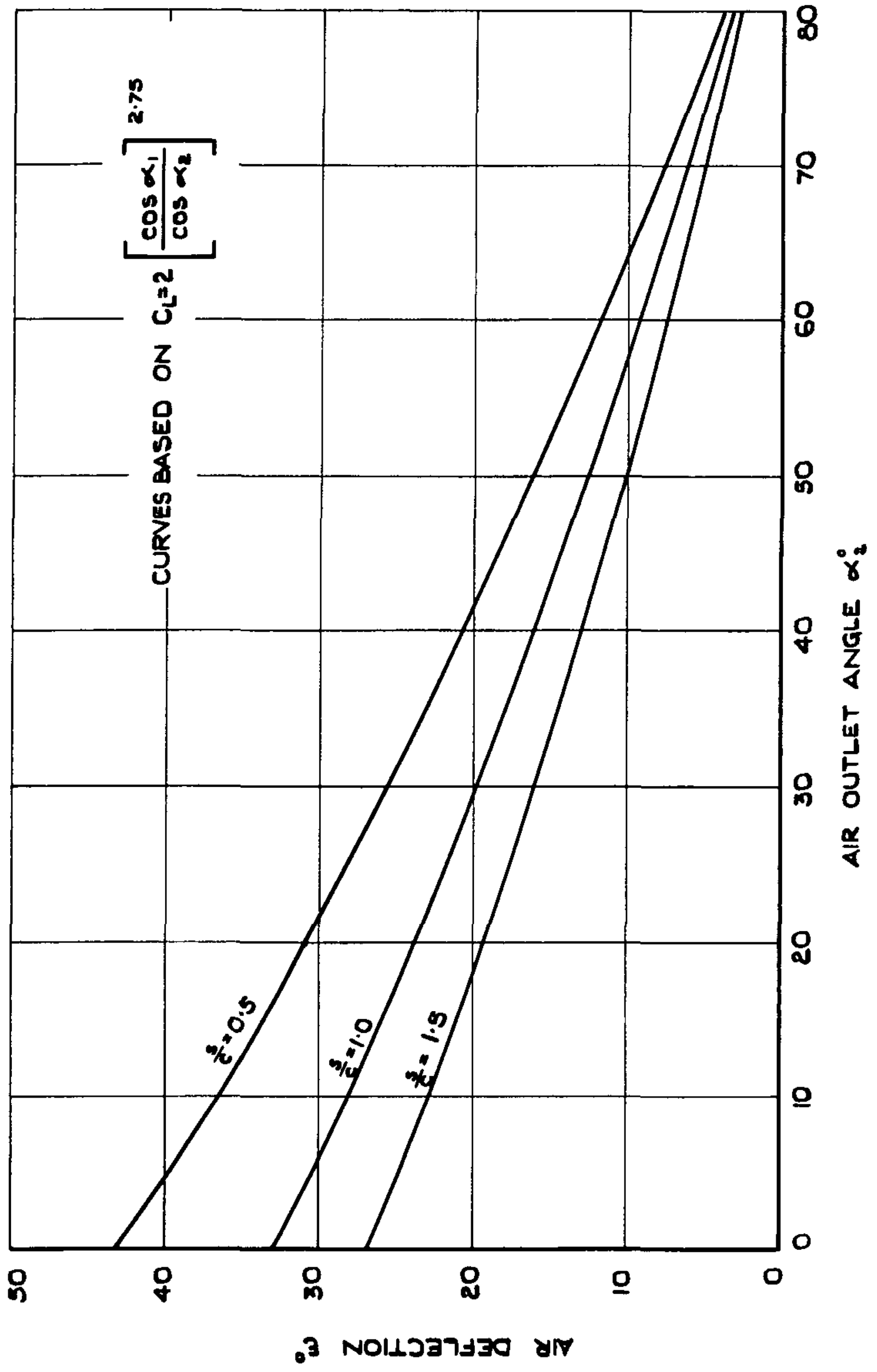
The remaining symbols are as defined in the text.

The lift coefficients are the theoretical values which neglect the drag term.

Angles are given in degrees.

D 76920/1/125875 K4 10/66 R & TXL

FIG.1

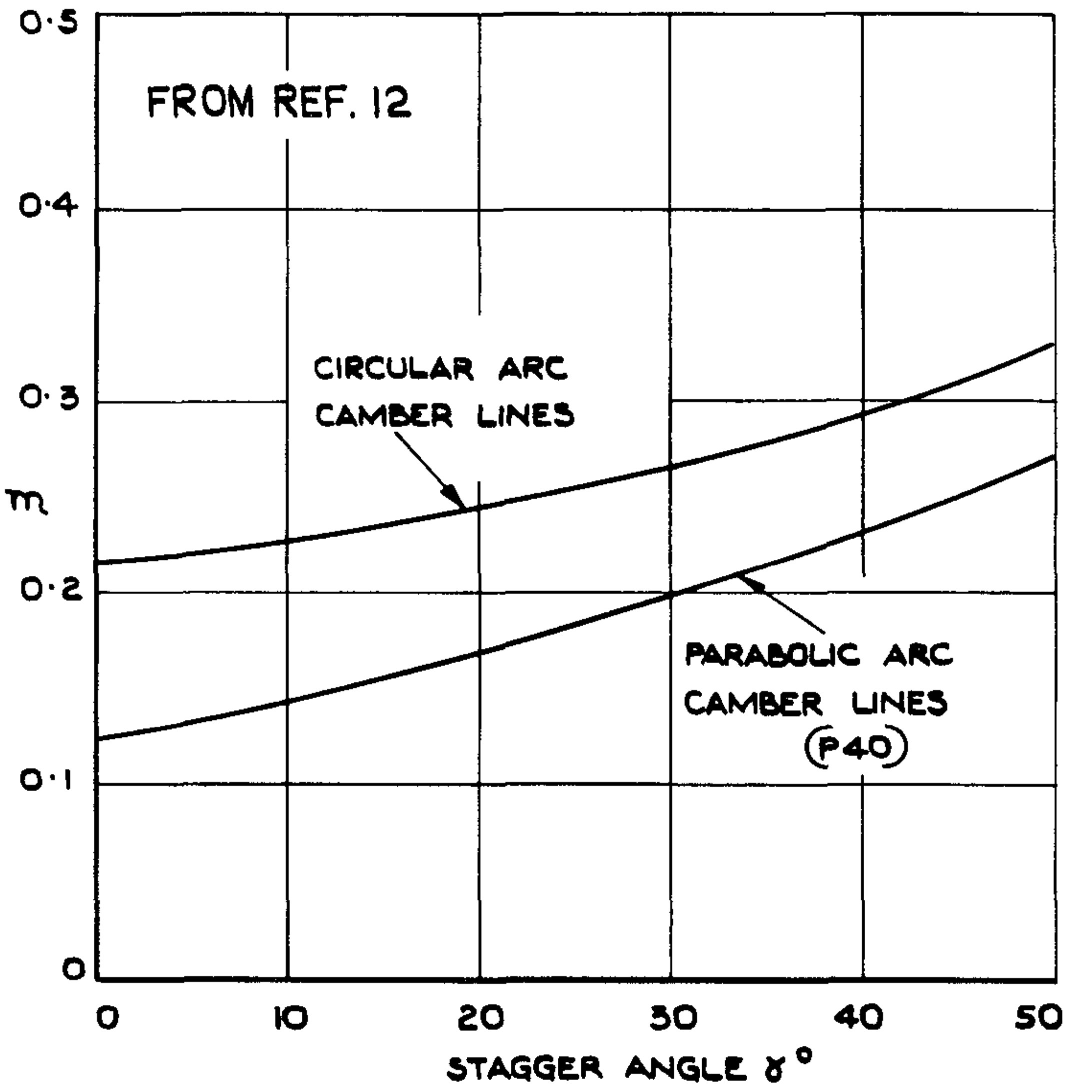


TYPICAL CASCADE DEFLECTION DESIGN RULE

FIG. 2

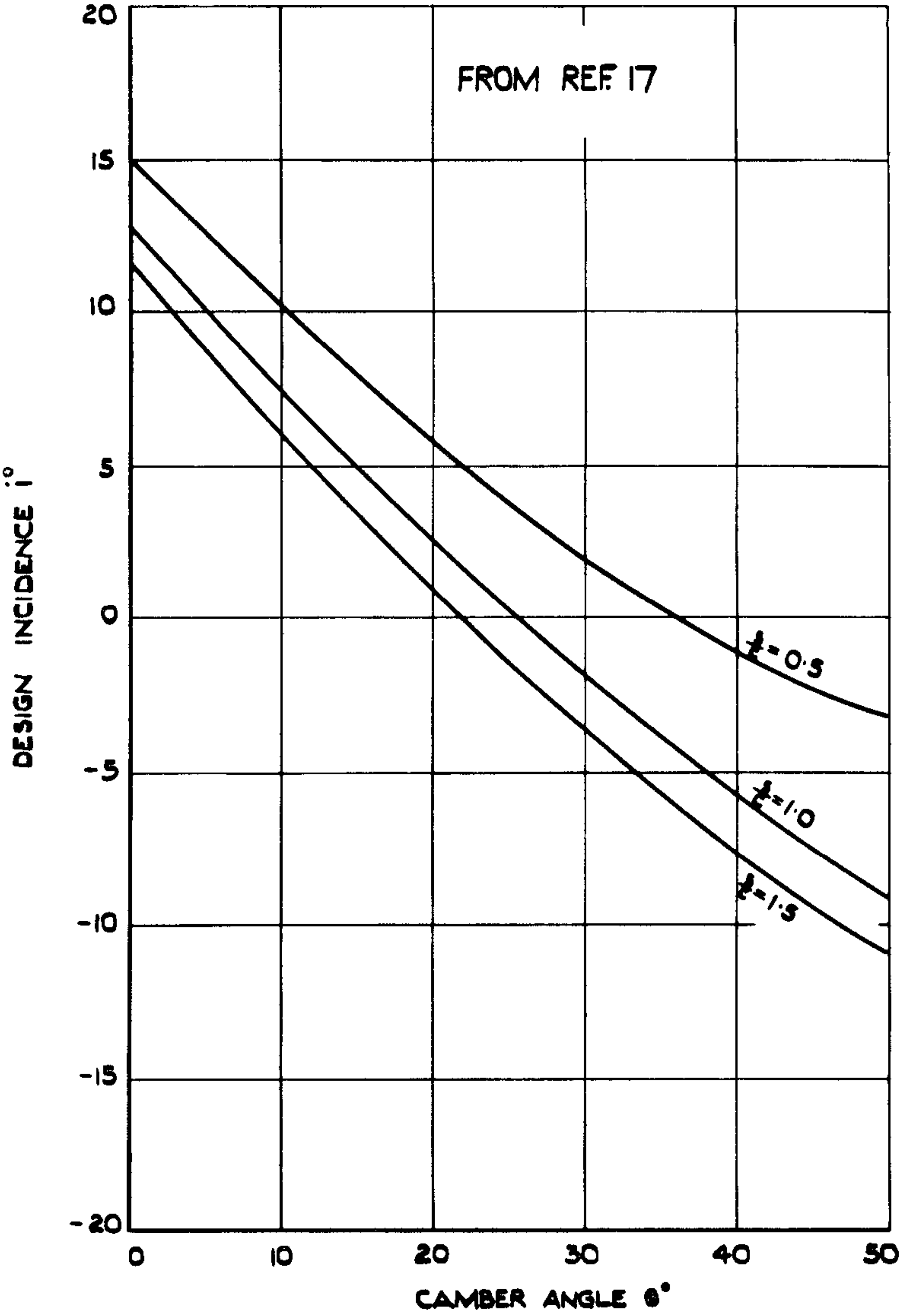
$\delta \; m\theta\sqrt{\frac{s}{c}}$

WHERE δ = DEVIATION ANGLE
θ = BLADE CAMBER ANGLE
$\frac{s}{c}$ = PITCH / CHORD RATIO

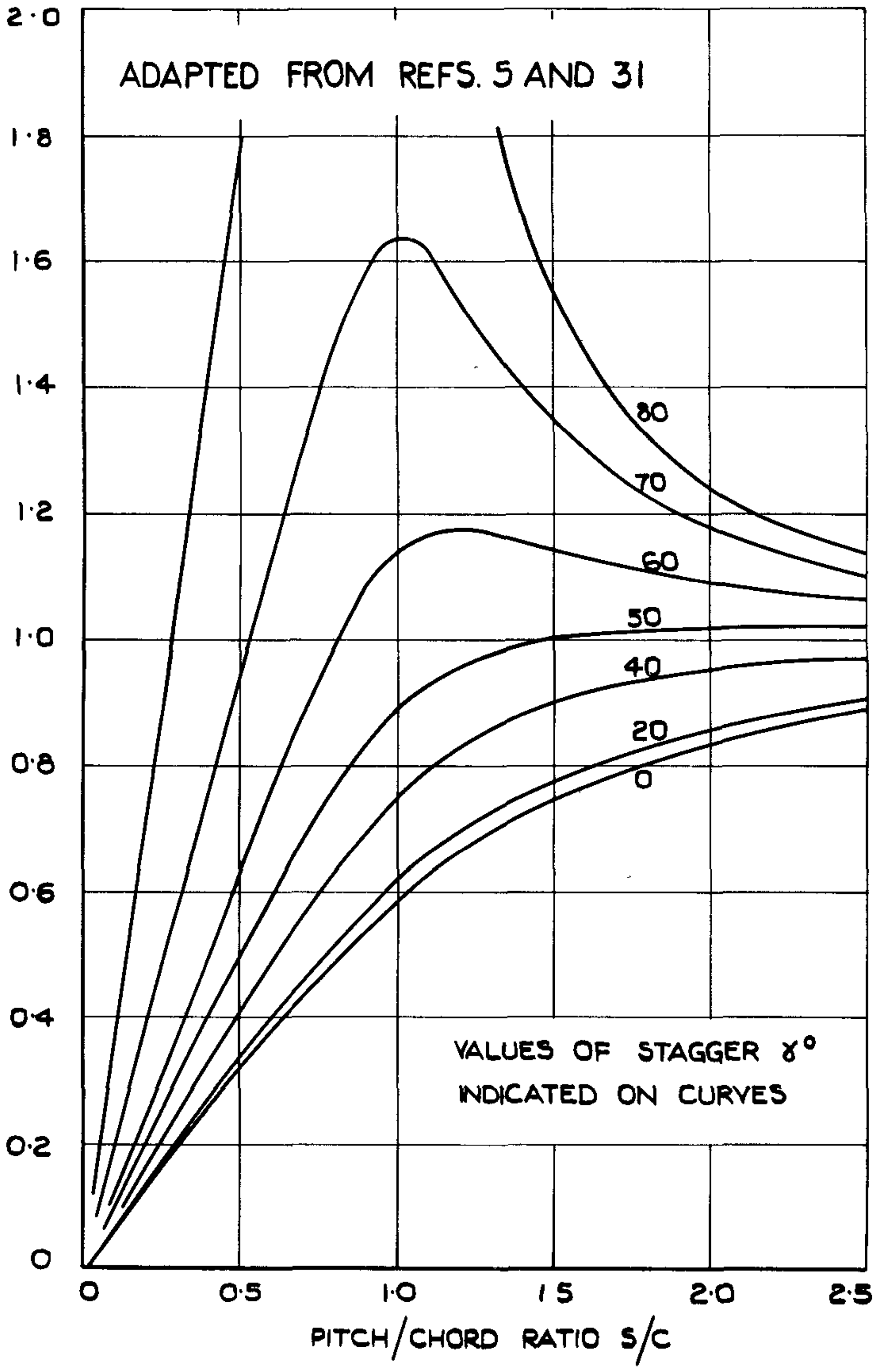


CASCADE DEVIATION RULE

FIG. 3

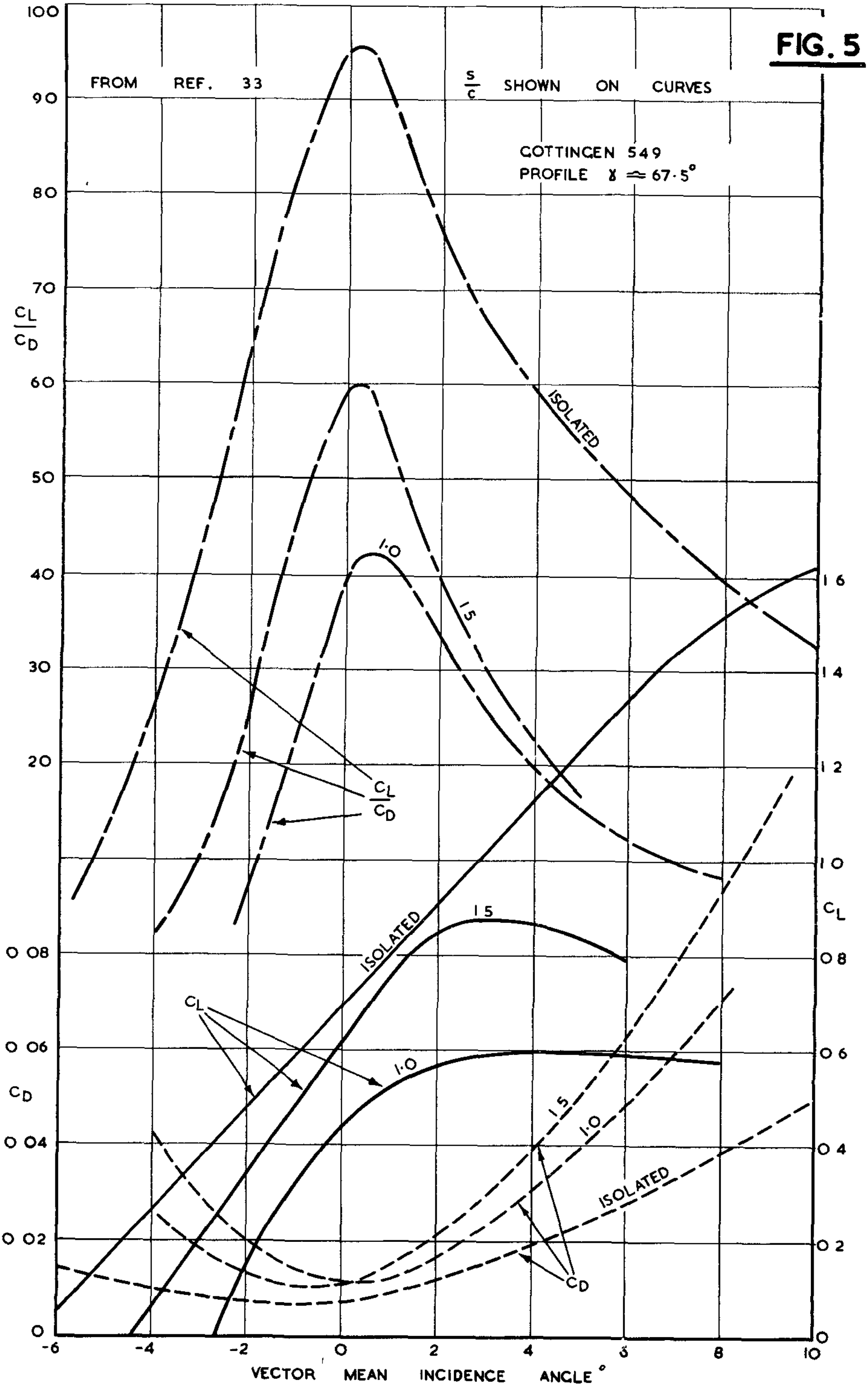


CASCADE INCIDENCE DESIGN RULE



THEORETICAL LIFT FACTORS FOR FLAT PLATE CASCADES

FIG. 5

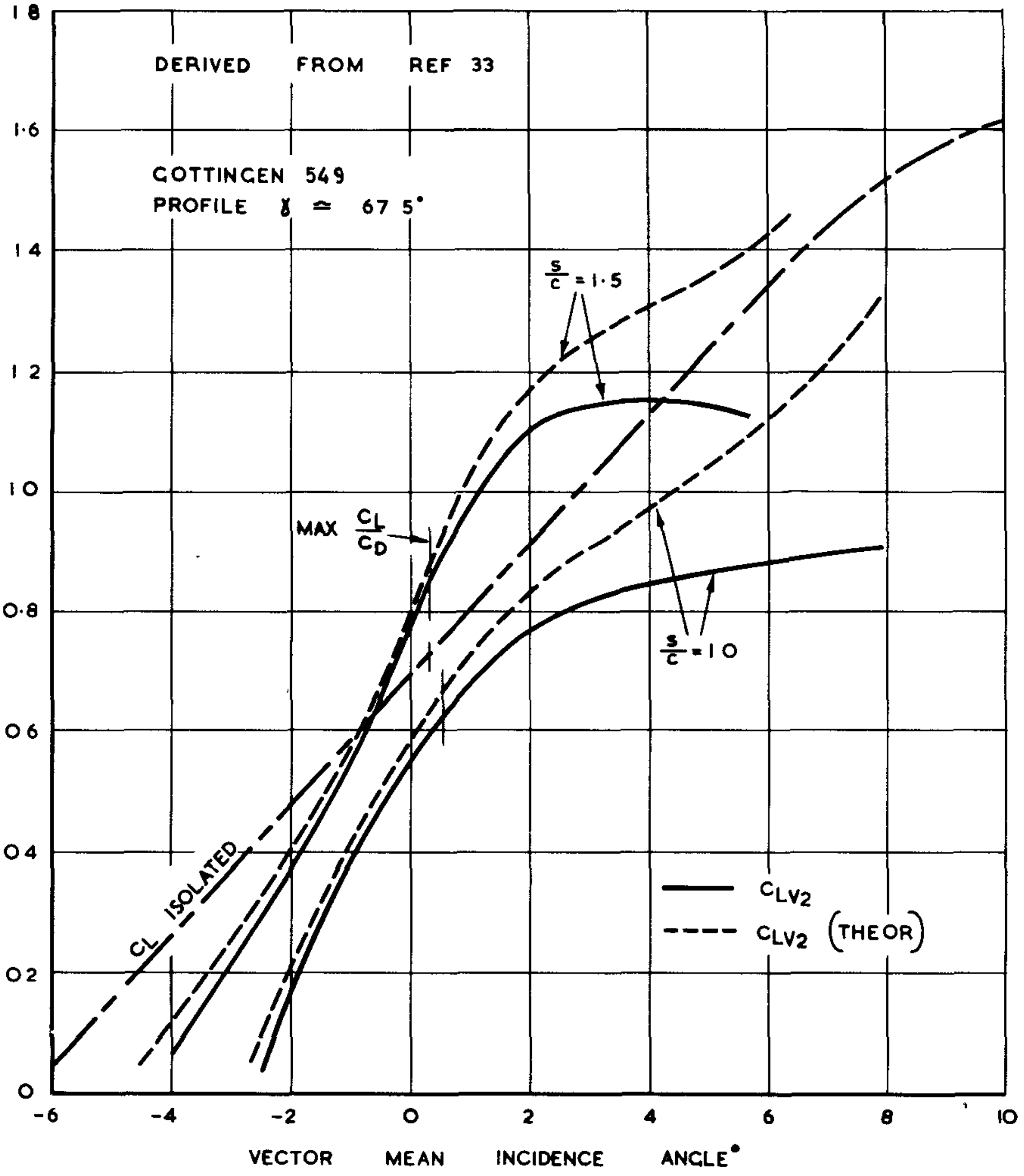


LIFT AND DRAG COEFFICIENTS FOR ISOLATED AND CASCADED AEROFOILS

FIG. 6.

$$C_{LV_2} = 2\frac{s}{c}\left(\tan\alpha_1 - \tan\alpha_2\right)\frac{\cos^2\alpha_2}{\cos\alpha_m} - C_D \tan\alpha_m \frac{\cos^2\alpha_2}{\cos^2\alpha_m}$$

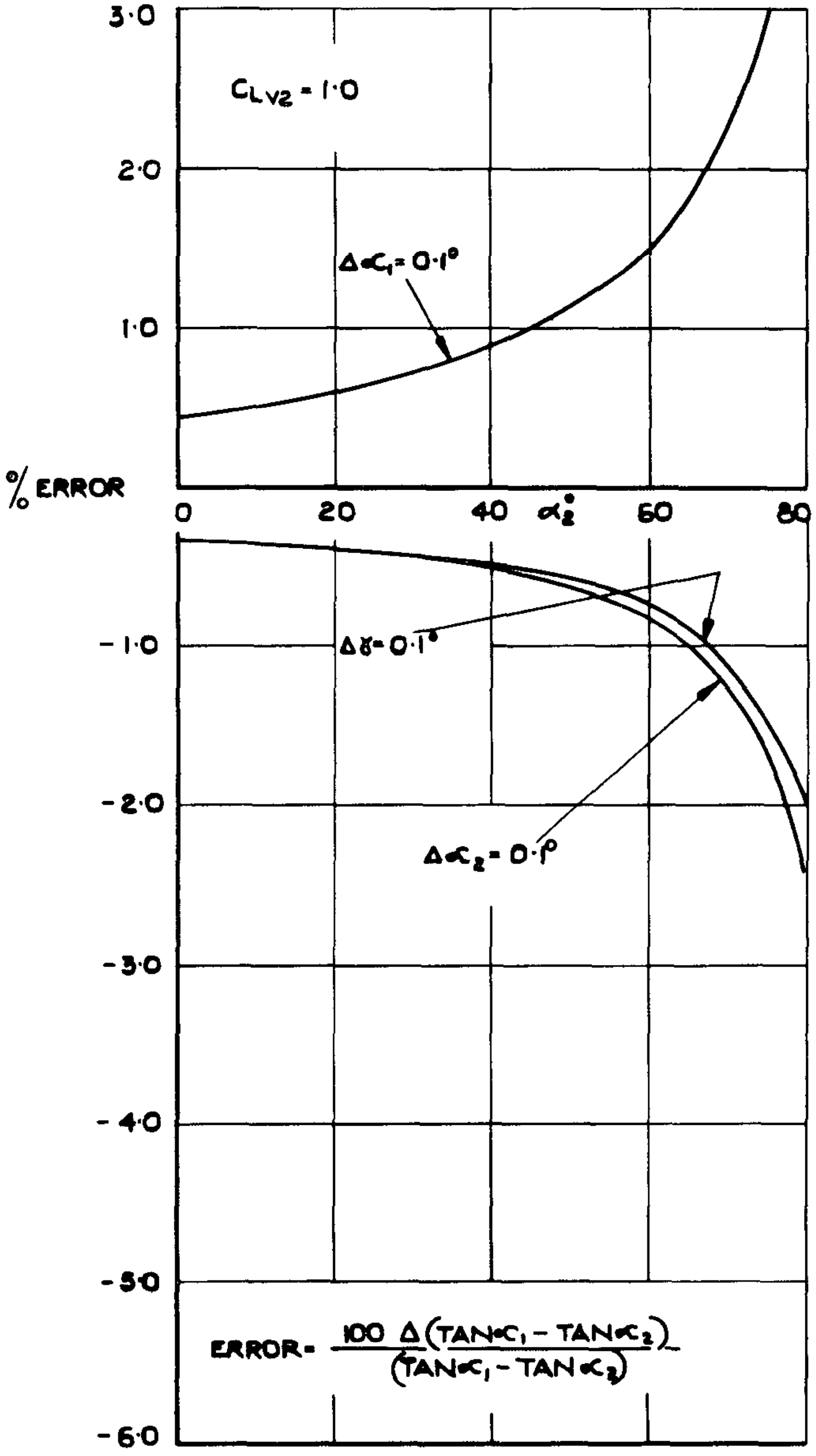
$$C_{LV_2}\ (\text{THEOR}) = 2\frac{s}{c}\left(\tan\alpha_1 - \tan\alpha_2\right)\frac{\cos^2\alpha_2}{\cos\alpha_m}$$



LIFT COEFFICIENTS FOR CASCADED AEROFOILS

BASED ON OUTLET VELOCITY

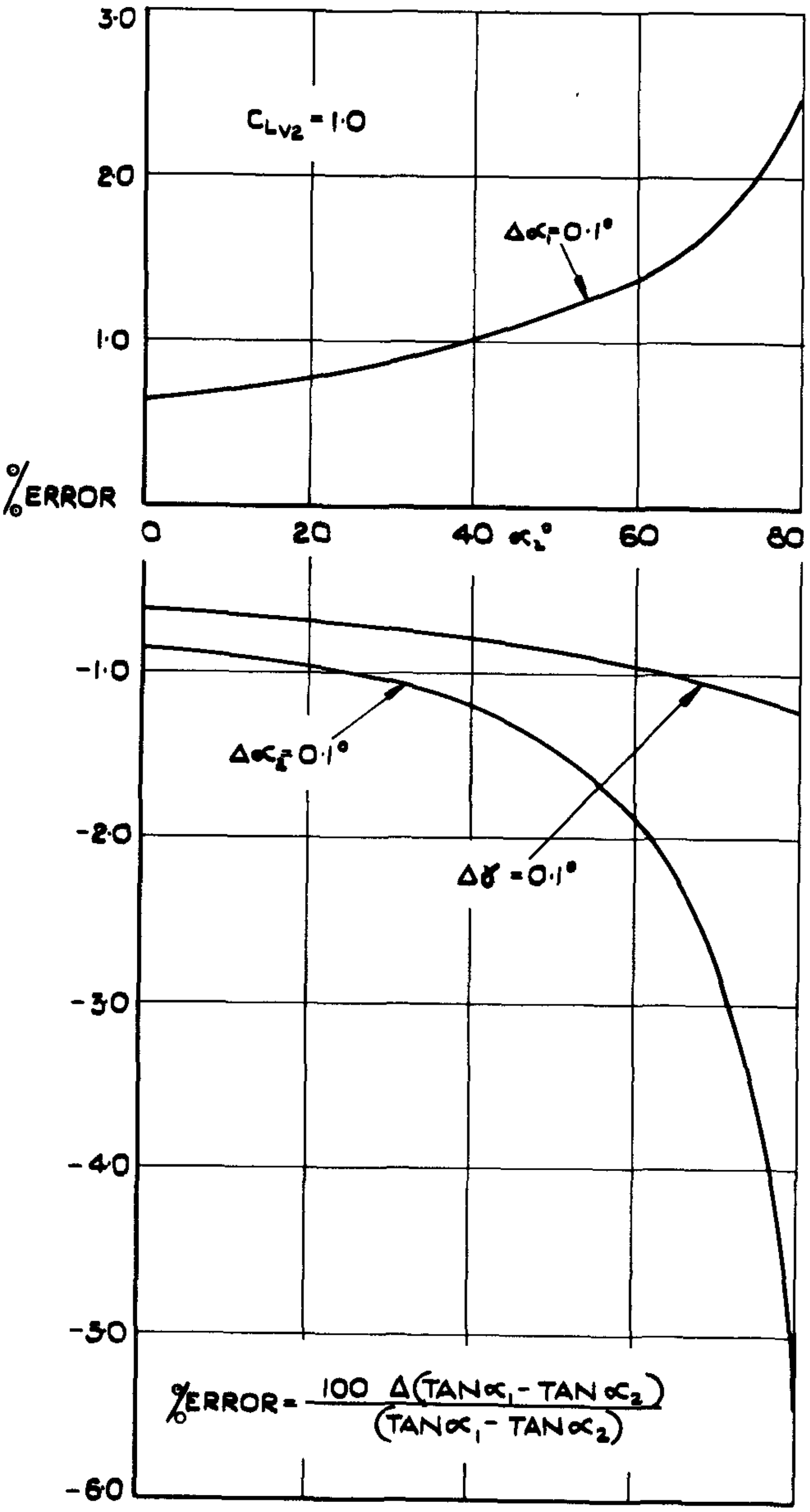
FIG.7



FLAT PLATE CASCADE. $S/C = 1{\cdot}0$

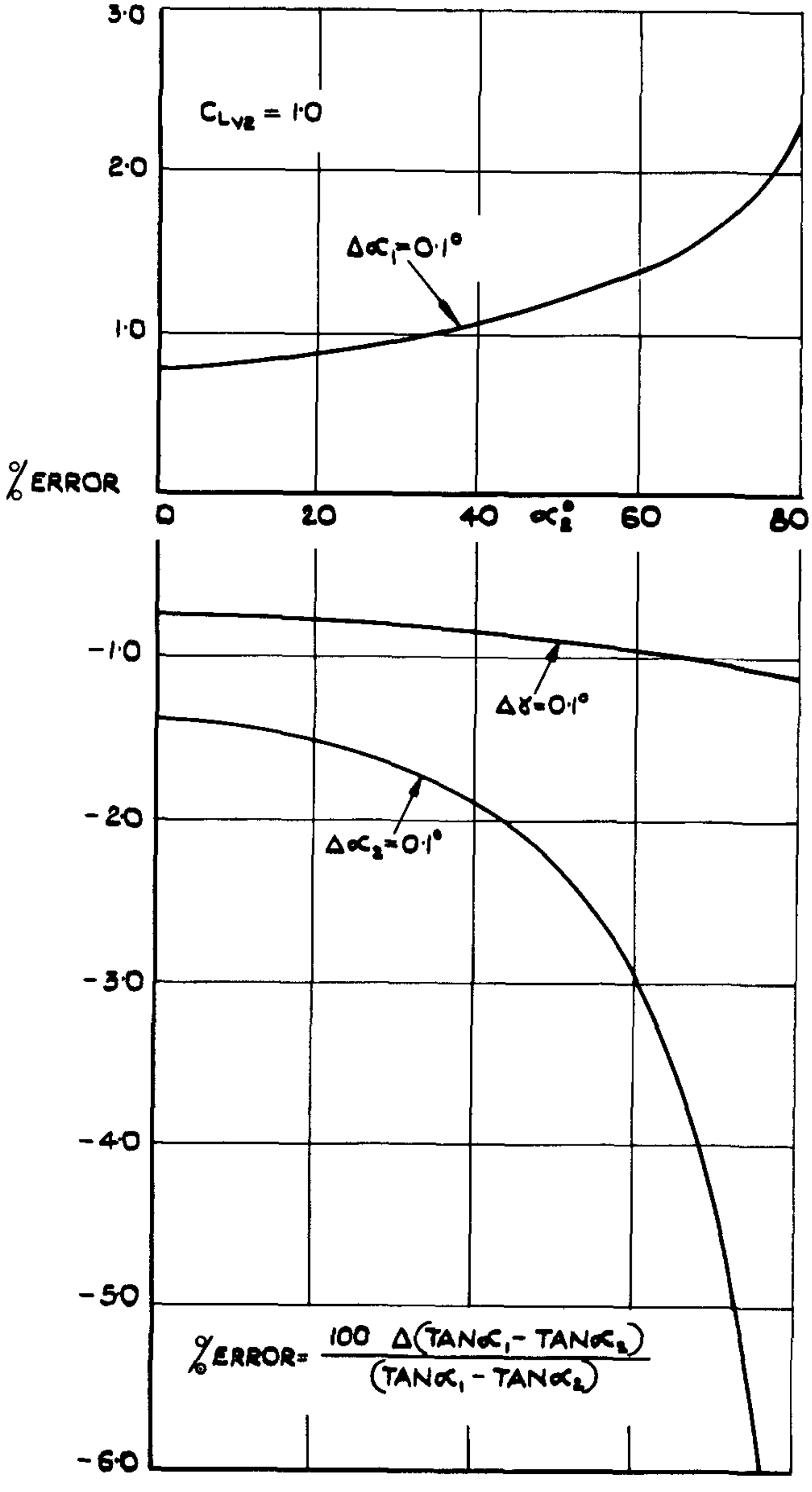
ERRORS IN $(\text{TAN}\,\alpha_1 - \text{TAN}\,\alpha_2)$

FIG. 8



FLAT PLATE CASCADE. $S/C = 2·5$

ERRORS IN $(\tan\alpha_1 - \tan\alpha_2)$



FLAT PLATE CASCADE. $S/C = 4{\cdot}0$

ERRORS IN $(\tan\alpha_1 - \tan\alpha_2)$

FIG. 10

DESIGN CONDITIONS

$\frac{V_a}{U} = 0\ 364$

$\frac{s}{c} = 4\ 0$

$\Omega = 1\ 0$

$\alpha_0 = -19\ 1^\circ$

$C_{L_{V_2}} = 1\ 0$

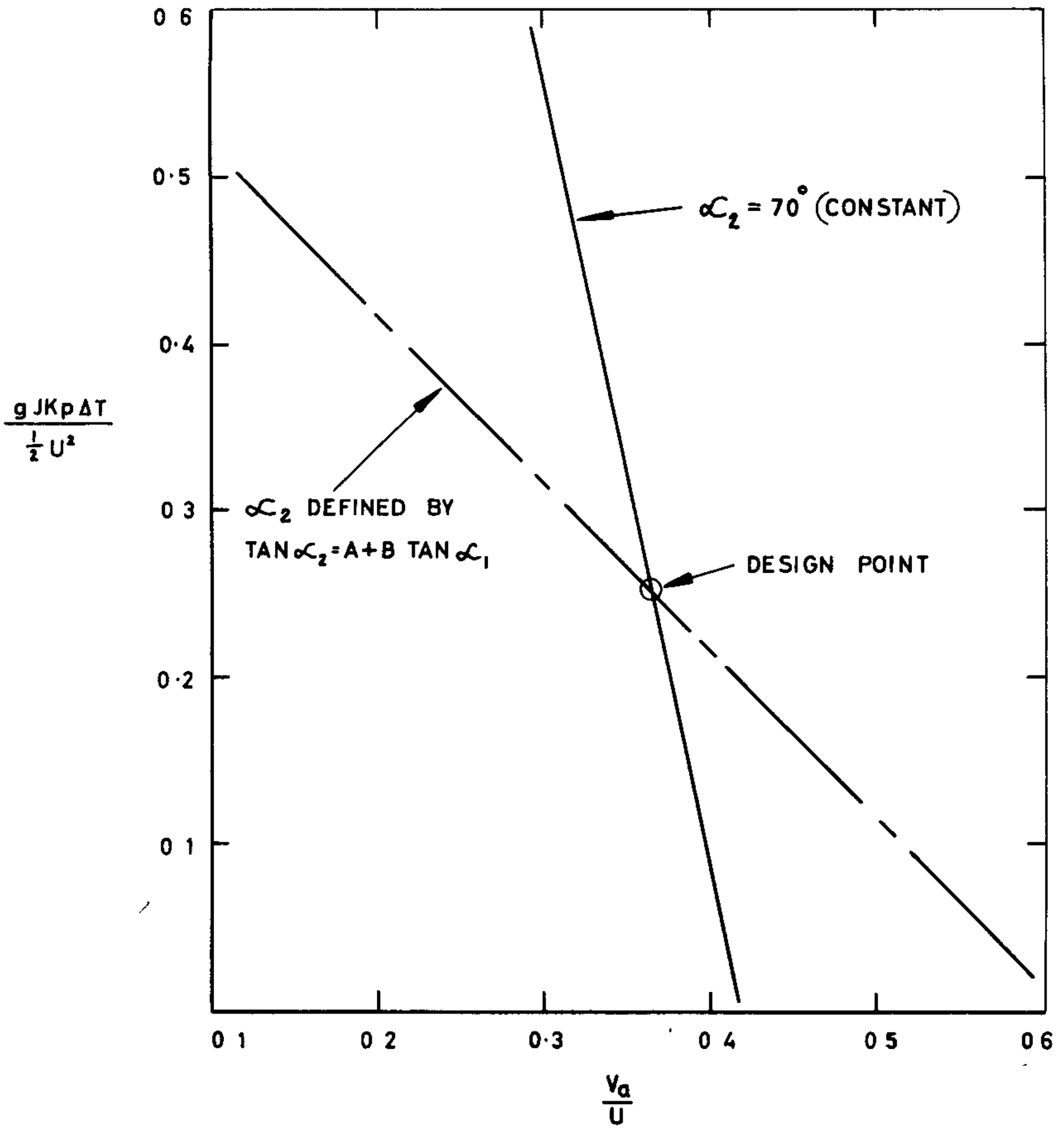
$\alpha_2 = 70^\circ$

$A = 0\ 611$

$\alpha_3 = 0$

$B = 0\ 691$

A AND B DERIVED FROM DATA FOR FLAT PLATE CASCADES IN POTENTIAL FLOW



FAN TEMPERATURE RISE CHARACTERISTIC

EFFECT OF VARIATION OF α_2 WITH α_1

A.R.C. C.P. No. 895 629.13.038.23
August, 1964
Turner, R. C.

NOTES ON DUCTED FAN DESIGN

In general, conventional compressor stages are designed by the cascade method, while high stagger low solidity ducted fans are designed on modified isolated aerofoil theory. The purpose of these notes is to provide a basis for discussion on the relative merits of the two methods and on the desirability of extending one method to cover the whole range of blading likely to be required in compressors and fans. Attention has been mainly confined to low speed two-dimensional considerations.

It is suggested that the cascade approach could provide a basis for the formulation of a unified design method.

A project of this nature would necessitate a programme of testing and performance analysis of typical fans; high stagger cascade tests might also provide supporting data, although there could be doubts as to their significance.

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DETACHABLE ABSTRACT CARDS

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C.P. No. 895

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