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By

W.J. Graham and Miss B.M. Davis

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The Change of Pitot Pressure across Oblique Shock Waves in a Perfect Gas

- By -
W. J. Graham and Miss B. M. Davis

December, 1963

1. In this note the variation of pitot pressure across an oblique shock wave in the two-dimensional flow of a perfect gas is examined by means of explicit formulae. Results are presented over the full range of shock angles for Mach numbers from 1·0 to ∞. This information is not readily available from standard references on shock wave relationships, and is given here to assist in interpreting pitot tube measurements in flows involving oblique shocks.

2. By manipulation of the usual oblique shock relations and the Rayleigh pitot formula (see for example Ref. 1) the ratio of pitot pressure across such a shock is given by

$$\frac{P_2}{P_1} = \left(\frac{M_2^2}{M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \left[\frac{2\gamma M_1^2 - (\gamma-1)}{2\gamma M_2^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}} \left[\frac{2\gamma M_1^2 \sin^2\theta - (\gamma-1)}{\gamma + 1}\right], \quad \ldots (1)$$

if $M_2 > 1$.

The pressure, P, is that indicated by a pitot tube: in supersonic flow this is the total pressure behind a normal shock; in subsonic flow it is just the stream total pressure. Suffices '1' and '2' denote conditions ahead of and behind the oblique shock respectively, M is the Mach number, θ is the shock angle, and γ the ratio of the specific heats (assumed constant).

M_2 is related to M_1 and $\sin\theta$ by the equation (Ref. 1)

$$M_2^2 = \frac{(\gamma+1)^2 M_1^4 \sin^2\theta - 4(M_1^2 \sin^2\theta - 1)(\gamma M_1^2 \sin^2\theta + 1)}{[2\gamma M_1^2 \sin^2\theta - (\gamma-1)][(\gamma-1)M_1^2 \sin^2\theta + 2]} . \quad \ldots (2)$$

For the case in which $M_2 < 1$, the pitot pressure behind the shock is identically equal to the total pressure, and the pitot pressure ratio becomes

$$\frac{P_2}{P_1} = \left[\frac{\sin^2\theta\{(\gamma-1)M_1^2 + 2\}}{(\gamma-1)M_1^2 \sin^2\theta + 2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{2\gamma M_1^2 - (\gamma-1)}{2\gamma M_1^2 \sin^2\theta - (\gamma-1)}\right]^{\frac{1}{\gamma-1}} . \quad \ldots (3)$$

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If M_1 is sufficiently large these equations become, respectively,

$$\frac{P_2}{P_1} = (M_2)^{\frac{2\gamma}{\gamma-1}} \left[\frac{2\gamma}{2\gamma M_2^2 - (\gamma-1)} \right]^{\frac{1}{\gamma-1}} \left[\frac{2\gamma \sin^2\theta}{\gamma + 1} \right], \qquad \ldots (1a)$$

$$M_2^2 = \frac{(\gamma+1)^2 - 4\gamma \sin^2\theta}{2\gamma(\gamma-1) \sin^2\theta}, \qquad \ldots (2a)$$

and for $M_2 < 1$

$$\frac{P_2}{P_1} = (\mathrm{cosec}\ \theta)^{\frac{2}{\gamma-1}}. \qquad \ldots (3a)$$

3. Using the above relations, the ratio of the pitot pressure across an oblique shock wave in air ($\gamma = 7/5$) has been calculated over the full range of shock angles for various Mach numbers from unity to infinity and is shown in Fig. 1. The curves to the right of the dotted line correspond to subsonic flow downstream of the shock ($M_2 < 1$).

4. For the case in which $M_2 > 1$, P_1 is the total pressure behind the normal shock (at M_1) occurring at the mouth of the pitot tube; whereas P_2 is the total pressure behind both an oblique shock (at M_1) and a normal shock (at $M_2 < M_1$) again at the mouth of the pitot tube. The two-shock system leading to P_2 has less losses than the single shock giving P_1, so that P_2 is greater than P_1. As the oblique shock wave angle is decreased, for a given upstream Mach number, the strength of the oblique shock decreases and in the limit, when the shock angle equals the Mach angle, $M_2 = M_1$ and it follows, therefore, that P_2/P_1 is unity.

For the case in which $M_2 < 1$, P_1 is again the total pressure behind a normal shock at M_1, but P_2 is the total pressure behind a 'strong' oblique shock at the same Mach number. The latter shock has the smaller losses and so P_2 is greater than P_1. As the oblique shock angle approaches 90°, the oblique shock becomes a normal shock and P_2/P_1 is again unity.

Between the two limits (Fig. 1), for which $P_2 = P_1$, the ratio P_2/P_1 increases to a maximum value for a given value of upstream Mach number, M_1. With increasing M_1 this peak value of P_2/P_1 increases, whilst the shock angle at which it occurs decreases. A limiting value of P_2/P_1 equal to 6 occurs at infinite M_1 and small θ. In general, the rise of pitot pressure across an oblique shock increases with increasing Mach number for a given shock angle.

Reference

1. Equations, Tables and Charts for Compressible Flow. N.A.C.A. Rpt. 1135. 1953.

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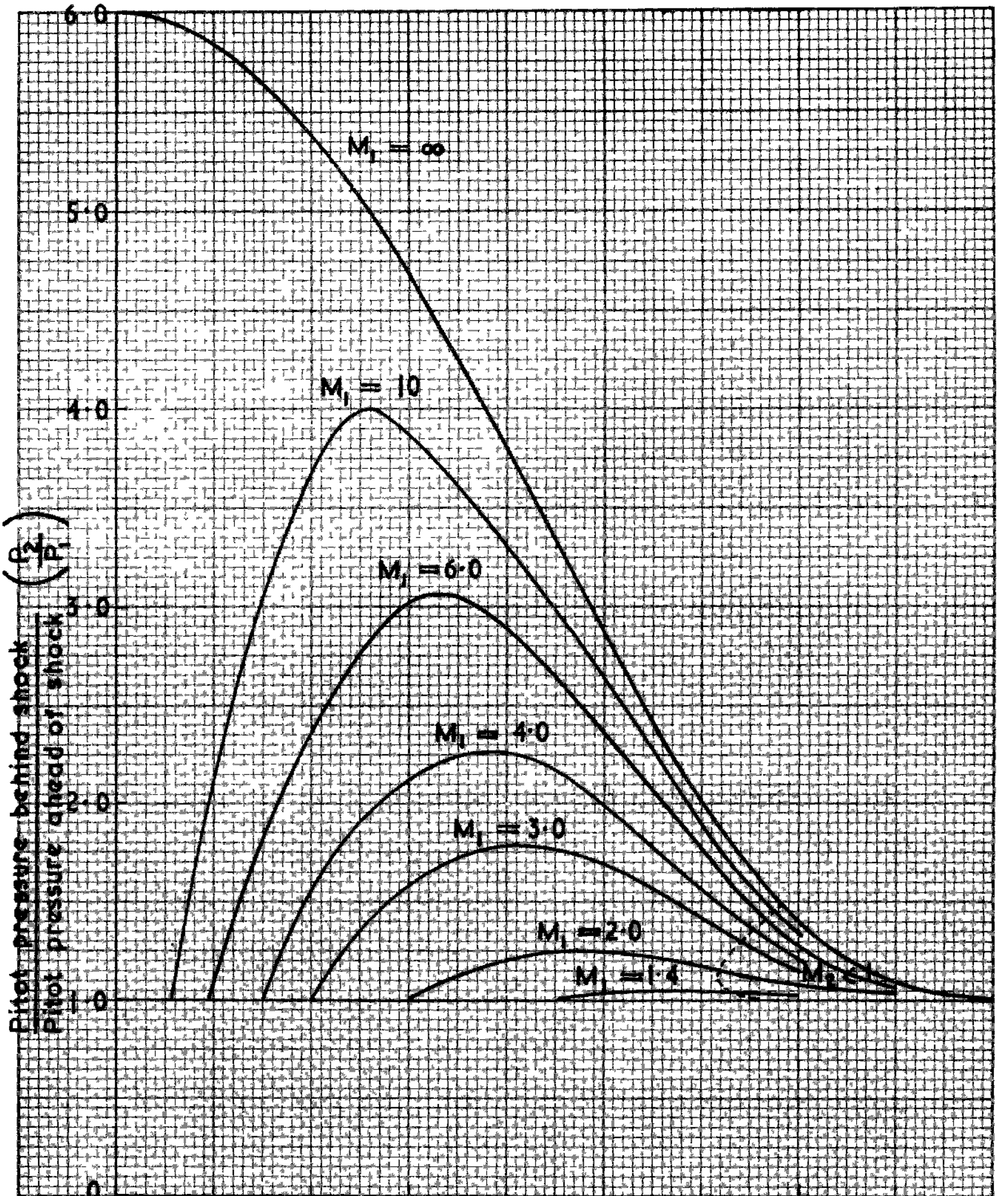
FIG. 1



The change of pitot pressure across oblique shock waves

$\gamma = 7/5$

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