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A Consideration of the Similarity Requirements for Aerothermoelastic Tests on Reduced Scale Models

by

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A CONSIDERATION OF THE SIMILARITY REQUIREMENTS FOR AEROTHERMOELASTIC TESTS ON REDUCED SCALE MODELS

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SUMMARY

The similarity requirements for model tests are established and different possibilities with regard to relaxations of these requirements are examined with the aim of devising an acceptable experimental technique for representative tests on models of reduced scale.

It appears that the representation of heating effects must of necessity be rather crude, but may nevertheless be adequate for aerothermoelastic research. Test techniques that provide a reasonably close representation of flight conditions require tunnels with heated flow and with subsidiary radiant heat in the working section. Failing this, models can be constructed with an effective stiffness representative of heated conditions for tests in conventional facilities.

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1 INTRODUCTION

The problem of the simulation of aerodynamic and structural parameters between an aircraft and a scale model for an adequate representation of aeroelastic effects is one with which the aeroelastic engineer is well familiar. In the absence of kinetic heating effects a degree of representation that is adequate for engineering purposes can generally be achieved without too much difficulty, and this, coupled with the inherent advantages of model tests in providing data in flow régimes where there are analytical difficulties, has led to an extensive use of scale representative models for aeroelastic work, particularly for flutter investigations. It is probably true to say that wind tunnel aeroelastic model tests are now accepted as an essential part of flutter clearance procedures for all aircraft with a supersonic capability. However, the effects of kinetic heating are not generally simulated using current techniques, and although this has imposed no serious limitation as yet it is apparent that some account must be taken of heating effects for future generations of high speed aircraft[1].

Kinetic heating will have its influence on aeroelastic properties primarily by modifying the structural stiffnesses and the effect on stiffness will be in two forms; namely, a stiffness loss due to a degradation of material properties with temperature, and a loss resulting from an unfavourable stress distribution due to thermal expansion. The effect of thermal stress may far exceed the effect of material degradation, it may be at its worst during the transient heating stage of the structure, and it generally results in a structural stiffness that is non-linear with displacement[2,3].

The aeroelastic engineer inevitably strives for the ideal in which these thermal effects, together with the aerodynamic and structural properties of the full scale aircraft are represented to model scale, since in this circumstance model test results and supporting calculations provide the best grounds for confidence in full scale behaviour. A number of investigations have been made of the similarity parameters that need to be satisfied for this ideal to be realised[4,5,6], but the outcome is far from encouraging. It appears that complete similarity can only be realised for a scale ratio of 1 : 1, and attempts to circumvent this difficulty by relaxations in some of the similarity requirements have not, as yet, led to a generally acceptable approach for aerothermoelastic model tests.

Accordingly, in what follows the similarity requirements are reconsidered with a view to establishing generally acceptable (though possibly fairly crude) techniques for aerothermoelastic model tests. This consideration shows that techniques are practicable in which either models in the same materials as the aircraft are tested in a gas other than air to the same scale of temperature as the aircraft, or models in different materials from the aircraft are tested in air to a different scale of temperature. In both cases an approximation to overall similarity can best be obtained by providing controlled radiant heat in the tunnel working section to supplement the heating provided by the gas flow.

In the absence of heated flow tunnels the best that can be done is to design models with reduced stiffness representative of heating effect for tests in 'cold' flow. However, a simple "effective" stiffness concept may not be adequate for all types of structure and work is required to establish the limitations of this approach.

2 DETERMINATION OF THE SIMILARITY RELATIONSHIPS

The similarity parameters that need to be satisfied for aerothermoelastic work can conveniently be established from a consideration of the general equations for stress, displacement and temperature distribution of a body immersed in a hot, flowing gas.

The boundary layer flow of a viscous compressible perfect gas is described by the equations of motion, energy, continuity and state[7].

A typical equation from the three equations of motion is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{2}{3}\frac{\partial \mu}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$+ 2\frac{\partial \mu}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial \mu}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) - \rho g_x\left(\frac{T-T_o}{T_o}\right). \tag{1}$$

The equation of energy is

$$\rho\left(\frac{\partial(c_pT)}{\partial t} + u\frac{\partial(c_pT)}{\partial x} + v\frac{\partial(c_pT)}{\partial y} + w\frac{\partial(c_pT)}{\partial z}\right)$$

$$= \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

$$+ \frac{\partial k}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial k}{\partial y}\frac{\partial T}{\partial y} + \frac{\partial k}{\partial z}\frac{\partial T}{\partial z} + \mu\left\{-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2\right.$$

$$\left.+ 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\} \tag{2}$$

The equation of continuity is

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{3}$$

and the equation of state may be written

$$p = \rho c_p T\left(\frac{\gamma-1}{\gamma}\right). \tag{4}$$

The temperature distribution in a body is determined by the heat input at the surface and the flow of heat internally by conduction[8].

The heat transfer at the surface is determined by the equation (neglecting inwards radiation)

$$k_B \frac{\partial T}{\partial n} = k \frac{\partial T}{\partial n} - \varepsilon \bar{\sigma} T^4 . \tag{5}$$

Heat flow by conduction is determined by the equation

$$\frac{\partial}{\partial x} k_B \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k_B \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k_B \frac{\partial T}{\partial z} = c\rho_B \frac{\partial T}{\partial t} . \tag{6}$$

The stress and deflections of an elastic body are determined by the stress-strain relationships, the equilibrium equations and the surface forces[9].

A typical equation from the six stress-strain equations is

$$\frac{\partial b}{\partial x} = \frac{1}{E}\left[\sigma_x - \nu\,(\sigma_y + \sigma_z)\right] + \alpha(T-T_o) . \tag{7}$$

A typical equation from the three equilibrium equations is

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho_B g_x - \rho_B \frac{\partial^2 b}{\partial t^2} = 0 . \tag{8}$$

At the surface the normal stress is equal to the applied pressure, i.e.

$$\sigma_n = p_n . \tag{9}$$

The above equations are adequate to describe the aeroelastic behaviour of a body in a hot flowing gas.

Now consider a second body in a similar gas flow, such that at all points in the field there is a constant ratio of corresponding properties, i.e. $E_2 = E_1 \lambda_E$, $\sigma_2 = \sigma_1 \lambda_\sigma$, $T_2 = T_1 \lambda_T$ etc. In particular $\lambda_x = \lambda_y = \lambda_z = \lambda_b = \lambda_L$ i.e. there is complete external geometrical similarity between the two bodies, including aerothermoelastic deflections. For this second body equations (1)-(9) above may be re-written

$$\frac{\lambda_\rho \lambda_V^2}{\lambda_L} \rho \left\{ \frac{\lambda_L}{\lambda_V \lambda_{t_1}} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right\}$$

$$= - \frac{\lambda_p}{\lambda_L} \frac{\partial p}{\partial x} + \frac{\lambda_\mu \lambda_V}{\lambda_L^2} \left\{ \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{\mu}{3} \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right.$$

$$- \frac{2}{3} \frac{\partial u}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + 2 \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\left. + \frac{\partial \mu}{\partial z} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right\} - \lambda_\rho \lambda_g \rho g_x \frac{(T - T_o)}{T_o} \qquad (10)$$

$$\frac{\lambda_\rho \lambda_{c_P} \lambda_T \lambda_V}{\lambda_L} \rho \left\{ \frac{\lambda_L}{\lambda_V \lambda_{t_1}} \frac{\partial (c_P T)}{\partial t} + u \frac{\partial (c_P T)}{\partial x} + v \frac{\partial (c_P T)}{\partial y} + w \frac{\partial (c_P T)}{\partial z} \right\}$$

$$= \frac{\lambda_p \lambda_V}{\lambda_L} \left\{ \frac{\lambda_L}{\lambda_V \lambda_{t_1}} \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right\}$$

$$+ \frac{\lambda_k \lambda_T}{\lambda_L^2} \left\{ k \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\partial k}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial k}{\partial y} \frac{\partial T}{\partial y} + \frac{\partial k}{\partial z} \frac{\partial T}{\partial z} \right\}$$

$$+ \frac{\lambda_\mu \lambda_V^2}{\lambda_L^2} \mu \left\{ - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + 2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 \right.$$

$$\left. + 2 \left(\frac{\partial w}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right\} \qquad (11)$$

$$\frac{\lambda_\rho \lambda_V}{\lambda_L} \left\{ \frac{\lambda_L}{\lambda_V \lambda_{t_1}} \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right\} = 0 \qquad (12)$$

$$\frac{\lambda_p}{\lambda_\rho \lambda_{c_p} \lambda_T} p = \rho c_p T \frac{(\lambda_\gamma \gamma - 1)}{\lambda_\gamma \gamma} \tag{13}$$

$$\frac{\lambda_{k_B} \lambda_T}{\lambda_L} k_B \frac{\partial T}{\partial n} = \frac{\lambda_k \lambda_T}{\lambda_L} k \frac{\partial T}{\partial n} - \lambda_\varepsilon \lambda_T^{\,4} \varepsilon \bar{\sigma} T^4 \tag{14}$$

$$\frac{\lambda_k \lambda_T}{\lambda_L^2} \left(\frac{\partial}{\partial x} k_B \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k_B \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k_B \frac{\partial T}{\partial z} \right) = \frac{\lambda_c \lambda_{\rho_B} \lambda_T}{\lambda_{t_2}} c \rho_B \frac{\partial T}{\partial t} \tag{15}$$

$$\frac{\lambda_b}{\lambda_L} \frac{\partial b}{\partial x} = \frac{\lambda_\sigma}{\lambda_E} \frac{1}{E} \left[\sigma_x - \lambda_\nu \nu (\sigma_y + \sigma_z) \right] + \lambda_\alpha \lambda_T \alpha (T - T_o) \tag{16}$$

$$\frac{\lambda_\sigma}{\lambda_L} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) + \lambda_{\rho_B} \lambda_g \rho_B g_x - \lambda_{\rho_B} \frac{\lambda_b}{\lambda_{t_1}^2} \rho_B \frac{\partial^2 b}{\partial t^2} = 0 \tag{17}$$

$$\lambda_\sigma \sigma_n = \lambda_p p_n \tag{18}$$

2.1 Similarity laws

For complete similitude in aerothermoelastic behaviour of the two bodies, corresponding equations in equations (1)-(9) and (10)-(18) must be identical. Recognising that $\frac{\mu V}{\rho c_p T L} = \left(\frac{\mu}{V \rho L}\right) \left(\frac{p}{\rho c_p T}\right) \left(\frac{V^2 \rho}{p}\right)$ the following equations must be satisfied.

$$\frac{\lambda_L}{\lambda_V \lambda_{t_1}} = 1 \tag{19}$$

$$\frac{\lambda_p}{\lambda_\rho \lambda_V^2} = 1 \tag{20}$$

$$\frac{\lambda_\mu}{\lambda_V \lambda_\rho \lambda_L} = 1 \tag{21}$$

$$\frac{\lambda_g \lambda_L}{\lambda_V^2} = 1 \tag{22}$$

$$\frac{\lambda_p}{\lambda_\rho \lambda_{c_P} \lambda_T} = 1 \tag{23}$$

$$\frac{\lambda_k}{\lambda_V \lambda_\rho \lambda_{c_P} \lambda_L} = 1 \tag{24}$$

$$\lambda_\gamma = 1 \tag{25}$$

$$\frac{\lambda_k}{\lambda_{k_B}} = 1 \tag{26}$$

$$\frac{\lambda_\varepsilon \lambda_T^3 \lambda_L}{\lambda_{k_B}} = 1 \tag{27}$$

$$\frac{\lambda_{k_B} \lambda_{t_2}}{\lambda_c \lambda_{\rho_B} \lambda_L^2} = 1 \tag{28}$$

$$\lambda_\nu = 1 \tag{29}$$

$$\frac{\lambda_\sigma \lambda_b}{\lambda_E \lambda_L} = 1 \tag{30}$$

$$\frac{\lambda_\alpha \lambda_T \lambda_L}{\lambda_b} = 1 \tag{31}$$

$$\frac{\lambda_{\rho_B} \lambda_g \lambda_L}{\lambda_\sigma} = 1 \tag{32}$$

$$\frac{\lambda_{\rho_B} \lambda_b \lambda_L}{\lambda_\sigma \lambda_{t_1}^2} = 1 \tag{33}$$

$$\frac{\lambda_p}{\lambda_\sigma} = 1 \qquad (34)$$

$$\frac{\lambda_b}{\lambda_L} = 1 \,. \qquad (35)$$

The times scales λ_{t_1} and λ_{t_2} are distinguished, t_1 relating to the time for oscillatory or accelerated motions and t_2 to heat flow times. It may be legitimate to regard these two times as independent on the assumption that heat flow is independent of factors such as the characteristic spectrum of the boundary layer or the vibrational environment of the body itself. This assumption is apparently of limited validity since it appears that boundary layer heat transfer can be influenced to some extent by certain frequency dependent effects such as the external noise environment.

The practice with regard to aeroelastic model tests in the absence of heating effects is to ignore Reynolds number simulation on the grounds that it has been found to be relatively unimportant, at least as regards the behaviour of main surfaces. This assumption cannot be supported when heating effects of the flow are included. The Reynolds number equations are equations (21) and (23) which may be written

$$\frac{\lambda_\mu}{\lambda_V \lambda_\rho \lambda_L} = \frac{1}{\lambda_{Re}} = 1$$

$$\frac{\lambda_k}{\lambda_V \lambda_\rho \lambda_{c_p} \lambda_L} = \frac{1}{\lambda_{Pr} \lambda_{Re}} = 1$$

3 COMPATIBILITY OF THE SIMILARITY EQUATIONS

In the main, little control can be exercised over the properties of structural materials, in the sense that once a particular material for model construction has been decided upon the properties E, ρ_B, c and ν take up a fixed relationship. The same is true for the properties α and k_B, for although significant variations in these properties can sometimes be achieved by slight changes in the constituent alloys of the material this is not a controllable variation. The relationship between the properties ρ, k, μ, c_p and γ for gases is similarly fixed, and a connecting relationship between material and gas properties is established by the need to satisfy equation (26). It follows that if the satisfaction of the similarity relationships depends upon a constant relationship between several of the properties of the structural material or the gas then compatibility is only likely to be achieved using the same materials or gas for both bodies, since to find a different material or gas with corresponding ratios of properties would be largely fortuitous.

A further feature to be borne in mind is that the same gravity field necessarily applies to both bodies so that λ_g is normally unity*.

For wind tunnel work a measure of independent control can be exercised over the quantities T, p and V, and these are termed the "disposable quantities".

Re-writing the similarity equations we obtain

$$\lambda_T = \lambda_\alpha^{-1} \tag{36}$$

$$\lambda_V = \lambda_{c_P}^{\frac{1}{2}} \lambda_\alpha^{-\frac{1}{2}} \tag{37}$$

$$\lambda_p = \lambda_E \tag{38}$$

$$\lambda_\sigma = \lambda_E \tag{39}$$

$$\lambda_b = \lambda_L \tag{40}$$

$$\lambda_{t_1} = \lambda_{c_P}^{-\frac{1}{2}} \lambda_\alpha^{\frac{1}{2}} \lambda_L \tag{41}$$

$$\lambda_{t_2} = \lambda_c \lambda_{\rho_B} \lambda_{k_B}^{-1} \lambda_L^2 \tag{42}$$

$$\lambda_g = \lambda_{c_P} \lambda_\alpha^{-1} \lambda_L^{-1} \tag{43}$$

$$\lambda_\varepsilon = \lambda_{k_B} \lambda_\alpha^3 \lambda_L^{-1} \tag{44}$$

$$\lambda_\rho = \lambda_E \lambda_{c_P}^{-1} \lambda_\alpha \tag{45}$$

$$\lambda_\gamma = 1 \tag{46}$$

$$\lambda_\nu = 1 \tag{47}$$

$$\lambda_{\rho_B} = \lambda_E \lambda_{c_P}^{-1} \lambda_\alpha \tag{48}$$

$$\lambda_\mu = \lambda_E \lambda_{c_P}^{-\frac{1}{2}} \lambda_\alpha^{\frac{1}{2}} \lambda_L \tag{49}$$

$$\lambda_k = \lambda_E \lambda_{c_P}^{\frac{1}{2}} \lambda_\alpha^{\frac{1}{2}} \lambda_L \tag{50a}$$

$$\lambda_k = \lambda_{k_B} . \tag{50b}$$

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*Gravity ratios other than unity might be feasible in some circumstances; for example a manoeuvre case for an aircraft provides a gravity field greater than unity, as do model tests on a whirling arm.

If the requirements defined by equations (43)-(50) can be satisfied for the two bodies then the disposable quantities λ_T, λ_V and λ_p can be adjusted to satisfy equations (36)-(38) and the stress, deflection and time scales are then defined by equations (39)-(42). However, since equations (45)-(50) all depend on achieving a constant relationship between different ratios of material and gas properties they are unlikely to be satisfied except by using the same materials and gases for both bodies.

It then follows from equation (36) that the temperature scale must be the same for both bodies. Equations (36)-(50) now reduce to

$$\lambda_T = 1 \tag{51}$$

$$\lambda_V = 1 \tag{52}$$

$$\lambda_p = 1 \tag{53}$$

$$\lambda_\sigma = 1 \tag{54}$$

$$\lambda_b = \lambda_L \tag{55}$$

$$\lambda_{t_1} = \lambda_L \tag{56}$$

$$\lambda_{t_2} = \lambda_L^2 \tag{57}$$

$$\lambda_g = \lambda_L^{-1} \tag{58}$$

$$\lambda_\varepsilon = \lambda_L^{-1} \tag{59}$$

$$\lambda_\rho = 1 \tag{60}$$

$$\lambda_\gamma = 1 \tag{61}$$

$$\lambda_\nu = 1 \tag{62}$$

$$\lambda_\mu = \lambda_L \tag{63}$$

$$\lambda_k = \lambda_L \tag{64a}$$

$$\lambda_k = 1 \ . \tag{64b}$$

The expressions for λ_μ and λ_k are incompatible with the assumptions already made except when λ_L is unity, i.e. complete similarity obtains only when the two bodies are the same in all respects, including size. This conclusion has been arrived at by many earlier investigators (e.g. Refs.4, 5, 6 and 17).

4 APPROXIMATIONS TO SIMILARITY

Although tests on full scale components have their merits the nature of existing facilities limits such tests to relatively small components.

For aerothermoelastic model work to be of anything like the value of the purely aeroelastic model work, and to make the best use of available experience and facilities, the ability to make representative tests on small scale models of full scale components is essential. It is apparent that this cannot be achieved if a complete representation is attempted, and accordingly some possible approaches are examined that involve a relaxation of the similarity conditions. It is worth bearing in mind that relaxations of similarity requirements, both with regard to structural and aerodynamic parameters, have been necessary for all the purely aeroelastic models tested in the past but this has not prevented their making an indispensible contribution to aircraft flutter clearance programmes.

4.1 Relaxation of the Reynolds number requirement

The Reynolds number equation is the equation that is most difficult to satisfy for a reduced scale model. For tests in unheated flow the assumption is generally made that correct representation of Reynolds number is unnecessary provided the Reynolds number for the model is greater than about 10^6 (based on the mean chord of the surface) and this has generally proved acceptable, at least for main surface flutter investigations. However, for heated flow the Reynolds number is of first order importance in relation to heat transfer, which in turn influences structural stiffness and hence aeroelastic properties. The Reynolds number must therefore be taken into account for aerothermoelastic work, insofar as it is associated with heat transfer properties.

Now, as an approximation equation (5) can be replaced by the equation

$$k_B \frac{\partial T}{\partial n} = h\,(T_{aw} - T_w) - \varepsilon\,\bar{\sigma}\,T^4 . \tag{65}$$

Strictly speaking this formulation applies for an isothermal surface, but in practice it appears to provide a fair approximation for non-isothermal surfaces.

At the same time the heat transfer coefficient h in the above equation is given approximately by[10]

$$h = A\,\frac{k}{x}\left(\frac{x\,\rho\,V}{\mu}\right)^{r}\left(\frac{\mu\,c_P}{k}\right)^{1/3}\left(\frac{T_a}{T_w}\right)^{s} \tag{66}$$

where A, r and s are constants having different values in the laminar flow and turbulent regions, and x is measured from the leading edge in the laminar case, and from the transition point in the turbulent case*. In particular $r = 0.5$ for laminar flow and $r = 0.8$ for turbulent flow.

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*This formulation strictly applies only for incompressible flow, but there seems some justification for extending it to the compressible flow régime[11].

With these limitations in mind, on substituting for h in equation (65) the equation for the second body (replacing equation (14)) is:-

$$\frac{\lambda_{k_B}\lambda_T}{\lambda_L} k_B \frac{\partial T}{\partial n}$$

$$= \frac{\lambda_k\lambda_T}{\lambda_L}\left(\frac{\lambda_L\lambda_\rho\lambda_V}{\lambda_\mu}\right)^r\left(\frac{\lambda_\mu\lambda_{c_P}}{\lambda_k}\right)^{1/3} A\,\frac{k}{x}\left(\frac{x\rho V}{\mu}\right)^r\left(\frac{\mu c_P}{k}\right)^{1/3}\left(\frac{T_a}{T_w}\right)^s (T_{aw} - T)$$

$$- \lambda_\varepsilon\lambda_T{}^4\,\varepsilon\,\bar{\sigma}\,T^4$$

$$\ldots\ldots \quad (67)$$

Hence, the similarity equation (26) is replaced by

$$\frac{\lambda_k}{\lambda_{k_B}}\left(\frac{\lambda_L\lambda_\rho\lambda_V}{\lambda_\mu}\right)^r\left(\frac{\lambda_\mu\lambda_{c_P}}{\lambda_k}\right)^{1/3} = 1 \; . \qquad (68)$$

If this equation is satisfied then heating rates for the two bodies will be to scale. Assuming other effects of Reynolds number can be neglected it is then no longer necessary to satisfy the Reynolds number equations (21) and (24) independently. Accordingly, referring to equations (36)-(50), equations (49) and (50) are replaced by the single equation

$$\lambda_{k_B} = \lambda_k\left(\frac{\lambda_L\lambda_E\lambda_{c_P}^{-\frac{1}{2}}\lambda_\alpha^{\frac{1}{2}}}{\lambda_\mu}\right)^r\left(\frac{\lambda_\mu\lambda_{c_P}}{\lambda_k}\right)^{1/3}$$

and hence

$$\lambda_L = \lambda_{k_B}^{1/r}\,\lambda_E^{-1}\,\lambda_{c_P}^{-\frac{1}{2}}\,\lambda_\alpha^{-\frac{1}{2}}\,\lambda_k^{\left(\frac{r-1}{r}\right)}\,\lambda_{Pr}^{\left(\frac{3r-1}{3r}\right)} \; . \qquad (69)$$

It is apparent that if the same gas and materials are used for both bodies we again arrive at the conclusion that compatibility of the similarity equations obtains only when $\lambda_L = 1$. However, the number of property dependent relationships is reduced by the above procedure and the possibility of using different gases and materials for the two bodies merits further consideration. In the main, properties of both structural materials and gases are affected by temperature, and in some cases significant anomalies in thermal properties as functions of temperature are obtained. A completely general treatment is impracticable in these circumstances, and some simplifying assumptions defining variation of material properties with temperature must be made.

For the present purposes it is assumed that within a limited range of temperature all temperature dependent material properties can be assumed to vary according to the law:-

$$\phi = \phi_o \, T^{s_\phi} \tag{70}$$

where ϕ = value of temperature dependent property at temperature T

ϕ_o = property value at reference temperature

s_ϕ = temperature exponent related to property ϕ.

Furthermore, we assume that the possible gases that could be used are such that

$$\begin{aligned} \mu &= \mu_o \, T^{0.7} \\ k &= k_o \, T^{0.9} \\ c_P &= c_{P_o} \, T^{0.2} \\ \gamma &= \gamma_o \, . \end{aligned} \tag{71}$$

In this circumstance Prandtl number remains constant independent of temperature, which is in reasonable accordance with experimental data for most gases.

From equations (36) and (70) we obtain

$$\lambda_T = \lambda_{\alpha_o}^{-\left(\frac{1}{1+s_\alpha}\right)} \tag{72}$$

and from equation (69) we then have

$$\lambda_L = \lambda_{c_{P_o}}^{-\frac{1}{2}} \, \lambda_{k_o}^{\left(\frac{r-1}{r}\right)} \, \lambda_{Pr}^{\left(\frac{3r-1}{3r}\right)} \, \lambda_{k_{B_o}}^{-\frac{1}{r}} \, \lambda_{E_o}^{-1} \, \lambda_{\alpha_o}^{-\{s_k - s_{k_B} + r(s_k - s_E - s_{c_P} + 0.5)\}/r(1+s\alpha)} \tag{73}$$

The main aim is to further the use of small scale models, i.e. we require $\lambda_L > 1$, where λ is the ratio full scale property: model scale property.

4.1.1 Wind tunnel tests in a gas other than air

In considering the possible use of a gas other than air for wind tunnel tests the basic gas properties required are

$$\lambda_\rho = \lambda_E \lambda_\alpha \lambda_{c_P}^{-1} \tag{45}$$

$$\lambda_\gamma = 1 \tag{46}$$

$$\lambda_{c_{P_o}}^{-\frac{1}{2}} \lambda_{k_o}^{\left(\frac{r-1}{r}\right)} \lambda_{Pr}^{\left(\frac{3r-1}{3r}\right)} > 1 . \tag{74}$$

The inequality (74) follows from equation (73) if we assume that the gas is to further the aim $\lambda_L > 1$.

The properties of a number of gases are given in Table 1 and the air : gas property ratios are given in Table 2. It can be seen that inequality (74) is satisfied by all the gases considered whose density is significantly less than that of air. For hydrogen in particular quite large values are obtained.

If it is now assumed that the model is constructed in the same materials as the aircraft, equations (45), (46) and (73) reduce to

$$\lambda_\rho \lambda_{c_P} = 1$$

$$\lambda_\gamma = 1$$

$$\lambda_{c_P}^{-\frac{1}{2}} \lambda_k^{\left(\frac{r-1}{r}\right)} \lambda_{Pr}^{\left(\frac{3r-1}{3r}\right)} = \lambda_L . \tag{75}$$

It is apparent from Table 2 that for tests in hydrogen not only are the first two of these equations closely satisfied but the values of λ_L are adequate for a wide range of model tests. For tests in pure hydrogen the aircraft : model scale is fixed at 24.9 : 1 for a laminar flow model and 5.94 : 1 for a turbulent flow model. However, the model designer will generally want to work with the maximum scale of model that is practicable for a given facility, as this generally eases fabrication and handling problems, and hence some control over the model scale parameter is desirable. A possible way in which this might be achieved (though less ideally than for tests in hydrogen) is to use mixtures of gases. For example, assume that the properties of gases vary linearly with mixture ratio* and suppose that we require the relationship $\lambda_\rho \lambda_{c_P} = 1$ to be satisfied for a binary mixture. Then for two gases mixed in the ratio 1 : f we require

$$\frac{(\rho_1 + f\,\rho_2)\ (c_{P_1} + f\ c_{P_2})}{(1 + f)^2} = \rho\ c_{P(air)}$$

i.e.

$$(\lambda_{\rho_1}^{-1} + f\lambda_{\rho_2}^{-1})\ (\lambda_{c_{P_1}}^{-1} + f\lambda_{c_{P_2}}^{-1})\ (1 + f)^{-2} = 1\ . \tag{76}$$

This equation has been solved for a mixture of methane and helium, for which $f = 5.19$, and the values of the various property ratios are given in Table 2. Unfortunately, although this mixture enables tests on quite small models to be made, and satisfies the requirement $\lambda_\rho \lambda_{c_P} = 1$, it does not satisfy the requirement $\lambda_\gamma = 1$. On the other hand it may not be necessary to satisfy $\lambda_\gamma = 1$ identically. Aeroelastic tests at transonic speeds in Freon 12 ($\lambda_\gamma = 1.21$) show quite good correlation with tests in air, so that the effect of γ is not too significant at these speeds. At high supersonic speeds ($M > 2$) there is evidence that the pressure distribution is often provided fairly accurately by piston theory[13] and in this theory γ occurs only in the form $(1 + \gamma)$ and in association with the thickness term. Hence, it may be permissible to neglect small errors in γ for thin surfaces (without excessive leading edge blunting) and for panels, and the majority of surfaces for aeroelastic investigations fall into this category.

Referring to the similarity equations (36)-(48) it is now apparent that the similarity equations that cannot be satisfied by laminar or turbulent flow models in the same materials as the aircraft and tested in hydrogen are:-

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*Investigations by Chapman[12] show that this assumption is unjustified for some gas mixtures. For example helium-argon mixtures have lower values of Pr than for either gas in the pure state.

$$\lambda_g = \lambda_{c_P} \lambda_L^{-1}$$

$$\lambda_\varepsilon = \lambda_L^{-1}$$

$$\lambda_{\rho_B} = \lambda_{c_P}^{-1} .$$

These imply that for tests in hydrogen a model of reduced density is required in an increased gravitational field and with emissivity increasing as the reciprocal of model size. Since the aircraft surfaces will be designed for maximum emissivity significantly higher emissivities for the model are unlikely to be achieved; since λ_g is unity, model deflections under gravitational load, and free convective heat transfer will not be to scale; and since λ_{ρ_B} is unity for model and aircraft in the same materials the frequency parameter for structural oscillations in the gas will not be to scale. However, the effects of frequency parameter on flutter characteristics are generally small so that failure to satisfy $\lambda_{\rho_B} = \lambda_{c_P}^{-1}$ may not be important - it can of course be ignored completely for static aerothermoelastic tests.

Effects of deflection under gravitational load are generally negligible but the effects of free convective heat transfer and of emissivity may be of some significance. They must be ignored for present purposes.

Within these limitations, models can be constructed in the same materials as the aircraft to provide representative aerothermoelastic effects in laminar flow or in turbulent flow regions, but one single model does not provide for representative investigations in both régimes of flow simultaneously. A model constructed to a laminar flow scale will have less than true scale rate of heating in turbulent flow regions*, and vice-versa. Some possible ways of overcoming this difficulty are considered in section 4.2.

4.1.2 Wind tunnel tests in air

The similarity requirements for wind tunnel tests in air cannot possibly be satisfied unless the model is constructed in different materials from the aircraft, the temperature scale then being defined by equation (72) namely

$$\lambda_T = \lambda_{\alpha_o}^{-1/(1+s_\alpha)} . \qquad (72)$$

For tests in air equation (73) becomes

$$\lambda_L = \lambda_{k_{B_o}}^{1/r} \lambda_{E_o}^{-1} \lambda_{\alpha_o}^{-\{s_k - s_{k_B} + r(s_k - s_E - s_{c_P} + 0.5)\}/r(1+s_\alpha)} . \qquad (77)$$

---

*Since Reynolds number for the model will be lower than full scale, natural transition from laminar to turbulent flow may not occur in the correct location. Leading edge roughness or some other device will be necessary to "trigger" the flow so that it changes from laminar to turbulent in the correct region for transition on the full scale aircraft.

As an approximation it is assumed that

$$-s_E = s_\alpha = 0.2$$

$$s_{k_B} = 0$$

therefore

$$\lambda_L = \lambda_{k_{B_o}}^{1/r} \lambda_{E_o}^{-1} \lambda_{\alpha_o}^{-(0.8+1.3r)/1.2r} \qquad (78)$$

Property values for a number of materials are given in Table 3 and ratios of property values for full scale aircraft in duralumin and in stainless steel are given in Tables 4 and 5. It can be seen that to obtain values of λ_L greater than unity the model material must have a conductivity and expansion coefficient less than that of the aircraft material. The conductivity of duralumin is very high and hence most of the materials considered in Table 4 provide values of $\lambda_L > 1$ for a duralumin aircraft. The reverse is true for a nickel steel aircraft, and for the particular steel considered in Table 5 only glass and a low conductivity copper alloy provide values of $\lambda_L > 1$.

However, it appears that heating requirements in laminar or turbulent flow regions can be satisfied using models constructed in different materials from the aircraft and tested in a hot air tunnel at a temperature scale different from that for the aircraft.

Similarity parameters from equations (36)-(48) that are not satisfied identically are

$$\lambda_\nu = 1$$

$$\lambda_{\rho_B} = \lambda_{E_o} \lambda_{\alpha_o}^{0.1667}$$

$$\lambda_g = \lambda_{\alpha_o}^{-1} \lambda_L^{-1}$$

$$\lambda_\varepsilon = \lambda_{k_{B_o}} \lambda_{\alpha_o}^{2.5} \lambda_L^{-1} .$$

λ_ν is close to unity for nearly all the materials considered, and the requirement can therefore be regarded as satisfied. The values of $\lambda_{\alpha_o}^{0.1667}$ are also close to unity, and hence we require $\lambda_{\rho_B} \lambda_{E_o}^{-1} \simeq 1$, i.e. the model material must be equally as efficient as the aircraft material in terms of stiffness : weight ratio. Magnesium, duralumin, glass, titanium, carbon steel and nickel all satisfy this requirement approximately, and for models in these materials the density requirement may be regarded as satisfied. For the remaining materials the requirement cannot be satisfied and hence errors in frequency parameter will arise in oscillatory tests (see section 3.1.1). The λ_g requirement is fortuitously satisfied quite closely by a low conductivity copper alloy model of a steel aircraft, but the requirement is not satisfied in most cases and must be ignored. On the other hand for a duralumin aircraft the emissivity requirement is such that model emissivity must generally be of the same order or lower than that for the aircraft, and this requirement can probably be satisfied. For example if carbon steel is used for a model of a duralumin aircraft then the emissivity is 1.1 times that of the aircraft for a laminar flow model and 0.36 times that of the aircraft for a turbulent flow model.

A further feature to be noted is that models constructed in materials for which $\lambda_{\alpha_o} < 1$ require testing under a reduced temperature as compared with the aircraft. This is generally an attractive feature, since it reduces the demand on the absolute operating temperature of the tunnel heat exchanger. On the other hand refrigeration of the model may be required to provide correct starting conditions. Model materials for which $\lambda_{\alpha_o} > 1$ are less attractive by the same token; for some of these, representative high speed flight conditions are unattainable because of degradation in model material properties due to a too close approach to the melting point.

4.2 Approximations to overall similarity

It appears from sections 3.1.1 and 3.1.2 that provided the heat transfer coefficient satisfies equation (66) models can be constructed in the same materials as the aircraft for tests in a gas other than air at unity temperature scale, or in different materials for tests in air at a different temperature scale. The models will provide complete representation in laminar or turbulent regions, but heating effects are not represented in both régimes of flow simultaneously. Such models are of value for investigations of local effects (e.g. panel flutter) or for investigations where the major load carrying structure lies wholly in the laminar or turbulent flow régime.

Since a laminar flow model has less than true scale heating rate in the turbulent regions (and vice-versa for a turbulent flow model) it follows that if some method of providing a local increase or decrease in heating rate can be devised, one single model will provide representative conditions in both flow régimes. The possibilities in this direction merit consideration.

4.2.1 Turbulent scale model with insulation in the laminar flow regions

A possible method of reducing the heating rate in local areas is to cover the surface with a thin film of low conductivity, whose heat capacity relative to the wing structure, stiffness, and main contributions can be neglected.

For a film of thickness d_x at x from the leading edge and of conductivity k_i, the effective heat transfer $h_{e_{li}}$ at the wing surface at x is

$$h_{e_M}^{-1} = h_M^{-1} + \frac{d_x}{k_i}$$

where h_M is the heat transfer coefficient for the film surface at x and M refers to the model.

We require

$$\left(\frac{h_F}{h_{e_M}}\right)_{\text{laminar}} = \left(\frac{h_F}{h_M}\right)_{\text{turbulent}}$$

i.e.

$$\left(\lambda_h + h_F \frac{d_x}{k_i}\right)_{\text{laminar}} = \lambda_{h_{\text{turbulent}}} \tag{79}$$

where F refers to full scale.

Now from equation (66)

$$\lambda_h = \lambda_{c_p}^{\frac{r}{2}} \lambda_k^{(1-r)} \lambda_{Pr}^{\left(\frac{1-3r}{3}\right)} \lambda_L^r \tag{80}$$

<u>For a model built in aircraft materials and tested in a different gas</u>

$$\lambda_L = \lambda_{c_p}^{-\frac{1}{2}} \lambda_k^{\left(\frac{r_1-1}{r_1}\right)} \lambda_{Pr}^{\left(\frac{3r_1-1}{3r_1}\right)} \tag{75}$$

and hence for a model built to turbulent flow scale where $r = r_1 = 0.8$

$$\lambda_{h_{\text{turbulent}}} = 1 \text{ .}$$

In the laminar flow region of the model $r = 0.5$, $r_1 = 0.8$, therefore

$$\lambda_{h_{\text{laminar}}} = \lambda_k^{0.375} \lambda_{Pr}^{0.125}$$

therefore

$$d_x = \frac{k_i}{h_{F\ \mathrm{laminar}}} \left(1 - \lambda_k^{0.375}\,\lambda_{Pr}^{0.125}\right) . \tag{81}$$

<u>For a model built of different materials from the aircraft and tested in air</u>

$$\lambda_L = \lambda_{k_B}^{\frac{1}{r_1}}\,\lambda_E^{-1}\,\lambda_\alpha^{-\frac{1}{2}}\,\lambda_{c_P}^{-\frac{1}{2}}\,\lambda_k^{\left(\frac{r_1-1}{r_1}\right)}\,\lambda_{Pr}^{\left(\frac{3r_1-1}{3r_1}\right)} \tag{69}$$

and hence for a model built to turbulent flow scale where $r = r_1 = 0.8$

$$\lambda_{h_{\mathrm{turbulent}}} = \lambda_{k_B}\,\lambda_E^{-0.8}\,\lambda_\alpha^{-0.4} = \lambda_{k_{B_o}}\,\lambda_{E_o}^{-0.8}\,\lambda_{\alpha_o}^{-0.467}$$

In the laminar flow region of the model $r = 0.5$, $r_1 = 0.8$, therefore

$$\lambda_{h_{\mathrm{laminar}}} = \lambda_{k_B}^{0.625}\,\lambda_E^{-0.5}\,\lambda_\alpha^{-0.25}\,\lambda_k^{0.375}\,\lambda_{Pr}^{0.125} = \lambda_{k_{B_o}}^{0.625}\,\lambda_{E_o}^{-0.5}\,\lambda_{\alpha_o}^{-0.542}$$

therefore

$$d_x = \frac{k_i}{h_{F\ \mathrm{laminar}}}\,\lambda_{k_{B_o}}\,\lambda_{E_o}^{-0.8}\,\lambda_{\alpha_o}^{-0.467}\left(1 - \lambda_{k_{B_o}}^{-0.375}\,\lambda_{E_o}^{0.3}\,\lambda_{\alpha_o}^{-0.075}\right) . \tag{82}$$

It follows from equations (81) and (82) that d_x should increase in thickness from zero at the leading edge and inversely proportional to full scale heat transfer coefficient.

Equation (66) for the laminar flow heat transfer coefficient may be written[10]

$$h_{F_{\mathrm{laminar}}} = 0.332\ k_F\ x_F^{-0.5}(Re')^{0.5}(Pr)^{1/3} \tag{83}$$

where Re' is Reynolds number/ft. For a high speed aircraft assuming $Re' = 10^7$, $Pr = 0.7$ the laminar heat transfer is

$$h_{F_{\mathrm{laminar}}} \simeq 10^3\ k_F\ x_F^{-0.5} .$$

The insulant is unlikely to have a lower conductivity than air, and hence it follows the required values of d_x are excessive, except for values of λ_L not greatly different from unity.

4.2.2 Laminar scale model with radiant heating in the turbulent flow regions

A possible way of providing an increased heating rate in the turbulent flow regions of a model built to a laminar flow scale is to provide radiant heating lamps in the walls of the wind tunnel working section. In this circumstance the effective heat transfer in the turbulent flow region at a point distance x aft of the transition point is:-

$$h_{e_M} = h_M + \frac{R_x}{(T_{aw} - T_w)}$$

where R_x is the contribution to heating rate at x due to absorption of radiant heat.

We require

$$\left(\frac{h_F}{h_M}\right)_{laminar} = \left(\frac{h_F}{h_{e_M}}\right)_{turbulent}$$

therefore

$$\lambda_{h_{laminar}} = \left\{\lambda_h^{-1} + \frac{R_x}{(T_{aw} - T_w)h_F}\right\}^{-1}_{turbulent} . \qquad (84)$$

Following the procedure of section 3.2.1 we obtain; <u>for a model built in aircraft materials and tested in a different gas</u>

$$\frac{R_x}{(T_{aw} - T_w)} = \left(1 - \lambda_k^{0.6}\,\lambda_{Pr}^{0.2}\right) h_{F_{turbulent}} \qquad (85)$$

and <u>for a model built of different materials from the aircraft and tested in air</u>

$$\frac{R_x}{(T_{aw} - T_w)} = \lambda_{k_{B_o}}^{-1}\,\lambda_{E_o}^{0.5}\,\lambda_{\alpha_o}^{0.292}\left(1 - \lambda_{k_{B_o}}^{-0.6}\,\lambda_{E_o}^{0.3}\,\lambda_{\alpha_o}^{-0.225}\right) h_{F_{turbulent}}$$

...... (86)

Equation (66) for the full scale turbulent flow heat transfer may be written[10]

$$h_{F_{turbulent}} = 0.0296\, k_F\, x_F^{-0.2} (Re')^{0.8} (Pr)^{1/3} \left(\frac{T_a}{T_w}\right)^{0.44} \qquad (87)$$

where T_a is the free stream temperature of the air and T_w is the temperature at the surface. For a maximum heat transfer it is assumed that $T_a = T_w$. Following the same reasoning as in section 3.2.1 it follows that

$$h_{F_{turbulent}} \simeq 10^4\, k_F\, x_F^{-0.2} .$$

A consideration of Tables 2, 4 and 5 in relation to equations (85) and (86) indicates that the maximum rate of heating is

$$\frac{R_{x_{max}}}{(T_{aw} - T_w)} \simeq h_{F_{turbulent}} ,$$

i.e. the radiant heater must be capable of providing heating rates equivalent to full scale turbulent flow values (approximately 40 Kw/ft^2 for $(T_{aw} - T_w) = 1000°F$). It appears from Ref.14 that there is some prospect of providing heating rates of the required order using banks of quartz-tube lamps lining the walls of the working section, and it would also seem feasible to use a uniform heating system and to control the absorption in the different areas of the model by varying the surface finish. The heating rate provided by the lamps must tend to zero as $T_{aw} \to T_w$, and this requires control of the lamp supply voltage in relation to a measurement of temperature on the model, so as to follow a precalculated temperature-time history. Very rapid control is implied since the time scale for heating varies as the square of the length scale, and this may prove the limiting factor in relation to model scale.

However, the method appears to have attractive possibilities as a means of providing overall similarity for an aerothermoelastic model, and merits further investigation.

4.3 Models of reduced effective stiffness tested in 'cold' flow

Although the previous sections have indicated that an acceptable approach to aerothermoelastic similarity might be achieved using models tested in a hot gas stream the unfortunate feature is that suitable heated flow tunnels of the type envisaged (see section 5) are conspicuous by their absence. In these circumstances some attempt must be made to design models with a reduced effective stiffness representative of thermal effects, which can be tested in existing facilities (which generally provide little more than atmospheric stagnation temperatures).

This approach can never be wholly satisfactory, for it presupposes that an "effective" reduced stiffness, corresponding to heated conditions for the aircraft, can be calculated. In fact the effects of transient heating lead to thermal stress distributions producing effects on stiffness that are highly non-linear with displacement of the structure. Hence the effective stiffness for a 1g loading condition for the aircraft may be quite different from the zero g load condition. Furthermore, certain effects of heating cannot be represented by a simple "effective" stiffness concept. For example Broadbent[15] considers an effect of thermal stress which leads to an anticlastic curvature (camber) of the wing chord causing a loss in flexural stiffness*. Flutter then arises from the aerodynamic coupling due to change of wing camber, and not from the loss of stiffness.

However, assuming that an effective stiffness approach is acceptable the approach to be followed is then the conventional one for 'cold' model tests. The similarity equations to be satisfied (neglecting gravitational effects) are equations (19), (20), (23), (25), (29), (30), (33), (34) and (35). From these we obtain, for tests in air,

$$\lambda_V = \lambda_T^{0.6}$$

$$\lambda_p = \lambda_\sigma = \lambda_{E_o} \lambda_T^{-0.2}$$

$$\lambda_\rho = \lambda_{\rho_B} = \lambda_{E_o} \lambda_T^{-1.4} .$$

If these relationships are satisfied then the Mach number for the model will be the same as for the aircraft. Unfortunately existing wind tunnels are such that to satisfy these relationships generally presents some difficulty. The problem has been discussed in detail by Lambourne and Scruton[16], and it is apparent that the main difficulty results from the limited stagnation pressure generally available in existing tunnels. For a model in the same materials as the aircraft, with skin thickness to the same scale as the geometric scale, this implies that the pressure relationship cannot be satisfied. This is generally overcome by assuming that the whole of the aircraft stiffness lies in the skin, and by varying the skin thickness scale relative to geometric scale the effective E for the model becomes $E\,\lambda_\delta^{-1}$ where λ_δ is the ratio of skin thickness scale to length scale. By using a model skin thickness less than the geometric scale, models can then be tested in tunnels with lower than full scale stagnation pressure. The requirement $\lambda_{\rho_B} = \lambda_\delta^{-1} \lambda_{E_o} \lambda_T^{-1.4}$ cannot generally be satisfied, but this affects only the frequency parameter and it is assumed that this can be ignored.

The limitations of aeroelastic model tests in 'cold' flow are well known[16], and little further need be said about them. Obviously, in the absence of heated flow tunnels, the best use must be made of existing

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*Anticlastic curvature due to bending occurs for an unheated wing, due to mid plane stress, but the effect of thermal stress is markedly to increase this curvature - as if Poisson's ratio for the material were several times its true value.

facilities, and this necessitates the use of the effective stiffness concept so far as heating effects are concerned. However, more work is required to determine the types of structure for which an effective stiffness concept is likely to provide an adequate representation.

5 WIND TUNNEL FACILITIES

There are very few existing tunnels that provide for heating the flow. Those that do exist are generally of the intermittent type running from compressed air storage through a pre-heated exchanger into a fixed Mach number working section, but even these rarely provide stagnation conditions representative of high speed flight at low altitude. Neither do they have provision for radiant heater installations in the tunnel sidewalls.

The stagnation pressure and temperature appropriate to free flight conditions are shown in Figs.1 and 2. Free flight conditions are appropriate for the conditions of section 3.1.1, but for the conditions of sections 3.1.2 temperatures and pressures greater than free flight may be required*. Quite obviously, in these circumstances, formidable engineering problems are involved simply in providing a wind tunnel of reasonable dimensions (say 2' x 2') that will withstand the temperatures and pressures involved.

Ideally of course, the tunnel conditions should be capable of controlled variation during a run so as to simulate the aircraft flight plan, and this requires control of stagnation pressure, stagnation temperature, Mach number and radiant heat. At the same time these quantities may need to be varied rapidly, since the time scale for model heating varies as the square of the length scale. For such a tunnel to be operated effectively, complete automation of tunnel control would probably be required. A block layout of a tunnel of this kind is shown in Fig.3.

Returning to reality, there may be some possibility of covering parts of the aircraft flight plan using models in existing tunnels, with the added provision of radiant heat in the working section. The problem of generally low stagnation pressure conditions in existing tunnels might be met to some extent by reducing model skin thickness scale relative to linear scale (as in 'cold' model work) though this will invalidate panel flutter results and results in an increased size of model being required**.

6 MODEL CONSTRUCTION

There is, of course, little point in demonstrating that an approximation to aerothermoelastic similarity can be achieved for small scale models if it should then prove that the difficulties of model construction are insurmountable. If the tunnel size is large enough for models to be constructed using conventional riveting, welding and shaping procedures the problem is simplified, but this will rarely be the case.

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*Wind tunnel requirements are alleviated if a model material can be used having a higher coefficient of expansion, a lower elastic modulus and a lower conductivity than the aircraft material (e.g. magnesium in relation to an aircraft in dural). A search through the metal alloys and plastics may be rewarding in this respect.

**The quantities λ_{k_B}, λ_E, λ_{ρ_B} and λ_{c_B} are replaced by $\lambda_{k_B}\lambda_\delta^{-1}$, $\lambda_E\lambda_\delta$, $\lambda_{\rho_B}\lambda_\delta$ and $\lambda_{c_B}\lambda_\delta^{-1}$ respectively and only conduction in the skin plane is considered. Values of λ_L in Tables 2, 4 and 5 then require multiplying by a factor $\lambda_\delta^{\frac{-(1+r)}{r}}$.

The construction must be a closer replica of that for the aircraft than is usually the case for purely aeroelastic models, and the construction of the latter is formidable enough. Furthermore, few of the established techniques for the construction of small scale aeroelastic models are likely to be applicable in the thermal case.

Investigations of alternative methods of model construction are required, and in this respect a technique at present being developed at R.A.E. may ultimately be of value. This technique takes cognisance of the fact that several of the materials suitable for small scale aerothermoelastic models can be deposited electrolytically (e.g. nickel, cadmium, tin and zinc).

Nickel model wings of a duralumin aircraft have been made successfully by this process, a nickel skin of the required thickness being deposited directly onto a prepared former.

7 CONCLUDING REMARKS

The foregoing considerations indicate that an approach to complete aerothermoelastic similarity can be achieved for small scale models, but models in the same materials as the aircraft must be tested in a different gas, while models in different materials can be tested in air. In both cases the wind tunnels required must have provision for heating the stream to full scale stagnation conditions (or higher), and controlled radiant heat in the working section is also necessary.

Areas where experimental investigations and development work are required are in the effectiveness of radiant heat in a supersonic tunnel and its control in relation to aerothermoelastic models, and in techniques of construction for these models. A survey of existing hot tunnels would also be of value in indicating their limitations with regard to the representation of full scale flight conditions, thus providing a datum for model similarity requirements.

LIST OF SYMBOLS

a,b,c	displacements
B	suffix relating to body properties
c_B, c_P	specific heat
d_x	insulation thickness
E	Young's modulus
F	suffix denoting full scale
g	acceleration due to gravity
g_x	component of gravity in x direction
h	heat transfer coefficient
k, k_B	thermal conductivity
L	length

LIST OF SYMBOLS (Cont'd.)

M	Mach number; suffix denoting model scale
p	pressure
Pr	Prandtl number
r	exponent of Reynolds number in heat transfer formulation
Re	Reynolds number
R_x	radiant heating rate
s	exponent of temperature ratio in heat transfer formulation
s_ϕ	exponent of temperature for temperature dependent property ϕ
t	time
t_1	time for oscillatory or accelerated motion
t_2	time for heat flow
T	temperature
(T_o-T)	temperature difference; temperature change
T_{aw}	adiabatic wall temperature
T_a	free stream temperature
T_w	skin temperature
u,v,w	velocities
V	flow velocity
x,y,z	rectangular co-ordinates
α	coefficient of thermal expansion
γ	ratio of specific heats for gases
δ	skin thickness : typical length
ε	emissivity
λ	ratio aircraft property : model property
μ	viscosity
ν	Poisson's ratio
ρ, ρ_B	density

LIST OF SYMBOLS (Cont'd.)

σ, σ_x, σ_y, σ_z	stress components
$\bar{\sigma}$	Stefan-Boltzmann radiation constant
τ_{xy}, τ_{xz}, τ_{yz}	shearing stress components

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TABLE 1

Average gas properties at 32°F and a pressure of one atmosphere

Gas	Density lb/ft³ ρ	Specific heat BTU/lb °F c_p	Specific heat ratio γ	Dynamic viscosity × 10^7 slugs/ft sec μ	Conductivity BTU/hr ft °F k	Speed of sound ft/sec	Boiling point °K
Hydrogen	0.0054	3.41	1.41	1.73	0.0950	4500	20
Helium	0.0108	1.26	1.67	3.80	0.0800	3200	4
Methane	0.0435	0.59	1.32	2.17	0.0175	1410	109
Ammonia	0.048	0.51	1.31	1.95	0.0123	1360	
Neon	0.0534	0.25	1.67	6.2	0.0270	1460	27
Nitrogen	0.076	0.245	1.40	3.5	0.014	1095	79
Air	0.078	0.24	1.40	3.6	0.014	1090	90
Argon	0.107	0.13	1.67	4.41	0.0094	1050	87
Carbon Dioxide	0.120	0.20	1.30	2.90	0.0082	850	195

TABLE 2

Property ratios λ; (air : gas)

Gas	λ_{ρ_o}	$\lambda_{c_{P_o}}$	λ_γ	λ_{μ_o}	λ_{k_o}	$\lambda_{Pr} = \dfrac{\lambda_{c_{P_o}} \lambda_{\mu_o}}{\lambda_{k_o}}$	$\lambda_{\rho_o} \lambda_{c_{P_o}}$	$\lambda_L = \dfrac{\lambda_{k_o}^{\left(\frac{r-1}{r}\right)} \lambda_{Pr}^{\left(\frac{3r-1}{3r}\right)}}{\lambda_{c_{P_o}}^{\frac{1}{2}}}$ r=0.5 Lam.	r=0.8 Turb.
Hydrogen	14.5	0.07	0.99	2.07	0.15	0.97	1.01	24.9	5.94
Helium	7.2	0.19	0.84	0.95	0.18	1.0	1.36	12.1	3.54
Methane	1.79	0.41	1.06	1.66	0.80	0.85	0.73	1.85	1.50
Ammonia	1.63	0.47	1.07	1.85	1.14	0.76	0.77	1.18	1.21
Neon	1.47	0.93	0.84	0.58	0.52	1.07	1.36	1.96	1.22
Nitrogen	1.03	0.98	1.00	1.03	1.00	1.00	1.01	1.00	1.00
Air	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Argon	0.73	1.86	0.84	0.82	1.49	1.02	1.36	0.49	0.67
Carbon Dioxide	0.65	1.20	1.08	1.24	1.69	0.88	0.78	0.51	0.74
Methane + 5.19 (Helium)	4.83	0.207	0.87	0.94	0.206	0.95	1.00	10.5	3.14

TABLE 3

Average material properties at 68°F

Material	Temp. coeff. of expansion $\times 10^6$ $°F^{-1}$ α	Young's modulus $\times 10^{-6}$ lb/in^2 E	Modulus ratio ν	Conducty. $\frac{BTU}{Hr\ Ft\ °F}$ k_B	Specific heat BTU/lb °F c_B	Density lb/ft^3 ρ_B	Melt. point °F
Duralumin	12.5	10.0	0.38	95	0.21	174	1250
Nickel Steel	5.5	29.5	0.39	15	0.11	490	2500
Carbon Steel	7	29.5	0.39	30	0.11	490	2500
Titanium	5	16	0.38	15	0.13	280	3270
Magnesium Alloy	16	6.5	0.37	40-87	0.24	112	1200
Copper Alloy	9.3	15.0	0.37	15-200	0.095	540	1940
Glass	4.7	8.0	0.40	0.44	0.20	169	2000
Cadmium	16.5	5.0	0.38	53	0.055	536	610
Nickel	7.1	28.5	0.38	52	0.11	550	2600
Silver	10.6	11.1	0.37	242	0.056	650	1760
Tin	11.7	7.7	0.375	37	0.054	453	446
Zinc	16.5	8.7	0.436	65	0.092	440	785

TABLE 4

Property ratios for a duralumin aircraft

Model material	λ_{ρ_B}	λ_{α_o}	λ_{E_o}	$\lambda_{k_{B_o}}$	$\lambda_{c_{B_o}}$	λ_{ν}	$\lambda_L = \lambda_{k_{B_o}}^{1/r} \lambda_{E_o}^{-1} \lambda_{\alpha_o}^{\frac{-(0.8+1.3r)}{1.2r}}$		$\lambda_\varepsilon = \lambda_{k_{B_o}} \lambda_{\alpha_o}^{2.5} \lambda_L^{-1}$	
							r=0.5 Lam.	r=0.8 Turb.	r=0.5 Lam.	r=0.8 Turb.
Magnesium	1.55	0.78	1.55	$\begin{cases} 2.4 \\ 1.09 \end{cases}$	0.88	1.03	$\begin{cases} 6.75 \\ 1.39 \end{cases}$	$\begin{cases} 3.06 \\ 1.07 \end{cases}$	$\begin{cases} < 1 \\ < 1 \end{cases}$	$\begin{cases} < 1 \\ < 1 \end{cases}$
Glass	1.04	2.70	1.35	216	1.05	0.95	3120	88	0.84	29.9
Titanium	0.62	2.5	0.62	6.3	1.61	1.0	6.96	2.79	8.95	22.3
Zinc	0.40	0.76	1.15	1.46	2.3	0.87	3.58	2.39	< 1	< 1
Tin	0.38	1.07	1.30	2.57	3.89	0.99	4.31	2.20	3.24	6.35
Nickel Steel	0.35	4.20	0.32	6.30	1.91	0.98	3.84	1.88	23.4	47.2
Carbon Steel	0.35	1.8	0.32	6.30	1.61	0.98	30.0	9.90	0.91	2.76
Cadmium	0.33	0.76	2.00	1.79	3.82	0.98	2.94	1.78	< 1	< 1
Copper	0.32	1.35	0.68	$\begin{cases} 6.30 \\ 0.48 \end{cases}$	2.33	1.03	$\begin{cases} 28.3 \\ < 1 \end{cases}$	$\begin{cases} 8.16 \\ < 1 \end{cases}$	$\begin{cases} 0.47 \\ - \end{cases}$	$\begin{cases} 1.66 \\ - \end{cases}$
Nickel	0.32	1.76	0.35	1.83	1.91	0.98	2.44	2.01	3.07	3.73
Silver	0.27	1.18	0.90	0.39	3.86	1.03	< 1	< 1	-	-

TABLE 5

Property ratios for a nickel steel aircraft

Model material	λ_{ρ_B}	λ_{α_o}	λ_{E_o}	$\lambda_{k_{B_o}}$	$\lambda_{c_{B_o}}$	λ_{ν}	$\lambda_L = \lambda_{k_{B_o}}^{1/r} \lambda_{E_o}^{-1} \lambda_{\alpha_o}^{\frac{-(0.8+1.3r)}{1.2r}}$ r=0.5 Lam.	r=0.8 Turb.
Magnesium	4.40	0.34	4.45	$\begin{cases}0.38\\0.17\end{cases}$	0.46	1.05	$\begin{cases}< 1\\< 1\end{cases}$	$\begin{cases}< 1\\< 1\end{cases}$
Duralumin	2.82	0.44	2.95	0.16	0.53	1.03	< 1	< 1
Glass	2.90	1.17	3.7	34.0	0.55	0.98	456	165
Titanium	1.75	1.10	1.85	1.0	0.85	1.03	< 1	< 1
Zinc	1.11	0.33	3.4	0.23	1.20	0.90	< 1	< 1
Tin	1.08	0.47	3.84	0.41	2.04	1.04	< 1	< 1
Carbon Steel	1.00	0.79	1.00	0.50	1.00	1.00	< 1	< 1
Cadmium	0.92	0.33	5.9	0.28	2.00	1.03	< 1	< 1
Copper	0.91	0.55	1.97	$\begin{cases}1.0\\0.075\end{cases}$	1.22	1.05	$\begin{cases}2.15\\< 1\end{cases}$	$\begin{cases}1.62\\< 1\end{cases}$
Nickel	0.89	0.77	1.03	0.29	1.00	1.03	< 1	< 1
Silver	0.75	0.52	2.66	0.06	1.97	1.05	< 1	< 1

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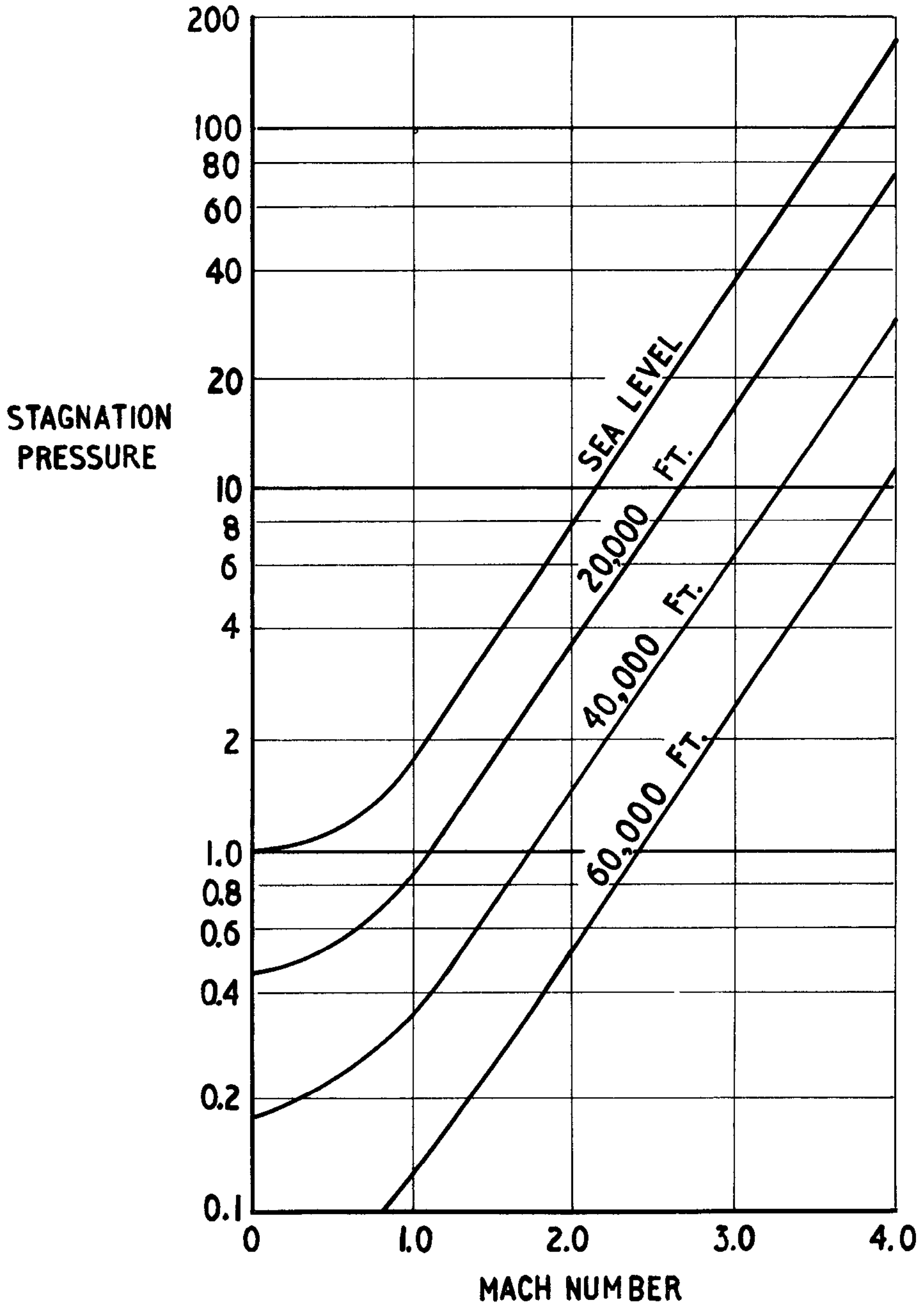

FIG. 1. STAGNATION PRESSURE REQUIREMENTS FOR FREE FLIGHT SIMULATION.

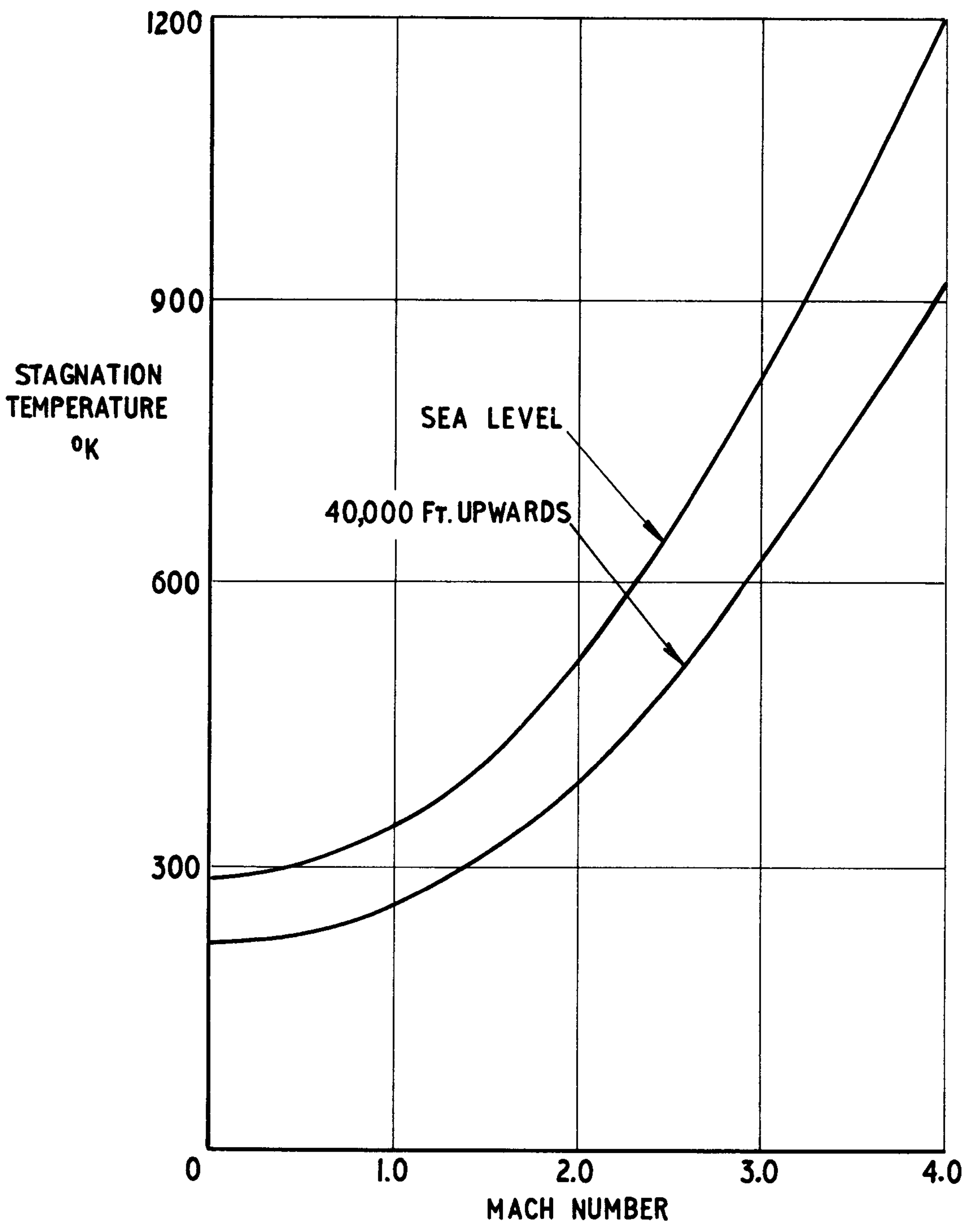

FIG. 2. STAGNATION TEMPERATURE REQUIREMENTS FOR FREE FLIGHT SIMULATION.

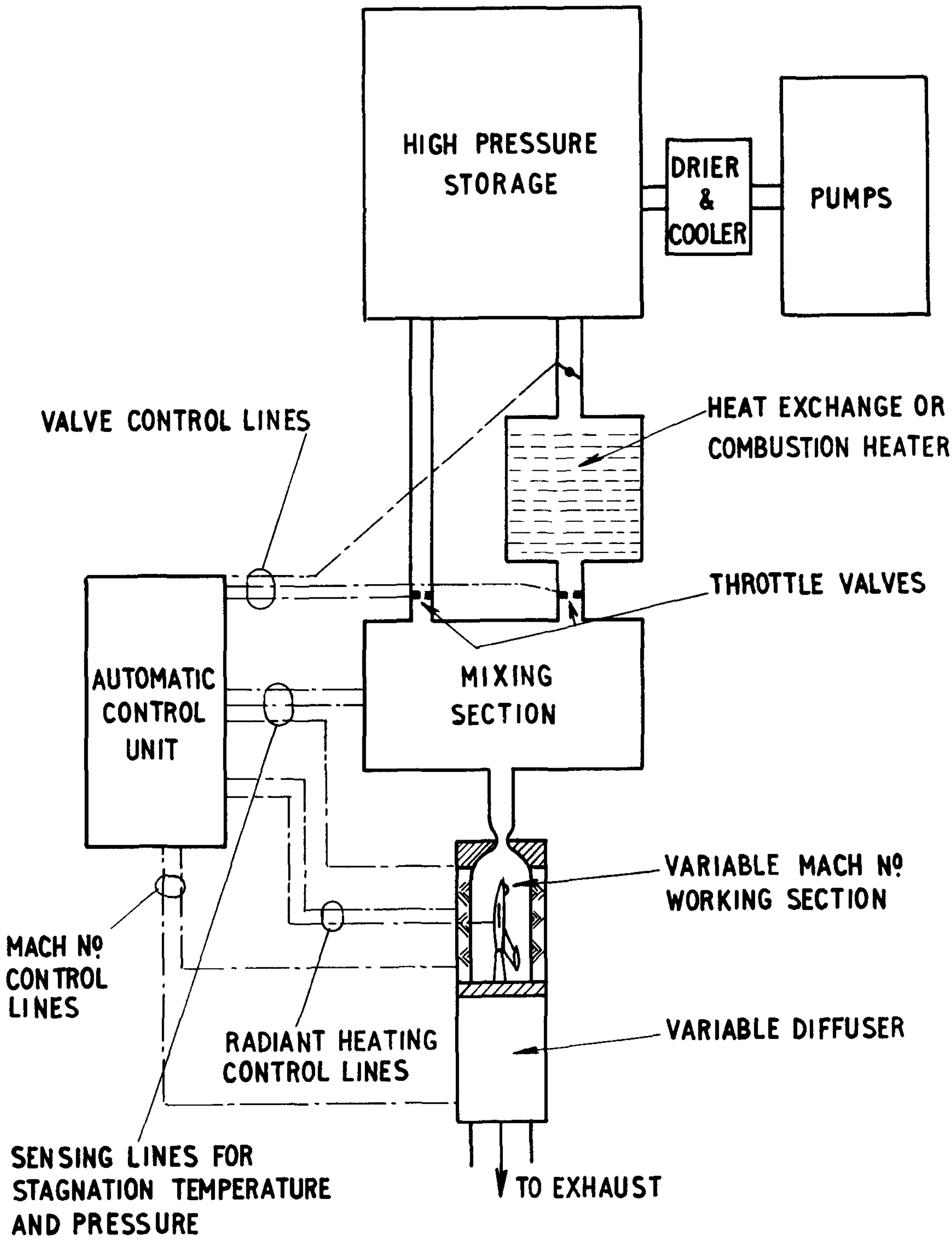

FIG. 3. POSSIBLE TUNNEL LAYOUT.

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