C. P. No. 727

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Simple Theoretical and Experimental Studies of the Flow Through a Three-Shock System in a Corner.

By

E. Eminton

LONDON: HER MAJESTY'S STATIONERY OFFICE

1966

PRICE 4s 6d NET

U.D.C. No. 533.6.011.5 : 533.6.011.72 : 532.526.5

3 8006 10038 6971

C.P. No. 727

September, 1961

(FARNBOROUGH)

SIMPLE THEORETICAL AND EXPERIMENTAL STUDIES OF THE FLOW THROUGH A THREE-SHOCK SYSTEM IN A CORNER

by

E. Eminton

SUMMARY

In the hope of finding the three-shock system in a given corner appropriate to flow at a given Mach number a very simple theoretical model is considered. It assumes that shocks and streamlines are all straight and concentrates on the flow in the region of the point where the main shock branches into two. A shear layer is allowed to originate there and conditions imposed to match both pressures and flow directions on either side of it. The results of the calculations suggest that a more sophisticated theory is needed and a few experiments are made to substantiate this. Theory and experiment together lead to the conclusion that the shock system in a corner is determined not by the way the flow behaves around the branch point but by its behaviour around the feet of the branches which lie within the boundary layer. Thus viscous effects dominate the flow field and the external inviscid part of the flow appears to be able to accommodate itself readily by small deviations from the simplified model considered here.

Replaces R.A.E. Tech. Note No. Aero 2784 - A.R.C. 23 516.

LIST OF CONTENTS

LIST OF ILLUSTRATIONS

1 INTRODUCTION

The mainspring of this note is Fig.1. This shows a very simple conception of what happens when a uniform supersonic stream encounters a sudden change of slope in a plane boundary. It is a fact of experience that a kind of branching shock system does sometimes exist and that between its feet the flow separates from the wall. The simplicity of this two-dimensional model lies in the assumption that the shocks are straight and so is the boundary of the separation which joins the two feet.

We confine our attention to the flow outside the viscous region and ignore the presence of both wall and separation except in so far as they define the direction of the flow there. If we insist that the part of the flow which passes through the two branches must suffer the same rise in pressure and the same change in direction as that which passes through the main shock, we can use oblique-shock relations to find the unique three-shock system appropriate to any given corner angle at any given Mach number - at least in principle we can. In fact, we discover that such a solution exists only for a very limited range of Mach numbers for each angle. If we relax the restriction that matches the flow directions behind the main shock and its two branches, retaining only the restriction that matches the pressure rises, then for any angle at any Mach number above a certain minimum there is an infinity of solutions and in all of these the flow directions, now free to differ, do so by a surprisingly small angle.

Just how the flow adjusts itself to such discrepancies is difficult to predict, and in order to help clarify the situation a few experiments have been made. The wind tunnel most readily available - the R.A.E. 9 × 9 inch tunnel - has a Mach number range which lies entirely below the theoretical range of unique solutions. This is not such a handicap as it may seem, for what appear to be essentially three-shock systems still occur in practice at these Mach numbers; so the results of the tests may throw some light on the ways in which a real flow can differ from the model assumed in Fig.1.

The problem of the intersection of three shock waves has, of course, been treated before. Early work by Weise, Eggink and Wuest has been reported by Wecken[1], and a discussion of some of the difficulties involved has been given by Courant and Friedrichs[2]. A brief account with references to more recent work appears in a note by Sanders and Crabtree[3].

This note is part of a more general investigation into separated flows involving bubbles. In this general context, the present work may be regarded as an attempt to find out to what extent the external inviscid stream is a determining factor in the whole flow field.

2 FLOW THROUGH A SINGLE SHOCK

This section and the one that follows are based on three simple shock relations. Suppose M is the upstream Mach number and x is the ratio of the pressure ahead of the shock to the pressure behind it. The angle θ through which the flow is deflected is given by

$$\tan^2\theta = \left(M^2\frac{(1+k)x}{1+kx} - 1\right)\Bigg/\left(M^2\frac{\gamma x}{1-x} - 1\right)^2 \quad ; \qquad (1)$$

the inclination α of the shock to the stream by

$$\sin^2 \alpha = \frac{1}{M^2} \frac{1+kx}{(1+k)x} \quad ; \tag{2}$$

and the Mach number M_1 behind the shock by

$$M_1^2 = \frac{M^2 (1+kx) x - (1-k)(1-x^2)}{k+x} \quad . \tag{3}$$

These relations are exact for a perfect, inviscid gas. They may be deduced from the equations of continuity in mass, momentum and energy and are given explicitly by Wecken[1]. The constant γ is the ratio of specific heats - about 1.4 for air - and $k = (\gamma-1)/(\gamma+1)$.

Fig.2 shows the notation and Fig.3 the variation of each of these three quantities with x for different values of M. For each Mach number M there is a minimum pressure ratio

$$(x)_{min} = \frac{1}{M^2 (1+k) - k} \quad . \tag{4}$$

As the pressure ratio increases from $(x)_{min}$ to 1 the shock changes from a normal shock to a Mach line, α decreases from $\pi/2$ to $\sin^{-1}(1/M)$, M_1 increases from a minimum

$$(M_1)_{min} = \frac{1 + (M^2-1)k}{M^2 + (M^2-1)k} \tag{5}$$

to M, and θ increases from 0 to a maximum and decreases again to 0. The values of α and M_1 which correspond to the maximum deflection are shown on the curves.

3 FLOW THROUGH A THREE-SHOCK SYSTEM

Returning to the three-shock system of Fig.1, let us use suffices 0, 1 and 2 to denote respectively the main shock and its upstream and downstream branches as shown in Fig.2. For the pressure rise through the two branches to equal the pressure rise through the main shock,

$$x_0 = x_1 x_2 \quad ; \tag{6}$$

and for the flow behind the two branches to be parallel to the flow behind the main shock,

$$\theta_0 = \theta_1 + \theta_2 \quad . \tag{7}$$

We know from section 2 that the flow through a single shock is completely defined by the two parameters x and M. If therefore we specify x_0, x_1 and M, we can use the condition (6) to find x_2 and equations (1), (2) and (3) to determine all the other parameters. Now, allowing x_1 to vary from x_0 to 1, we can plot the variation of $\theta_1 + \theta_2$ and discover which of all possible systems also satisfies the condition (7).

In calculations x_0, x_1 and M are the simplest parameters to use as variables since in these all the other parameters are single valued*. However, in practice θ_0, θ_1 and M are the more significant parameters and it is in terms of these that the results are presented. At any M two values of x_0 correspond to each θ_0 (see Fig.3) but we have always chosen the larger of these since the other corresponds to a nearly normal shock and this is seldom met with in practice.

It is well known (and can be seen from Fig.3) that a uniform stream can be turned through an angle θ by a single shock provided M is greater than a certain minimum shown in Fig.4. It appears from the calculations that there are two such limits, also shown in Fig.4, for the existence of a three-shock system in a given corner. The lower limit is approached as the upstream shock tends to a Mach line and all the turning is accomplished through the downstream shock. The upper limit is approached as the downstream shock degenerates to a Mach line in a similar way. Fig.5 shows the variation of θ_1 with M between these limits; this angle, which varies from 0 to θ_0, is physically the angle through which the flow turns as it separates from the wall. Beyond the limits the only possible solutions are the trivial ones of a single shock.

Experimental evidence[4,5] shows that three-shock systems appear to occur over a very much wider range of Mach numbers than are predicted in Fig.4 including Mach numbers both above and below the limit curves. We can only conclude, therefore, that this model of the flow is somehow oversimplified. If we retain the skeleton of the flow shown in Fig.1 then we must relax either the pressure condition (6) or the restriction on flow direction (7). In either case this gives an infinity of possible solutions and no idea of their plausibility. When the pressure condition is retained but the flow directions are no longer constrained to be parallel, Fig.6 shows the maximum angles by which they differ, a divergence being shown positive and a convergence negative.

However, it may be that Fig.1 itself must be rejected, being too simple to represent adequately the true state of affairs - for example, the shocks may not be straight or, perhaps, other phenomena occur as well. In order to advance further a more sophisticated theory is needed. Meanwhile the results of a few simple experiments may help to suggest a suitable line of attack.

---

* We know $M_1 > 0$ and since we are concerned with flow in a concave corner we need only the values of θ and α which satisfy $0 \leq \theta \leq \pi/2$, $0 \leq \alpha \leq \pi/2$

4 AN EXPERIMENT IN CORNER FLOW

Some tests were made during April, 1961 in the R.A.E. 9 x 9 inch tunnel - a continuous flow supersonic tunnel with fixed liners which is described in Refs.6 and 7. It was run at a nominal Mach number of 1.9 and stagnation pressures between 5 and 40 inches of Mercury* giving Reynolds numbers between 0.06×10^6 and 0.44×10^6 per inch.

The models tested each consisted of a brass wedge on a horizontal steel plate with sharp leading edge spanning the tunnel. Model 1 was a $7\frac{1}{2}^\circ$ wedge with 24 pressure holes along its centre line. Initially it spanned the full width of the tunnel but interaction between the boundary layer on the wedge and that on the tunnel wall so confused the Schlieren pictures that it was advisable to reduce the dimensions of the wedge as shown in Fig.7. Model 2 was a 15° wedge with 28 extra pressure holes making 52 in all. These holes were connected to a set of mercury manometers which could be read to an accuracy of 0.02 inches.

For most of the tests the boundary layer on the plate was laminar but on the wedge it was turbulent. To show this, azo-benzene was used as an indicator. The stagnation pressure was held at eight different values to discover the effect of Reynolds number on the flow. At each of these values pressure measurements were recorded and Schlieren pictures taken. A mixture of titanium dioxide and cylinder oil was used to show the flow pattern on the surface - in particular the position of the flow separation and reattachment lines. The mixture was also painted on the window of the tunnel to identify one of the shocks in the Schlieren pictures.

A selection of Schlieren photographs** for the two models is reproduced in Figs.8 and 9. The separation and reattachment shocks can be seen clearly in most of the pictures and in some the edge of the boundary layer can be imagined although it is unwise to attach too much importance to this as its shape could be changed by slightly adjusting the Schlieren screen. The third shock, just downstream of the reattachment shock, is the trace on the window of the separation shock as it curves round the sides of the wedge; although confusing at first glance, this extraneous shock is always well clear of the main shock system and once identified can be ignored. Shocks appearing across the corners of the photographs are from the leading edge of the plate.

The oil flow patterns proved difficult to photograph mainly because the best patterns obtained with the tunnel running were spoilt during shut down. However, they were quite good enough to locate the mean position of both separation and reattachment lines to about 0.1 inches and to show that the flow was reasonably two-dimensional over an appreciable part of the model. A photograph of one typical pattern on model 2 is reproduced in Fig.10. It shows clearly the forward flow in the separated region and the accumulation of oil along the separation line; at an earlier stage in its formation the oil could be seen flowing away from the reattachment line in both directions.

The surface pressure distributions measured on models 1 and 2 for eight values of the stagnation pressure H are shown in Figs.11 and 12. On the same

---

*At atmospheric stagnation pressure, the Mach number in the working section is actually 1.91 ± 0.01.

**The values of H given in these figures are those of the stagnation pressure in inches of mercury. Corresponding values of Reynolds number are given e.g. in Fig.11.

figures are marked the positions of separation and reattachment lines measured from oil flow patterns, and two theoretical pressures: the pressure far upstream $p_{-\infty}$ given by

$$\frac{p_{-\infty}}{H} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\gamma/(\gamma-1)} \quad ; \qquad (8)$$

and the pressure far downstream p_{∞} given by

$$\frac{p_{-\infty}}{p_{\infty}} = x_0 \quad , \qquad (9)$$

with x_0 given implicitly by equation (1). Fig.13 shows the same pressure distributions replotted in terms of p/H and Fig.14 the variation of separation and reattachment positions with Reynolds number.

The results for both models show an initial pressure rise across the separation line followed by a region of near-constant pressure and a much larger pressure rise across the reattachment line. This is consistent with the three-shock system sketched in Fig.1. However, on model 1 the pressure rise through the downstream shock overshoots the value appropriate to an angle of $7\frac{1}{2}°$ and a region of pressure reduction occurs behind this shock. Unfortunately there are not quite enough experimental points to define this region properly. The additional pressure holes on model 2 were intended to remedy this but there the phenomenon did not appear!

When the boundary layer ahead of the corner was made turbulent with a trip wire across the leading edge of the plate, the flow pattern was significantly altered - the separation region in the corner shrank to about 0.25 inches and the two branches of the shock almost coalesced. No Schlieren photographs were taken and no pressure measurements recorded as it was impossible to distinguish between the two branches of the shock or between their corresponding pressure rises. However, it served to confirm that the flow on the plate had hitherto been laminar and Fig.15 shows an amusing oil flow pattern of both flows together.

5 DISCUSSION

The theoretical results obtained in section 3 for the very simple model of the flow illustrated in Fig.1 show that, if the conditions (6) and (7) matching flow directions and pressures behind the shocks are to be satisfied exactly, solutions are possible for only a very small range of Mach numbers in a given corner. On the other hand, a slight relaxation of the matching conditions admits an infinite range of possible solutions at any Mach number.

Even in the simplest case, when the pressures and flow directions are matched exactly, the velocities differ and a shear layer of non-zero thickness must exist extending downstream from the point where the main shock divides. Such a shear layer would tend to thicken like a boundary layer and might in itself provide a means of accommodating a small divergence in flow directions behind the shocks, as suggested by Liepmann[2]. However, for Mach numbers below the range of non-trivial solutions in Fig.4, the results require a convergence of the two streams (see Fig.6) and the growth of the shear layer cannot explain this away.

At this stage we must admit the inadequacy of Fig.1 and the need for a more sophisticated model of the flow. One possibility is that the flow behind the shock system is not uniform, for example there may be an expansion behind the foot of the downstream branch. This is apparently what happens in the experiments on the $7\frac{1}{2}^{o}$ wedge where an appreciable pressure reduction is observed. Similarly, for Mach numbers above the range of non-trivial solutions, where the theory predicts a divergence of the two streams, an extra shock or a compression region might be postulated. Johannesen[8], who performed experiments similar to those reported here, considered such an additional shock and remarked that the shock would be extremely weak in most cases (see also Fig.6) and therefore difficult to detect. Finally we must remember that in reality the shocks may not be straight but slightly curved or the flow may be curved with additional simple waves.

It is difficult to see which of these suggested generalisations it is necessary to incorporate in any more elaborate theoretical approach to the problem but there is one important conclusion to be drawn. Broadly speaking, the model is of the right general form but it is not in itself enough to determine how the main shock is divided into two. It merely indicates that if there is such a division then any division very nearly satisfies the equations. To insist on an exact solution is to ask for a refinement which the model is not capable of supplying. It seems reasonable to deduce that the shock system is determined not by the way the flow behaves in the region of the branch point but by its behaviour around the feet of the branches - that is at the points of separation and reattachment - and it is impossible to study these regions without considering the flow within the boundary layer and viscous effects generally. Thus the inviscid external flow does not determine the flow to any extent but readily accommodates itself to what is required by the inner viscous regions. It is these that require further study.

LIST OF SYMBOLS

Symbols	
H	tunnel stagnation pressure
k	$(\gamma-1)/(\gamma+1)$
M	upstream Mach number
M_{suffix}	Mach number behind a shock
p	pressure
R	Reynolds number
x	pressure ahead of shock/pressure behind shock
α	inclination of shock to stream direction
γ	ratio of specific heats (1.4 for air)
θ	angle through which shock deflects the flow

LIST OF SYMBOLS (Cont'd)

Suffices

0	main shock
1	upstream branch
2	downstream branch
$-\infty$	far upstream
∞	far downstream

LIST OF REFERENCES

Ref. No.	Author	Title, etc.
1	Wecken, F.	Grenzlagen gegabelter Verdichtungsstöβe ZAMM Band 29 Heft 5, p.147, May, 1949.
2	Courant, R., Friedrichs, K.D.	Supersonic flow and shock waves. Interscience publishers. New York, 1948 (see p.332f).
3	Sanders, F., Crabtree, L.F.	A preliminary study of large regions of separated flow in a compression corner. ARC. 23,006. March, 1961.
4	Chapman, D.R., Kuehn, D.M., Larson, H.K.	Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transition. NACA Report 1356.. 1958.
5	Kuehn, D.M.	Experimental investigation of the pressure rise required for the incipient separation of turbulent boundary layers in two-dimensional supersonic flow. NASA Memo 1-21-59A TIL 6209. February, 1959.
6	Lord, W.T., Beastall, D.	Calibration of the RAE No.18 (9in. x 9in.) supersonic wind tunnel. Part 1 Preliminary investigations ARC C.P.162. September, 1953.
7	Lord, W.T., Hunt, G.K., Pallant, R.J., Turner, J.	Calibration of the RAE No.18 (9in. x 9in.) supersonic wind tunnel. Part 2 Tests at atmospheric stagnation pressure. ARC C.P. 163 September, 1953.
8	Johannesen, N.H.	Experiments on two-dimensional supersonic flows in corners and over concave surfaces. Phil. Mag., Vol.43, p.568, 1952.

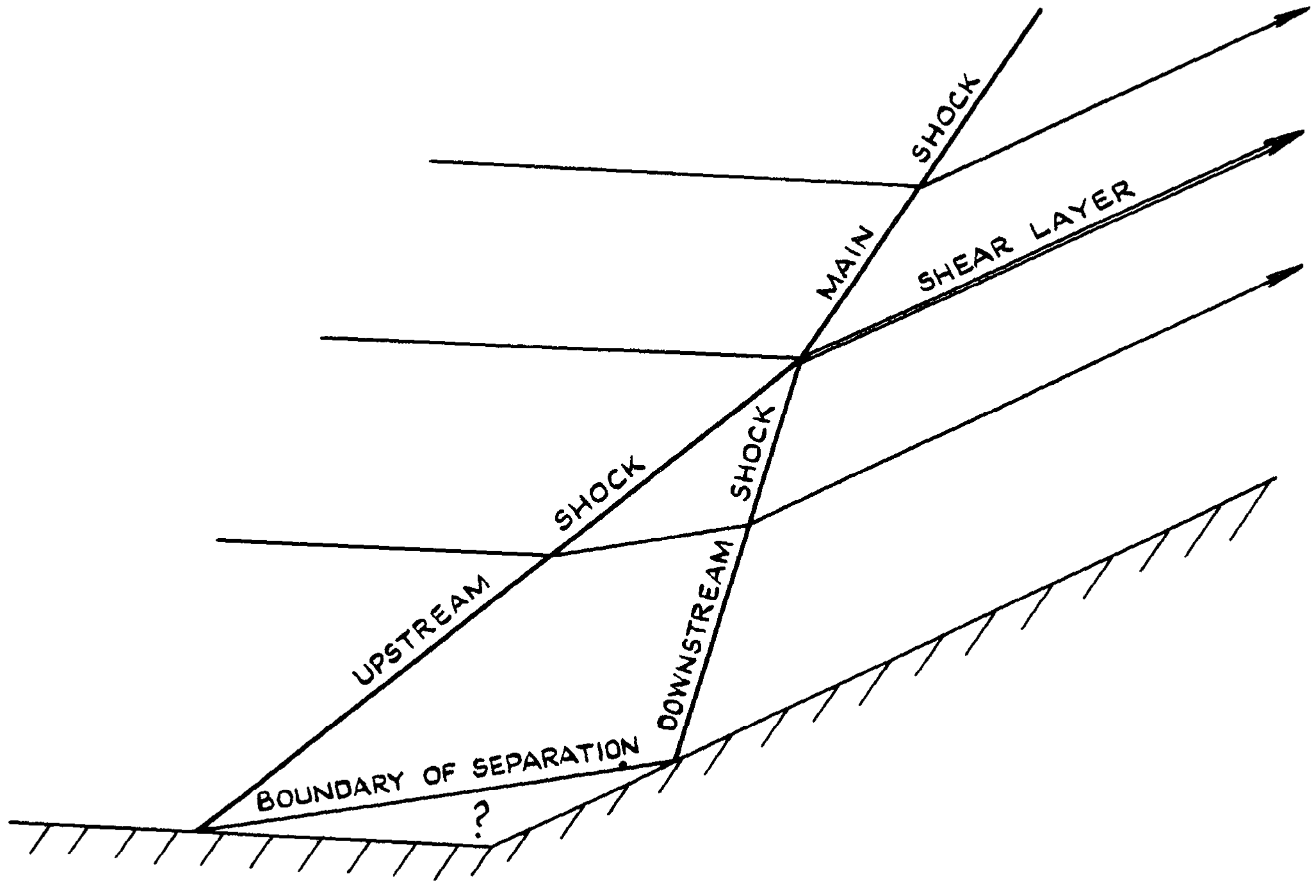


FIG. 1. A THEORETICAL MODEL OF FLOW IN A CORNER

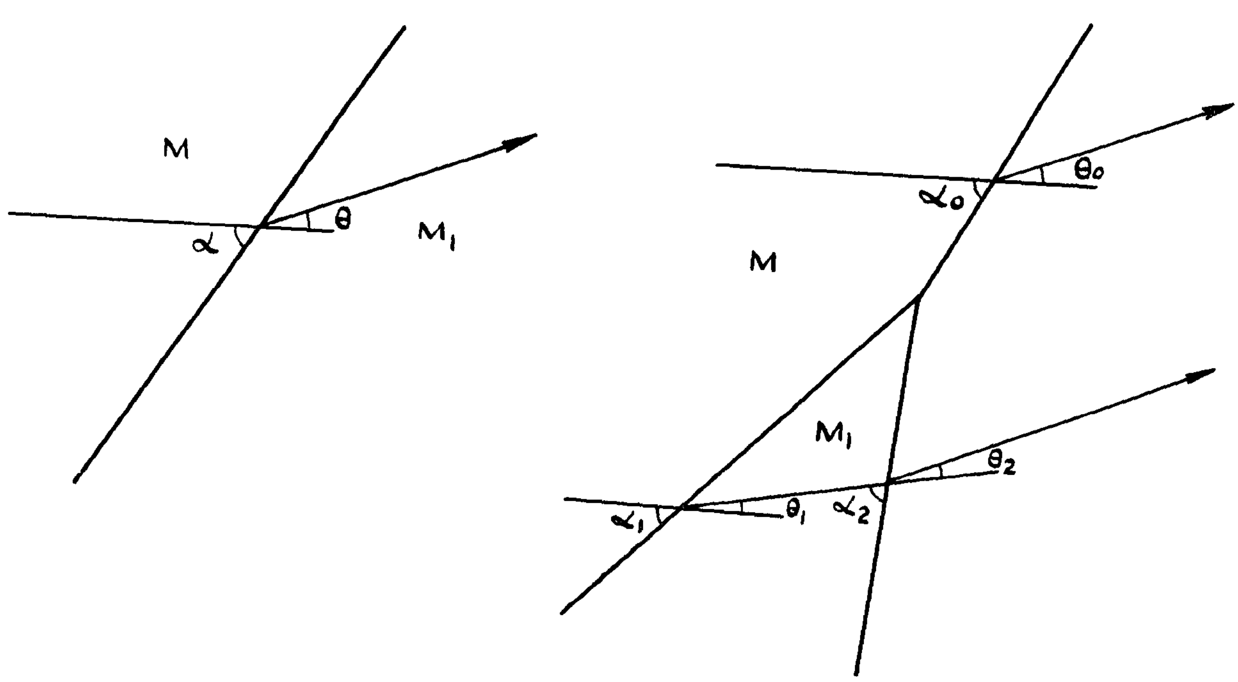


FIG. 2. NOTATION

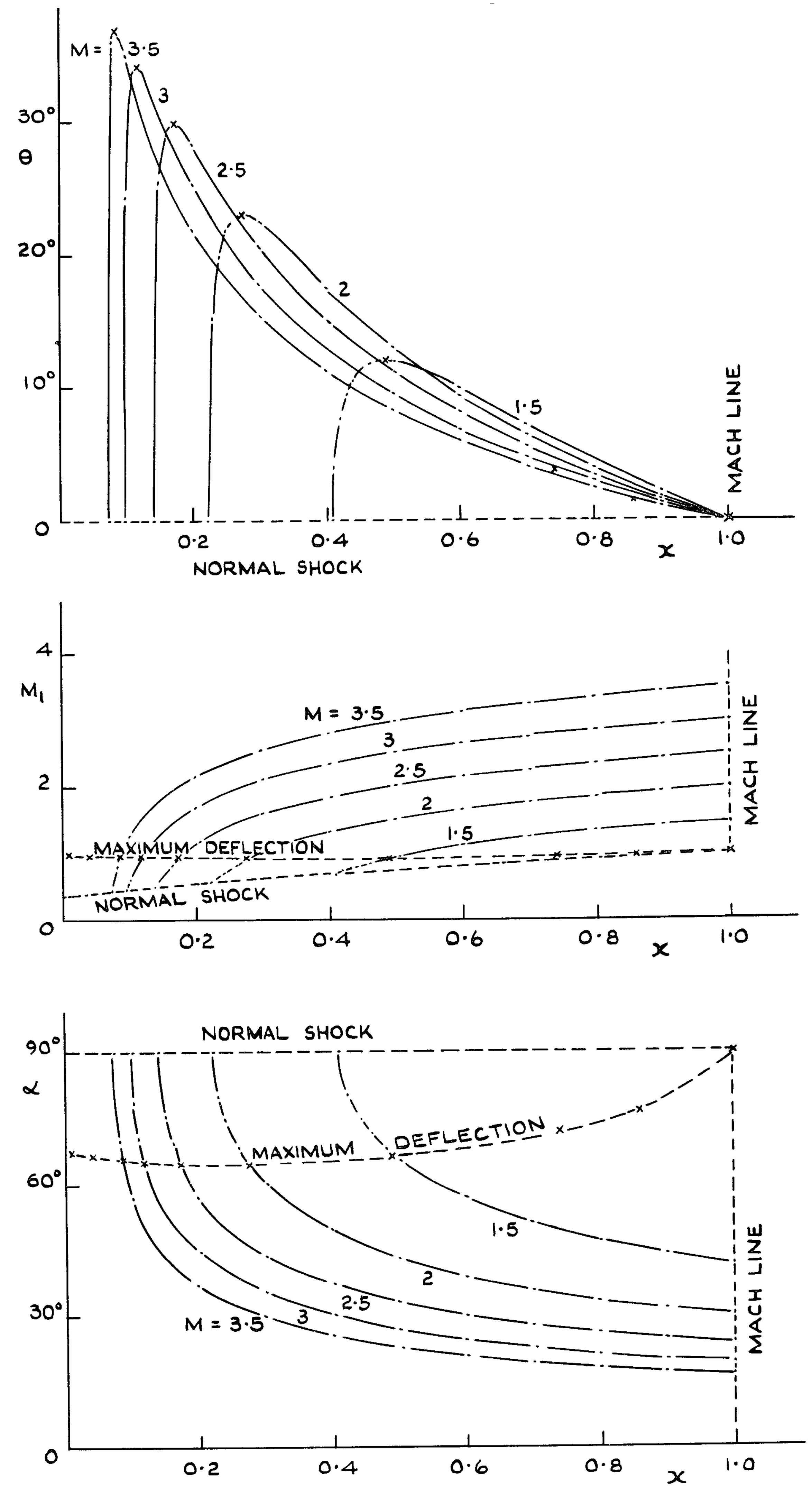


FIG. 3. FLOW THROUGH A SINGLE SHOCK

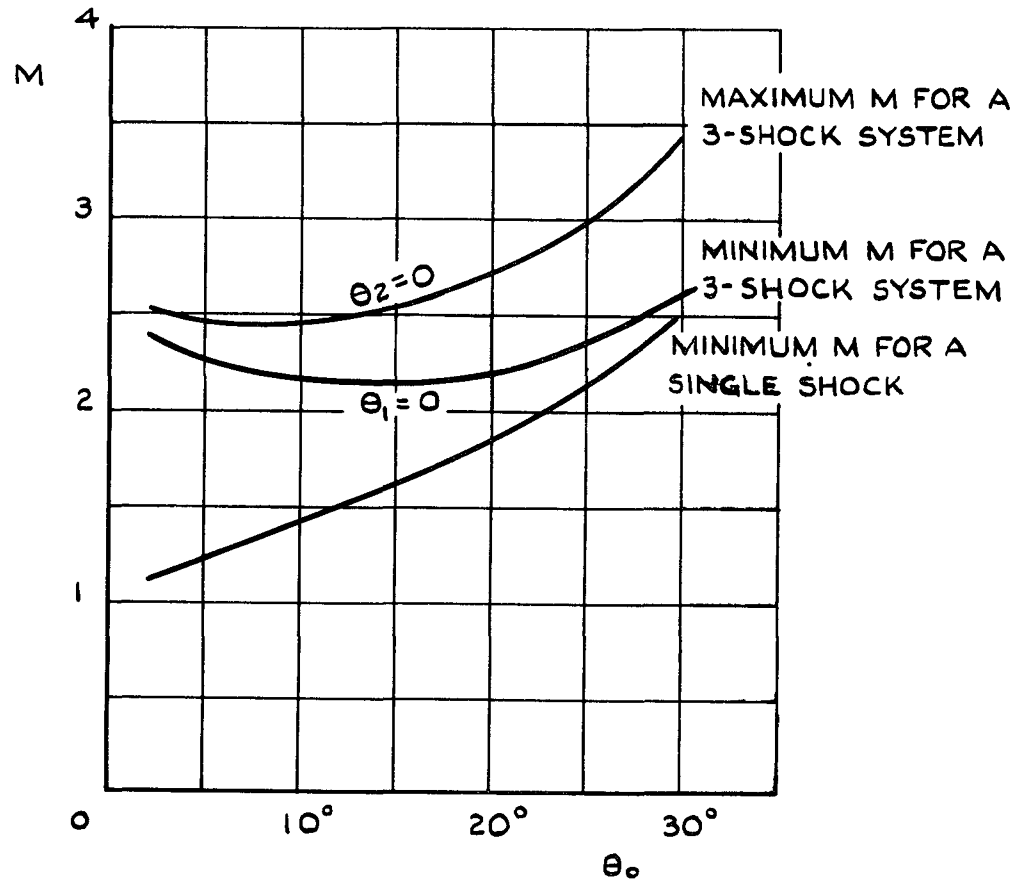


FIG. 4. THEORETICAL LIMITS ON MACH NUMBER

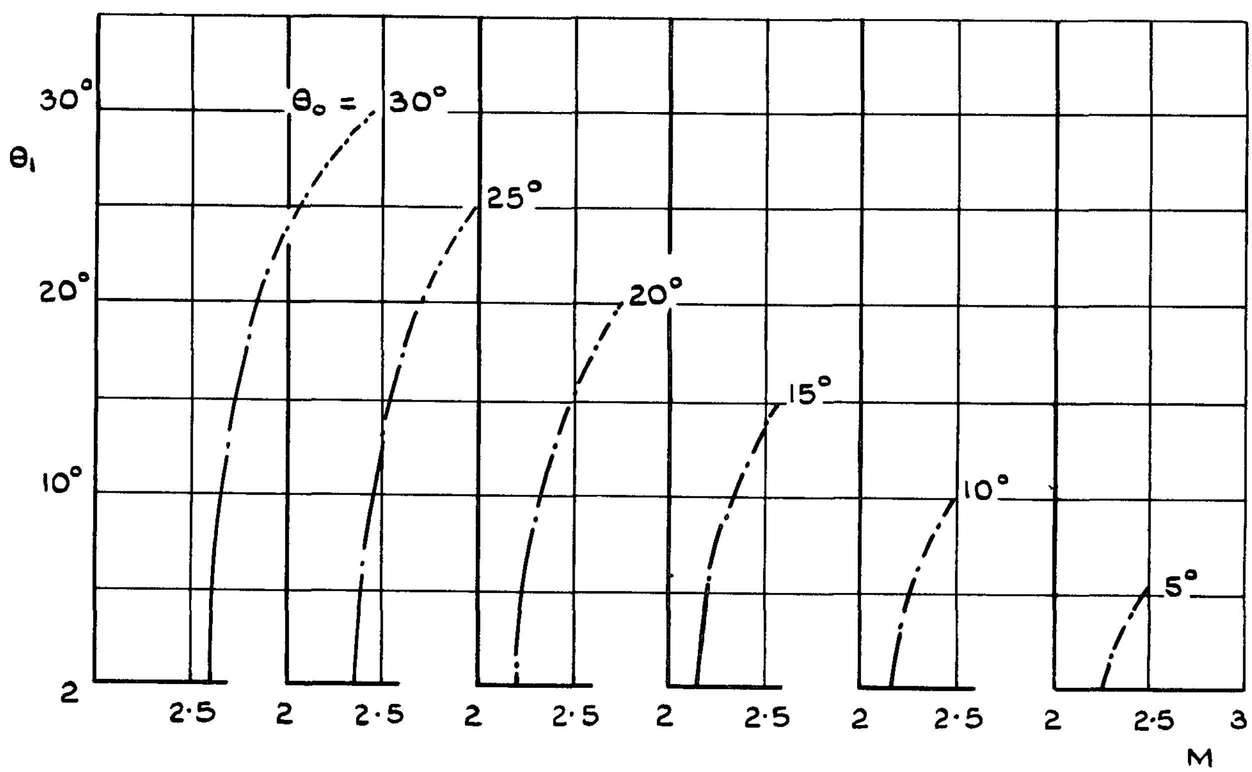


FIG. 5. VARIATION OF SEPARATION ANGLE WITH MACH NUMBER

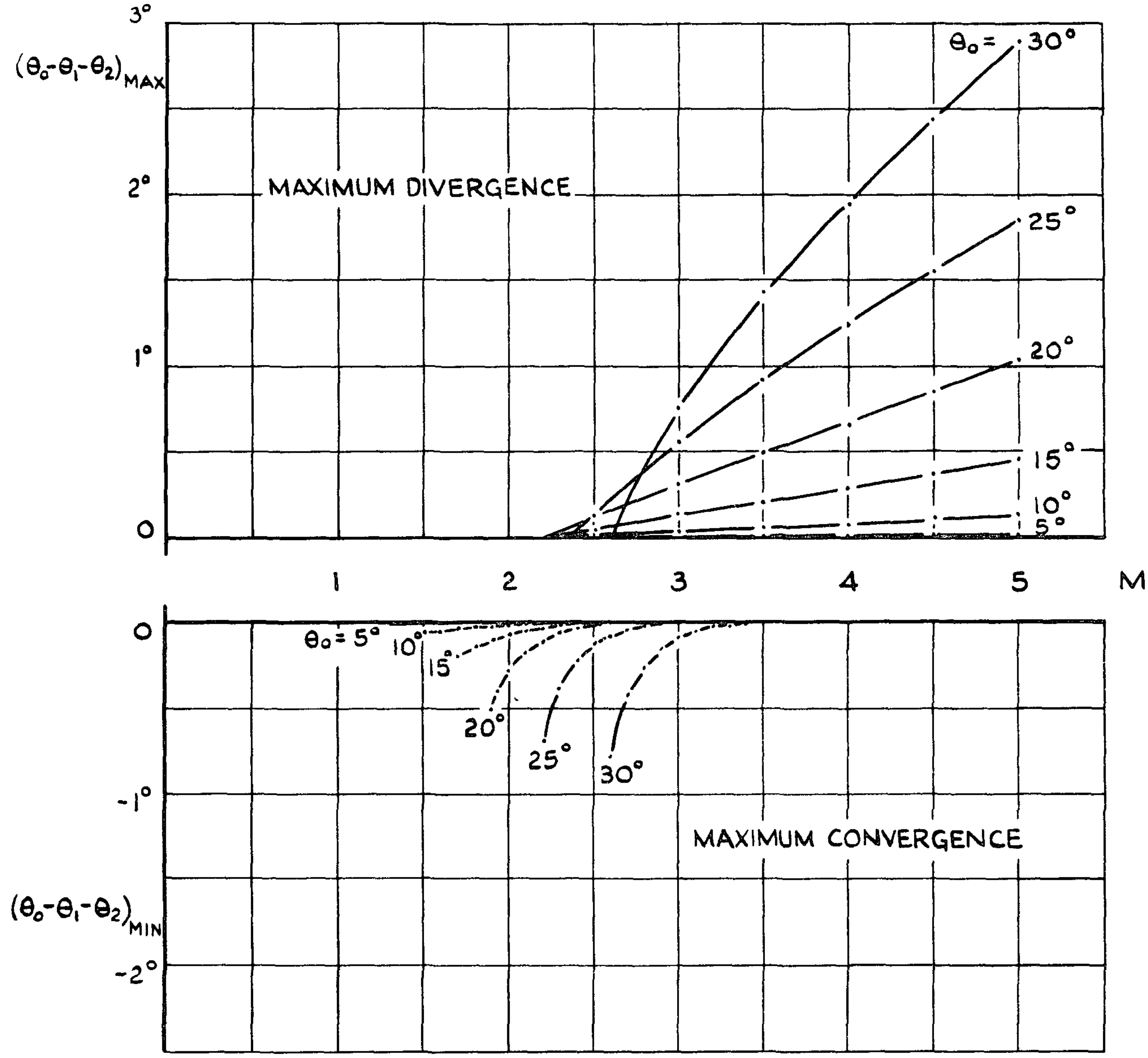


FIG. 6. MAXIMUM DIVERGENCE OF FLOW DIRECTIONS

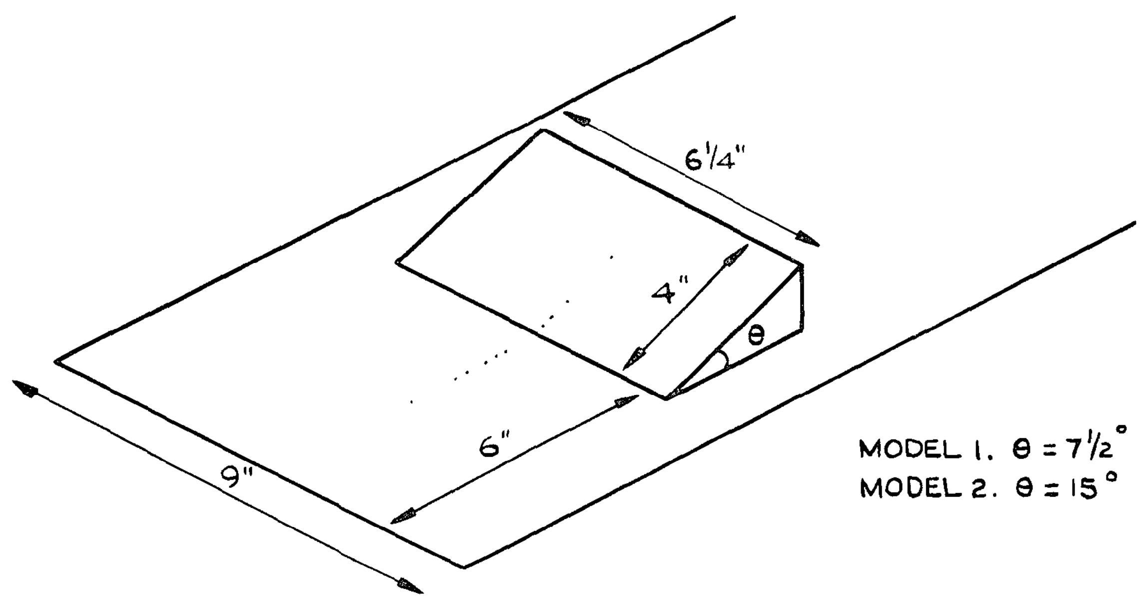


FIG. 7. SKETCH OF EXPERIMENTAL MODELS.

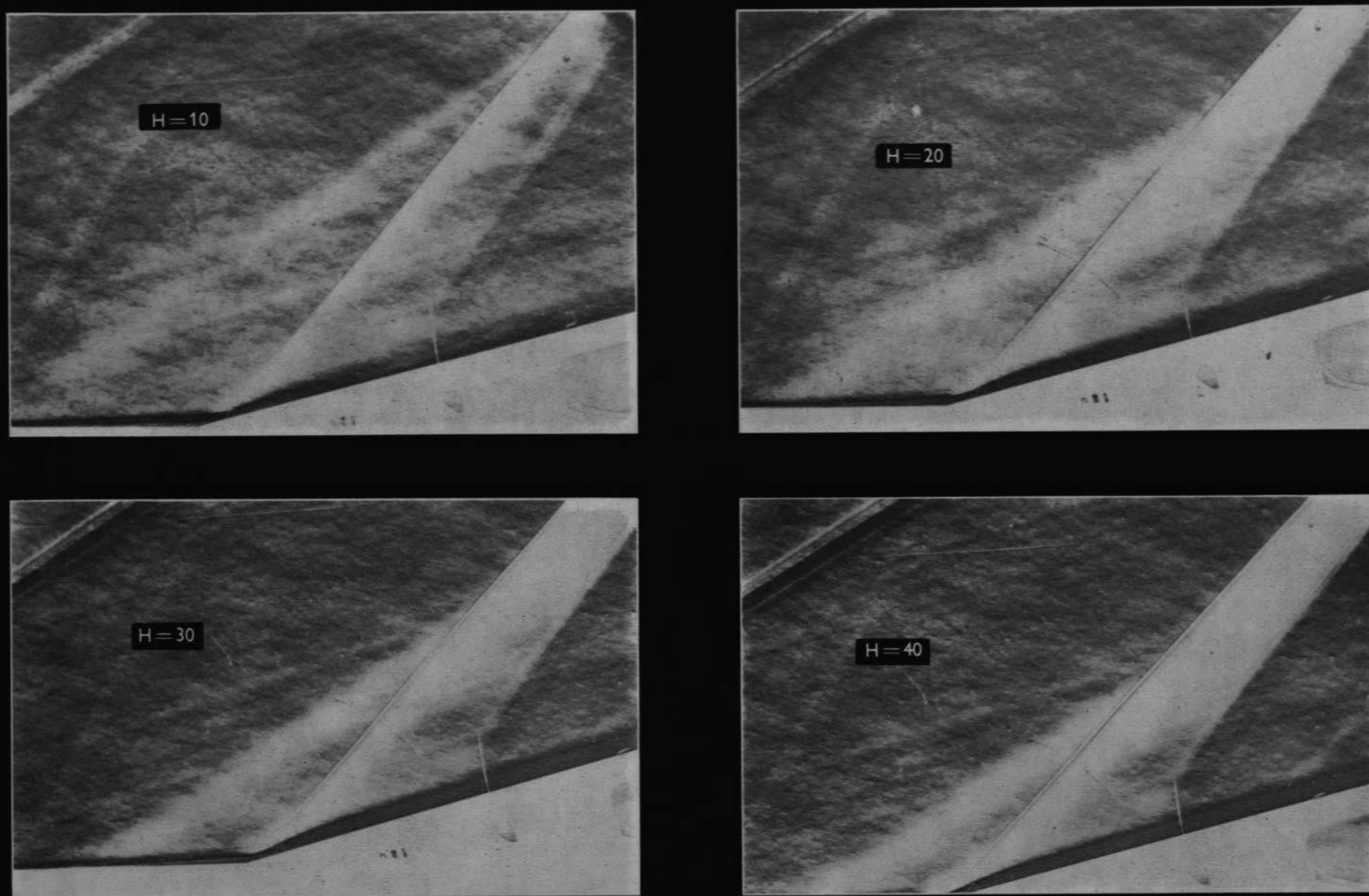
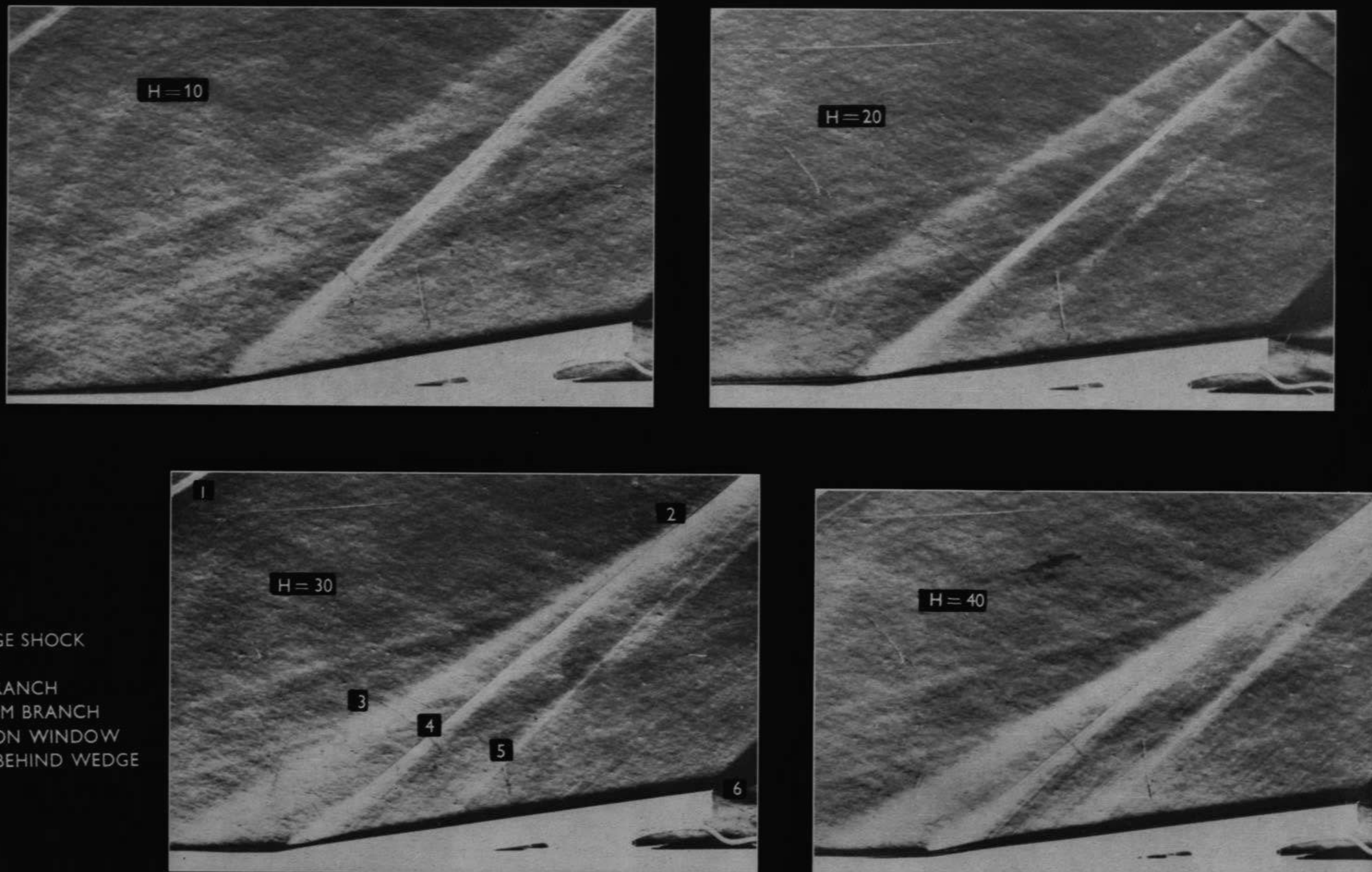


FIG.9. SCHLIEREN PHOTOGRAPHS FOR MODEL 2



1. LEADING EDGE SHOCK
2. MAIN SHOCK
3. UPSTREAM BRANCH
4. DOWNSTREAM BRANCH
5. TRACE OF 3 ON WINDOW
6. EXPANSION BEHIND WEDGE

FIG.8. SCHLIEREN PHOTOGRAPHS FOR MODEL I

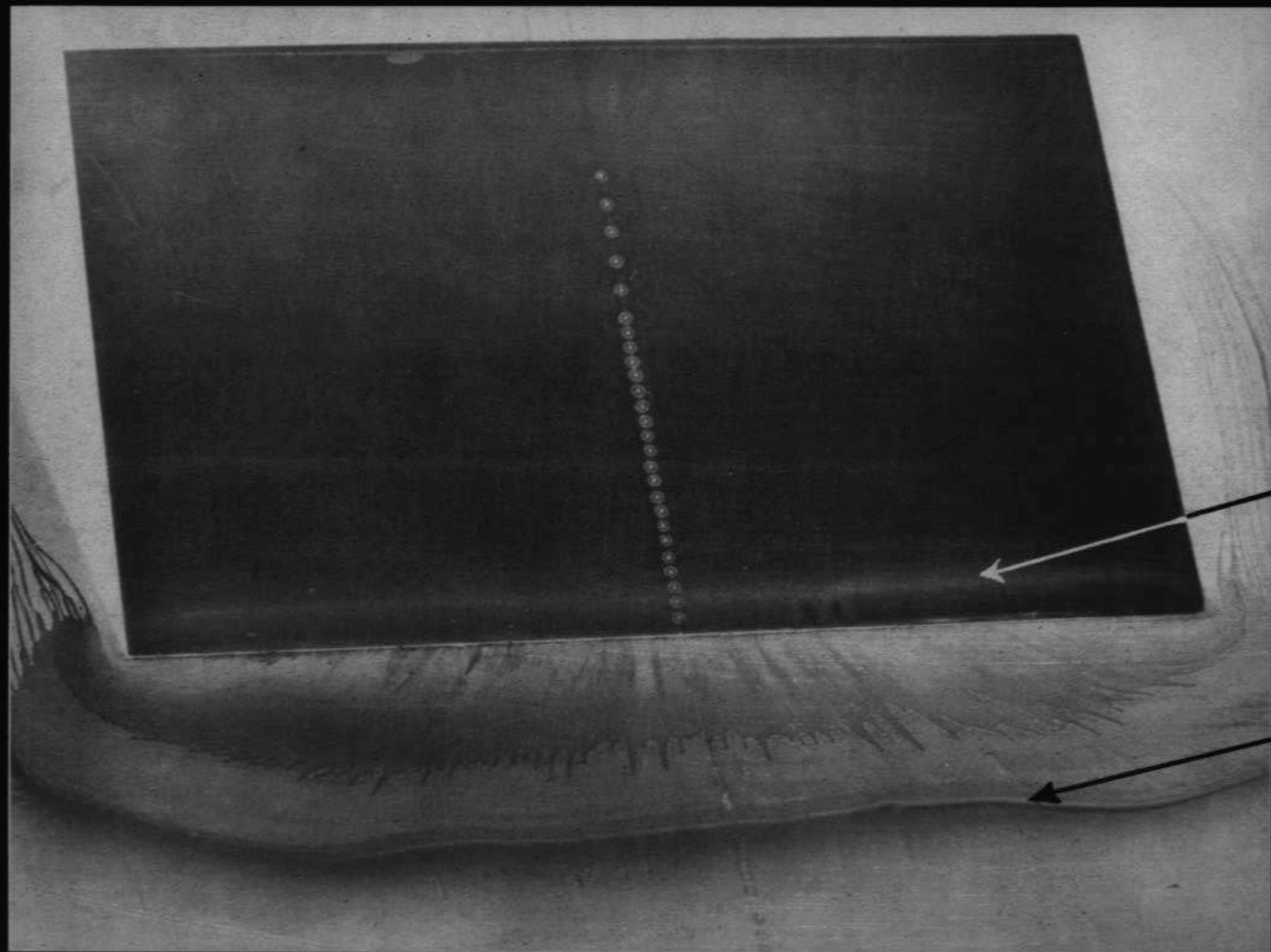


FIG.10. A TYPICAL OIL FLOW PATTERN FOR MODEL 2

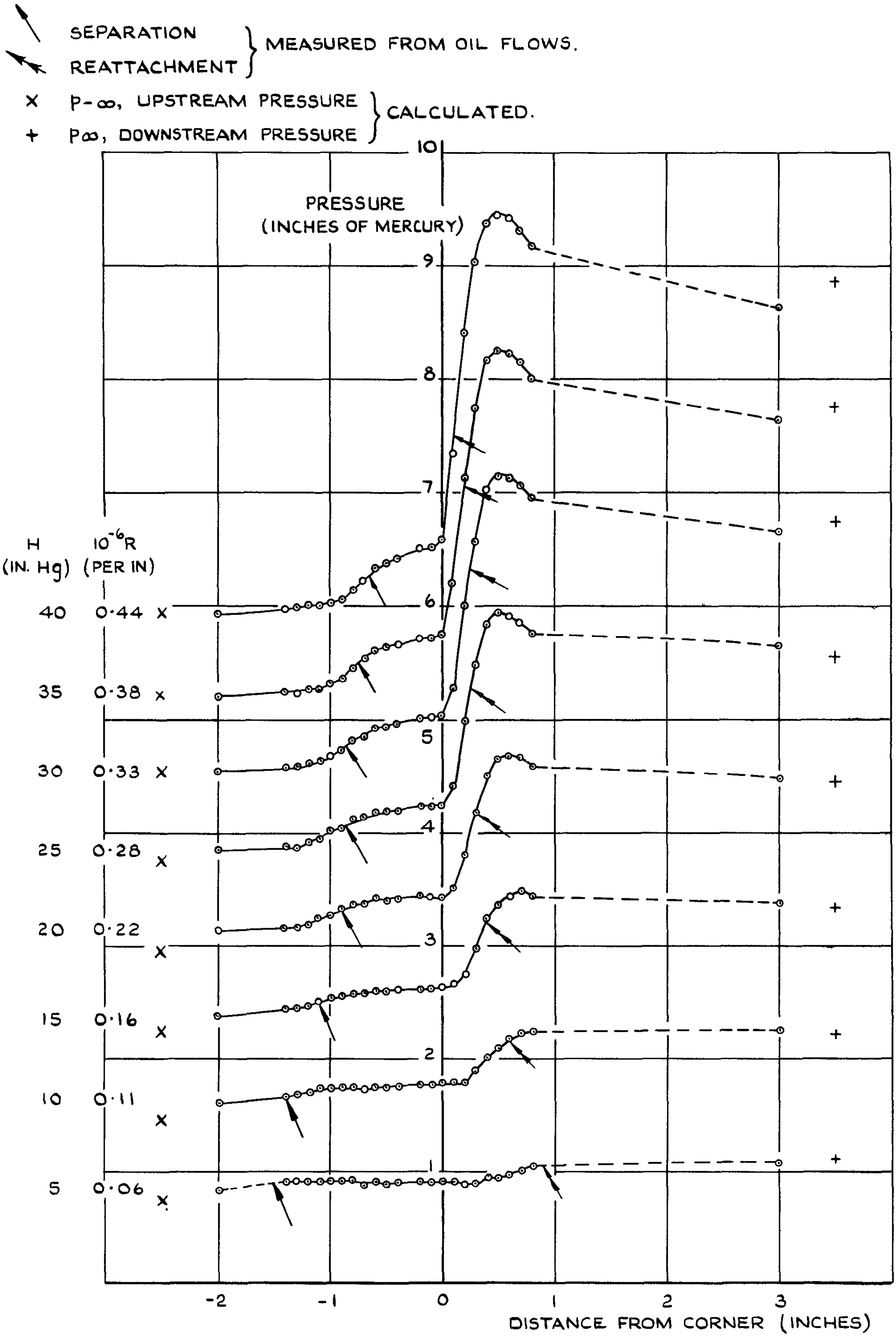


FIG. 11. SURFACE PRESSURE DISTRIBUTIONS FOR MODEL 1.

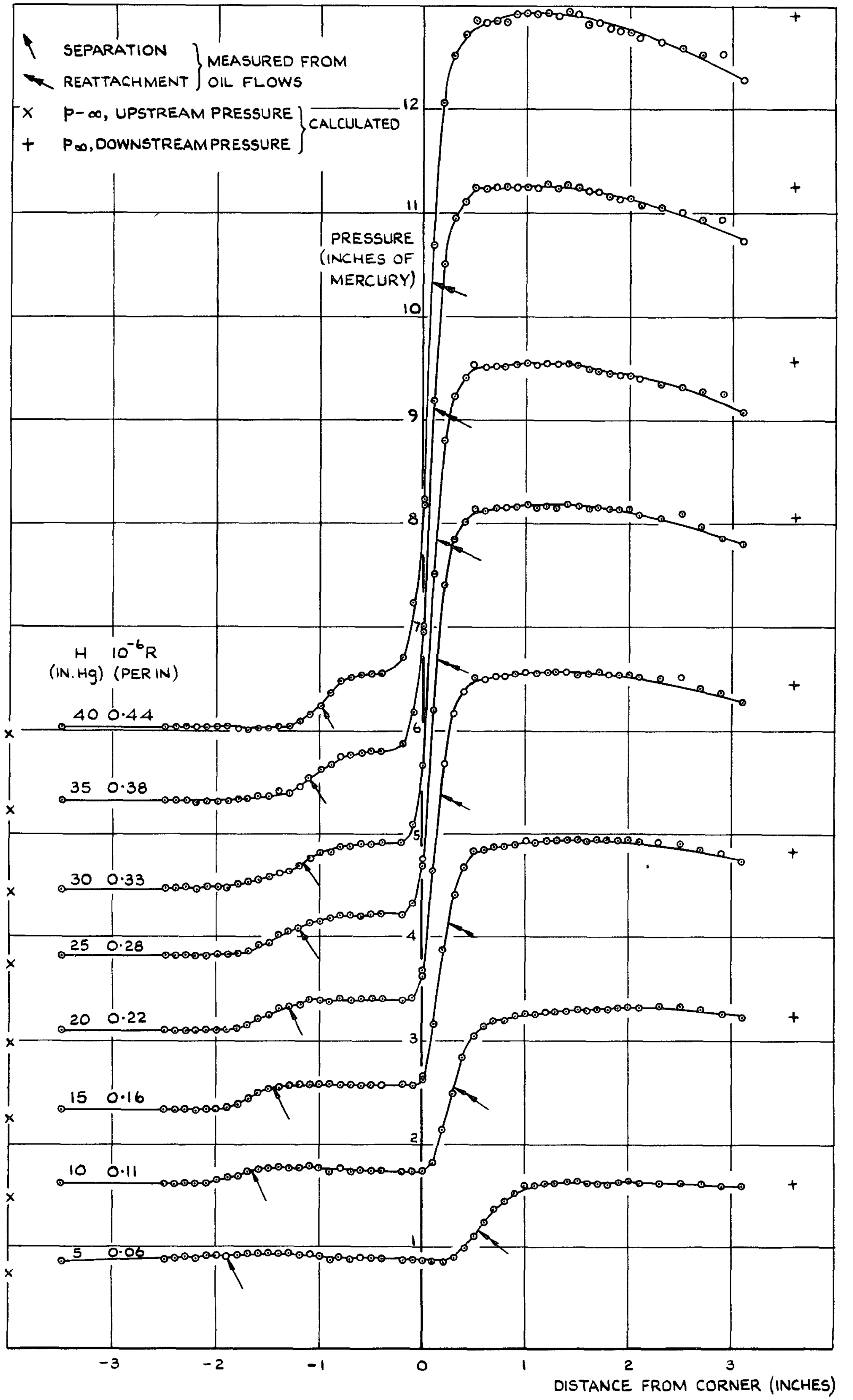
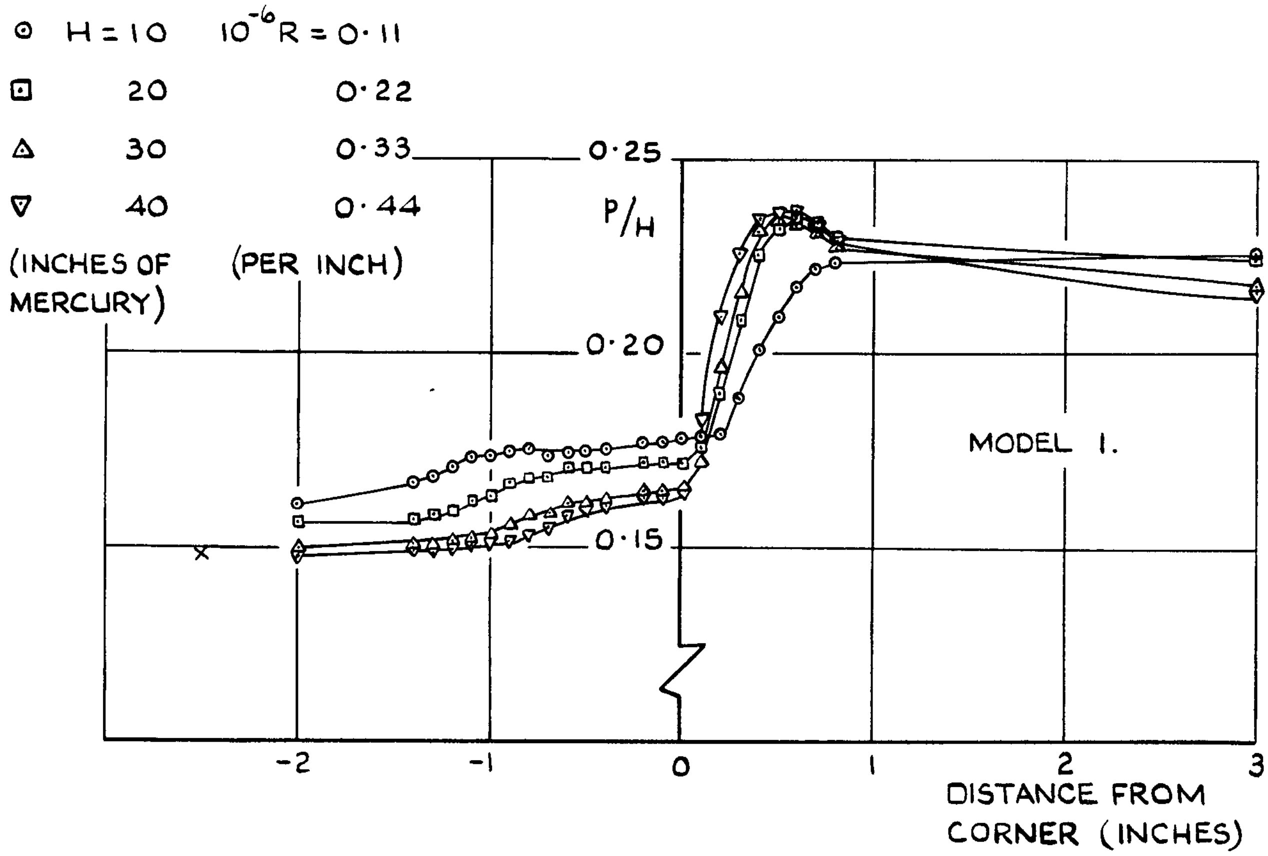


FIG. 12. SURFACE PRESSURE DISTRIBUTIONS FOR MODEL 2.



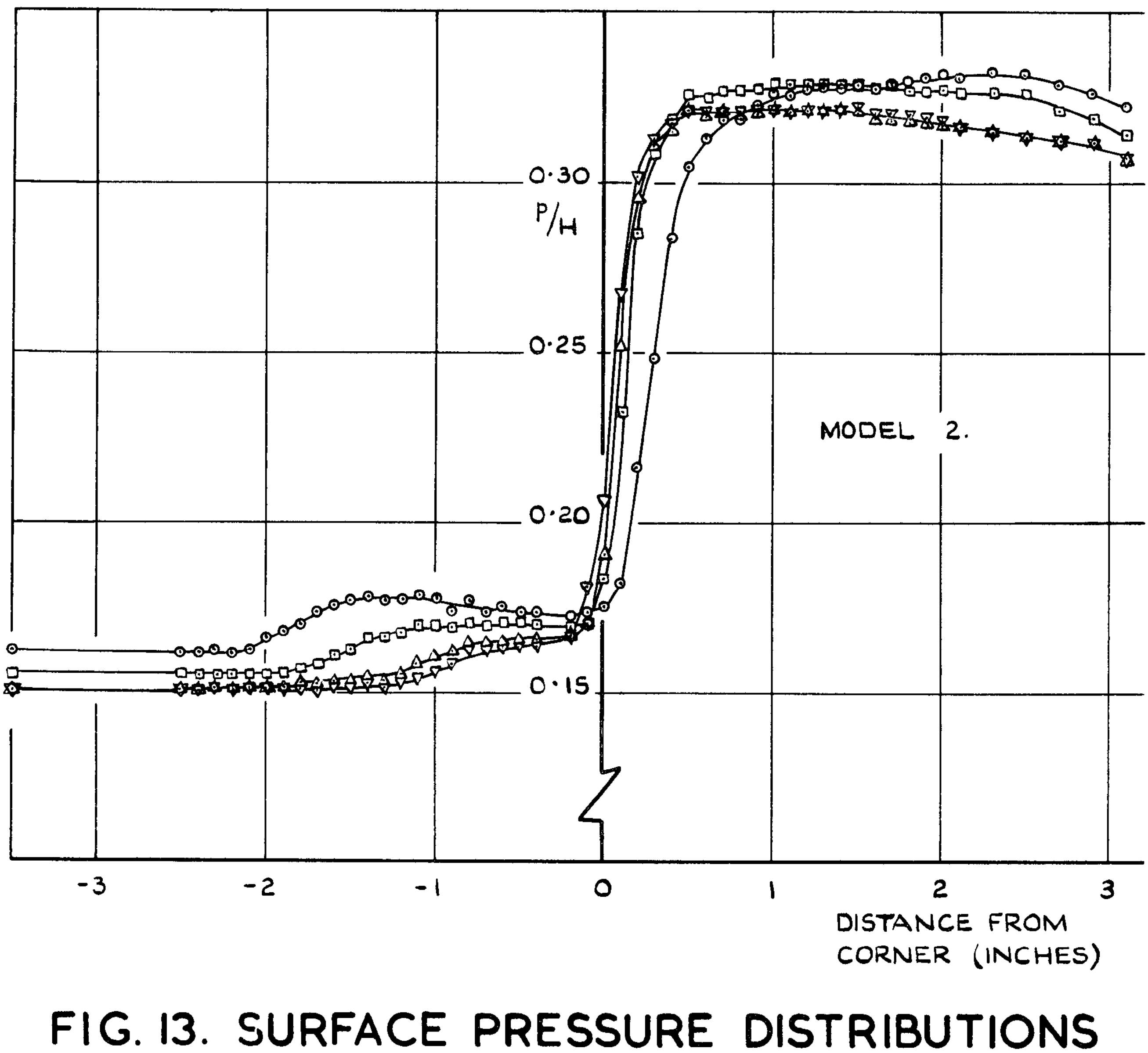


FIG. 13. SURFACE PRESSURE DISTRIBUTIONS IN COEFFICIENT FORM.

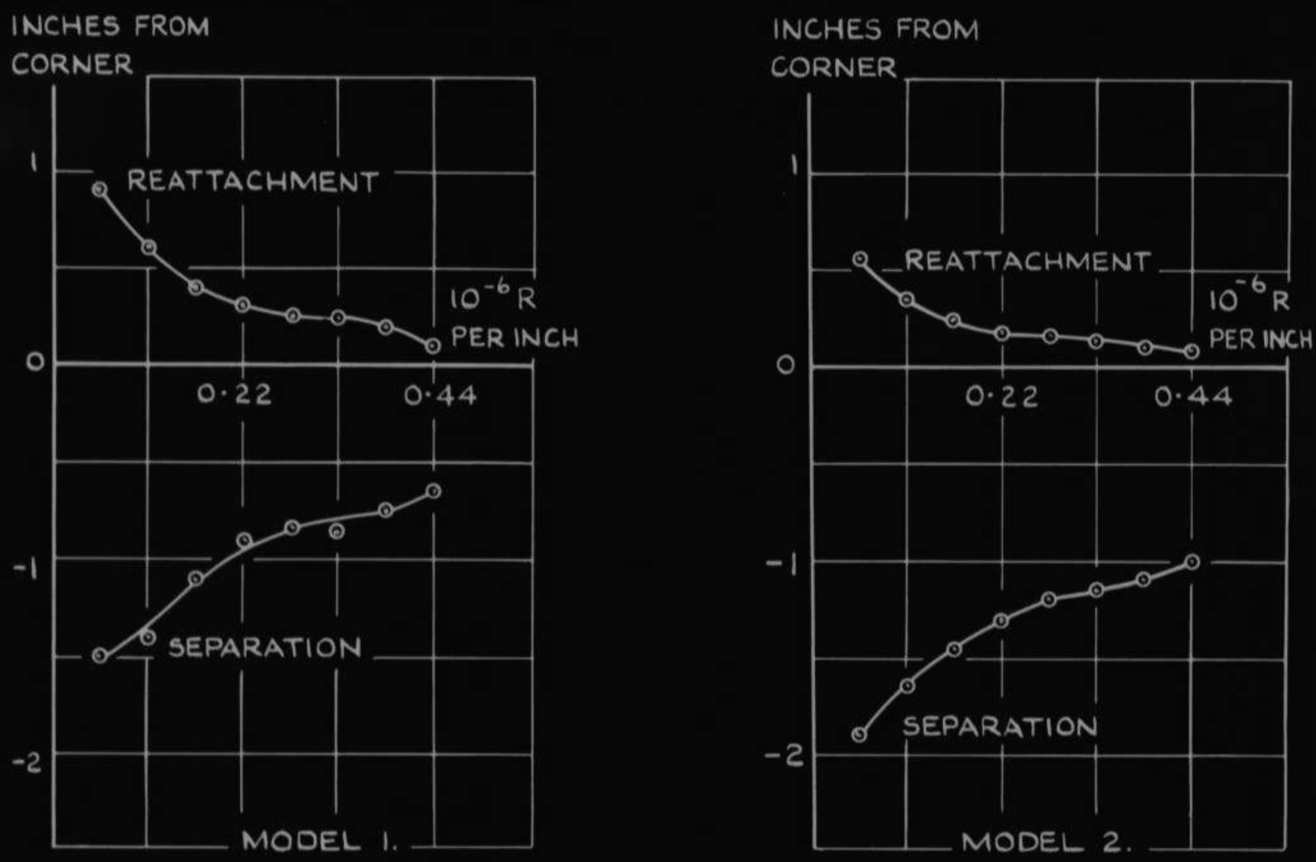


FIG. 14. VARIATION OF SEPARATION AND REATTACHMENT POSITIONS WITH REYNOLDS NUMBER.

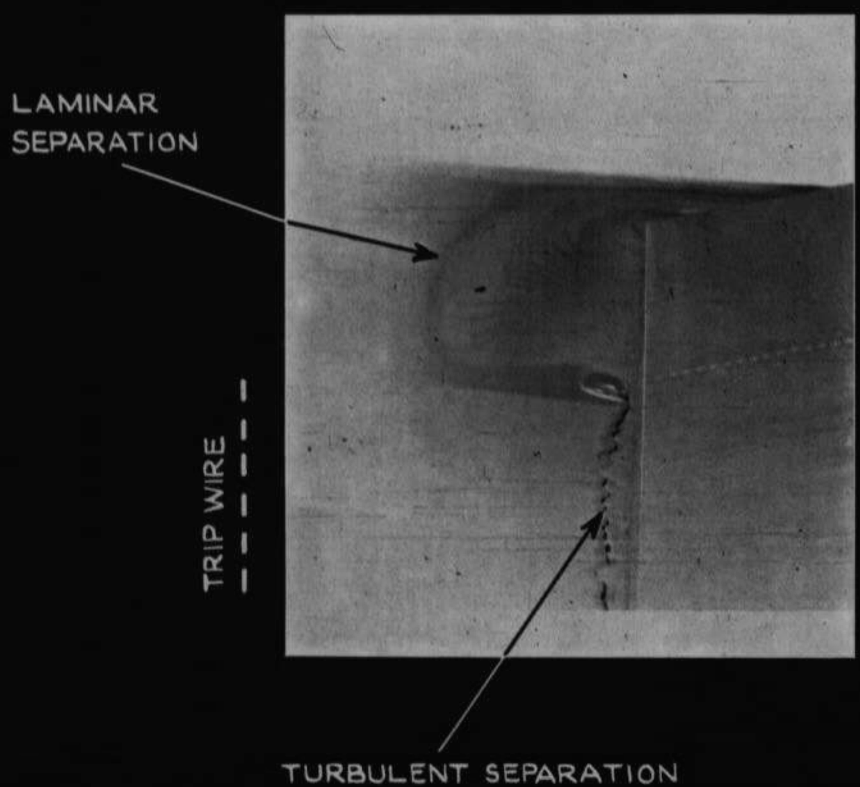


FIG. 15. AN OIL FLOW PATTERN OF LAMINAR AND TURBULENT FLOW TOGETHER

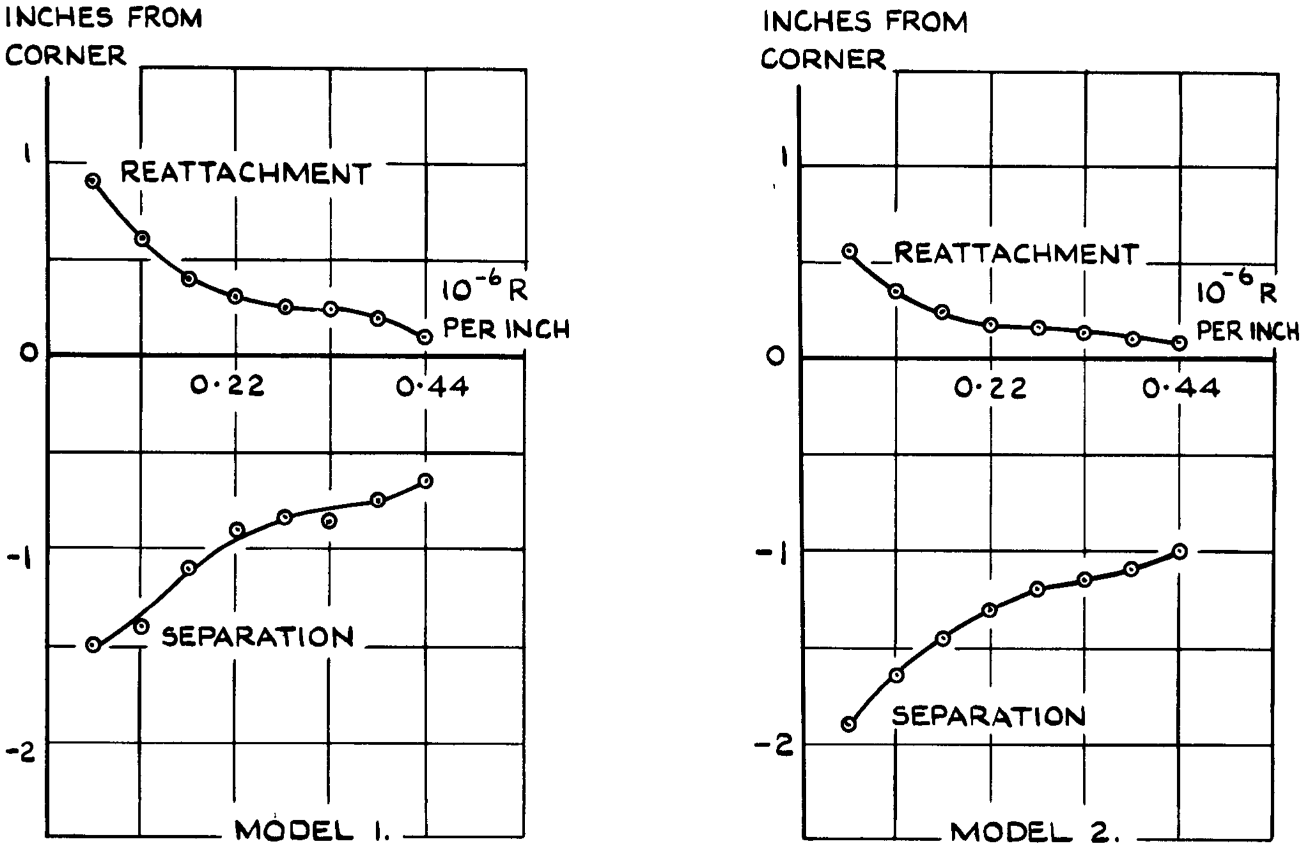


FIG. 14. VARIATION OF SEPARATION AND REATTACHMENT POSITIONS WITH REYNOLDS NUMBER.

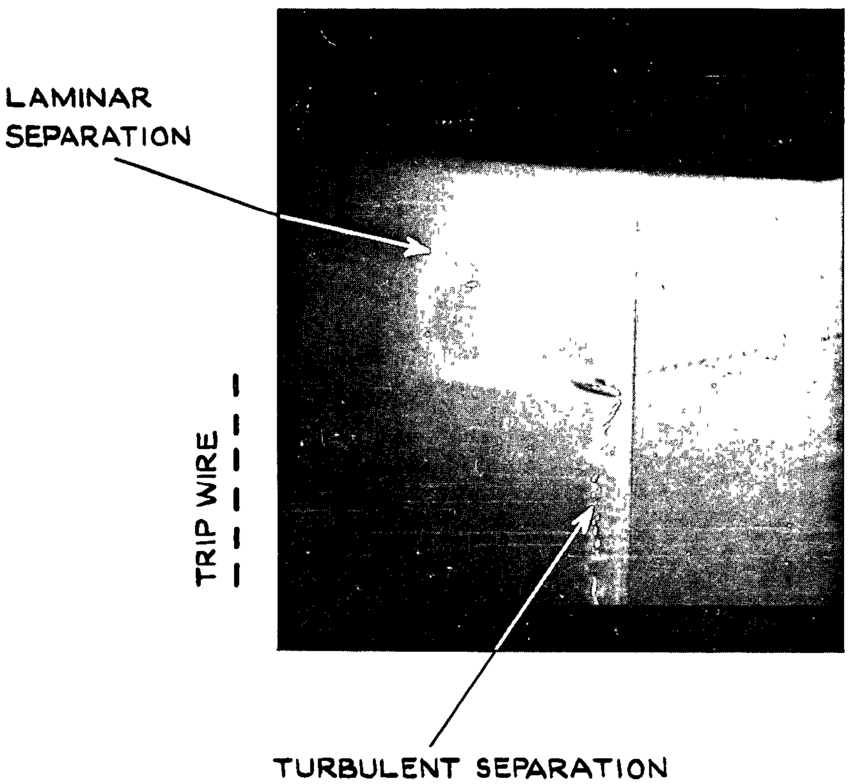


FIG. 15. AN OIL FLOW PATTERN OF LAMINAR AND TURBULENT FLOW TOGETHER

A.R.C. C.P. No.727

533.6.011.5:
533.6.011.72:
532.526.5

SIMPLE THEORETICAL AND EXPERIMENTAL STUDIES OF THE FLOW THROUGH A THREE-SHOCK SYSTEM IN A CORNER.
Eminton, E. September, 1961.

In the hope of finding the three-shock system in a given corner appropriate to flow at a given Mach number a very simple theoretical model is considered. It assumes that shocks and streamlines are all straight and concentrates on the flow in the region of the point where the main shock branches into two. A shear layer is allowed to originate there and conditions imposed to match both pressures and flow directions on either side of it. The results of the calculations suggest that a more sophisticated theory is needed and a few experiments are made to substantiate this. Theory and experiment together lead to the conclusion that the shock system in a corner is determined not by the way the flow

(Over)

A.R.C. C.P. No.727

533.6.011.5:
533.6.011.72:
532.526.5

SIMPLE THEORETICAL AND EXPERIMENTAL STUDIES OF THE FLOW THROUGH A THREE-SHOCK SYSTEM IN A CORNER.
Eminton, E. September, 1961.

In the hope of finding the three-shock system in a given corner appropriate to flow at a given Mach number a very simple theoretical model is considered. It assumes that shocks and streamlines are all straight and concentrates on the flow in the region of the point where the main shock branches into two. A shear layer is allowed to originate there and conditions imposed to match both pressures and flow directions on either side of it. The results of the calculations suggest that a more sophisticated theory is needed and a few experiments are made to substantiate this. Theory and experiment together lead to the conclusion that the shock system in a corner is determined not by the way the flow

(Over)

A.R.C. C.P. No.727

533.6.011.5:
533.6.011.72:
532.526.5

SIMPLE THEORETICAL AND EXPERIMENTAL STUDIES OF THE FLOW THROUGH A THREE-SHOCK SYSTEM IN A CORNER.
Eminton, E. September, 1961.

In the hope of finding the three-shock system in a given corner appropriate to flow at a given Mach number a very simple theoretical model is considered. It assumes that shocks and streamlines are all straight and concentrates on the flow in the region of the point where the main shock branches into two. A shear layer is allowed to originate there and conditions imposed to match both pressures and flow directions on either side of it. The results of the calculations suggest that a more sophisticated theory is needed and a few experiments are made to substantiate this. Theory and experiment together lead to the conclusion that the shock system in a corner is determined not by the way the flow

(Over)

A.R.C. C.P. No.727

533.6.011.5:
533.6.011.72:
532.526.5

SIMPLE THEORETICAL AND EXPERIMENTAL STUDIES OF THE FLOW THROUGH A THREE-SHOCK SYSTEM IN A CORNER.
Eminton, E. September, 1961.

In the hope of finding the three-shock system in a given corner appropriate to flow at a given Mach number a very simple theoretical model is considered. It assumes that shocks and streamlines are all straight and concentrates on the flow in the region of the point where the main shock branches into two. A shear layer is allowed to originate there and conditions imposed to match both pressures and flow directions on either side of it. The results of the calculations suggest that a more sophisticated theory is needed and a few experiments are made to substantiate this. Theory and experiment together lead to the conclusion that the shock system in a corner is determined not by the way the flow

(Over)

behaves around the branch point but by its behaviour around the feet of the branches which lie within the boundary layer. Thus viscous effects dominate the flow field and the external inviscid part of the flow appears to be able to accommodate itself readily by small deviations from the simplified model considered here.

behaves around the branch point but by its behaviour around the feet of the branches which lie within the boundary layer. Thus viscous effects dominates the flow field and the external inviscid part of the flow appears to be able to accommodate itself readily by small deviations from the simplified model considered here.

behaves around the branch point but by its behaviour around the feet of the branches which lie within the boundary layer. Thus viscous effects dominates the flow field and the external inviscid part of the flow appears to be able to accommodate itself readily by small deviations from the simplified model considered here.

behaves around the branch point but by its behaviour around the feet of the branches which lie within the boundary layer. Thus viscous effects dominates the flow field and the external inviscid part of the flow appears to be able to accommodate itself readily by small deviations from the simplified model considered here.

C.P. No. 727

Published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
49 High Holborn, London, W.C.1
423 Oxford Street, London, W.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
Brazennose Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England for Her Majesty's Stationery Office by Keyswift Ltd., Edgware, Middx. Wt. No. 59 K4 3/66

C.P. No. 727

S.O. Code No. 23-9015-27