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The Theoretical Stability Derivatives for a Symmetrically Tapered Wing of Aspect-Ratio 3 at Supersonic Speeds

By

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SUMMARY

A symmetrically tapered wing of aspect ratio 3 and taper ratio $\frac{5}{11}$ with streamwise tips describes simple harmonic oscillations of low frequency in supersonic flow. Heaving and pitching derivatives are calculated to first order in frequency for Mach numbers in the range $1{\cdot}031 \leq M \leq 1{\cdot}875$. Some comparisons are made with transonic and supersonic experimental results and with other theoretical values.

1. Introduction

The aerodynamic forces acting on oscillating wings of hexagonal planform are considered on the basis of linearized thin-wing theory in R. & M.3298[1]. To first order in frequency, the lift and pitching-moment derivatives are formulated exactly for simple harmonic heaving and pitching oscillations. The formulae of Ref.1 apply to wings having supersonic, or sonic, leading and trailing edges with a lower limit on Mach number imposed by the condition for non-interacting tips.

In this note, the stability derivatives are evaluated for a wing planform of aspect ratio 3 with symmetrical taper and streamwise tips (Table 1). This planform is included as Wing G in a programme of low-frequency wind-tunnel measurements at transonic and supersonic speeds, which has been carried out by Hawker Siddeley Dynamics Ltd.[2]. The effect of higher frequency has been estimated theoretically in Ref.3, where heaving and pitching derivatives are given for three values of the frequency parameter at Mach numbers $M = 1{\cdot}25$, $1{\cdot}875$ and $2{\cdot}5$. The present low-frequency calculations for the range $1{\cdot}031 \leq M \leq 1{\cdot}875$ provide some results by linearized theory for lower supersonic speeds.

2./

Replaces N.P.L. Aero Note 1026 - A.R.C.25 889.

2. Results

The present application of Ref.1 is similar to that outlined in Ref.4 for a symmetrically tapered wing of aspect ratio 4. The planform of the wing of aspect ratio 3 is defined in Table 1. In the notation of Refs.1 and 4, the heaving and pitching derivative coefficients are defined in terms of the wing root-chord c_0 (Table 2). These derivatives are often required with geometric mean chord $\bar{c}$ or aerodynamic mean chord $\bar{\bar{c}}$ as representative length; the appropriate conversion factors are given in Table 3.

The derivatives are calculated to first order in frequency from the formulae given in Sections 3 and 5 of Ref.1, for the pitching axis $h_0 = 0$ through the root leading-edge. The necessary condition for non-interacting tips is satisfied for the present planform when $\beta \geq \frac{5}{24}$ by equation (20) of Ref.1. The exact solutions are evaluated for the six Mach numbers corresponding to

$$\beta = [M^2 - 1]^{\frac{1}{2}} = \tfrac{1}{4},\ \tfrac{11}{24},\ \tfrac{2}{3},\ \tfrac{3}{4},\ 1,\ \tfrac{1}{8}\sqrt{105}\ ,$$

for the first of which the leading and trailing edges are sonic. The lift and pitching-moment derivatives are tabulated in Table 4 as "exact theory" for the frequency parameter $\nu_0 = \omega c_0/U_\infty \to 0$.

To conform with the notation of this note, it appears necessary to change the sign of all heaving derivatives from Ref.2 and all pitching moments from Ref.3. The experimental results[2] obtained for $1{\cdot}0 \leq M \leq 1{\cdot}90$ and a frequency $\omega/2\pi = 10$ cyles/sec ($\nu_0 \approx 0{\cdot}07$) are included in Table 4 as derivative values referred to the axis $h_0 = 0$; likewise the theoretical results[3] are given for $M = 1{\cdot}250$ and $1{\cdot}875$ at $\nu_0 = 0{\cdot}1375$.

3. Discussion

It can be seen from Table 4 that at $M = 1{\cdot}875$ the values from Ref.3 for $\nu_0 = 0{\cdot}1375$ are close to those from "exact theory"for $\nu_0 \to 0$. At the lower Mach number $M = 1{\cdot}250$, no values of $\ell_{\dot\theta}$ and $-m_{\dot\theta}$ are given in Ref.3 but the other derivatives show important frequency effects.

In Table 4, the experimental values of the pitching derivatives for supersonic Mach numbers $M = 1{\cdot}56$ and $M = 1{\cdot}90$ correlate quite well with the theoretical results; experiment shows less effect of M on the damping derivatives $\ell_{\dot\theta}$ and $-m_{\dot\theta}$. For both Mach numbers, experimental values of the heaving derivatives $\ell_{\dot z}$ and $-m_{\dot z}$ appear to be too large. Such discrepancies are probable, since the heaving derivatives are derived from measurements for pitching oscillations about the two axis positions $h_0 = 0{\cdot}5$ and $h_0 = 1{\cdot}045$. At all Mach numbers the measurements of Ref.2 were subject to effects of wing distortion, especially for the rear axis, and no corrections could be applied to the damping derivatives. Although all tests reported in Ref.2 correspond to low values of the frequency parameter ν_0, there are large and unsystematic frequency effects on the values of the

transonic/

transonic damping derivatives. At transonic speeds $M \leq 1{\cdot}2$, the experimental damping data are thought to have been affected by unknown slotted-wall interference effects. The values for $M \leq 1{\cdot}2$ in Table 4 show that theory and experiment give very different results for the axis position $h_o = 0$; however, the stiffness derivatives ℓ_θ and $-m_\theta$ compare quite well at $M = 1{\cdot}2$.

Fig.1 shows plots of ℓ_θ against Mach number for the axis position $h_o = 0{\cdot}5$. At each Mach number the experimental value for steady flow is in good agreement with the oscillatory test result; for $M \geq 1{\cdot}20$ these are reasonably close to the theoretical values. Also illustrated for $\nu_o \to 0$ are values of $\ell_\theta = 2/\beta$ from strip theory and additional values by "exact theory" not given in Table 4. A similar comparison between the values of m_θ for $h_o = 0{\cdot}5$, plotted against M, is made in Fig.2. While the agreement between "exact theory" for $\nu_o \to 0$ and experiment is not good, the difference can largely be attributed to thickness effect. "Exact theory" with correction for the 5% thick biconvex section, estimated on a strip-theory basis as in Ref.1, gives a curve in Fig.2 which agrees well with experiment for $M > 1{\cdot}56$. The experimental results for $\nu_o = 0$ and $\nu_o \approx 0{\cdot}07$ show a small frequency effect which is opposite to that indicated by theory when $M > 1{\cdot}2$. In Figs.1 and 2, the theoretical values for $M < 1{\cdot}2$ appear to correlate with sonic theory as formulated in equation (5) of Ref.4.

The variation of $-m_{\dot\theta}$ with axis position h_o is expressed in terms of the damping derivatives for $h_o = 0$ by equation (46) of Ref.1. Curves from "exact theory" for $\nu_o \to 0$ are shown in Fig.3 for the four Mach numbers $M = 1{\cdot}875$, $1{\cdot}250$, $1{\cdot}100$ and $1{\cdot}031$. It is remarkable how rapidly the axis position for minimum damping moves upstream with decreasing Mach number. Values from strip theory, depending only on Mach number, planform taper ratio and axis position, give a fair accuracy for $M = 1{\cdot}875$. Strip theory becomes a poorer approximation to "exact theory" as Mach number and aspect ratio decrease. For example, in Fig.6 of Ref.4 strip theory is regarded as a useful rough estimate at $M = 1{\cdot}250$ for aspect ratio 4, whereas the present Fig.3 for aspect ratio 3 shows that the discrepancy is nearly doubled.

In Fig.4, comparison is made between the theoretical and experimental values of $-m_{\dot\theta}$ against h_o for $M = 1{\cdot}875$. The theoretical values are in satisfactory agreement for all axis positions. The value (x) obtained from measurements about the axis $h_o = \frac{1}{2}$ compares well with theory, but the experimental curve of $-m_{\dot\theta}$ indicates a more forward position for minimum damping and large variations from the theoretical values for rearward axis positions. A likely explanation is that this curve is derived from unreliable measurements about the pitching axis $h_o = 1{\cdot}045$.

4. Acknowledgement

The numerical results given in this note were calculated by Mrs. S. Lucas of the Aerodynamics Division, N.P.L.

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4	Doris E. Lehrian and Gillian Smart	Theoretical stability derivatives for a symmetrically tapered wing at low supersonic speeds. A.R.C. C.P.736 April, 1963.

Table 1 /

Table 1

Definition of Symmetrically Tapered Wing

Semi-span/root chord	=	$\frac{12}{11}$
Tip chord/root chord	=	$\frac{5}{11}$
Semi-apex angle	=	75·96°
Aspect ratio	=	3
Geometric mean chord	=	$\frac{8}{11}$ root chord
Aerodynamic mean chord	=	$\frac{67}{88}$ root chord

Table 2

Definition of Heaving and Pitching Derivatives

Motion	Heaving	Pitching
Upward Displacement of Wing	$-z_0 e^{i\omega t}$	$-(x - x_0)\theta_0 e^{i\omega t}$
$\dfrac{\text{Lift}}{\rho_\infty U_\infty^2 S}$	$(\ell_z + i\nu_0 \ell_{\dot{z}}) \dfrac{z_0}{c_0} e^{i\omega t}$	$(\ell_\theta + i\nu_0 \ell_{\dot{\theta}})\theta_0 e^{i\omega t}$
$\dfrac{\mathcal{M}}{\rho_\infty U_\infty^2 S c_0}$	$(m_z + i\nu_0 m_{\dot{z}}) \dfrac{z_0}{c_0} e^{i\omega t}$	$(m_\theta + i\nu_0 m_{\dot{\theta}})\theta_0 e^{i\omega t}$

$\mathcal{M}$ = Nose-up pitching moment about axis $x = x_0 = h_0 c_0$

c_0 = Representative length = root chord

ν_0 = Frequency parameter = $\omega c_0/U_\infty$

Table 3/

Table 3

Conversion Factors for Representative Length d

Derivative	ℓ_z	$\ell_{\dot z}$	ℓ_θ	$\ell_{\dot\theta}$	$-m_z$	$-m_{\dot z}$	$-m_\theta$	$-m_{\dot\theta}$
Factor	K^{-1}	1	1	K	1	K	K	K^2

$K = c_o/d$

$= 1.3750$ when d is geometric mean chord

$= 1{\cdot}3134$ when d is aerodynamic mean chord

Table 4

Stability Derivatives for Pitching Axis $h_o = 0$

		M	ℓ_z	$\ell_{\dot z}$	ℓ	ℓ	$-m_z$	$-m_{\dot z}$	$-m$	$-m$
Exact Theory	→ 0	1·031	0	2·780	2·780	-2·791	0	0·717	0·717	0·662
		1·100	0	2·788	2·788	-1·806	0	1·141	1·141	-0·828
		1·202	0	2·377	2·377	-0·385	0	1·092	1·092	-0·213
		1·250	0	2·205	2·205	-0·086	0	1·033	1·033	-0·050
		1·414	0	1·773	1·773	0·331	0	0·854	0·854	0·193
		1·875	0	1·184	1·184	0·445	0	0·582	0·582	0·265
Theory Ref.3	0·1375	1·250	0·020	2·041	2·049	-	0·010	0·859	0·864	-
	0·1375	1·875	0·003	1·156	1·141	0·416	0·002	0·564	0·557	0·255
Expt. Ref.2	0·082	1·00	-0·18	1·98	1·76	0·10	-0·21	-0·15	0·41	-0·38
	0·077	1·10	-0·04	3·56	2·07	0·94	-0·02	1·05	0·78	+0·38
	0·072	1·20	+0·22	1·75	2·37	-1·42	+0·07	0·86	0·94	-0·52
	0·059	1·56	-0·12	2·08	1·43	0·41	-0·06	0·99	0·65	+0·23
	0·052	1·90	-0·01	1·50	1·15	0·40	+0·03	0·74	0·54	+0·25

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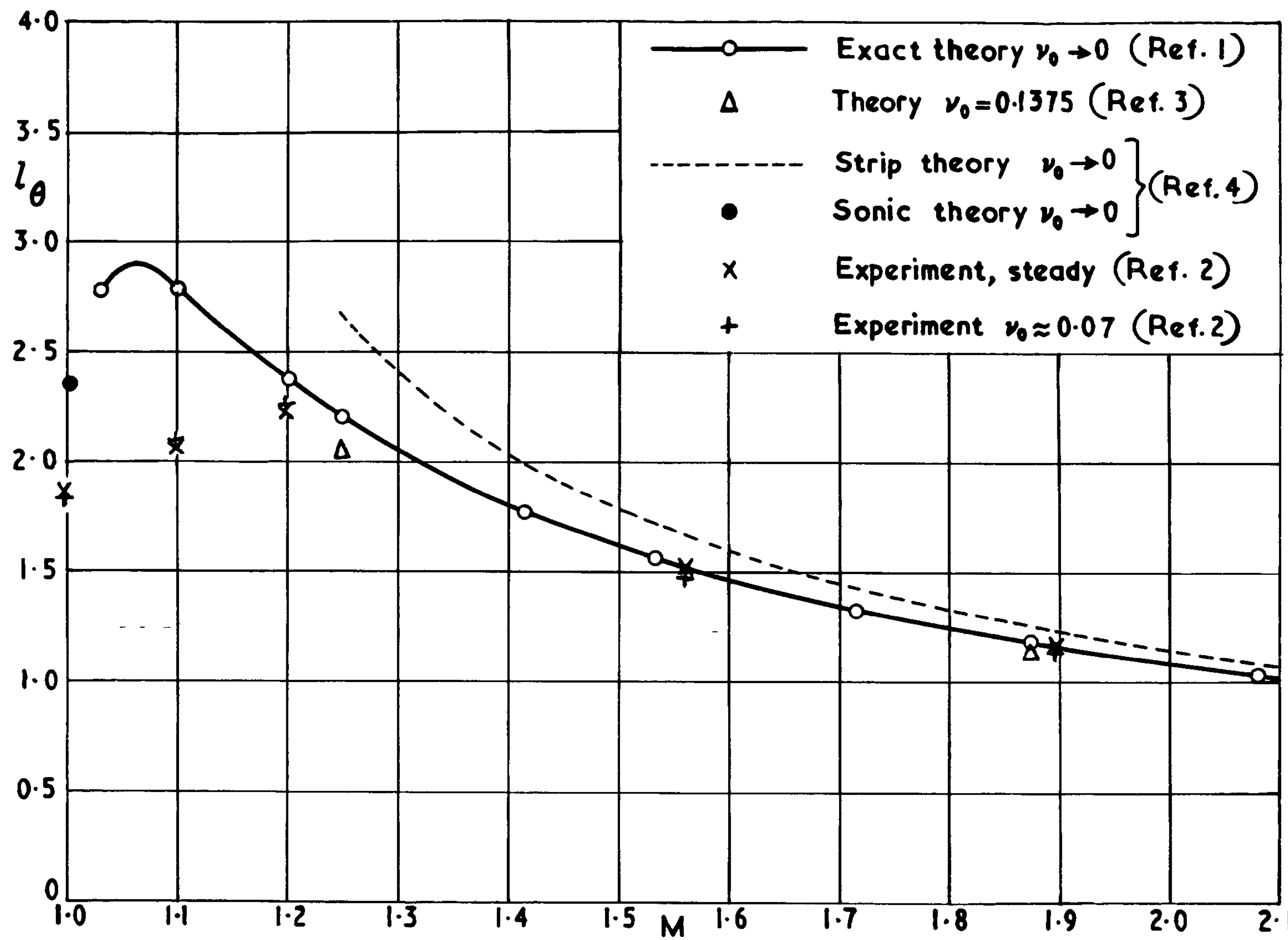
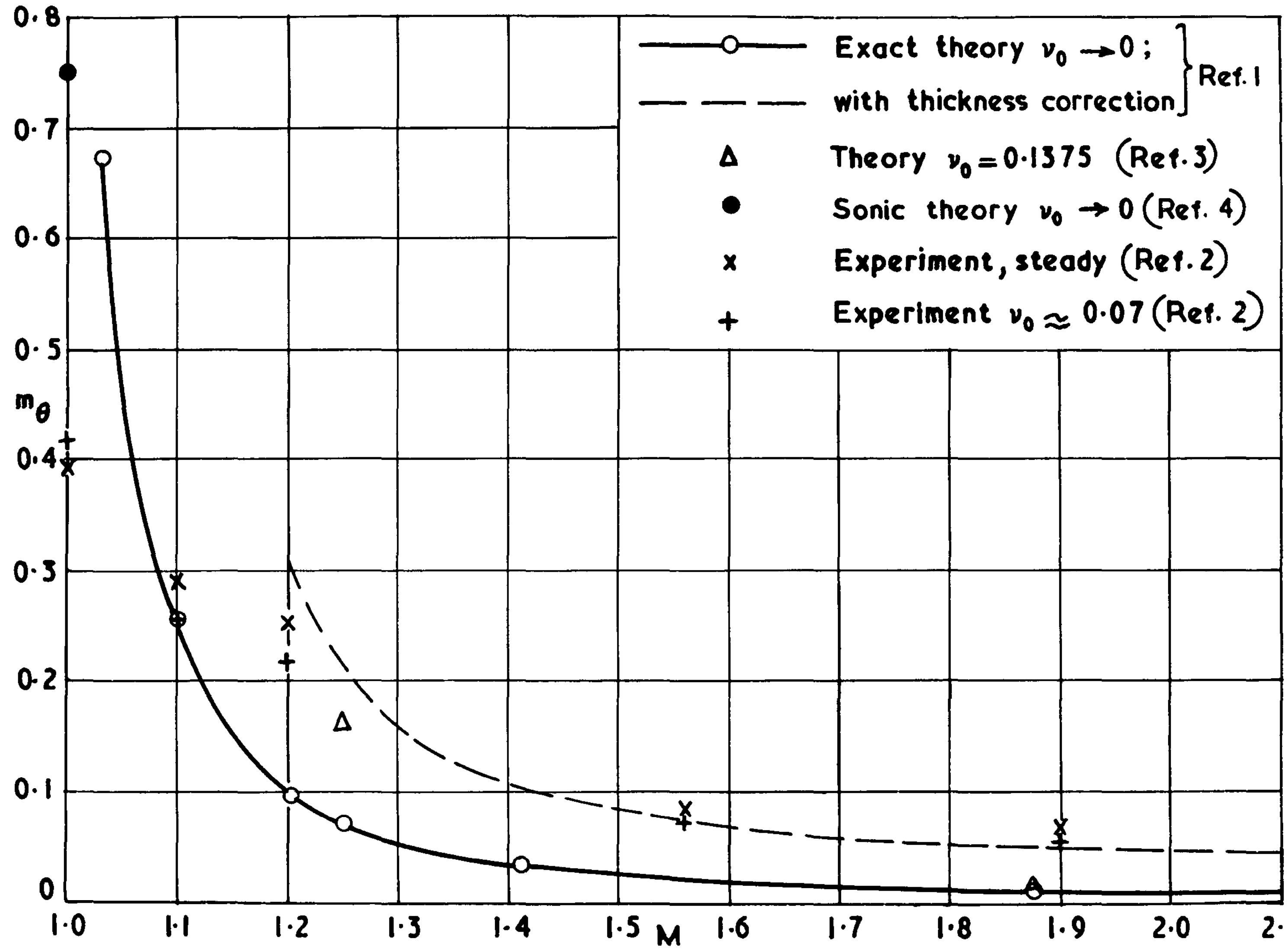


Lift derivative l_θ against Mach number ($h_o = 0{\cdot}5$)

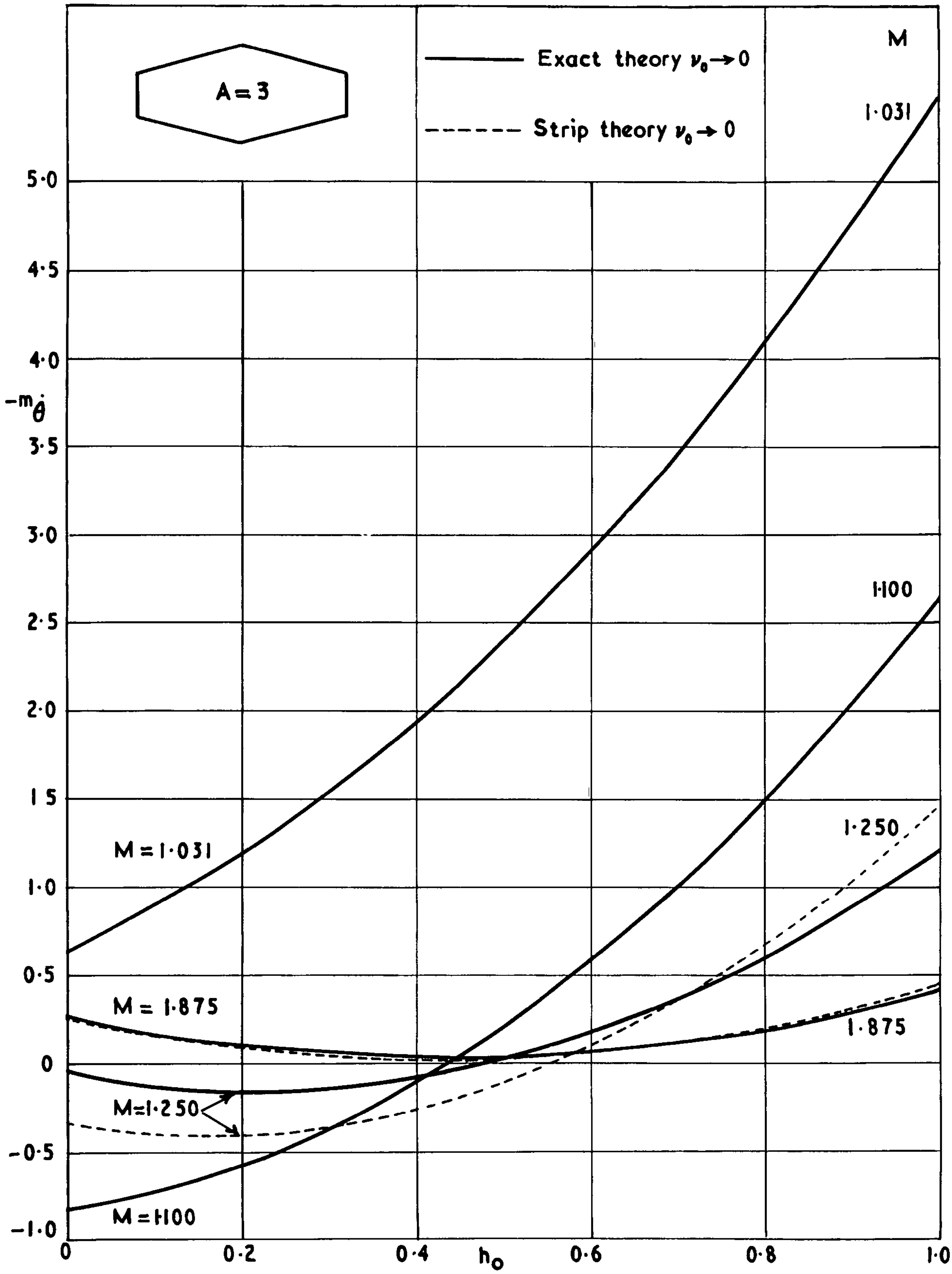
FIG. 1



Pitching moment derivative m_θ against Mach number ($h_0 = 0{\cdot}5$)

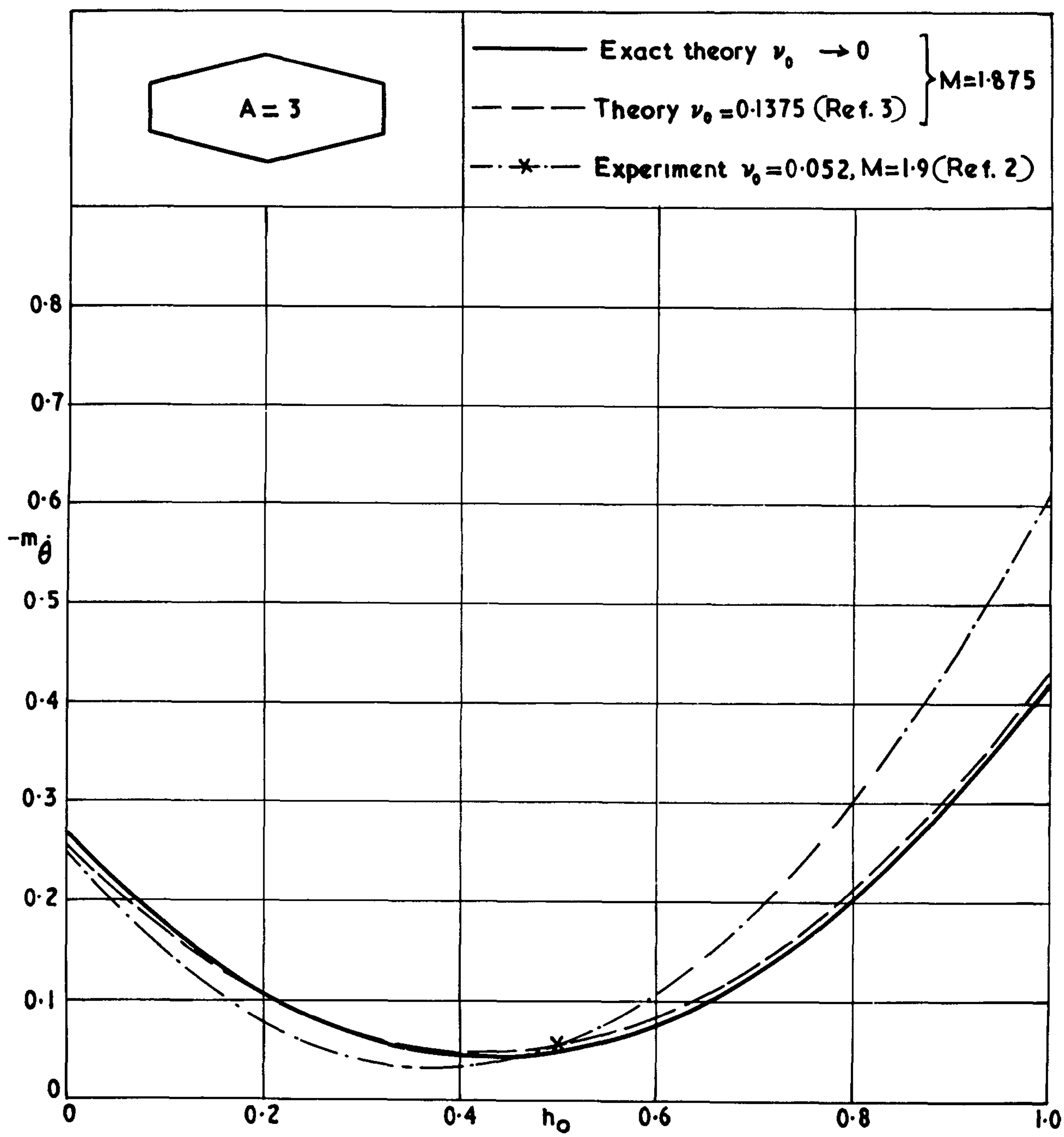
FIG. 2

FIG. 3



Effect of Mach number on the pitching damping $-m_{\dot{\theta}}$ against h_0

FIG.4



Pitching damping $-m_{\dot\theta}$ against h_o at M = 1·875

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