



# MINISTRY OF AVIATION

# AERONAUTICAL RESEARCH COUNCIL CURRENT PAPERS

Progress Report on an Experiment on the Effect of Surface Flexibility on the Stability of Laminar Flow

By

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1962

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December, 1961

#### SUMMARY

This paper describes the flexible surfaces whose properties have been examined and which have been tested on an aerofoil in a wind tunnel. The experiment has been rather inconclusive as no drag reductions have been found in turbulent flow, whilst the only rearward movements of transition occurred in conditions where the alteration has been inhibited by the onset of laminar separation. The limitations of the experiment are discussed carefully in order to clarify the next steps which are to be taken with more flexible surfaces with less damping.

#### 1. Theoretical Background

These experimental investigations into the effect of surface flexibility on the stability of laminar flow in air were initiated a year ago at a time when the state of knowledge was as summed up by Gregory in Ref. 1. The most important theoretical paper was that by Brooke Benjamin<sup>2</sup>, who suggested two ways in which flexibility might be used in attempts to stabilize the two-dimensional boundary layer against infinitesimal two-dimensional disturbances (Tollmien-Schlichting waves).

One technique suggested was to use an elastic medium which was both soft (to have a large response to surface pressures) and also light (to keep the speed of free surface waves in the medium greater than the external flow velocities) and which at the same time suffered little internal friction, since when these other conditions were satisfied, internal friction was thought to be destabilising to infinitesimal disturbances. It was pointed out, however, that the stiffness might need to be kept above a certain margin in order to avoid a kind of "static" instability (so-called Kelvin-Helmholtz type). In this case the change of pressure distribution associated with the introduction of a wave (either stationary or moving) might exceed the restoring forces generated by the wave in the elastic medium so that the amplitude would rapidly diverge.

The other technique, which was thought to be that acting in Kramer's underwater experiments, was to use a medium with a "fairly high" overall stiffness, but tuned so that the velocity of propagation of surface waves of the frequency of the most dangerous Tollmien-Schlichting waves coincided with the Tollmien-Schlichting wave velocity. In these circumstances the Tollmien-Schlichting mechanism of instability was completely disrupted, as the wall friction layer disappeared, and it was hinted that the rôle of stabilization was taken over by the damping in the material, which might, if it was of some appropriate but unspecified amount, result in the raising of the critical Reynolds number to some extent.

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It has not yet been conclusively proved, as far as the present authors are aware, that the drag reduction reported by Kramer was due to the stabilization of infinitesimal two-dimensional Tollmien-Schlichting waves. It is possible that stabilization of three-dimensional waves (or vortex loops) of finite amplitude was occurring, or even that the skin friction in a turbulent boundary layer was reduced by the flexibility of the surface. If this is the case, the appropriate surface compliance is not necessarily that suggested by Brooke Benjamin's theories<sup>2</sup>.

When the present experiment was being considered, it was realised that if a homogeneous elastic medium were used, its depth would be restricted to about 1/4 in., as this would be the largest convenient thickness if the flexible skin were to be wrapped around an existing rigid aerofoil. A sufficiently low stiffness in compression was then thought to necessitate the use of foam rubber materials, and it seemed likely that the high internal damping inherent in foam materials would completely inhibit any stabilization associated with the use of non-dissipative flexible surfaces. The experiment was therefore designed to accommodate a tuning device (other than change of wind speed) to facilitate the matching of the speed of free surface waves to the expected Tollmien-Schlichting wave speed which appeared to be the requisite technique when damping was present. It was hoped that this matching could be done by adjusting the tension in the outer impervious layer with which any foam materials would have to be clad, or even adjusting the tension of the whole elastic medium. The response of such a taut skin to travelling pressure waves is discussed in the Appendix.

During the course of the experiments, a fresh theoretical treatment by Nonweiler appeared. This treatment relates to an undamped elastic medium and pays special attention to the boundary conditions at the interior surface of the medium, which may be either fixed, or free of stress, or exposed to a fluid which at the extremes could vary between a heavy inviscid fluid and a light viscous fluid. This detailed treatment of the problem emphasizes the importance of the parameter  $\sigma$ , the ratio of the density of the external fluid stream to that of the elastic medium, and reveals the possibility of a variety of modes of oscillation (apart from Tollmien-Schlichting waves) which have propagation speeds largely determined by the surface properties. It is Nonweiler's contention that the simple assumptions made by Brooke Benjamin in his treatment of the stability equation are inadequate as they do not lead to the right singular behaviour in the regions of interest.

From Nonweiler's arguments it appears that stabilization can only be attained with thick skins, whatever their inner boundary conditions, if the value of  $\sigma$  is large. This implies the use of elastic materials having densities the same as, or smaller than, the surrounding fluid, and this is clearly impossible in the aeronautical application. One is therefore obliged to use thin skins.

A thin flexible skin is therefore required (according to Nonweiler) mounted with some freedom at its inner boundary, and to avoid various neutral oscillations that the surface could perform in the absence of the boundary layer, it again appears to be necessary to ensure that the lowest speed of propagation of elastic waves, that of Rayleigh surface waves, should exceed the free-stream speed. Amplification of the flexural wave mode can be reduced by increasing its speed of propagation until it exceeds the stream speed\*, or by restraining the normal displacement of the boundary by exposing it to a heavy fluid. Provided the skin thickness is less than a certain maximum, for which a relationship is given in this paper (Fig.6) it would appear that the Tollmien-Schlichting mode might be absent.

Nonweiler's/

<sup>\*</sup>Note that Nonweiler is referring here to flexural waves associated with bending of the thin (but stiff) skin. He does not consider the case of the tensioned skin.

Nonweiler's paper does not consider the effect of damping, and its inclusion would render his mathematical approach extremely difficult. He has subsequently suggested that damping is undesirable in all cases. Increasing damping from zero would gradually reduce the response of the skin until with large damping there would be no response. The stabilization achieved would be correspondingly reduced. Discontinuities or resonances would not be expected, but it must be emphasized that these intuitive ideas are unsupported by mathematical argument.

#### 2. Experimental Programme

It was appreciated in Ref. 1 that the ideal experiment to illuminate the theories should be carried out on a flat plate in zero pressure gradient in a wind tunnel of very low turbulence. Disturbances could be introduced into the boundary layer with a vibrating ribbon and their downstream behaviour, over both rigid and flexible surfaces could be examined with the aid of a traversing hot-wire probe. This would be a fairly complicated experiment to set up and it would be necessary to measure velocity correlations in order to differentiate between the effects on a probe fixed in space in the boundary layer, of surface oscillations on the one hand and of the passage of fluid waves on the other.

Before embarking on such a programme it was thought desirable and expedient to conduct a limited experiment with more practical objectives. This was to discover whether the transition position and the drag coefficient of a low drag aerofoil (the flow over much of which could be arranged to be at or near zero pressure gradient) was in any way affected by the presence of a flexible surface. This experiment could thus cater also for the possibility that drag reductions might be achieved in turbulent flow. Only if some effect of flexibility was observed, or if the zero pressure gradient experiment of A.R.L. yielded favorable results\*, would consideration be given to carrying out the detailed investigation on a flat plate in air.

The experiments were carried out on an available low drag aerofoil of 5 ft chord whose characteristics were well known 4,5,6. The aerofoil shape is illustrated in Fig.1 together with the velocity distribution (taken over the flexible surface) at various angles of incidence. The aerofoil chord was reduced by 3% at the rear and the concave surface associated with the original cusped trailing edge eliminated on the working side of the aerofoil. A slot was cut through the section 5% ahead of the new trailing edge and the flexible surfaces were fed through this slot and anchored to the aerofoil on The step-down aft of the flexible surface was eliminated the lower surface. by sticking and stretching an 0.010 in. thick rubber sheet between the flexible surface and the trailing edge. The flexible surface was wrapped over the upper surface and round the leading edge of the aerofoil and onto a roller inserted in a cavity in the lower surface. The tension in the skin was calculated from measurements of the moment acting on the roller and of the radius of the outer turn of flexible skin on the roller. When adjusting the tension in the skin by winding the roller it is necessary to eliminate friction between the flexible skin and the aerofoil surface. This is achieved by floating the skin on compressed air supplied by perforated pipes inlaid on the surface of the aerofoil. The same piping has on occasion been connected to a source of suction in order to reduce the pressure between the flexible skin and the rigid aerofoil surface and so to delay the windspeed at which the complete skin develops bulges and lifts off the aerofoil.

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<sup>\*</sup>This remark was made before the work of Nonweiler emphasized the essential difference between the flows in air and water. Although it is now seen that results from water experiments cannot be transferred to aerodynamic experiments as the parameter  $\sigma$  is inevitably different in the two cases, the aerodynamic experiment is the harder one, and success in the hydrodynamic case would certainly encourage attempts to reproduce the phenomenon in air.

The drag coefficients of the flexible surface side of the aerofoil and of a comparison portion of rigid surface beyond the limited span (30 in.) of the flexible surface were obtained from combs of pitot tubes held at the trailing edge of the aerofoil.

Transition on the aerofoil was indicated by means of the sublimation of napthalene sprayed on the surface; at the lower windspeeds the acoustic stethoscope was also used occasionally. Hot wires were also used qualitatively.

Movement of the flexible surface was occasionally examined with the aid of a 1 cm diameter galvanometer mirror affixed to the surface reflecting a beam of light to form an image 3 metres away. This optical lever allowed very small amplitudes of movement of the surface to be detected, but the mirror had a mass equal to that of 7 additional skin thicknesses, so that its response at high frequency was probably somewhat reduced. This system of indicating surface movement is not ideal, but no very easy way could be seen for doing this using much lighter reflecting tape which was not even optically flat.

#### 3. Mechanical Properties of Foam Materials

Knowledge of the appropriate stiffnesses of the flexible surface is required for correlation with the boundary layer stabilizing properties of the surface. With the use of non-homogeneous skins, these constants must of course be measured with the skin subject to the same kind of loading as it will receive from the boundary layer disturbances. This experiment is not easily performed, but as is explained in the Appendix, the constants can be inferred from the knowledge of the response of the skin to oscillations imposed by a knife edge. Furthermore, the Appendix presents an approximate treatment showing how the response is likely to be determined in terms of the more fundamental elastic constants of the materials used. As a first step, in order to allow some comparison to be made between the possible materials proposed for flexible surface construction, this section describes the simple measurements made of Young's modulus (unfortunately not at the frequency of interest) and of the first measurements of free-surface wave speeds that came from a vibration test rig.

The zero-frequency stress/strain curve for a fairly soft polyurethane foam under compression is shown in Fig. 2. The foam sheet was nominally 1/4 in. thick and the sample tested was large enough for edge effects to be neglected. Apart from the toe to the curve, due probably to an inadequately flat sample, it will be seen that the material behaves linearly at first, but its stiffness is considerably reduced when compressed beyond a strain of about 0.15 (as measured) owing to buckling of the fibres The material stiffens up again at much higher strains as it of the foam. "goes solid". This shape of curve is typical of all foam materials, though the degree of non-linearity after "buckling" can be varied by the chemical composition of the foam. Advantage was taken of the presence of the non-linear stress/strain relation by sealing the foam between two airtight Hence by altering the pressure between the skins it was possible to use the sandwich in either the stiff or soft regions as seemed appropriate.

The stress-strain curve of Fig.2 shows a very large hysteresis loop. Similar results were obtained under dynamic conditions by Messrs. Dunlop Rubber Co. Research Centre. Tests in their Davies machine at frequencies between 1 and 12 cycles per second show for this particular foam a dissipation per cycle of about 1.9 times the strain energy associated with the stress amplitude. The strain amplitude of these tests was ±10% about a 20% mean strain, thus taking the material into its buckled state (as can be seen from Fig.2). When the strain amplitude was reduced to ±5% about an unstrained mean position the relative damping fell to between 1/3 and 1/5 of its former value. This information, however, is not very relevant to frequencies in the range 200 - 1500 cycles/sec where air damping

within the foam might be expected to be greater, although the material itself might behave better. It must also be remembered that the mode of vibration is different, being progressive surface waves, and these will involve combinations of normal and shear modes of distortion.

The flexible surfaces which were tried first consisted of sheets of 0.013 in. rubber membrane on either side of a 1/4 in. layer of polyurethane foam. The sandwich was sealed at its edges and the pressure inside the sandwich could be adjusted. The whole sandwich, including the foam, could be stretched in the chordwise direction in order to control the speed of free surface waves. Fig.3 shows the stress/strain relation for the thin rubber membrane in tension. This led to a rather considerable extent of flexible surface being wrapped onto the roller when the appropriate tension was attained, but this did allow the roller technique to be used, since the minimum rotation of the roller did not result in a large change in tension. However it did not seem too satisfactory to have to use the foam in so highly a stretched state. Other rubber sheet which was examined showed strains at a stress level of 10<sup>4</sup> lb/sq ft which varied from 0.3 to 1.6 compared with the value of 0.9 to 1.0 for the rubber used.

The foam material had a fairly linear stress/strain relation with increasing tension, and a modulus that was rather greater than the maximum value obtained in compression. Typical values are quoted on Fig.2 for a particular material. With decreasing tension, the material still exhibited considerable hysteresis. Both rubber and foam crept slightly under tension so that in the experiment the tension was frequently checked by floating the flexible surface off the aerofoil. In order to calculate the tension in the outer membrane, the foam and the membrane were assumed to creep at the same rate.

A vibration test rig was constructed in order to measure the speed of propagation and attenuation of free surface waves on the flexible medium. In the preliminary form of the rig the waves were impressed by the head of a 2 BA screw which was pressed onto the surface and was oscillated by means of Surface waves spread out in all directions and their a Goodman vibrator. passage in the direction in which the flexible skin was stretched was followed by means of a capacity probe and associated electronic equipment. By traversing the probe along the surface and noting the distances at which the signal was in phase with that of the vibrator it was possible to measure the wavelength, and hence to calculate the speed of propagation of the waves. This is shown in Fig.4 for the skin already referred to. Notice that the skin is slightly dispersive, in that the shorter waves propagate faster, but that the speed of propagation is primarily controlled by the surface tension, and only to a smaller extent by the foam backing. This is qualitatively in agreement with the theory of the Appendix. The attenuation of the waves was much higher than would be accounted for by the spread from a point source, and was greater for the low frequency (long) waves than for the high It was possible to follow the vibration for only 3 to 8 frequency waves. The attenuation was not precisely measured in the preliminary wavelengths. rig as this was not fitted with the necessary precision vertical traverse necessary for this purpose. This defect is being overcome in the final form of the test rig in which, also, two-dimensional waves will be propagated by means of an oscillating knife edge. In comparison, waves were found to be propagated with very little attenuation when the top membrane was lifted off the foam by inflating the sandwich. It should be pointed out that results were not obtained for tensions less than the minimum value shown in Fig. 4. This was because the attenuation was probably even more severe, and at the lower frequencies, especially, the situation was confused by spurious vibrations transmitted through the wooden structure on which the flexible surfaces were mounted for test.

Although Fig.4 suggests that the wave propagation speed is largely controlled by the outer skin tension, some tests were carried out in which the internal pressure in the sandwich was varied, the tension of the sandwich

remaining constant. These tests yielded the curious results shown in The variation in wave speed with the compression modulus of the foam sandwich is now seen to depend markedly upon the frequency. Qualitatively, the behaviour at 320 cycles/sec is consistent with a variation in E of the foam of the kind suggested by Fig. 2, although calculation does not suggest a curve that bears any close agreement with the experimental one. The result is not fully understood, and coupled with the possibilities of differential creep also affecting the tension distribution between foam and membrane the conclusion is drawn that it may be difficult to control the free surface wave speeds within close limits For this reason, the with this form of flexible surface construction. vibrating knife-edge rig has been constructed so that it can be placed over the flexible surface in position on the aerofoil so that the characteristics can be measured, if required, immediately following a wind tunnel run.

#### 4. Wind Tunnel Results

### 4.1 Foam sandwich with chordwise tension

In Fig.6 are shown the variation of transition position and drag coefficient with windspeed at  $1\frac{1}{2}^{\circ}$  and  $2^{\circ}$  incidence for both the rigid surface and for the foam sandwich, which was fitted first with just sufficient tension to keep it taut. It was intended to concentrate on the behaviour of the model at speeds in the range 150 ft/sec to 180 ft/sec for which as is shown in Table 1 the unstretched skin had ratios of zero frequency compression modulus to tunnel dynamic pressure of the same order of magnitude as those used by Kramer in his underwater tests (5 to 13). At these speeds, however, transition turned out to be well forward and this was attributed to the rough surface finish of the rubber membrane which had been manufactured in contact with a fine woven fabric and possessed the texture of this latter fabric. In these circumstances it was thought that a spotty mode of transition would be manifest and that Tollmien-Schlichting waves would be difficult to trace above the background "noise". Furthermore, the surface was close to the critical windspeed above which it became statically unstable and a standing bulge completely upset the local pressure distribution. More attention was therefore paid to obtaining results at lower wind speeds where the surface roughness was relatively less, and transition on the flexible surface as fitted was more closely the same as on the rigid surface. Unfortunately it was not appreciated that at  $1\frac{1}{2}$ ° and  $2\frac{1}{2}$ ° of incidence, the calculated laminar separation positions on the aerofoil are in the region of 65% chord so that records obtained with the sublimation technique showing "transition" well behind these positions are probably indicating the position of re-attachment of a turbulent boundary layer, the length of the separated bubble increasing with decreasing Reynolds number.

The effect of altering the tension in the foam sandwich is illustrated by the results obtained at  $1\frac{1}{2}$ ° incidence at windspeeds of 90, 140 and 160 ft/sec shown in Fig.7. At the higher speeds where transition on the lightly tensioned skin was well forward, transition is seen to move back with increasing tension. This is thought to be because with increasing strain in the outer rubber sheet, the rough texture of the sheet is much reduced. Transition, however, is only further aft than that on the rigid surface at the lowest speed 90 ft/sec, where as already pointed out, turbulent re-attachment is in fact being indicated. The abscissae in Fig.7 are also marked with theoretical wave propagation speed derived from the straight line of Fig.4. According to Brooke Benjamin's theory for surfaces with damping one might hope for enhanced stabilizing effects when the wave propagation speed is roughly one half the tunnel speed (i.e., the Tollmien-Schlichting speed 0.41  $U_1$  where  $U_1 \neq 1.2 U_0$ ). There is no suggestion in Fig.7 that any such enhanced effects are occurring.

Further measurements were made over a wide speed range at  $1\frac{1}{2}$ ° incidence with a medium value of the tension Fig.8 and over a range of incidence, Fig.9. It is not clear whether the maximum delay in turbulent

reattachment of 3 to 4% chord found at low speeds should be attributed to an effect of flexibility of surface (which with 8 in. of water suction compressing the foam has a suitable ratio of compressible modulus to dynamic pressure for these low speeds (Table 1)). It is possible that turbulent reattachment may be affected by the chordwise variation in the external pressure distribution resulting in a varying thickness distribution along the length of the sandwich which so modifies the aerofoil contour as to alter the pressure distribution and modify the position of laminar separation. The pressure distributions of Fig.1 were taken with the aid of a  $2\frac{1}{2}$  mm bore plastic tubing wrapped round the flexible surface with 6 static holes pierced through it at one particular station. The distribution was measured at a windspeed of 140 ft/sec, although laminar separations are affecting the start of the turbulent boundary layer at much lower speeds where the thickness of the flexible skin would be different.

During the tests at  $1\frac{1}{2}^{\circ}$  incidence, with a tension in the outer layer of about 3.5 lb/ft span, the galvanometer mirror was fixed at various positions along the chord and the movements of the reflected light spot observed at various speeds. The vibration of the surface increased rapidly as the mirror approached closely to the position of transition and then decreased somewhat when well into the turbulent layer. However, assuming that the wavelength of the motion was appropriate to that of Tollmien-Schlichting disturbances, the amplitude of the motion just ahead of transition at 160 ft/sec was only approximately 1/10,000 in. Furthermore, the vibration was predominantly in a spanwise direction.

Although, theoretically, only infinitesimal surface movements are required to stabilise the boundary layer to infinitesimal disturbances, one imagines that much larger movements would occur if the rate of amplification of finite disturbances just ahead of transition were being affected. was thought, therefore, that the small motions observed (which at Tollmien-Schlichting frequencies implied correspondingly small normal-velocity amplitudes) were unlikely to have much effect on the flow. It was therefore concluded that the response of the foam sandwich was greatly inhibited by the high damping of the foam material, so fresh surfaces were constructed with less damping. In addition, it was appreciated that the outer skin, tensioned in the chordwise direction only, resulted in a much softer surface to spanwise travelling waves. On this account, with coupling between the two modes, the predominant vibrations were spanwise moving waves, which was not what was required for stability purposes. In all subsequent tests the skin was therefore tensioned in the spanwise direction as well, until the skin possessed the same strain in the spanwise direction (as measured by the distance between gauge marks on the skin) as in the chordwise direction where the tension was directly measured at the roller. When this was done (on the softer surfaces whose performance is discussed below) the surface waves were found to move predominantly in the chordwise direction. No further tests were carried out on the foam sandwich as the outer rubber skin was found to have perished and to possess large numbers of pin-hole leaks which tended to produce spurious wedges of turbulence.

#### 4.2 Soft surfaces with equal chordwise and spanwise tensions

An 0.01 in. thick rubber membrane was stretched both spanwise and chordwise over 9 spanwise strips of 1/4 in. thick foam as used in the previous skin sandwich. The foam strips were lightly stuck to the rigid aerofoil surface. The width of the strips was 10% of the strip spacing and the widths (and spacings) were reduced towards the front of the aerofoil so that the strip width was always less than the wavelength of the expected disturbances. Uniformity of tension in the membrane was judged by the equality of strain which was obtained by hand adjustment as it was no longer possible to float the membrane on compressed air.

During the course of these tests a 4 in. wide strip of foam was stuck round the leading edge of the aerofoil, butting up to the first support

strip at 3% of the chord. Though this improved the shape of the nose, transition still moved rapidly forward towards the nose from the laminar separation position as the speed was increased between 40 and 60 ft/sec and this was ascribed to the adverse pressure gradients associated with the now polygonal shaped aerofoil section. Furthermore, the membrane would not support negative loading (it was not stuck to the foam strips) so that at windspeeds between 60 and 140 ft/sec depending on the tension a large standing bulge appeared, upsetting the prescribed pressure distribution.

An attempt was made to recover a smooth pressure distribution by supporting the membrane on chordwise foam strips but standing bulges appeared at even lower speeds than described above. This occurred because the membrane tended to be blown off the aerofoil at the position of minimum pressure since the space beneath the membrane was now subjected to some mean value of the pressure between nose and tail instead of being vented at the ends of the span of the membrane to the local static pressure on the rigid aerofoil.

It was also found that the amplitude of the standing bulge was not greatly reduced when the thin rubber membrane was replaced by a relatively inextensible thin plastic sheet. A complete sandwich was therefore constructed consisting of two layers of 0.0065 in. thick polythene sheeting separated by 10% area of foam stuck to both surfaces, as shown inset in Fig.11, in the form of  $\frac{1}{2}$  in.  $\times$   $\frac{1}{10}$  in. strips at  $\frac{1}{2}$  in. spanwise pitch. This distributed support allowed the outer membrane to withstand negative loading without long-wavelength bulging since the complete sandwich could be sucked down onto the rigid aerofoil using the compressed-air line in reverse. However, the relatively wide spacing of the supports (in terms of boundary-layer thickness) allowed short-wavelength bulging or quilting of the surface in some parts (depending on the internal pressure level in relation to the external distribution) and wedges of turbulence were troublesome above 90 ft/sec although the complete sandwich was free of bulges up to much higher speeds. Such observations of "transition" (or rather turbulent reattachment) as could be obtained by the use of naphthalene and were free from spurious turbulence wedges are shown in Fig. 10. the foam rubber sandwich, the improvements over the rigid surface results are confined to speeds where the phenomenon is turbulent reattachment and not transition, and one does not know whether the alteration is due to static or dynamic effects of the compliance of the surface.

#### 5. A Flexible Surface in Turbulent Flow

A 24 s.w.g. transition was fixed at 5.0% chord on both the rigid and foam strip supported flexible surface of the previous section. The variation of surface drag coefficient with windspeed is shown in Fig.11 for two values of the skin tension. Although the curves are not identical, the flexible surface in these two conditions did not yield drag coefficients less than those obtained for the rigid surface.

#### 6. <u>Discussion</u>

The detailed account of the present experiments reveals many reasons why the results so far have been inconclusive. It seems probable that the general lack of effect of the foam supported skins has been due to excessive damping, and in the light of Nonweiler's theoretical treatment it seems reasonable to undertake further work on the existing aerofoil with lighter, thinner skins with less damping. The flexural stiffness of the outer skin itself could be increased over that of the present rubber membrane without disadvantage but a softer support is required. Further work might prove fruitful provided the skins are smooth enough to allow laminar flow to be obtained at the higher wind-tunnel speeds and the tests are then restricted (as was originally intended) to incidences of  $2\frac{1}{2}$ ° and upwards where the flow over the first half of the aerofoil is in zero or weakly adverse pressure

gradients (Fig.1) so that transition is a rapidly varying function of wind speed. Only in these conditions does transition follow from the processes initiated by Tollmien-Schlichting waves rather than as a result of laminar separation. Furthermore, as transition in these conditions is sensitive to Reynolds number, it is reasonable to hope that if surface flexibility can influence the position of transition at all, the changes would be large enough not to be confused with any small changes arising from alterations in the basic pressure distribution due to the addition of the flexible layer itself.

Although it is possible in tests on a flat plate to use thin skins supported only by their edges and relying on surface tension and flexural stiffness to resist deformation, the discussion in the Appendix shows that such skins suffer from static instability at very low speeds. instability leads first to a standing bulge of large wavelength which distorts the external pressure distribution. The amplitude of such bulges could be greatly diminished by the provision of local normal elastic stiffness which could stand tensile loading if such stiffness could be arranged to increase in non-linear fashion (provided the material were strong enough) as soon as the amplitude became comparable with the depth of the flexible layer. This might limit the amplitude sufficiently to allow the skin to be used with standing bulges present. In any case, the value of the normal stiffness required to eliminate the instability is shown to be Normal stiffness is of course essential very small for a tensioned skin. for a skin that is to be wrapped round a cambered surface, so no consideration will be given to entirely unsupported skins. This discussion emphasizes the necessity for the normal stiffness to withstand negative loading; and to prevent the complete sandwich from lifting, it will be necessary to provide the present aerofoil with further grooving for an additional suction system. The small perforations at the compressed—air piping used in reverse did not allow sufficient suction flow to overcome the leakage at the edges of the sandwich and did not hold the sandwich down very effectively as the pressure could not be reduced very much.

The proposed new skins will again be of the sandwich type in which deformation of the outer skin will be resisted by its own flexural stiffness, and if necessary by surface tension, and local normal stiffness will be provided not by foam filling, but by the flexural distortion of closely pitched spanwise intercostal members.

The present investigation has so far provided little encouragement for undertaking the detailed fundamental experiment of forcing Tollmien-Schlichting oscillations on a flat plate and observing how their rate of amplification or decay is affected by flexibility of the surface. The stability of infinitesimal disturbances is the subject with which the existing theoretical papers have dealt and an experiment is obviously necessary to complete our understanding of this mechanism once it has been established that this was the mechanism responsible for Kramer's drag If in fact these drag reductions took place in turbulent flow, forcing Tollmien-Schlichting oscillations in a laminar layer might not prove very fruitful. Even if the theories are correct, it is still necessary to ascertain whether a small gain in the value of the stability critical Reynolds number will reflect in a corresponding alteration in transition Reynolds number. If the position of transition is more controlled by the extremely large amplification rates of the two- or three-dimensional disturbances just ahead of transition and these happen to be adversely affected by flexibility, no drag reductions would be found despite a gain in the critical Reynolds number. A large improvement in this, however, which allowed the flow over the surface to remain entirely at sub-critical Reynolds numbers, would achieve the desired result.

The aeronautical application of flexible surfaces is perhaps not so attractive as was at first supposed. For as far as two-dimensional wings are concerned, flight experiments have shown that if the aerofoil

contour is accurately made and carefully maintained, the transition point can be delayed to the position of the pressure minimum without auxiliary means. Hence if flexible surfaces are found to extend laminar flow, they will make their main contribution in regions of zero pressure gradient or weak adverse gradient, for they are unlikely to be able to extend the laminar layer beyond the calculated separation point: for this purpose control by suction is required, since this, in maintaining boundary layer stable by thinning it, also defers the position of separation. Since laminar boundary layers only permit a relatively small pressure rise before separation, flexible surfaces would only be able to entirely replace suction in the case of wings of very small thickness/chord ratio, or more generally on axisymmetrical bodies where the super-velocities are much less than on wings and separation is confined to close to the tail. But in both cases it may be necessary to specify the absence of cross-flow as flexible surfaces may not be able to cope with the stationary vortex instabilities associated with three-dimensional boundary layers. This may prove a rather serious limitation.

In view of all these comments the authors propose only to continue the present experiment with lighter skins, although any flow mechanism that may be found may not be fully understood until the fundamental experiment is undertaken. The decision whether to proceed with this latter work at N.P.L. will be still further postponed until favourable results are obtained either from the present work or from the A.R.L. experiment on an axisymmetrical cylinder under water.

#### Acknowledgments

Acknowledgments are due for the helpful co-operation of Mr. L. Jenkins and the staff of the Dunlop Rubber Co. Research Centre in testing various foam materials and making the foam sandwiches.

Acknowledgments are also due for the assistance rendered in the experimental work by Miss S. Fermor and by Mr. T. W. Brown (vacation student).

APPENDIX/

#### APPENDIX

#### The Response of the Flexible Medium

The values of the basic elastic constants of a dissipative flexible medium obviously determine its response to a travelling pressure wave system (e.g., Tollmien-Schlichting waves), but the response is most simply expressed in terms of an effective stiffness and phase lag (or complex stiffness) for the disturbances in question. Further damping coefficients may need to be introduced to deal with systems whose amplitude of motion is not constant in both space and time.

For a homogeneous medium, the appropriate stiffnesses are in theory calculable from the basic elastic constants, but in practice, flexible surfaces are not of homogeneous construction, and are of such limited depth (introducing difficult boundary conditions), that direct determination of the appropriate stiffnesses or compliances appears to be necessary. However, it is not easy to simulate the travelling pressure wave system artificially. Fortunately, Boggs and Tokita have described a technique by which the surface compliances appropriate to the travelling pressure wave system may be inferred by means of Fourier transforms from the measured response of the skin to oscillations imposed by a single vibrating knife edge.

Knowledge of the appropriate stiffnesses or compliances is of course required for correlation with the boundary-layer stabilizing properties of the flexible surface which emerge from wind-tunnel experiment.

At the present stage in the experiments, when various forms of construction of flexible surfaces are being tried in order to cover a wide range of flexible surface properties in the attempt to find one that has marked stabilizing effects, it is helpful to consider the following approximate analysis which relates the response of the fabricated surface to various basic properties of its component parts for two-dimensional conditions.

A taut, thin but flexurally stiff elastic membrane is supported over a layer of foam material. For simplicity, we neglect the mass of the foam compared with that of the skin\*, the flexural stiffness of the foam compared with its stiffness in compression, and the curvature of the surface which, if present, will result in an initial foam layer of varying depth. The effects of rotary inertia of the outer skin have also been neglected in this theory. The following notation applies:

T = tension in outer skin, per unit span

m = mass of outer skin, per unit span and length

E = Young's modulus of outer skin

I = sectional moment of inertia of outer skin about transverse axis

=  $t^3/12$  per unit span

t = outer skin thickness

 $\eta$  = normal displacement of skin

 $\ell_0$  = undisturbed foam thickness

 $E_0$  = compression modulus of foam

 $E = E_0/\ell_0$ 

K = a damping coefficient for the foam.

Then/

<sup>\*</sup>Even when this is not true, the results might still be applicable if allowance can be made for the weight of foam by an alteration to its effective compression modulus.

Then, extending Brooke Benjamin's analysis, the equations governing the response of the flexible skin to an applied pressure  $p_S = f(x,t)$  is, within the approximations of the present analysis,

$$p_{S} = T \frac{\partial^{2} \eta}{\partial x^{2}} - m \frac{\partial^{2} \eta}{\partial t^{2}} - E_{1} I \frac{\partial^{4} \eta}{\partial x^{4}} - E \eta - K \frac{\partial \eta}{\partial t} \qquad ... (A1)$$

where the damping has been assumed to be viscous.

Under static conditions where a uniform pressure is applied over a large area so that  $\partial\eta/\partial t$  and  $\partial\eta/\partial x$  are both zero, we see that  $p_s=-E\eta$ , or the static stiffness, defined as the modulus of  $p_s/-\eta$ , is equal to E, as is obvious. This result also applies to a steady sinusoidal distribution of pressure if the tension and either  $E_1$  or I are also zero. In the general case where none of the three parameters is zero, however, if we take and the response  $\eta = ae^{ikx}$ , we see upon substitution in the equation that the effective stiffness is equal to E + Tk2 + E Ik4. In other words, as the wavelength of the disturbance is reduced and the wave number k increased, the deflection of the surface is reduced, and the stiffness is increasingly controlled by the tension in the skin; and eventually for the shortest waves of all, by the flexural stiffness of the This is illustrated by the figures given in the first 6 columns of Table 2 which relate to the foam sandwich with chordwise tension whose wind-tunnel tests are discussed in  $\S_{+}$ .1, and refer to its use in the buckled state with the lower value of  $E_0$ , the compression modulus of the foam. A wind-tunnel speed of 140 ft/sec has been assumed and a value of the tension has been taken that gives a free surface wave speed (Fig.4) that roughly matches the expected speed of Tollmien-Schlichting waves. Columns (viii) to (x) of Table 2 give the ratio of the effective modulus to the local dynamic pressure for the complete surface and its components. windspeed of the example, the dangerous wavelengths on a rigid surface would vary from ½ to 1 in. at 1 ft back from the leading edge to 1 to 3 in. at 5 ft back from the nose if laminar flow could be maintained to this point. These wavelengths are large enough so that the flexural stiffness of the thin rubber membrane adds a negligible contribution to the effective stiffness which is seen still to vary considerably with the wavelength on account of the dominance of the tension term, and which is probably too large near the leading edge (see below). If, however, it is not necessary to match the Tollmien-Schlichting wavespeed to the speed of free surface waves and the tension can be greatly reduced, the effective stiffness would then be much more nearly constant over the wavelength range of interest.

So far, the argument relates to the response of the surface to a stationary sinusoidal pressure distribution. For the case of a travelling system the response is still further modified by the damping in the foam.

If the applied distribution is  $p_s = \rho Pe^{ik(x-ct)}$  where c is the speed of propagation of the waves, and the response is  $\eta = ae^{ik(x-ct)}$  where a is complex, we see that  $\rho P_s = -aTk^2 + amk^2c^2 - ak^4E_1I - aE + aKikc$ , so that the dynamic stiffness, or ratio of pressure amplitude to deflection amplitude is equal to

$$\sqrt{(E + Tk^2 + E, Ik^4 - mk^2c^2)^2 + K^2k^2c^2}$$
.

This stiffness is composed of the sum of two essentially positive terms and is minimised by minimising each term separately. As far as the terms in brackets is concerned, this can readily be made to vanish, at least for any particular frequency, by suitably adjusting the tension on the skin. Column (vii) of Table 2 shows that the tension has been adjusted to make this term vanish for the longest wavelength of interest (3 in.). The dispersive

nature of the skin, illustrated in Fig.4, shows that for other frequencies the residue of the term in brackets is less than 50% of the value of Tk<sup>2</sup> (Table 2).

The value of Kkc can be determined from the decay of the waves propagating from an oscillating knife edge. Far away from the knife edge we might expect an oscillation of the form

$$ik_{1}(x - c_{1}t) - rx$$

$$\eta = ae$$

to satisfy the right-hand side of equation (A1) put equal to zero. This is so if

$$mk_{1}^{2}c_{1}^{2}-k_{1}^{2}T+r^{2}T-E-E_{1}Ik_{1}^{4}+6E_{1}Ik_{1}^{2}r^{2}-E_{1}Ir^{4}=0$$
 ... (A2)

and  $Kk_1 c_1 - 2Tk_1 r - 4E_1 Tk_1^3 r + 4E_1 Tk_1 r^3 = 0$ . (A3)

The second of these equations may be solved for r, and if  $E_1$  I may be neglected, the equations simplify and we see that

$$r = \frac{Kc_1}{2T} .$$

If D is the logarithmic decrement, the natural logarithm of the ratio of the amplitude of two successive waves, then

$$D = \frac{2\pi r}{k_1} = \frac{\pi K c_1}{T k_1} .$$

Hence, if the logarithmic decrement in the knife edge experiment is measured at the Tollmien-Schlichting frequency, so that  $k_1\,c_1=kc$ , we see that

$$\text{Kke} = \frac{\text{DTk}_1 \text{ ke}}{\pi c_1} = \frac{\text{DTk}^2 e^2}{\pi c_1^2} .$$

Although values of D were not determined in the original experiments, we can see that if the wave amplitude decays to about 1/10 of its value over the length of 3 waves, so that D = 0.75, then the value of Kkc for the tuned skin turns out to be small compared with E for 3 in. waves but comparable with E for the shorter waves. It thus appears that for the taut skin the considerable damping associated with foam backing somewhat limits the response of the skin to shorter waves at the resonant condition. If however the skin is untensioned, it can be seen from (A3) that

Kke 
$$\neq$$
  $2E_1 Ik^4 \left(\frac{c}{c_1}\right)^4 D/\pi$ 

unless the damping is very heavy. To the same approximation, equation (A2) suggests that the speed of free surface waves is given by

$$mk_1^2 c_1^2 = E + E_1 T c_1^4$$
. . . . (A4)

The skin is now dispersive, the speed of free waves depending markedly upon the wavelength. However, this relation does imply that the dynamic stiffness of the skin may be written as

$$\sqrt{\left(E_{\underline{1}} \operatorname{Ik}^{4} \left[1 - \left(\frac{c}{c_{\underline{1}}}\right)^{4}\right]^{2} + \left(2E_{\underline{1}} \operatorname{Ik}^{4} \left(\frac{c}{c_{\underline{1}}}\right)^{4} \frac{D}{\pi}\right)^{2}} .$$

This stiffness may again be kept small if and only if c is not greatly different from  $c_1$  for the waves of interest. This is not true for the present example. Equation (A4) shows that the condition requires E to be equal to  $mk_1^2c^2$  if  $E_1lk^4$  remains negligible in comparison, and implies a much stiffer foam backing than was used in the example of Table 2.

The discussion so far has had in mind a skin that conforms to Brooke Benjamin's ideas for a damped skin, i.e., one which can be tuned so that the speed of free surface waves matches the speed of Tollmien-Schlichting waves of those wavelengths which would be found over the corresponding rigid surface and shows the contributions to the effective stiffness of the skin arising from its various components - flexural stiffness of the outer skin, tension in outer skin, and compressive elasticity of a backing layer - and shows moreover that these three stiffnesses are proportional to the fourth, second and zeroth power of the wave number, respectively. The effective dynamic stiffness is increased by the damping, and indeed at the resonant condition is controlled entirely by the damping. The theory suggests, however, that the damping associated with the foam backing may be excessive and this is borne out by the minute amplitude of vibration observed, even at The theory will readily show the extent to which the stiffness may be reduced by alteration to the various parameters.

Turning to Nonweiler's ideas on the other hand, his more detailed analysis suggests that for the aeronautical application, a 'free' thin surface is required whose stiffness results entirely from its flexural stiffness. On this assumption, we discover that Nonweiler's criterion for the elimination of the Tollmien-Schlichting mode,

$$\theta^3 c_4^2 / \sigma u_\delta^2 < 2,000$$
 (Nonweiler's notation)

is roughly equivalent to

$$\frac{E_1 I}{\frac{1}{2} \rho u_{S}^2} < 1310 (\delta^*)^3$$

or for comparison with Table 2

$$\frac{E_1 \text{ Ik}^4 \ell_0}{\frac{1}{2} \rho U_1^8} < 1900 (\delta^*)^3 k^4 \ell_0.$$

The values of this maximum stiffness are given in columns (xi) and (xii) of this Table for  $\delta^*$  equal to 0.020 and 0.040 in. which are roughly the appropriate displacement thicknesses 1 ft and 4 ft downstream from the leading It will be seen that the flexural stiffness of the rubber outer skin of Table 2 is well below that demanded by Nonweiler's theory. outer skin if used alone could have been 5 to 10 times thicker, or made of a more rigid material. But used in conjunction with tension and backing the stiffnesses are low enough near the nose only for waves under  $\frac{1}{2}$  in., and further back only for waves shorter than 1 in. It is clear that the modulus of the foam backing would require to be less than  $\frac{1}{2}$  the dynamic pressure for the combined skin to satisfy the requirements for waves up to 2 in. length near the nose or 3 in. further back. As the Tollmien-Schlichting mode is eliminated according to theory, it is not obvious over what wavelength range it is necessary to satisfy the criterion. One might imagine that not satisfying it for long flexural waves might not affect the boundary layer stability at the point in question.

Presence of a backing to the outer skin is essential if the outer skin is to be wrapped over a cambered surface, and a value of the compression modulus  $E_0$  (or  $E\ell_0$ ) of about  $\frac{1}{2}\rho U^2$  has been thought to be necessary to avoid excessive variations in thickness of the layer due to the static pressure variation round the surface of the aerofoil. If these thickness variations can be permitted or allowed for, the minimum value of  $E_0$  can be considerably reduced. For in order to avoid the static bulge which has always appeared first in the experiments and for which linearized theory suggests the peak pressure coefficient is equal to -2ak, it is merely necessary to ensure that

$$E_0 \frac{a}{\ell_0} > 2ak \cdot \frac{1}{2} \rho U_1^2$$

i.e., 
$$\frac{\mathbb{E}\ell_{0}}{\frac{1}{2}\rho U_{1}^{2}} > 2k\ell_{0}.$$

Column (xiii) of Table 2 gives  $2k\ell_0$  and at 140 ft/sec it can be seen that this inequality is satisfied for the complete skin of the example\*. Comparison of the last three columns shows that a skin with flexural stiffness that just satisfies Nonweiler's criterion would only avoid static bulges 1 ft from the nose (A), with wavelength less than about  $1\frac{1}{2}$  in., and 4 ft from the nose (B), with wavelength less than about  $2\frac{1}{2}$  in. The addition of quite a small value of compression modulus  $E_0$  coupled with some reduction in the flexural stiffness of the outer skin would enable static bulges to be eliminated at all wavelengths, yet not greatly reduce the wavelength range over which Nonweiler's criterion is satisfied. If the flexural stiffness is reduced too much, the bulge inequality might not be satisfied over a limited range of the shorter wavelengths. This might not matter for wavelengths of the same order as the boundary layer thickness since the full pressure peak indicated by potential flow theory may not be realised owing to the effective amplitude of the bulges being reduced by the lagging oscillatory growth of the boundary layer.

The discussion thus suggests that on the basis of the present theories, more suitable combinations of parameters can be chosen for composite skins than were possessed by the skins already tested on the wind-tunnel model: it is hoped that correspondingly more favourable results will ensue.

References/

<sup>\*</sup>The skin actually tested was not bonded to the foam and hence could not withstand external suction forces when its internal pressure was not low enough to precompress the foam. If E is zero, the tension term alone does not satisfy the inequality for wavelengths greater than 10 in. This minimum wavelength is greater than the chord of the model (for the same tension) only at windspeeds less than 60 ft/sec, which is therefore the speed below which static bulges would not occur even without reduced internal pressure. These features were demonstrated in the experiments.

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Table 1

Variation with windspeed of ratio of compression modulus of unstretched foam sandwich to local dynamic pressure

Free stream windspeed Uo ft/sec	Mean velocity between 0.3 and 0.6 chord, U, ft/sec = 1.2 U,	Foam in stiff state* $\frac{E_0}{\frac{1}{2}\rho U_1^2}$	Foam in buckled state $\frac{E_0}{\frac{1}{2}\rho U_1^2}$		
40	40 48		18•5		
60	72	133	8.2		
90	108	59	3.6		
120	144	33	2.0		
140	<b>1</b> 68	24	1.5		
180	216	15	0.9		

$$E_o = 50 \text{ lb/sq ft.}$$

<sup>\*</sup>Stiff state given by a suction of 2 in. water:  $E_0 = 814$  lb/sq ft.

<sup>\*</sup>Buckled state given by a suction of 8 in. - 11 in. water:

Table 2 Variation of the effective stiffness of a foam sandwich with the wavelength of a sinusoidal pressure distribution (see Appendix)

(i)	(ii)	(iii)	(iv)	(∀)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)
Wavelength	k 2π	E	$\mathrm{Ll}^{\zeta}_{\mathbf{S}}$	E <sub>1</sub> Ik <sup>4</sup>	(E+Tk²+E <sub>1</sub> Ik⁴)	mk <sup>2</sup> c <sup>2</sup>	Tk² lo	E <sub>1</sub> Ik <sup>4</sup> l <sub>0</sub>	$(E+Tk^2+E_1 Ik^4)\ell_0$	E, Ik <sup>4</sup> l <sub>o</sub>		
λ in.	$=\frac{2N}{\lambda}$	× 10 <sup>-3</sup>	× 10 <sup>-3</sup>	× 10 <sup>-3</sup>	× 10 <sup>-3</sup>	× 10 <sup>-3</sup>	- <u>1</u> ρυ <sup>2</sup>	$\frac{1}{2}\rho U_1^2$		1 <sub>A</sub> 1	·βι 1Βι	2kl 0
0	8	2.4	<b>∞</b>	∞	∞	<b>«</b>						
0.25 in. 0.5 in. 1 in. 2 in. 3 in. 10 in. 20 in. 30 in. 60 in.	302 151 75 37.6 25 7.5 3.75 2.5 1.25	11 11 11 17 15 15	764 191 47.2 11.9 5.3 0.47 0.12 0.053 0.013	13.6 0.85 0.052 0.0033 0.00064 0.0000052	780 194 49•7 14•3 7•7 2•9		29 7•3 3•4		480 120 31 8.8 4.7 1.8	1500 95 5•8 0•36 0•0715 0•0005 <sub>8</sub>	12,200 760 46 2.9 0.57 0.0046	12 6 3 1.5 1.0 0.3 0.15 0.10 0.05
<b></b>	0	tt	0	0	2•4		0	0	1.5			0

Note:

- (ii)  $k = 2\pi/\lambda$  where  $\lambda$  is wavelength in ft.
- (iii)  $E = E_0/\ell_0$  where  $E_0$  of buckled foam is 50 lb/sq ft and  $\ell_0 = \frac{1}{4}$  in. = 0.0208 ft.
- (iv) T = 8.4 lb/ft span in order to match approximately the Tollmien-Schlichting wavespeed, c = 84 ft/sec appropriate for  $U_0 = 140$  ft/sec.

  (v) E<sub>1</sub> rubber = 15.4 × 10<sup>3</sup> lb/sq ft.

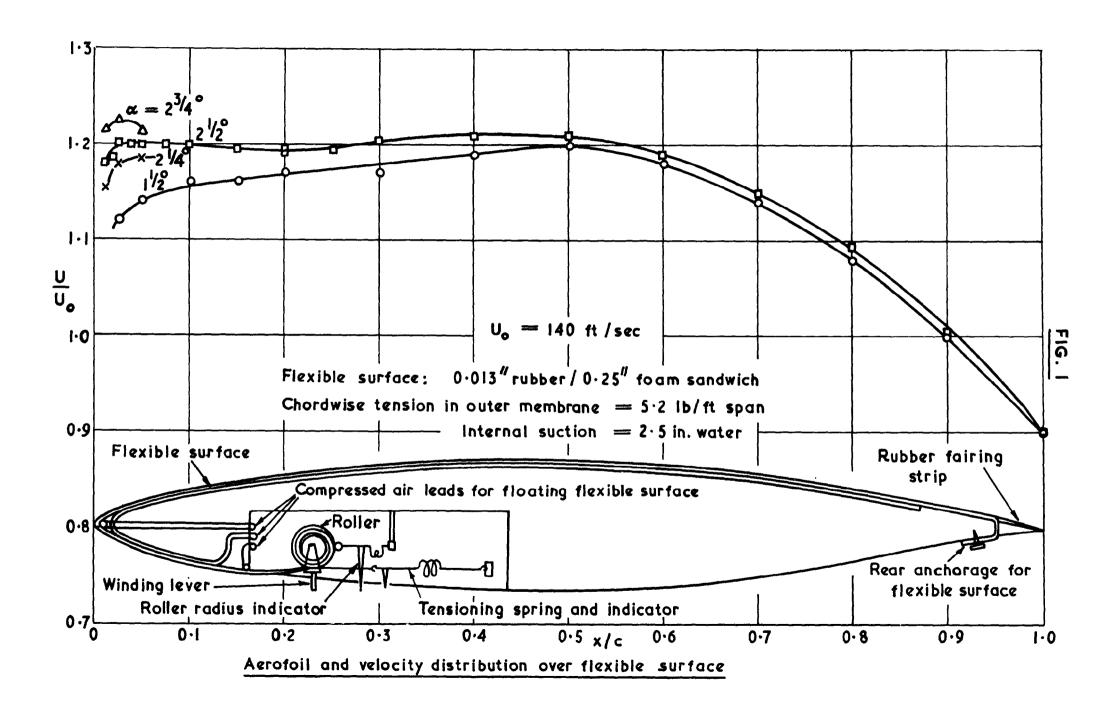
  I =  $t^3/12$  where t = skin thickness = 0.013 in. = 0.00108 ft.

  (vii) m = skin mass = 0.055 lb/sq ft = 1.72 × 10<sup>-3</sup> slugs/sq ft.

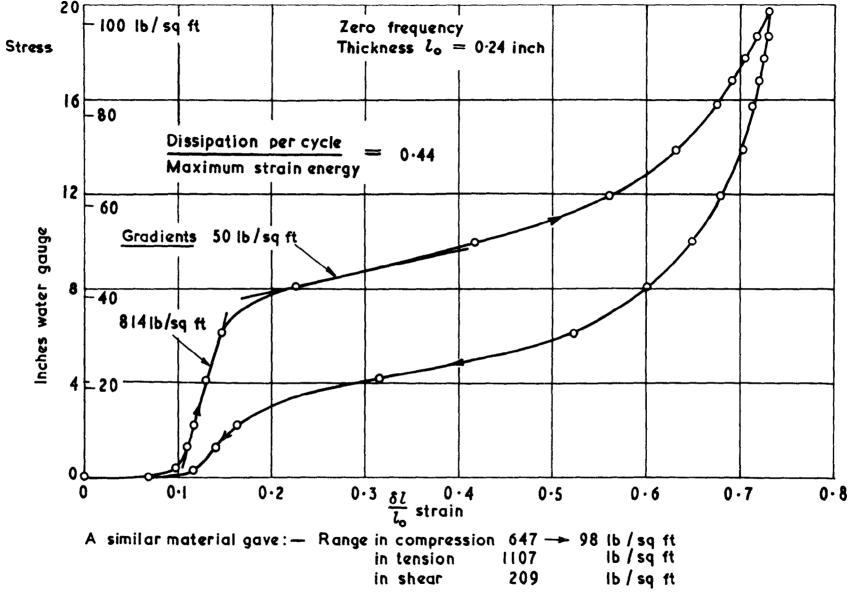
- (viii-xii)  $U_1 = 1.2 U_0$ ,  $U_0 = 140 \text{ ft/sec}$ ,  $\frac{1}{2}\rho U_1^2 = 33.8 \text{ lb/sq ft}$ .

  (xi) 'A' Nonweiler's criterion (= 1900  $(\delta^*)^3 k^4 \ell_0$ ) for  $\delta^* = 0.020$  in. as appropriate 1 ft from leading edge.

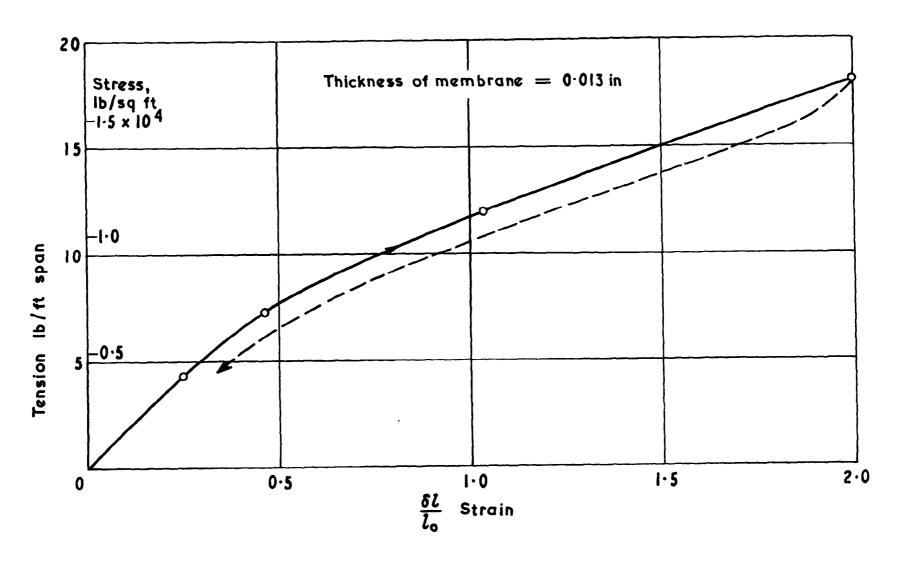
  (xii) 'B' Nonweiler's criterion for  $\delta^* = 0.040$  in. as appropriate 4 ft from leading edge.



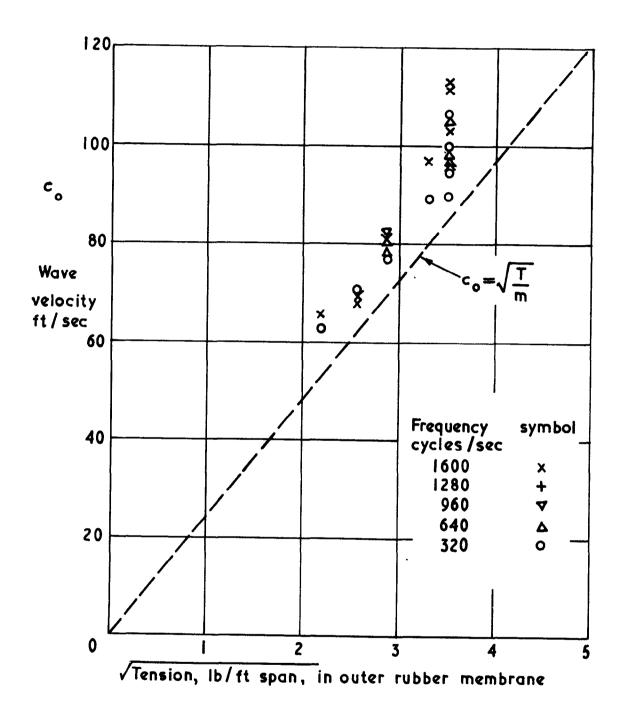




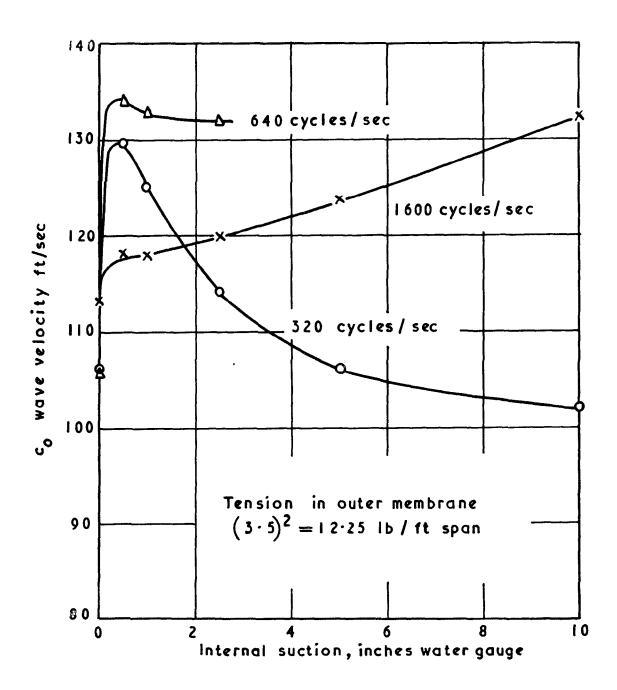
Stress-strain curves for polyurethane foam in compression



Stress ~ strain curves for rubber surface membrane in tension

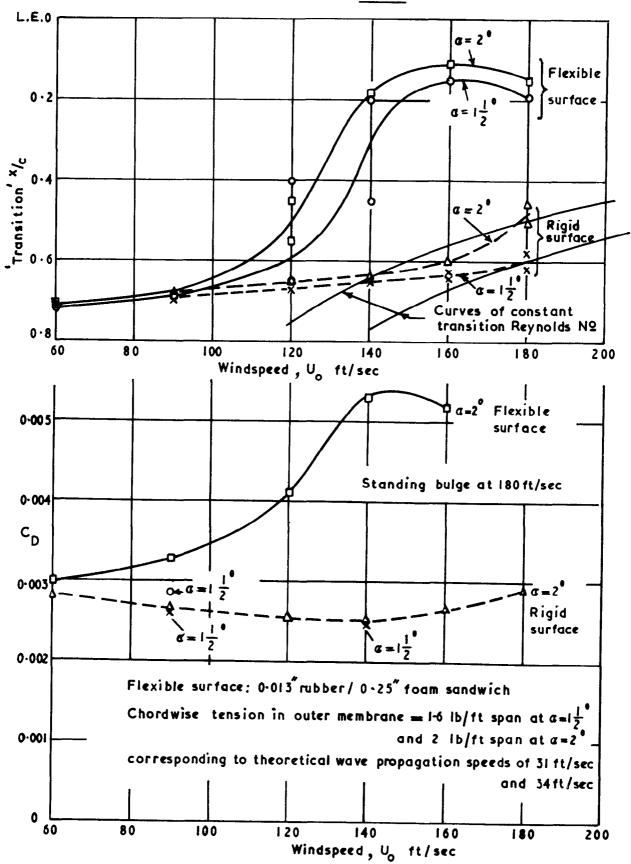


Variation of speed of free surface waves with tension and frequency

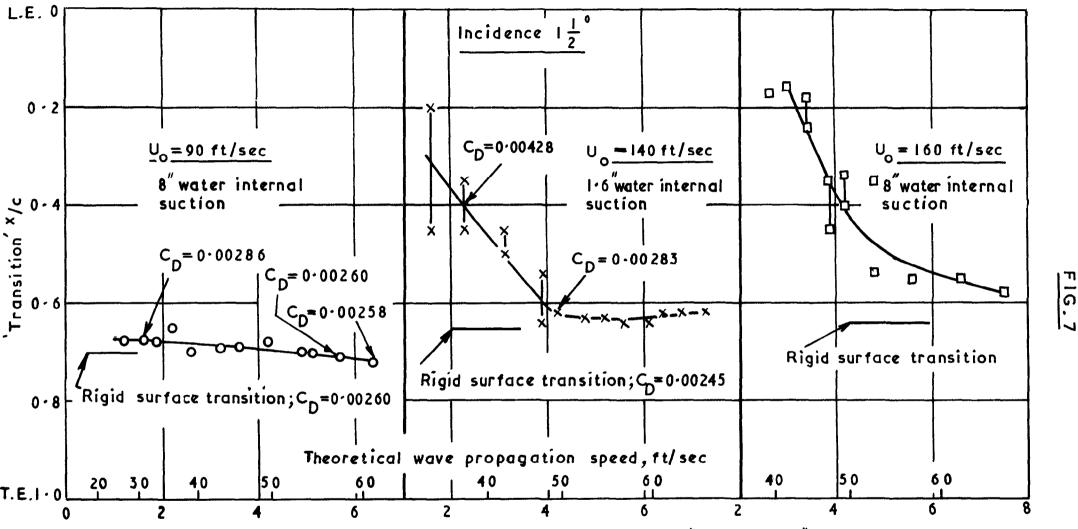


Effect of distributed load on free surface wave speed.



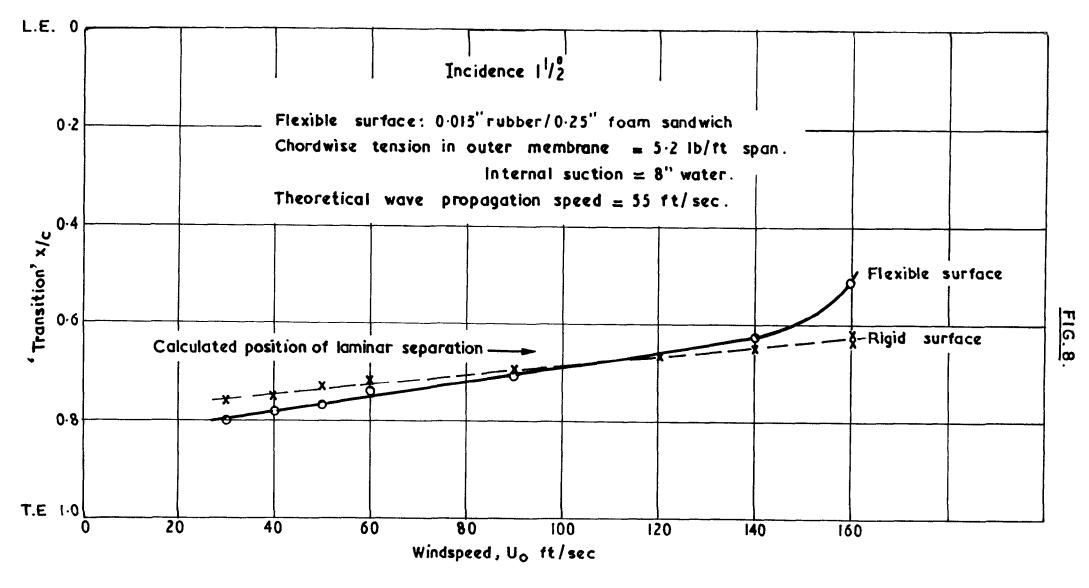


Variation of transition and drag coefficient with windspeed.



Chordwise tension, 1b/ft span, in outer rubber membrane of 0.013" rubber/0.25" foam sandwich

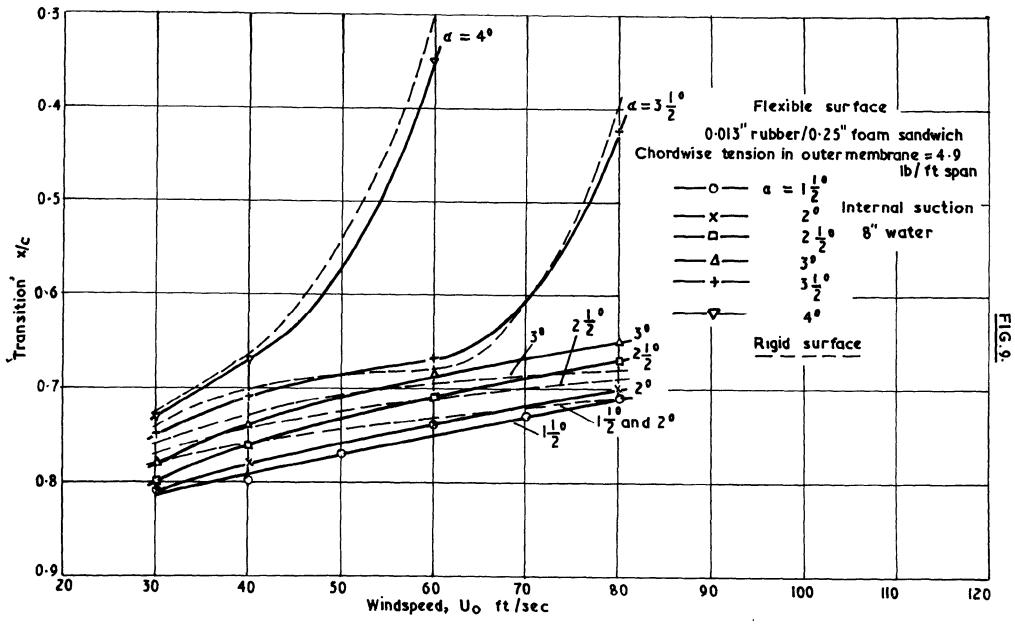
Variation of transition with tension in flexible surface at  $1\frac{1}{2}$  incidence.



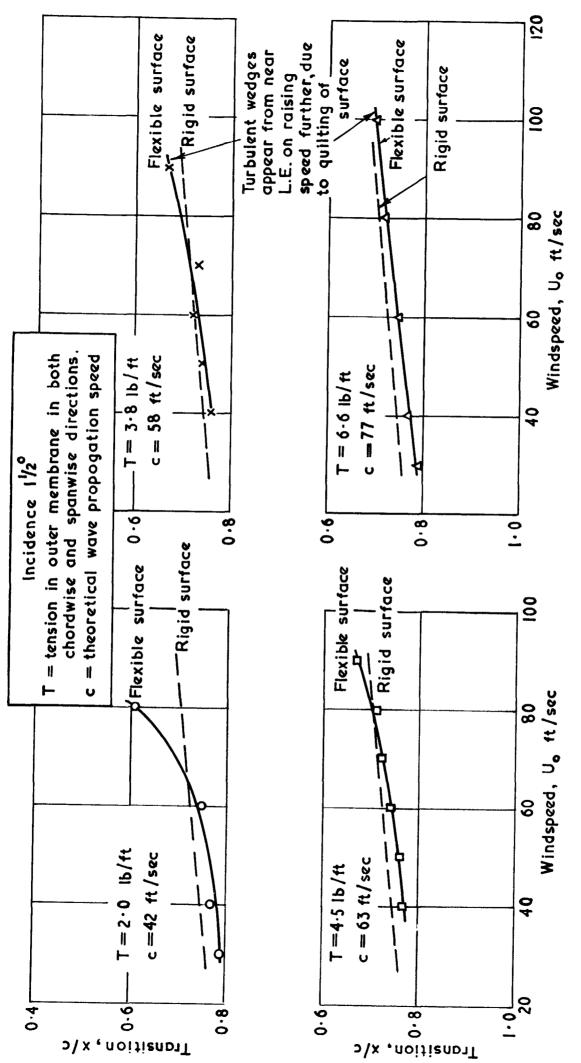
Variation of 'transition' with windspeed at 11/2° incidence

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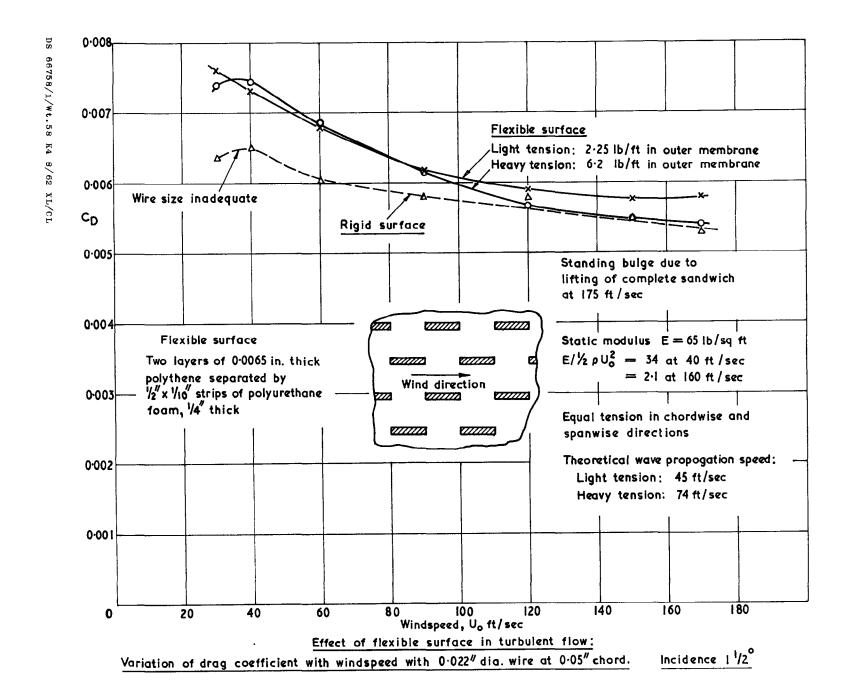


Variation of 'transition' with windspeed at constant incidence



Variation of transition with windspeed for studded polythene sandwich under various tensions





A.R.C. C.P. No.602. December, 1961 Gregory. N. and Love, Edna M. - Nat. Phys. Lab.

PROGRESS REPORT ON AN EXPERIMENT ON THE EFFECT OF SURFACE FLEXIBILITY ON THE STABILITY OF LAMINAR FLOW

This paper describes the flexible surfaces whose properties have been examined and which have been tested in the wind tunnel. The limitations of the experiment are discussed in the light of the inconclusive results so far obtained both for the information of others, and in order to clarify the next steps to be taken.

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