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The Comparison of Theory and Experiment for Oscillating Wings

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1. Introduction

The importance of an adequate knowledge of the forces acting on wings in oscillatory motion hardly needs stressing since it plays a vital part in the study both of flutter and of the prediction of the motion of an aircraft as a whole. The subject has been studied for many years but naturally the early investigators were mainly concerned with incompressible flow and unswept wings. With the rise in aircraft speeds and changes in planform it became necessary to investigate the effects of compressibility and sweepback and this began about twenty years ago. There has been subsequent steady development of experimental techniques and theoretical methods, the former being greatly advanced by the introduction of ventilated tunnels for transonic speeds while the approach to the latter was radically altered by the use of electronic computers. Even so the range of parameters which now has to be covered is so large that the amount of information available is sometimes quite inadequate.

Some experimental information has been obtained from flight tests of aircraft, or from models mounted on rockets or aircraft, but most has been obtained in wind tunnel measurements using rigid models of wings or rigid controls. Measurements with distorting models, although not unknown, are uncommon, due to experimental difficulties. Thus for a knowledge of the forces due to modes involving distortion we are dependent on theory and it is therefore essential that any theory used should have been checked by comparison with whatever experimental evidence is available. The experimental information usually takes the form of overall forces and moments and we shall for the most part be concerned with the comparison of their non-dimensional values. Measurements of pressure distribution over the wing also tend to raise experimental difficulties but a few experiments of this sort have been performed.

Almost/

Almost all theoretical work lies within the framework of linearised theory, so that it applies to small oscillations of very thin wings in a fluid of negligible viscosity. This means that much of the experimental data can only be expected to agree approximately with theory. In only a few cases has an attempt been made to take account of the thickness of the wing or the viscosity of air. Even for linearised theory the calculations required may be very lengthy, especially in the later developments dealing with three-dimensional wings, and because of this it has not always been possible to give a theoretical comparison with an experiment.

Experimental work on oscillatory wings tends to be difficult and inaccurate, moreover the calculation of wind-tunnel interference is often a formidable task, so that many values are uncorrected. Coupled with the approximations of theory this means that the standard for what constitutes "good" agreement has to be set fairly low; errors of 10% might be considered small in this context.

The general procedure in this chapter is to take each flow régime in turn, to consider briefly first the relevant theories, and then see what experimental data can be found to check them. The survey of experimental work is not intended to be exhaustive. The quantities compared are usually the derivatives for rigid modes, defined below, which strictly apply only to sustained simple harmonic motion, but the error due to using experimental values obtained from the decaying oscillation technique are probably not serious. Where sufficient evidence exists for conclusions to be drawn they are given at the end of the relevant sub-section, and summarised in Section 8.

Notation

a	speed of sound in free stream
A	aspect ratio of wing, $A = 2s/\bar{c}$
B	width of wind-tunnel working section
$\bar{c}$	geometric mean chord of wing
c_r	root chord of wing
C_m	pitching moment coefficient; pitching moment $= \frac{1}{2}\rho V^2 S\bar{c}\, C_m$
C_{m_T}	value of C_m calculated by free stream linearised theory
d	width of slot in a slotted wall wind tunnel
f	frequency of oscillation (cycles per unit time)
$\bar{h}$	distance of pitching axis downstream of the leading edge of the centre section, non-dimensionalised with respect to $\bar{c}$
h_z, h_θ, h_ξ; $h_{\dot{z}}, h_{\dot{\theta}}, h_{\dot{\xi}}$	non-dimensional derivative coefficients for hinge moment, see under "Definitions"

$\tilde{h}_z$,/

$\tilde{h}_z, \tilde{h}_\theta, \tilde{h}_\xi$	see under "Definitions, C"
H	height of wind-tunnel working section
ℓ	local loading coefficient, (pressure difference) × $(\frac{1}{2}\rho V^2)^{-1}$
$\ell_z, \ell_\theta, \ell_\xi$ $\ell_{\dot{z}}, \ell_{\dot{\theta}}, \ell_{\dot{\xi}}$	non-dimensional derivative coefficient for lift, see under "Definitions"
$\tilde{\ell}_z, \tilde{\ell}_\theta, \tilde{\ell}_\xi$	see under "Definitions, C"
m_z, m_θ, m_ξ $m_{\dot{z}}, m_{\dot{\theta}}, m_{\dot{\xi}}$	non-dimensional derivative coefficients for pitching moment, see under "Definitions"
$\tilde{m}_z, \tilde{m}_\theta, \tilde{m}_\xi$	see under "Definitions, C"
M	free stream Mach number
N	number of slots on roof or floor of a wind tunnel with longitudinally slotted roof and floor
R	Reynolds number, $R = \rho V c/\mu$ where μ is the coefficient of viscosity
s	semi-span of wing
S	area of wing planform
t	time
V	free stream velocity
w	component of perturbation velocity in the z direction
x, y, z	rectangular Cartesian co-ordinates, x in the direction of the flow of air relative to the undisturbed position of the wing, y to starboard, z upwards. It is assumed that the wing always lies near the plane $z = 0$
x_o	value of x at the pitching axis
x_h	value of x at the axis of rotation of a control surface (at the junction between wing and controls unless stated to be elsewhere)
α	mean incidence in a pitching oscillation
β	$\lvert(1 - M^2)\rvert^{\frac{1}{2}}$

ζ/

ζ $\bar{c}z_o e^{i\omega t}$ is the downward displacement of the reference axis (see Fig.1)

$\theta_o e^{i\omega t}$ oscillatory angle of incidence, positive trailing edge down (see Fig.1)

$\bar{\nu}$ frequency parameter, $\bar{\nu} = \omega\bar{c}/V$

$\bar{\nu}_{res}$ critical frequency parameter for tunnel resonance

$\xi_o e^{i\omega t}$ angle of deflection of a trailing edge control, positive trailing edge down (see Fig.1)

ρ free stream density

τ thickness ratio of wing section

ϕ velocity potential of flow round wing

$\bar{\phi}$ $\phi = \bar{\phi}e^{i\omega t}$ for oscillatory flow

ψ_m phase angle of pitching moment, arc tan $(\bar{\nu}m_{\dot{\theta}}/m_\theta)$

ω angular frequency of oscillation, $2\pi f$

Definitions

(A) Modes of Oscillation

The modes considered in this chapter can be specified sufficiently accurately by giving the position of the wing surface in the form $z = f(x, y, t)$.

"heaving"* is defined by $z = -\bar{c}z_o e^{i\omega t}$

"pitching" is defined by $z = -(x - x_o)\theta_o e^{i\omega t}$

"control rotation" is defined by $z = -(x - x_h)\xi_o e^{i\omega t}$ on the control

and $z \equiv 0$ off the control

"rolling" is defined by $z = y\phi_o e^{i\omega t}$

"flapping" is defined by $z = |y|\phi_o e^{i\omega t}$

Unless otherwise stated we shall be considering only controls hinged at the junction between the wing and control. A typical section of a wing heaving and pitching with control rotation is shown in Fig.1.

(B)/

*Often referred to as "plunging".

(B) Derivatives

The representative length and area for the wing are taken to be $\bar{c}$ and S. Then for a rigid pitching wing fitted with a hinged aileron

$$\text{Lift} = \rho V^2 S e^{i\omega t}\{(\ell_z + i\bar{\nu}\ell_{\dot{z}})z_o + (\ell_\theta + i\bar{\nu}\ell_{\dot{\theta}})\theta_o + (\ell_\xi + i\bar{\nu}\ell_{\dot{\xi}})\xi_o\}$$

Pitching Moment (positive if it tends to raise the leading edge)

$$= \rho V^2 S\bar{c} e^{i\omega t}\{(m_z + i\bar{\nu}m_{\dot{z}})z_o + (m_\theta + i\bar{\nu}m_{\dot{\theta}})\theta_o + (m_\xi + i\bar{\nu}m_{\dot{\xi}})\xi_o\}$$

Control Hinge Moment (positive if it tends to depress the trailing edge

$$= \rho V^2 S\bar{c} e^{i\omega t}\{(h_z + i\bar{\nu}h_{\dot{z}})z_o + (h_\theta + i\bar{\nu}h_{\dot{\theta}})\theta_o + (h_\xi + i\bar{\nu}h_{\dot{\xi}})\xi_o\}$$

The configuration envisaged in these definitions is that the wing has a symmetrical planform, and that there are two symmetrically situated trailing edge controls, one on the starboard half-wing and one on the port half-wing, and that these controls are oscillating in phase. The control hinge moment is the total moment exerted on the wing by the controls, and the lift and pitching moments are also taken to include the effect of both controls.

The general scheme of defining derivative coefficients for other modes should be apparent from these formulae.

For two-dimensional wings S must be replaced by $\bar{c}$ (which is of course equal to c the constant chord) and the forces and moments then have their values per unit span.

As defined above both the modes of oscillation and the forces and moments are assumed to be simple harmonic. In practice of course the latter will not be simple harmonic, and the expressions above represent merely the first terms in their Fourier series expansions.

(C) Acceleration Derivatives

Some authors divide the stiffness derivatives into two parts, thus

$$\ell_z = \tilde{\ell}_z - \bar{\nu}^2\tilde{\ell}_{\ddot{z}}$$

and similarly for ℓ_θ, ℓ_ξ, m_z, m_θ, m_ξ, h_z, h_θ and h_ξ. Here the "acceleration derivatives" $\tilde{\ell}_{\ddot{z}}$, etc., are taken to define the still-air virtual inertias, so that in still air Lift $= -\rho S\bar{c}^2\omega^2\tilde{\ell}_{\ddot{z}} z_o e^{i\omega t}$; they are thus independent of M and $\bar{\nu}$, in fact $-\tilde{\ell}_{\ddot{z}} = \lim_{\bar{\nu}\to\infty} (\ell_z/\bar{\nu}^2)$. Tabulated values for two-dimensional flow may be found in Ref.1.

2./

2. Two-Dimensional Incompressible Flow

2.1 Rigid aerofoils

The principal sources of data for comparison in incompressible flow are the theories developed by Theodorsen, Kussner and others which are described in Chapter 2 of Part I of this Manual. Early work in this field is reviewed in Ref.2. Modified forms of the theory which allow for the presence of wind tunnel walls are also available, for example Refs.3 and 4.

The theory may be expected to apply when viscosity is negligible, the aerofoil is thin, and its amplitude of oscillation is small. When these restrictions are borne in mind the degree of agreement with experiment may be regarded as satisfactory. As an example Fig.2 shows the direct pitching derivatives, m_θ and $m_{\dot\theta}$, obtained for an aerofoil of conventional section, 7.3% thick, pitching about the quarter-chord axis, during the extensive experimental investigation by Greidanus, van de Vooren, and Bergh (Refs.5 to 8). The experimental points plotted were obtained from Table 5 of Ref.8. The systematic differences between theory and experiment are obvious; the other derivatives measured (ℓ_θ, $\ell_{\dot\theta}$, m_z, $m_{\dot z}$, ℓ_z and $\ell_{\dot z}$) also show systematic differences especially at the higher end of the frequency range.

Apart from experimental scatter, the data shown in Fig.3 show a similar sort of comparison. These were obtained from Table II of Ref.9, and apply to a wing of NACA 0012 section pitching, with amplitude 6.74°, about an axis $0.37\bar{c}$ downstream of the leading edge. To avoid overcrowding Fig.3 some of the measured values have been omitted. Ref.9 also contains values of the derivatives for heaving oscillations and other amplitudes of pitching oscillation. The lifts and moments show much the same sort of agreement as that in Fig.3.

Other experimental investigations (Refs.10 to 13) lead to the same conclusion, that theory will give a fair approximation to the truth for conventional aerofoils with moderately small thickness and amplitude of oscillation. If these conditions are not satisfied the theory may differ widely from experiment. Bratt and Wight[11] found that for a model with elliptic section the pitching damping varied with frequency in a way radically different from that for conventional aerofoils. The same authors also found that mean incidence and amplitude of oscillation could lead to wide divergences if the aerofoil approached its stalling incidence. The agreement may also be adversely affected (Ref.5) if the Reynolds number is such that the boundary-layer laminar-turbulent transition point moves during the oscillation; in fact the experimental values plotted in Fig.2 were obtained with a transition wire near the leading edge. A discussion of these effects may be found in Chapter 5 of Part V.

2.2 Controls

The theory also applies to aerofoils with controls, although its accuracy is much less satisfactory. Experimental data for controls is scanty but points to the conclusion that theory overestimates control stiffness and damping derivatives by a factor which may be as large as 2.5.

Fig.4/

Fig.4 shows theory compared with the hinge moment derivatives $\tilde{h}_{\xi}$ and $h_{\dot{\xi}}$ measured by Wight[14]. The control chord was 20% of the chord of the wing which was 15° thick. At the test Reynolds numbers the transition point was well forward of the hinge line. The changes with frequency parameter are relatively small and experiment and theory are roughly in the ratio 0.6 for $\tilde{h}_{\xi}$ and $h_{\dot{\xi}}$. Further measurements described in Part II of Ref.14 for a small tab with chord 4.2% of the wing chord gave even smaller ratios, about 0.5 for the direct tab hinge moment derivatives and all the cross derivatives except the aileron hinge stiffness due to tab oscillation for which it was about 0.4. The effect of varying the Reynolds number is small but there is a perceptible effect on the damping. It was found that varying the position of transition could have effects of similar magnitude, but these are negligible compared with the discrepancy with theory. Inclusion of the acceleration derivatives would be unlikely to affect the comparison significantly.

Ref.15 describes experiments on a thinner aerofoil (NACA 0010 profile) with a larger (40%) aileron and larger (10%) tab. The measured values of the aileron hinge moments tended to be only slightly smaller than theory but the measured tab derivatives and cross derivatives were considerably smaller although the ratios were not as small as for the measurements of Ref.12.

Presumably the different ratios of theory and experiment in the two sets of experiments reflect different profiles, thicknesses and control sizes. The tendency seems to be the smaller the control the smaller the ratio of experiment to theory but there is not enough experimental evidence to make it possible to give with any confidence a rule by which the right relation could be predicted. It is in any case unlikely to be simple, since for steady flow, for which much more data is available, the effects on control derivatives of control chord, incidence, profile and Reynolds number are large and complicated, see for example Ref.16. In particular if the trailing edge angle is small, as opposed to the fairly large angles used in the experiments cited above, the experimental hinge moments, $-h_{\xi}$, tend to be higher than the theoretical.

2.3 Semi-Empirical Methods

Various devices have been adopted to improve the theoretical estimates of derivatives by incorporating experimental results to account for the effects of thickness and viscosity. A discussion of these has been given by van de Vooren[17].

The most straightforward is the "equivalent profile" or "skeleton line" technique in which the thick aerofoil is replaced by one of zero thickness whose mode of oscillation is determined from the measured forces acting on it in steady flow or from charts such as those in Ref.16. An application of this method to the calculation of control derivatives is described in Ref.18; the resulting values have been plotted in Fig.4. The improvement in the agreement for $-h_{\dot{\xi}}$ in Fig.4(b) is striking.

3. Two-Dimensional Compressible and Transonic Flow

3.1 Theoretical background

The theory which has to be checked in this régime consists of the solution of the linearised equation of subsonic compressible flow or of the integral equations by which it may be replaced, e.g., Possio's equation. (See Part I, Chapter 2.) Many authors have worked on this problem and extensive tables of derivatives are available. Methods are also extant for calculating tunnel interference[19,20], but the results have not been tabulated as extensively as for incompressible flow, although relatively simple formulae exist for low-frequency oscillations[20]. Aerofoils oscillating in wind tunnels may be affected by "tunnel resonance" in which the model and the air in the tunnel form a resonating system so that the interference effect becomes very large[21]. The critical frequency for resonance is given by

$$\bar{\nu} = \frac{\bar{c}}{H} \cdot \frac{\pi\beta}{M} = \bar{\nu}_{res} \qquad \ldots(3.1)$$

and the phenomenon is therefore most serious when M is near to one and β is small.

For sonic flow solutions of the "linearised transonic flow equation" (equation 6.8), are available; see, for example, Refs.22 and 23. The calculation of interference effects for M near to one is a practically unknown subject, complicated by the fact that the tunnels used have slotted or perforated walls.

3.2 Rigid aerofoils at subcritical Mach numbers

To illustrate the phenomenon of tunnel resonance and to give an idea of the sort of agreement found in the compressible but subcritical flow régime we may quote the results given in Ref.19. In Fig.5, which is in fact Fig.4(c) of that report, the experimental points refer to NACA experiments on a 10% thick aerofoil (NACA 65-010 section) pitching about its mid-chord axis at a Mach number $M = 0.6$, in a tunnel for which $H = 3.8\bar{c}$. The moment ratio is obtained by dividing the modulus of the pitching moment by its theoretical free-stream value, and the frequency ratio by dividing the frequency parameter by its critical value for resonance. By equation (3.1) for the parameters just given, $\bar{\nu}_{res} = 1.10$. The full-line curves represent the theoretical pitching moment for the wing in the tunnel calculated by the method put forward in Ref.19. The loss of pitching moment corresponding to resonance is obvious. The phase angle is well predicted by theory but the actual magnitude is overestimated, but near resonance there is a large wall effect on the phase angle corresponding to large discrepancies in m_θ. Similar effects were found for $M = 0.35, 0.5$ and 0.7, for both lift and pitching moment.

One inference is that tunnel interference on forces and moments is appreciable for values of $H/\bar{c}$ as large as 3.8 and that allowance should be made for it; indeed it is known (Ref.20) that the interference corrections

to/

to $\ell_{\dot{\theta}}$ and $m_{\dot{\theta}}$ tend to infinity as $\bar{\nu}$ tends to zero, however large $H/\bar{c}$ may be although the actual forces of course remain finite.

It might also be expected from consideration of Fig.5 that the magnitudes of the lift and pitching moment would generally be overestimated by theory, but the evidence from other experimental investigations is that the relationship is by no means so simple. Since according to theory the centre of pressure for a wing at steady incidence is the quarter-chord point, comparison of theoretical and experimental pitching moments for this axis may be one between small quantities; it was in fact found (Ref.24), for subcritical M and $\bar{\nu}$ up to 0.9, that the ratio of the experimental and theoretical values of $|m_\theta + i\bar{\nu}m_{\dot{\theta}}|$ was nearer two than one, the amount of the discrepancy depending on the thickness and thickness distribution without any obvious system. The ratio for $|\ell_\theta + i\bar{\nu}\ell_{\dot{\theta}}|$ was found to be near to, and usually greater than, one, but again with no obvious system. The experimental phase angle showed only rough agreement with theory. In the experiments reported in Ref.25, also for oscillation about the quarter-chord axis, the ratio for lift was less than one, and again there was rough agreement on phase angle.

When the axis of oscillation is not near the quarter-chord point and the frequency parameter is small, the phase angles will also be small and the in-phase derivatives, ℓ_θ and m_θ, will not differ much from $|\ell_\theta + i\bar{\nu}\ell_{\dot{\theta}}|$ and $|m_\theta + i\bar{\nu}m_{\dot{\theta}}|$. The damping derivatives $\ell_{\dot{\theta}}$ and $m_{\dot{\theta}}$ will be approximately proportional to the phase angle and therefore liable to greater experimental error. Fig.6 shows the comparison of derivatives taken from Ref.26 for a 10% thick aerofoil of conventional section (RAE 104), pitching about an axis $0.445\bar{c}$ downstream of the leading edge. The theoretical $-m_\theta$ is in good agreement with experiment up to the critical Mach number (about $M = 0.78$ for this section) after which the experimental value undergoes rapid fluctuations, but the theoretical $-m_{\dot{\theta}}$ is much too large. This comparison is not materially affected by tunnel interference. Somewhat similar results were obtained for a $7\frac{1}{2}\%$ thick biconvex aerofoil in the experiments described in Ref.27.

In the tests of Ref.28 measurements were made with a 6% thick wing for translational motion and pitching about three different axes of rotation, in the range $M = 0.3$ to 0.9 and $\bar{\nu}$ up to about 0.5 for $M = 0.3$ and 0.3 for $M = 0.9$. Fair agreement, allowing for experimental scatter, was obtained with theory for the derivatives ℓ_z, $\ell_{\dot{z}}$, ℓ_θ, $m_{\dot{z}}$ and m_θ, but there were serious discrepancies in $\ell_{\dot{\theta}}$ and $m_{\dot{\theta}}$ especially for the higher Mach numbers and lower frequencies. These differences were ascribed partly to tunnel interference and partly to experimental error.

It may be concluded that the in-phase derivatives will be fairly well predicted by theory, but the out-of-phase derivatives can only be relied on to be a rough approximation.

A semi-empirical approach can of course be used in compressible flow as well as incompressible and Fig.6 includes points showing how greatly the prediction of damping can be improved by the use of the equivalent profile method. (See Section 2.3.)

3.3 Non-linear effects and supercritical Mach numbers

The experiments already cited show that theory may be applicable for small oscillations about low mean incidences. If the mean incidence approaches the stalling angle the agreement rapidly breaks down. The effects of high mean incidence have been examined in Refs.29 and 30. Again as may be seen from Fig.6 the agreement breaks down when the Mach number exceeds the critical value. There appears to be no theory by which derivatives may be calculated satisfactorily under these conditions.

The data necessary for an adequate check on the transonic theory are not available.

3.4 Controls

As is well known, trailing-edge controls are particularly liable to oscillatory instability in the transonic speed range generally described as "control surface buzz". Theory predicts that for M greater than or equal to one the hinge moment damping may become negative, so that it might be expected to be negative for M less than but near to one. However the instabilities are known to be caused in many cases by mechanisms involving shock-wave movement and boundary-layer shock-wave interaction so they are outside the scope of linearised theory. Some theoretical work has been done (Ref.22) but as yet the subject is really only tractable by experiment.

Since incompressible theory gives forces on a control which are of the right order of magnitude, this very rough agreement should persist for some part of the subsonic Mach number range, although obviously it should not be assumed for M near the critical value. Published data for checking this view is very scanty as far as purely two-dimensional experiments are concerned, but we may refer to some measurements made by Wyss and Sorensen[31] for a 25% control on a 13% thick aerofoil. Fig.7 shows a comparison between the theoretical hinge-moment derivatives and corresponding experimental values derived from Table I of Ref.31. There is rough agreement on h_ξ, although the experimental values for $M = 0.2$ are higher than theory presumably because of the slightly cusped trailing edge. Apart from those for $M = 0.2$ the experimental values of the damping coefficient, $-h_{\dot{\xi}}$, differ widely from theory as the frequency increases and in fact $-h_{\dot{\xi}}$ is negative for some frequencies when $M = 0.6$. This loss of damping persists at the higher Mach numbers. Since the critical Mach number was $M = 0.7$, the theory breaks down for the damping at Mach numbers well below the critical. Although the data were not corrected for tunnel interference it seems unlikely that it could account for all the discrepancy. For thinner aerofoil sections theory is likely to give better results. For example the experiments of Ref.32 give some agreement up to $M = 0.9$ for a 4% thick profile with a 25% control. Although the theory and experiment referred to slightly different configurations this does show that the agreement is not always so bad as might be supposed from Fig.7.

4./

4. Two-Dimensional Supersonic Flow

4.1 Theoretical background

Linearised two-dimensional supersonic flow theory is formally applicable for any frequency and mode of oscillation at any Mach number greater than one, but for any particular profile it will lose accuracy as M tends to one. For M near to one the sonic solution is to be preferred. The relation between the sonic and supersonic solutions is discussed by Jordan[33], who gives formulae and tables of derivatives. Further tables may be found in Ref.34.

Van Dyke[35] has extended linearised theory to account for first order thickness effects; the theory is restricted to small oscillations superimposed on flows for which the leading-edge shock wave is attached. Finally piston theory[36] is available provided the Mach number is sufficiently high.

4.2 Rigid aerofoils

One of the first points observed from the linearised theory was that it predicted that the pitching damping could be negative for a range of forward positions of the pitching axis, depending on the Mach number and frequency parameter. Thus single degree of freedom flutter was theoretically possible.

Measurements by Bratt and Chinneck[27] of the pitching moment derivatives for a $7\frac{1}{2}\%$ thick biconvex model pitching about its mid-chord axis gave values of the damping much higher than those predicted by either linearised theory or Van Dyke's theory, but these measurements may be regarded as superseded by the supersonic tests in Ref.26. In this later work also biconvex models were used, and the pitching derivatives measured for $M = 1.42$ and $M = 1.61$, for several positions of the pitching axis, and a range of small frequency parameters ($\bar{\nu}$ up to about 0.04). Fig.8, taken from Ref.26, shows how the pitching derivatives varied with axis position, specified by the parameter $\bar{h}$, for $M = 1.42$. The stiffness derivative, $-m_\theta$, is overestimated by linearised theory, but the agreement is much improved by the use of Van Dyke's theory. The picture for the damping derivative, $-m_{\dot{\theta}}$, is more complicated; Van Dyke's theory again produces an improvement except for a small range of $\bar{h}$ near the quarter-chord axis. It should be added that for the $7\frac{1}{2}\%$ thick biconvex profile the leading-edge shock wave is detached for M less than about 1.38, so the theory can be regarded as only marginally applicable. At $M = 1.61$ similar comparisons were obtained, and in this case the curve of experimental $-m_{\dot{\theta}}$ was much nearer the parabola predicted by theory although a considerable difference remained. From these results it appears that the theoretical damping is not reliable for the forward axis positions, but the stiffness is approximately correct except near the mid-chord axis position where it is small.

A further series of experiments, by Scruton et al[37], used models of double wedge section, with thickness ratios 8%, 12% and 16%, and involved the measurement of the pitching moment derivatives for pitching with small frequency parameter about a range of pitching axes, $-0.25 \leq \bar{h} \leq 1.25$, for a

range/

range of Mach numbers $1.37 \leq M \leq 2.43$. The conclusion is that Van Dyke's theory gives good agreement with experiment for the higher Mach numbers. As M decreases this agreement deteriorates until that value of M for which the leading-edge shock becomes detached is approached, and then theory and experiment differ widely, especially as regards the damping derivative, $-m_{\dot{\theta}}$. Piston theory agrees well for M greater than two.

Martucelli[38] measured the pressure distribution on thin single-wedge profiles, pitching at low-frequency parameters, for $\bar{h} = 0.6$ and $M = 1.4$ and 1.8. He found that the amplitude of the measured pressures was near to, or slightly above, that predicted by linearised theory, but the phase angle, theoretically just less than 180°, was found experimentally to be much less, especially towards the trailing edge. Comparison with Van Dyke's theory gives only slightly better agreement.

Most of these experiments dealt with profiles for which the leading-edge shock wave was attached. Pugh and Woodgate[39] measured the pitching moment derivatives for pitching single-wedge profiles of angles 14° and 9° with rounded leading edges. For $M = 1.75$ and 2.47 $-m_{\theta}$ was predicted very well by first order piston theory. The agreement on damping was less good but as the difference decreased markedly as M was increased to its higher value it appears that the theory would be satisfactory for M above about three.

We may conclude that for sharp-nosed sections performing small, low-frequency parameter pitching oscillations Van Dyke's theory will be fairly satisfactory provided M is well above its value for shock detachment. First order piston theory will give rough agreement for round-nosed sections for high Mach numbers. Lack of experimental information prevents an assessment of theory except for low-frequency parameters. There is no theory which is adequate for sharp-nosed sections if M is so low that the shock is detached, or for round-nosed sections in the lower supersonic range.

4.3 Controls

Since there is no upstream influence in supersonic flow an oscillating trailing-edge control on a two-dimensional wing would be expected to behave like a two-dimensional wing pitching about an axis at its leading edge, and therefore to be liable to negative damping at low supersonic Mach numbers. Purely two-dimensional evidence on this point is not available but the measurements described in Ref.40 are for conditions sufficiently close to two-dimensional to shed some light on the subject. In this work the main wing models were two-dimensional, spanning the tunnel, but the controls were rectangular with chord equal to one third of the wing chord, and span equal to 1.45 wing chords, so that the control surface was a rectangle of aspect ratio 4.35. The experimental values of the hinge moment derivatives for $M = 1.3$ and $M = 1.6$ were compared with those from two-dimensional theory and from two-dimensional theory corrected for end effects by three-dimensional lifting surface theory. Both theories predict negative damping for $M = 1.3$ in the frequency range covered by the experiments, $\bar{\nu} = 0.2$ to 0.45, but the end effects halve the amount of negative damping. In fact the damping for $M = 1.3$ was found to be either positive or, for one test only, marginally negative; the disagreement decreased with increasing frequency. For $M = 1.6$ both theories predict only slightly different amounts of positive damping, and the experimental values agree well with both estimates. Again the values of the stiffness derivative given by the theories differ very little for either $M = 1.3$ or 1.6, and the experimental values were in good agreement, although consistently smaller than theory. In these tests the wings were fairly thin, either of 5% hexagonal section or NACA 65A004 profile. The evidence therefore indicates that for thin wings theory becomes satisfactory if M is high enough ($M \simeq 1.6$ in this case), but for M nearer to one it predicts negative damping which either does not occur or is less severe than predicted. These conclusions must be regarded as only tentative as they are based solely on one series of experiments.

5. Three-Dimensional Low-Speed and Subsonic Flow

5.1 Theoretical background

The theoretical estimates with which the experimental data considered in this section are compared are derived from linearised subsonic theory, that is essentially the solution of the equation

$$\nabla^2 \bar{\phi} = \frac{1}{a^2}\left(V \frac{\partial}{\partial x} + i\omega \right)^2 \bar{\phi}, \qquad \ldots(5.1)$$

with appropriate boundary conditions. The mathematical details of its solution are treated at length elsewhere in this manual, and it is only necessary to say here that the solutions are in fact rarely obtained directly from the differential equation but more commonly from one or other of the integral equations which may be derived from it. For example ℓ, the unknown lift distribution over the wing, and w, the known vertical velocity distribution prescribed by the wing's motion are related by the equation

$$\frac{w(x,y,0)}{V} = \iint \ell(x',y') \, . \, K(x-x',\ y-y',\ M,\omega)dx'dy' \qquad \ldots(5.2)$$

where the integration is over the wing planform and K is a rather complicated kernel function, which is discussed in Ref. 41. Several systematic numerical procedures have been devised for obtaining ℓ from equation (5.2). These differ in detail but all are of the type known as the "kernel function" or "collocation" method, which involves replacing equation (5.2) by a set of simultaneous linear equations. Refs. 42, 43 and 44 are examples; the special case of small frequency parameters is treated in Ref. 45. Similar methods may be applied to the alternative integral equation in which ℓ in equation (5.2) is replaced by $\bar{\phi}(x',y',+0)$ and K is a different kernel, but this equation is not used as often as (5.2).

The practicability of these solutions depends on the availability of electronic computers, whose use has made unnecessary some of the simplifying assumptions used in earlier theories. For the present purpose we shall describe any solutions of equations (5.1) or (5.2) as "lifting surface theory" without distinguishing between the techniques used, provided that the solution has a satisfactory (mathematical) accuracy for the configuration for which it has been obtained.

The parametric restrictions on lifting surface theory are discussed in Refs. 22 and 46. It may be expected to become inaccurate for M near to one, depending on the thickness distribution of the wing, and, since it is essentially a small perturbation method, for thick wings, high mean incidence and high amplitudes of oscillation. The theory envisages a flow which is continuous except on the wing and in the wake, and must therefore be regarded with suspicion if leading-edge separation and vortex formation occurs. Since this phenomenon is associated especially with slender planforms the oscillatory version of slender wing theory, e.g., Ref. 47, requires investigation as regards its physical assumptions as well as for its validity as a mathematical approximation.

5.2 Rectangular and unswept wings at low speeds

Fig. 9 contains the theoretical curves of the stiffness and damping derivatives $-m_\theta$ and $-m_{\dot\theta}$ for rectangular wings pitching about the mid-chord axis. These curves were obtained from the theories of Refs. 45, 48 and 49. The general trend of their variation with A and $\bar{\nu}$ is obvious. This particular pitching axis was selected as being the most convenient for comparison with experiment but the theoretical data could be plotted for any other axis.

There is a large amount of experimental data available and only an outline of it will be given here.

Ashley, Zartarian and Neilson[50] carried out an extensive experimental investigation for rectangular wings of conventional section 15% thick of aspect ratio 10 and 6 pitching about the mid-chord axis and in a flapping oscillation, and of aspect ratio 4, 2 and 1 pitching about the mid-chord axis and in a plunging oscillation. The frequency range covered was up to about $\bar{\nu} = 0.7$, and the Reynolds number was about 0.9×10^6. Their plots of pitching moment amplitude and phase angle are subject to considerable experimental scatter and the curves in Figs. 9(a) and 9(b) must be regarded as rough means; the stiffness derivative follows roughly the trend of theory, but the damping derivatives tend to be much lower. For $A = 4$, 2 and 1 the degree of scatter in the phase angle is such that all that can be said is that $-m_{\dot\theta}$ is much smaller than theory predicts, and is even negative for small $\bar{\nu}$ when $A = 1$. The amplitude of the lift due to pitching, and the lift and moment due to plunging are in good agreement with theory; the corresponding phase angles have a large experimental scatter but allowing for this scatter there is rough agreement for the higher aspect ratios, but this cannot be relied on for $A = 2$ or 1. Much the same is true of the forces due to flapping; there is good agreement on amplitude, and the phase angle shows rough agreement when allowance is made for experimental scatter. The theories used for comparison in Ref. 50 were due to Kussner, Biot and Wasserman and are probably satisfactory for the higher aspect ratio wings.

Guyett and Poulter[51] measured the pitching moment for a series of rectangular wings with aspect ratio ranging from 2 to 8 oscillating about axes at the leading and trailing edges for frequency parameters $\bar{\nu} = 0.13$ to 0.4 and Reynolds numbers 0.38×10^6 to 0.13×10^6. The agreement with theory was generally good, although at the higher end of the frequency range the damping for pitching about the leading edge tended to be lower than theory but this frequency parameter of course corresponded to a very low Reynolds number. A further series of measurements by Guyett and Curran[52] concentrated on the rectangular wing of aspect ratio 3.35 with 10% RAE 101 section, but now, as well as pitching, included rolling of the half-model about an axis about one tenth of the span inboard of its root, that is, a motion effectively flapping superimposed on heaving. The Reynolds number

varied/

varied from 1.5×10^6 for $\bar{\nu} = 0.4$ to 0.4×10^6 at $\bar{\nu} = 1.3$. The full set of derivatives was measured, so that since two pitching axes were used, plunging was effectively included. Theoretical comparisons have not been worked out for the lift and moment due to flapping but the direct derivatives are in good agreement with theory (Ref. 49). Values of m_θ and $-m_{\dot\theta}$ calculated from Ref. 52 are plotted in Figs. 9(a) and 9(b); m_θ fits in well with theory, but $-m_{\dot\theta}$ is rather high.

A mass of data on pitching rectangular wings was accumulated by Bratt and his collaborators in the experiments of Refs. 10, 11, 53, 54 and 55. No attempt will be made to describe it all in detail as much of the work was concerned with oscillations of large amplitude or oscillations about high mean incidence for which theory is not adequate. It appears that Reynolds number can have a significant effect on the air forces, particularly the pitching damping, and especially for the higher aspect ratios and frequency parameters when the Reynolds number is small, say less than 0.5×10^6. As examples of the sort of agreement found Figs. 10(a) and 10(b) contain some derivatives found in Ref. 11 for a Reynolds number $R = 0.283 \times 10^6$; both stiffness and damping agree quite well with theory although the theory tends to be too small. Also plotted are values of damping from Ref. 55 for a Reynolds number $R = 2 \times 10^6$, in this case theory is slightly high.

Refs. 56 and 57 report measurements of the flapping moment stiffness and damping derivatives for a 15% thick rectangular wing of aspect ratio 6 in a flapping oscillation; both are in good agreement with theory (Ref. 49) for $\bar{\nu}$ in the range 0 to 1.5 ($R = 1.26 \times 10^6$ to 0.42×10^6). Measurements of $\ell_{\dot z}$ for 20% thick rectangular wings of aspect ratio 3, 4 and 5 described in Ref. 58 gave values slightly lower than theory for $\bar{\nu}$ up to 0.5 ($R = 0.35 \times 10^6$ to 0.1×10^6).

The lift, pitching moment, and flapping moment on a 10% thick rectangular wing oscillating in a flapping mode were measured by Woolston et al[59], for $\bar{\nu} = 0.4$ to 1.8 and $R = 2.85 \times 10^6$ to 0.65×10^6 and compared with lifting surface theory. Their amplitudes were in good agreement with theory, with only small differences which varied systematically with frequency parameter, but the phase angles showed differences which although also systematic were up to about 40% of the theoretical value.

All the experimental work mentioned above has been concerned with the measurement of overall forces and moments. The pressure distribution over an oscillating wing has received less attention but some information exists. Molyneux and Ruddlesden[60] measured the pressure distribution on a pitching rectangular wing with $R = 2 \times 10^6$ and found fairly good agreement with theory for the integrated overall forces, though no comparison was made with lifting surface theory for the pressures. Laidlaw[61,62] measured pressures on pitching and plunging rectangular wings of aspect ratio 1 and 2 and found reasonable agreement with a theory he developed for rectangular wings of moderate aspect ratio. Slender wing theory, Lawrence and Gerber's theory[48], and high aspect ratio theories were found to be unsatisfactory for predicting lift distributions. Finally Lessing, Troutman and Menees[63] measured pressures on a rectangular wing ($A = 3$)

oscillating/

oscillating in its first symmetrical bending mode and found good agreement with lifting surface theory for M = 0.24. The somewhat less good agreement for the spanwise distribution of lift and pitching moment was attributed to unsatisfactory treatment by theory of lift distributions near the leading edge.

The chief impression obtained from the comparisons summarised above is of the absence of any obvious regularity. While some of the larger discrepancies may probably be explained by experimental error or low experimental Reynolds numbers it does appear that for unswept wings of moderate or large aspect ratio theory cannot be relied on with any certainty to be in more than rough agreement with experiment, say within 30%. Again theory does not seem to be consistently too large or too small.

5.3 Unswept wings in compressible subsonic flow

The amount of data available here is comparatively small. Fig. 11 shows the pitching moment derivatives for a rectangular wing of aspect ratio 4 compared with low-frequency and finite-frequency theory. The experimental values were taken from Ref. 26 and refer to a 10% thick section with critical Mach number M = 0.78 approximately. Low-frequency theory slightly overestimates the stiffness and seriously overestimates the damping. A similar trend is apparent for subsonic Mach number in Fig. 17 (see Ref. 64). Including the effect of the experimental frequency in the theory removes the discrepancy for the stiffness but only partially removes it for the damping.

In Ref. 42 lifting surface theory is compared with the experimental results reported in Ref. 65 for a rectangular wing of aspect ratio 2 pitching about the mid-chord axis in the range M = 0.2 to 0.7; the experimental lift and pitching moment show a considerable experimental scatter but it can be seen that they follow the variations predicted by theory fairly well. Ref. 42 also compares theory with the lift and pitching moment for the same wing in a flapping oscillation, obtained in Ref. 59, for M = 0 to 0.4; the experimental scatter is relatively small and the agreement with theory is good for both amplitude and phase angle.

Lessing[63] measured pressures on a rectangular wing of 5% thick biconvex section in its first bending mode for M = 0.24, 0.7 and 0.8. The agreement with theory was good for M = 0.24 but at M = 0.7 the phase angles were poor, although the amplitudes of the pressures were in good agreement; this was ascribed to tunnel resonance. At M = 0.8 the agreement was poor for both amplitude and phase angle but at this Mach number the flow contained shock waves.

From this evidence one might expect that for subcritical Mach number just as for incompressible flow theory for unswept wings would give at least a rough approximation to the derivatives although as appears from Fig. 11 the error may be as large as 30%.

5.4 Swept wings in low-speed flow

Although several investigations (Refs. 50, 58, 66, 67, 68) have been made on this type of planform at low speeds, not all have had the

comparable/

comparable theory worked out; these are listed for reference only and not discussed below.

Ashley, Zartarian and Neilsen[50] measured the lift and pitching moments on constant chord wings of aspect ratio 4 and angles of sweep 35° and 50°, pitching and plunging with frequency parameters $\bar{\nu} = 0$ to 0.6, and compared their values with a theory of their own for untapered swept wings which is probably sufficiently accurate for the present purpose. Except for the quantities corresponding to ℓ_z and m_z which are theoretically small the agreement is fairly good although the experimental values show considerable scatter.

Fig. 12 shows the pitching damping and stiffness derivatives measured by Scruton, Woodgate and Alexander[66] for a wing of arrowhead planform. The trends of the variations with frequency are predicted fairly well by the theory of Ref. 69 although for m_θ, which is small the actual magnitude is not good. The low-frequency theory of Ref. 45 gives slightly better agreement. The lift derivatives gave discrepancies of the same order of magnitude, but as these are not small the percentage difference is only of order 10%.

Flapping oscillations have been investigated by Bratt and Wight[57] for a constant chord wing of aspect ratio 6 with 41.3° sweepback. The stiffness and damping derivatives for the rolling moment on the half-wing model used are in good agreement with theory (Ref. 70) for frequency parameters up to $\bar{\nu} = 1.5$. The Reynolds number varied from 0.42×10^6 to 2.2×10^6 and it appeared that its influence on the derivatives was negligible.

One concludes that for swept wings at low speeds theory is likely to give a fairly good estimate of derivatives (say within 20%) except those which are small.

5.5 Swept wings in compressible flow

The few experimental results available show that theory can predict the pitching derivatives with some degree of success. Ref. 64 gives the comparison between low-frequency theory (method of Ref. 45) and experimental pitching derivatives for a series of swept wings of aspect ratio 2.64 and taper ratio 7/18 with leading-edge angles of sweepback 33.7°, 49.4° and 59.0°. The comparison for 49.4° is shown in Fig. 13. The stiffness derivative, $-m_\theta$, is correctly predicted by theory for the forward pitching axis but consistently overestimated for the rearward pitching axis. The damping derivative, $-m_{\dot{\theta}}$, is approximately correct for $M = 0.6$ but shows some discrepancy for other Mach numbers. This comparison was duplicated to a remarkable extent by the other two wings.

Pitching derivatives for a much more unconventional swept wing, of the "M-wing" type, are discussed in Refs. 71 and 72. Although theory overestimates the damping derivative by up to 25% the agreement is good for such a complicated planform.

For/

For swept wings the approximate agreement of theory and experiment persists up to much higher Mach numbers than for unswept. This is of course consistent with the fact that the onset of transonic effects is delayed by sweepback.

5.6 Delta wings in low-speed flow

As already mentioned the steady flow round a wing with highly swept leading edges often contains strong vortices in the flow over the upper surfaces when the wing is at incidence, especially when the leading edges are sharp. If such a wing is oscillating about a non-zero mean incidence or through a large enough amplitude or with a high frequency, the resulting vortices might be expected to influence the oscillatory forces; then both ordinary lifting surface theory and the slender wing approximation would be suspect. There is however some evidence to show that lifting surface theory can correctly predict the pitching damping for slender wings provided the mean incidence and amplitude of oscillation are small. Fig. 14 shows the damping derivative, $-m_{\dot{\theta}}$, as a function of axis position for a triangular wing of aspect ratio 1.0. The experimental values (unpublished) were measured by the Bristol Aircraft Co. and the Royal Aircraft Establishment and for zero mean incidence agree very well with those calculated by low-frequency lifting surface theory. This agreement breaks down when the mean incidence is raised to non-zero values. Fortunately the effect of positive mean incidence appears to be an increase in damping. Slender wing theory (Ref. 47) seriously overestimates the damping; evidently the planform is not sufficiently slender to justify the mathematical approximations in the theory. Somewhat similar comparisons also hold for a gothic wing of aspect ratio 0.75. (See Ref. 64.)

Laidlaw[62] found that for a delta wing of aspect ratio 1.07, heaving or pitching about the mid-root-chord axis slender wing theory overestimated the magnitudes of the lift force and pitching moment by a factor of 2 or more; the lifting surface theory of Lawrence and Gerber[48] gave much better agreement. For a delta wing of aspect ratio 2.31 the same author again found very poor agreement with slender wing theory; Lawrence and Gerber's theory was in good agreement for the amplitudes but much less so for phase angles. Evidently slender wing theory cannot be relied on for aspect ratios as large as $A = 1$; Laidlaw suggests that this is due to the fact that it does not satisfy the Kutta-Joukowski condition at the trailing edge, and in fact he obtains much better agreement by introducing a simple modification which ensures that it does.

Scruton, Woodgate and Alexander[66] measured the lift and pitching moment derivatives for a delta wing of aspect ratio 1.6 and a cropped delta wing (taper ratio 1/7) of aspect ratio 1.2, for pitching about axes near the mid-chord point for frequencies up to $\bar{\nu}$ about 0.6. The thickness ratio was 6% for both wings and the Reynolds number 1.0×10^6 to 1.5×10^6. Apart from small values of $\bar{\nu}$, when the experimental values were sometimes uncertain, the agreement with theory was good, the discrepancies being of order 10% or less. Again slender wing theory was in poor agreement.

Finally/

Finally in this section we will refer to some measurements by Moss[67] on the pitching derivatives of a cropped delta wing (taper ratio 1/7), leading-edge sweep 45°, and aspect ratio 3.02. (The tips were slightly curved; for straight tips A would have been 3 exactly.) The model had 10% thick conventional section (RAE 102) and the Reynolds number was of order 1×10^6 to 3×10^6. In the frequency range covered by the tests, $\bar{\nu} < 0.2$, the agreement with theory was good for $m_{\dot{\theta}}$, m_θ and ℓ_θ; there were discrepancies of order 20% for $\ell_{\dot{\theta}}$, but all the derivatives except $m_{\dot{\theta}}$ were measured when the wing was fitted with a body so that exact agreement could not be expected.

One may conclude that lifting surface theory may be expected to give the pitching derivatives reasonably well for small oscillations about zero mean incidence, unless of course the derivative concerned is small. If the flow contains leading-edge separation vortices theory is unreliable. Slender wing theory should not be used for wings of aspect ratios greater than 0.5. A further observation is that, since the pitching derivatives tend to vary very little with frequency parameter in the range $0 < \bar{\nu} < 1$, when A is less than about 3, calculations for small $\bar{\nu}$ may often be sufficiently accurate.

5.7 Delta wings in compressible flow

It seems reasonable to expect that as lifting surface theory is satisfactory for delta wings in incompressible flow it will continue to be satisfactory for at least part of the subsonic Mach number range. Fig. 15 shows that this expectation is at least sometimes justified. Here the theoretical curves are those obtained by the method of Ref. 45 for a cropped delta wing of aspect ratio A = 2, and the experimental values were measured in N.P.L. experiments. We may note the relative insensitivity of the derivatives to Mach number for the forward axis position for M up to about 0.9, and the general similarity to Fig. 13. Very similar diagrams were obtained for delta wings with the same taper ratio (1/7) but aspect ratio 1.5 and 3.0 (Ref. 64). Some measurements for the delta wing of the same series of aspect ratio A = 1.8 (Ref. 73) gave damping derivatives for low frequency in good agreement with theory, but again the theoretical stiffness derivative, $-m_\theta$, was too negative for the rearward axis ($\bar{h} = 1.2$) by about 30% and too positive for the forward axis ($\bar{h} = 0.55$) by about 15%.

The insensitivity to M had previously been observed by Leadbetter and Clevenson[74] who measured lift and pitching moments on complete delta wings of aspect ratio 2 and 4 pitching about their mid-chord axes for $0.2 < M < 0.8$ and $0.08 < \bar{\nu} < 0.81$. Their experimental scatter was sufficiently large to mask the variation with Mach number in the range M = 0 to 0.8, and they confined themselves to comparisons with Lawrence and Gerber's incompressible flow theory (Ref. 48), and, for A = 2 only, slender wing theory. The latter greatly overestimated both in-phase and out-of-phase parts of the pitching moment, indeed its only success was for the amplitude of the lift. Lawrence and Gerber's theory overestimated the

pitching/

pitching damping by about 20%, and very much underestimated the pitching stiffness which is small for this axis position. For $A = 4$ the lift was fairly well predicted but for $A = 2$ its amplitude was underestimated by about 30% to 50% and its phase angle overestimated by about 30°.

Ref. 75 reports an extensive series of measurements of the pitching derivatives of a cropped delta wing of aspect ratio 2.045 and taper ratio 0.1. This planform is close to the cropped delta wing of aspect ratio 2 shown in Fig. 15. Points were therefore read off the mean curves of derivatives given in Ref. 75 and the pitching moment derivatives calculated for pitching axes in the same position relative to the root chord as those of the wing with $A = 2$; these values have been plotted in Fig. 15. In view of the way in which the experimental points plotted were obtained from the report it would be unwise to attach much importance to the details of the comparison but it may be observed that for the rearward axis both $-m_\theta$ and $-m_{\dot\theta}$ lie very close to the theoretical curves, while for the forward axis they are both consistently higher than theory but only by about 10% to 20%. This comparison may be regarded as reasonably satisfactory.

Tobak, Reese and Beam[76] and Beam[77] carried out experiments using a delta wing model of aspect ratio 4 fitted with a slender body. Tobak found that for M in the range 0.2 to 0.9 the pitching damping for an axis near the root mid-chord was doubled when the Reynolds number was increased from about 0.4×10^6 to 0.8×10^6, and was in fairly good agreement with theory for the higher value of R, being about 20% too low at $M = 0.2$ and in almost exact agreement for $M = 0.9$. At about $M = 0.9$ the damping suddenly dropped to negative values. The frequency in these experiments was small. The theory constructed by Tobak in Ref. 76 appears to overestimate the damping slightly for low Mach numbers. Beam was mainly concerned with effects of amplitude of oscillation and mean incidence but his results for zero mean incidence support Tobak's.

In general then lifting surface theory gives correct values of pitching derivatives for delta or cropped delta wings up to high subsonic Mach numbers, although the agreement cannot be relied on completely, discrepancies of 20% being quite typical.

5.8 Controls on three-dimensional wings in subsonic flow

Although there has been no lack of experimental investigation of three-dimensional control derivatives, for example, Refs. 78 to 85, the principal difficulty here is the absence of any rigorous lifting surface theory for direct control derivatives. The boundary condition in linearised theory has discontinuities at the edges of the control and the shape of the wing cannot be correctly represented by the finite number of conditions assumed in "kernel function" ("collocation") theories. Two methods of overcoming this difficulty have been proposed although neither can be regarded as completely satisfactory. The first is the inclusion in the assumed lift distribution functions of one having singularities at the wing-control junction of the type indicated by two-dimensional theory;

this/

this approach has not yet been exploited for oscillatory flow. The second is to replace the control oscillation mode by an equivalent smooth mode constructed in some plausible manner. For example we may calculate the sectional lifts, pitching moments and hinge moments by two-dimensional theory, and then take the equivalent smooth mode to be that which gives the same sectional lifts and moments. Comparisons may of course be made with two-dimensional theory, but their significance must obviously depend on the planform.

The indirect control derivatives, for example, total lift or pitching moment due to control oscillation may be calculated from linearised theory for smooth modes by the reverse flow theorem, but for trailing-edge controls this procedure involves an integration over a region near the leading edge of the reversed wing, where existing lifting surface theories are least satisfactory.

It should however be remembered that a trailing-edge control operates in that part of the flow most affected by viscous effects so that even if mathematically correct solutions of the linearised equations were available they might well give derivatives different from those occurring in either tunnel experiments or in full scale. As in two-dimensional flow, and for similar reasons, there is a tendency for a loss of damping to occur, often quite suddenly, as the Mach number increases above the critical value.

From the evidence of two-dimensional flow it may be expected that for low speeds experimental hinge moment derivatives for controls of high aspect ratio on unswept wings would also be low compared with two-dimensional theory, and that there would be only small variations with frequency parameter. This expectation is in the main supported by experiment. For example the experiments of Ref. 78 on an outboard trailing-edge control on a high aspect ratio unswept wing gave h_{ξ} and $h_{\dot{\xi}}$ about 0.6 of two-dimensional theory. Somewhat similar comparisons were found by Lambourne, Chinneck and Betts[79] for a horn-balanced elevator, and by Molyneux[80] for a full-span control on a rectangular wing of aspect ratio 4.

Some rough measure of agreement would be expected to persist for higher Mach numbers up to the point at which transonic effects become dominant. This view is supported for an unswept wing by the experiments of Ref. 32. Trailing-edge controls on delta wings have been investigated in experiments described in Refs. 81, 82, 83 and 84. Bratt, Miles and Johnson[84] found fairly good agreement with lifting surface theory of the equivalent smooth mode type described above for a full-span trailing-edge control on a cropped delta wing of aspect ratio 1.8 and 6% thick RAE 102 section. The frequency range covered was $\bar{\nu} = 0.15$ to 0.58 at $M = 0.4$ falling to $\bar{\nu} = 0.07$ to 0.26 at $M = 1$, but in fact the derivatives h_{ξ} and $h_{\dot{\xi}}$ varied only slightly with frequency parameter. Fig. 16 shows the comparison with theory for the lowest frequency. On this evidence one would expect theory to be fairly good for Mach numbers up to that for which the sudden drop in damping occurs, in this instance for M less than 0.95. From the remaining papers this appears likely to be the characteristic behaviour.

6./

6. Three-Dimensional Transonic and Low-Supersonic Flow

6.1 Theoretical background

The chief difficulty in the theoretical treatment of oscillating wings in transonic flow is that when M is near to one the linearised partial differential equation for the velocity potential which is commonly used for subsonic and supersonic flow, that is

$$(1-M^2)\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{2M^2}{U}\phi_{xt} - \frac{M^2}{U^2}\phi_{tt} = 0, \qquad \ldots(6.1)$$

becomes inaccurate. Its ranges of validity as given by Landahl[22] are

$$|1-M| >> \tau^{\frac{2}{3}} \qquad \ldots(6.2)$$

for two-dimensional flow, and

$$|1-M| >> A\,\tau\,\log(A^{-1}\,\tau^{-\frac{1}{3}}) \qquad \ldots(6.3)$$

for slender planforms, where τ is the thickness ratio of the wing. It may be helpful to put some numbers into the formulae (6.2) and (6.3); thus for a wing having $\tau = 0.03$, (6.2) becomes

$$|1-M| >> 0.097, \qquad \ldots(6.4)$$

and taking $\tau = 0.03$ and $A = 1$, (6.3) becomes

$$|1-M| >> 0.035. \qquad \ldots(6.5)$$

It is important to observe that the symbol >> means "is very much larger than", so that even taking this phrase to imply a factor as small as two, (6.4) becomes

$$M < 0.806 \quad \text{or} \quad M > 1.194, \qquad \ldots(6.6)$$

and (6.5) becomes

$$M < 0.93 \quad \text{or} \quad M > 1.07. \qquad \ldots(6.7)$$

For thicker wings these restrictions on M will of course be more severe. It may be observed from Figs. 6, 11, 13 and 15 that linearised theory based on equation (6.1) does in fact break down at a lower Mach number for the two-dimensional and rectangular wings than for the swept or delta wings.

This breakdown of ordinary linearised theory may to some extent be overcome by using the equation

$$\phi_{yy} + \phi_{zz} - \frac{2M^2}{U}\phi_{xt} - \frac{M^2}{U^2}\phi_{tt} = 0 \qquad \ldots(6.8)$$

instead/

instead of (6.1) when the Mach number is near to one. Landahl[22] has shown that a sufficient, though not always necessary, condition for equation (6.8) is that the frequency parameter is large compared with the ratio of the maximum steady longitudinal perturbation velocity to the free-stream velocity. For two-dimensional flow this condition becomes

$$\bar{\nu} \gg \tau^{\frac{2}{3}}, \qquad \ldots(6.9)$$

and for a slender wing

$$\bar{\nu} \gg A\,\tau\,\log(A^{-1}\,\tau^{-\frac{1}{3}}). \qquad \ldots(6.10)$$

If $M = 1$ equations (6.1) and (6.8) are identical, but if M is not equal to one and the mode of oscillation is $z = g(x,y,t)$ then

$$\phi(x,y,z,t;\;M) \equiv M^{-1}\,\phi(x,My,Mz,t;\;1) \qquad \ldots(6.11)$$

where in the right hand side ϕ is the velocity potential for $M = 1$ for the planform altered in the spanwise direction by the factor M, (shrunk if $M < 1$, stretched if $M > 1$), and oscillating in the mode $z = g(x,M^{-1}y,t)$. This line of attack has been extensively treated by Landahl[22] who gives solutions for various planforms.

The alternative approach is via the integral equation obtained by letting M tend to one in equation (5.2). A method of determining this limit is described in Ref. 41 and its application to a collocation method in Ref. 86. The solutions obtained by this method should be identical with those of (6.1) but in fact there are differences (Ref. 22) due to the arithmetical approximations in the collocation method and algebraic approximations in the analytical solutions of the differential equation.

It should also be added that Mangler[87] has derived an integral equation for $M = 1$ and obtained solutions for low frequency pitching of delta wings.

One of the characteristics of the oscillatory transonic solutions available is that near $M = 1$ the effect of frequency parameter is very much greater than elsewhere. As will appear below this prediction is to some extent confirmed by experiment, but in view of the complicated phenomena occurring in transonic flow it seems unlikely that any linearised theory will be sufficiently accurate unless some empirical corrections are introduced.

6.2 Unswept wings

There has been comparatively little experimental work on unswept wings in transonic flow. We may however cite the data plotted in Figs. 17a and 17b which show respectively $-m_\theta$ and $-m_{\dot\theta}$ for a rectangular wing of aspect ratio 2 pitching about an axis 42% of the chord downstream of the leading edge. The experimental results were obtained in some

unpublished/

unpublished experiments at the N.P.L. and refer to a model of 10% RAE 102 section. Except near M = 1 there is only a small effect of varying the frequency parameter, but at M = 1 the damping shows a distinct decrease as $\bar{\nu}$ increases. The subsonic theoretical curves were obtained by the method of Ref. 45 for $\bar{\nu} \to 0$ and by that of Ref. 43 for $\bar{\nu} = 0.3$ and 0.6. The low-frequency supersonic curve labelled "Miles" was obtained from that author's solution of equation (6.1) for the low aspect ratio rectangular wing, given in Ref. 46 and mathematically valid for small values of βA. The other supersonic curves were obtained from Ref. 88. The curves labelled "Landahl" are those given by his solution of the transonic differential equation (6.8) (Ref. 22, Chapter 6).

As regards the damping (Fig. 17b) subsonic and transonic theory agree fairly well especially for the highest frequency parameter, but both overestimate the damping near M = 1 although, as predicted by theory, the error decreases as the frequency parameter increases. In Fig. 17a the sign of the frequency effect near M = 1 is correctly predicted but the actual magnitude of $-m_\theta$ is very different. It must be pointed out that the experimental results were not corrected for tunnel interference, and for the highest frequency may be affected by distortion of the model. Even so the comparison is generally poor in the transonic region. Experimental derivatives have also been determined for the same planform by Gourbil[89] but no theoretical comparison is given. Emerson and Robinson[90] measured the pitching damping for an unswept tapered wing of aspect ratio 3 for M = 0.6 to 1.18 but again no comparison was made with theory.

On this basis one would not expect transonic theory to give more than the order of magnitude of the forces in the transonic region, but to improve rapidly as M increases from one.

6.3 Delta wings

Most experimental work with these planforms consists of measurements for rigid pitching, in particular of the damping derivative. This quantity is characterised by a steady rise with M, followed by a very sudden drop in the range M = 0.9 to 1, often to negative values, followed by a rise to positive damping for supersonic flow.

Emerson and Robinson[90] also measured the pitching damping for a triangular wing of aspect ratio 2 mounted on a slender body. In his book Landahl[22] compares their values with transonic theory. The agreement is not good since theory fails to reproduce the curiously sharp maximum in the curve of damping versus Mach number which occurs at about M = 0.98. Nevertheless as M is increased from M = 0.98 the experimental damping decreases so rapidly that for M about 1.05 the agreement is good. Evidently some transonic phenomena violate the assumptions of theory until M has reached values greater than one. Low supersonic theory is in rough agreement with the experiments of Ref. 90.

Landahl also compares his transonic theory for a delta wing with some values of the pitching damping measured by Orlik-Rückemann and Olsson[91] for a triangular wing of aspect ratio 1.45 pitching with $\bar{h} = 1.2$. In Fig. 18 experimental values of the pitching damping from Ref. 91, (actually those for $f = 70$ c.p.s. from Fig. 9 corresponding to $\bar{\nu} \simeq 0.07$ for M near to 1), are compared with theory for a slightly different aspect ratio - $A = 1.5$. The measured values fit in fairly well up to $M = 0.95$, but the transonic theories for $M = 1$ of Landahl[22] and Mangler[87] overestimate the damping. For M greater than one theory fails to reproduce the low values of $-m_{\dot{\theta}}$ in the range $1 < M < 1.1$. Simple slender-wing theory[47] gives a gross overestimate of damping.

In Ref. 92, Rose reports wing-flow measurements of $-m_{\dot{\theta}}$ for a delta wing of aspect ratio 4 mounted on a slender body and pitching about an axis near the mid-root chord with frequency parameter about $\bar{\nu} = 0.05$. Agreement with subsonic theory was good up to $M = 0.9$ when the characteristic sudden reduction of damping occurred. This behaviour was confirmed by flight tests and the results of Ref. 77. Sonic theory (Ref. 87) predicted strongly negative damping at $M = 1$ but the observed values were negative only in a small range near $M = 0.93$ and rose again to positive values at $M = 1$ and remained positive up to $M = 1.1$, the maximum experimental Mach number. Supersonic theory predicted negative damping up to $M = 1.4$.

D'Aiutolo in Ref. 93 gives values of $-m_{\dot{\theta}}$ for delta wings of aspect ratio 2 and 3 mounted on slender bodies measured in experiments with rocket-powered models. No comparison is made with transonic theory although the low supersonic values of M at which $-m_{\dot{\theta}}$ recovers to positive values after becoming negative in the transonic range are fairly well predicted by theory. As these are so near to one ($M = 1.01$ and 1.05 respectively) and as $-m_{\dot{\theta}}$ varied so rapidly with M when M was near to one it is possible that this agreement was purely fortuitous.

Again Miles, Bratt and Bridgeman[73] found generally poor agreement between measured values of $\ell_{\dot{\theta}}$ and $m_{\dot{\theta}}$ for a cropped delta wing with $A = 1.8$ and taper ratio 1/7 and low supersonic theory ($M \simeq 1.1$). The in-phase derivatives agreed slightly better.

6.4 Tunnel interference in transonic flow

Taken together the experimental evidence is not favourable for transonic or low-supersonic linearised theory particularly the latter. Although good agreement with experiment is sometimes found, it appears that theory is not capable of reliably predicting the rapid changes with M which occur near $M = 1$. It must however be added that there is reason to suppose that derivative measurements for some planforms may be strongly affected by tunnel interference, which is not yet adequately understood in the transonic régime particularly for slotted tunnels. As evidence for this consider Fig. 19 which shows the direct pitching derivatives for a model of M-type planform as measured in the N.P.L. 25 in. by 20 in. wind tunnel (see Ref. 71). This tunnel has a rectangular working section with longitudinal slots in its roof and floor. T, one of the parameters commonly used in connection with slotted tunnels, is defined by

$$\frac{1-T}{1+T} /$$

$$\frac{1-T}{1+T} = \frac{B}{\pi NH} \log \operatorname{cosec} \frac{\pi Nd}{2B} \qquad \ldots(6.12)$$

where B is the width of the tunnel, H is its height, d is the width of a slot and N is the number of slots in the roof or floor. Fig. 19 (taken from Ref. 64) shows the large variation in the derivatives which occurs when N is varied. (N is shown at the top of the figure.) The effect is largest for $-m_{\dot{\theta}}$ at the higher Mach numbers, and is then large enough to mean that there is a large uncertainty in the measured values. It must be admitted that this is a very unusual planform and added that similar experiments with a model of delta planform gave a much smaller effect. Even so Fig. 19 shows that in some cases at least tunnel interference may have a significant effect, and it would therefore be unwise to dismiss linearised theory as unsatisfactory until much more experimental evidence of known reliability has been accumulated.

6.5 Controls

As in two-dimensional flow controls on three-dimensional wings have a strong tendency to instability in the transonic range and for the same reasons. Evidence of the non-linear nature of the phenomena involved may be found in Refs. 32, 83 and 94. Landahl[22] has given a theoretical investigation of control surface buzz, based on equation (6.8), for a rectangular trailing-edge control surface which predicts negative damping for certain combinations of frequency and aspect ratio when M is near to one. Comparison may also be made with two-dimensional linearised theory but the agreement with experiment, although sometimes good, is not to be relied on. In fact there seems little point in adding to the discussion of Section 3.4.

7. Three-Dimensional Supersonic Flow

7.1 Theoretical background

Linearised potential theory for supersonic flow is governed by equation (6.1) with M greater than one. The restrictions on thickness and aspect ratio necessary for this equation to be applicable are discussed in detail in Refs. 22 and 46. The principal restriction is on the thickness ratio; the smaller τ is, the wider is the range of Mach number. A rough rule might be stated as follows:

$$\tau^{\frac{2}{3}} \ll (M-1) \ll \frac{1}{\tau}, \qquad \ldots(7.1)$$

with some relaxation at the lower end of the range for wings of slender planform.

As in subsonic flow solutions are often obtained by the use of integral relations corresponding to equation (6.1). There are in fact equations of the form

$$\frac{w(x,y,0)}{V} = \iint \ell(x',y')\, K(x-x',\, y-y',\, M,\omega)dx'dy', \qquad \ldots(7.2)$$

and

$$\frac{w(x,y,0)}{V} = \iint \phi(x',y'+0)\, K(x-x',\, y-y',\, M,\omega)dx'dy', \qquad \ldots(7.3)$$

corresponding to those in subsonic flow such as equation (5.2), but there is in supersonic flow an additional integral relation of the form

$$\phi(x,y,0) = \iint \frac{w(x,y)}{U} \cdot K(x-x',\, y-y',\, M,\omega)dx'dy'. \qquad \ldots(7.4)$$

Since there is no upstream influence in supersonic flow the region of integration in equations (7.2), (7.3) and (7.4) is the Mach wedge upstream of the point (x,y). The kernels, K, have of course different forms in each equation; expressions for them may be found in Refs. 95, 96 and 97. The chief advantage of these integral relations lies in their use for irregular planforms for which solutions of the differential equation are hard to obtain. In this connection the modifications to equation (7.4) due to Evvard[98] and Stewartson[97] are of great importance. An extensive account of solutions which have been worked out for supersonic flow may be found in Miles' book[46]. By using these together with equations (7.2) to (7.4) the forces on almost any wing in any mode of oscillation can be calculated given sufficient computation.

Controls on wings present no particular difficulty for supersonic speeds, since the discontinuous boundary condition may now be allowed to

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cause discontinuities in the flow. Moreover if the hinge line is supersonic the control has no upstream influence on the wing.

There is no theory claiming to account for thickness effects in three-dimensional supersonic flow, although as we shall see, improved agreement with experiment can often be obtained by applying Van Dyke's two-dimensional method to modify the three-dimensional linearised solution, or by using piston theory in the same way for the higher Mach numbers.

7.2 Unswept wings

From the results for two-dimensional wings it could be anticipated that theory would be fairly successful for wings of moderate aspect ratio and small sweepback provided they were sufficiently thin. This view is supported by the measurements reported in Ref. 99. These experiments were carried out with a series of models of the planforms shown in Fig. 20 for rigid pitching about various axes. The basic profile was a 5% thick double wedge section; when the tip was cut off at an angle, it was in some cases machined down so that the section was a double wedge over the whole span and in others left with a vertical face. The latter type of model is designated "blunt raked edge" in Fig. 20, which shows the variation of the pitching damping derivative with the angle of rake for pitching with low frequency parameter, $\bar{\nu} \simeq 0.02$, about the mid-chord axis at $M = \sqrt{2}$. The agreement with lifting surface theory is only moderate but the addition of a thickness correction, calculated from Van Dyke's theory by strip theory, improves it greatly, especially for the two larger spans (see Ref. 100). It may be noted that sharpening the side edge can have a significant effect on the derivative. This series of experiments in fact covered the Mach number range $M = 1.4$ to 2.5 and the improved agreement with theory was maintained at the higher values of M. Fig. 21 shows the variation of $-m_\theta$ and $-m_{\dot{\theta}}$ with M for the largest model, and the comparison with linearised lifting surface theory, and with theory corrected for thickness effect by Van Dyke's theory and also by piston theory. The degree of agreement is very satisfactory.

The success of theory appears however to depend on the fact that the wing is thin. Similar measurements for rectangular planform models, $A = 1, 2, 3, 4$ and 5, with 12% thick double wedge section pitching about the mid-chord axis again with small frequency parameter in the Mach number range $M = 1.8$ to 2.4 are reported in Ref. 101. Theory was much less successful in predicting the pitching derivatives, underestimating both thickness and damping by as much as 50%.

Mention must also be made of the experiments of Tobak[102] who measured the pitching damping for an unswept tapered wing of aspect ratio 3 and thickness parameter 0.03 in the range $M = 1.2$ to 1.9, and found fairly good agreement with the theoretical value for a rectangular wing of the same aspect ratio for M greater than 1.4. For M less than 1.4 the agreement was poor presumably because a different planform was used for the theory. Again the frequency parameter was very small.

It may be concluded that for thin, slowly oscillating unswept wings theory is likely to be satisfactory for M well above one, M greater

than/

than 1.4 say, especially if some allowance is made for thickness effects. Increasing the thickness may cause this agreement to be lost. Little information is available for higher frequency parameters or M nearer to one. However Fig. 17 shows that the experimental pitching derivatives plotted there tend to fair agreement with theory at M = 1.2. (This figure is more fully discussed in Section 6.2.)

7.3 Swept wings

Not much experimental information is available for swept wings other than those of delta and similar planforms. (See Section 7.4.) However Tobak[102] found fairly good agreement with theory for the pitching damping of a model with swept tapered planform (leading-edge sweep 45°, root chord 12.4 in., tip chord 5 in., A = 3), and thin section (biconvex 3% thick), pitching about axes near the mid-root chord for Mach numbers in the range 1.2 to 1.9. Theory overestimated the damping for the lower Mach numbers, but this may have been due to the fact that the wing was mounted on a slender body.

Measurements by Moore[103] on a thin model with the same aspect ratio and leading-edge sweep, but taper ratio 0.3 pitching about axes near 0.7 root chord, showed that at a Mach number of 3.92 the damping for low frequency pitching was again overestimated slightly by theory. In this investigation increasing the mean incidence through angles up to 10° was shown to increase the damping and increasing the frequency to reduce it.

7.4 Delta wings

A larger amount of experimental work has been done for this type of planform and the general conclusion is that supersonic linearised theory is fairly satisfactory for slow pitching oscillations for Mach numbers greater than about M = 1.4.

The theory predicts that for Mach numbers less than $2^{\frac{1}{2}}$ the pitching damping of a triangular wing may be negative for low frequency oscillations about any axis in a range depending on M and A (see Ref. 46). In comparing theory with experiment we shall be concerned mostly with axis positions near to the middle of the root chord. For these axes the Mach number below which the damping is negative decreases rapidly with A, from about 1.4 for A greater than 4 to M ≈ 1.1 for A = 3. Although this occurrence of negative damping is confirmed by experiment the precise Mach number at which the transition occurs cannot be expected to be the same in theory and experiment. For example, Tobak, Reese and Beam[76] in experiments on triangular wings with A = 4 found that according to theory (including a correction for the slender body on which the wing was mounted) the damping became negative for M less than 1.13 when $\bar{h}$ was 1.27 whereas experiment gave M about 1.15 for a model having a 6% thick section with sharp leading edge and M about 1.2 for a model of the same thickness ratio but with a rounded leading edge. For $\bar{h}$ = 1.13 the corresponding figures were M = 1.27, 1.36 and 1.39. Another prediction of linearised theory is that cutting off the tips of a triangular wing will reduce its tendency to have negative pitching damping.

Experiments,/

Experiments, also reported in Ref. 76, support this conclusion; in these the model was a delta wing with the same leading-edge sweep but having a tip-chord 1/5 of the root chord.

In a further series of experiments[102] Tobak extended these measurements to triangular wings of aspect ratio 2, 3 and 4 all with 3% thick sections. The frequency parameter was again small. The agreement with theory in the range $M = 1.2$ to 1.9 was generally good for the pitching damping. For $A = 2$ he found little change with Reynolds number for $R = 1.2 \times 10^6$ to 1.9×10^6. Henderson[104] found that for thin triangular wings of aspect ratio 1.865, 2.309 and 2.801 both the pitching stiffness and damping derivatives were predicted satisfactorily for $M = 1.6$, 1.9 and 2.4 and that increasing the amplitude of oscillation from 0° to 3°, generally increased $-m_{\dot{\theta}}$ slightly while having only a very small effect on $-m_{\theta}$. Moore[103] found that for triangular wings of aspect ratio 2 and 3, 4% thick, at Mach numbers 2.96 and 3.92 the pitching damping was smaller than theory by about 15% for zero mean incidence but increased as the mean incidence was increased. Measurements by Orlik-Rückemann[91] gave pitching damping only about 60% of its theoretical value in the range $M = 1.4$ to 2 for a triangular wing of aspect ratio 1.45.

Taken together this evidence justifies the conclusion stated at the beginning of this section. For Mach numbers less than 1.4 theory can become very inaccurate although it seems rather less so for the lower aspect ratios. The effect of frequency variation has not been investigated sufficiently for any conclusions to be given.

There is much less experimental data for modes other than pitching, but we may refer to some experiments by Conlin and Orlik-Rückemann[105] on the damping in rolling oscillations of a triangular wing of aspect ratio 2 and NACA 0003-63 section mounted on a slender body. The measurements were carried out for $M = 1.35$, 1.57, 1.78 and 2.03 and two frequency parameters for each M, $\bar{\nu} = 0.32$ and 0.45 for $M = 1.35$ to $\bar{\nu} = 0.25$ and 0.35 for $M = 2.03$. The experimental values of the rolling damping derivative quadratically extrapolated to zero frequency parameter were consistently about 30% less than the theoretical value.

7.5 Controls

As already mentioned the linearised theory of control surfaces for supersonic speeds contains no special difficulties, but in fact only a few direct experimental comparisons are available, moreover comparison is made with theory for configurations not exactly the same as those used in the experiment but resembling them more or less closely.

Orlik-Rückemann[106] measured the hinge moment stiffness and damping derivatives for an inboard constant chord trailing-edge control on a 3% thick model of roughly delta planform (the trailing edge was swept back 11°), for $M = 1.35$, 1.57 and 2.02 and $\bar{\nu}$ of order 0.2. Two-dimensional theory greatly overestimated the stiffness and underestimated the damping; the theory for a trailing-edge rectangular control overestimated the stiffness by

about/

about 50% at $M = 1.35$ decreasing to about 20% at $M = 2.02$ but was in good agreement on the damping. Modifying the latter theory by taking the Mach number as its component normal to the trailing edge improved the agreement slightly for the stiffness and made it slightly worse for the damping.

Reese[81, 82] measured the hinge moment for oscillating full span controls on the trailing edge of a triangular wing of aspect ratio 2 and NACA 0005 section. For each series of experiments the hinge line was unswept but the control-chord/root-chord ratio was 0.107 in the first and 0.067 in the second. The Mach number varied from $M = 1.3$ to 1.9 and the frequency parameter based on control chord was small. Comparison was made with two-dimensional theory and also some allowance was made for the tip effect by using the theory of a pitching rectangular wing. In both cases the agreement on damping was very poor; theory predicted that it should be negative for $M = 1.3$ and rise to a positive value as M increased, but the experimental values while sometimes agreeing roughly at $M = 1.3$ persisted in remaining small or even negative as M increased. This type of comparison was affected only in detail by increasing the angle of attack up to 10°, the amplitude of oscillation up to 5°, or the frequency parameter up to $\bar{\nu}$ of order one. Theory overestimated the stiffness by up to 20%.

On this evidence linearised theory cannot be relied on for control damping derivatives, and is likely to give a considerable overestimate of stiffness derivatives.

8. Summary of Conclusions

In this section the success or failure of theory in predicting derivatives is summarised for the two commonest experimental situations, namely pitching of a rigid wing and rotation of a rigid trailing-edge control. Unless otherwise stated theory means linearised thin wing potential theory (equation (5.1)). The amplitude of oscillation and the mean incidence are assumed to be small.

These conclusions are to be regarded as general impressions, sometimes based on tenuous or even conflicting evidence. The terminology is necessarily rather vague; "satisfactory" agreement means within about 20% when applied to quantities which are not small.

8.1 Two-dimensional flow - rigid pitching aerofoils

(i) Low-speed flow — Theory is satisfactory for aerofoils of conventional section and moderate or small thickness (say up to 15%), for $\bar{\nu}$ up to 2 and R large enough to avoid large movements of the transition point during the oscillation.

(ii) Compressible flow — As for incompressible flow, except that theory may be less accurate for out-of-phase derivatives, and the range of $\bar{\nu}$ for which evidence is available is smaller, up to $\bar{\nu} = 1$.
In both incompressible and compressible flow theory may be improved by introducing a semi-empirical approach.

(iii) Transonic flow — Ordinary linearised theory breaks down. "Transonic" linearised theory may be better but cannot be checked for lack of evidence.

(iv) Supersonic flow — If the leading-edge shock wave is detached theory is unreliable. For low $\bar{\nu}$ Van Dyke's theory gives good results provided M is well above its value for shock detachment. There is no information for high frequency parameters.

8.2 Two-dimensional flow - controls

(i) Low-speed flow — For wedge-shaped trailing edges theory consistently overestimates hinge moment derivatives by a factor as large as 2 or 3, but this relationship cannot be relied on for cusped trailing edges.
Theory can be much improved by a semi-empirical approach.

(ii)/

(ii)	Compressible flow	Little data is available, but it indicates that the degree of agreement found for incompressible flow may soon be lost as M increases and fails completely well below critical M.
(iii)	Transonic flow	No reliable theory is available.
(iv)	Supersonic flow	For thin wings theory is satisfactory for high M, but for low supersonic M (M = 1.3 say) theory tends to predict damping more negative than occurs in practice.

8.3 Three-dimensional flow - rigid pitching wings

(A) Rectangular and unswept wings

(i)	Low-speed flow	Theory should be at least roughly correct for wings of conventional section for $\bar{\nu}$ up to about 2, and large enough R.
(ii)	Compressible flow	As for incompressible flow but agreement breaks down as M approaches its critical value.
(iii)	Transonic flow	No theory of known reliability is available.
(iv)	Supersonic flow	For M high enough for leading-edge shock waves to be attached theory is fairly good for thin wings and low $\bar{\nu}$ especially if allowance is made for thickness effect. Agreement deteriorates as M decreases, failing altogether for M near to one.

(B) Swept wings

(i)	Low-speed flow	Theory should be fairly good for small mean incidences and frequencies up to $\bar{\nu}$ = 1.
(ii)	Compressible flow	Theory should be fairly good up to some M just less than 1 (depending on sweepback and profile), and better than for unswept wings.
(iii)	Transonic flow	No reliable theory is available.
(iv)	Supersonic flow	Theory is fairly good for low $\bar{\nu}$ except for M near to 1.

(C) Delta wings

(i)	Low-speed flow	Theory is good for small mean incidence but agreement deteriorates if leading-edge vortices occur in the flow. Slender wing theory should not be used for $A > \frac{1}{2}$.

(ii) Compressible flow — Theory is fairly good, and there is some evidence for systematic differences with experiment.

(iii) Transonic flow — No reliable theory is available.

(iv) Supersonic flow — For thin wings, low $\tilde{\nu}$ and high M theory is good, but it becomes inaccurate for $M < 1.4$ especially for low A.

8.4 Three-dimensional flow - controls

(i) Low-speed flow — For controls of high aspect ratio on high aspect ratio unswept wings two-dimensional theory overestimates the derivatives (as for two-dimensional flow).
For controls of high aspect ratio on swept wings two-dimensional theory may give satisfactory agreement.
There is no mathematically satisfactory lifting surface theory for controls, although the approach using equivalent smooth modes can give good agreement.

(ii) Compressible flow — The sort of agreement described in (i) persists up to some M depending on wing profile and planform, but near $M = 1$ control derivatives are usually highly non-linear.

(iii) Transonic flow — There is no theory of known reliability.

(iv) Supersonic flow — Theory is not reliable.

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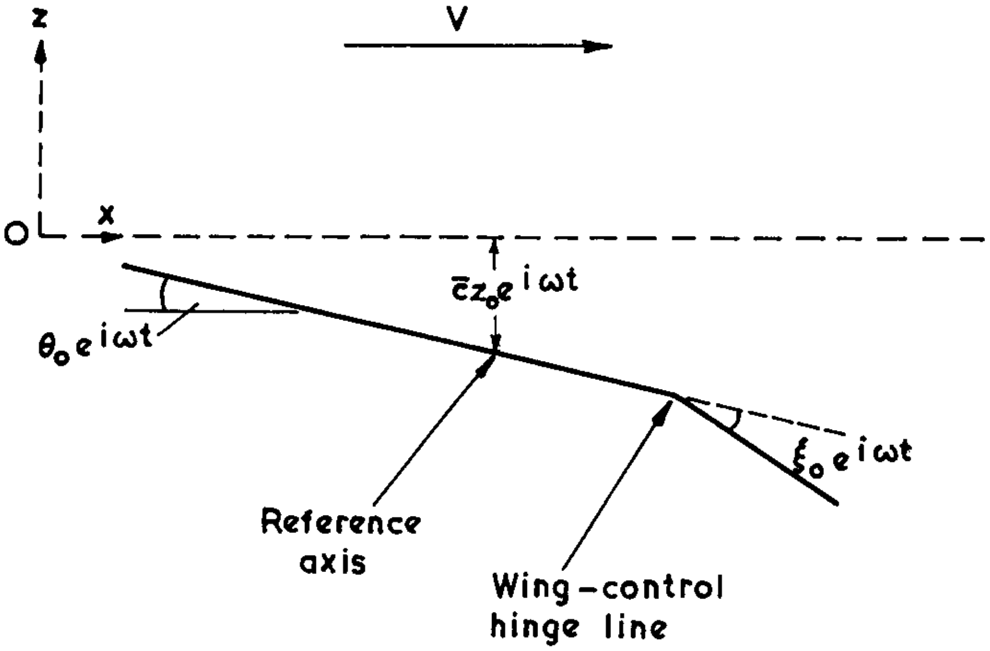
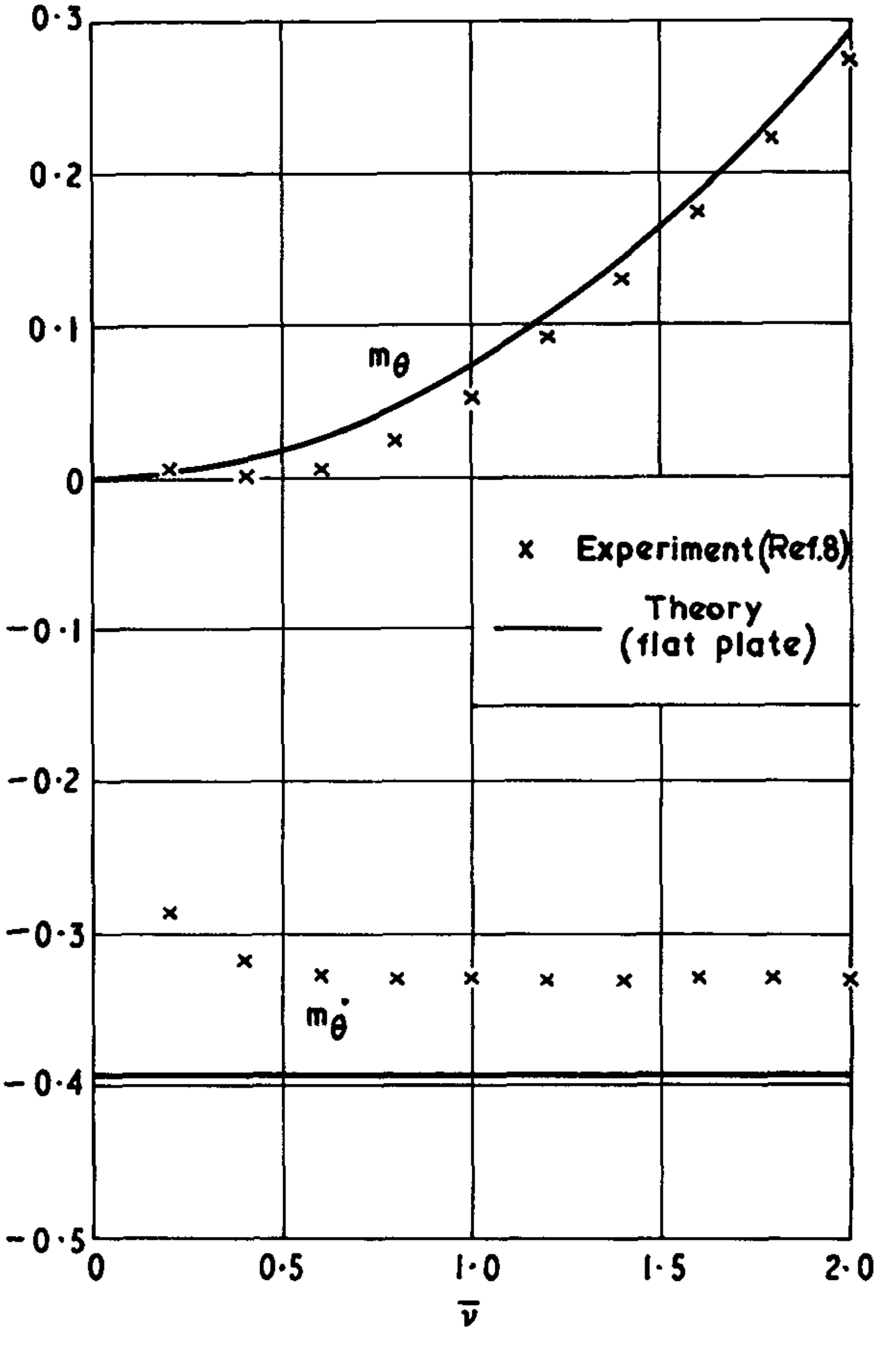
FIG. 1



Quantities defining the oscillatory motion of a rigid wing with a control

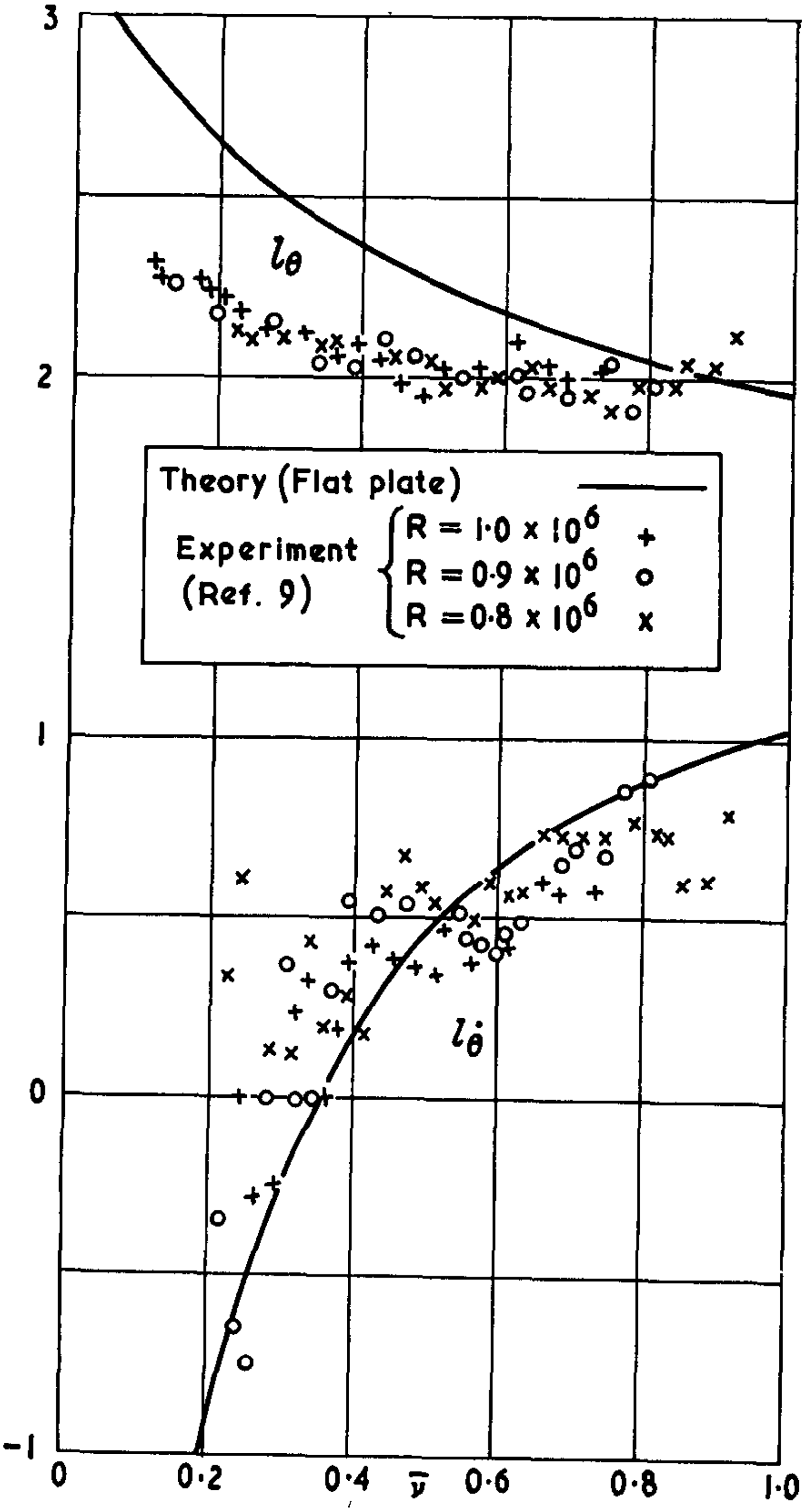
FIG. 2



Direct pitching derivatives for a 7·3% thick aerofoil

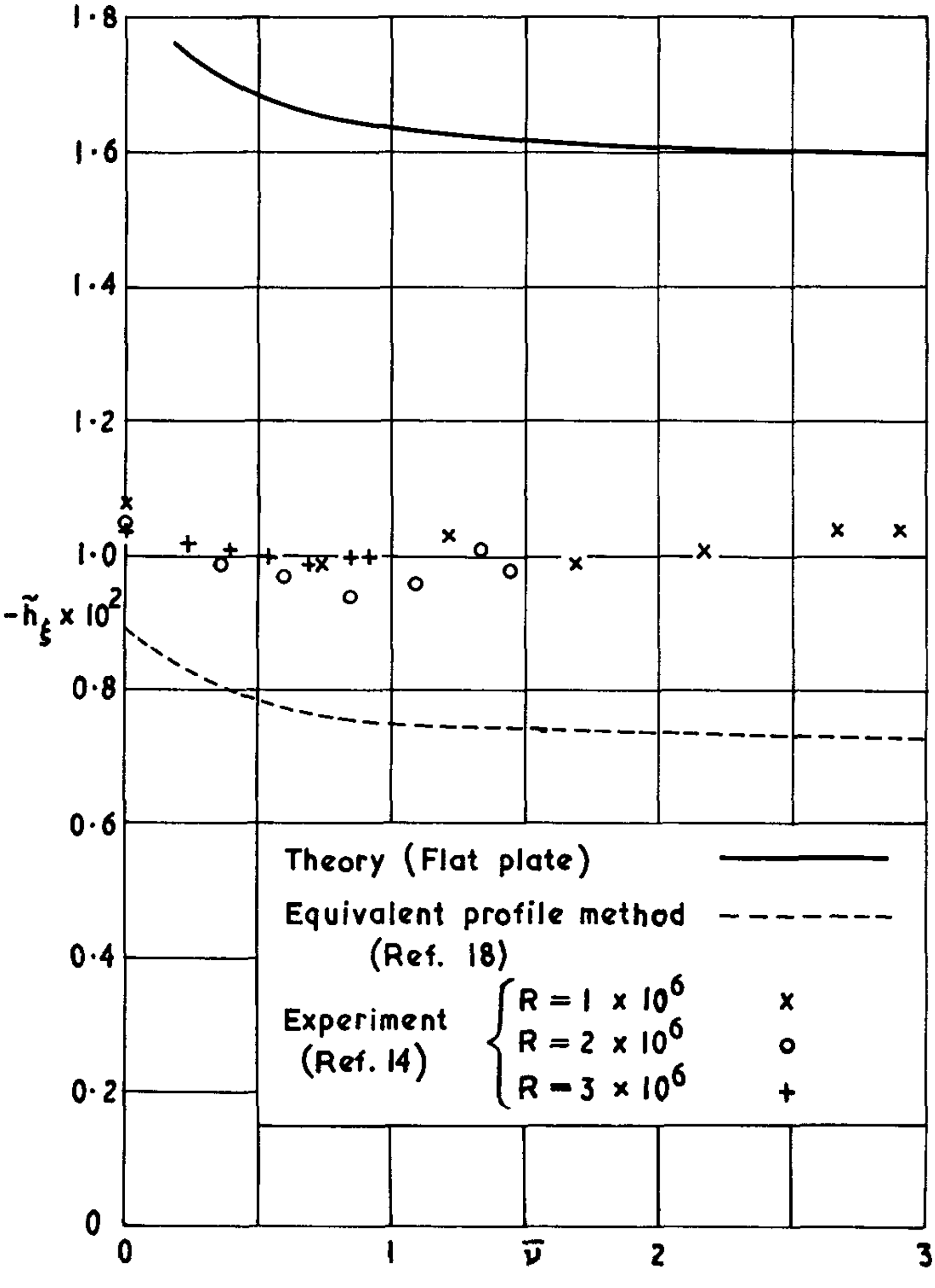
$(\bar{h} = 0{\cdot}25)$

FIG. 3.



Lift derivatives for a pitching aerofoil ($\bar{h} = 0{\cdot}37$)

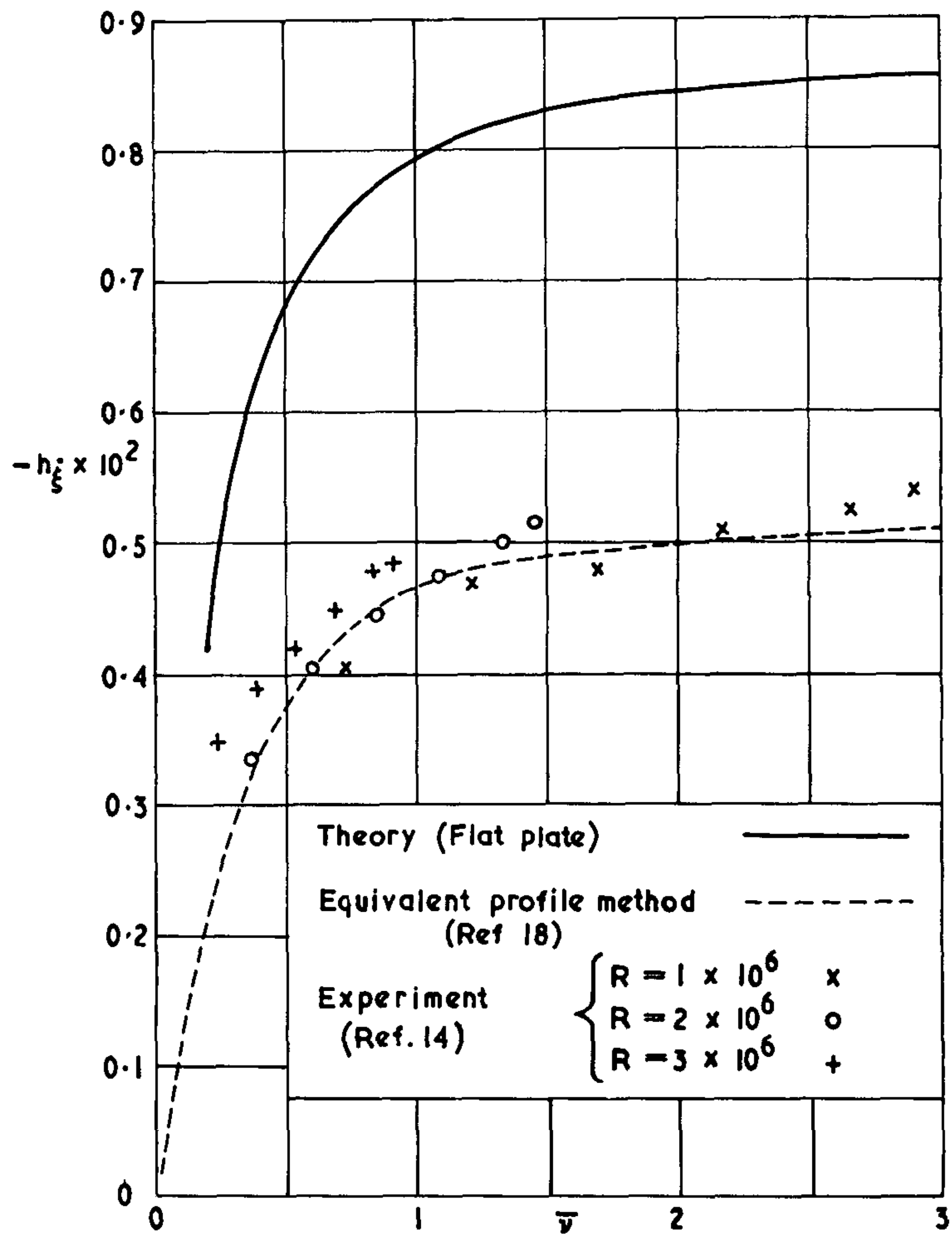
FIG. 4 (a)



Hinge moment derivatives for a 20% control on a 15% thick aerofoil

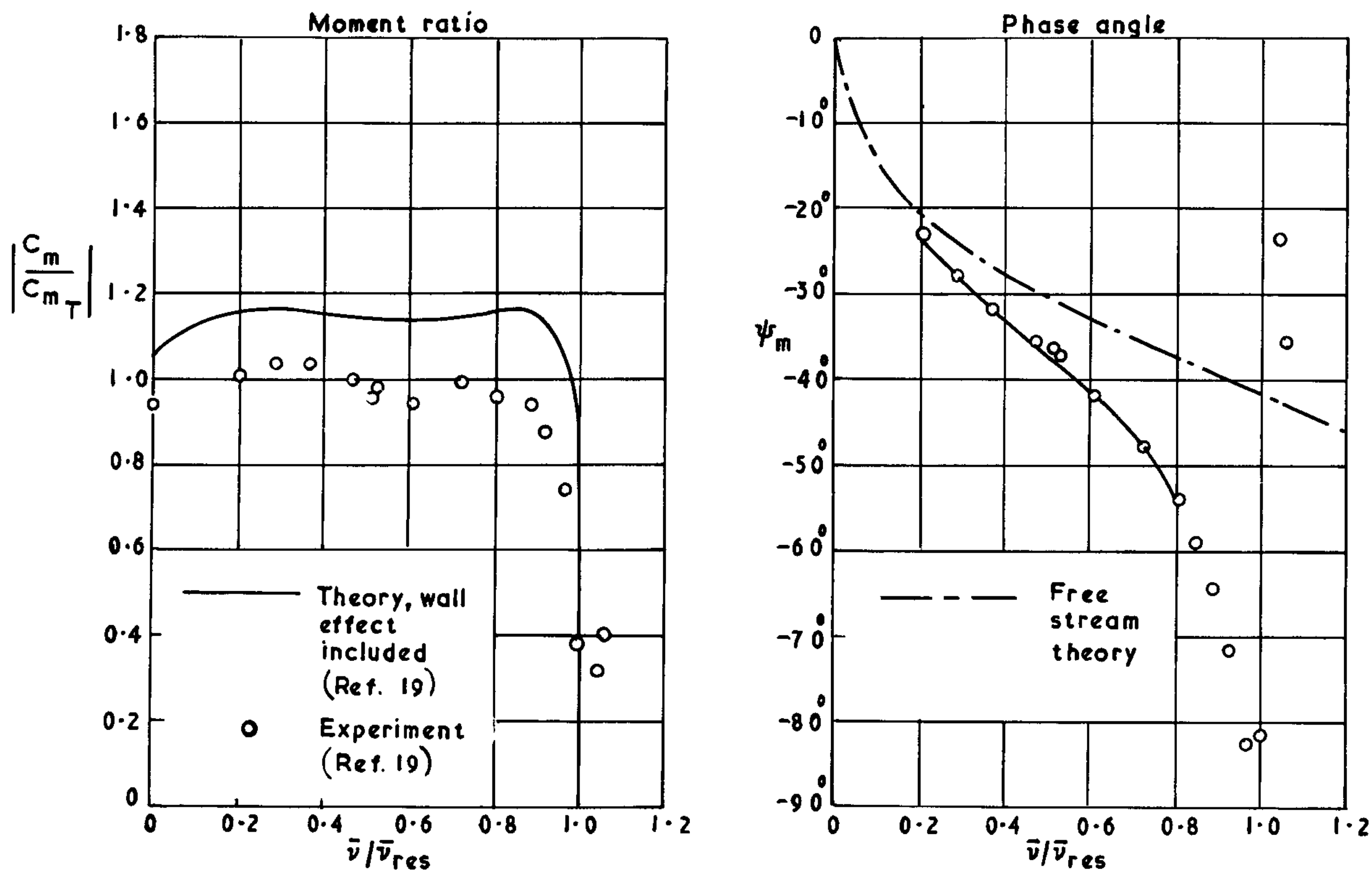
(a) Stiffness derivative, $-\tilde{h}_\xi$

FIG. 4(b)



Hinge moment derivatives for a 20% control on a 15% thick aerofoil

(b) Damping derivative, $-h_{\dot{\xi}}$

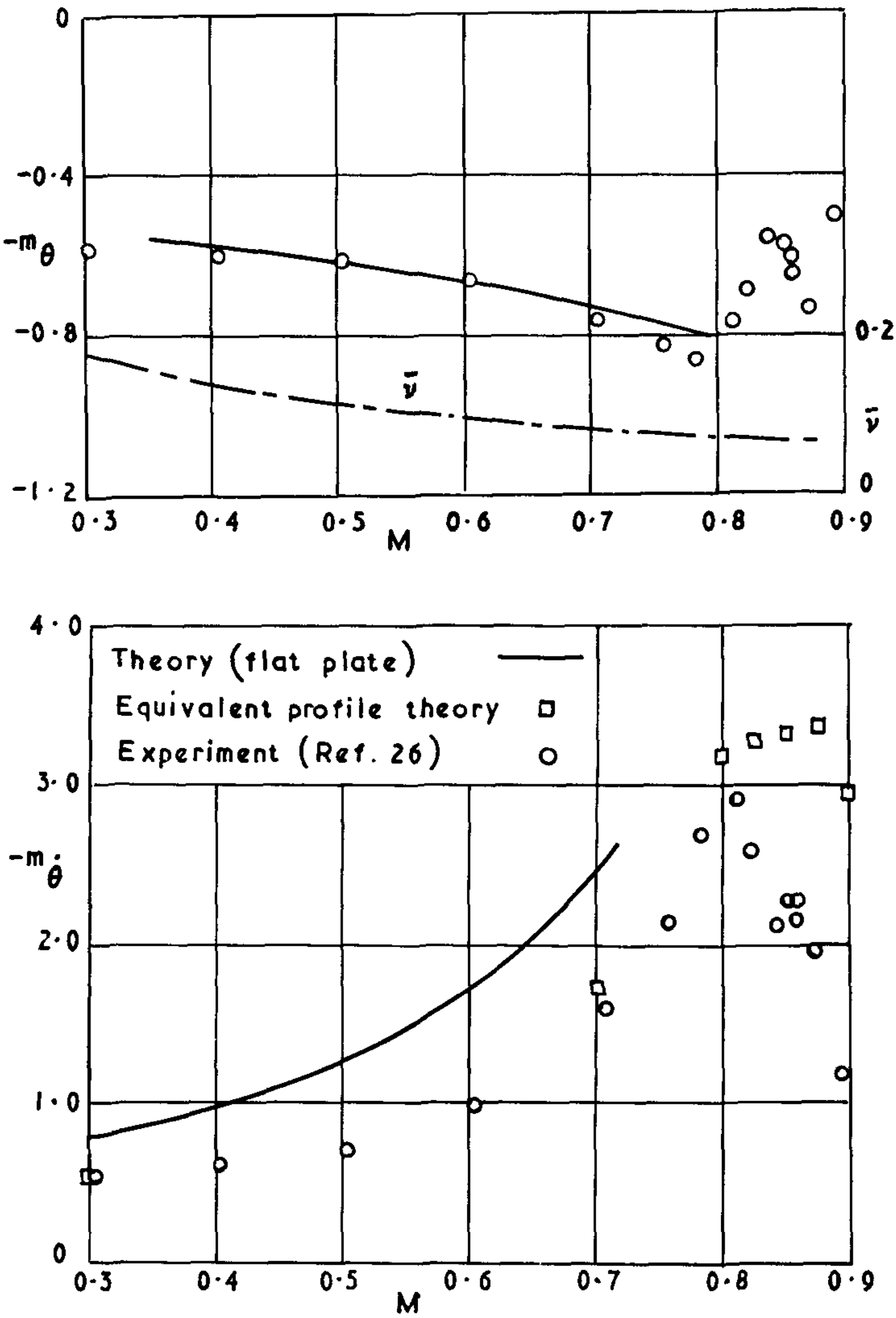


Direct pitching moment for an aerofoil at frequencies through tunnel resonance

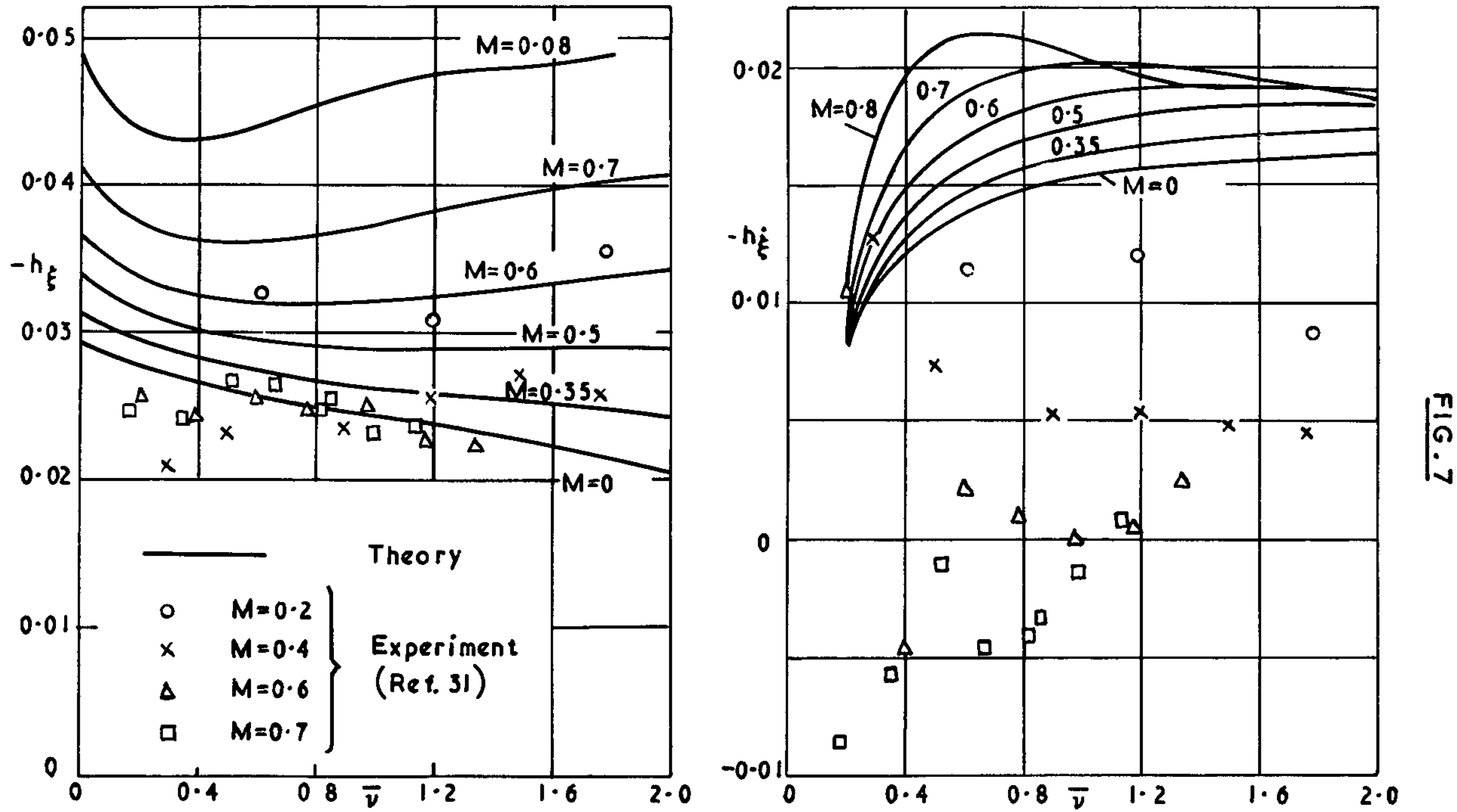
$(\bar{h} = 0\cdot5,\ M = 0\cdot6)$

FIG. 5

FIG. 6



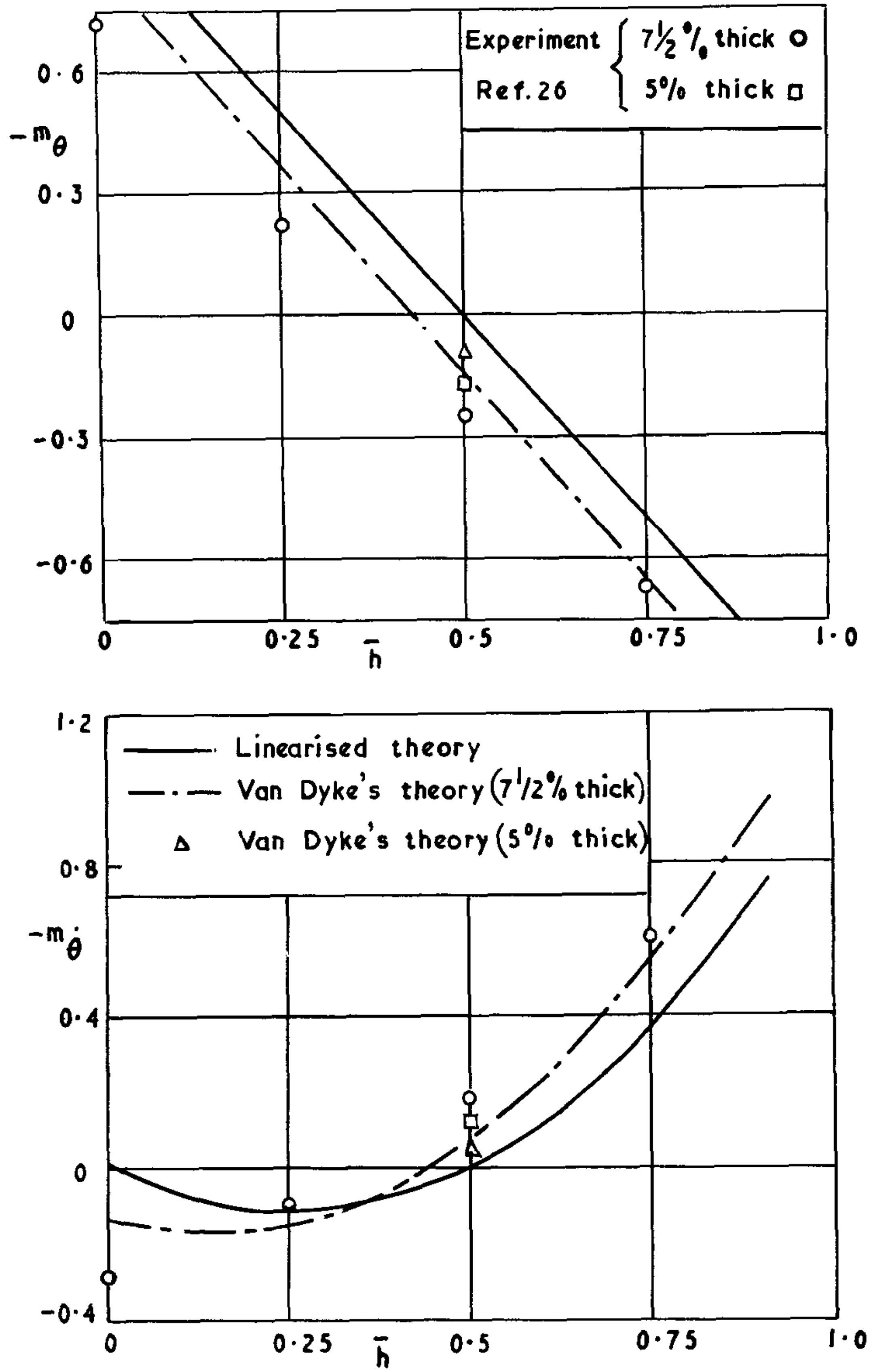
Pitching moment derivatives for a 10% thick aerofoil ($\bar{h} = 0{\cdot}445$)



Hinge moment derivatives for a 25% control on a 13% thick wing

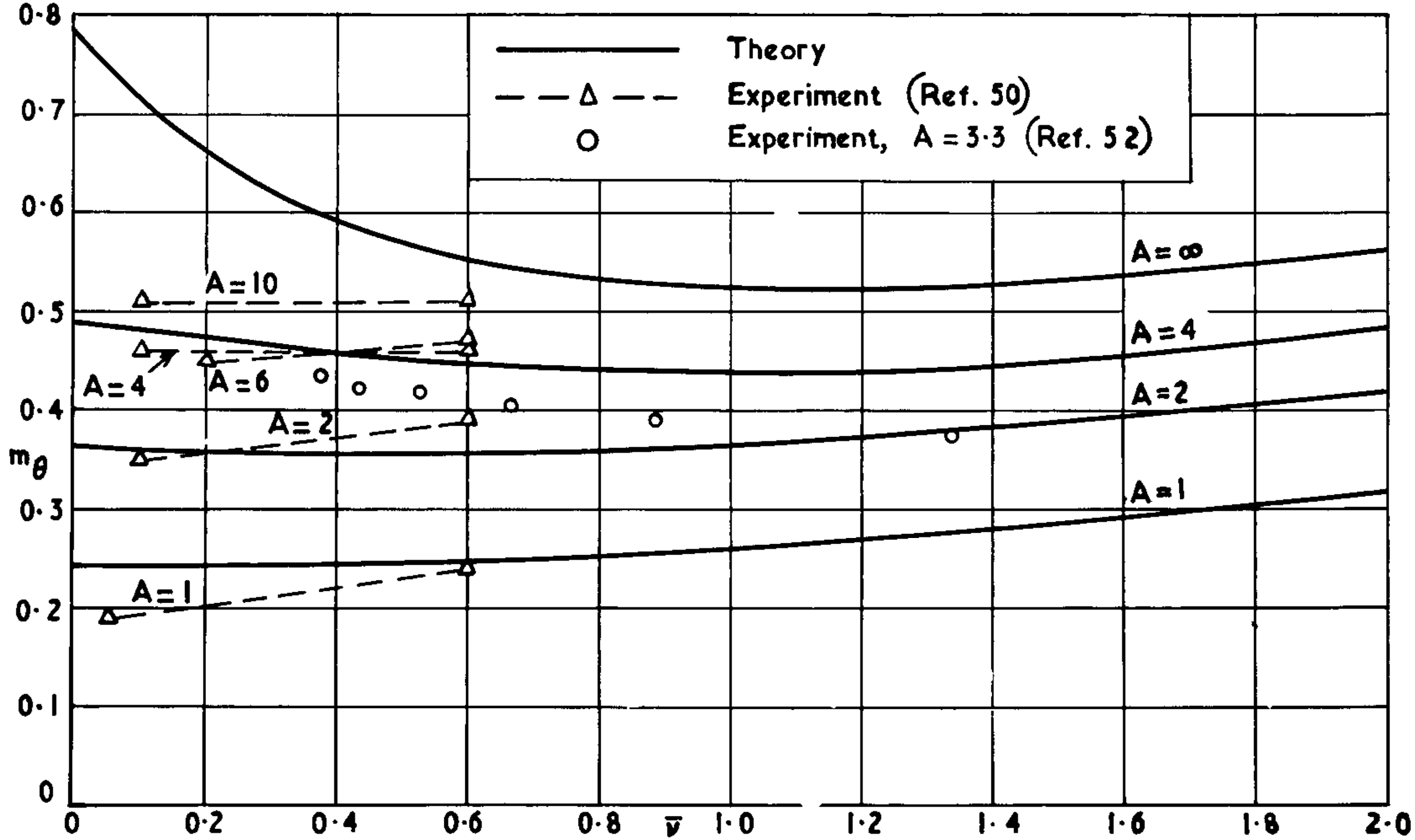
FIG. 7

FIG. 8



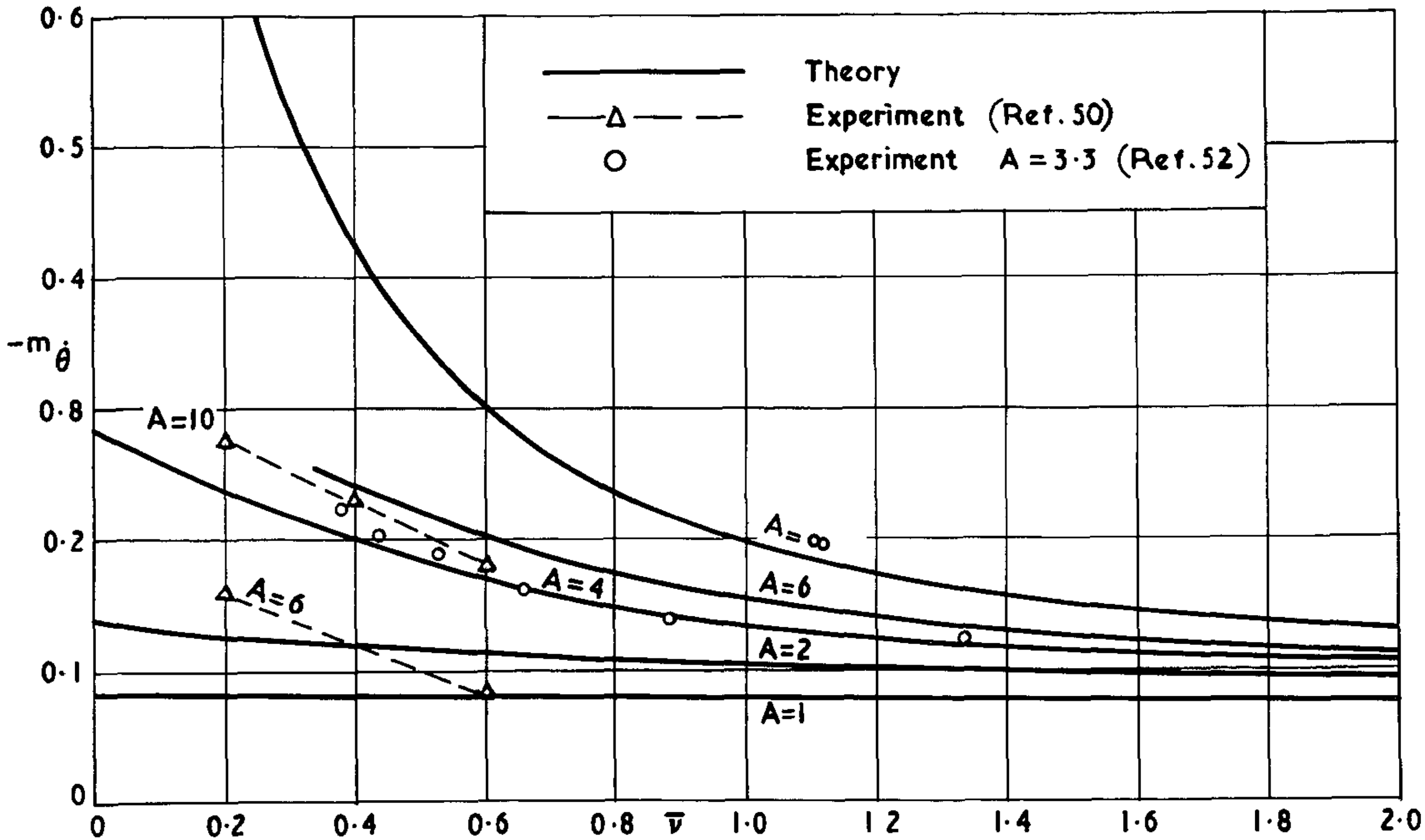
Direct pitching derivatives for biconvex aerofoils

$(M = 1{\cdot}42,\ \bar{\nu} = 0{\cdot}025,\ \theta_0 = 2^\circ)$



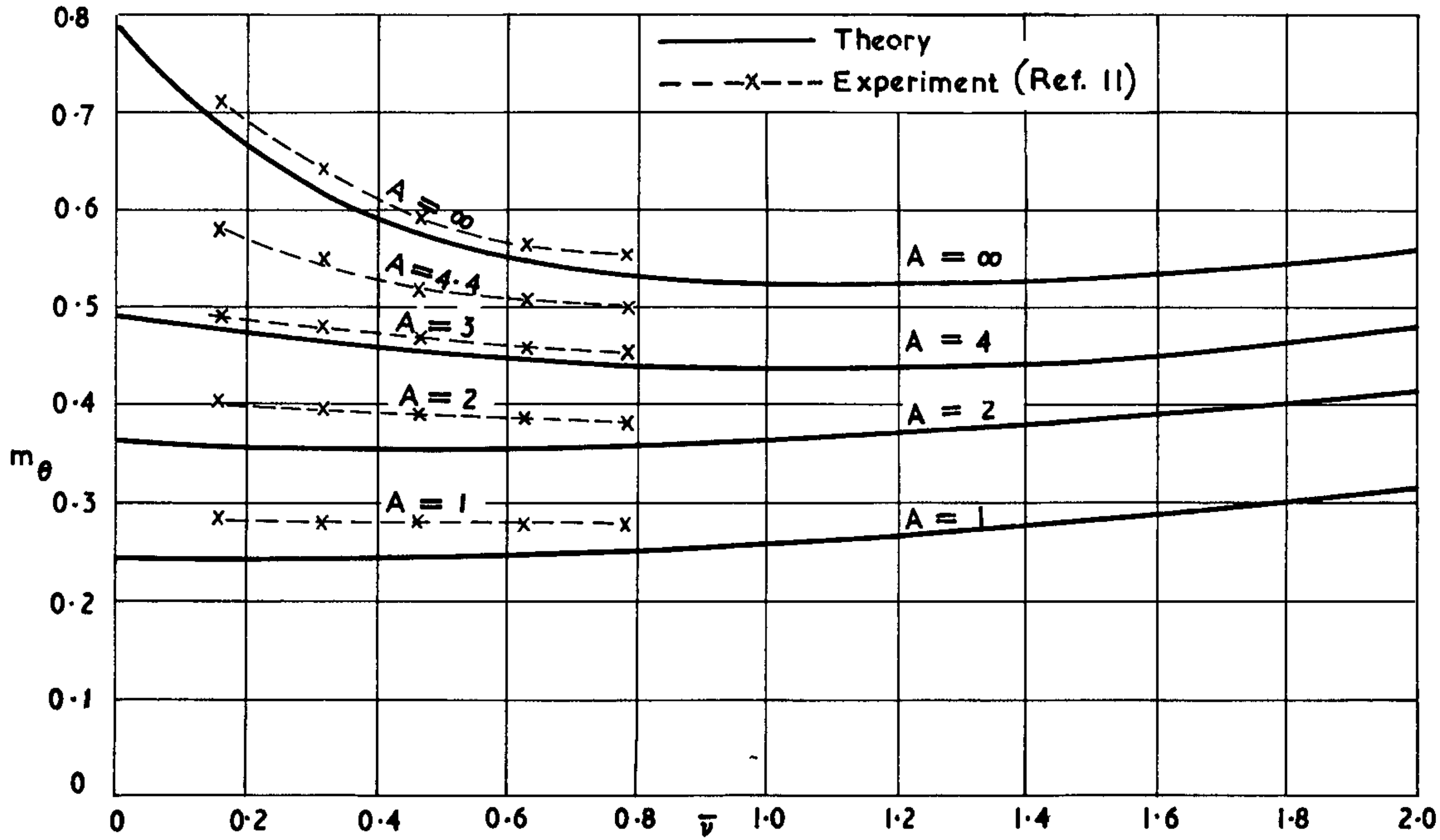
Direct pitching derivatives for rectangular wings ($M = 0$, $\bar{h} = 0{\cdot}5$) (a) Stiffness derivative m_θ

FIG. 9(a)



Direct pitching derivatives for rectangular wings ($M = 0$, $\bar{h} = 0{\cdot}5$) (b) Damping derivative, $-m_{\dot\theta}$

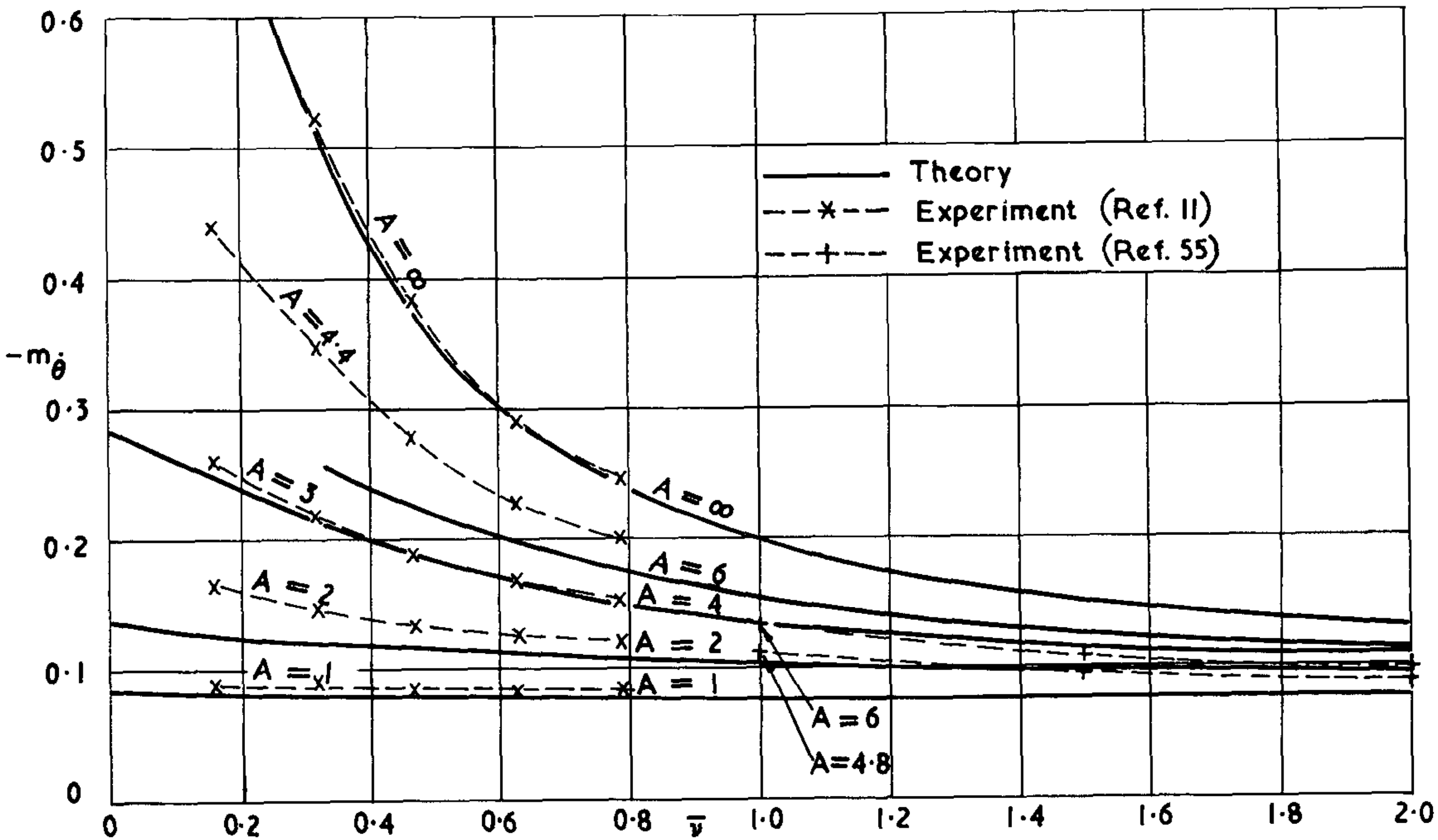
FIG. 9 (b)



Dėrect pitching derivatives for rectangular wings $(M = 0, \bar{h} = 0{\cdot}5)$

(a) Stiffness derivative, m_θ

FIG. 10.a.

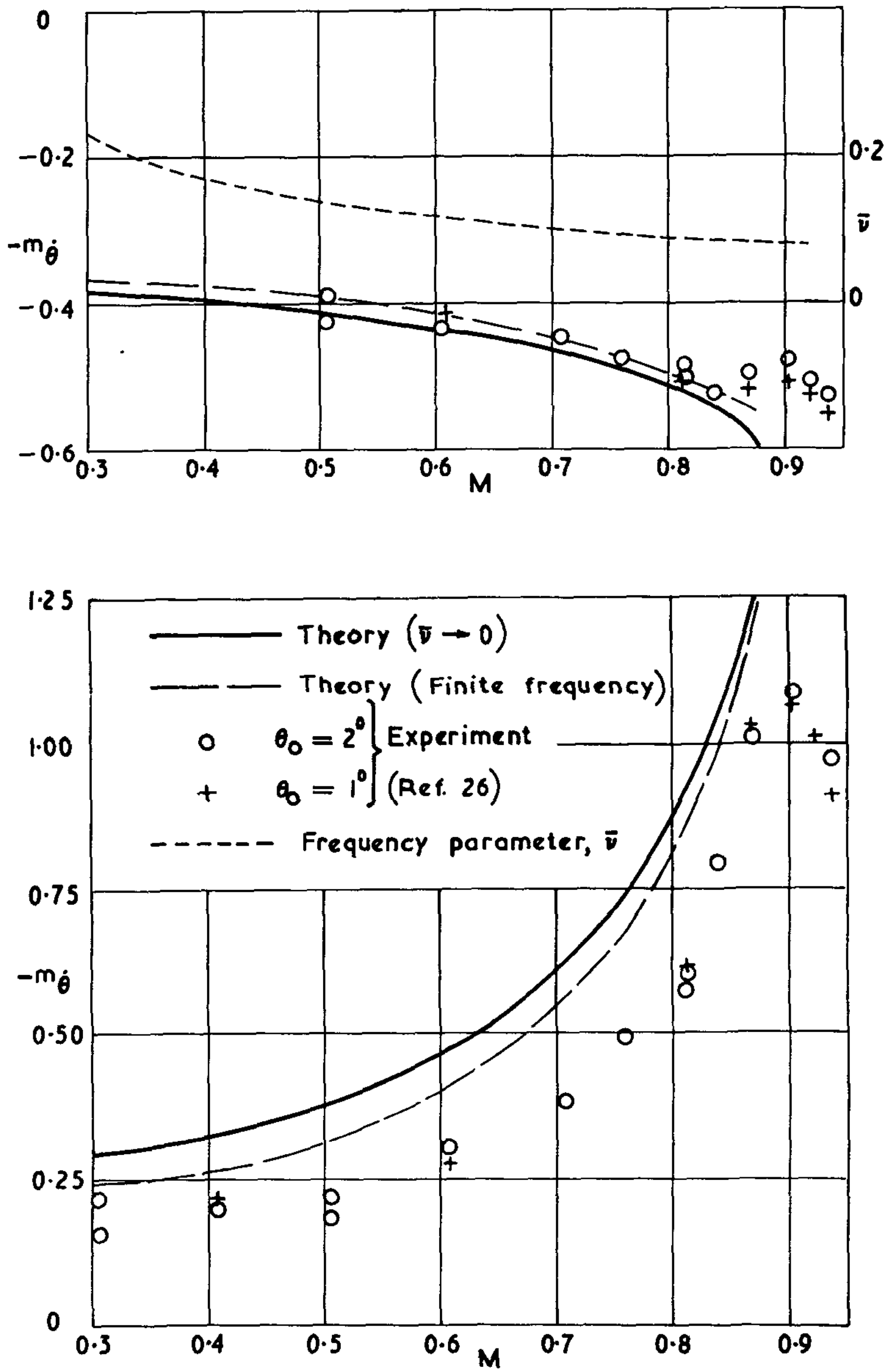


Direct pitching derivatives for rectangular wings $(M = 0, \bar{h} = 0{\cdot}5)$

(b) Damping derivative, $-m_{\dot{\theta}}$

FIG. 10b.

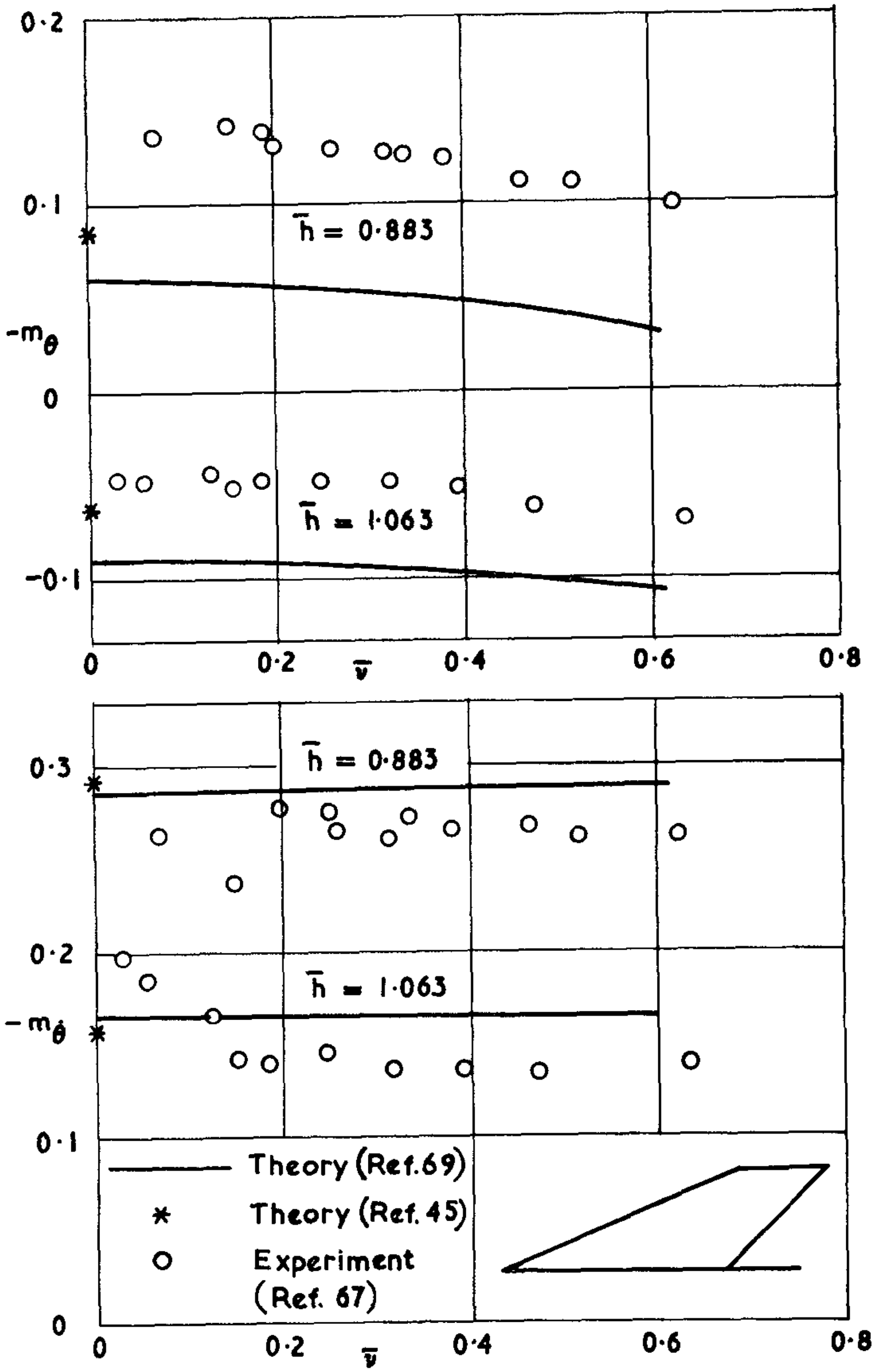
FIG. 11.



Direct pitching derivatives for a rectangular wing

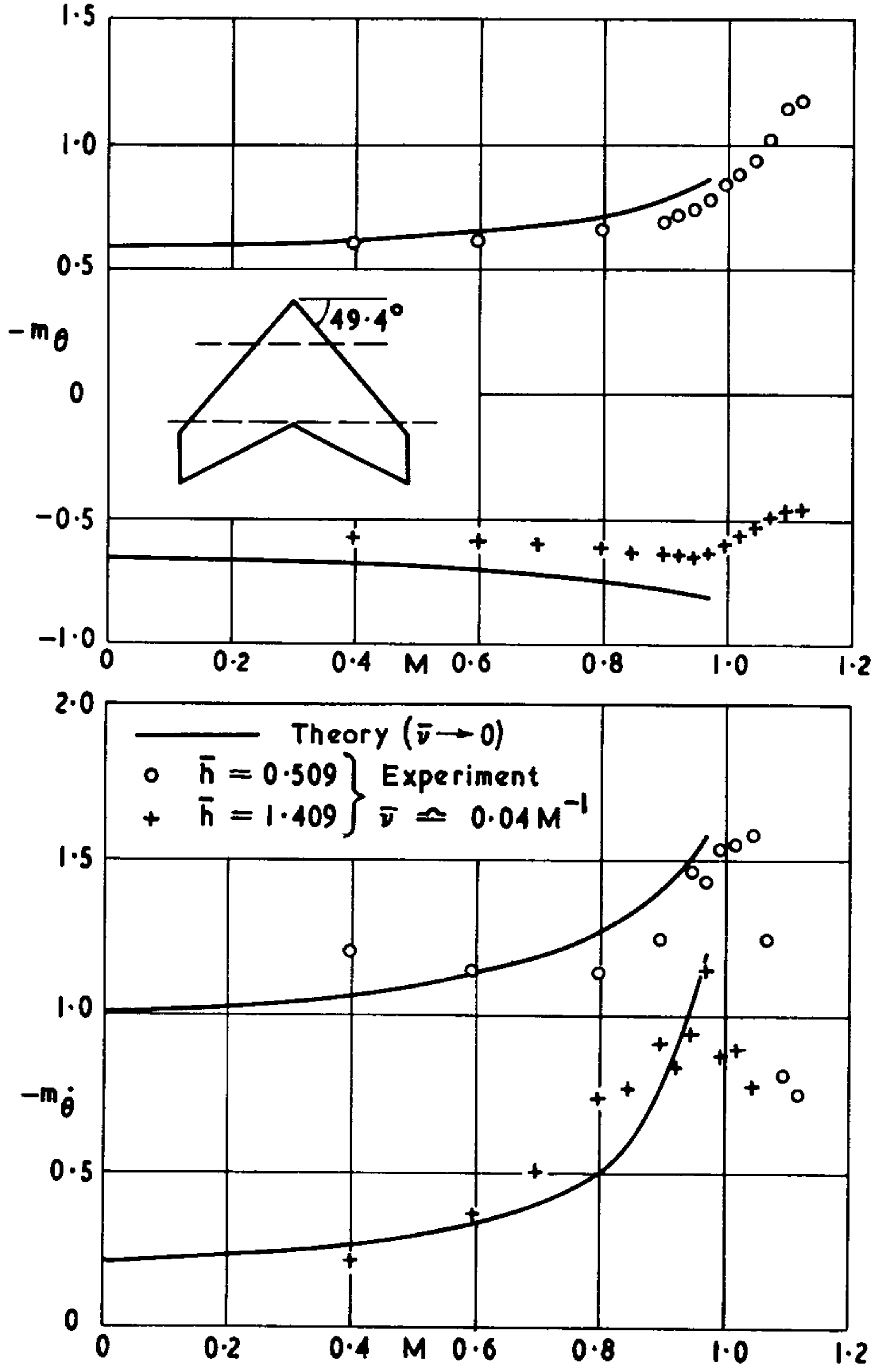
($A = 4, \bar{h} = 0\ 445$)

FIG. 12.



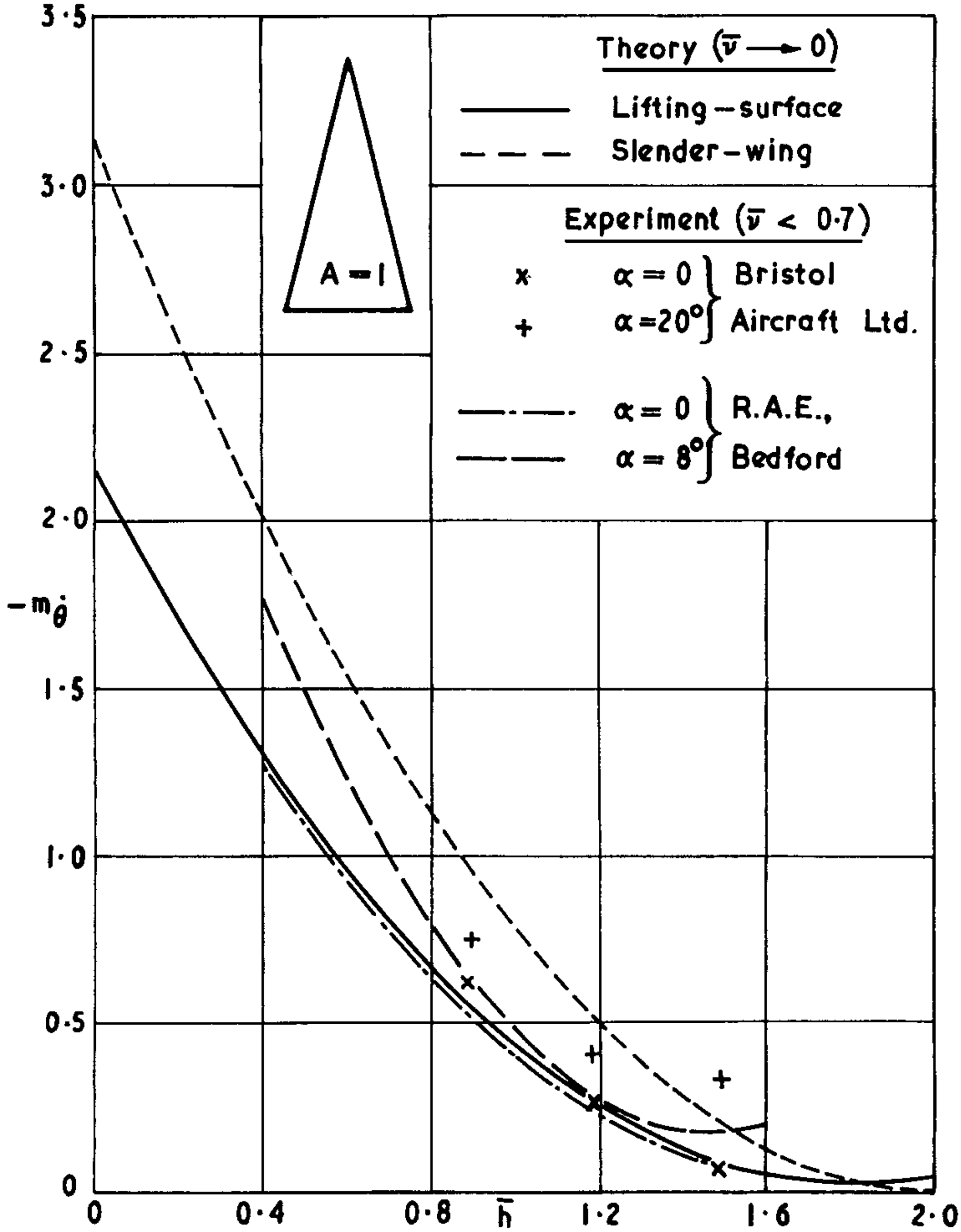
Direct pitching derivatives for a tapered swept wing
(A = 1·32, M = 0)

FIG. 13



Direct pitching derivatives for a tapered swept wing (A = 2·64)

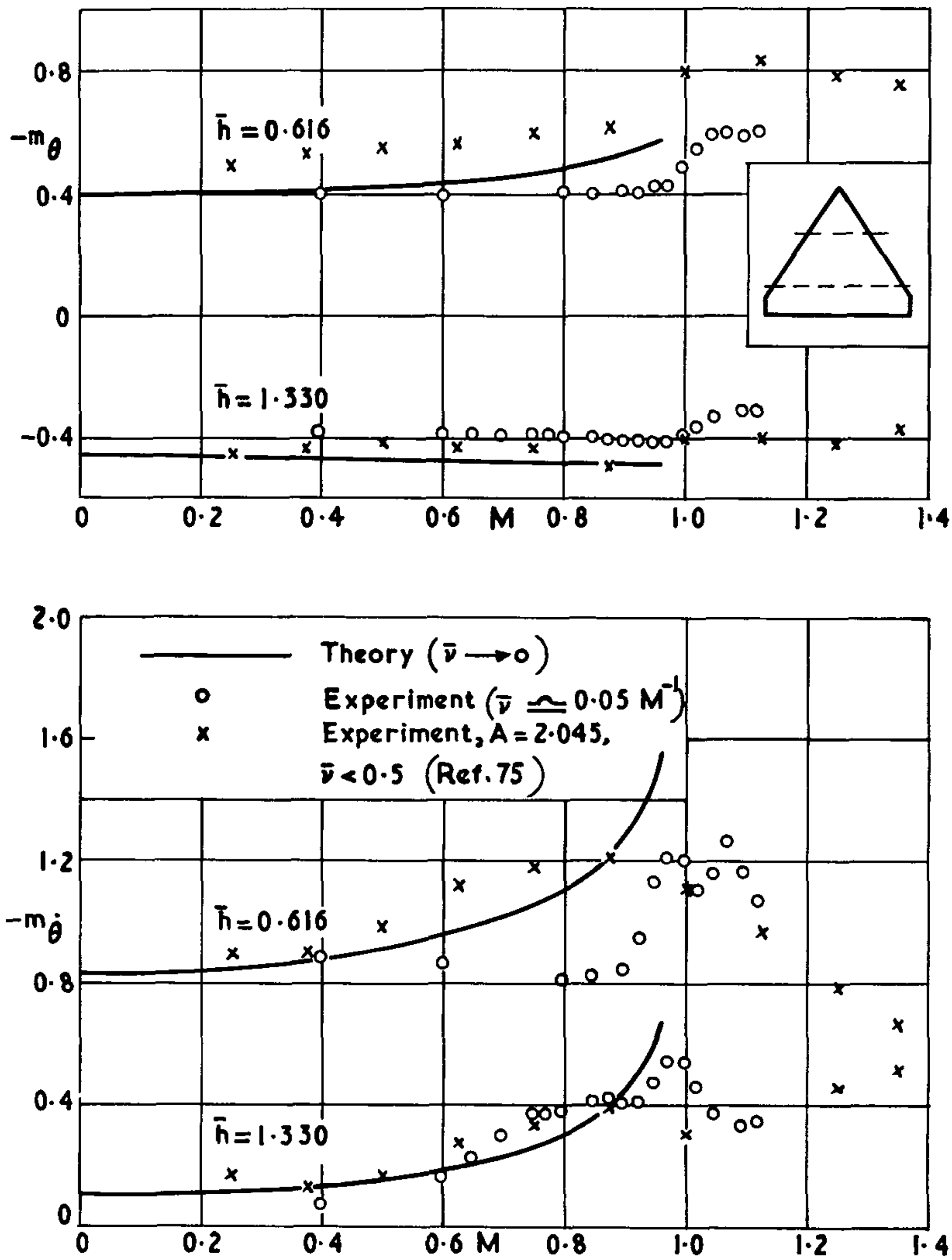
FIG. 14



Effect on mean incidence of pitching damping for a triangular wing

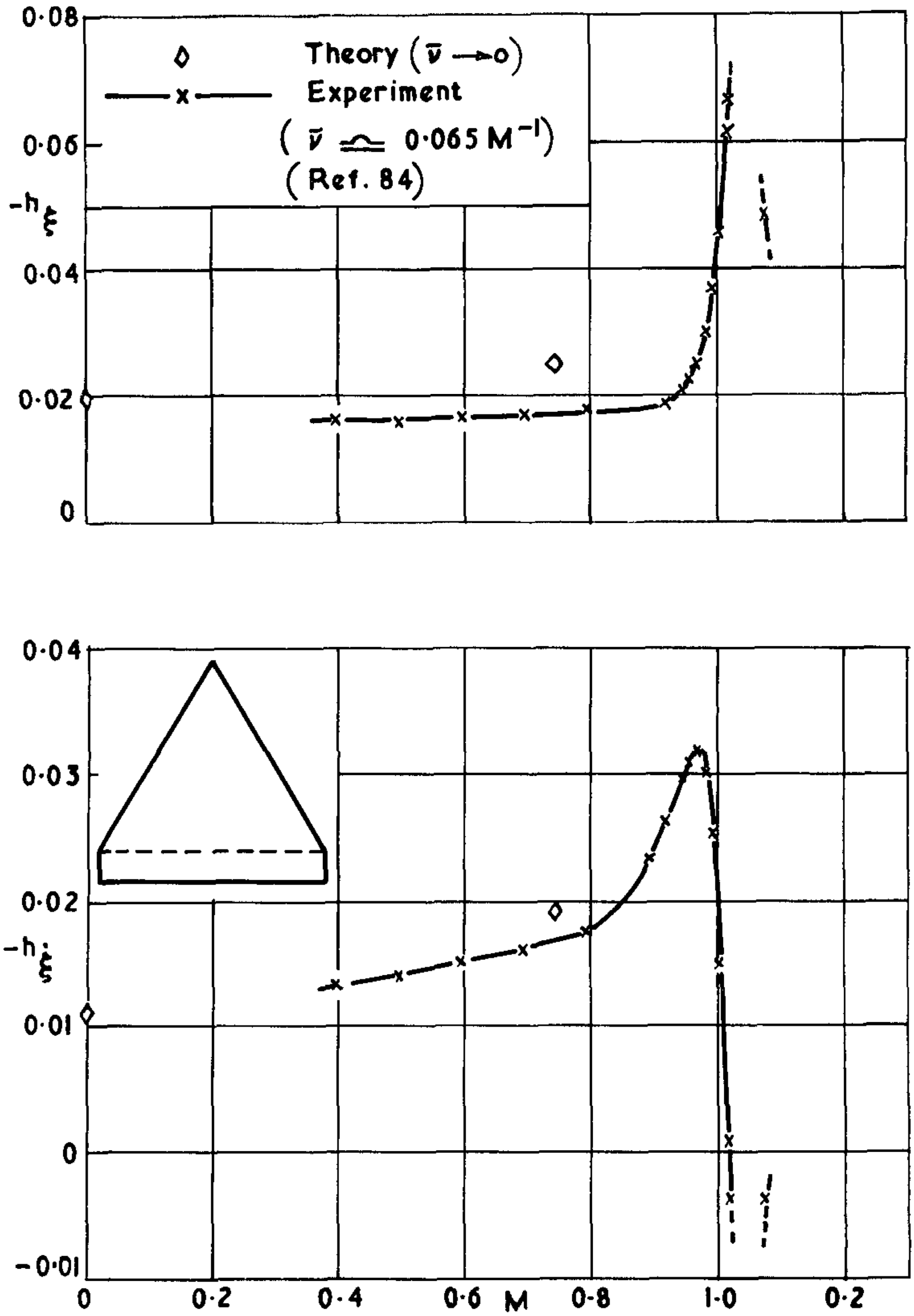
(A = 1, M = 0)

FIG. 15.



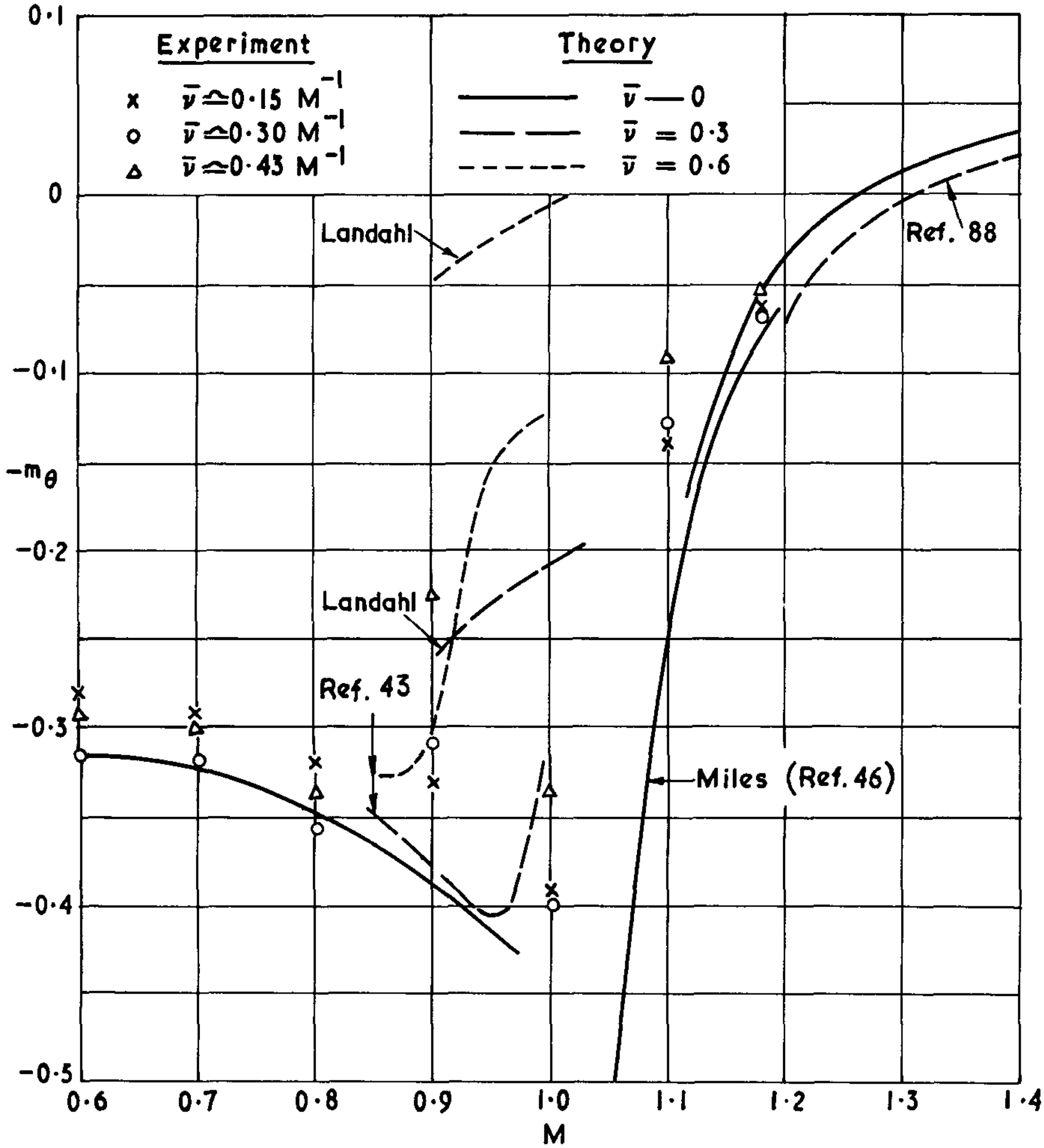
Direct pitching derivatives for a cropped delta wing (A = 2.0)

FIG.16.



Direct hinge moment derivatives for a delta wing (A = 1·8) with a full span control

FIG. 17(a)

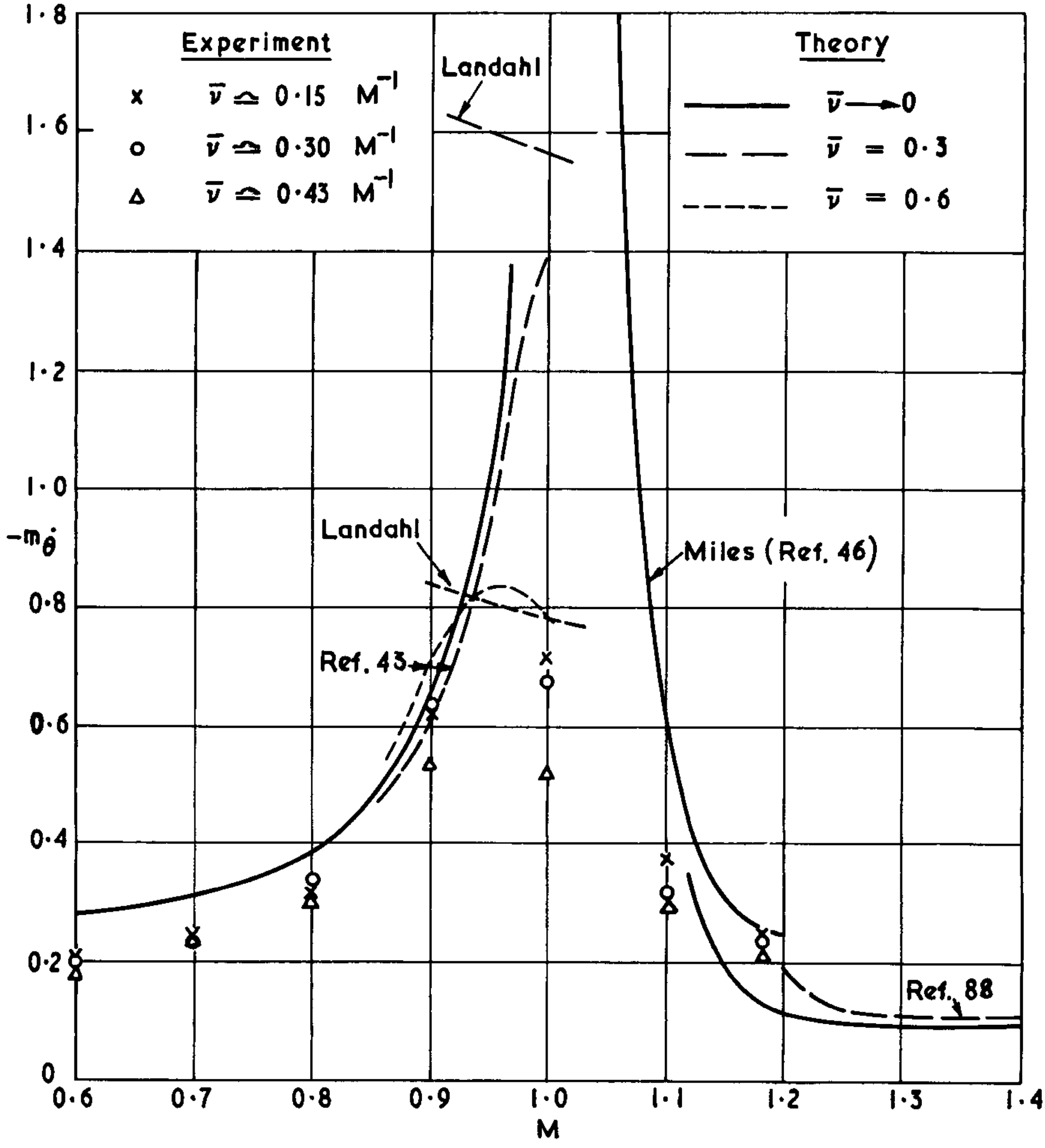


Direct pitching derivatives for a rectangular wing at transonic speeds

$(A = 2,\ \bar{h} = 0{\cdot}42)$

(a) Stiffness derivative, $-m_\theta$

FIG. 17 (b)

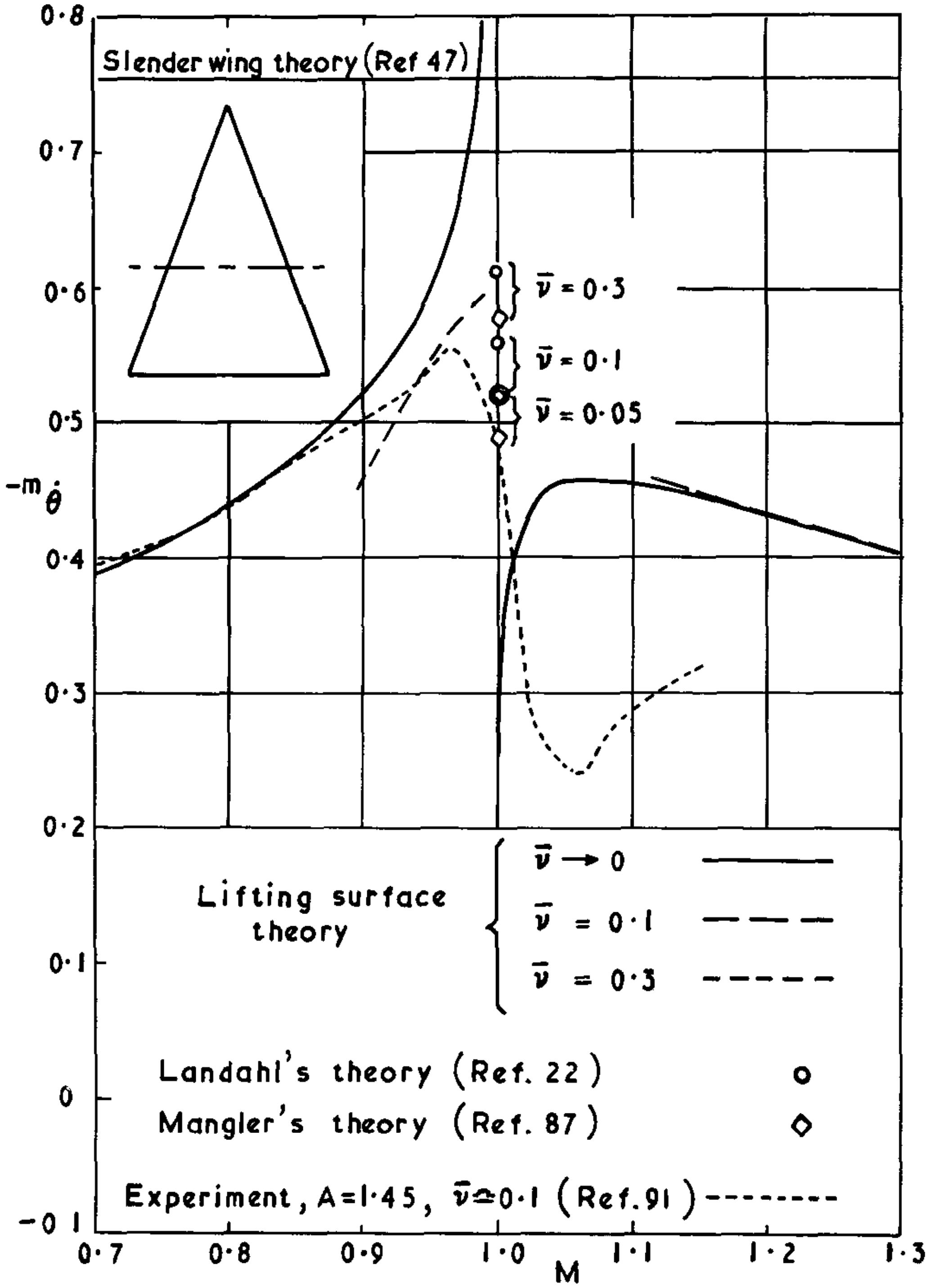


Direct pitching derivatives for a rectangular wing at transonic speeds

($A = 2$, $\bar{h} = 0{\cdot}42$)

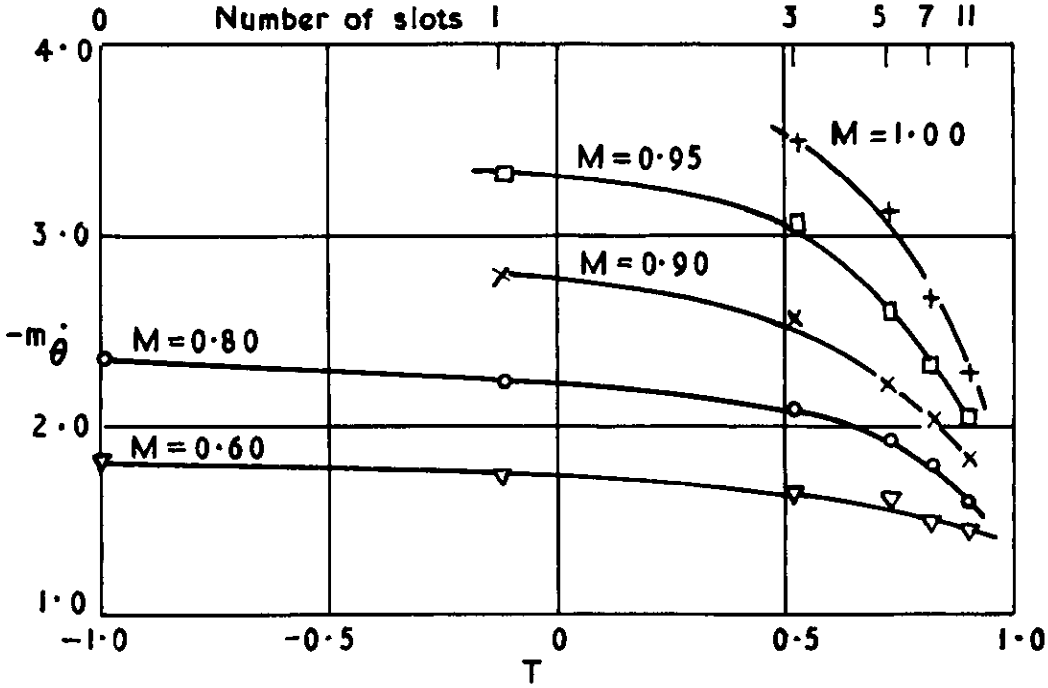
(b) Damping derivative, $-m_{\dot{\theta}}$

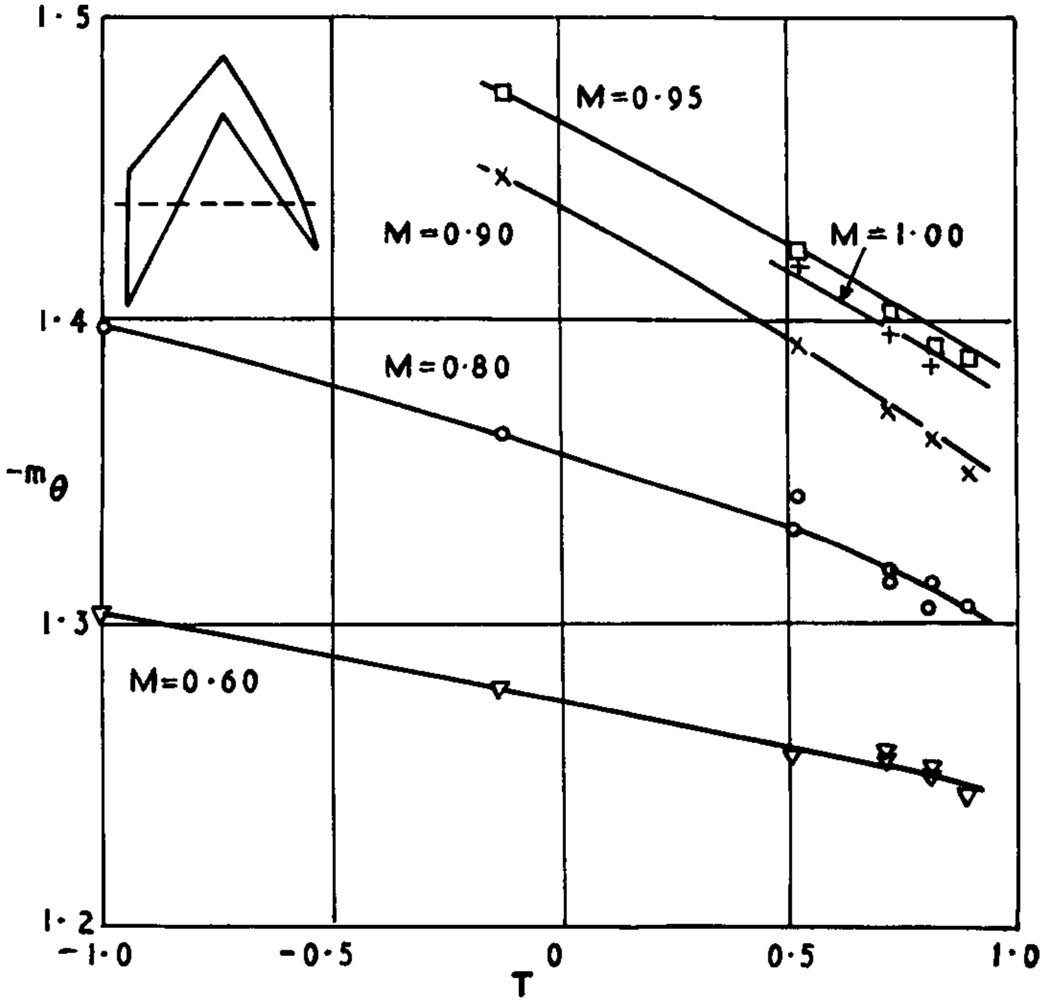
23,640
FIG. 18



Pitching damping for a triangular wing at transonic speeds $(A=1·5, \bar{h}=1·2)$

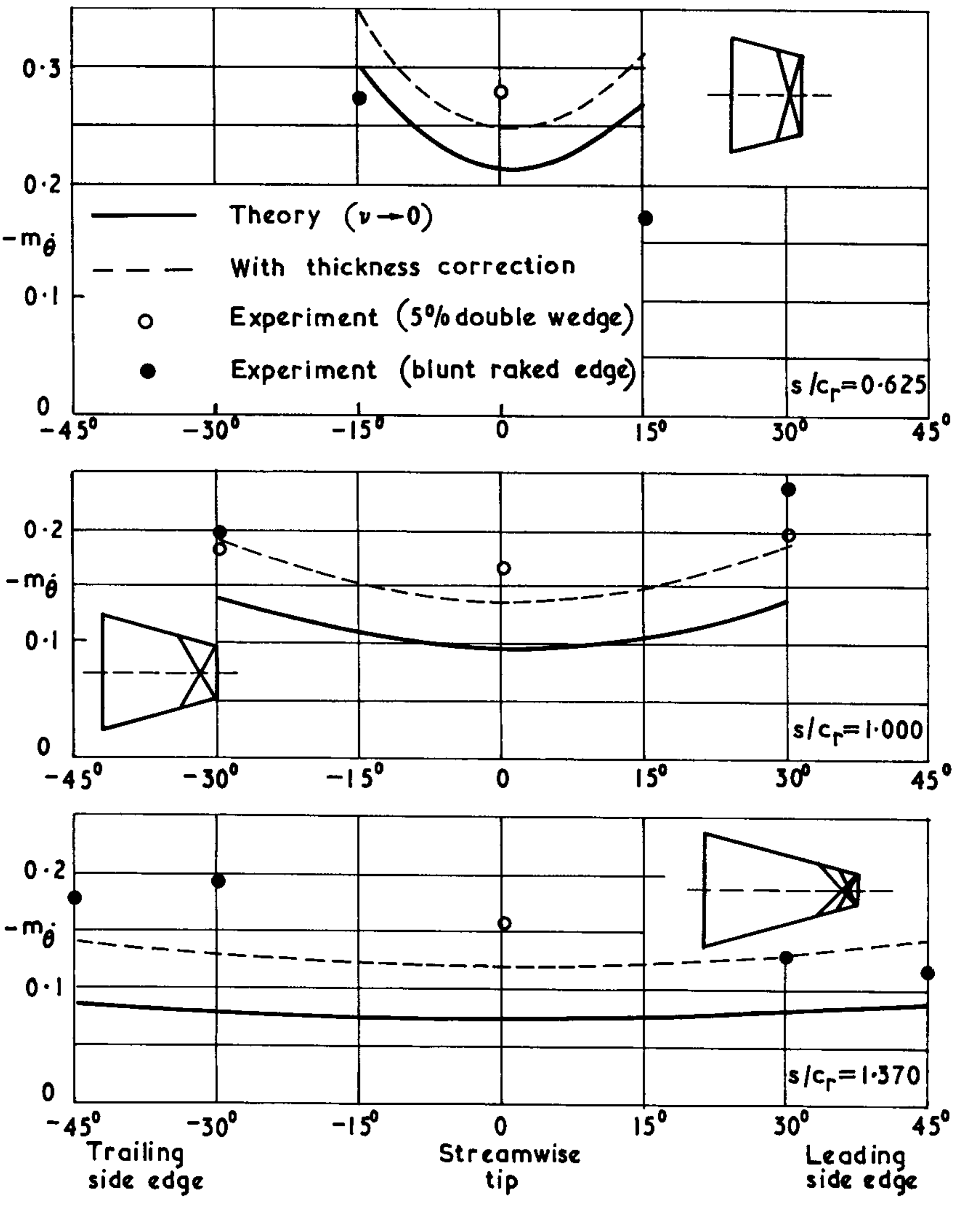
FIG. 19





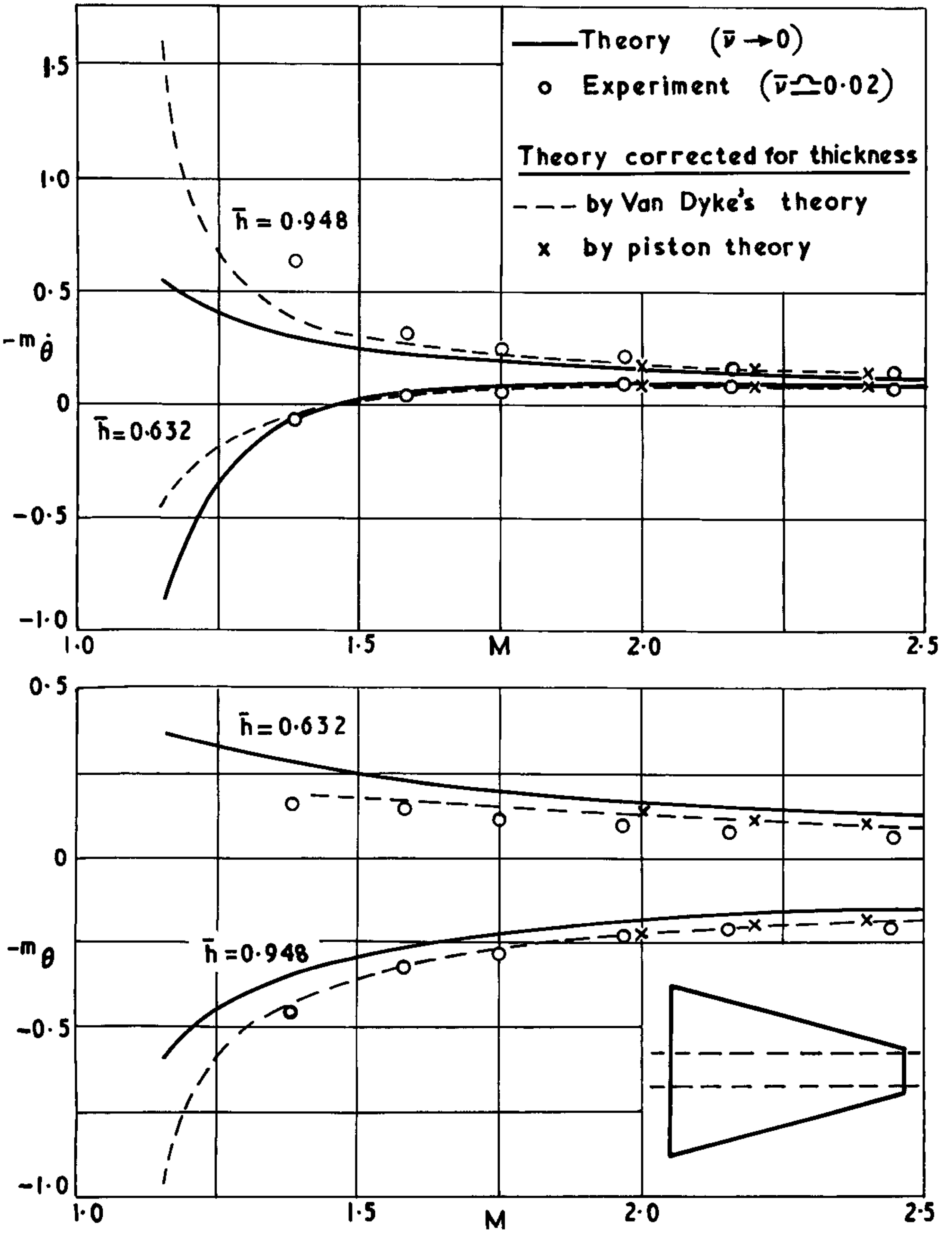
Direct pitching derivatives for an M-wing measured in a rectangular tunnel with longitudinally slotted roof and floor ($A = 5{\cdot}02$, $\bar{h} = 0{\cdot}4644$)

FIG. 20.



Effect of tip rake on pitching damping for unswept tapered wings pitching about the mid chord axis (M = 1·41)

FIG. 21



Direct pitching derivatives for a 5% thick unswept wing (A = 4·33) in supersonic flow.

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