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by

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VERTICAL ACCELERATIONS DUE TO STRUCTURAL VIBRATIONS OF A SLENDER AIRCRAFT FLYING IN CONTINUOUS TURBULENCE

by

J. K. Zbrozek

SUMMARY

A numerical study is made of the vertical accelerations due to structural vibrations of a slender aircraft flown in continuous turbulence. The basic aircraft configuration considered is that used in previous aeroelastic studies, but the analysis is extended to cover the effects of aircraft size, airframe stiffness, wing loading, lift slope and speed and altitude of flight.

Only the first structural mode is studied in some detail, but the order of the effects of the rigid body mode and higher structural modes are investigated also.

It is concluded, that for a configuration approximating to a future supersonic transport design, the vertical vibrations should be quite acceptable for passengers but, in some conditions may impair crew efficiency.

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1 INTRODUCTION

The aeroelastic properties of a slender aircraft, a configuration envisaged for the supersonic transport aircraft, have already received considerable attention[1-10]. The effects of aeroelasticity on this aircraft's static and manoeuvre margins, elevator power, flutter characteristics etc. have been studied to some extent and after an initial and rather confused period, those properties are now reasonably well understood. It is believed, that if the aeroelasticity is seriously considered in the design stage, it should not lead to any special difficulties, or weight penalties. Also a recent, preliminary study of the aeroelastic effects of gust loads[10] has shown that this configuration should not be worse, and might be even better than more conventional aircraft of a comparable size.

One aspect of the aeroelastic properties of a slender aircraft which has not as yet received any consideration, is structural vibration, and the effect of the vibrational characteristics on passenger and crew comfort.

The normal modes and frequencies of vibration, of a slender aircraft are not dissimilar from the corresponding modes of a conventional wing, except that the modes of a slender aircraft involve mainly lengthwise bending, while the modes of a conventional unswept wing are predominantly in a spanwise direction. Also the aerodynamic loading is predominantly distributed along the length of a slender aircraft, and along the span of a conventional wing. Thus, the slender wing might be looked upon as a conventional wing, but flying "sideways", and hence, the vibrational properties of a slender aircraft in turbulent air might be compared to those of a conventional wing. It should be noticed however, that the crew is situated very close to the nose of the slender aircraft, in a position corresponding to one situated near the tip of a conventional wing, and we know from experience that the vibrational environment of the wing tip is definitely not suitable for the crew compartment.

In addition it can be argued, that because the direction of flight is along the main vibrational modes of a slender aircraft, the excitation of these modes by the turbulent air should be larger than the corresponding excitation of a conventional wing. Further, for one particular mode, the conventional wing has only one resonant frequency, namely the frequency corresponding to the natural frequency of this particular mode, say f_1 c.p.s., and this resonant frequency will correspond to a resonant turbulent wavelength $\lambda_1 = \frac{V}{f_1}$, where V is the forward true speed in ft sec^{-1}. The slender aircraft however has apparently two "resonant" wavelengths corresponding to one vibrational mode. One, is the self-evident wavelength $\lambda_1 = \frac{V}{f_1}$ corresponding to the natural frequency of this mode f_1, and the other arises from the mode of excitation. This phenomena is illustrated in Fig.1. Let us assume that the slender aircraft is flying through continuous turbulence, which can be regarded as composed of all possible wavelengths. The distribution of energy, or gust velocities, among the wavelengths, is defined by the power spectrum density of turbulence. It can be seen from Fig.1 that the shape of, say, the first fundamental mode is very similar to part of a sinusoidal wave; it is possible therefore, that there is a range of turbulence wavelengths which will "fit best" to the modal shape (and associated lift distribution), and so lead to much larger excitation in this particular wavelength band, than for any other wavelengths. It would then follow that the forcing function of the turbulence for a given mode will have well defined maxima at some particular wavelengths of the order of multiples of the aircraft, length ℓ. Therefore, the response of a particular mode to the atmospheric turbulence, plotted against frequency (or wavelength) e.g. power spectral density of acceleration, may have two peaks, one corresponding to true resonance frequency f_1, and the other corresponding to maximum excitation, say f_{exc}. Assuming that the maximum excitation occurs at a wavelength $\lambda_{exc.} = k \times \ell$, where k is some numerical constant, the second peak frequency is $f_{exc.} = \frac{V}{k \times \ell}$. Hence there will exist some speed where the two apparent resonance frequencies will coincide, $f_{exc.} = f_1$, leading to larger vibrations, than would have been experienced in the same turbulence, if the slender aircraft was flying "broadside", as a conventional wing. This is the same phenomenon, which leads to a large "dynamic overshoot factor" on the gust loads of the order of 3. (Ref.10.)

It can be seen from the above argument that one may expect the vibration amplitudes of a slender aircraft when flown in turbulent air, to be of such magnitude, that they may prove to be prohibitive not only from the point of view of comfort, but even from safety[11] considerations.

The object of this note is to assess theoretically the order of magnitude of the vibrations expected in turbulent air. At this stage there is no point in making extensive and very accurate studies, as we do not know the aeroelastic properties of the actual design. For the purposes of this study, the "Broadbent" aircraft which was used in previous studies,[1,2,4], appears most suitable, because, we know approximately its first few normal modes, and second, the results of this study could be compared with other studies made already on this aircraft.

2 RESULTS OF NUMERICAL STUDIES

2.1 Theory and assumptions

Evaluation of the accelerations due to the first structural mode of a slender aircraft flown in continuous turbulence has been made under the following assumptions.

(i) Each mode, rigid or elastic, can be investigated separately, i.e. there is no coupling between modes.

(ii) Lift distribution can be found assuming piston theory, irrespective of Mach number.

(iii) Atmospheric turbulence is one-dimensional, i.e. only variation of gust velocity along the chord is taken into account.

(iv) Structural damping is neglected.

A more detailed discussion of the assumptions, together with the development of the theoretical approach are given in the Appendix.

The numerical studies were based on the "Broadbent" model of a slender aircraft, because (i) this is the only model for which the normal modes have been computed, (ii) the results of calculations can be compared with other studies made already for that model.

It is known from Ref.10, that the maximum aeroelastic response of the first structural mode occurs at low speeds, of the order of 400 ft sec^{-1}; at the same time the atmospheric turbulence intensity reaches, on average, its maximum at low altitude above the ground. Therefore it is to be expected that from a vibrations point of view, the critical flight regime is low speed, low altitude conditions. It is desirable however to investigate as large a range of forward speeds as practicable, at the same time keeping the dynamic pressure, $\frac{1}{2}\rho V^2$, compatible with a specified design value, say $M = 2{\cdot}2$ at 50,000 ft. Keeping the above considerations in mind, a compromise altitude of 10,000 ft was chosen for the numerical calculations.

The parameters defining the basic aircraft used in calculations are summarised in Table 1.

As the actual supersonic transport design will differ considerably from the aircraft model used in the present study, an attempt is made to generalise the results in such a way that they could, perhaps, be applied to a different but similar configuration.

2.2 Effect of forward speed

The calculations have been made for a range of forward speeds, keeping all other parameters, including the lift slope, constant. The natural frequency of the first fundamental mode, f_1, which defines the structural stiffness, was taken to be 2·14 c.p.s., which enables the results to be compared directly with the results of Ref.10. Fig.6 shows the results in terms of R.M.S. of acceleration at the wing apex, plotted against forward speed, for a turbulence R.M.S. of 1 ft sec^{-1}.

Fig.6a shows the R.M.S. of acceleration due to the first structural mode only, $\sigma_{\ddot{z}_1}$. It can be observed that the maximum of the response is at low speed, some 480 ft sec^{-1}, and decreases with further increase in speed. This trend is directly opposite to that normally expected for the excitation of any mode is proportional to forward speed and thus the response is expected to increase with increasing speed. The trend illustrated in Fig.6a might be typical for any slender design, resulting from "tuning" between the modal shape and turbulence pattern. To illustrate the magnitude of the accelerations due to structural oscillation, the R.M.S. value $\sigma_{\ddot{z}_1}$, is compared in Fig.6b with the R.M.S. of acceleration due to the rigid body response σ_n. For the calculations of σ_n, one degree of freedom, heaving, was assumed, (see Appendix 1, section 3). The neglected pitching degree of freedom can appreciably modify the value of σ_n, and thus the rigid body response values quoted are only an approximate indication of the order of magnitude of the accelerations to be expected. The total acceleration R.M.S. at the wing apex is shown also and is computed $\left(\sigma^2_{\ddot{z}_1} + \sigma^2_n\right)^{\frac{1}{2}}$ i.e. it is assumed that there is no interaction between heaving and the structural mode *. It can be demonstrated that in this particular case this assumption results in relatively small errors.

It can be seen from Fig.6b that at low speeds the accelerations at the wing apex (pilot's cockpit) originate almost entirely from structural oscillations. Of course in other positions along the aircraft axis this contribution decreases rapidly, being zero at nodal points and on average some 15% of the apex value in the neighbourhood of the passenger cabin.

Fig.7 shows the spectral densities of vertical accelerations due to combined rigid body and first structural modes for two forward speeds and two positions along the aircraft axis, $\xi = 0$ and $\xi = 0{\cdot}7$. The two positions, $\xi = 0$ and $\xi = 0{\cdot}7$, may represent crew cabin, and passenger compartment or aircraft C.G., respectively. The curves of Fig.7 actually refer to an aircraft half the size of the "Broadbent" aircraft, i.e. $\ell = 113{\cdot}4$ ft, which is more representative of current ideas. It can be seen that, at the aircraft nose, Fig.7a, and especially at the lower speed, the peak due to structural oscillations is very large and contributes far more to the general level of oscillations than the rigid body mode. In the passenger compartment, $\xi = 0{\cdot}7$, and at higher speed, Fig.7b, the spectral densities of acceleration look quite conventional. To illustrate the last point, measured spectral densities of acceleration for a Comet aircraft are shown in Fig.8. It can be seen that there is a great similarity between the measured Comet spectrum at the aircraft C.G. and the computed spectrum for a slender aircraft at $\xi = 0{\cdot}7$ and high speed, indicating that in the passenger cabin area of a slender aircraft, the contribution of structural vibrations to the overall accelerations level should not be dissimilar from that of a conventional aircraft.

The significance of the magnitude of accelerations predicted for a slender aircraft is discussed in section 3.2.

2.3 The effect of airframe stiffness

The effect of airframe stiffness on turbulence induced accelerations was investigated by repeating the calculations for two additional natural frequencies of the first structural mode, i.e. 1·5 c.p.s. and 2·5 c.p.s.; other parameters were kept unchanged.

The results of the calculations are summarised in Table 2, and plotted in terms of acceleration R.M.S. against forward speed, in Fig.9. It can be seen that the airframe stiffness has a very large effect on the structural accelerations in turbulent air.

Theoretical considerations, (Appendix 1, section 4,) suggest that the R.M.S. of structural accelerations should be approximately proportional to forward speed V and inversely proportional to the root of damping ratio ζ_1,

* This expression for R.M.S. response takes account of phase relations between the modes only in an average sense but it is thought to be a reasonable assumption for this treatment of the problem.

for a given value of reduced frequency $\Omega_1 \ell$ (expression A.26). To check this approximation, and also in an attempt to generalise the numerical calculations, a "reduced" R.M.S. of acceleration

$$\frac{\sigma_{\ddot{z}}}{V}\sqrt{\zeta}$$

is plotted against reduced frequency $\Omega_1 \ell$ in Fig.10.

The remarkable collapse of the computed reduced acceleration R.M.S's, as shown in Fig.10, suggests that the theoretical approximate relationship is valid, at least for the range of parameters considered.

2.4 The effect of airframe size

The overall length of the aircraft used as a model in the present calculations is 226·8 ft. The actual Anglo-French supersonic transport design is expected to be considerably smaller, and it was thought worthwhile to extend the calculations to a smaller size. A half scale of the "Broadbent" aircraft has been assumed, i.e. ℓ = 113·4 ft, other parameters remaining unchanged. Calculations have been made for one airframe stiffness only, and the natural frequency of the first structural mode was chosen to be f_1 = 2·5 c.p.s., the highest frequency of the previous set of calculations, more appropriate, perhaps, to the smaller airframe.

The spectral density of the "acceleration producing input", Φ_1, was recalculated for the new length, and is shown in Fig.4. Actually, the computation of Φ_1 for $\ell_2 = \frac{1}{2}\ell_1$ required only shifting the original curve plotted on log-log scale along the Ω axis by log 2 and lifting it by log 4.

The results of the computations are shown in Fig.11 plotted against forward speed. The accelerations due to structural vibrations are considerably smaller for the half-size airframe, except at impracticably low speeds.

An attempt was made again to collapse the computed results into one curve, taking into account airframe stiffness and damping, forward speed and airframe size (Appendix 1, expression A.28).The reduced acceleration response due to 1 f.p.s. turbulence R.M.S. in the form:

$$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\frac{\zeta}{\ell}} \quad \sec^{-1} \mathrm{ft}^{-\frac{1}{2}}$$

is plotted against reduced frequency, $2\pi \frac{f_1 \ell}{V}$, in Fig.12. It can be seen that the results for the smaller airframe do not collapse with the previous results into a unique curve. However the collapse is probably good enough for the study of trends.

2.5 The effects of wing loading, lift slope and altitude of flight

It was shown in section 2.4 that the structural mode accelerations can be approximately expressed in terms of forward speed V, damping ratio of the mode, ζ and also of aircraft length, ℓ. It is also shown in Appendix 1, section 4, that the damping ratio can be approximately expressed in terms of aircraft parameters, and that the R.M.S. value of the structural accelerations varies in the manner,

$$\sigma_{\ddot{z}_1} \sim \left[\frac{g\rho a}{W/S} V \cdot \ell \cdot f_1\right]^{\frac{1}{2}} . \tag{A.30}$$

To check the validity of expression (A.30) the computed values of $\sigma_{\ddot{z}_1}$, ft sec^{-2}, were reduced by dividing them by the right hand side of equation (A.30) and plotting against reduced frequency $\Omega_1\ell = 2\pi f_1\ell/V$, Fig.13. It can be seen that the formula (A.30) gives a very good collapse of computed points, predicting the effect of aircraft parameters on the acceleration response, except, perhaps, the effect of aircraft size. For comparison, the modulus of the integral, $|I(\Omega\ell)|$*, is plotted in the same Fig.13. The curve of reduced acceleration R.M.S's follow quite closely the shape of the integral $|I(\Omega\ell)|$. The derivation of the expression (A.30) took into account the dynamic properties of the airframe and of atmospheric turbulence, except the effect of integral $|I(\Omega\ell)|$, thus the similarity of the curves of reduced acceleration and integral $|I(\Omega\ell)|$ in Fig.13 suggests that the expression (A.30) might be valid for a quite wide range of aircraft parameters, as long as the resonant frequency $\Omega_1 = \omega_1/V$ stays within the $1/\Omega^2$ range of turbulence spectrum.

For a given value of $\Omega_1\ell$, the expression (A.30) in conjunction with Fig.13 defines approximately the effects of altitude, ρ, lift slope a, wing loading W/S, forward speed, V, aircraft size, ℓ, and airframe stiffness, f_1, on the value of accelerations R.M.S. due to structural vibration of the first structural mode in continuous turbulence.

On the basis of expression (A.30) an interesting observation can be made about the effect of altitude on airframe vibration accelerations. It has been suggested in Ref.14 that on the average, the atmospheric turbulence intensity expressed as true, not indicated, gust velocities is approximately independent of altitude, for altitudes above say 5000 ft. Therefore, assuming the true gust velocities invariant with altitude, a simple relationship between acceleration R.M.S. at altitude H and acceleration R.M.S. at sea level can be obtained using expression (A.30). In fact for a flight at constant indicated speed, thus constant C_L, the ratio of acceleration R.M.S. at height, H, to that at sea level can be written:

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* Integral $|I(\Omega\ell)|$ is proportional to the first structural mode excitation function due to turbulence at non-dimensional frequency, $\Omega\ell = 2\pi\,\ell/\lambda$, see Appendix 1, section 2.2.

$$\frac{\left(\sigma_{\ddot{z}_1}\right)_H}{\left(\sigma_{\ddot{z}_1}\right)_{S.L.}} = \left(\frac{\rho}{\rho_o}\right)^{\frac{1}{4}} .$$

The above relationship is valid outside the earth boundary layer turbulence i.e. for altitudes in excess of the first few thousand feet. It might be observed that for altitudes up to some 60,000 ft, the air densities ratio can be expressed approximately as:

$$\left(\frac{\rho}{\rho_o}\right)^{\frac{1}{4}} \simeq 1 - \frac{H}{135,000} ;$$

with an error of less than 1%.

2.6 The contribution of other modes to the level of vibrations

The present calculations of accelerations due to structural vibrations in turbulent air were made for the first fundamental mode only. It is usually believed that the contribution of that mode exceeds by far the contribution from other modes. The slender aircraft is however a new species in the aero-elastic family of aircraft and it would be advantageous to assess the order of contribution from other modes to the overall level of vibrations. This is possible, as a consistent, though approximate set of normal modes have been computed by Smith[5] for an aircraft similar to the model used in the present calculations except for a small modification in planform. The modal shapes and their frequencies are shown in Figs.14 to 17 for the first four symmetrical modes. It can be seen that the first, second and even fourth modes contain very little spanwise bending; only the third mode is predominantly a tip bending. Where spanwise bending is absent, the configuration can be described as "structurally slender", and is one to which the approximate analysis taking account only of longitudinal bending, accurately applies.

Fig.18 shows a comparison between the shape of the centre line of the first mode from Fig.14[5] and the analytical approximation used in the present study. It can be seen that the agreement is not much worse than in Fig.2, where the analytical approximation was compared with the modal shape obtained from Ref.4. The frequency of the fundamental mode is $3{\cdot}38\sqrt{\tau}$ c.p.s.[5], compared with $3{\cdot}5\sqrt{\tau}$ c.p.s.[4], where τ is the skin thickness in inches. It is concluded therefore that the set of modes shown in Figs.14 to 17 is, for practical purposes, consistent with the first fundamental mode used in the present study. The primary object of this paragraph is to assess the relative magnitudes of accelerations due to structural modes in relation to the first fundamental mode, which has already been discussed in some detail. For that purpose, some simple, approximate, easy to calculate analytical approach should suffice. It is shown in Appendix 1, section 5, that under certain simplifying assumptions, the acceleration response R.M.S. of each mode, $\sigma_{\ddot{z}_i}$, is proportional to:

$$\frac{\int_0^1 \int_0^1 w_i \, d\eta \, d\xi}{\int_0^1 \int_0^1 w_i^2 \, d\eta \, d\xi} \qquad \text{(A.32)}$$

The integration is now along the chordwise and spanwise axes (ξ and η respectively) as some of the modes contain considerable spanwise deflections. The above integrals were evaluated and the results are presented in Table 3, below.

TABLE 3

Acceleration response

Relative contribution of higher modes at $\xi = 0$, $\eta = 0$

Mode No.	$\int_0^1 \int_0^1 w_i \, d\eta \, d\xi$	$\int_0^1 \int_0^1 w_i^2 \, d\eta \, d\xi$	$\sigma_{\ddot{z}_i} \times$ const.	$\frac{\sigma_{\ddot{z}_i}}{\sigma_{\ddot{z}_1}}$	$\frac{(\sigma_{\ddot{z}})_{TOT}}{\sigma_{\ddot{z}_1}}$	f_i/f_1
1	$3 \cdot 82 \times 10^{-2}$	$5 \cdot 09 \times 10^{-2}$	0·752	1·000	1·000	1·00
2	$0 \cdot 92 \times 10^{-2}$	$3 \cdot 16 \times 10^{-2}$	0·291	0·387	1·073	2·16
3	$-4 \cdot 55 \times 10^{-2}$	$9 \cdot 49 \times 10^{-2}$	0·479	0·637	1·247	3·73
4	$0 \cdot 31 \times 10^{-2}$	$0 \cdot 88 \times 10^{-2}$	0·355	0·473	1·333	4·17

It can be seen that at the wing apex, none of the modes is likely to contribute as much as the first fundamental mode. Including the first four modes increases the value of the R.M.S. acceleration, $(\sigma_{\ddot{z}})_{TOT}$, by some 33%, when compared with the accelerations due to the first mode only.

The level of vibrations in the neighbourhood of the passenger cabin due to the first structural mode is considerably less than in the aircraft nose, so it was interesting to assess the contribution of other modes to the vibration at a representative point, chosen to be $\xi = 0 \cdot 7$, $\eta = 0$. By simple scaling of the results of Table 3 by appropriate modal deflections, the following results were obtained:

TABLE 4

Relative contribution of higher modes at $\xi = 0{\cdot}7,\ \eta = 0$

Mode No.	$\sigma_{\ddot{z}_i} \times$ const.	$\dfrac{\sigma_{\ddot{z}_i}}{\sigma_{\ddot{z}_1}}$	$\dfrac{(\sigma_{\ddot{z}})_{TOT}}{\sigma_{\ddot{z}_1}}$	f_i/f_1
1	$10{\cdot}18 \times 10^{-3}$	1·000	1·000	1·00
2	$2{\cdot}62 \times 10^{-3}$	0·258	1·033	2·16
3	$12{\cdot}5 \times 10^{-3}$	1·23	1·605	3·73
4	$0{\cdot}41 \times 10^{-3}$	0·04	1·605	4·17

It can be seen that only the 3rd mode contributes appreciably to the total level of vibrations. At this particular point along the structure ($\xi = 0{\cdot}7$, $\eta = 0$) the contribution of the 3rd mode appears to be larger than the contribution of the first fundamental.

It should be emphasised that these estimates of the vibration level due to modes other than the first are essentially qualitative. Nevertheless one is forced to a conclusion that a mode involving, large, though local deflection of the lifting surface (3rd mode, Fig.16) may contribute considerably to the overall level of vibrations and thus cannot à priori be neglected.

3 DISCUSSION

3.1 The range of application of the present study

The present study of vibrations of a slender aircraft when flown in turbulent air covers a limited range of parameters. The model used for the study is a slender,integrated delta aircraft of overall length of 226·8 ft; the full calculations have been made for one altitude, 10,000 ft, one value of lift slope, a = 2 and for one structural mode only.

Because the aircraft model used for the present study differs considerably from the current supersonic transport design, an attempt has been made to extend the calculations to cover a range of parameters. It was shown, in Figs.10, 12 and 13 that the results obtained can be applied to other altitudes, lift slopes, wing loadings and structural stiffness and approximately to other aircraft sizes. The only parameter which was unchanged was the generalised gust forcing function, Fig.3. This function is an integral of a product of the modal shape, of turbulence amplitude distribution and of associated lift distribution. In the calculations, the simplest aerodynamic theory, namely piston theory, was used. This theory, without doubt, overestimates the generalised gust forcing function.

Strictly speaking, the results of the present note can be applied to another design only if the gust forcing function is the same, which is very unlikely in practice. Nevertheless it is believed, that once the structural and aerodynamic properties of a new, similar design are known, the results can be approximately corrected for a new gust forcing function, and thus the order of accelerations due to structural vibrations can be estimated from available results without resorting to rather lengthy numerical calculations. If such rough and ready estimates indicate that vibrations might be of dangerous amplitude, then of course, more exact calculations will have to be made.

The present study also assesses the order of magnitude of the contributions of modes higher than the first fundamental mode. This assessment can probably be used for another design, at least to establish the order of magnitudes, as long as the modal shapes of a new design are not too widely different from the shapes assumed in this work.

3.2 Numerical example of application - crew and passenger comfort

It has been shown that the magnitude of the structural vibrations depends very markedly on aircraft speed, size and stiffness, and also on modal shape, lift distribution, wing loading, overall lift slope and altitude of flight. Therefore, it is impossible, at the present time, to give estimates of vibrational environment for the actual supersonic transport design. However we can study this problem on a fictitious aircraft of a configuration nearer to the actual design than the basic, "Broadbent" aircraft.

Let us make the following assumptions.

(i) The generalised gust forcing function is the same as used in this study, Fig.3. It is believed that the actual SST design will have a wing of much shorter chord than that used here, thus the excitation due to turbulence will be smaller. However the long, overhanging fuselage may produce deflections near the nose of larger amplitude, thus it is not inconceivable that the vibrations in pilot's cockpit will be similar to those based on the present model. On the basis of the same argument, the vibrations in the passenger compartment should be smaller.

(ii) The other parameters defining the S.S.T. aircraft are as follows:

H =	1000 ft	altitude
ℓ =	150 ft	aircraft length
W/S =	50 lb ft^{-2}	wing loading, light conditions
a =	3·4	aircraft lift slope; low speed condition
f_1 =	2 c.p.s.	frequency of the first structural mode.

Referring to Fig.13; the maximum "reduced" response will occur at reduced frequency, $2\pi f_1 \ell/V$ of about 6·7, which for assumed values of f_1 and ℓ gives the forward speed for maximum "reduced" response of 281 ft sec^{-1}.

As the actual, non-reduced, response depends on speed, the maximum response will occur at speed slightly higher than 281 ft sec^{-1}. Let us assume then that the forward speed is $V = 300$ ft sec^{-1}, 178 kts true or 175 kts indicated, probably the lowest indicated speed at which this aircraft would normally fly. The value of the reduced frequency at 300 ft sec^{-1} is then 6·28 and the reduced response from Fig. 13 is

$$\sigma_{\ddot{z}_1} \left[\frac{W/S}{g\rho a} \quad \frac{1}{V\ell f_1} \right]^{\frac{1}{2}} \simeq \quad 6{\cdot}05 \times 10^{-2} \quad . \qquad (3.1)$$

Substituting the appropriate values for aircraft parameters into expression (3.1), the value of R.M.S. of acceleration at the wing apex is obtained:

$$\sigma_{\ddot{z}_1} = 1{\cdot}29 \text{ ft sec}^{-2} \simeq 4 \times 10^{-2} \text{ in } g \text{ units} . \qquad (3.2)$$

Adding 33% for the contribution of other elastic modes the total value of the R.M.S. of vertical acceleration at the wing apex due to 1 ft sec^{-1} turbulence R.M.S. is about:

$$(\sigma_{\ddot{z}})_{TOT} \simeq 5{\cdot}3 \times 10^{-2} \text{ in } g \text{ units at } \xi = 0 \ ;$$

with a predominant frequency at 2 c.p.s.

The rigid body contribution, being at this speed about $0{\cdot}5 \times 10^{-2}$ in g, can be neglected.

Performing similar calculation for the point $\xi = 0{\cdot}7$, representing the passenger cabin, we obtain:

$$(\sigma_{\ddot{z}})_{TOT} \simeq 1{\cdot}4 \times 10^{-2} \text{ in } g \text{ units } \quad \xi = 0{\cdot}7 \ ;$$

with two predominant frequencies, 2 c.p.s. and about 7 c.p.s.

In order to appreciate the significance of the values of R.M.S. of vertical acceleration calculated above, we have to specify both the expected intensity of atmospheric turbulence and the acceptable limits of the vibrational accelerations.

Using data from Ref.13, the following values for turbulence intensity at low altitude can be assumed:

average turbulence $\sigma_{w_g} = 3{\cdot}8$ ft sec^{-1}, wind speed 10-15 kts.

heavy turbulence $\sigma_{w_g} = 7{\cdot}0$ ft sec^{-1}, wind speed ~ 30 kts.

The above values apply to heights of the order of 200 ft above the terrain, corresponding to the last phase of landing approach.

Using the above values of atmospheric turbulence intensity, the values of acceleration R.M.S. have been estimated and are tabulated below.

TABLE 5

Estimated levels of vibration for an S.S.T. design

Turbulence	R.M.S. of vertical acceleration in "g" $V = 300$ ft sec^{-1}, $H \sim 1000$ ft	
	Pilot's cabin $\xi = 0$	Passenger cabin $\xi = 0{\cdot}7$
average	0·20 g	0·053 g
heavy	0·37 g	0·098 g

The endurance limits of the human body to vertical vibrations has been studied extensively in connection with land transport, though the available data are not readily applicable to aircraft. A good summary of vibration studies applicable to ground vehicles can be found in Ref.15, from which it can be deduced that at 2 c.p.s., amplitudes of 0·08 to 0·16 g are at the upper limit of comfort and amplitudes in excess of 0·5 - 0·6 g are intolerable.

Flight experience with a rigid aircraft at high speed[14] suggests the view that R.M.S. values of vertical accelerations of the order of 0·1 g can be regarded as "slight", 0·2-0·3 g as "heavy" and above 0·3 g as "very heavy" and probably unacceptable.

Notess, Ref.11, suggests boundaries of the time of exposure against intensity of vibrations and some are reproduced from that reference in Fig.19.

The vibrational tolerance data from all three sources are reasonably consistent, and remembering the dislike of the human body to frequencies around 4·5 c.p.s.[14], a rough boundary of the vibrational tolerance can be stipulated. For the purpose of the present study let us define these boundaries as follows: for the duration of a few minutes, vibrations with R.M.S. values up to 0·3 g can be tolerated, probably without any deterioration in pilot's performance, although they will be objectionable to passengers. Vibrations with R.M.S. values of 0·5 g define the upper boundary, beyond which a pilot may not be able to operate satisfactorily.

Referring to Table 5, we can see that at approach speeds, when the vibrational environment is expected to be worst, the comfort of passengers should be satisfactory even in heavy turbulence (0·1g R.M.S.). The environment of the pilot however is not so satisfactory. In heavy turbulence, the R.M.S. of normal accelerations may well exceed the stipulated limit of 0·3 g, and thus may impair his capacity for handling the aircraft in this critical manoeuvre.

Increasing the approach speed, at least in the case considered in here, is doubly beneficial, as not only does it decrease the handling difficulties in turbulent air, but it also reduces the level of vibrations experienced by the pilot.

4 CONCLUDING REMARKS

A study has been made of the aeroelastic response to turbulent air of a slender aircraft, the responses of main interest being the vertical accelerations experienced by passengers and crew.

The "Broadbent" aircraft was used, as a basic model, as this is the only configuration for which the elastic properties are sufficiently known. An attempt was made to generalize the results, so that the effects of forward speed, airframe stiffness, wing loading, lift slope, flight altitude and of aircraft size can be investigated.

It has been shown, that for a slender aircraft configuration, there exists a forward speed for which the accelerations due to vibrations in first structural mode are maximum. Increasing or decreasing speed from this critical speed decreases considerably the amplitudes of vibrations in contrast to a conventional aircraft.

The effects of the rigid body and 3 higher symmetric structural modes were approximately evaluated by further calculations.

The results have been applied to an aircraft with parameters similar to those of possible SST design, and the intensity of vibrations for this aircraft estimated. For this particular aircraft, the speed for the maximum vibrational response in the first structural mode (2 c.p.s.) is very low being about 180 kts. It should be realized that reducing aircraft size and airframe stiffness reduces also the speed for maximum response. For example assuming the aircraft length to be 120 ft, and structural frequency 1·6 c.p.s. the speed of maximum response is about 100 kts and is thus outside the flight envelope.

The estimated values of vibration levels indicate that although passenger comfort should not be noticeably affected by structural vibration, the vibrations in the pilot's cabin are of some concern. In heavy turbulence, the estimated R.M.S. value of vertical accelerations in the pilot's cabin is in excess of 0·3 g which, it is thought, may affect pilots efficiency, already strained by the handling difficulties in turbulent conditions.

It should be emphasied that the present calculations indicate only trends and the order of the vibration intensity on the actual SST design. Relatively small changes in the assumed values of parameters, such as airframe stiffness, may considerably modify the whole picture.

Nevertheless, the present results should serve as a warning, that the vibrational properties of the SST design, may, in some circumstances, affect the operational utilisation of the aircraft.

It is strongly advised, that calculations similar to those on the present paper, be made on the actual design as soon as the required structural information is available.

The present study discusses only the vertical accelerations. It is known that the lateral accelerations play a very important part in defining the acceptable environment of passengers and crew. A study of lateral vibrations should be made as soon as the necessary information is available.

LIST OF SYMBOLS

a — lift slope

$$A = \int_0^1 \cos \Omega \ell \xi \times w_1 \times \xi \, d\xi$$

$$B = \int_0^1 \sin \Omega \ell \xi \times w_1 \times \xi \times d\xi$$

integrals defining excitation of the structural mode due to sinusoidal gust

$b = \frac{1 \cdot 774}{\mu_g} \frac{V}{\ell}$ — damping coefficient in the characteristic equation of the first structural mode sec^{-2}

$c = \omega_1^2 + \frac{3 \cdot 84}{\mu_g} \left(\frac{V}{\ell}\right)^2$ — stiffness coefficient in the characteristic equation of the first structural mode, sec^{-1}

$d = \frac{34 \cdot 88}{\mu_g} \frac{V}{\ell}$ — coefficient defining the gust excitation of the first structural mode, sec^{-1}

f_i — undamped natural frequency of the ith mode, c.p.s.

EI — stiffness (equal to modulus of elasticity × moment of inertia)

g — acceleration due to gravity, ft sec^{-2}

$G(\Omega)$	spectral density of atmospheric turbulence
H	altitude, ft
$\lvert I(\Omega\ell)\rvert = [A^2 + B^2]^{\frac{1}{2}}$	modulus of the integral defining the excitation of the structural mode due to sinusoidal gust
k	a constant
K_{SP}	spectral gust alleviation factor
K_{α}	incidence function
ℓ	aircraft length, ft, in this paper being equal to wing centre chord
L	turbulence scale, ft
M_1	generalised mass
$m(\xi)$	mass per unit length of aircraft, slugs
S	wing area, ft^2
t	time, sec
V	forward speed, true, ft sec^{-1}
W	aircraft weight, lb
w_g	gust velocity, ft sec^{-1}
$w_i(\xi,\eta)$	normalised function defining ith modal shape, $w_i(0,0) = 1$
$Z_i(\xi,\eta,t) = w_i(\xi,\eta) \times z_i(t)$	deflection in ith mode
$z_i(t)$	time dependent deflection of ith mode
α	incidence
ζ_i	damping ratio of ith structural mode (Appendix 1, expressions A.16a and A.16b)
η	non-dimensional spanwise coordinate
$\lambda = \frac{2\pi}{\Omega}$	turbulence wavelength, ft
$\mu_g = \frac{2\,W/S}{g\rho a \ell}$	mass parameter

LIST OF SYMBOLS (Contd.)

ξ	non-dimensional chordwise coordinate
ρ	air density, slug ft^{-3}
σ	R.M.S. value of given quantity specified by subscript
τ	skin thickness, inches
Φ	spectral density of a quantity specified by subscript
$\Phi_1(\Omega\ell)$	spectral density of "acceleration producing input" for first structural mode
$\Omega = \frac{\omega}{V} = \frac{2\pi}{\lambda}$	space frequency of turbulence, ft^{-1}
ω	frequency, sec^{-1}
$\omega_i = 2\pi f_i$	undamped natural frequency of ith structural mode in vacuum, sec^{-1}
ω_i^*	undamped natural frequency of ith structural mode including aerodynamic stiffness, sec^{-1} (Appendix 1, expressions A.16a and A.16b)

Subscripts

z_i	pertaining to deflection of ith mode
$\ddot{z}_i$	pertaining to acceleration of ith mode
n	pertaining to acceleration in rigid body mode
i	pertaining to ith structural mode
TOT	total, sum of first ith components

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APPENDIX

THEORY

1 THEORETICAL APPROACH

The complete linear equations of motion for an aircraft with many degrees of freedom due to vertical gust velocity w_g can be written as:

$$[A]\{\ddot{q}\} + [B]\{\dot{q}\} + [E]\{q\} + [C]\{q\} = [F]\{w_g\} \qquad (A.1)$$

where [A] is a matrix of inertia coefficients

[B] " aerodynamic damping coefficients

[C] " aerodynamic stiffness coefficients

[E] " elastic stiffness coefficients (may include out of phase components due to internal friction)

[F] " aerodynamic coefficients due to gust

and q_i is a generalised co-ordinate of ith mode.

The solution of (1) requires large computational effort and can only be justified if the inertia, elastic and aerodynamic coefficients are known or can be estimated with reasonable accuracy.

At the present moment no aerodynamic and structural properties of the actual slender aircraft design are known, thus it is not possible to study the vibrational properties of this particular aircraft. To by-pass these difficulties and, at the same time, to retain continuity in the aeroelastic studies, we will use a model as studied originally by Broadbent[1]. Furthermore, to facilitate a comparison with the results of other works[4,10], we will retain the simplicity of aerodynamic piston theory, irrespective of Mach number. Under the above simplifying and crude assumptions, an attempt to obtain the solutions of the full equations (1) is probably not justified and some simple, less laborious method will suffice.

The following general assumptions are made:

(1) Each mode, rigid body or elastic, can be investigated separately i.e. there is no coupling between modes. It was shown in Ref.4 that the frequency and damping of the first structural mode are only slightly altered by the rigid body mode.

(2) In order to simplify the computation, piston theory is used, irrespective of Mach number. It is assumed that the incremental local lift is proportional to the local incidence, either due to structural distortion or due to gust. The constant of proportionality, say a, is the same for all wing stations and equal to the overall lift slope.

(3) Atmospheric turbulence is one dimensional i.e. only the variation of turbulence velocities along the flight axis will be considered. This is equivalent to an assumption that the aircraft span is small comparing with turbulence scale. This assumption can be justified when remembering that the span of the aircraft in question is less than 90 ft.

(4) The damping due to friction within the structure is neglected.

(5) The vibrations due only to one elastic mode will be studied in some detail. It can be shown, that in general, the level of vibrations is defined mainly by response of the first fundamental mode. The order of the effect of other modes on the vibrational environment will be investigated also.

The parameters defining the basic aircraft used in numerical studies are shown in Table 1. The numerical values of these parameters are those used in previous aeroelastic studies[1,2,4,10]. The planform, modal shape, mass and stiffness distributions are illustrated in Fig.2. The values of the integrals defining the generalised mass and generalised aerodynamic forces for the first fundamental mode are also presented in Table 1.

2 RESPONSE OF THE FIRST FUNDAMENTAL MODE

2.1 Equation of motion

The equation of motion for ith fundamental mode alone and neglecting structural damping in response to vertical gust w_g is:

$$M_i \ddot{z}_i + \left[\tfrac{1}{2}\rho V a \ell^2 \int_0^1\int_0^1 w_i^2 \, d\eta \, d\xi\right] \dot{z}_i + \left[M_i \omega_i^2 + \tfrac{1}{2}\rho V^2 a \ell \int_0^1\int_0^1 \frac{dw_i}{d\xi} w_i \, d\eta \, d\xi\right] Z_i =$$

$$= \tfrac{1}{2}\rho V a \ell^2 \int_0^1\int_0^1 w_g w_i \, d\eta \, d\xi ; \qquad \ldots\ldots(A.2)$$

where, in general, the gust velocity w_g is a function of non-dimensional spanwise, η, and chordwise ξ coordinates and of time t.

The modal displacement, w_i, is also in general a function of η and ξ coordinates.

M_i is a generalised mass in ith mode defined as:

$$M_i = \int_0^1\int_0^1 m(\eta,\xi) \, w_i^2 \, d\eta \, d\xi \qquad (A.3)$$

where $m(\eta,\xi)$ is a mass distribution.

The equation (A.2) is derived under an assumption that the deflection of structure, in ith mode, say $Z_i(\eta,\xi,t)$ can be split into time dependent, $z_i(t)$, and time independent, modal shape, $w_i(\eta,\xi)$ parts. It follows that:

$$Z_i(\eta,\xi,t) = w_i(\eta,\xi)\; z_i(t) \quad . \tag{A.4}$$

In the application to the first fundamental mode the equation (A.2) can be simplified. It is assumed that for this mode the spanwise deflections can be neglected, thus the modal shape is a function of chordwise coordinate only. The modal shape for the first fundamental mode is shown in Fig.2 and was obtained from Ref.4. To simplify the computation an analytical approximation for w_1 was found:

$$w_1(\xi) = 1 - 2{\cdot}15\,\xi - 2{\cdot}3\,\xi^2 + 4{\cdot}15\,\xi^3 \; . \tag{A.5}$$

Circled points in Fig.2 define the approximated modal shape.

For the triangular planform used in present calculations, Fig.2, the non-dimensional elemental area $d\eta\; d\xi$ can be expressed as follows:

$$d\eta\; d\xi = \frac{1}{\ell^2}\, 2\; S\xi\; d\xi \; . \tag{A.6}$$

Thus for first fundamental mode the equation of motion is expressed in terms of chordwise coordinate ξ only, and substituting equation (A.6) into (A.2) the following equation is obtained:

$$M_1\; \ddot{z}_1 + \left[\rho\, S\, a\, V \int_0^1 w_1^2\; \xi\; d\xi\right] \dot{z}_1 + \left[M_1\; \omega_1^2 + \rho\, S\, a\, V^2 \frac{1}{c} \int_0^1 \frac{dw_1}{d\xi}\; w_1\; \xi\; d\xi\right] z_1 =$$

$$= \rho\, S\, a\, V \int_0^1 w_g\; w_1\; \xi\;\; d\xi \; ; \tag{A.7}$$

Dividing the equation (A.7) by the generalised mass M_1 and substituting the numerical values for integrals as given in Table 1, the equation of motion for first fundamental mode including numerical constants is obtained.

$$\ddot{z}_1 + 0{\cdot}887 \frac{g\rho a}{W/S} V \dot{z}_1 + \left(\omega_1^2 + \frac{4{\cdot}42}{\ell} \frac{g\rho a}{W/S} V^2\right) z_1 = \frac{1}{0{\cdot}0574} \frac{g\rho a}{W/S} V \int_0^1 w_g w_1 \xi \, d\xi . \qquad \ldots (A.8)$$

2.2 Generalised gust force

The right hand side of equation (A.8) gives an expression for first mode excitation force due to turbulence. To compute this force we need to evaluate the integral

$$\int_0^1 w_g w_1 \xi \, d\xi \qquad (A.9)$$

where w_g is a gust velocity. In general the gust velocity is a function of time, t, and η and ξ coordinates, but under the present assumption it is a function of ξ and t only.

For the study of responses to continuous turbulence we need to evaluate the integral (A.9) for a range of frequencies (or wavelengths). For an aircraft flying with forward speed V, the expression for sinusoidal gust of space frequency Ω and amplitude w_{g_0} for any point along its ξ axis can be written:

$$w_g(\xi, t) = w_{g_0} \sin \Omega (\ell\xi - Vt) . \qquad (A.10)$$

The space frequency Ω, is related to circular frequency ω, sec^{-1} or to gust wavelength λ, ft by the following:

$$\Omega = \frac{\omega}{V} = \frac{2\pi}{\lambda} \text{ rad ft}^{-1} . \qquad (A.11)$$

Thus for a sinusoidal gust the integral (A.9) can be written as:

$$w_{g_0} \int_0^1 \sin \Omega(\ell\xi - Vt) \, w_1 \xi \, d\xi = w_{g_0} I \sin(\Omega V t + \varphi) . \qquad (A.12)$$

where φ is a phase angle.

Expanding $\sin \Omega(\ell\xi - Vt)$ we can split the integral into two parts:

$$I \sin(\Omega Vt + \varphi) = - A(\Omega\ell) \sin \Omega VT + B(\Omega\ell) \cos \Omega Vt \tag{A.13}$$

where

$$A(\Omega\ell) = \int_0^1 \cos(\Omega\ell\xi)\, w_1\, \xi\, d\xi \quad ;$$

$$B(\Omega\ell) = \int_0^1 \sin(\Omega\ell\xi)\, w_1\, \xi\, d\xi \quad ;$$

It can be seen from (A.13) that the generalised gust force has two components, one in phase with the gust at the reference point (in this case wing apex), the other out of phase.

For the present study of response of one mode only, we are interested only in the modulus squared of the integral I, thus in the value of

$$|I(\Omega\ell)|^2 = [A(\Omega\ell)]^2 + [B(\Omega\ell)]^2 \ . \tag{A.14}$$

The evaluated values of integrals $A(\Omega\ell)$ and $B(\Omega\ell)$ and of the modulus of integral $I(\Omega\ell)$ are shown in Fig.3. The two components $A(\Omega\ell)$ and $B(\Omega\ell)$ of the integral $I(\Omega)$ are presented, as they might be useful for further more comprehensive studies. In present application only values of $|I(\Omega\ell)|$ are required.

It should be pointed out that for turbulence wavelengths λ, smaller than the aircraft dimension, say ℓ, the assumption of one dimensional turbulence is losing its validity. Thus for higher frequencies (shorter wavelengths) the present calculations overestimate the turbulence excitation forces. The curves of values of integrals shown in Fig.3 are drawn as interrupted curves for frequencies where, it is believed, these values are substantially in error, due to the assumption of one dimensional turbulence.

2.3 Solution of the equation of motion

Substituting the expression (A.13) for the integral of the generalised gust forcing function into equation (A.8) and rearranging terms, an equation of motion of the first fundamental mode due to sinusoidal gust of amplitude w_{g_o} and space frequency Ω is obtained:

$$\ddot{z}_1 + b\dot{z}_1 + cz_1 = d(- A \sin \Omega Vt + B \cos \Omega Vt)\, w_{g_o} \tag{A.15}$$

where

$$\left.\begin{aligned} b &= \frac{1 \cdot 774}{\mu_g} \frac{V}{\ell} ; \quad sec^{-1} \\ c &= \omega_1^2 + \frac{8 \cdot 84}{\mu_g} \left(\frac{V}{\ell}\right)^2 ; \quad sec^{-1} \\ d &= \frac{34 \cdot 88}{\mu_g} \frac{V}{\ell} ; \quad sec^{-1} \ . \end{aligned}\right\} \qquad (A.16)$$

The equation (A.15) can be written in terms of damping ratio ζ_1 and natural frequency ω_1^* of the first structural mode

$$\ddot{z}_1 + 2 \zeta_1 \omega_1^* \dot{z}_1 + (\omega_1^*)^2 z_1 = d(- A \sin \Omega Vt + B \cos \Omega Vt) \, w_{g_0} \qquad (A.15a)$$

where

$$\left.\begin{aligned} 2 \zeta_1 \omega_1^* &= b = \frac{1 \cdot 774}{\mu_g} \frac{V}{\ell} ; \\ (\omega_1^*)^2 &= c = \omega_1^2 + \frac{8 \cdot 84}{\mu_g} \left(\frac{V}{\ell}\right)^2 \ . \end{aligned}\right\} \qquad (A.16a)$$

In some applications, the aerodynamic contribution to the natural frequency of the structural mode can be neglected and then the frequency and damping ratio can be approximately expressed as:

$$\left.\begin{aligned} \omega_1^* &\simeq \omega_1 \\ \zeta_1 &= \frac{b}{2\sqrt{c}} \simeq \frac{1 \cdot 774}{\mu_g} \frac{V}{2 \ell \omega_1} = \frac{0 \cdot 887}{\mu_g} \frac{V}{\omega_1 \ell} \ . \end{aligned}\right\} \qquad (A.16b)$$

Comparison of the full solutions given in Table 2 with the values of damping given by the approximate expression (A.16b) indicates that this expression overestimates the damping, with an error of the order of 10%.

The non-dimensional mass parameter

$$\mu_g = \frac{2\,W/S}{g\,\rho\,a\,\ell} \tag{A.17}$$

is analogous to the gust mass parameter defining rigid body response of a conventional aircraft[12], except that for slender aircraft the reference length is the aircraft length ℓ and not the mean wing chord $\bar{c}$.

The steady state solution of equation (A.15) gives the modulus squared frequency response function of displacement z_1:

$$\left| \frac{z_1}{w_{g_o}} \right|^2 = \frac{d^2}{[c - (\Omega V)^2]^2 + (bV\Omega)^2} \, |I(\Omega\ell)|^2 \tag{A.18}$$

where $|I(\Omega\ell)|^2$ is defined by expression (A.14).

The spectral density of the accelerations of z_1 coordinate for 1 ft sec^{-1} turbulence R.M.S. $\Phi_{\ddot{z}_1}(\Omega)$, is obtained by multiplying equation (A.18)by $\omega^4 = (\Omega V)^4$ and by the spectral density of atmospheric turbulence, taken as:

$$\frac{G(\Omega)}{\sigma^2_{w_g}} = \frac{L}{\pi} \, \frac{1 + 3(\Omega L)^2}{[1 + (\Omega L)^2]^2} \tag{A.19}$$

and the following expression results:

$$\Phi_{\ddot{z}_1}(\Omega) = V^4 \, \frac{d^2}{[c - (\Omega V)^2]^2 + (bV\Omega)^2} \left\{ \frac{G(\Omega)}{\sigma^2_{w_g}} \, \Omega^4 \, |I(\Omega\ell)|^2 \right\} \tag{A.20}$$

(dimensions: (ft sec^{-2})^2 per rad ft^{-1} per ft sec^{-1} = ft sec^{-2})

The above expression for spectral density of accelerations of the first mode is written as a product of two terms. The first term may be regarded as an aircraft admittance and is a function only of aircraft parameters, such as forward speed V, frequency of the mode, ω_1 (stiffness), and mass parameter, μ_g. The second term, in curly brackets, defines the input of atmospheric turbulence and is related to the aircraft only through the function $I(\Omega\ell)$ which in turn depends on the modal shape, lift distribution and aircraft size. In the study of the effects of forward speed, airframe stiffness, lift slope, wing loading, the expression in curly brackets is constant and can be regarded as a "spectrum density of acceleration producing input" of the first mode say $\Phi_1(\Omega\ell)$.

$$\Phi_1(\Omega\ell) = \frac{G(\Omega)}{\sigma^2_{w_g}} \Omega^4 \left|I(\Omega\ell)\right|^2 . \tag{A.21}$$

This, so defined function Φ_1, is plotted in Fig.4 for two sizes of the aircraft defined by its mode shown in Fig.2. The curves of Fig.4 were computed assuming the scale of turbulence, L = 1000 ft. The choice of turbulence scale is not critical in the present application, as the vibrational response covers the wavelengths range of turbulence well inside the inertial subrange, where the spectrum shape is independent of the assumed turbulence scale, L.

The expression (A.20) was actually modified at low frequencies to allow for rigid body response, as discussed in the following section. This effect of rigid body response although noticeable for the structural deflection response, z_1, is negligible for the acceleration response, $\ddot{z}_1$.

It can be seen from Fig.4 that the "acceleration producing input" for the first structural mode has peaks corresponding to a "tuning" between the mode deflection shape and turbulence waves, thus the spectral density of vibrations will have at least two peaks, one corresponding to a true resonance at structural frequency f_1 c.p.s. and other (or others) at frequency $f_{exc.} = \frac{V}{2\pi} \Omega_{peak}$, where Ω_{peak} is the space frequency corresponding to a peak in Fig.4.

From integration of the spectral density function, equation (A.20), with respect to Ω, a value of R.M.S. acceleration at reference point (wing apex in here) for 1 ft sec^{-1} turbulence R.M.S. can be found. By varying values of parameters in the first term of equation (A.20) the effects on vibration level of forward speed, structural stiffness, lift slope and wing loading can be investigated.

3 THE EFFECT OF RIGID BODY RESPONSE

The analysis of structural response to atmospheric turbulence neglecting the rigid body response, overestimates the excitation at low frequencies. Neglecting rigid body degrees of freedom is equivalent to restraining the structure at the nodal lines of the structural mode in question. The rigid body response alleviates the actual incidence experienced by the wing and for frequencies approaching zero, the excitation of the structural modes approaches zero also.

It was thought worthwhile to assess the order of the effect of rigid body response on the structural vibrations response. At the same time, the accelerations due to turbulence in the rigid body mode could be estimated for comparison with the accelerations due to structural vibration.

As only an order of magnitude is required, simplified calculations were made, neglecting pitching and unsteady aerodynamics. It should be pointed out, that the pitching degree of freedom has an appreciable effect on the calculated accelerations in rigid body modes of a slender aircraft. The pitching effect on the accelerations of structural modes is negligibly small, as the overall effect of rigid body response on structural vibration is very small.

The relationship between the R.M.S. of normal acceleration of rigid body, σ_n, and between turbulence R.M.S., σ_{w_g} can be written:

$$\frac{\sigma_n}{\sigma_{w_g}} = \frac{V}{g\, \ell \mu_g} \; K_{SP} \quad ; \tag{A.22}$$

where K_{SP} is a spectral gust alleviation factor.

Making the following assumptions:

(i) pitching degree of freedom is neglected

(ii) unsteady aerodynamics are neglected

(iii) the aircraft size is small in comparison with turbulence scale.

The spectral gust alleviation is defined:

$$K_{SP} = \left[\int_0^{\infty} |K_\alpha|^2 \frac{1}{\sigma_{w_g}^2} \; G(\Omega) \; d\Omega \right]^{\frac{1}{2}} \tag{A.23}$$

where $|K_\alpha|^2$ is modulus square of incidence function given by:

$$|K_\alpha|^2 = \frac{\Omega^2}{\Omega^2 + \left(\frac{1}{\ell \mu_g}\right)^2} \quad . \tag{A.24}$$

It is seen that under the above assumptions the spectral gust alleviation factor K_{SP} is a function of turbulence spectrum shape and of aircraft parameter

$$\ell\mu_g = \frac{2\ W/S}{g\ \rho\ a}$$

and thus is independent of forward speed.

The incidence function (A.24) defines the incidence experienced by the wing in turbulence as alleviated by aircraft heaving motion, thus can be used to estimate the effect of rigid body response on the excitation of structural mode. Multiplying the spectral density of turbulence, $G(\Omega)$, by the modulus square of incidence function $|K_\alpha|^2$, we obtain a modified spectrum, spectrum of the actual vertical air velocity as experienced by the wing surface. This treatment is only approximate, and can only be allowed when there is no appreciable coupling between rigid and structural modes.

The effects of the rigid body response on the excitation of structural mode is small, being noticeable for the deflection response, but negligible for the acceleration response. These effects are illustrated in Fig.5.

4 DISCUSSION OF THE SOLUTION OF EQUATION OF MOTION

The expression for the spectral density of accelerations of z_1 coordinate, equation (A.20) indicates that the value of acceleration R.M.S. of z_1 will depend on values of aircraft parameters defining the admittance function and on value of the excitation function, Φ_1, equation (A.21). For a given modal shape, lift distribution and turbulence scale, the values of excitation function Φ_1 depend on the aircraft size, ℓ, only.

It can be shown that integration of the spectral density of second order system response to input spectrum as used in present paper (equation A.19) gives the R.M.S. value proportional to $1/\sqrt{\zeta}$ hence:

$$\sigma_{\ddot{z}_1} \sim \frac{d}{\sqrt{\zeta_1}} \tag{A.25}$$

where parameter d is defined by expression (A.16) and ζ_1 is damping ratio of structural oscillations (A.16b). For given values of μ_g and ℓ, the expression (A.25) can be written as:

$$\sigma_{\ddot{z}_1} \sim \frac{V}{\sqrt{\zeta_1}} \ . \tag{A.26}$$

The validity of approximation (A.26) was demonstrated by numerical calculations, results of which are shown in Fig.10.

The approximate effect of aircraft size, ℓ, could not be expressed so simply, but it was found empirically that for the range of parameters used in present study the value of acceleration R.M.S. is roughly proportional to the root of aircraft length ℓ:

$$\sigma_{\ddot{z}_1} \sim \sqrt{\ell} \ . \tag{A.27}$$

Combining the expressions (A.26) with (A.27) we obtain the following expression for acceleration R.M.S.

$$\sigma_{\ddot{z}_1} \sim V \sqrt{\frac{\ell}{\zeta_1}} \ . \tag{A.28}$$

The degree of validity of approximation (A.28) is illustrated in Fig.12.

The approximation (A.28) is not convenient for practical applications as it contains damping ratio ζ the value of which is usually not known.

Neglecting aerodynamic stiffness contribution the damping ratio can be approximately written as:

$$\zeta_1 \simeq \frac{0{\cdot}887}{\mu_g} \frac{V}{\omega_1 \ell} \tag{A.16b}$$

Combining the approximations (A.16b) with (A.25) and (A.27) a further approximation for R.M.S. acceleration of the structural oscillations is obtained

$$\sigma_{\ddot{z}_1} \sim \sqrt{\ell} \sqrt{\frac{V\omega_1}{\mu_g \ell}} \sim \sqrt{\frac{g \rho a}{W/S} V \ell f_1} \ . \tag{A.30}$$

The approximation (A.30) gives a relationship between the acceleration R.M.S. and relevant aircraft parameters. The degree of approximation given by (A.30) is shown in Fig.13. The results of calculations presented in Fig.13 allow an approximate extrapolation of the numerical results given on this paper to values of aircraft parameters other than those used in here.

5 AN ESTIMATE OF THE RESPONSES OF HIGHER MODES TO ATMOSPHERIC TURBULENCE

A method was required, by which the order of magnitude of the response of modes other than first to turbulent air can be estimated with a minimum of computational effort.

The method was developed by which a response in ith mode relative to response in first mode can be estimated. The following simplifying assumptions were made:

(i) The overall gust excitation of any mode is proportional to the excitation of this mode at zero frequency. Thus only one integral

$$\int_0^1 \int_0^1 w_i \, d\eta \, d\xi$$

has to be evaluated.

(ii) The R.M.S. value of response is proportional to the value of the appropriate transfer function at resonant frequency $\omega_i = \Omega_i V$ and to the value of the turbulence spectral density at this frequency, Ω_i.

Under these assumptions and using equation (A.2), the R.M.S. of z_i coordinate acceleration is proportional to:

$$\sigma_{\ddot{z}_i} \sim \frac{\frac{1}{2}\rho V a \ell^2 \int_0^1 \int_0^1 w_i \, d\eta \, d\xi}{\frac{1}{2}\rho V a \ell^2 \int_0^1 \int_0^1 w_i^2 \, d\eta \, d\xi} \; \frac{1}{V\Omega_i} \; [G(\Omega_i)]^{\frac{1}{2}} \; \Omega_i^2 \; . \tag{A.31}$$

At a given speed V and remembering that at structural frequencies $[G(\Omega_i)]^{\frac{1}{2}} \sim \frac{1}{\Omega_i}$, the above expression reduces to:

$$\sigma_{\ddot{z}_i} \sim \frac{\int_0^1 \int_0^1 w_i \, d\eta \, d\xi}{\int_0^1 \int_0^1 w_i^2 \, d\eta \, d\xi} \; . \tag{A.32}$$

The expression (A.32) states that the R.M.S. of acceleration response of ith mode due to continuous turbulence is proportional to the turbulence excitation of this mode (at zero frequency) and inversely proportional to the damping of that mode.

By comparing the value of expression (A.32) for ith mode with corresponding value for the first mode, the relative contribution to the overall vibration level of this particular mode can be estimated, by evaluating the ratio $\sigma_{\ddot{z}_i}/\sigma_{\ddot{z}_1}$.

TABLE 1

Parameters used in numerical study defining the basic aircraft

weight	W	=	695,000 lb
wing area	S	=	10,000 ft^2
aircraft length	ℓ	=	226·8 ft
max. wing depth	d_{max}	=	12 ft
wing loading	W/S	=	69·5 lb ft^{-2}
lift slope	a	=	2
altitude of flight	H	=	10,000 ft
mass parameter	μ_g	=	5·43

$$\ell\mu_g = \frac{2}{g}\frac{W/S}{\rho a} = 1230 \text{ ft}$$

frequency of first mode f_1 = 2·14 c.p.s. (varied)

Parameters defining the generalised forces of first structural mode

longitudinal mass distribution $\quad m(\xi) = 10{\cdot}57 \frac{W}{g} (1 - \xi)\, \xi^2$

plus concentrated mass of $0{\cdot}1195 \frac{W}{g}$ at $\xi = 0{\cdot}889$

longitudinal stiffness distribution $\quad EI = \frac{8}{15} \frac{S}{\ell}\, d^2_{max}\, E\, \tau\, \xi(4{\cdot}63\xi^3 - 10{\cdot}63\xi^2 + 6{\cdot}00\xi)^2$

frequency of first mode $\quad f_1 = 3{\cdot}5\sqrt{\tau}$ c.p.s.

modal shape $\quad w_1(\xi) = 1 - 2{\cdot}15\xi - 2{\cdot}3\xi^2 + 4{\cdot}15\xi^3$

generalised mass $\quad M_1 = \int_0^1 m(\xi)\, w_1^2\, d\xi = 0{\cdot}0574\ W/g$

aerodynamic stiffness $\quad \int_0^1 \xi\, w_1 \frac{dw_1}{d\xi}\, d\xi = 0{\cdot}2534$

TABLE 1 (Contd.)

aerodynamic damping	$\int_0^1 \xi\, w_1^2\, d\xi$	$= 0{\cdot}0509$
turbulence excitation (at zero frequency)	$\int_0^1 \xi\, w_1\, d\xi$	$= 0{\cdot}0382$

TABLE 2

Numerical results of response calculations

	ℓ = 226·8 ft, f_1 = 1·5 c.p.s.					ℓ = 226·8 ft, f_1 = 2·14 c.p.s.					ℓ = 226·8 ft, f_1 = 2·5 c.p.s.				
V ft sec^{-1}	$\frac{\omega_1\ell}{V}$	ζ%	$\sigma_{\ddot{z}_1}$ in "g"	$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\zeta}$ sec^{-1}	$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\frac{\zeta}{\ell}}$	$\frac{\omega_1\ell}{V}$	ζ%	$\sigma_{\ddot{z}_1}$ in "g"	$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\zeta}$ sec^{-1}	$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\frac{\zeta}{\ell}}$	$\frac{\omega_1\ell}{V}$	ζ%	$\sigma_{\ddot{z}_1}$ in "g"	$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\zeta}$ sec^{-1}	$\frac{\sigma_{\ddot{z}_1}}{V}\sqrt{\frac{\zeta}{\ell}}$
250						12·16	1·34	$1·08\times10^{-2}$	$1·61\times10^{-4}$	$1·07\times10^{-5}$					
300	7·12	2·27	$2·41\times10^{-2}$	$3·9\times10^{-4}$	$2·59\times10^{-5}$	10·12	1·60	$1·94\times10^{-2}$	$2·64\times10^{-4}$	$1·75\times10^{-5}$	11·87	1·37	$1·39\times10^{-2}$	$1·75\times10^{-4}$	$1·16\times10^{-5}$
350							1·86				10·18	1·60	$2·32\times10^{-2}$	$2·69\times10^{-4}$	$1·79\times10^{-5}$
400	5·35	2·97	$2·46\times10^{-2}$	$3·41\times10^{-4}$	$2·27\times10^{-5}$	7·60	2·12	$3·28\times10^{-2}$	$3·83\times10^{-4}$	$2·48\times10^{-5}$	8·90	1·82	$3·11\times10^{-2}$	$3·38\times10^{-4}$	$2·24\times10^{-5}$
500	4·27	3·68	$2·12\times10^{-2}$	$2·62\times10^{-4}$	$1·74\times10^{-5}$	6·08	2·64	$3·68\times10^{-2}$	$3·85\times10^{-4}$	$2·55\times10^{-5}$	7·12	2·27	$4·03\times10^{-2}$	$3·93\times10^{-4}$	$2·61\times10^{-5}$
600	3·56	4·33	$1·83\times10^{-2}$	$2·01\times10^{-4}$	$1·33\times10^{-5}$	5·07	3·13	$3·36\times10^{-2}$	$3·18\times10^{-4}$	$2·11\times10^{-5}$	5·93	2·70	$4·26\times10^{-2}$	$3·75\times10^{-4}$	$2·49\times10^{-5}$
700		4·96				4·35	3·61	$3·04\times10^{-2}$	$2·65\times10^{-4}$	$1·76\times10^{-5}$	5·09	3·12	$3·94\times10^{-2}$	$3·20\times10^{-4}$	$2·13\times10^{-5}$
800	2·67	5·54	$1·70\times10^{-2}$	$1·62\times10^{-4}$	$1·07\times10^{-5}$	3·80	4·08	$2·79\times10^{-2}$	$2·27\times10^{-4}$	$1·51\times10^{-5}$	4·45	3·54	$3·61\times10^{-2}$	$2·73\times10^{-4}$	$1·81\times10^{-5}$
1000	2·135	6·58	$2·10\times10^{-2}$	$1·74\times10^{-4}$	$1·15\times10^{-5}$	3·04	4·97	$2·38\times10^{-2}$	$1·70\times10^{-4}$	$1·13\times10^{-5}$	3·56	4·38	$3·04\times10^{-2}$	$2·04\times10^{-4}$	$1·36\times10^{-5}$
1200		7·48					5·76				2·97	5·06	$2·80\times10^{-2}$	$1·69\times10^{-4}$	$1·12\times10^{-5}$

V ft sec^{-1}	ℓ = 113·4 ft, f_1 = 2·5 c.p.s.				
200	8·90	0·96	$1·59\times10^{-2}$	$2·52\times10^{-4}$	$2·37\times10^{-5}$
250	7·12	1·14	$2·00\times10^{-2}$	$2·75\times10^{-4}$	$2·58\times10^{-5}$
300	5·93	1·36	$2·05\times10^{-2}$	$2·56\times10^{-4}$	$2·41\times10^{-5}$
400	4·45	1·80	$1·65\times10^{-2}$	$1·77\times10^{-4}$	$1·66\times10^{-5}$
500	3·56	2·23	$1·24\times10^{-2}$	$1·19\times10^{-4}$	$1·12\times10^{-5}$
600	2·97	2·74	$1·08\times10^{-2}$	$0·95\times10^{-4}$	$0·93\times10^{-5}$
800	2·22	3·41	$1·44\times10^{-2}$	$1·07\times10^{-4}$	$1·01\times10^{-5}$
1000	1·78	4·10	$2·14\times10^{-2}$	$1·39\times10^{-4}$	$1·30\times10^{-5}$

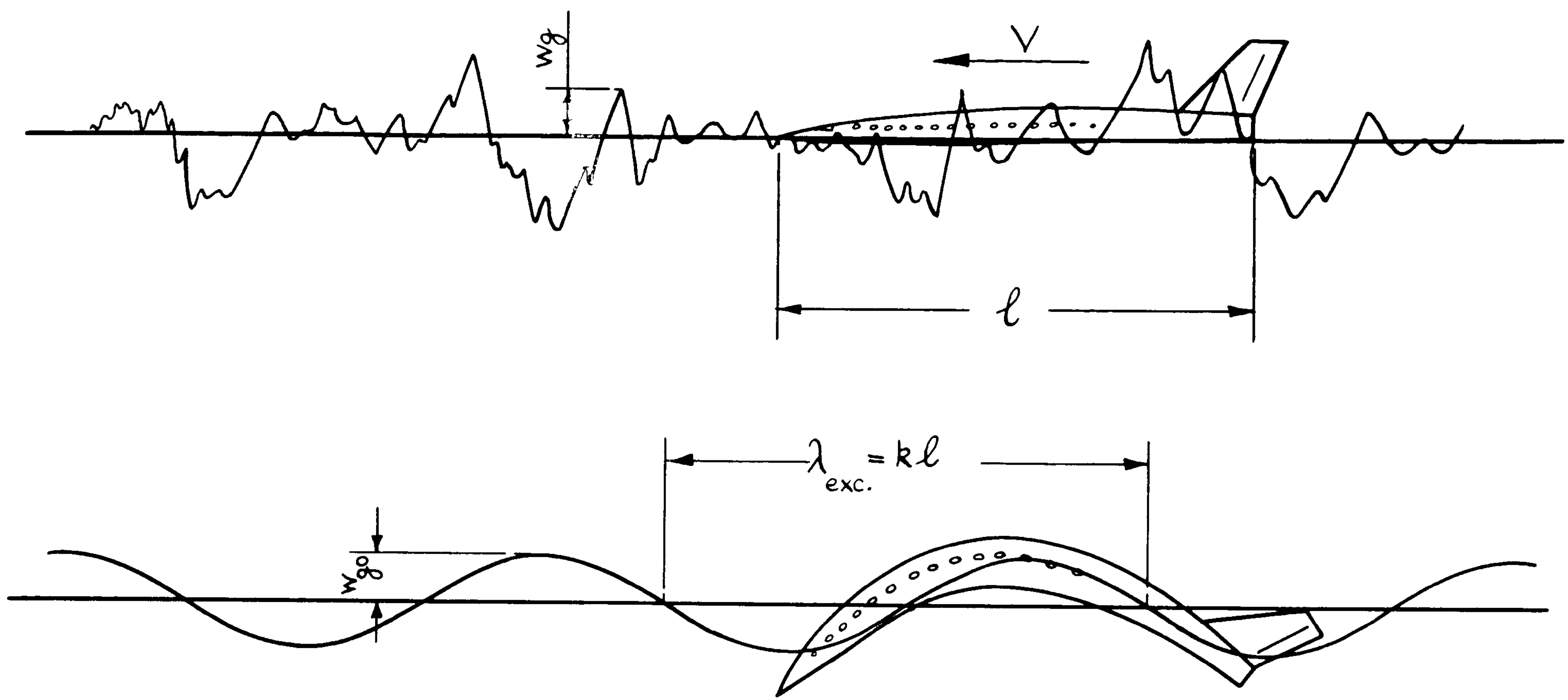


FIG. 1. EXCITATION OF ELASTIC MODE OF A SLENDER AIRCRAFT BY A CONTINUOUS TURBULENCE. EXCITATION "TUNING" BETWEEN PARTICULAR WAVELENGTH AND MODAL SHAPE.

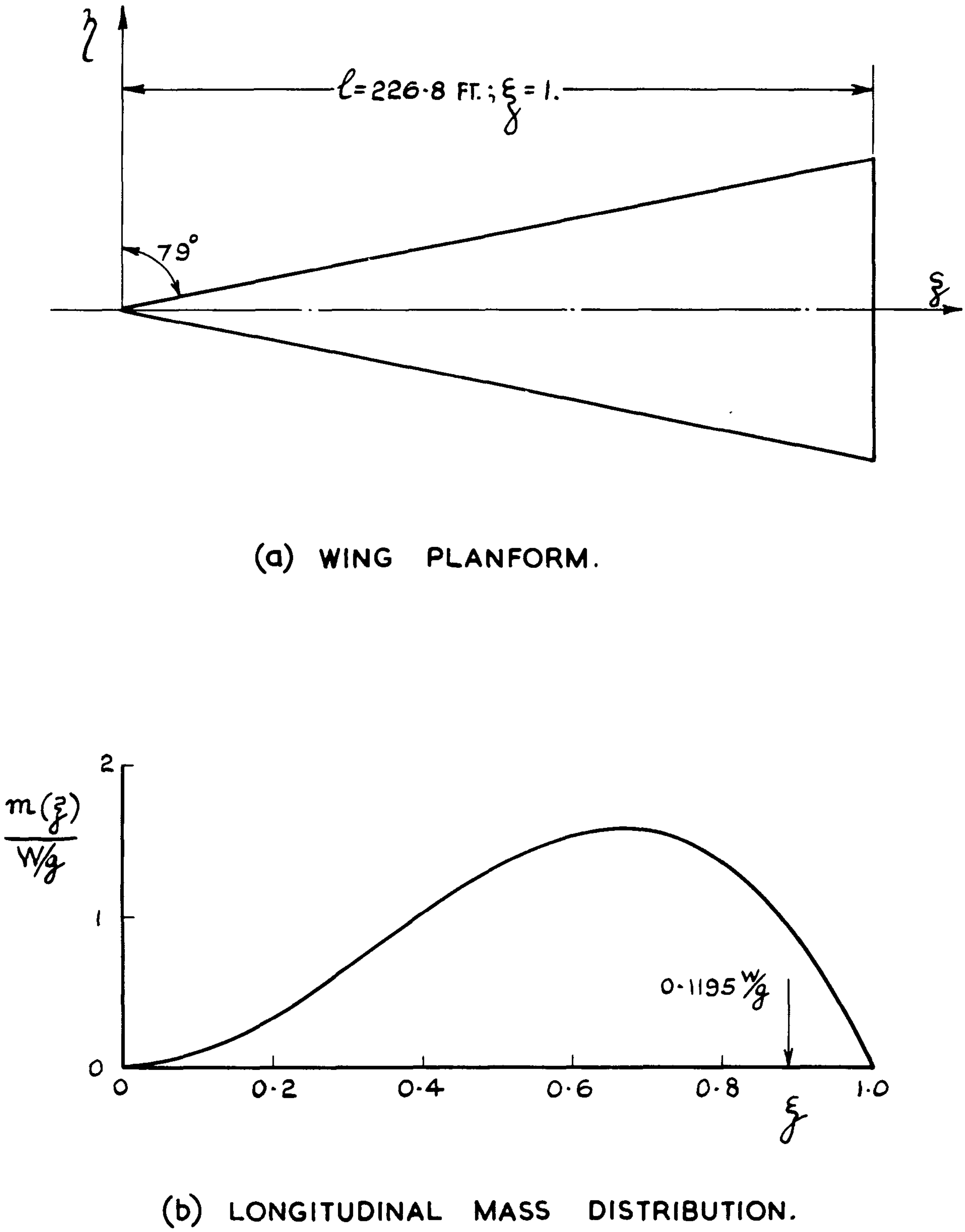
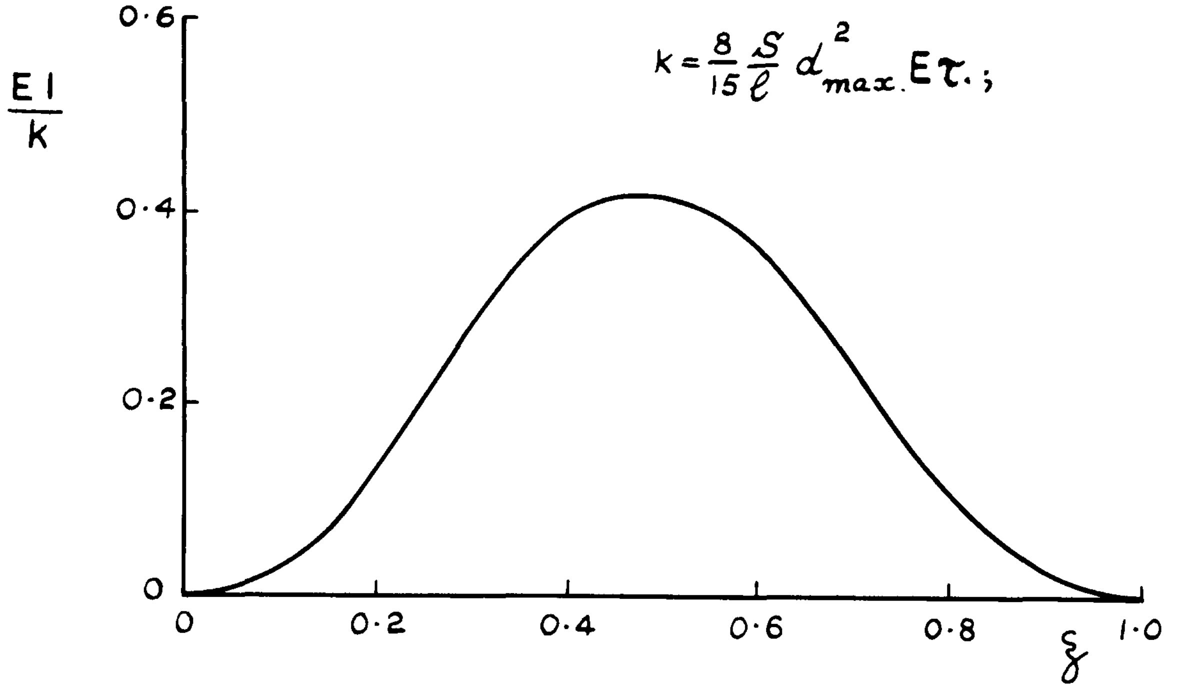
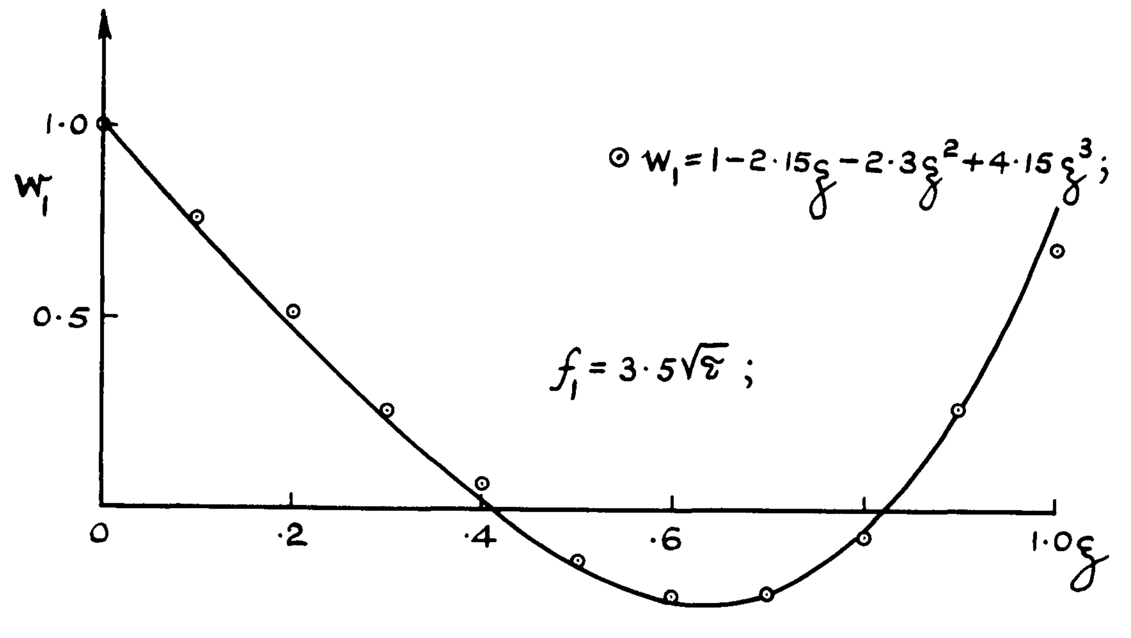


FIG. 2. WING PLANFORM, LONGITUDINAL MASS AND STIFFNESS DISTRIBUTIONS AND THE SHAPE OF FIRST FUNDAMENTAL MODE.

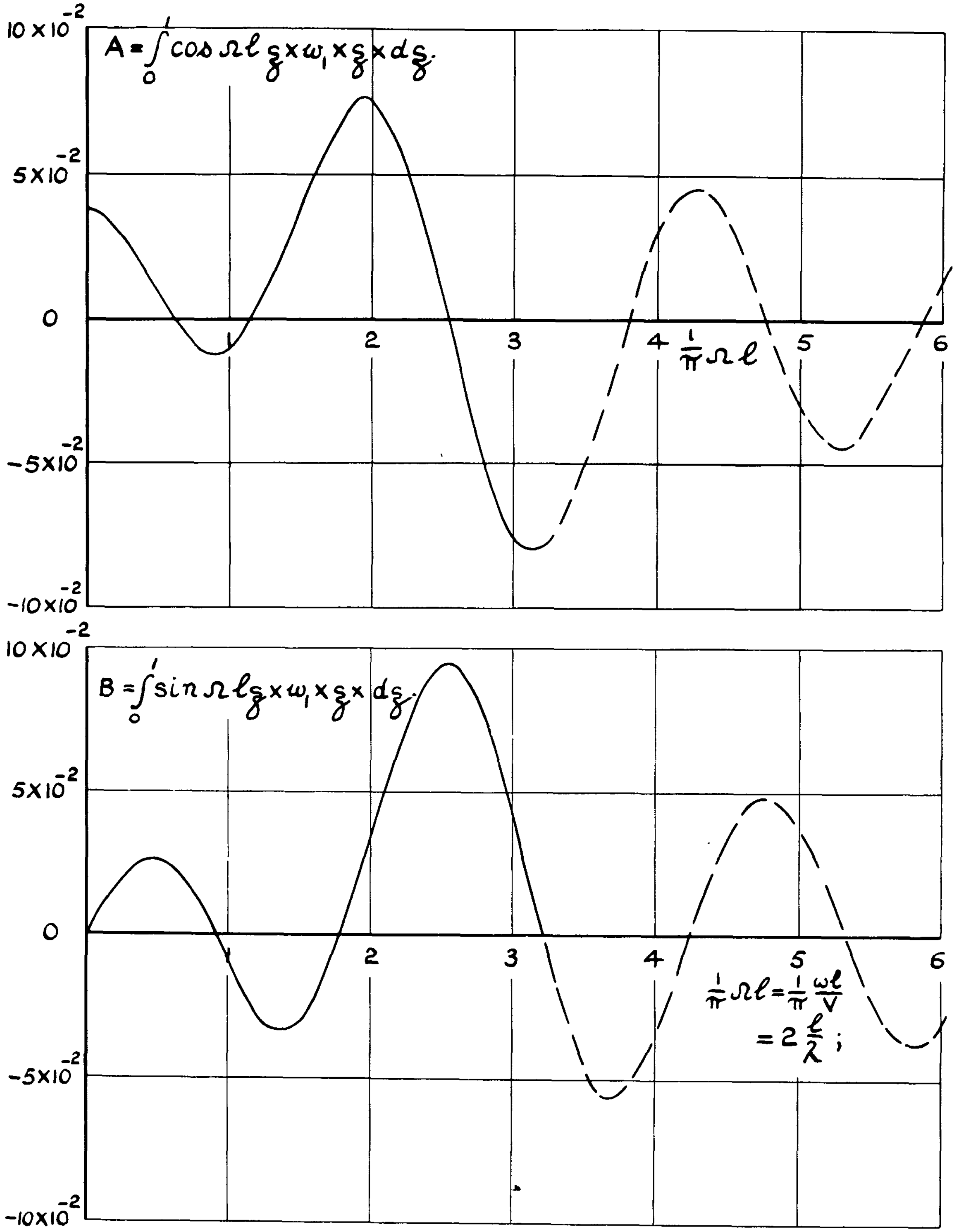


(c) LONGITUDINAL STIFFNESS DISTRIBUTION.



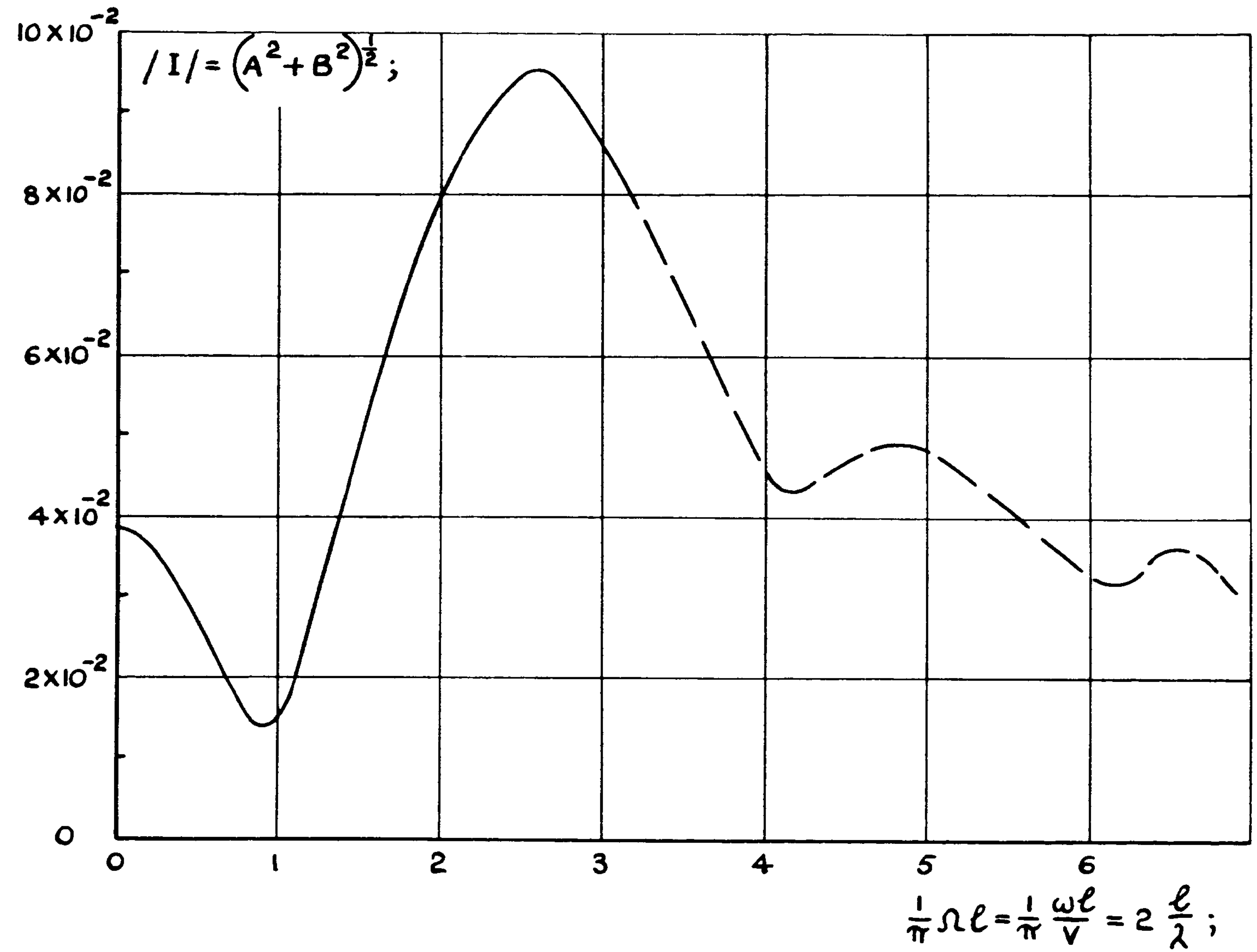
(d) THE SHAPE OF FIRST FUNDAMENTAL MODE.

FIG. 2 (CONTD.)



(a). COMPONENTS IN AND OUT OF PHASE WITH SINUSOIDAL GUST AT REFERENCE POINT (WING APEX)

FIG. 3. INTEGRALS DEFINING GENERALISED GUST FORCING FUNCTION FOR THE FIRST FUNDAMENTAL MODE PLOTTED AGAINST REDUCED FREQUENCY, $\omega\ell/V$. (PISTON THEORY)



(b) MODULUS OF THE INTEGRAL.

FIG. 3 (CONTD.)

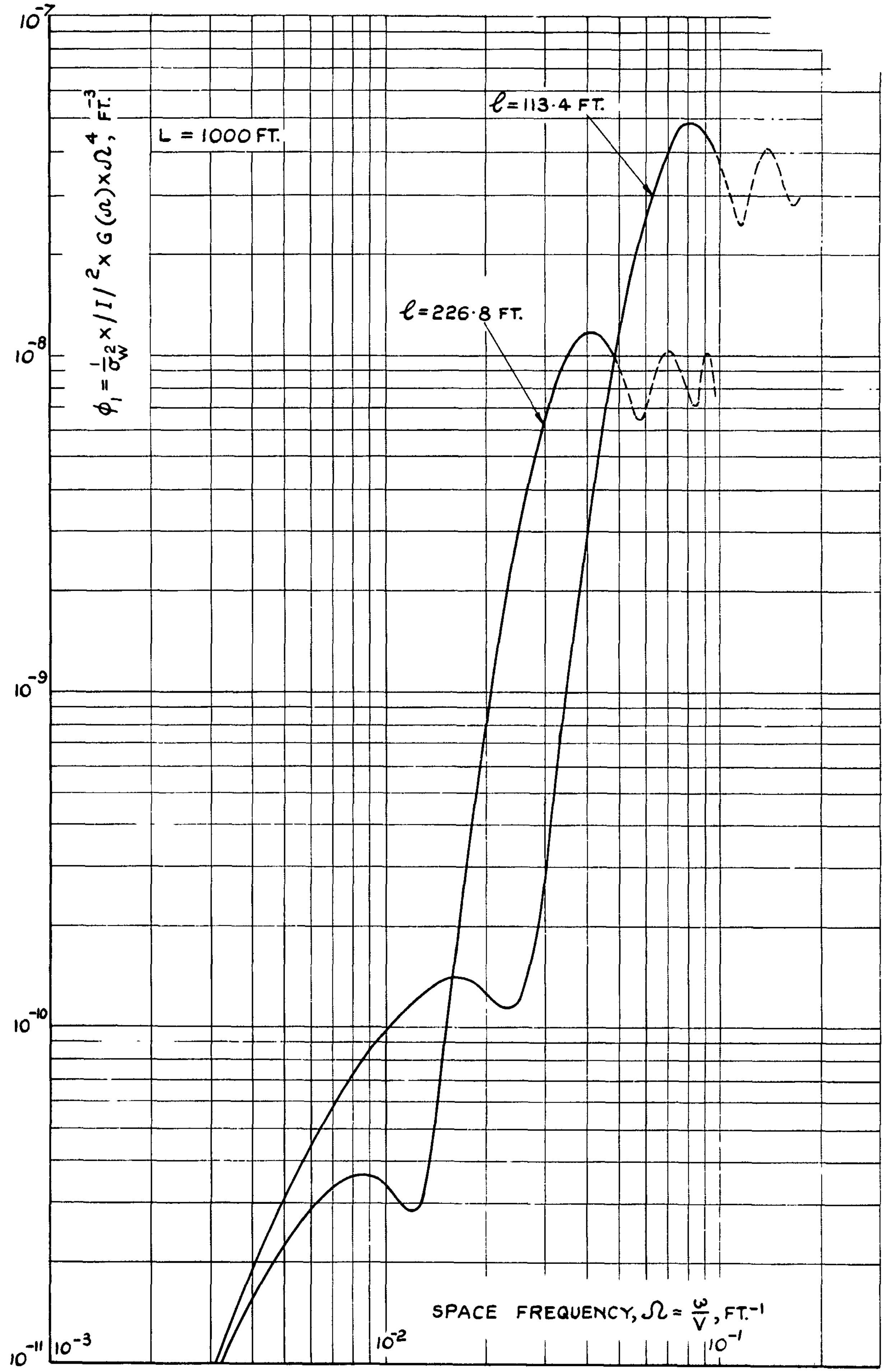


FIG. 4. SPECTRAL DENSITY OF "ACCELERATION PRODUCING INPUT," ϕ_1, FOR FIRST FUNDAMENTAL MODE.

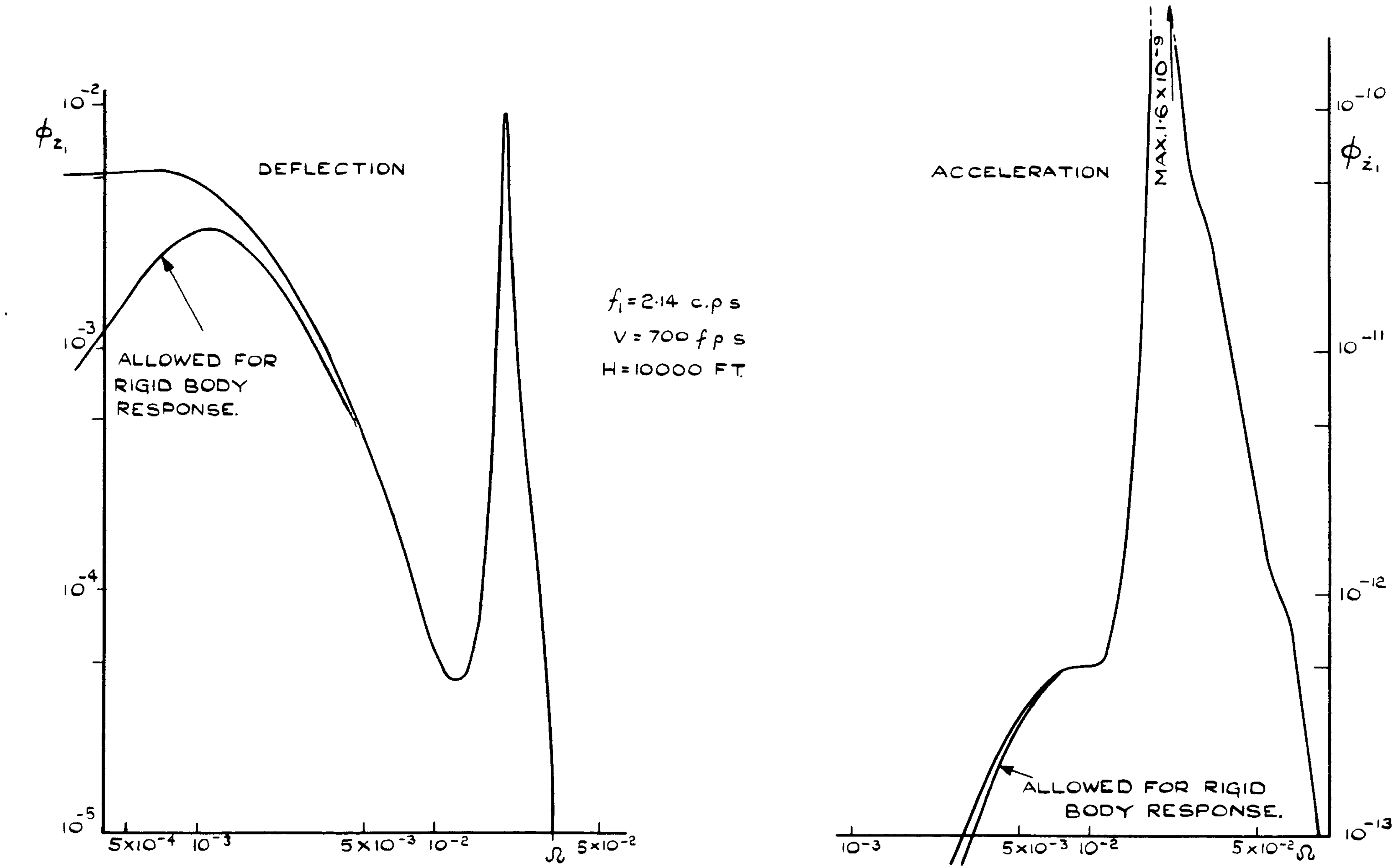
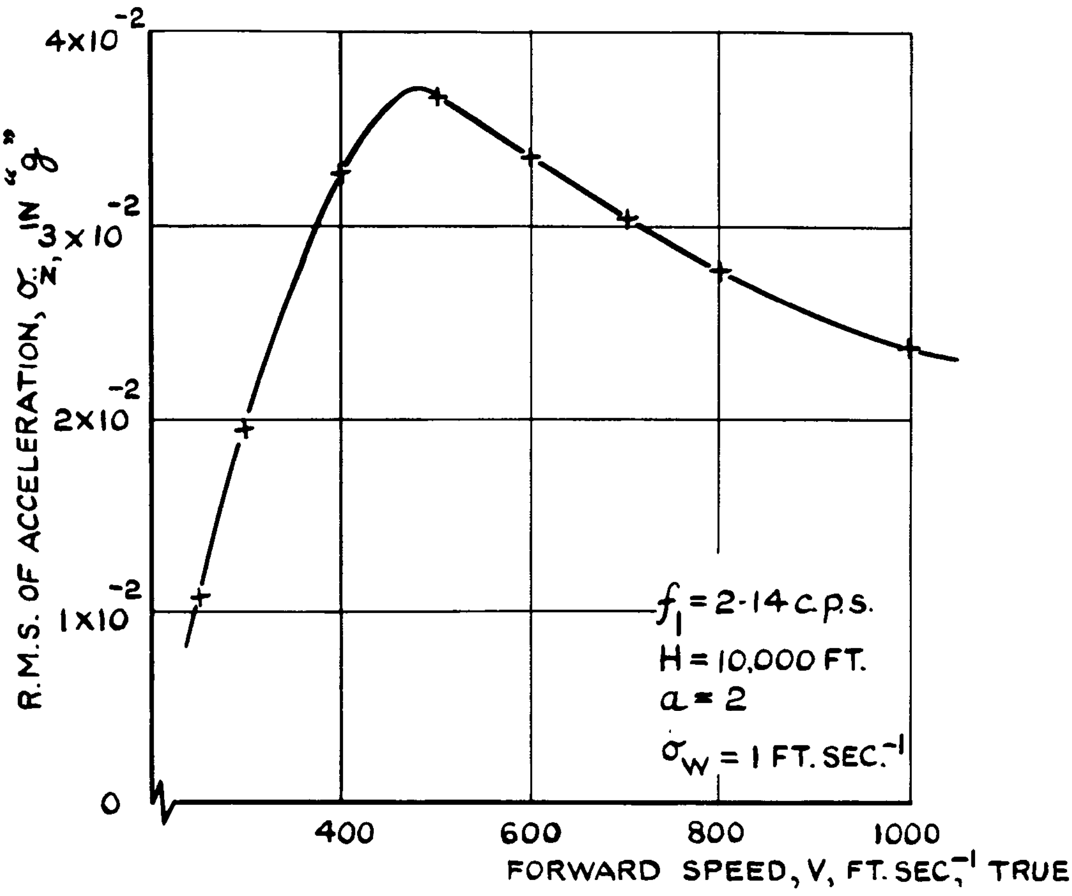
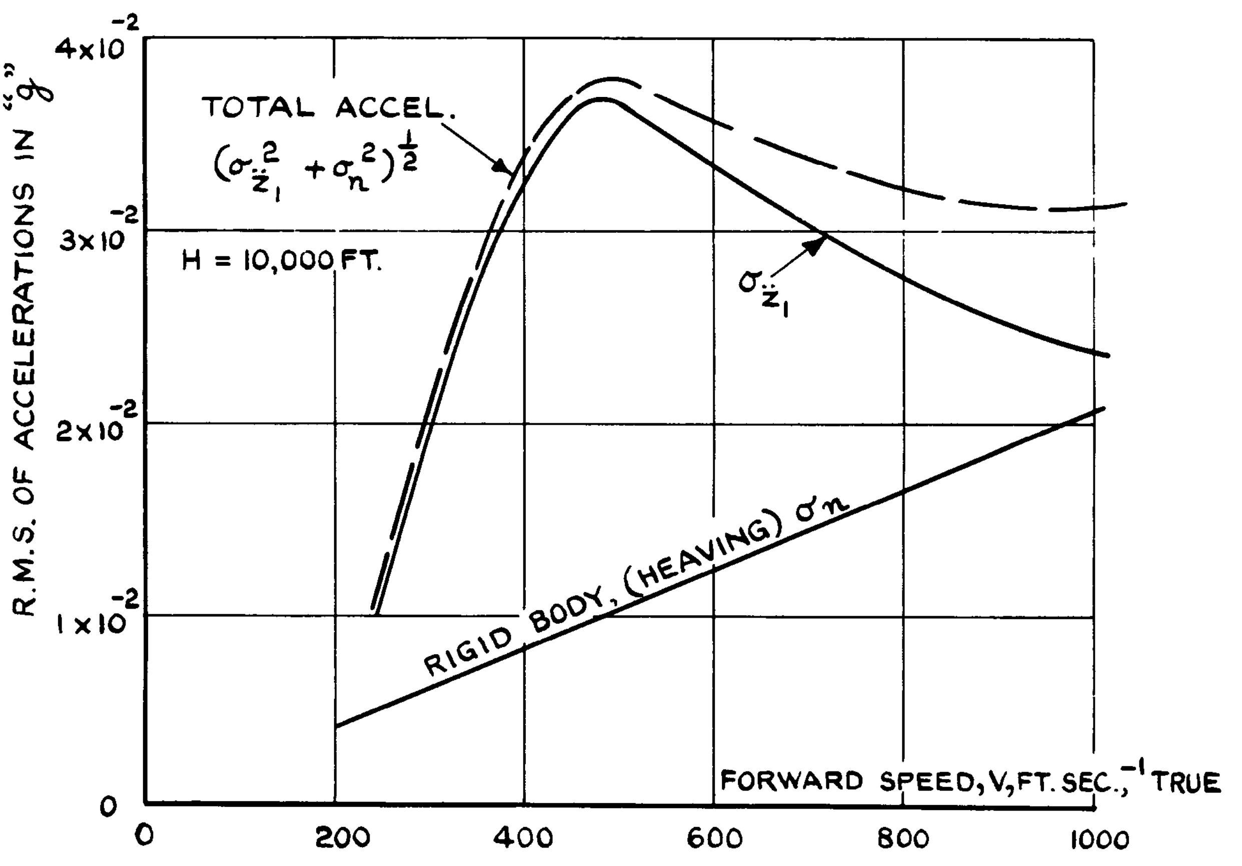


FIG. 5. THE SPECTRAL DENSITY OF THE RESPONSE OF FIRST STRUCTURAL MODE, THE EFFECT OF RIGID BODY MODE.

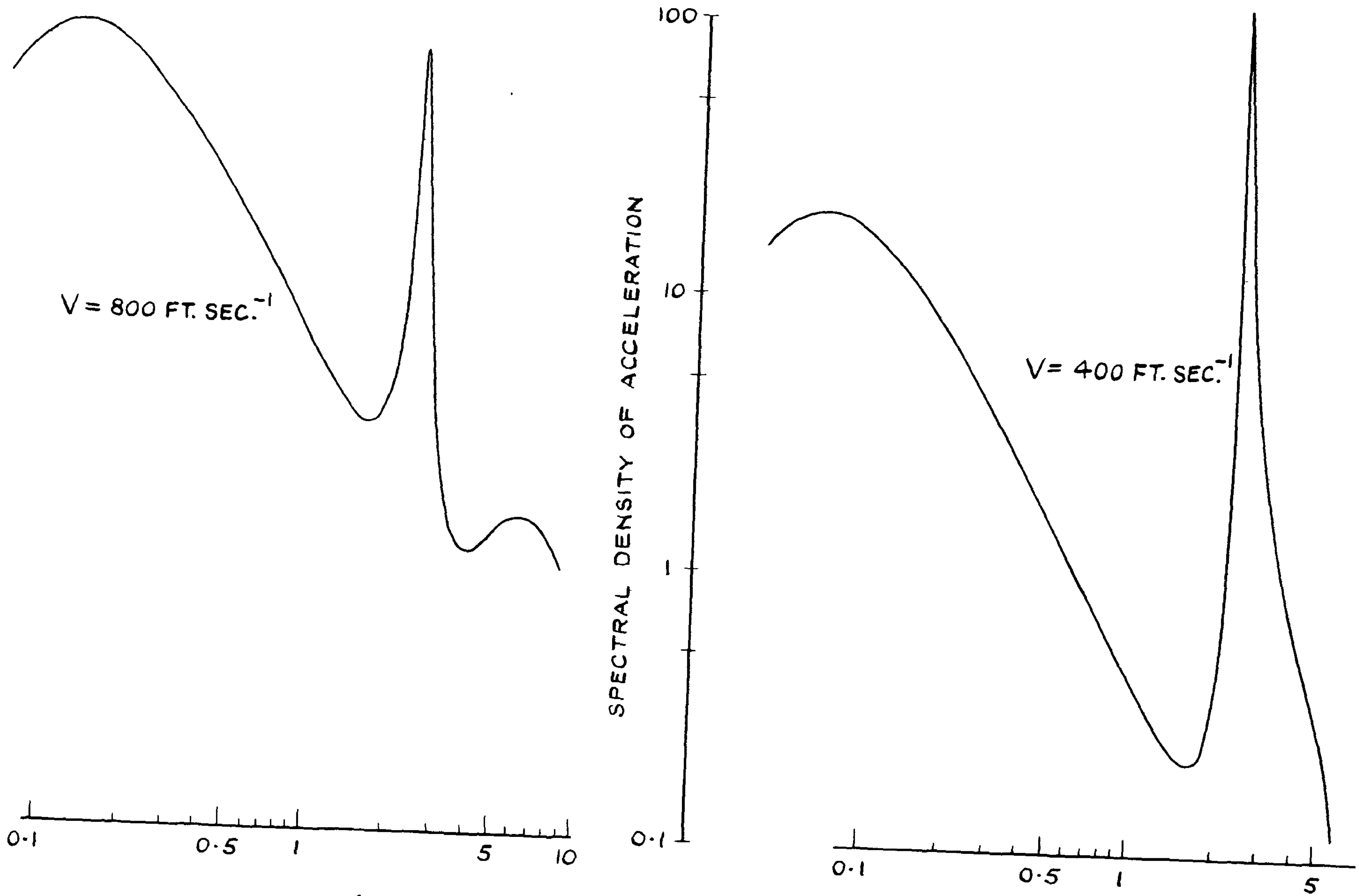


(a.) ACCELERATIONS DUE TO FIRST STRUCTURAL MODE.



(b.) ACCELERATIONS DUE TO STRUCTURAL AND RIGID BODY MODES.

FIG. 6. THE EFFECT OF FORWARD SPEED ON THE ACCELERATION R.M.S. AT THE WING APEX.



(a) ACCELERATIONS AT $\xi = 0$, AIRCRAFT NOSE.

FIG. 7. SPECTRAL DENSITY OF ACCELERATION FOR TWO SPEEDS AND TWO LOCATIONS ALONG THE AIRCRAFT AXIS ($\ell = 113{\cdot}4$ FT., $f_1 = 2{\cdot}5$ c.p.s., H = 10,000 FT.)

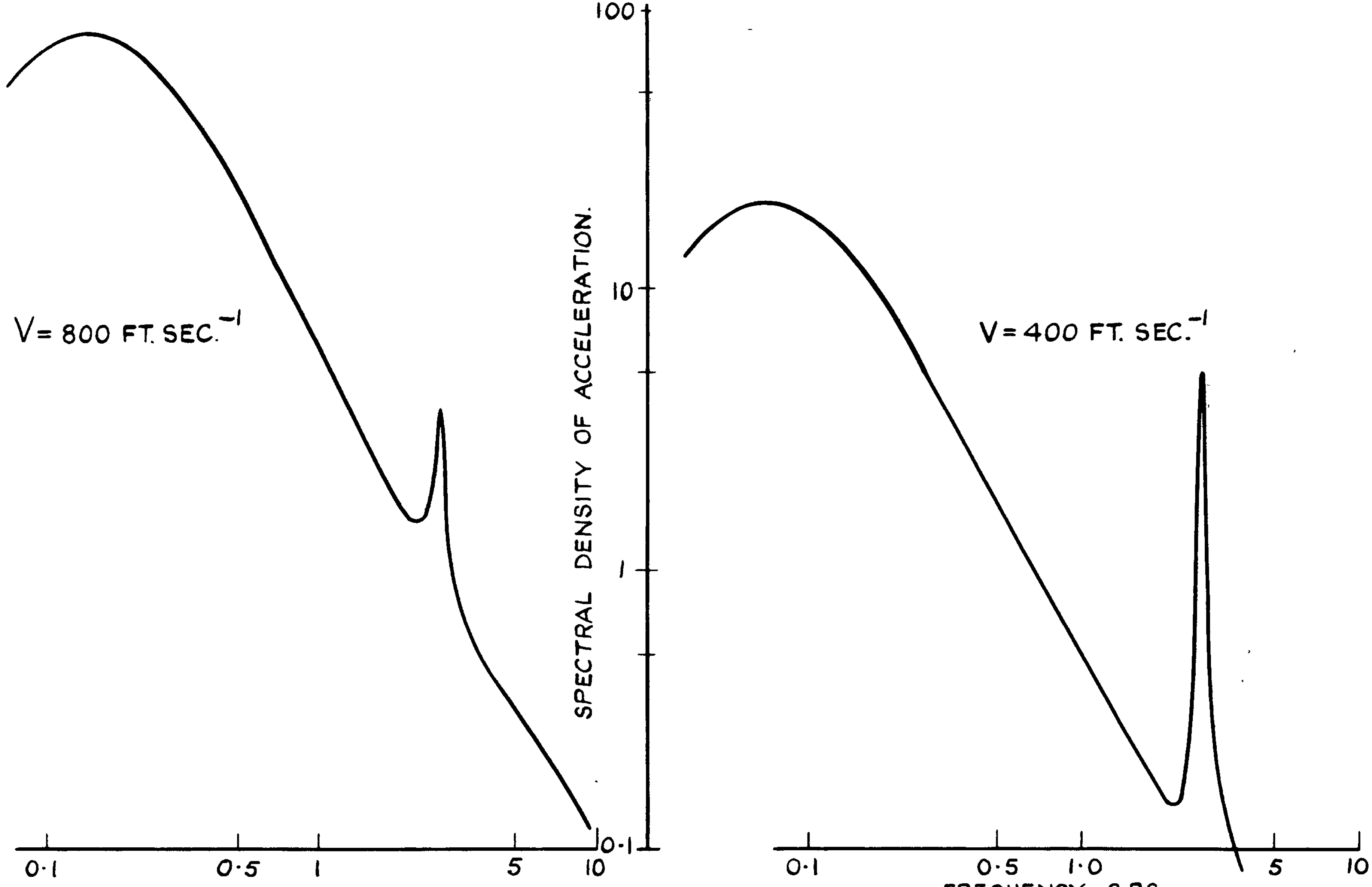


FIG. 7(b.) ACCELERATIONS AT ξ = 0·7, APPROX. AIRCRAFT C. OF G.

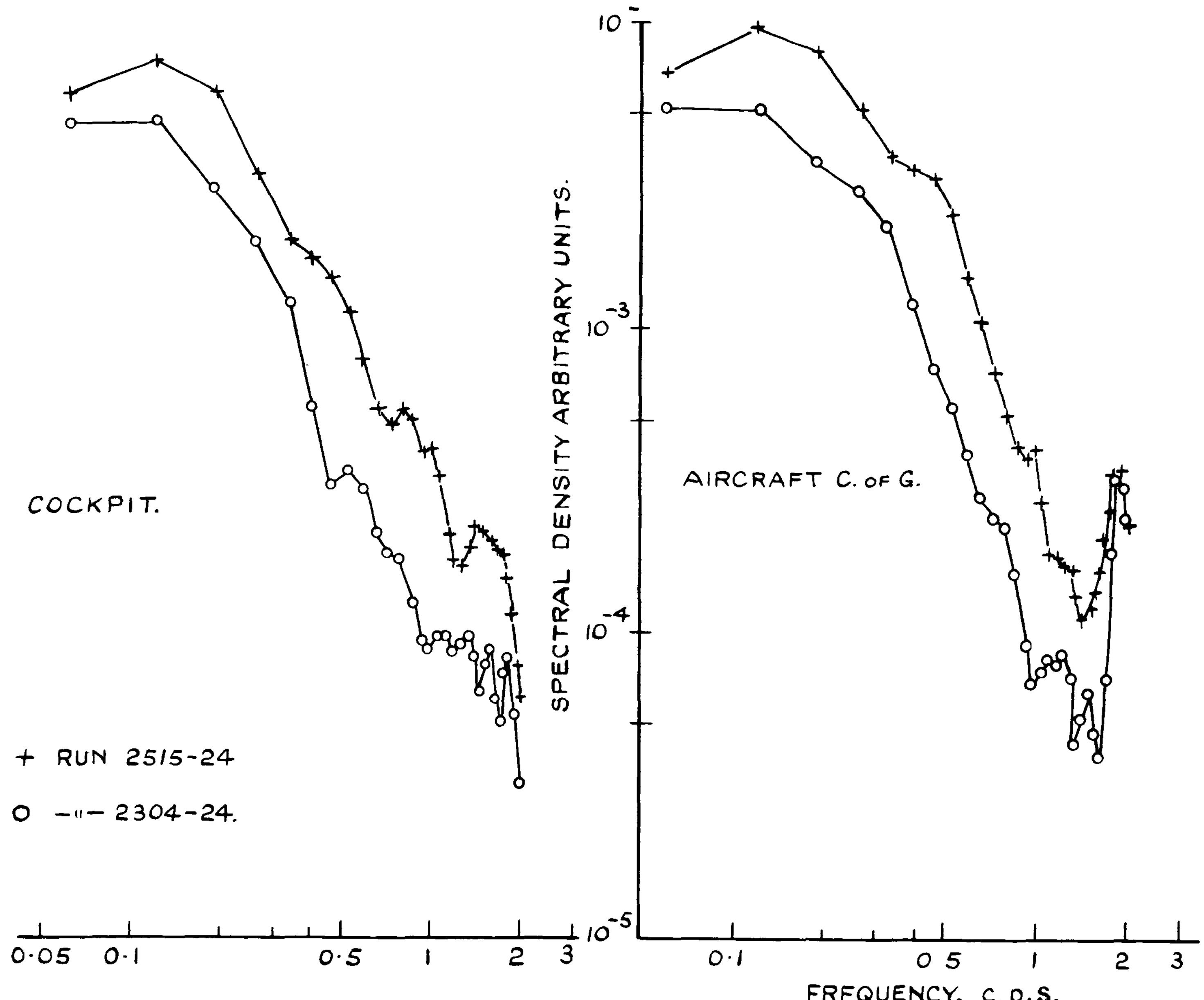


FIG. 8. MEASURED SPECTRAL DENSITIES OF VERTICAL ACCELERATIONS OF A COMET AIRCRAFT. (H=1000 FT; V=220 FT. SEC.$^{-1}$)

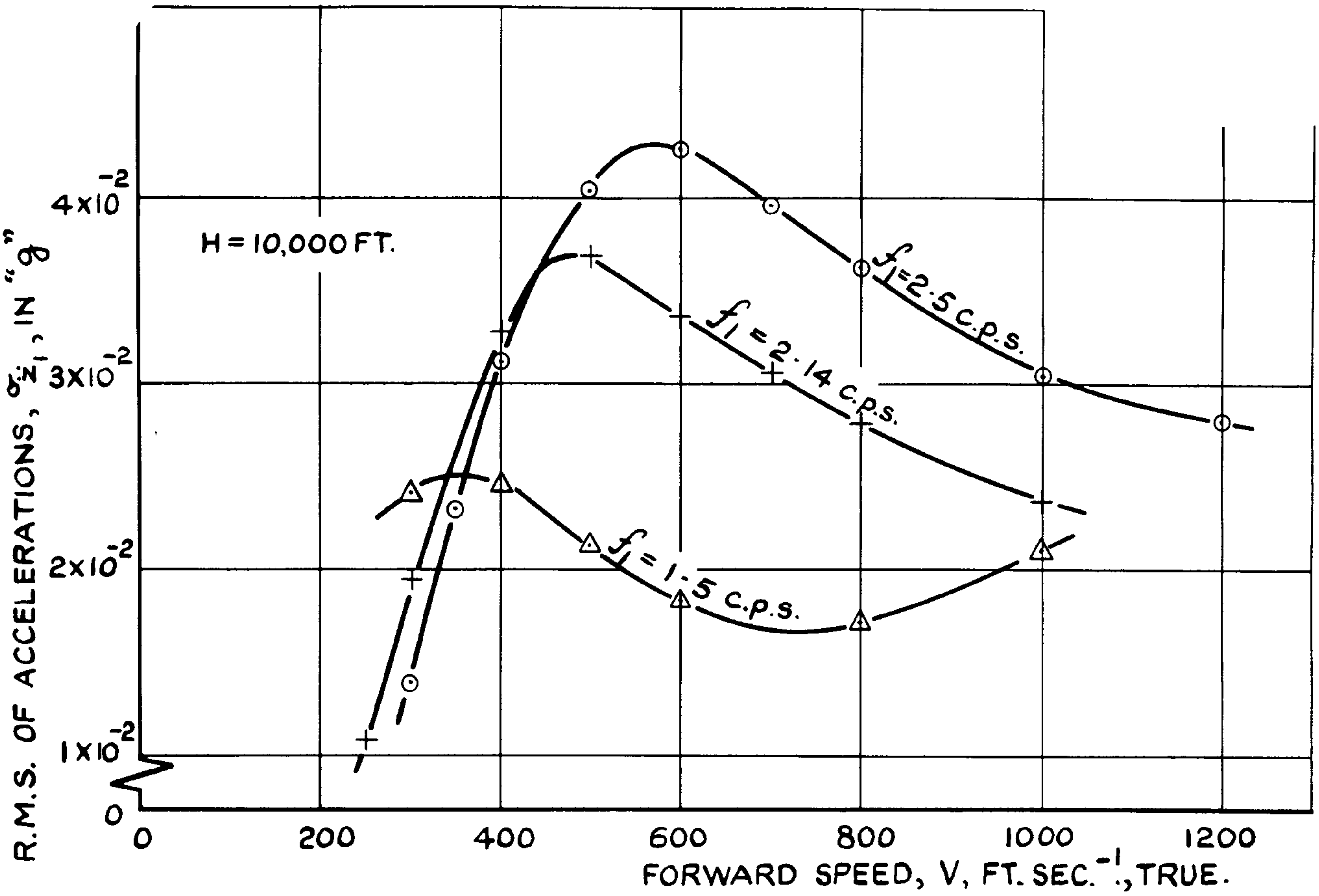


FIG.9. THE EFFECT OF AIRFRAME STIFFNESS ON THE VERTICAL ACCELERATIONS OF FIRST STRUCTURAL MODE DUE TO TURBULENT AIR, $\sigma_{w_g} = 1$ FT. SEC.$^{-1}$ (AT WING APEX, $\xi = 0$)

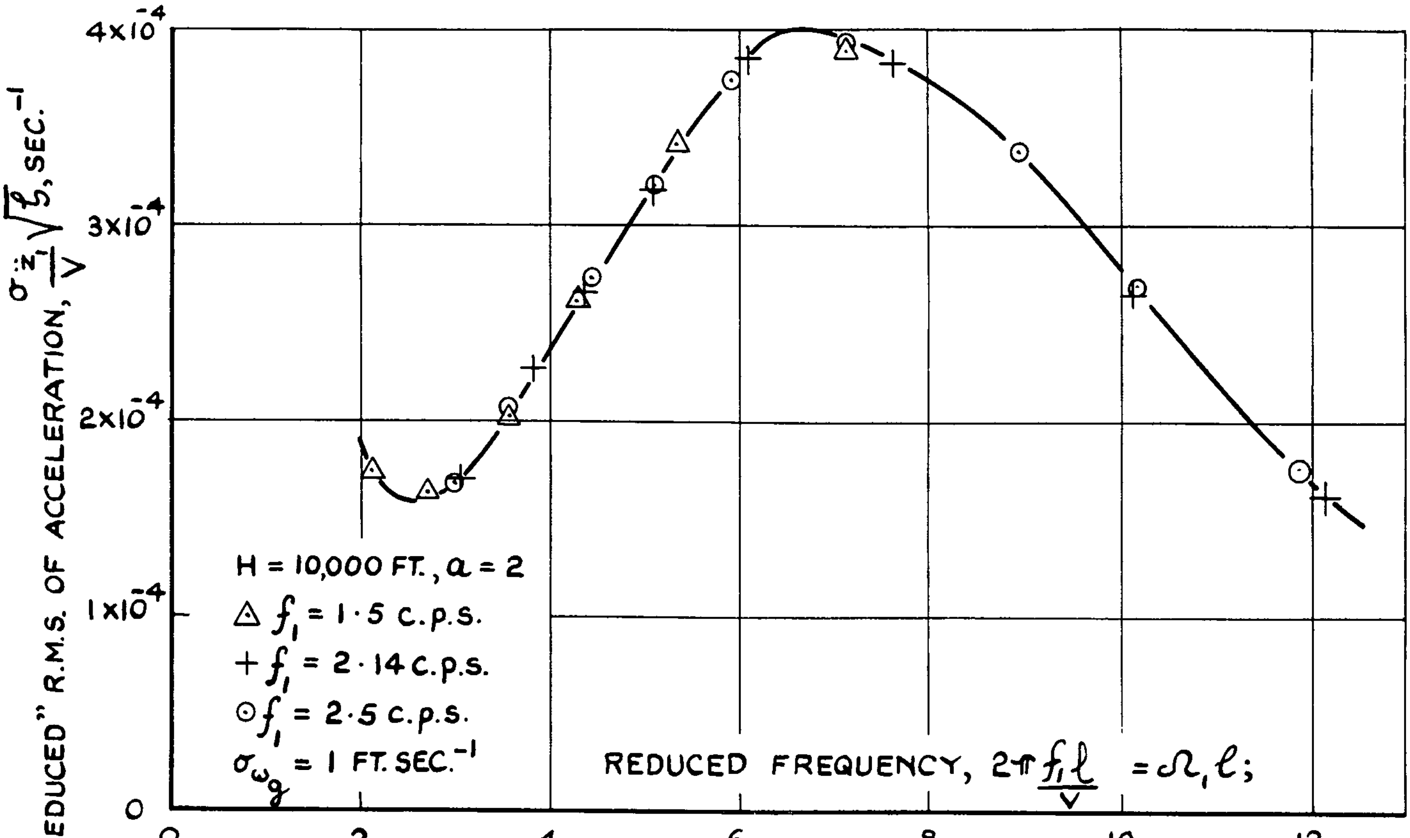


FIG. 10. "REDUCED" ACCELERATION OF FIRST STRUCT. MODE DUE TO TURBULENT AIR. COMBINED EFFECTS OF FORWARD SPEED AND AIRFRAME STIFFNESS.

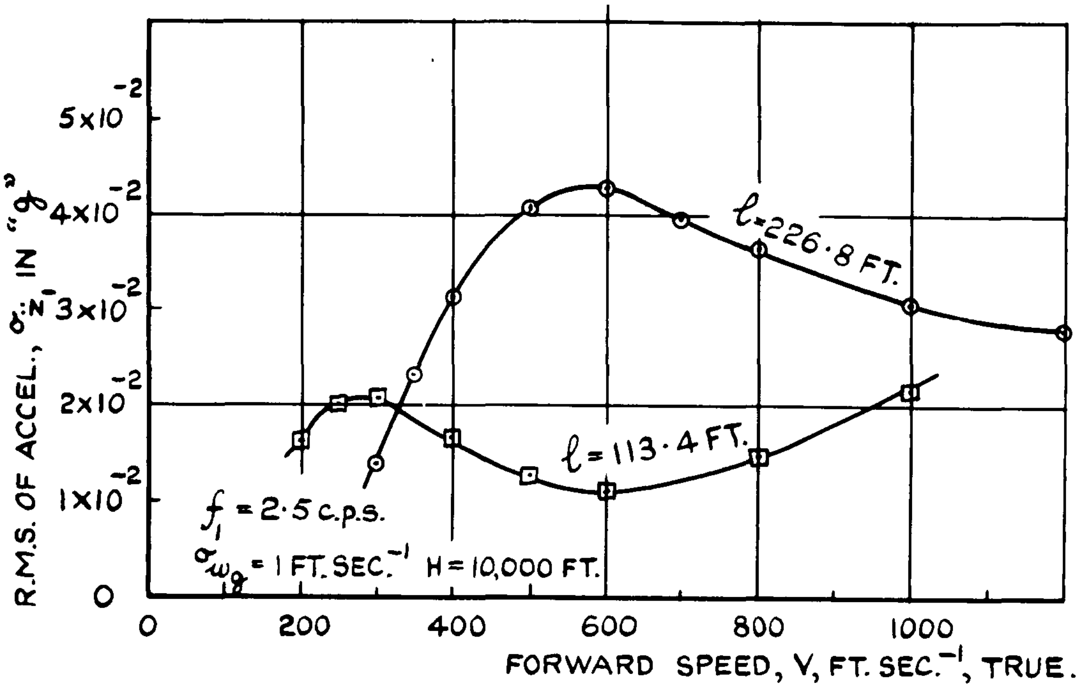


FIG. 11. THE EFFECT OF AIRCRAFT SIZE ON THE ACCELERATIONS OF FIRST STRUCT. MODE IN TURBULENT AIR.

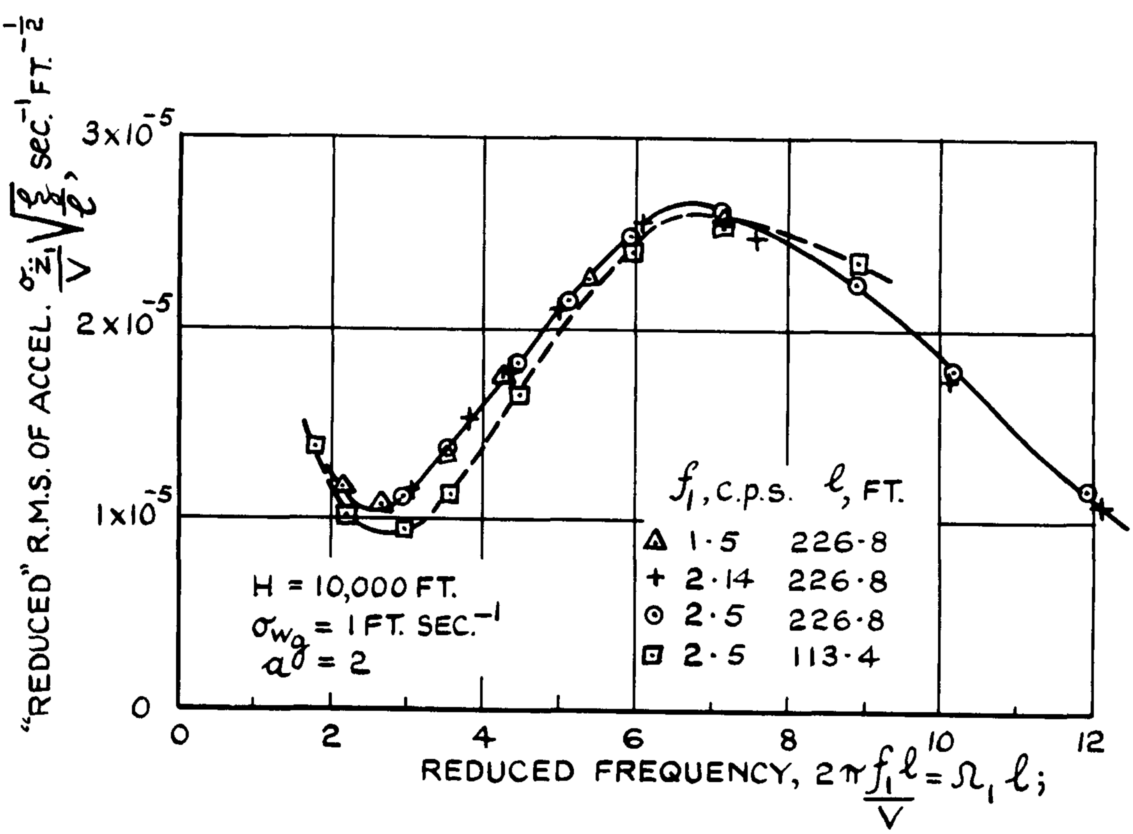


FIG. 12. "REDUCED" ACCELERATION OF FIRST STRUCT. MODE DUE TO TURBULENT AIR. COMBINED EFFECTS OF:- FORWARD SPEED, AIRFRAME STIFFNESS & DAMPING, & OF AIRCRAFT SIZE.

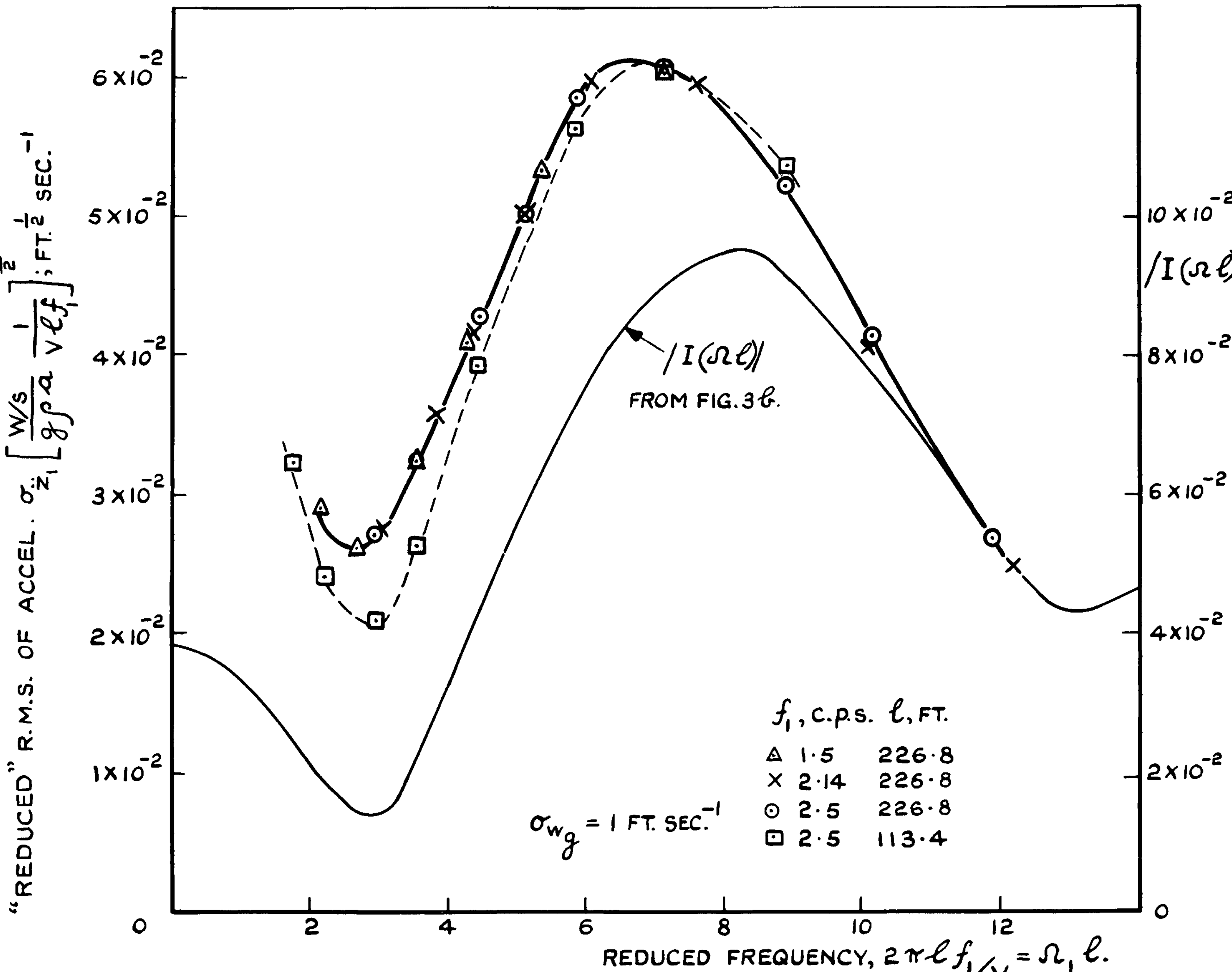


FIG. 13. "REDUCED" ACCELERATION OF FIRST STRUCT. MODE DUE TO TURBULENT AIR.

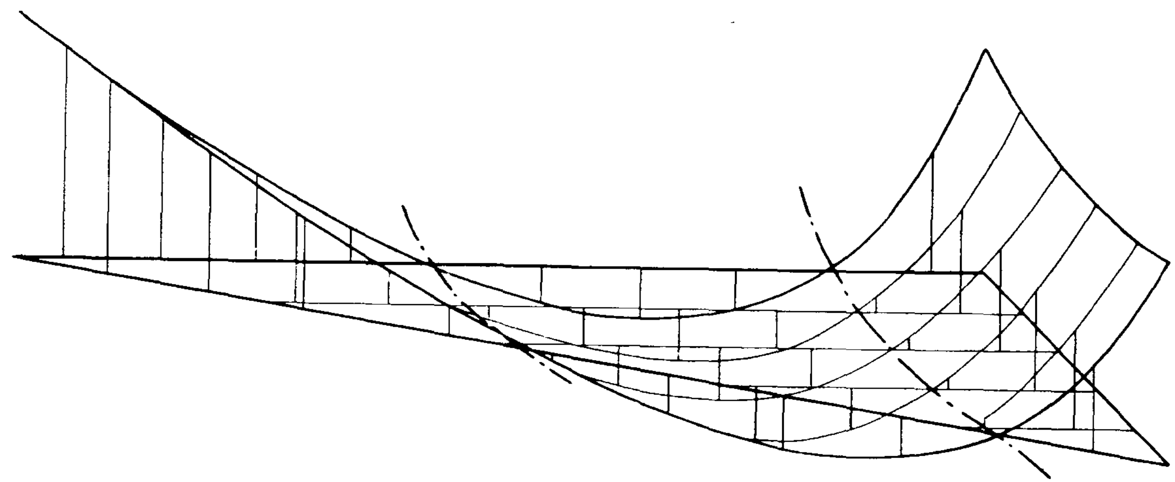


FIG. 14. FIRST NORMAL MODE BASED ON INFLUENCE COEFFICIENTS
$f_1 = 3 \cdot 38 \sqrt{\tau}$ c.p.s. REF. 5.

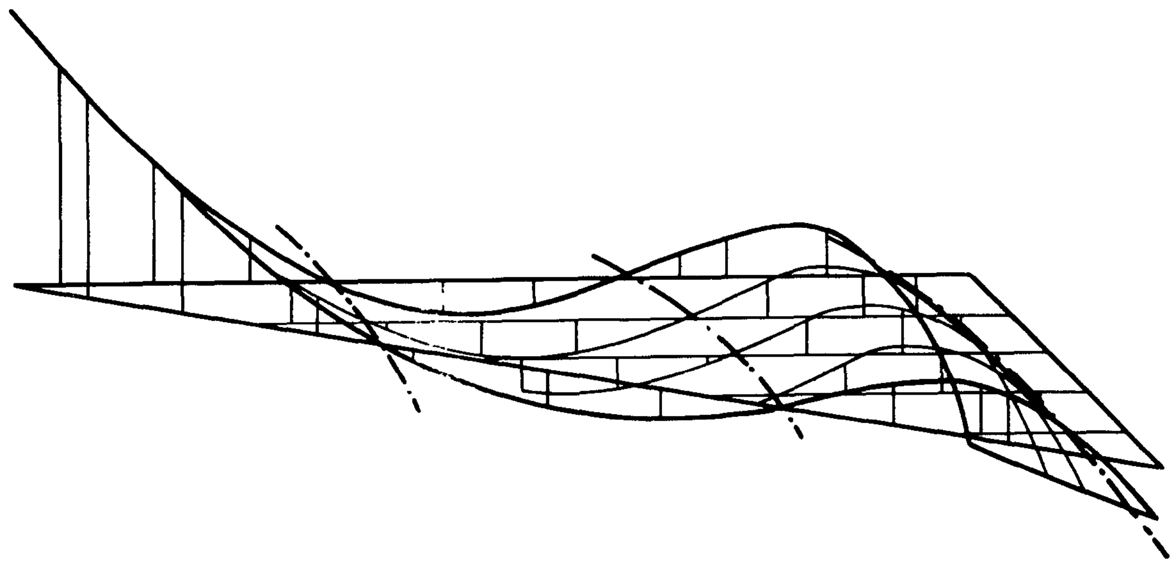


FIG.15. SECOND NORMAL MODE BASED ON INFLUENCE COEFFICIENTS.
$f_2 = 7 \cdot 30 \sqrt{T}$ c.p.s. REF. 5

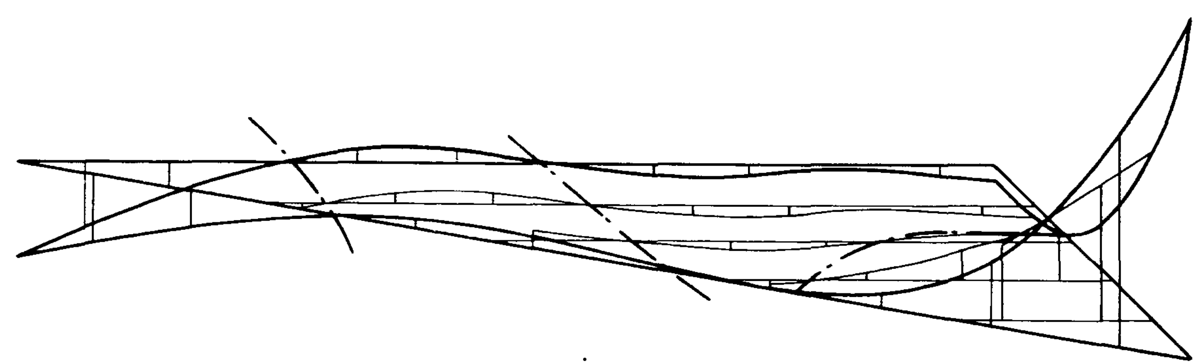


FIG.16. THIRD NORMAL MODE BASED ON INFLUENCE COEFFICIENTS.

$f_3 = 12\cdot60\ \sqrt{\tau}$ c.p.s. REF. 5.

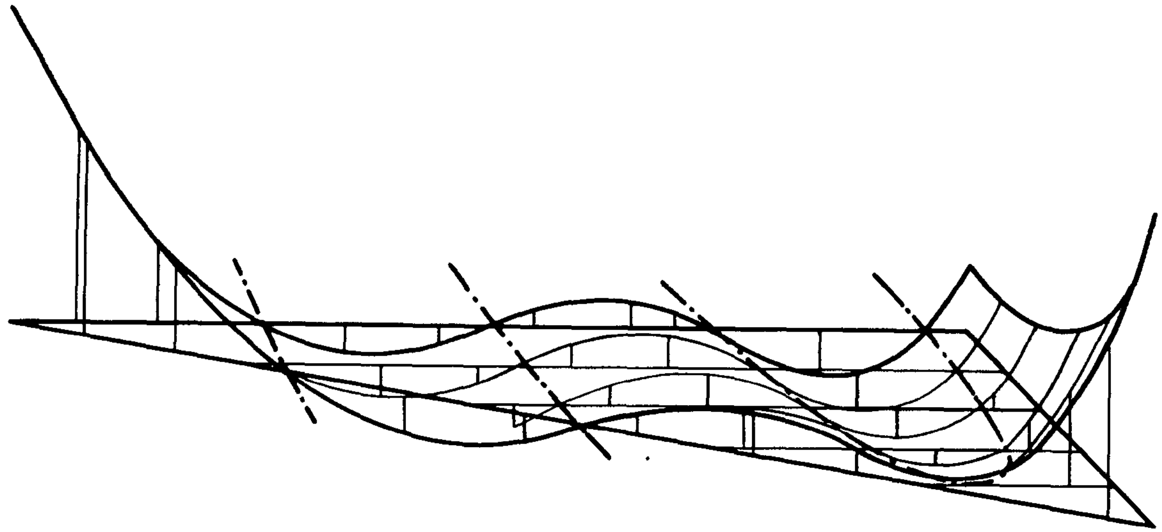


FIG.17. FOURTH NORMAL MODE BASED ON INFLUENCE COEFFICIENTS.
$f_4 = 14 \cdot 12\sqrt{T}$ c.p.s. REF.5.

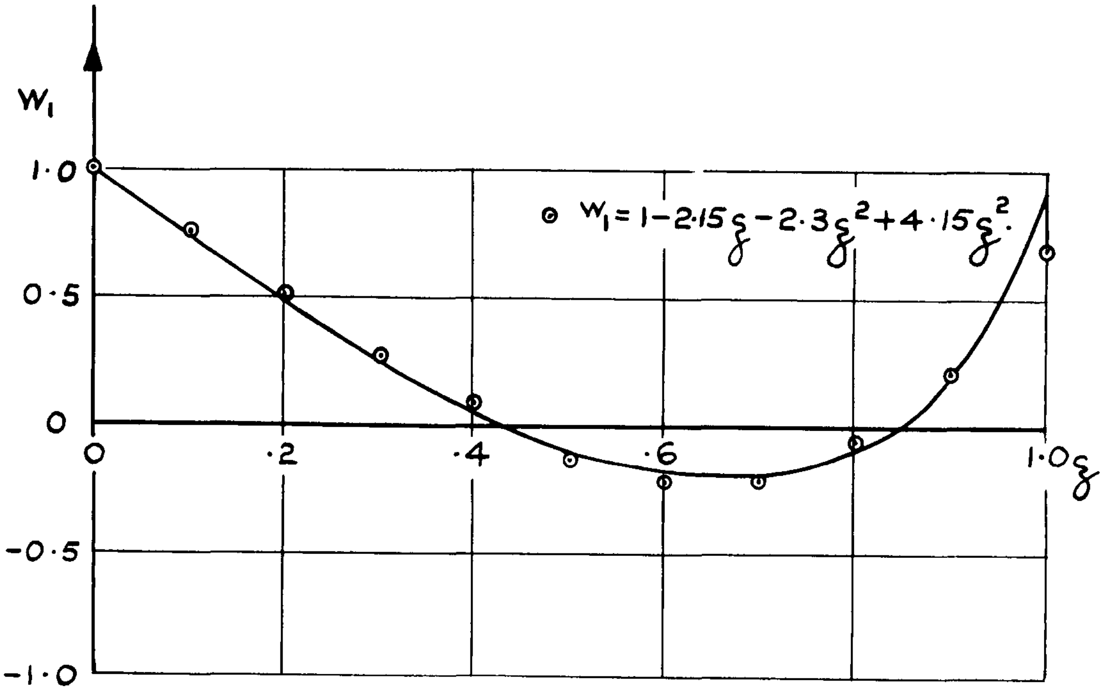


FIG. 18 COMPARISON BETWEEN MODAL SHAPE FROM REF. 5 AND ANALYTICAL APPROXIMATION.

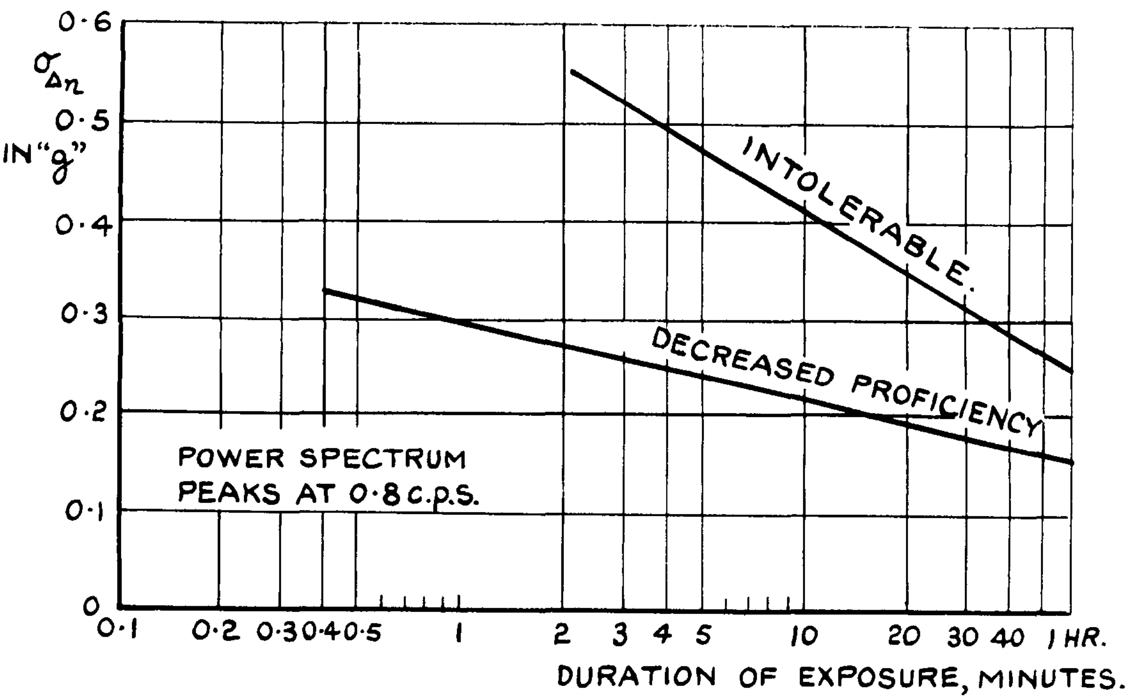


FIG. 19. TIME-INTENSITY VIBRATION BOUNDARIES, REF. 11.

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A numerical study is made of the vertical accelerations due to structural vibrations of a slender aircraft flown in continuous turbulence. The basic aircraft configuration considered is that used in previous aeroelastic studies, but the analysis is extended to cover the effects of aircraft size, airframe stiffness, wing loading, lift slope and speed and altitude of flight.

Only the first structural mode is studied in some detail, but the order of the effects of the rigid body mode and higher structural modes are investigated also.

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