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Methods and Charts for Estimating Skin Friction Drag in Wind Tunnel Tests with Zero Heat Transfer

by

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1 INTRODUCTION

In estimating the skin friction drag of aircraft it is commonly assumed that pressure gradients and the geometry of the flow do not significantly affect the skin friction, so that experimental results for skin friction on flat plates may be used. The development of low drag shapes suitable for economic supersonic flight has led to types of flows which may be strongly three dimensional in character with strong pressure gradients, so that the use of flat plate results needs further examination.

The calculation of laminar boundary layers in three dimensional flow is adequately catered for by existing methods (see for example Cooke and Hall[1]). However most boundary layers occurring in practice will be turbulent. Cooke, in a series of papers[2,3,4,5], has considered the calculation of the growth of turbulent boundary layers in three dimensional flow, and has shown[6,7] that in supersonic flow the frictional drag may be deduced from the boundary layer profiles at the trailing edge. The practical use of Cooke's methods is quite difficult and tedious. Before boundary layer calculations can be made the geometry of the external flow needs to be known quite accurately. At present it is not possible to calculate the flow fields about general bodies in any detail, and experimental investigation of flow fields is both laborious and lengthy. The various assumptions which different workers have made about velocity profiles and shearing stress distributions have been considered in some detail by Cooke[4]. These assumptions have all been based on the small amount of low speed data available, and the validity of their extension to compressible flow has not been adequately checked. As an example of the uncertainty which exists, all the methods which have been proposed take the variation of surface shearing stress along a streamline to be the same as in two dimensional flow. However measurements by Ashkenas and Riddell[8] showed that the rate of boundary layer growth (in the stream direction) over a yawed flat plate was slightly higher than the rate of growth over an unyawed plate, so that this assumption is only approximately correct. In view of these uncertainties it seems that, at least until a much greater body of experimental data is available, some simpler method will give adequate results.

Over the past few years experience in the 8 ft × 8 ft wind tunnel at Bedford has shown that flat plate formulae are capable of giving quite accurate estimates of the skin friction drag of models. For two slender wings for which the overall drag and wave drag are available separately Evans[9] has shown that estimation of friction drags by the methods given here are correct within the experimental accuracy of the other measurements. These wings had fairly small pressure gradients, but even for quite general types of flow the errors are relatively small. For a particular cambered delta wing with fairly strong pressure gradients and three dimensional effects Winter and Smith[10] have shown that the difference between overall drag and wave drag agreed with estimates of skin friction within about 10% over a range of Mach numbers and a fairly wide range of incidence. For more complex types of flow fields and pressure distributions the use of flat plate formulae may be less accurate, but the prediction of scale effect may still be sufficiently accurate for many purposes.

The pressure drag of a model is obtained by subtracting the estimated friction drag from the measured total drag. The total drag of the full scale aircraft is then obtained by adding the estimated full scale friction drag to the deduced pressure drag. An implicit assumption here is that the pressure drag of a model does not change with Reynolds number. The possibility of small changes in pressure drag with change in scale should not be overlooked, but this effect is not considered in the present note.

The charts given here were produced in order to make estimating friction drags a routine matter, and it is felt worth while making them more widely available. The curves have been calculated for zero heat transfer and a tunnel total temperature of 30°C. They cover Mach numbers up to 5, and Reynolds numbers between 10^5 and 10^7 for laminar boundary layers, and between 10^5 and 10^8 for turbulent boundary layers. Variations of total temperature between 0°C and 60°C have an insignificant effect on the calculated skin friction coefficients, and even raising the total temperature to 150°C, as may be necessary at a Mach number of 5 in order to avoid liquefaction of air, changes the skin friction coefficient (at constant Reynolds number) by less than 3%.

The skin friction formulae used are given in Section 3. Section 4 describes the charts, and Section 5 describes methods of using the charts. The charts are strictly applicable only to wind tunnel tests within the above range of total temperatures. Little error would be incurred, however, by using the charts for estimation of full scale friction drag at Mach numbers up to about 2.5. Above this speed the higher temperatures encountered in flight would lead to significant errors, and in any case heat transfer effects are likely to become important in this range of speeds.

2 LIST OF SYMBOLS

c_f local skin friction coefficient

C_F mean skin friction coefficient

M Mach number

Re_x Reynolds number based on distance from effective start of boundary layer

Re_θ Reynolds number based on momentum thickness of boundary layer

T temperature (degrees absolute)

u velocity

x distance from effective start of boundary layer

θ momentum thickness of boundary layer

μ viscosity

ρ density

τ surface shearing stress

Subscripts

1 free stream conditions

r recovery conditions (i.e. conditions at wall for zero heat transfer)

w wall conditions

Superscripts

* intermediate enthalpy conditions

3 SKIN FRICTION FORMULAE

Eckert[11] and Monaghan[12] have shown that the formulae for skin friction in incompressible flow may be used to give the skin friction in compressible flow, provided the physical properties of air (density and viscosity) are evaluated at a point in the boundary layer at a temperature T^* corresponding to an "intermediate enthalpy" i^*. For the low temperatures occurring in wind tunnels the specific heat does not vary with temperature, so that the "intermediate enthalpy" may be directly replaced by "intermediate temperature". Thus we have for Reynolds number

$$\begin{aligned} Re_x^* &= \frac{\rho^* u_1 x}{\mu^*} \; , \\ &= \frac{\rho^*}{\rho_1} \frac{\mu_1}{\mu^*} \frac{\rho_1 u_1 x}{\mu_1} \; , \\ &= \frac{T_1}{T^*} \frac{\mu_1}{\mu^*} Re_x \; , \end{aligned} \tag{1}$$

as the static pressure is assumed constant through the boundary layer. For the local skin friction coefficient we have

$$\begin{aligned} c_f^* &= \frac{\tau_w}{\frac{1}{2}\rho^* u_1^2} \; , \\ &= \frac{\rho_1}{\rho^*} \frac{\tau_w}{\frac{1}{2}\rho_1 u_1^2} \; , \\ &= \frac{T^*}{T_1} c_f \; , \end{aligned} \tag{2}$$

and similarly for the mean skin friction coefficient

4 DESCRIPTION OF CHARTS

4.1 Laminar boundary layers

From equations (1), (2) and (4) we have

$$c_f = \frac{T_1}{T^*} \times 0.664 \left(\frac{T_1}{T^*} \frac{\mu_1}{\mu^*}\right)^{-\frac{1}{2}} Re_x^{-\frac{1}{2}}$$

$$= 0.664 \left(\frac{T_1}{T^*} \frac{\mu^*}{\mu_1}\right)^{\frac{1}{2}} Re_x^{-\frac{1}{2}} \quad , \tag{13}$$

and similarly

$$C_F = 1.328 \left(\frac{T_1}{T^*} \frac{\mu^*}{\mu_1}\right)^{\frac{1}{2}} Re_x^{-\frac{1}{2}} \quad . \tag{14}$$

Assuming a recovery factor[12] of 0.85, with the viscosity given by Sutherlands formula

$$\mu = 3.045 \times 10^{-8} \left(\frac{T^{3/2}}{T + 110.4}\right) \text{slug/ft sec} \quad ,$$

it is found that the factor $[(T_1 \mu^*)/(T^* \mu_1)]^{\frac{1}{2}}$ varies only between 0.967 and 1.014 for Mach numbers less than 5. This factor may reasonably be ignored, as any errors incurred thereby are likely to be less than errors arising from neglect of pressure gradients and three dimensional effects. Thus we have

$$c_f = 0.664 \, Re_x^{-\frac{1}{2}} \quad , \tag{15}$$

shown in Fig.1, and

$$C_F = 1.328 \, Re_x^{-\frac{1}{2}} \quad , \tag{16}$$

shown in Fig.2. Equations (12) and (16) give

$$Re_\theta = 0.664 \, Re_x^{\frac{1}{2}} \quad , \tag{17}$$

shown in Fig.3. Substituting (17) into (15) and (16) gives

$$c_f = 0.441 \, Re_\theta^{-1} \quad , \tag{18}$$

shown in Fig.4, and

$$C_F = 0.882 \, Re_\theta^{-1} \tag{19}$$

shown in Fig.5. The upper limit of Reynolds number has been taken as 10^7 for Re_x, as it is unlikely that a laminar boundary layer will exist at any higher Reynolds number in wind tunnel tests with zero heat transfer.

4.2 Turbulent boundary layers

For turbulent boundary layers a recovery factor of 0.89 has been assumed[12]. Equations (1), (2) and (9) give

$$c_f = 0.288 \frac{T_1}{T^*} \left[\log_{10} \left(\frac{T_1}{T^*} \frac{\mu_1}{\mu^*} Re_x \right) \right]^{-2.45} , \tag{20}$$

which is plotted in Fig.6 at intervals of 0.2 in Mach number from 0.4 to 5.0, and for incompressible flow (M = 0). For the mean skin friction coefficient equations (1), (2) and (7) give

$$C_F = 0.455 \frac{T_1}{T^*} \left[\log_{10} \left(\frac{T_1}{T^*} \frac{\mu_1}{\mu^*} Re_x \right) \right]^{-2.58} , \tag{21}$$

which is plotted in Fig.7 for the same Mach numbers. From (12) and (21) we have

$$Re_\theta = 0.228 \frac{T_1}{T^*} Re_x \left[\log_{10} \left(\frac{T_1}{T^*} \frac{\mu_1}{\mu^*} Re_x \right) \right]^{-2.58} , \tag{22}$$

which is plotted in Fig.8 for M = 0,1,2,3,4 and 5. By cross plotting from Figs.6 and 8 (equations (20) and (22)) c_f is obtained as a function of Re_θ, shown in Fig.9. Similarly C_F as a function of Re_θ is shown in Fig.10 obtained by cross plotting from Figs.7 and 8 (equations (21) and (22)).

Equations (7) and (9), on which Figs.6 to 10 are based, are empirical formulae based on a large number of low speed experiments. For values of Re_x below 10^6 the experimental data has a large amount of scatter, and for Re_x below 10^5 there is very little data available (see for example Smith and Walker[17]). Clearly there is some limit to Re_x below which the use of equations (7) and (9) is not justified. In this connection Preston[18] has suggested that there exists a minimum value of Re_θ for a turbulent boundary layer, which he places at around 320 (for incompressible flow) on the basis of various experimental results. From equations (7) and (12) this corresponds to $Re_x = 0.9 \times 10^5$. Thus it seems reasonable to take $Re^*_x = 10^5$ as a lower limit to the validity of equations (7) and (9). Curves are given on Figs.6 to 10 for $Re^*_x = 10^5$. It is worth pointing out that the values of Re_x corresponding to $Re^*_x = 10^5$ (see for example Fig.6) agree quite closely with those that Van Driest and Blumer[19] found were needed for the establishment of a fully developed turbulent boundary layer downstream of three dimensional transition trips.

5 USE OF CHARTS IN ESTIMATING FRICTION DRAG OF WIND TUNNEL MODELS

Measurements on wind tunnel models may be made with boundary layer transition either free or fixed, and the methods of calculation will differ accordingly. If transition is allowed to occur naturally regions of laminar and turbulent flow are normally detected by flow visualisation. On the other hand if transition is fixed (normally near the leading edge) supplementary flow visualisation tests are not required. These two techniques each have advantages and disadvantages. If regions of laminar flow are present on the model the external flow may be different from that occurring at full scale, as separations and shock wave/boundary layer interactions may be different. On the other hand if transition is fixed allowance must be made for the drag of the tripping device. Evans[9] has shown that at moderate Reynolds numbers ($5 - 10 \times 10^6$ based on mean chord) it is possible to fix transition with only a small penalty which can be estimated sufficiently accurately.

5.1 Free transition

Transition fronts are usually detected by a sublimation technique[20]. Before calculations can be made the relationship between the sublimation pattern and the surface shear needs to be known. It is commonly assumed that the boundary shown on the sublimation pattern indicates the beginning of the fully turbulent boundary layer, and that the flow ahead of this point is laminar, with a sudden rise in surface shear at this point from the value appropriate to a laminar boundary layer to that for a turbulent boundary layer of the same momentum thickness. Tests on a cone reported by Winter, Scott-Wilson and Davies[21] have shown that in general this assumption is incorrect. They show that the sublimation front indicates the end of wholly laminar flow and is followed by a transition region in which the shear gradually rises to that characteristic of a turbulent boundary layer. This conclusion is consistent with the type of flow which Schubauer and Klebanoff[22] have shown to exist in the transition region. Recent tests by Pate and Brillhart[23] on swept wings show that the sublimation front may occur closer to the middle of the transition region, so that the sublimation pattern may need different interpretations in different types of flows.

The extent of the transition region and the variation of surface shear through the transition region still need to be known. Dhawan and Narasimha[24] have collected and analysed most of the available information on flow in the transition region. From their Fig.5, it can be seen that the extent of the transition region is between about $\frac{1}{4}$ and $\frac{1}{2}$ of the length of laminar flow preceding it, so that a reasonable approximation would be to take the length of the transition region as $^1/_3$ of the length of laminar flow preceding it. From their Figs.12 and 13 it can be seen that a reasonable approximation for the distribution of local skin friction is to take a linear increase of local skin friction coefficient with distance through the transition region.

It should be pointed out that the above arguments are derived from experiments in two dimensional flow, where breakdown of laminar flow arises from amplification of Tollmien-Schlichting waves. For three dimensional flow (e.g. swept or slender wings) the flow breakdown is more likely to occur from instability of the secondary flow in the boundary layer, and the details of the flow in the transition region may be quite different.

There may be regions on some models where the transition region is extremely short or almost non-existent. If the boundary layer on the body of a wing-body combination is turbulent, the boundary layer at the root of the wing will be contaminated by lateral spread of turbulence. In these circumstances it would be reasonable to assume that the local skin friction coefficient increases almost instantaneously from that for a laminar boundary layer to that for a turbulent boundary layer (of the same momentum thickness). Such regions may be found by close examination of transition patterns (see e.g. Fig.4 of Ref.9).

It should be pointed out that at moderate Reynolds numbers (5 to 10 million based on mean chord) the various assumptions which can be made about the transition region will lead to variations of less than about 3% in the overall friction drag.

5.2 Fixed transition

The calculations for fixed transition are simpler. The momentum thickness of the boundary layer at the transition trip may be calculated from the extent of laminar flow ahead of it. The flow is assumed to become fully turbulent at this point, with the same momentum thickness, and calculations may then be continued using the turbulent flow charts.

If more accurate results are needed, the effects of the delay in reaching a fully turbulent boundary layer after a transition trip (see Van Driest and Blumer[19]) may be considered. No measurements of local skin friction immediately downstream of transition trips have been made, but it would not be unreasonable to assume a linear growth of local skin friction coefficient with distance, as assumed for free transition.

A simple approximation, which gives quite accurate results in general, is obtained by ignoring the small length of laminar flow ahead of the transition trip, and assuming the flow to be turbulent from the leading edge.

5.3 Details of calculation

The method of calculation suggested here is that which experience with slender wings in the 8 ft × 8 ft wind tunnel at Bedford (see Refs.9 and 10) has shown to give accurate results. If the model has a discrete body (e.g. Model C of Ref.9) the body and wing are treated separately.

(a) Wings

The wing is split up into a number of chordwise strips, the variation of transition Reynolds number and chord between neighbouring strips being small. From the extent of wholly laminar flow we obtain θ_ℓ, the momentum thickness of the laminar boundary layer at the start of the transition region, from Fig.3. We also obtain the local skin friction coefficient at this point, c_{f_ℓ}, from Fig.1. Knowing (or assuming) the extent of the transition region, ΔRe_x, (which may be zero), we obtain the momentum thickness θ_t and local skin friction coefficient c_{f_t} at the end of the transition region by the following procedure.

(1) Assume a value of momentum thickness $\theta_t^{(1)}$ at the end of the transition region.

(2) Find $c_{f_t}^{(1)}$ from $Re_{\theta_t}^{(1)}$ from Fig.9.

(3) Assuming a linear growth of c_f with distance the momentum equation may be integrated to give

$$\theta_t^{(2)} = \theta_\ell + \frac{1}{4}\left(c_{f_\ell} + c_{f_t}^{(1)}\right)\Delta x \quad ,$$

or

$$Re_{\theta_t}^{(2)} = Re_{\theta_\ell} + \frac{1}{4}\left(c_{f_\ell} + c_{f_t}^{(1)}\right)\Delta Re_x \quad . \tag{23}$$

(4) Repeat steps (2) and (3) until two successive values of Re_{θ_t} agree to the accuracy required.

From this value of Re_{θ_t} we obtain, from Fig.8, the value, Re_{x_t}, of the effective Reynolds number of the turbulent boundary layer at this point. The effective Reynolds number of the boundary layer at the trailing edge, Re_c, is obtained by adding the Reynolds number based on the length of chord remaining between the point concerned and the trailing edge, and we obtain Re_{θ_c} from Re_c and Fig.8. The mean skin friction coefficient for this strip is then obtained from

$$C_F = 2\theta_c/c \quad , \tag{24}$$

where $\bar{c}$ is the mean chord of the strip. The friction drag of the complete wing is then obtained by spanwise integration over the various strips.

The above procedure gives accurate results for wings whose surface slopes are small in all directions. However some models (e.g. Models A and B of Ref.9) have a distinct bulge along the centre line, with large surface slopes in the spanwise direction. For these more accurate answers may be obtained by multiplying the friction drag obtained above by the ratio of wetted area to planform area. Caution should be exercised in applying this correction factor arbitrarily, as it is likely to give incorrect results for models with large surface slopes in the streamwise direction, such as two dimensional wings with round leading edges. If the equation of the wing surface is

$$z = z(x,y) \quad ,$$

then the effective wetted area, as far as skin friction drag is concerned, is given by

$$A_e = \iint \sqrt{1 + \left(\frac{\partial z}{\partial y}\right)^2} \, dA \quad , \qquad (25)$$

where the integration is made over the whole surface, and the correction factor is obtained by dividing A_e by the planform area.

(b) Bodies

The calculation for a body is the same as for a strip of wing. If the transition front does not occur at the same streamwise position around the circumference of the body little error is generally incurred by taking a mean position. The frictional drag coefficient is obtained by multiplying C_F from equation (24) by the ratio of effective body wetted area (from equation (25)) to reference area for the model.

(c) Fins, etc.

The frictional drag of fins or any other components over which the flow is approximately two dimensional may be calculated in the same manner as for wings.

5.4 Effects of separations, pressure gradients and three dimensional effects

Provided the flow is attached everywhere, and pressure gradients and three dimensional effects are small, the above procedure will give skin friction drags accurate to 2 or 3% (see Ref.9). However some consideration needs to be given to estimates of skin friction in more general flows.

Measurements of skin friction in the separated flow on the upper surface of a slender biconvex cone[25] have shown wide variations of local skin friction coefficient, suggesting that it would be unwise to rely on flat plate formulae to better than about 10% for the overall friction drag.

Measurements of skin friction have also been made on a cambered delta wing[10] with fairly strong pressure gradients and three dimensional effects. The measurements covered a range of incidence and although the local skin friction coefficients were at some points on the surface quite different from flat plate formulae, the integrated friction drag varied only a little with incidence and was within about 10% of the flat plate estimates.

There may be regions on models where the flow is clearly three dimensional in character (e.g. conical or ogival noses or boat-tailed after-bodies). Bradfield[26] has made a comprehensive survey of measurements of turbulent boundary layers on cones, and concludes that the local skin friction coefficient on a cone is 18% higher than that on a flat plate at the same Mach number and Reynolds number. For the range of Reynolds numbers likely in wind tunnel tests this implies (see Appendix IV of Ref.27) that the mean skin friction coefficient on a cone is about 7% higher than that on a flat plate at the same Mach number and Reynolds number, so that an approximate correction for nose effects should be applied. Usually this correction will be found to be insignificant. The only measurements in converging flow of the type which occurs on after-bodies seem to be those of Winter and Smith[10]. The local skin friction coefficients they measured were as low as half the flat plate values and can be explained, at least qualitatively, by the combined effects of converging flow with an adverse pressure gradient.* In any case any body with a boat-tailed after-body is likely to have a conical or ogival nose. The two effects are of opposite sign and would therefore at least partially cancel, so that little error should be incurred by assuming that they cancel completely.

6 CONCLUSIONS

Formulae and methods for estimating the skin friction drag of wind tunnel models at zero heat transfer have been discussed. Charts are provided to make the estimation simple and rapid.

Estimates which have been made over the past few years suggest that if the flow is attached everywhere, and pressure gradients and three dimensional effects are small, then the estimates may be accurate to 2 or 3%.** On the other hand if there are large regions of separated flow, or strong pressure gradients or three dimensional effects the estimates may be in error by 10%.

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*Recent tests on a waisted body of revolution[28] have also given low values of local skin friction coefficient. These results agree moderately well with theoretical calculations.

**Recent measurements at high Reynolds numbers[29] suggest that intermediate temperature hypothesis may only correlate compressible and incompressible skin friction to within about 5%.

REFERENCES

No.	Author	Title, etc.
1	Cooke, J.C. Hall, M.G.	Boundary layers in three dimensions. Progress in Aeronautical Sciences, Vol.2, pp.221-282. Pergamon Press, London, 1962.
2	Cooke, J.C.	A calculation method for three dimensional turbulent boundary layers. A.R.C. R. & M. 3199. October, 1958.
3	Cooke, J.C.	Boundary layers over infinite yawed wings. Aero. Quart. 11(1960), 333-347.
4	Cooke, J.C.	Three dimensional turbulent boundary layers. A.R.C. C.P.635. June, 1961.
5	Cooke, J.C.	Turbulent boundary layers on delta wings at zero lift. A.R.C. C.P.696. March, 1963.
6	Cooke, J.C.	Boundary layer drag in three dimensional supersonic flow. J. Fluid Mech. 17(1963), 6'-74.
7	Cooke, J.C.	The boundary layer drag of bodies with swept trailing edges in supersonic flow. A.R.C. C.P.699. February, 1963.
8	Ashkenas, H. Riddell, F.R.	Investigation of the turbulent boundary layer on a yawed flat plate. NACA Technical Note 3383. April, 1955.
9	Evans, J.Y.G.	Use of a wind tunnel to determine the performance of slender wings suitable for a supersonic transport aircraft. R.A.E. Technical Note No. Aero 2884 A.R.C. 24,818. 1963. A.R.C. C.P.742. March, 1963.
10	Winter, K.G. Smith, K.G.	Unpublished M.O.A. Report.
11	Eckert, E.R.G.	Survey on heat transfer at high speed. W.A.D.C. Technical Report 54-70. 1954.
12	Monaghan, R.J.	Formulae and approximations for aerodynamic heating rates in high speed flight. A.R.C. C.P.360. October, 1955
13	Schlichting, H.	Boundary layer theory, 4th Edition. McGraw-Hill, New York. 1960.

REFERENCES (Contd.)

No.	Author	Title, etc.
14	Monaghan, R.J.	A review and assessment of various formulae for turbulent skin friction in compressible flow. A.R.C. C.P.142. August, 1952.
15	Peterson, J.B.	A comparison of experimental and theoretical results for the compressible turbulent boundary layer skin friction with zero pressure gradient. N.A.S.A. T.N. D-1795. March, 1963.
16	Smith, K.G.	Unpublished M.O.A. Report.
17	Smith, D.W. Walker, J.H.	Skin friction measurements in incompressible flow. N.A.S.A. TR R-26. 1959.
18	Preston, J.H.	The minimum Reynolds number for a turbulent boundary layer and the selection of a transition device. J. Fluid Mech. 3(1957-58), 373-384.
19	Van Driest, E.R. Blumer, C.B.	Effect of roughness on transition in supersonic flow. A.G.A.R.D. Report 255. 1960.
20	Main-Smith, J.D.	Chemical solids as diffusible coating films for visual indication of boundary layer transition in air and water. A.R.C. R. & M. 2755. February, 1950.
21	Winter, K.G. Scott-Wilson, J.B. Davies, F.V.	Methods of determination and of fixing boundary layer transition on wind tunnel models at supersonic speeds. A.R.C. C.P.212. September, 1954.
22	Schubauer, G.B. Klebanoff, P.S.	Contributions on the mechanics of boundary layer transition. Boundary layer effects in Aerodynamics. Proceedings of a Symposium held at N.P.L. H.M.S.O., 1955.
23	Pate, S.R. Brillhart, R.E.	Investigation of boundary layer transition on swept wings at Mach numbers 2.5 to 5. AEDC-TDR-63-109. 1963.
24	Dhawan, S. Narasimha, R.	Some properties of boundary layer flow during the transition from laminar to turbulent motion. J. Fluid Mech. 3(1957-58), 418-436.
25	Smith, K.G. Gaudet, L. Winter, K.G.	The use of surface pitot tubes as skin friction meters at supersonic speeds. A.R.C. R. & M. 3351. June, 1962.

REFERENCES (Contd.)

No.	Author	Title, etc.
26	Bradfield, W.S.	Research on laminar and turbulent boundary layers at supersonic speeds. Final summary report. Minnesota University, Rosemount Aero. Lab. Report No. 131. 1957.
27	Davies, F.V. Cooke, J.R.	Boundary layer measurements on 10° and 20° cones at $M = 2.45$ and zero heat transfer. A.R.C. C.P.264. November, 1954.
28	Winter, K.G. Smith, K.G. Rotta, J.C.	Turbulent boundary layer studies on a waisted body of revolution. Proceedings of Agard Specialists Meeting "Recent Developments in Boundary Layer Research", Naples, Italy, 10-14th May, 1965.
29	Winter, K.G. Smith, K.G. Gaudet, L.	Measurements of turbulent skin friction at high Reynolds numbers at Mach numbers of 0.2 and 2.2. Proceedings of Agard Specialists Meeting "Recent Developments in Boundary Layer Research", Naples, Italy, 10-14th May, 1965.

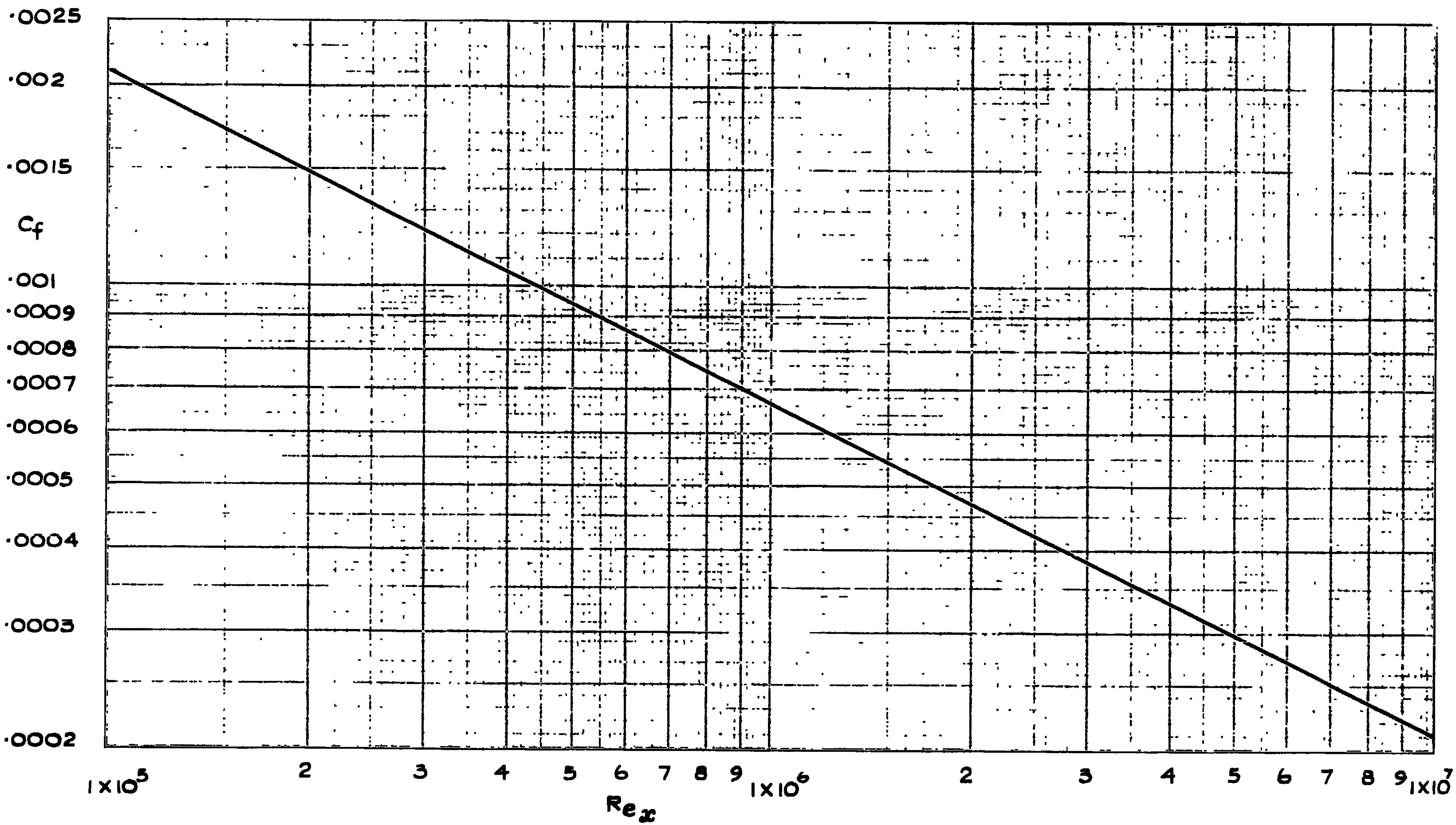

FIG. 1. LAMINAR BOUNDARY LAYERS: LOCAL SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON DISTANCE FROM START OF BOUNDARY LAYER.

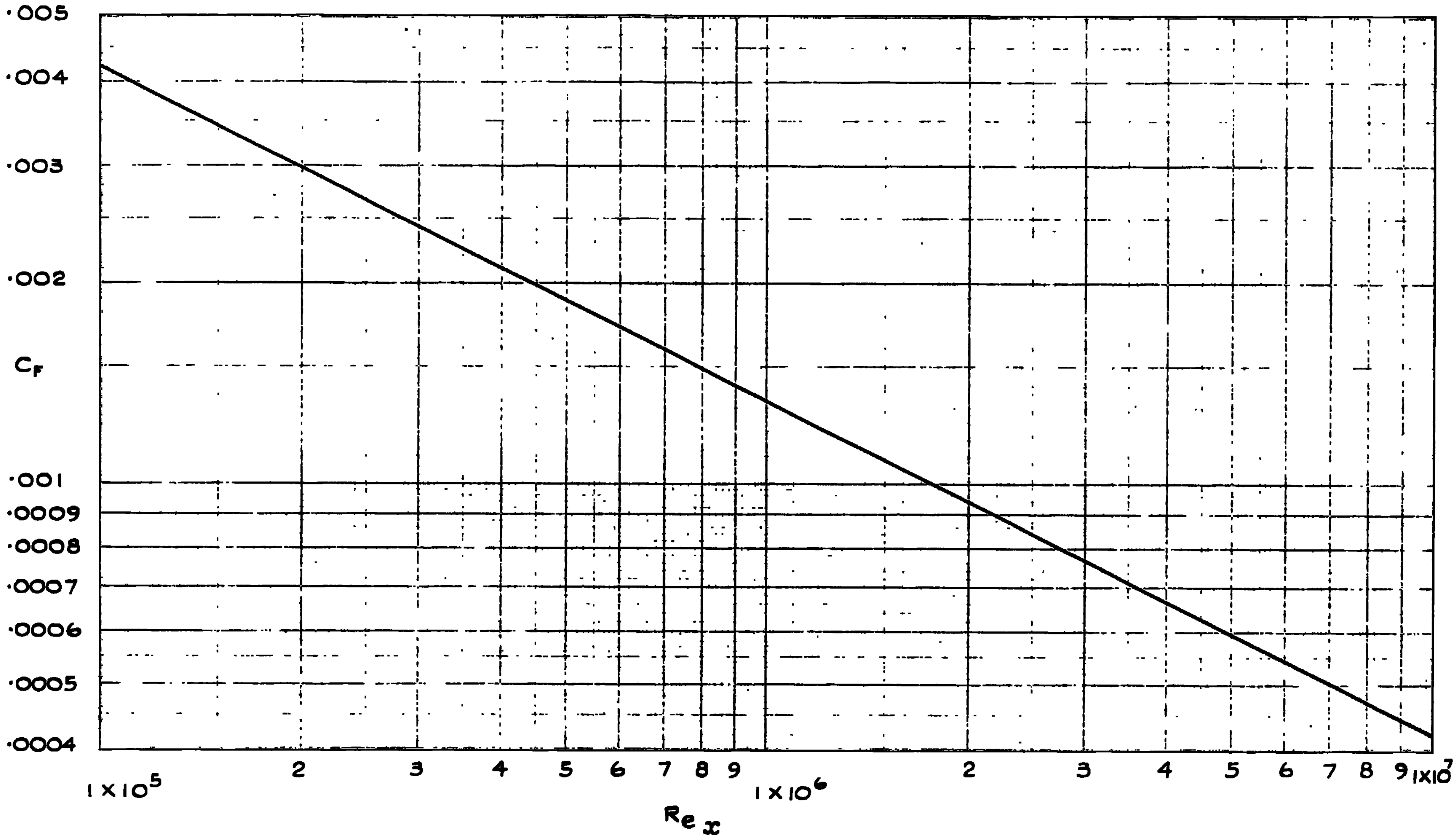

FIG. 2. LAMINAR BOUNDARY LAYERS: MEAN SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON DISTANCE FROM START OF BOUNDARY LAYER.

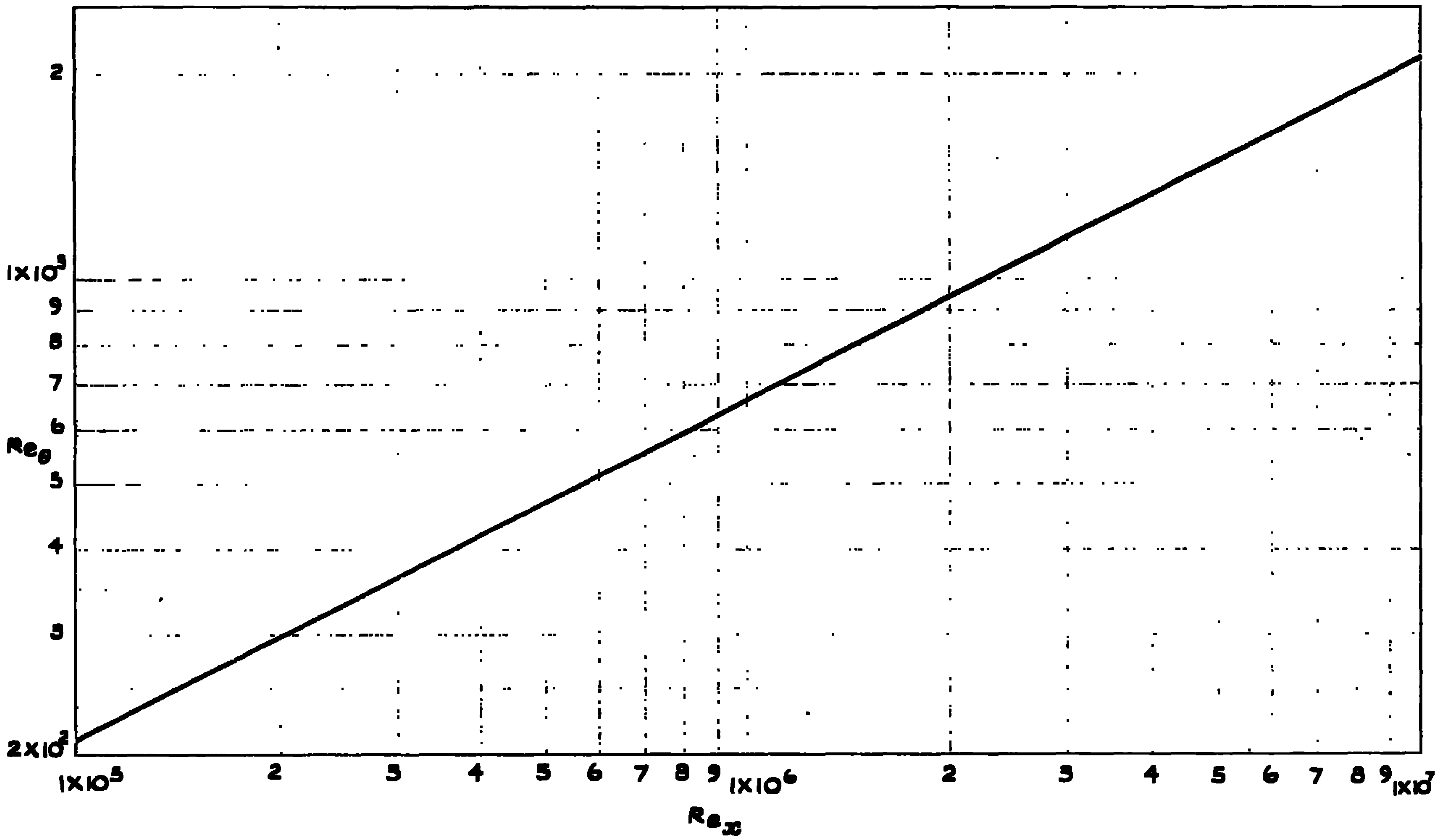

FIG. 3. LAMINAR BOUNDARY LAYERS: RELATIONSHIP BETWEEN REYNOLDS NUMBERS BASED ON MOMENTUM THICKNESS OF BOUNDARY LAYER AND DISTANCE FROM START OF BOUNDARY LAYER.

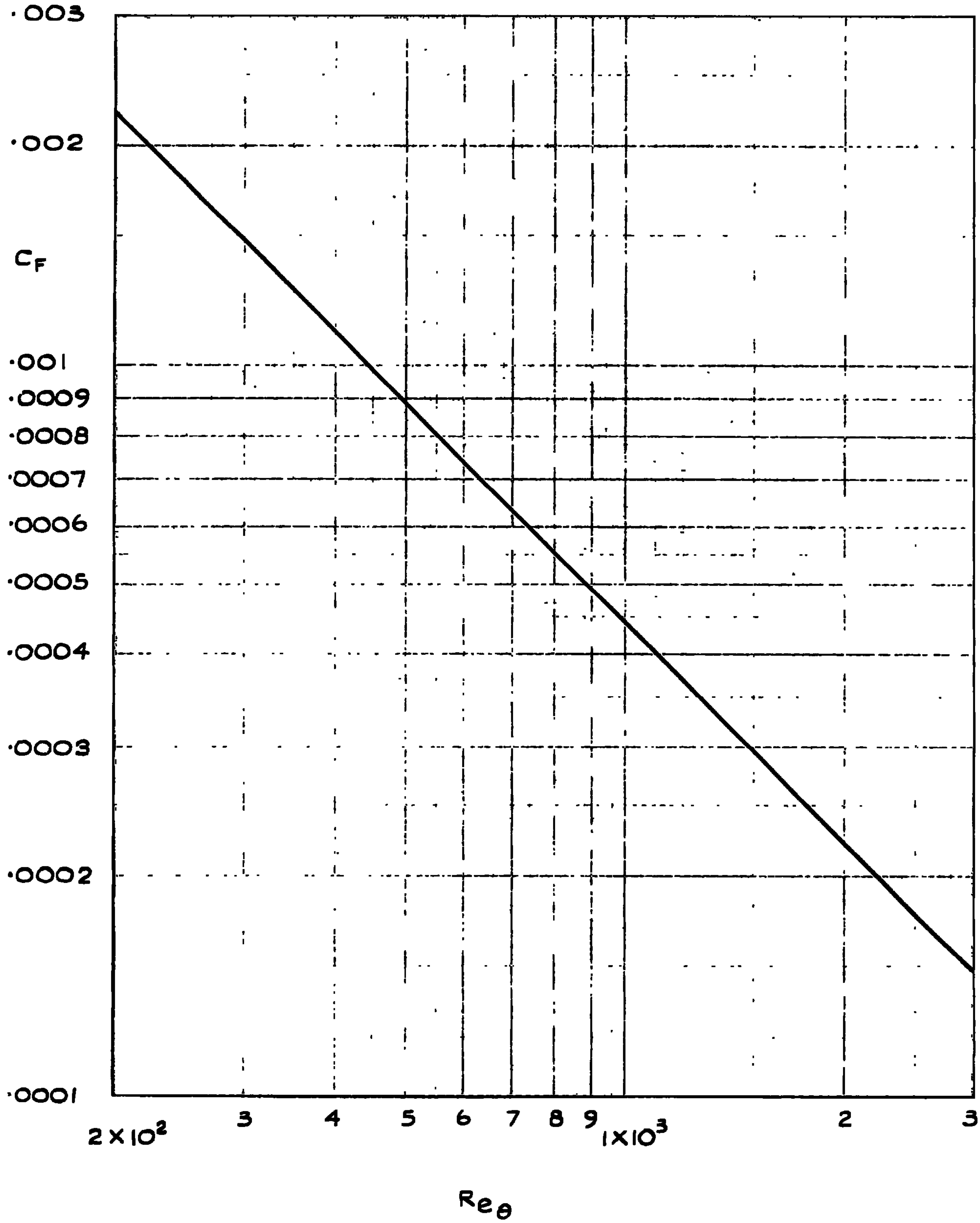

FIG. 4. LAMINAR BOUNDARY LAYERS: LOCAL SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON MOMENTUM THICKNESS OF BOUNDARY LAYER.

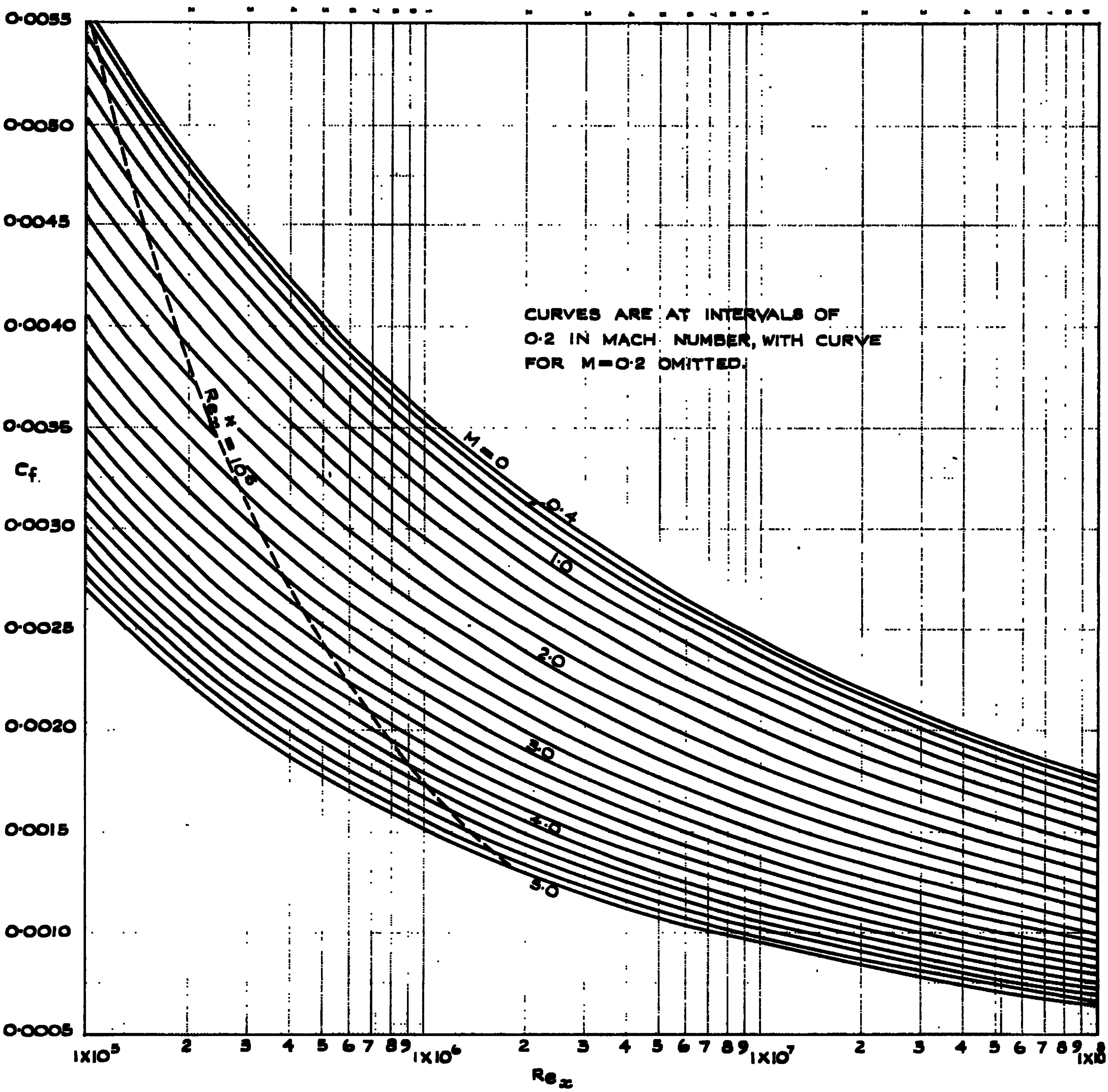

FIG. 6. TURBULENT BOUNDARY LAYERS: LOCAL SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON DISTANCE FROM EFFECTIVE START OF BOUNDARY LAYER.

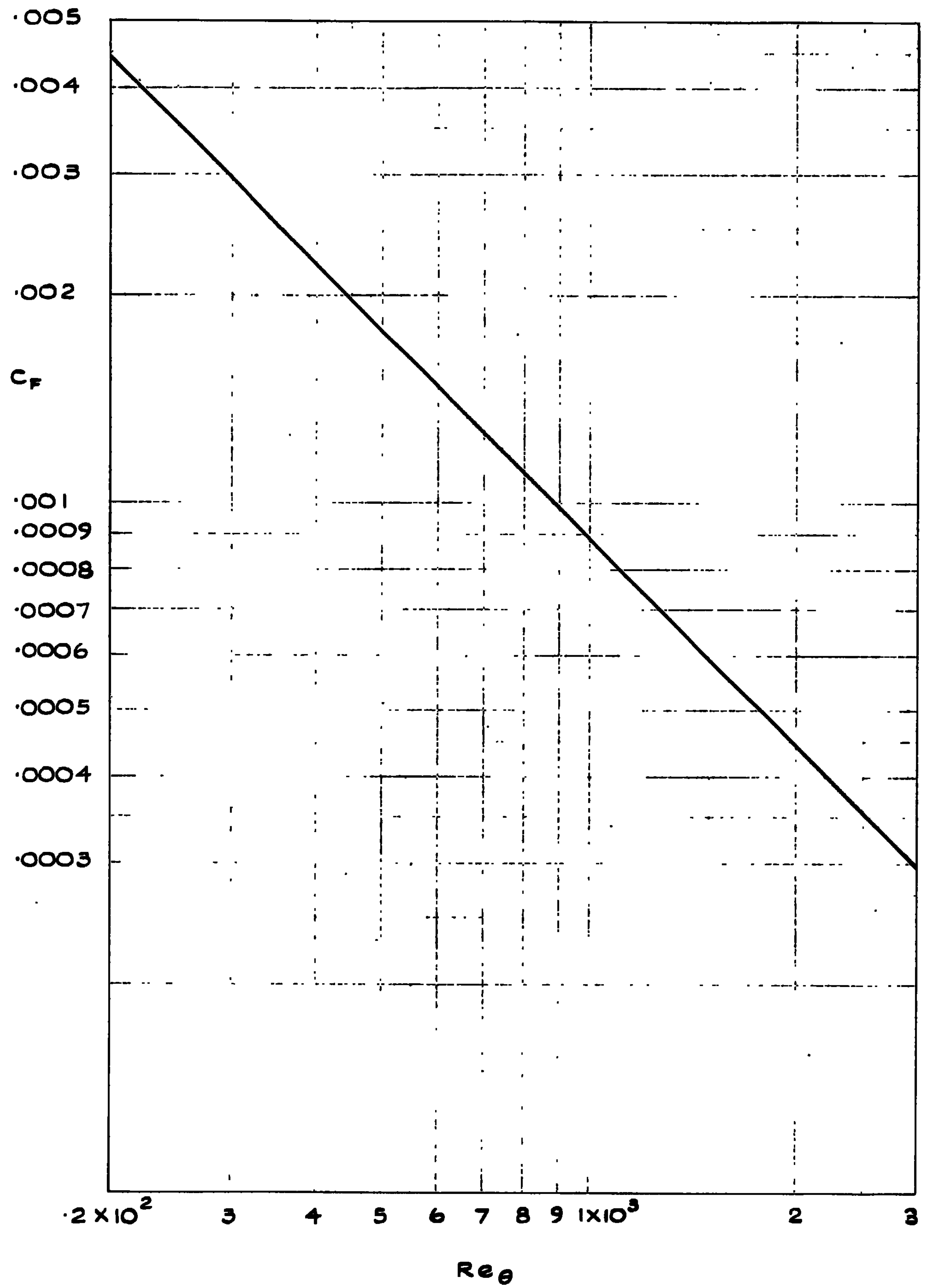

FIG. 5. LAMINAR BOUNDARY LAYERS: MEAN SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON MOMENTUM THICKNESS OF BOUNDARY LAYER.

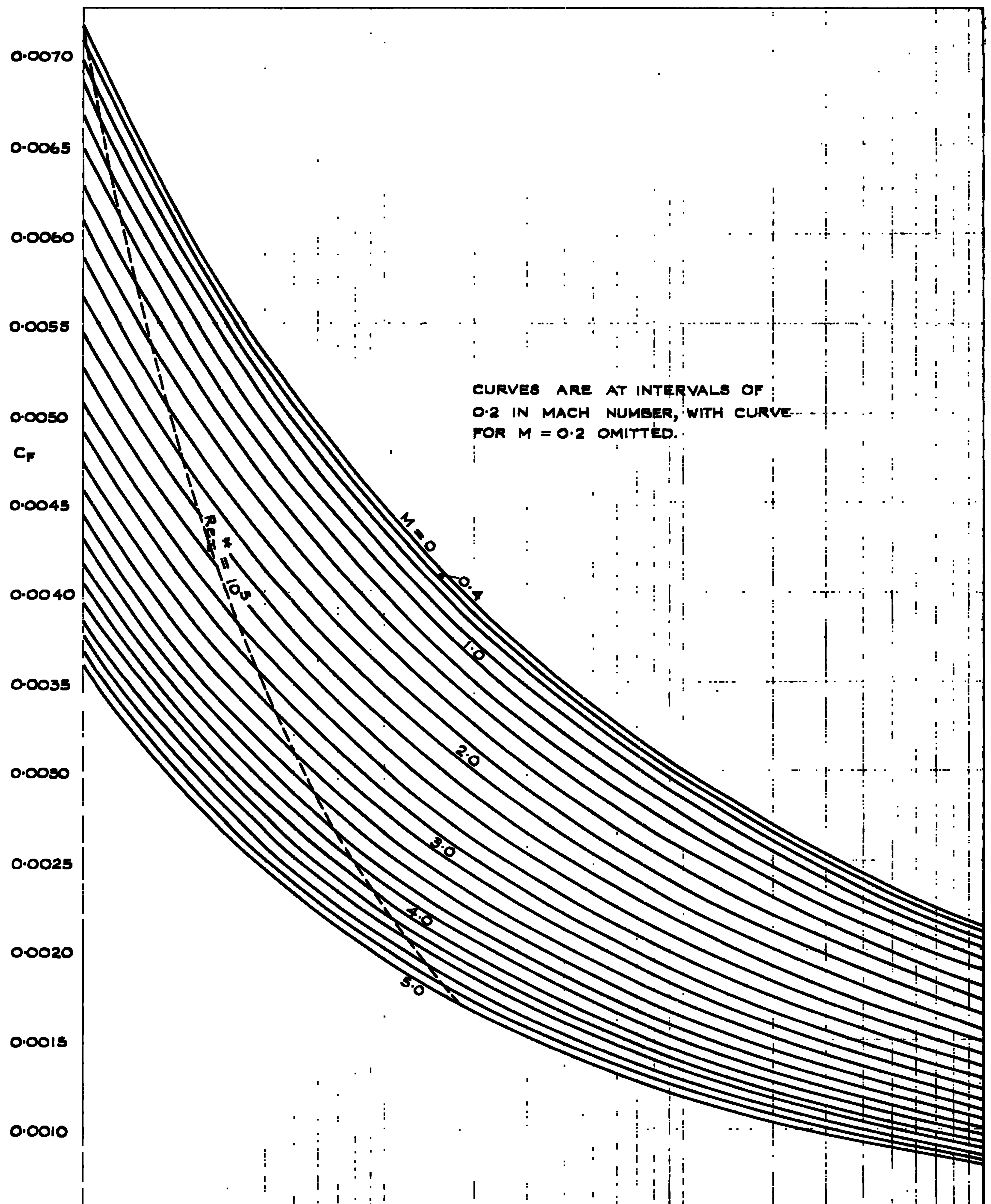

FIG. 7. TURBULENT BOUNDARY LAYERS: MEAN SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON DISTANCE FROM EFFECTIVE START OF BOUNDARY LAYER.

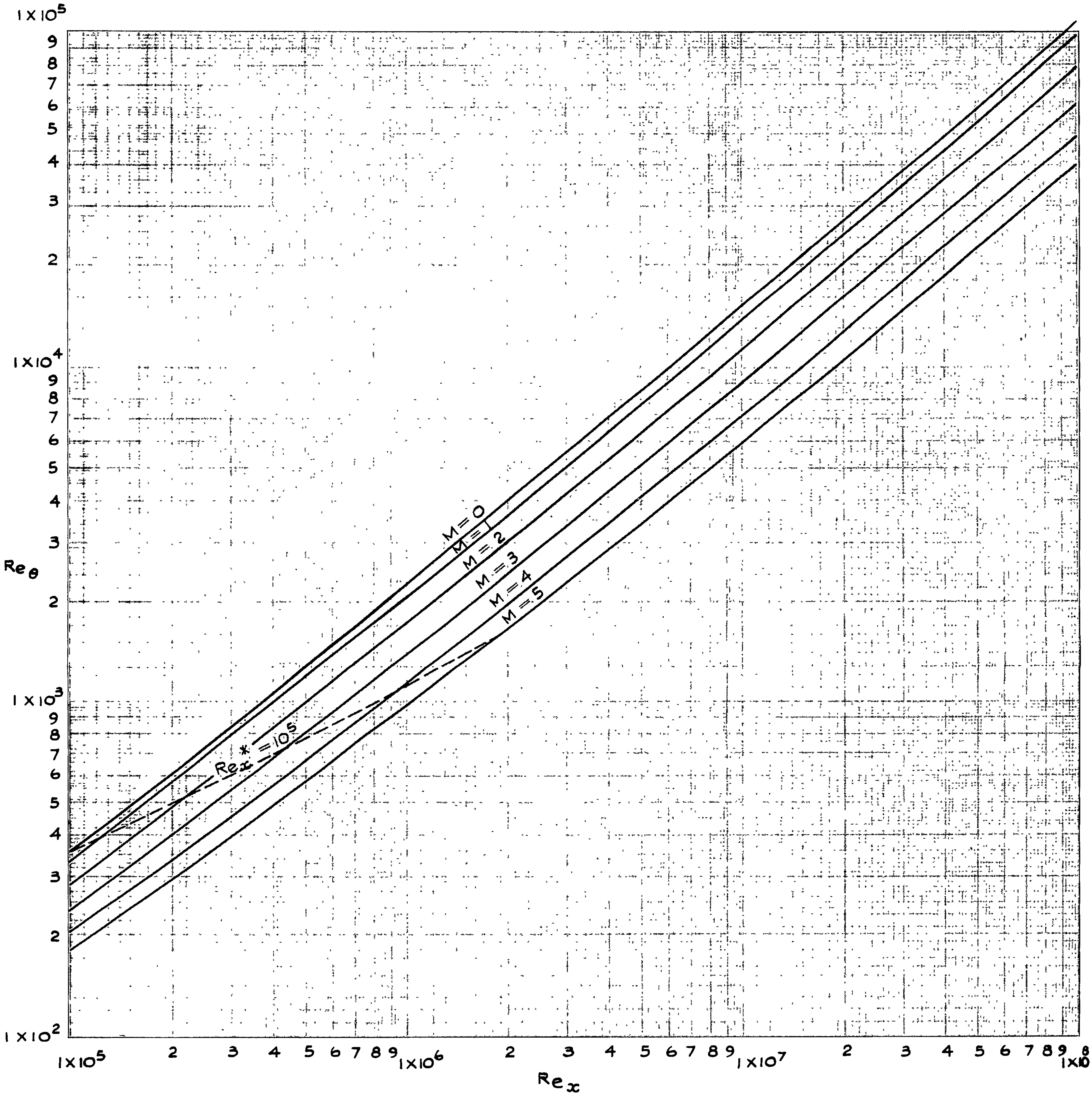

FIG. 8. TURBULENT BOUNDARY LAYERS: RELATIONSHIP BETWEEN REYNOLDS NUMBERS BASED ON MOMENTUM THICKNESS OF BOUNDARY LAYER AND DISTANCE FROM EFFECTIVE START OF BOUNDARY LAYER.

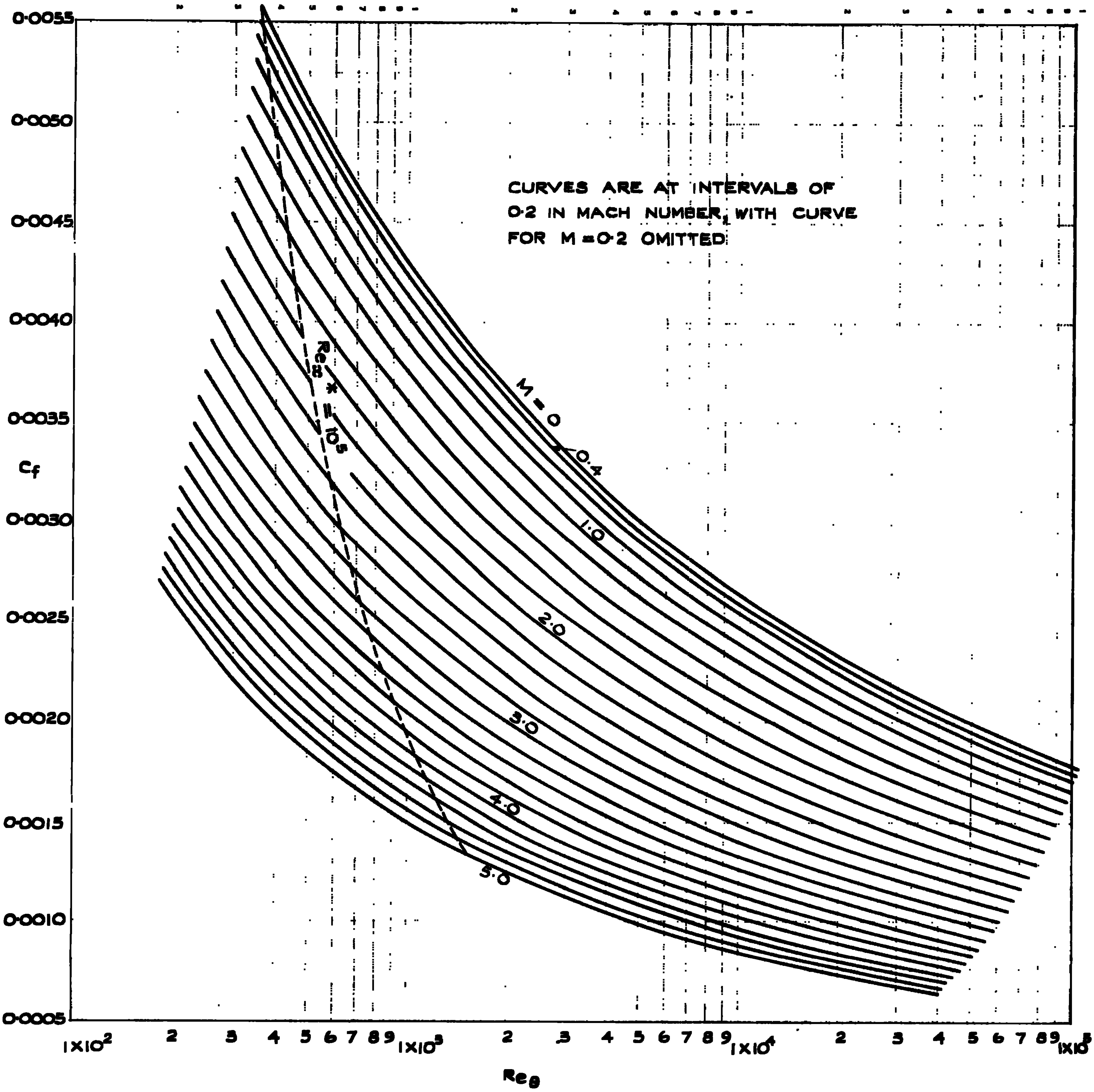

FIG.9. TURBULENT BOUNDARY LAYERS: LOCAL SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON MOMENTUM THICKNESS OF BOUNDARY LAYER.

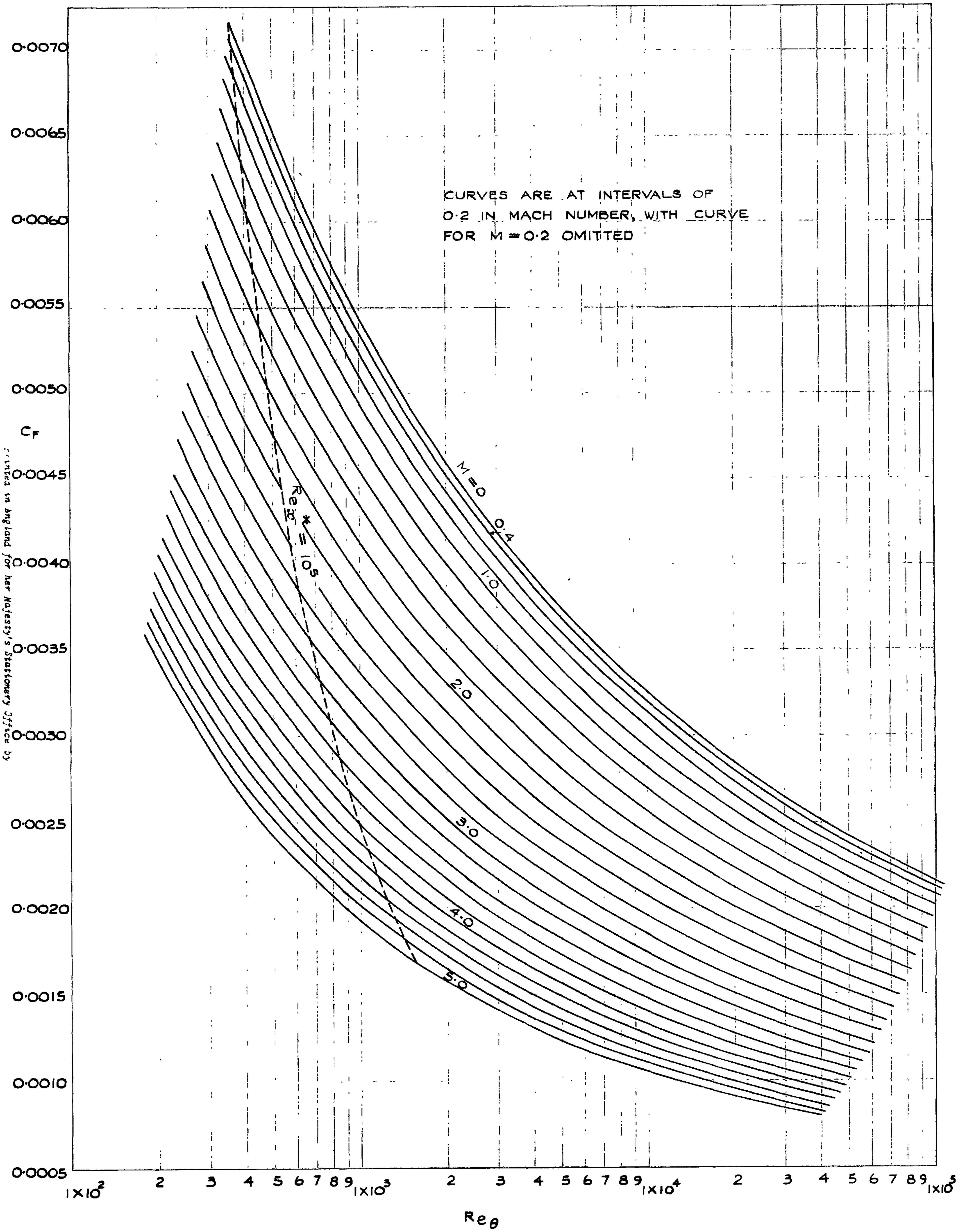

FIG. 10. TURBULENT BOUNDARY LAYERS: MEAN SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER BASED ON MOMENTUM THICKNESS OF BOUNDARY LAYER.

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533.6.011.6 :
532.526.2 :
532.526.4

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Smith, K.G. August, 1964.

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