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Comparative Calculations of Supersonic Pitching Derivatives over a Range of Frequency Parameter

By

H.C. Garner, M.A., A.F.R.Ae.S.
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of the Aerodynamics Division, N.P.L.

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April, 1961

SUMMARY

With the object of checking and comparing methods used by the aircraft industry to obtain supersonic flutter derivatives, calculations have been made for pitching and plunging modes at infinitesimal and finite frequencies. For both a symmetrically tapered planform of aspect ratio 4.329 and a modified one with a leading side-edge, generally satisfactory comparisons are found at Mach numbers of 1.155, 1.414 and 2 over a range of frequency parameter.

For finite frequencies the Pegasus programme used by Hawker Aircraft Ltd., is found to be marginally preferable to the BAC 11 programme for DEUCE. The effect of frequency is very small at M = 2 but becomes important at M = 1.155.

Comparison with low-frequency experiment shows that linearized theory for infinitesimal frequency predicts direct pitching derivatives in fair agreement. Simple corrections for aerofoil thickness have been applied, and in all cases these corrections improve the correlation between calculated and measured values.

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1. Introduction

A review of the flutter research programme under the auspices of the M.O.S. Flutter and Vibration Committee in 1959 included as one recommendation the evaluation of pitching and plunging derivatives for wings in supersonic flow. These calculations had previously been recommended by a former aerodynamic loading panel of the A.R.C. Computation Sub-Committee, in order to check and compare various mechanized computing methods based on linearized supersonic theory for infinitesimal and finite frequency.

The two wings selected for the present investigation are denoted as Wings E and F (Fig. 1). These planforms are included in the series of pitching-moment derivative measurements made at the N.P.L.[1] for low frequency ($\nu_r = 0.03$) and supersonic Mach numbers ($1.38 < M < 2.47$).

The theoretical methods considered are based on the analytical treatment given by Stewartson[2] for the loading in the region of a streamwise or subsonic tip of an oscillating wing. The method of calculation developed by Hunt[3] for finite frequencies can be applied to wings of arbitrary planform provided that the supersonic speed is sufficiently high. The method has been mechanized for the DEUCE by Bristol Aircraft Ltd.[4] as programme BAC 11 and for the Pegasus computer by Hawker Aircraft Ltd. (unpublished): for the case of infinitesimal frequency, Hawker Aircraft Ltd. have developed a distinct Pegasus programme (unpublished). Exact theoretical solutions to first order in frequency have been evaluated at the N.P.L.[5] from algebraic expressions for the velocity potential.

The method of Ref. 3 has greatly extended the types of planform and range of frequency for which supersonic flutter derivatives can readily be calculated. The most severe limitation is the restriction to Mach number such that an inboard portion of the leading edge and the whole trailing edge must be supersonic. A general restriction in the basic theory[2] precludes all cases where there is interaction between subsonic tips. There are also certain restrictions to the non-dimensional parameters accepted by a computing programme, e.g., Section 5 of Ref. 4; corresponding details of the Pegasus programmes have not been published. Not mentioned in Ref. 4 is the further limitation on Mach number such that no part of the wing is affected by both subsonic tips; this leads to the restrictions $M \geq 1.102$ for Wing E and $M \geq 1.235$ for Wing F. The calculations for Wing F at $M = \frac{2}{3}\sqrt{3} = 1.155$ are therefore subject to errors which, however, are considered to be fairly small (Section 4).

The calculations of the pitching and plunging derivatives for Wings E and F at Mach numbers $M = \frac{2}{3}\sqrt{3}, \sqrt{2}$ and 2 were carried out as follows.

Method (i) The derivatives by exact theory for infinitesimal frequency were evaluated in the Aerodynamics Division, N.P.L.

Method (ii) Results by the DEUCE programme BAC 11 for finite frequencies were obtained with the co-operation of the Mathematics Division, N.P.L.

Method (iii) The application of the Pegasus programme for infinitesimal frequency was carried out by Hawker Aircraft Ltd., for wing E.

Method (iv)/

Method (iv) The application of the Pegasus programme for finite frequency was carried out by Hawker Aircraft Ltd.

Method (v) One of the calculations by method (ii) ($N = 8$) was repeated with a finer mesh ($N = 12$).

The calculations by Hawker Aircraft Ltd. were covered by a M.O.S. contract.

The theoretical results are described in Section 3 and discussed in Section 4. Comparisons with the experiments of Ref. 1 are made in Section 5, where allowance for aerofoil thickness (Appendix C of Ref. 5) is also considered.

2. Notation

For a wing describing plunging and pitching oscillations, such that the upward displacement of a point (x, y) on the planform is

$$z = -[z_0 + (x - hc_r)\theta_0]e^{i\omega t},$$

the total lift and pitching moment are written as

$$\left.\begin{aligned} L &= [(\ell_z + i\nu_r\ell_{\dot z})(z_0/c_r) + (\ell_\theta + i\nu_r\ell_{\dot\theta})\theta_0]\rho U^2 S e^{i\omega t} \\ \mathcal{M} &= [(m_z + i\nu_r m_{\dot z})(z_0/c_r) + (m_\theta + i\nu_r m_{\dot\theta})\theta_0]\rho U^2 S c_r e^{i\omega t} \end{aligned}\right\} .$$

Other symbols are defined below.

c_r	root chord
h	value of x/c_r along pitching axis
ℓ_z, $\ell_{\dot z}$	direct plunging derivatives
ℓ_θ, $\ell_{\dot\theta}$	cross derivatives (lift due to pitching)
m_z, $m_{\dot z}$	cross derivatives (pitching moment due to plunging)
m_θ, $m_{\dot\theta}$	direct pitching derivatives
M	Mach number [= U/(speed of sound)]
N	(geometric mean chord)/(chordwise diagonal of mesh) as in Fig. 2
S	area of planform
t	time
U	velocity of free stream
x, y	co-ordinates defined in Fig. 1
z_0	amplitude of plunging motion

3. Results

The plunging and pitching derivatives have been evaluated by methods (i) to (iv) for Wings E and F defined in Fig. 1. The calculations cover the values of frequency parameter $\nu_r = 0(0.15)0.60$ for the Mach numbers $M = \frac{2}{3}\sqrt{3}(= 1.155), \sqrt{2}(= 1.414)$ and 2; in the case of Wing F these Mach numbers correspond respectively to subsonic, sonic and supersonic tips. The derivatives are presented as functions of the pitching axis $x = hc_r$; the results for Wings E and F are given in Tables 1 and 2 respectively.

The results denoted as method (ii) have been obtained by applying the BAC 11 programme with a mesh $N = 8$ based on the geometric mean chord as standard length. The effect of mesh size was investigated in one calculation with $N = 12$ for Wing F in the case $M = \frac{2}{3}\sqrt{3}$, $\nu_r = 0.15$; the meshes are illustrated in Fig. 2. The plunging and pitching derivatives obtained with $N = 12$ are denoted as method (v) in Table 2.

It will be seen from Tables 1 and 2 that at $M = 2$ none of the derivatives is influenced much by the change in planform from Wing E to Wing F. Larger effects are found at the lower Mach numbers due to the more extensive influence of the tips. The results for finite values of ν_r show that the effect of frequency is likewise small at $M = 2$ but becomes increasingly important as M decreases. To illustrate the effect of ν_r for Wing E at $M = \frac{2}{3}\sqrt{3}$, the derivatives $-m_\theta$ and $-m_{\dot\theta}$ are plotted in Fig. 3 against axis position for $\nu_r \to 0$ and $\nu_r = 0.30$, 0.45 and 0.60. Although some negative damping persists for $h < 0.37$ at the highest frequency, these curves show a consistent gain in damping as ν_r increases provided that $h < 0.7$. The minimum value of $-m_{\dot\theta}$ increases from -0.76 to -0.12 over the range of ν_r from 0 to 0.6.

4. Comparison of Methods

The pitching derivatives for $h = 0.5$ obtained for Wings E and F by methods (i) to (v) are plotted against ν_r in Figs. 4 to 7. For finite frequency $(\nu_r \neq 0)$, the results obtained by methods (ii) and (iv) compare satisfactorily over the range $0.15 \leqslant \nu_r \leqslant 0.60$. Agreement at $M = 2$ is very good, but the discrepancies are more significant at the lower Mach numbers. These discrepancies show little variation with frequency parameter. No calculations were made by method (iv) for Wing F at $M = \frac{2}{3}\sqrt{3}$, but for the higher Mach numbers the discrepancies between the two methods are not influenced by the change in planform near the tip.

For infinitesimal frequency $(\nu_r \to 0)$, the mid-chord pitching derivatives obtained for Wing E by method (iii) are in good agreement with exact theory. In Figs. 4 to 7 the results by method (iii) $(\nu_r \to 0)$ and method (iv) $(\nu_r = 0.15)$ are linked by dotted curves which indicate a fair correlation between the two distinct Hawker Pegasus programmes; the derivative $-m_\theta$ is least satisfactory in this respect, partly due to the large scale of Fig. 6. For Wing F, a similar correlation is seen between the results of exact theory $(\nu_r \to 0)$ and method (iv) $(\nu_r = 0.15)$.

Since/

Since exact theory is intractable for finite ν_r, the accuracy of the methods (ii) and (iv) cannot really be assessed. Some indication that the errors may be small is given by the following application of the reverse-flow theorem[6]. In the case of Wing E, the planform is identical in direct or reverse flow; it follows from Ref. 6 that exact solutions for the derivatives about the axis h = 0 would satisfy the relations

$$\left.\begin{aligned} -\ell_\theta + \ell_z + m_z + \ell_{\dot z} &= 0 \\ -\ell_{\dot\theta} + \ell_{\dot z} + m_{\dot z} - (\ell_z/\nu_r^2) &= 0 \end{aligned}\right\} \quad \ldots(1)$$

In the notation of Table 1, the left-hand sides of equations (1) may be written as

$$\left.\begin{aligned} \epsilon_1 &= B + D + F \\ \epsilon_2 &= (D + H) - (B/\nu_r^2) \end{aligned}\right\} \quad \ldots(2)$$

If the values given in Table 1 are inserted, then ϵ_1 and ϵ_2 give the errors in the quantities in equation (1). It is found that the error ϵ_1 is trivial for both methods (ii) and (iv); $|\epsilon_1|$ is greatest for $M = \frac{2}{3}\sqrt{3}$ when it does not exceed 0.005B. The error ϵ_2 does not vary consistently with M nor with ν_r. Its root-mean-square value is 0.0031 for method (ii) and 0.0040 for method (iv), which are less than the actual discrepancies between the two methods in Table 1. Thus the application of the reverse-flow theorem for Wing E does not reveal any major errors.

The effect of mesh size (Fig.2) in the BAC 11 programme has been investigated for Wing F at $M = \frac{2}{3}\sqrt{3}$ and $\nu_r = 0.15$. As mentioned in Section 1, these calculations are subject to error; the loading over the small region of the wing, shaded in Fig. 2, is inadequately treated by Ref. 4. An upper limit to the error has been calculated for $\nu_r \longrightarrow 0$ by means of the integrated functions in Appendix B of Ref. 5; the derivatives at worst involve errors in the third decimal place. The effect of mesh size may hardly be influenced thereby, and it is therefore worth including in Table 2 the results for two mesh sizes N = 8 and N = 12, denoted as methods (ii) and (v) respectively. The plunging and pitching derivatives for Wing F with axis h = 0 are altered by less than 1% when the mesh size is reduced. The numerically smaller values of $-m_\theta$ and $-m_{\dot\theta}$ about mid-chord axis for N = 8 correlate quite well with exact theory ($\nu_r \longrightarrow 0$), but it seems that a better correlation is obtained in Figs. 6 and 7 with the results (Δ) based on the finer mesh.

Inspection of the curves for ℓ_θ and $\ell_{\dot\theta}$ in Figs. 4 and 5 indicates that for each Mach number, the average of the results of methods (ii) and (iv) would correlate best with exact theory ($\nu_r \longrightarrow 0$). For $-m_\theta$ and $-m_{\dot\theta}$ in Figs. 6 and 7, method (iv) consistently shows the better correlation with the exact values for $\nu_r \longrightarrow 0$.

5. Experiment and Thickness Effect

Pitching-moment derivatives have been measured[1] in the N.P.L. 11 in. Supersonic Wind Tunnel for a family of half-wing models including Wings E and F. The experiments have been carried out by a free-oscillation technique and cover the range of Mach number $1.38 < M < 2.47$. The oscillations correspond to low values of the

frequency/

frequency parameter, $\nu_r < 0.03$, and a mean amplitude of $1°$. Use will be made of the results for Wing E which was tested for the three pitching axes h = 0.4, 0.5 and 0.6.

The models used in the tests had a basic 5% double-wedge section. An estimate of thickness effect when $\nu_r \to 0$ was made by applying Van Dyke's[7] two-dimensional aerofoil theory on a strip-theory basis, as formulated in Appendix C of Ref. 5. The thickness corrections are not large, but it is shown in Refs. 1 and 5 that they improve significantly the comparison between exact theory $(\nu_r \to 0)$ and experiment. In the case of Wing E, the thickness corrections to the derivatives $-m_\theta$ and $-m_{\dot\theta}$ are illustrated in Figs. 8 and 9 respectively for the Mach number range $1.2 \leq M \leq 2.2$.

The thickness correction to $-m_\theta$ is independent of h; the agreement between the calculated and measured values in Fig. 8 is enhanced for all values of M. Although the use of strip theory will become less and less accurate as Mach number decreases, it is clear that thickness effect becomes increasingly important. It is noteworthy that for each axis position the experimental points in Fig. 8 lie roughly parallel to the theoretical curve with thickness correction. In Fig. 9 exact theory gives values of $-m_{\dot\theta}$ that compare quite satisfactorily with experiment for the higher values of M; even so, the thickness correction gives a distinct improvement. The effect of thickness on $-m_{\dot\theta}$ is of major significance at the lower values of M; for M = 1.38, the thickness correction and the discrepancy between theory and experiment are both greatest for the rearmost axis h = 0.6. Values of $-m_\theta$ and $-m_{\dot\theta}$ at $M = \sqrt{2}$ are plotted against h in Fig. 10. Comparison with experimental values (interpolated for $M = \sqrt{2}$) further illustrates the merit of the thickness corrections; for each derivative the variation with axis position is similar to that measured.

In relation to experimental scatter which is of order ±0.01 (Figs. 8 and 9), the estimated effect of frequency on the derivatives $-m_\theta$ and $-m_{\dot\theta}$ is negligible when M = 2; when $M = \sqrt{2}$, it only becomes important if the frequency parameter ν_r exceeds about 0.45 (Figs. 6 and 7).

6. Conclusions

(1) For infinitesimal and finite frequencies alike the various methods of calculating pitching and plunging derivatives compare satisfactorily with each other, whether M = 1.155, 1.414 or 2. When the derivative m_θ is small, the two sets of results cannot be linked by a realistic curve against frequency parameter (see dotted portions in Fig. 6).

(2) The various comparisons are scarcely affected by cropping the wing tip to form a leading side-edge. For finite frequencies the Pegasus programme used by Hawker Aircraft Ltd., is marginally preferable to the BAC 11 programme for DEUCE; a single calculation by the latter programme with a finer mesh supports this conclusion. (See Figs.6 and 7.)

(3) Effects of frequency and wing thickness are very small at M = 2, but both increase as Mach number decreases and become of major importance at M = 1.155. Increases in either frequency or thickness reduce the range of axis position for which negative pitching damping is calculated.

(4) Linearized theory for infinitesimal frequency predicts direct pitching derivatives in fair agreement with experimental values obtained in the speed range $1.38 \leq M \leq 2.47$. In all the cases considered, corrections for aerofoil thickness improve significantly the comparisons between calculated and measured values.

References/

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Table 1/

Table 1

Calculated Pitching and Plunging Derivatives for Wing E

Method	M	ν_r	A	B	C	D	E	F	G	H
(i)	1.155	$\longrightarrow 0$	3.0271	0	− 1.2496	3.0271	1.4441	− 3.0271	− 0.7551	− 0.1945
(iii)		$\longrightarrow 0$	3.0326	0	− 1.2551	3.0326	1.4492	− 3.0326	− 0.7605	− 0.1941
(ii)		0.15	2.9751	0.0618	− 1.2011	2.9502	1.4236	− 3.0116	− 0.7193	− 0.2061
(iv)		0.15	3.0004	0.0626	− 1.2125	2.9743	1.4270	− 3.0372	− 0.7169	− 0.1973
(ii)		0.30	2.7983	0.2196	− 0.9808	2.7063	1.3069	− 2.9248	− 0.5626	− 0.2661
(iv)		0.30	2.8206	0.2220	− 0.9816	2.7245	1.3080	− 2.9475	− 0.5513	− 0.2637
(ii)		0.45	2.5636	0.4080	− 0.6707	2.3817	1.1569	− 2.7880	− 0.3453	− 0.3699
(ii)		0.60	2.3333	0.5587	− 0.3346	2.0617	1.0184	− 2.6187	− 0.1159	− 0.5144
(iv)		0.60	2.3593	0.5576	− 0.3174	2.0788	1.0239	− 2.6384	− 0.0901	− 0.5323
(i)	1.414	$\longrightarrow 0$	1.8928	0	0.3518	1.8928	0.9290	− 1.8928	0.1961	− 1.2808
(iii)		$\longrightarrow 0$	1.8964	0	0.3525	1.8964	0.9323	− 1.8964	0.1968	− 1.2848
(ii)		0.15	1.8807	0.0134	0.3524	1.8755	0.9329	− 1.8888	0.1985	− 1.2818
(iv)		0.15	1.9020	0.0142	0.3342	1.8963	0.9401	− 1.9106	0.1902	− 1.2706
(ii)		0.30	1.8671	0.0523	0.3600	1.8466	0.9234	− 1.8986	0.2041	− 1.2699
(iv)		0.30	1.8876	0.0550	0.3428	1.8653	0.9299	− 1.9207	0.1966	− 1.2576
(iv)		0.45	1.8399	0.1163	0.3513	1.7922	0.9001	− 1.9090	0.2032	− 1.2199
(ii)		0.60	1.8175	0.1869	0.3884	1.7413	0.8892	− 1.9271	0.2250	− 1.2278
(iv)		0.60	1.8353	0.1951	0.3745	1.7529	0.8935	− 1.9493	0.2202	− 1.2140

Table 1 continued/

Table 1 continued

Method	M	ν_r	A	B	C	D	E	F	G	H
(i)	2	$\to 0$	1.1270	0	0.4455	1.1270	0.5592	- 1.1270	0.2598	- 1.0047
(iii)		$\to 0$	1.1289	0	0.4472	1.1289	0.5610	- 1.1289	0.2614	- 1.0082
(ii)		0.15	1.1227	0.0027	0.4409	1.1217	0.5624	- 1.1243	0.2582	- 1.0026
(iv)		0.15	1.1260	0.0030	0.4339	1.1248	0.5639	- 1.1278	0.2574	- 0.9970
(ii)		0.30	1.1218	0.0107	0.4411	1.1178	0.5617	- 1.1283	0.2584	- 1.0002
(iv)		0.30	1.1255	0.0117	0.4342	1.1206	0.5634	- 1.1325	0.2576	- 0.9944
(ii)		0.60	1.1184	0.0409	0.4421	1.1028	0.5592	- 1.1429	0.2592	- 0.9910
(iv)		0.60	1.1233	0.0442	0.4352	1.1045	0.5616	- 1.1496	0.2584	- 0.9844

Methods of calculation

(i) Exact theory for $\nu_r \to 0$ (Ref.5)

(ii) Bristol programme BAC 11 (N = 8)

(iii) Hawker programme for $\nu_r \to 0$

(iv) Hawker programme for $\nu_r \neq 0$

Derivatives for pitching axis $x = hc_r$

$\ell_z = B \qquad \ell_\theta = A - Bh$

$\ell_{\dot z} = D \qquad \ell_{\dot\theta} = C - Dh$

$m_z = (A + F) + Bh \qquad -m_\theta = E + Fh + Bh^2$

$m_{\dot z} = (C + H) + Dh \qquad -m_{\dot\theta} = G + Hh + Dh^2$

Table 2/

Table 2

Calculated Pitching and Plunging Derivatives for Wing F

Method	M	ν_r	A	B	C	D	E	F	G	H
(i)	1.155	$\rightarrow 0$	3.0452	0	- 1.1841	3.0452	1.4591	- 3.0452	- 0.6920	- 0.2750
(ii)		0.15	2.9630	0.0601	- 1.1137	2.9384	1.4184	- 2.9981	- 0.6418	- 0.2885
(v)		0.15	2.9718	0.0603	- 1.1086	2.9470	1.4182	- 3.0069	- 0.6359	- 0.2915
(ii)		0.30	2.7889	0.2132	- 0.8906	2.6984	1.3044	- 2.9100	- 0.4834	- 0.3550
(ii)		0.45	2.5609	0.3935	- 0.5783	2.3821	1.1605	- 2.7731	- 0.2654	- 0.4680
(ii)		0.60	2.3420	0.5346	- 0.2434	2.0753	1.0318	- 2.6077	- 0.0383	- 0.6226
(i)	1.414	$\rightarrow 0$	1.9349	0	0.3611	1.9349	0.9585	- 1.9349	0.2071	- 1.3196
(ii)		0.15	1.9194	0.0139	0.3588	1.9140	0.9590	- 1.9278	0.2067	- 1.3141
(iv)		0.15	1.9383	0.0146	0.3461	1.9325	0.9647	- 1.9471	0.2019	- 1.3070
(ii)		0.30	1.9051	0.0539	0.3669	1.8839	0.9490	- 1.9375	0.2128	- 1.3019
(iv)		0.30	1.9232	0.0562	0.3551	1.9004	0.9541	- 1.9570	0.2086	- 1.2940
(ii)		0.60	1.8528	0.1921	0.3972	1.7745	0.9130	- 1.9656	0.2352	- 1.2588
(iv)		0.60	1.8686	0.1989	0.3885	1.7845	0.9162	- 1.9849	0.2335	- 1.2494
(i)	2	$\rightarrow 0$	1.1404	0	0.4532	1.1404	0.5712	- 1.1404	0.2680	- 1.0244
(ii)		0.15	1.1329	0.0027	0.4476	1.1319	0.5719	- 1.1346	0.2655	- 1.0188
(iv)		0.15	1.1381	0.0030	0.4445	1.1368	0.5737	- 1.1399	0.2664	- 1.0173
(ii)		0.30	1.1320	0.0110	0.4479	1.1278	0.5712	- 1.1386	0.2658	- 1.0163
(iv)		0.30	1.1374	0.0117	0.4448	1.1325	0.5731	- 1.1445	0.2667	- 1.0146
(ii)		0.60	1.1282	0.0418	0.4490	1.1122	0.5684	- 1.1534	0.2666	- 1.0067
(iv)		0.60	1.1345	0.0445	0.4460	1.1158	0.5707	- 1.1612	0.2676	- 1.0043

Table 2 continued/

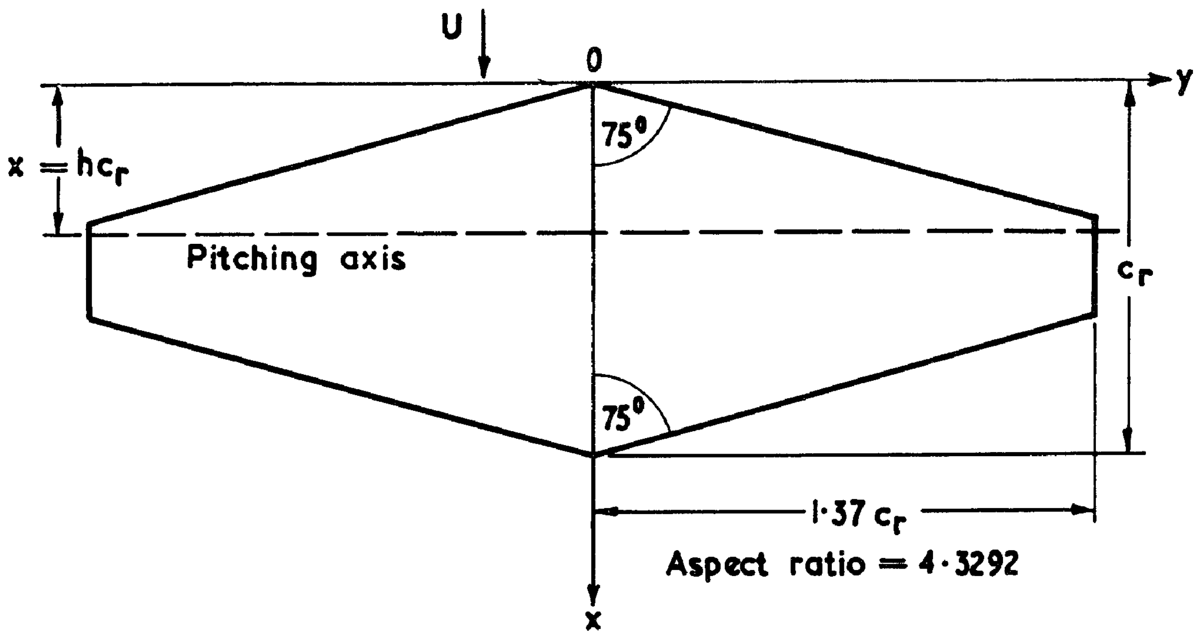
Table 2 continued

Methods of calculation	Derivatives for pitching axis $x = hc_r$	
(i) Exact theory for $\nu_r \to 0$ (Ref. 5)	$\ell_z = B$	$\ell_\theta = A - Bh$
(ii) Bristol programme BAC 11 ($N = 8$)	$\ell_{\dot{z}} = D$	$\ell_{\dot{\theta}} = C - Dh$
(iv) Hawker programme for $\nu_r \neq 0$	$m_z = (A + F) + Bh$	$-m_\theta = E + Fh + Bh^2$
(v) Bristol programme BAC 11 ($N = 12$)	$m_{\dot{z}} = (C + H) + Dh$	$-m_{\dot{\theta}} = G + Hh + Dh^2$

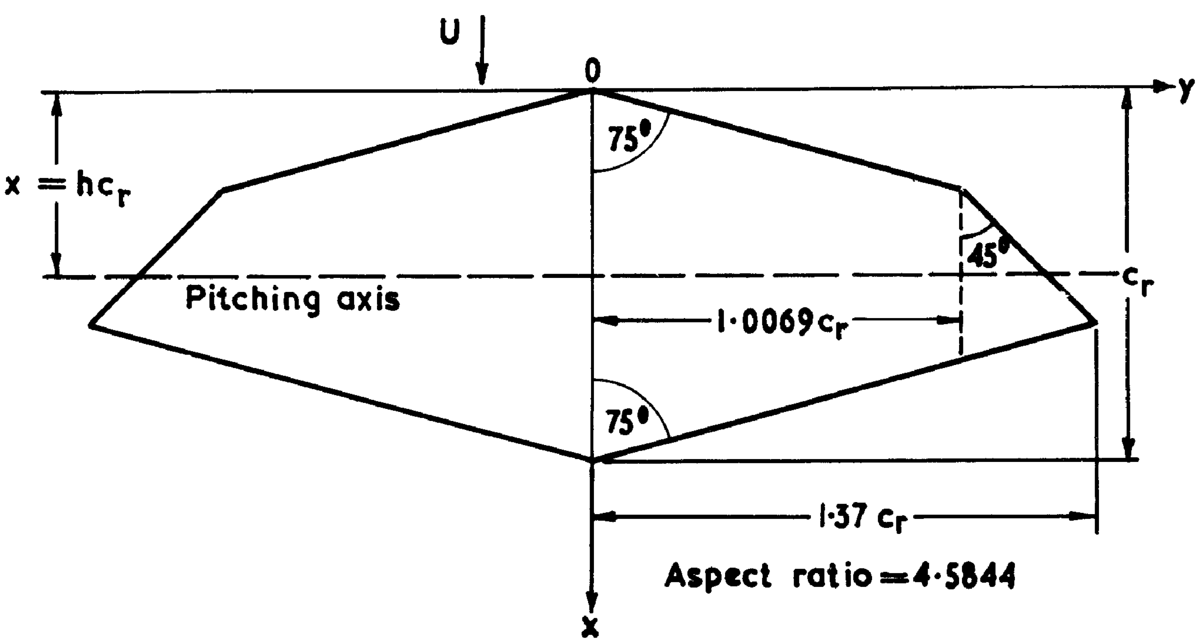
HD
CF

FIG. 1

WING E

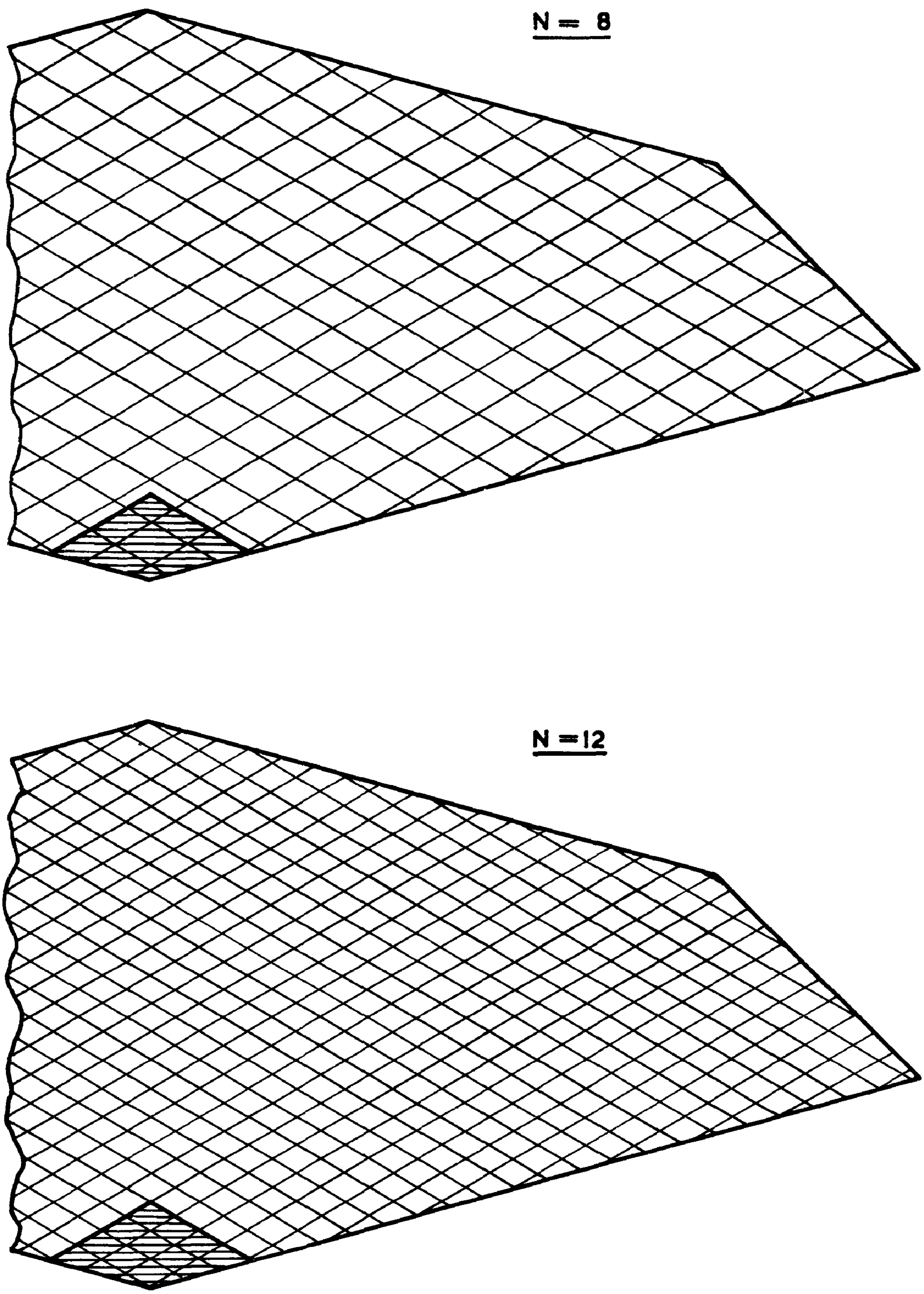


WING F



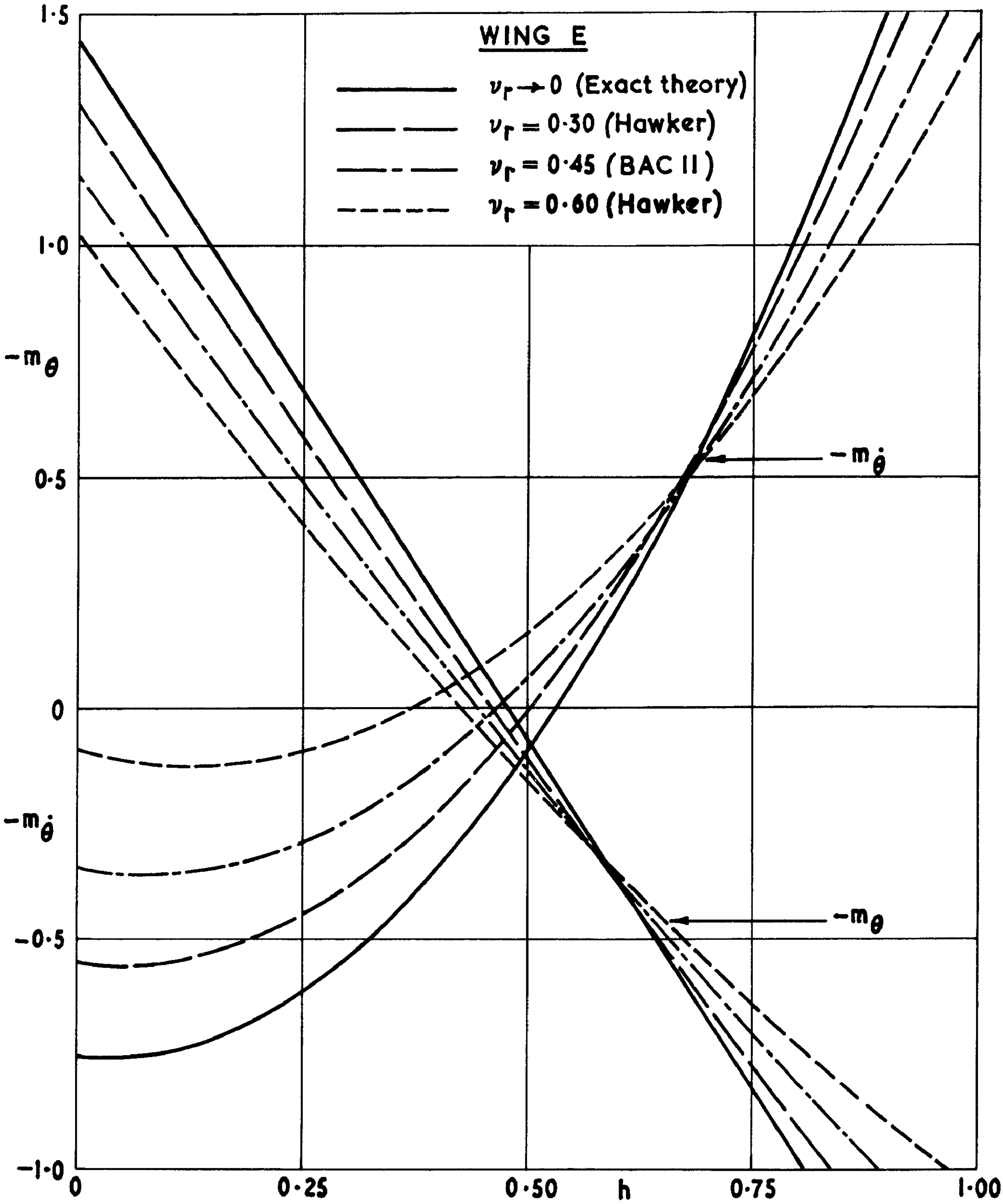
Details of symmetrically tapered planform and modified tip.

FIG. 2



Mesh size of BAC II programme for Wing F at $M = \frac{2}{3}\sqrt{3}$.

FIG. 3



Effect of ν_r on curves of $-m_\theta$ and $-m_{\dot\theta}$ against h $(M=\frac{2}{3}\sqrt{3})$.

FIG. 4

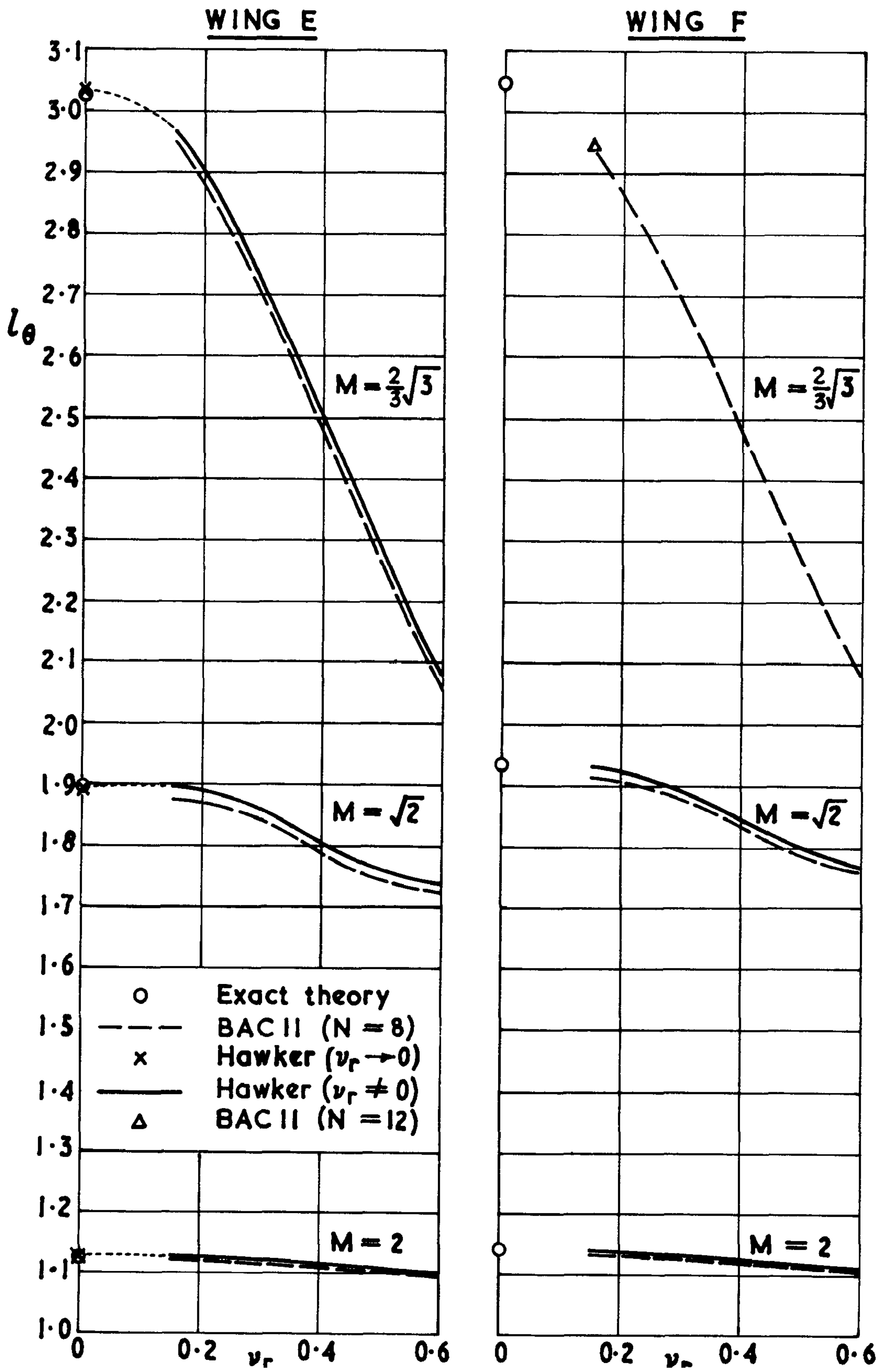
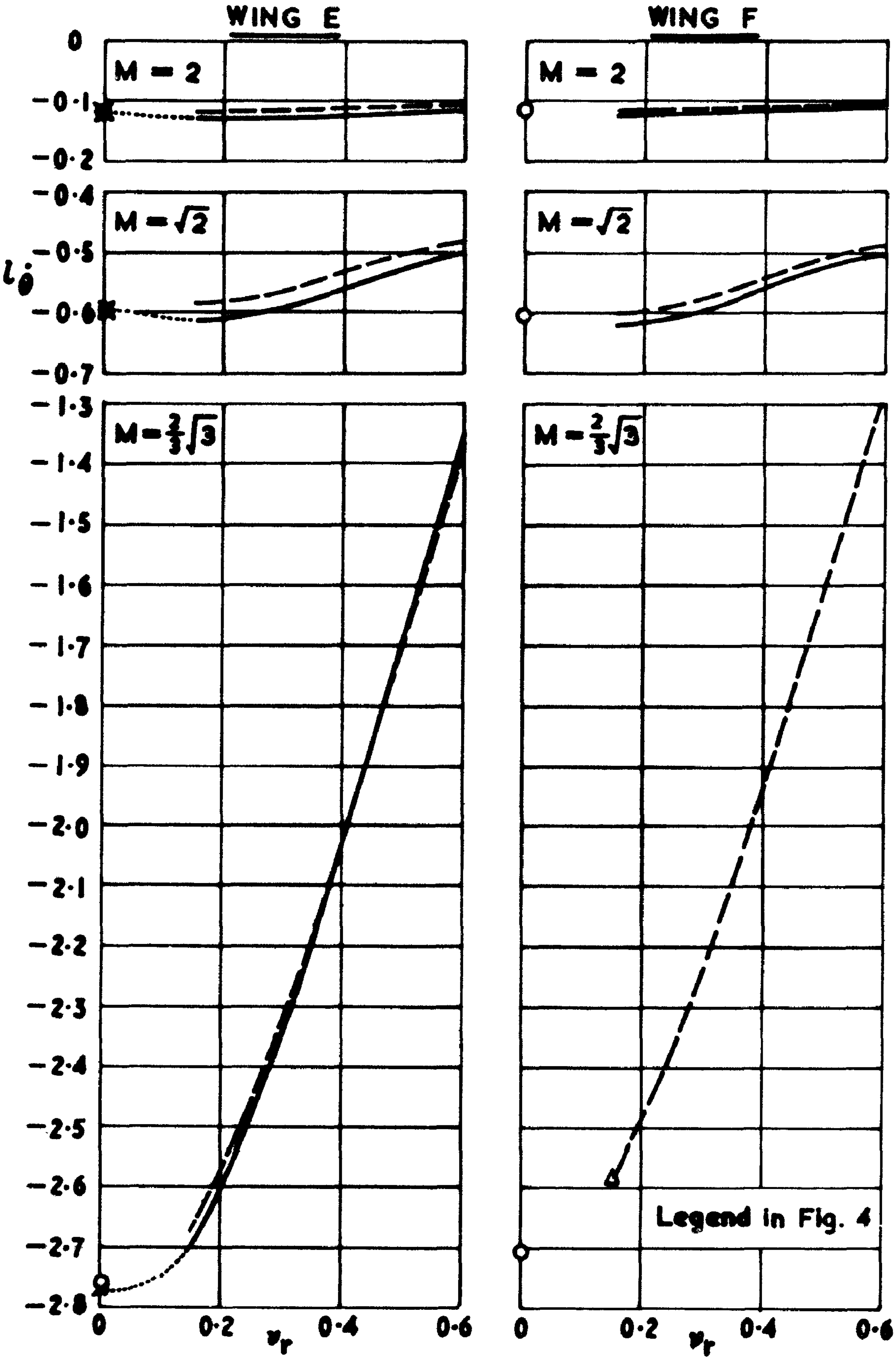
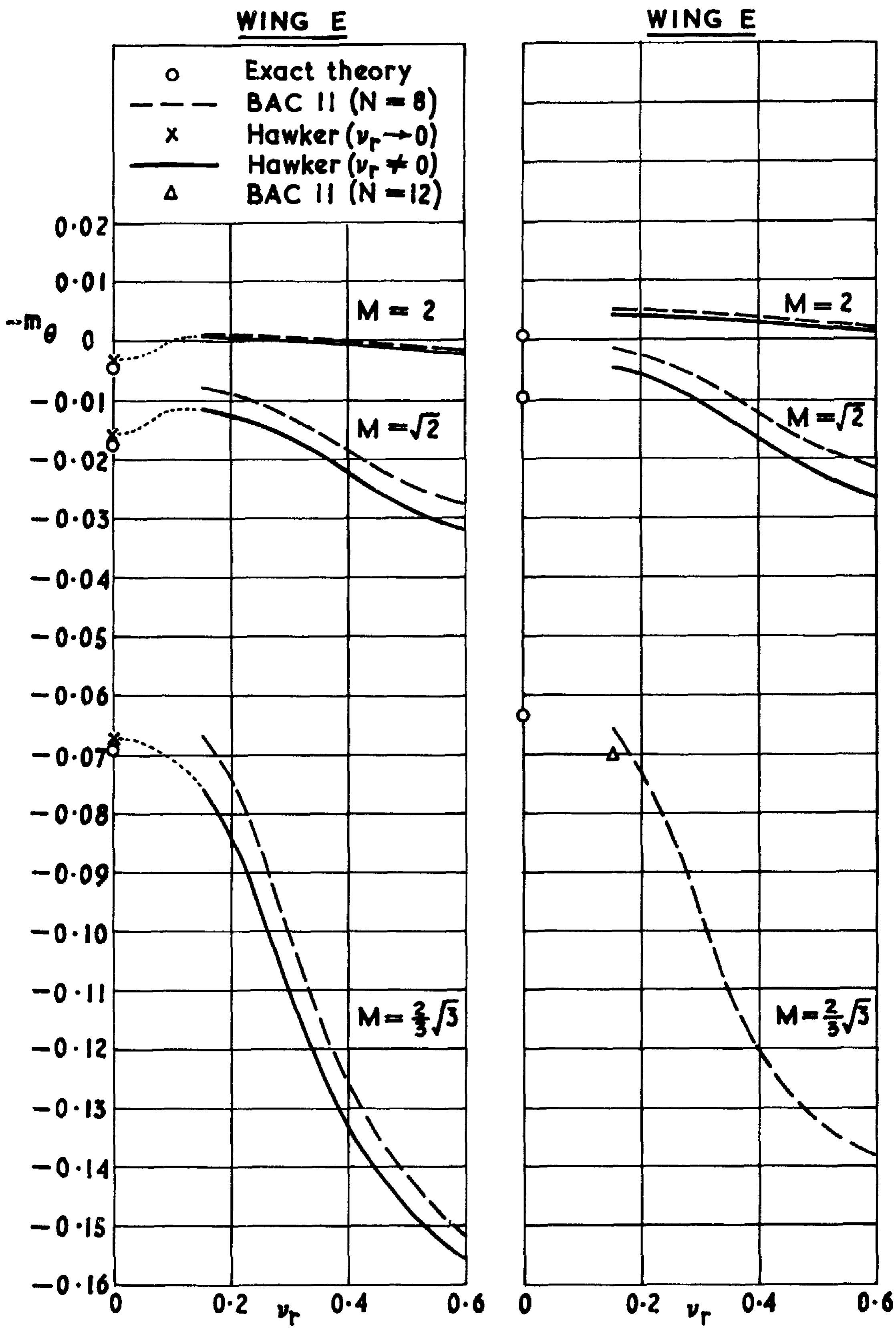

WING E
WING F
l_θ
3·1
3·0
2·9
2·8
2·7
2·6
2·5
2·4
2·3
2·2
2·1
2·0
1·9
1·8
1·7
1·6
1·5
1·4
1·3
1·2
1·1
1·0
0
0·2
ν_r
0·4
0·6
$M = \frac{2}{3}\sqrt{3}$
$M = \sqrt{2}$
$M = 2$
Exact theory
BAC 11 (N = 8)
Hawker ($\nu_r \to 0$)
Hawker ($\nu_r \neq 0$)
BAC 11 (N = 12)

FIG. 5



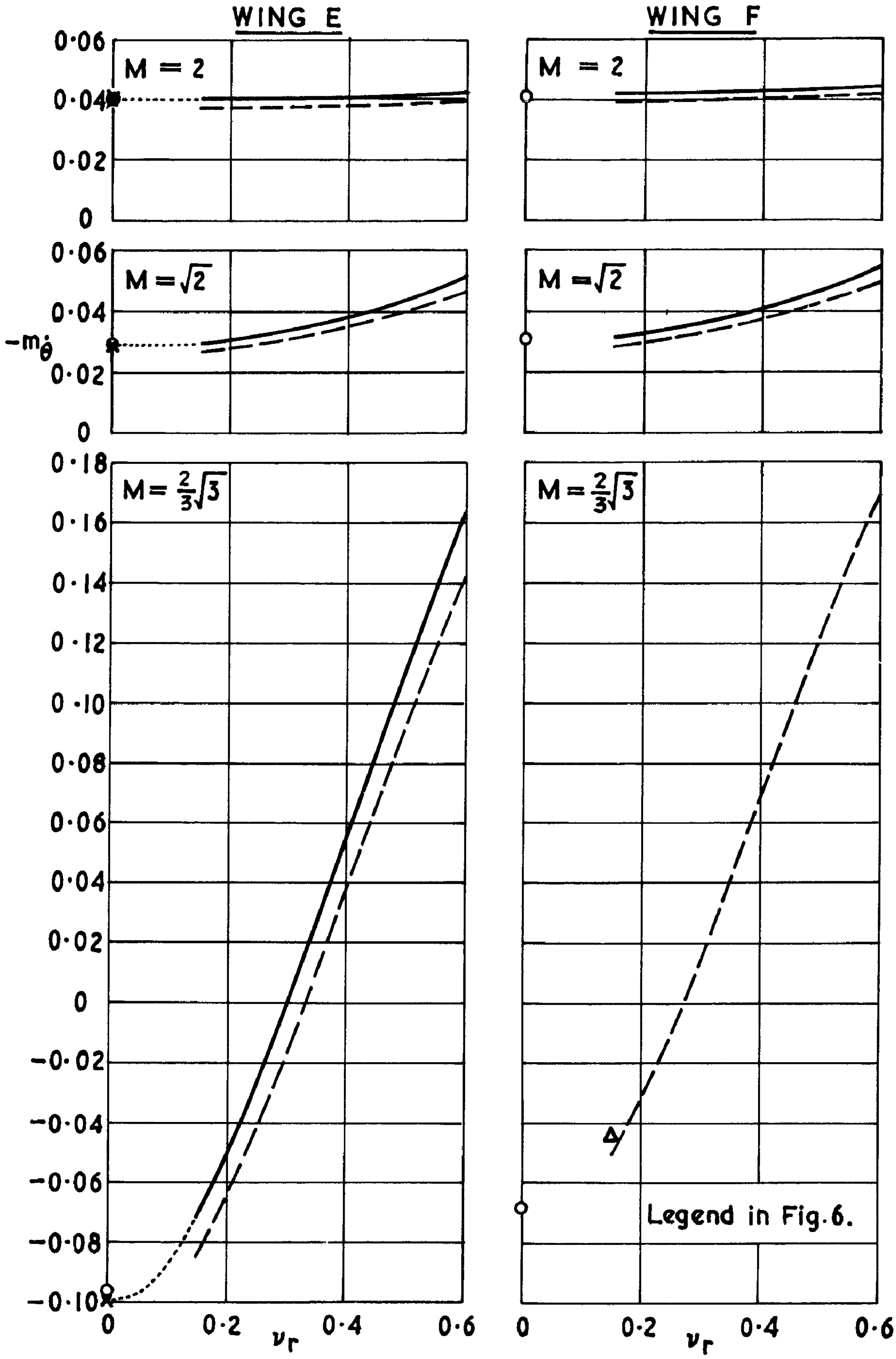
$l_{\dot{\theta}}$ against ν_r for pitching axis $h = 0\cdot5$.

FIG. 6



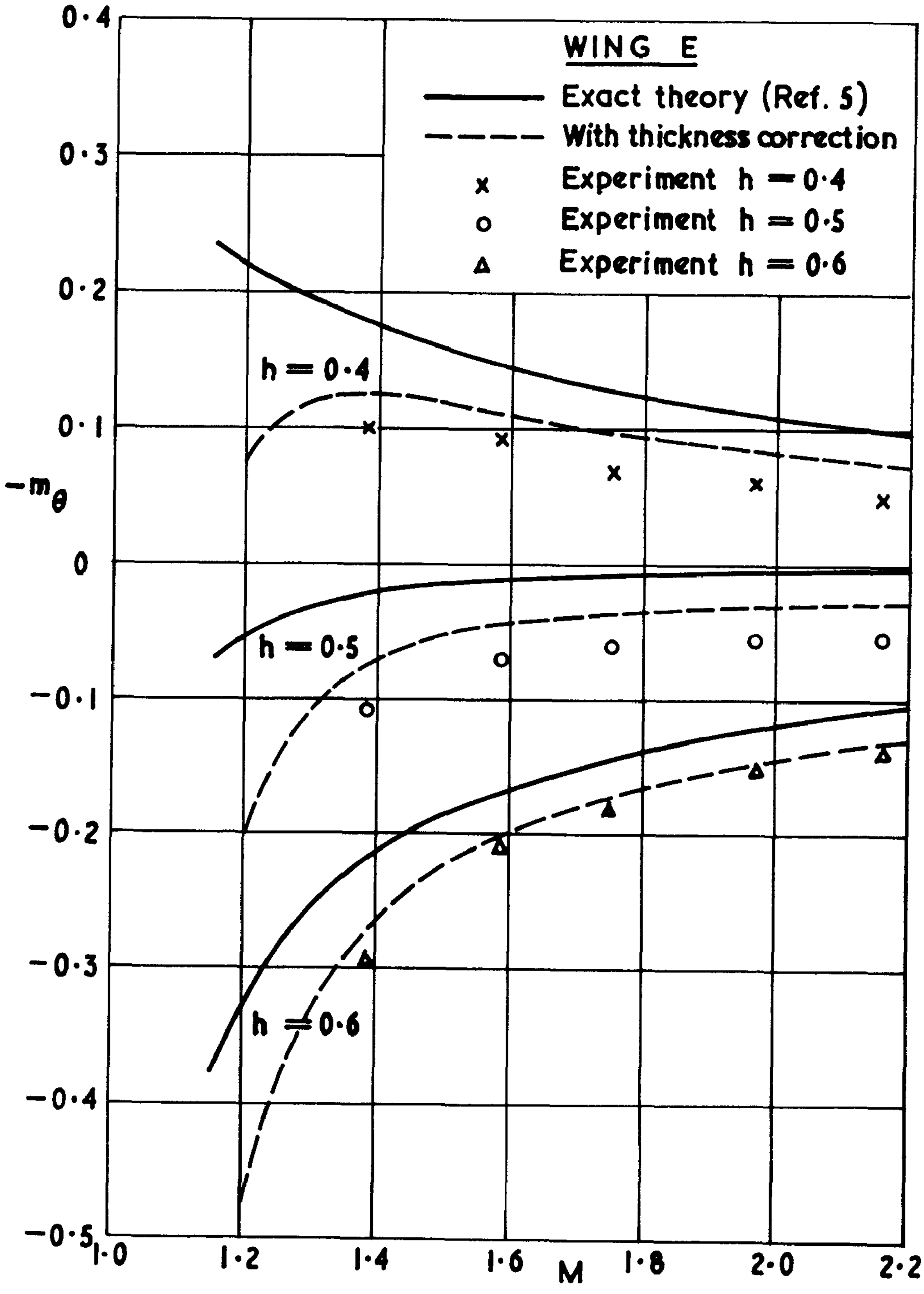
$-m_\theta$ against ν_r for pitching axis $h = 0{\cdot}5$.

FIG. 7



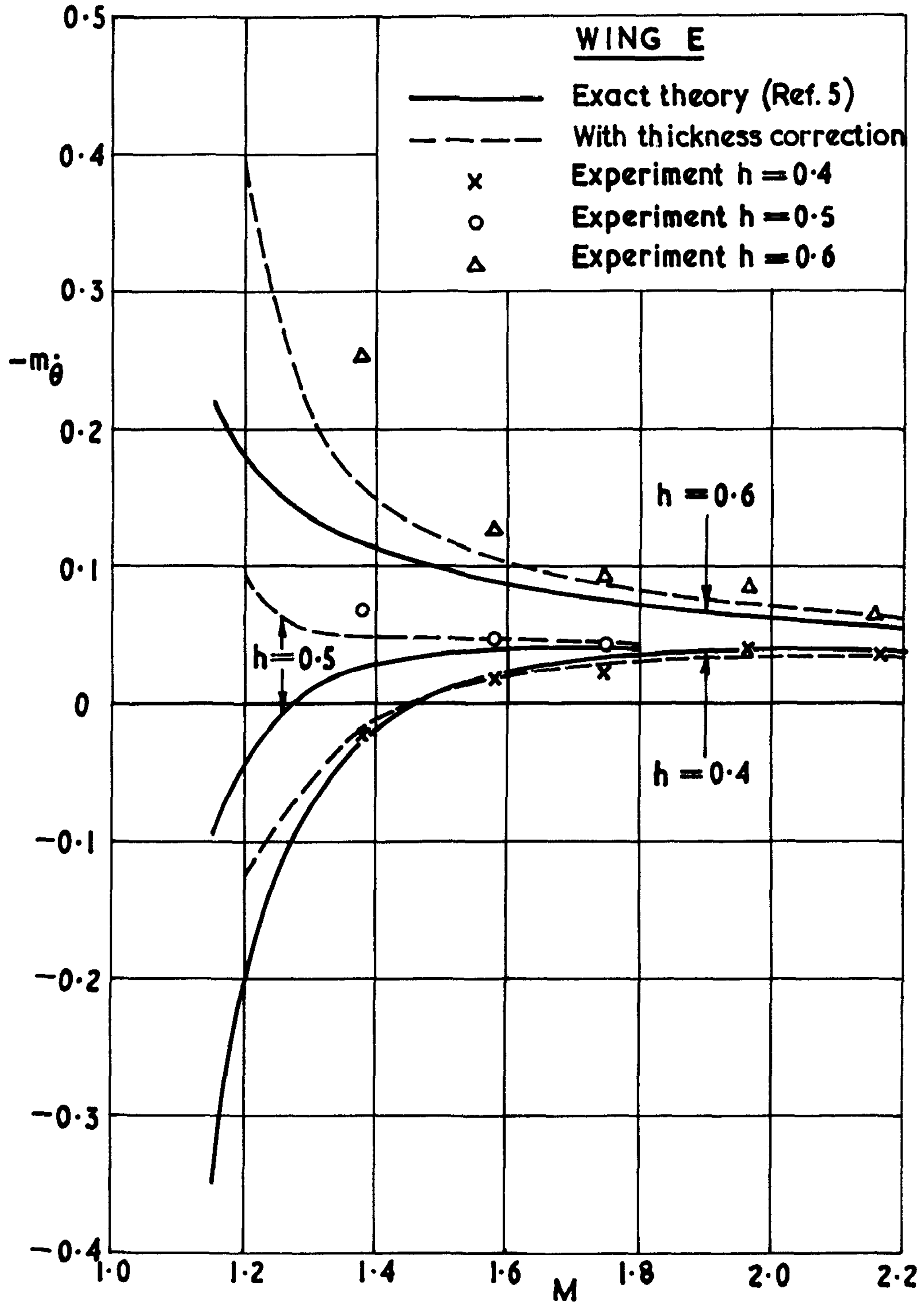
$-m_{\dot\theta}$ against ν_r for pitching axis $h = 0{\cdot}5$.

FIG. 8



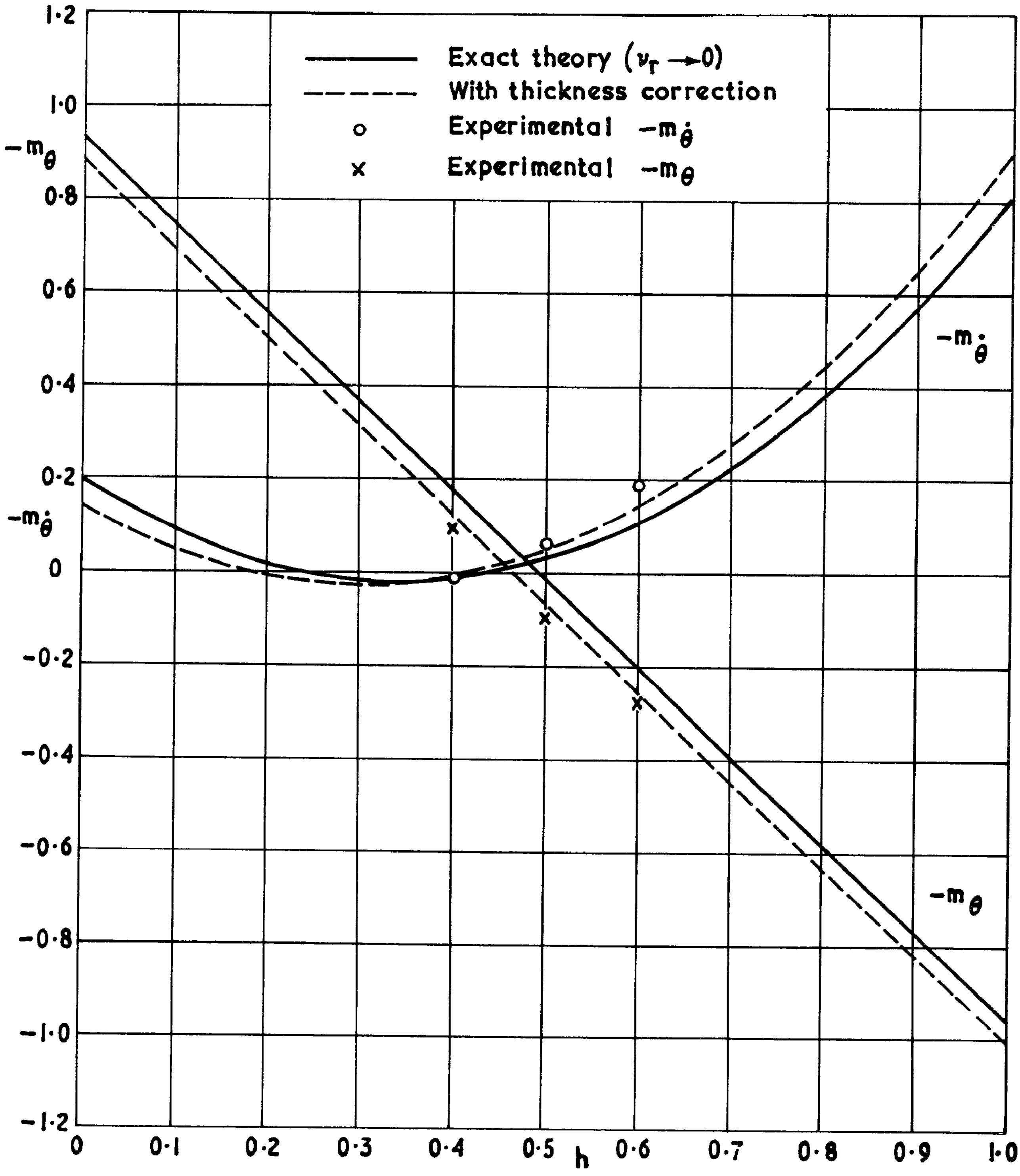
Experiment and thickness effect for $-m_\theta$ against M.

FIG. 9



Experiment and thickness effect for $-m_{\dot\theta}$ against M.

FIG. 10



Experiment and thickness effect on Wing E at $M = \sqrt{2}$.

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Nat. Phys. Lab.

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