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A Semi-Empirical Method for Estimating the Rotary Rolling Moment Derivatives of Swept and Slender Wings

by

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A SEMI-EMPIRICAL METHOD FOR ESTIMATING THE ROTARY ROLLING MOMENT DERIVATIVES OF SWEPT AND SLENDER WINGS

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SUMMARY

A method is derived for estimating the derivatives ℓ_p and ℓ_r of swept and delta wings based on theoretical data and steady six component wind tunnel results Good agreement is obtained with values of ℓ_p measured on a rolling balance for a series of narrow delta wings.

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1 INTRODUCTION

Recent wind tunnel tests on the rolling derivatives of slender wings[1] have shown variations of ℓ_p with incidence which cannot be predicted by existing methods. One particular shortcoming of existing work on derivatives is that it is based on the concept of attached potential flow. It is therefore unable to predict the aerodynamic loads resulting from the vortex type flow near the leading edge of slender wings. As wind tunnel tests for the rotary derivatives are much more rare than static six component tests, it appears desirable to derive an approximate method for the estimation of the rotary derivative of the aircraft, which makes the best use of steady tunnel data to correct basic theory for less predictable flow phenomena.

In this note such a method is given for the rolling moment derivatives ℓ_p and ℓ_r. Although in principle the same method appears suitable also for the estimation of the corresponding yawing moments, these are much more sensitive to relatively minor changes in the pressure distribution over the wing and need therefore more detailed analysis than it is proposed to apply here.

2 THE YAWING WING

In Fig.1 the geometry of the flow induced by a rate of yaw r on a swept wing moving with the velocity V_o through a stationary fluid is illustrated. It can be seen that yawing generates incremental velocities in the horizontal plane which can be split up into two components:-

$$u = -ry \qquad (1)$$

$$v = -rx \qquad (2)$$

where y is the spanwise and x the chordwise co-ordinate of a given point on the wing with respect to the centre of rotation. (In free flight this would be the C.G. of the aircraft). Within the usual restrictions to small perturbations these two components can be considered separately and their effects superimposed. When applied to the rolling moment due to rate of yaw ℓ_r, we can thus write:-

$$\ell_r = (\ell_r)_u + (\ell_r)_v \quad . \qquad (3)$$

The longitudinal velocity increment u results in a linear variation of the total velocity $V_o + u$ across the span as shown in Fig.2. As the local lift is proportional to the square of the local velocity one gets

$$L(\eta) = L_o(\eta)\left(1 + \frac{u}{V_o}\right)^2 = L_o(\eta)\left(1 + 2\frac{u}{V_o} + \left(\frac{u}{V_o}\right)^2\right) \qquad (4)$$

where $L_o(\eta)$ is the local lift in undisturbed steady flow. For small disturbances second order terms can be neglected, thus equation 4 reduces to:

$$L(\eta) = L_o(\eta)\left(1 + 2\frac{u}{V_o}\right) \quad . \tag{5}$$

The second term in the bracket represents incremental lift due to yawing which can be integrated into a rolling moment $(\ell_r)_u$. This phenomenon is considered in the conventional methods for calculating ℓ_r[2,3,4], giving values for ℓ_r in proportion to C_L. These calculations are based on the concept of either lifting line or lifting surface theory and can be expected to give good results if the actual flow at the wing under consideration corresponds to the assumptions of the theory, i.e. if there is attached flow. Normally conditions of separated flow are only of interest to the study of the stall and spin which are outside the scope of the present investigation. However, on highly swept leading edges the flow is known to separate and form a stable leading edge vortex, which is responsible for the generation of considerable additional lift on the outer portions of the wing. If this "vortex lift" is modified by flow asymmetrics such as exist in yawing motion, the resulting rolling moment (in proportion to the overall C_L of the wing) will be substantially greater than that predicted on the basis of the spanwise distribution of lift for attached flow.

Strictly it is impossible to separate readily the total lift acting on a wing into a contribution due to "attached flow" and one due to "vortex flow". However, a working estimate may be obtained by assuming that the contributions due to attached flow is defined by extrapolating the lift slope $(\partial C_L/\partial\alpha)_A$ pertaining at the incidence with attached flow α_A. If the wing is designed to have attached flow at $\alpha = 0$ as illustrated in Fig.3, we can define approximately the "vortex lift" as

$$\Delta C_{L_V} = C_L - \left(\frac{\partial C_L}{\partial\alpha}\right)_A \alpha \quad . \tag{6}$$

For warped wings where the incidence for flow attachment $\alpha \neq 0$ we get more generally:

$$\Delta C_{L_V} = C_L - \left(\frac{\partial C_L}{\partial\alpha}\right)_A (\alpha - \alpha_A) \quad . \tag{7}$$

A schematic diagram of the probable spanwise distribution of the two contributions to the total lift is given in Fig.4 both for symmetric flow and for the incremental lift due to yawing. In Appendix 1 the relative rolling moment arms of the incremental lift contributions of the assumed components of attached lift (η_A) and vortex lift (η_V) have been computed for a narrow delta wing of aspect ratio 1. The results are plotted in Fig.5 against incidence as η_V/η_A i.e. the ratio between the rolling moment contributions of the "vortex lift" and the "attached lift". The results given there may be considered generally representative of the family of slender wings. As a rough approximation within the range of flight incidences the use of a mean value $\eta_V/\eta_A \approx 1.3$ may be adequate.

Applying this increased moment arm of the contribution of the vortex lift to the estimated rolling moment due to rate of yaw one can now write

$$(\ell_r)_u = \ell_{r_{TH}} \frac{\left\{ C_{L_A} + \eta_V/\eta_A \, \Delta \, C_{L_V} \right\}}{C_L} \qquad (8)$$

or from equation 6

$$(\ell_r)_u = \ell_{r_{TH}} \frac{\left(\partial C_L/\partial_\alpha\right)_A \alpha\left(1 - \eta_V/\eta_A\right) + \eta_V/\eta_A \, C_L}{C_L} \qquad (9)$$

Equations 6 and 9 assume that flow attachment occurs at $\alpha = 0$. For wings designed with $\alpha_A \neq 0$ (warped wings) equation 9 can be readily modified according to equation 7 to take this into account.

In principle the procedure outlined above can also be applied to the yawing moments but this requires more detailed knowledge of the spanwise distribution of drag and also of the amount of L.E. suction established as the wing which will depend largely on wing thickness and section. Adequate general data are not available at the moment so that yawing moments are not considered in this present note.

The lateral velocity increment v on the yawing wing results in a chordwise distribution of sideslip as illustrated in Fig.1b. One must expect a rolling moment to result from this phenomenon which ought to be related to the steady sideslip derivative ℓ_v. If a point along the longitudinal axis can be found at which this sideslipping effect may be thought to be concentrated, say at a distance x_R from the centre of rotation, then a further contribution to ℓ_r would be defined as

$$(\ell_r)_V = \ell_v \frac{x_R}{b/2} \quad . \qquad (10)$$

In Appendix 2 numerical values have been computed by strip theory for the location of this "rolling moment centre" of swept wings and slender wings of this delta and "gothic" planform. The results for swept back wings are plotted against taper ratio in Fig.5. For the narrow delta wing the rolling moment centre is at approximately 75% of the root chord, for the gothic wing at 68.7%.

Adding the contributions due to u and v, equations 8 and 10 can be combined to give the total value of ℓ_r as

$$\boxed{\ell_r = \frac{\ell_{r_{TH}}}{C_L} \left\{ C_{L_A} + \frac{\eta_V}{\eta_A} \Delta \, C_{L_V} \right\} + \ell_v \frac{x_R}{b/2}} \qquad (11)$$

The data for the lift and ℓ_v may be taken from steady wind tunnel tests, $\ell_{r_{TH}}$, from existing theoretical data, e.g. from refs. 2-4.

3 THE ROLLING WING

A wing rolling about a longitudinal (x) axis will generate a vertical incremental velocity

$$w = py \qquad \text{or} \qquad \Delta\alpha = \frac{py}{V_o} \tag{12}$$

which varies linearly across the span. The resulting lift distribution can again be integrated to give the rolling moment derivative ℓ_p. This phenomenon is considered in the conventional methods for estimating this derivative[2-4].

As rolling causes incremental local changes of incidence, the resulting moments will be proportional to the resulting lift increments and thus to the lift slope of the airfoil. Any failure of the theory to predict the lift slope of a wing should therefore be reflected in a proportional error in the estimated rolling moments, i.e.

$$\frac{\ell_p}{\ell_{p_{TH}}} \equiv \frac{(\partial C_L/\partial\alpha)_{EXP}}{(\partial C_L/\partial\alpha)} \ .$$

If the theoretical value for ℓ_p is based on a theory complying with the actual flow on the wing, i.e. in general for wings with attached flow, the above expression can be used directly to obtain an empirical correction for ℓ_p as

$$\ell_p = \frac{\ell_{p_{TH}}}{a_{TH}} \left(\frac{\partial C_L}{\partial\alpha}\right)_{EXP} \ . \tag{13}$$

This expression will correct for an error in the overall lifting capacity of a given wing but not for differences between theory and actual flow in the spanwise distribution of lift, such as will occur on a wing with leading edge vortex flow.

Analogously to the procedure used in the previous section for the estimation of ℓ_p we again assume separate contributions by the "attached lift" and the "vortex lift" acting on the wing. As ℓ_p is proportional to the lift slope, rather than C_L itself we have now to determine two corresponding contributions to the lift slope. With the assumptions used previously, an "attached lift slope" can be defined by the tangent of $C_L(\alpha)$ at the incidence α_A where the flow is attached. Consequently the "vortex lift slope" is then given at any incidence by

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_V = \left(\frac{\partial C_L}{\partial\alpha}\right)_\alpha - \left(\frac{\partial C_L}{\partial\alpha}\right)_A \ . \tag{14}$$

As the lift generated by the leading edge vortex on a swept narrow wing acts at a greater spanwise moment arm than the lift associated with attached flow, its contribution to the rolling moment derivative ℓ_p will again be weighted by the factor η_V/η_A as derived in Appendix 1.

As an approximation the damping in roll derivative ℓ_p of a wing with leading edge vortex flow may be estimated as

$$\left(\ell_p\right)_W = \frac{\ell_{p_{TH}}}{a_{TH}} \left\{ \left(\frac{\partial C_L}{\partial \alpha}\right)_A + \left[\left(\frac{\partial C_L}{\partial \alpha}\right)_\alpha - \left(\frac{\partial C_L}{\partial \alpha}\right)_A \right] \frac{\eta_V}{\eta_A} \right\} . \qquad (15)$$

The term in square brackets is the portion of the total lift slope generated by the leading edge vortex. Ref.2 gives for slender delta wings

$$\frac{\ell_{p_{TH}}}{a_{TH}} = - \frac{1}{16} \qquad (16)$$

so that for slender wings one can write

$$\left(\ell_p\right)_W = - \frac{1}{16} \left\{ \left(\frac{\partial C_L}{\partial \alpha}\right)_A + \left[\left(\frac{\partial C_L}{\partial \alpha}\right)_\alpha - \left(\frac{\partial C_L}{\partial \alpha}\right)_A \right] \frac{\eta_V}{\eta_A} \right\} . \qquad (17)$$

If the wing is at an incidence α with respect to the rolling axis, as illustrated in Fig.6, rolling will also generate lateral velocities $v(x) = xp \sin \alpha$. x is the chordwise co-ordinate measured from the centre of rotation.

Thus there will be chordwise variation of sideslip

$$\beta(x) = \frac{xp}{V_o} \sin \alpha \qquad (18)$$

similar to that experienced by the wing in yawing motion (Fig.1). By analogy we obtain a contribution to the rolling moment derivative

$$\left(\ell_p\right)_V = \ell_v \frac{x_R}{b/2} \sin \alpha \qquad (19)$$

x_R is again the distance of the "centre of rolling moments" from the centre of rotation of the wing (the C.G. in free flight).

The total value of ℓ_p is then given by adding equations 15 and 19

$$\boxed{\ell_p = \frac{\ell_{p_{TH}}}{a_{TH}} \left\{ \left(\frac{\partial C_L}{\partial \alpha}\right)_A + \left[\left(\frac{\partial C_L}{\partial \alpha}\right)_\alpha - \left(\frac{\partial C_L}{\partial \alpha}\right)_A \right] \frac{\eta_V}{\eta_A} \right\} + \ell_v \frac{x_R}{b/2} \sin \alpha} . \qquad (20)$$

It is readily seen that the last term in equations 11 and 20 and therefore the value of ℓ_r and ℓ_p will vary considerably with the C.G. position if both ℓ_v and incidence are large. This applies in particular to highly swept wings at high C_L.

4 COMPARISON WITH WIND TUNNEL RESULTS

Values for the damping in roll derivative ℓ_p as computed from equation (20) have been compared with the results from a recent series of rolling balance tests[1] on four slender wings. The wings tested had all biconvex root sections and diamond cross-sections. Their geometry is given below:-

Wing A : Gothic, Aspect ratio = 0.75, root chord thickness $^t/c$ 5.0%

Wing B : Gothic, A = 0.75, $^t/c$ = 8.2%

Wing C : Gothic, A = 1.0, $^t/c$ = 8.2%

Wing D : Delta, A = 1.0, $^t/c$ = 8.2%.

The values for the lift slopes and ℓ_v as extracted from ref.1 and used in the present computations are plotted against incidence in Fig.8. For the rolling balance tests the centre of rotation (and the reference for the moments) was at 48.3% of the root chord for the gothic wings and at 59.3% for the delta. The chordwise centre of rolling moment for the gothic was taken to be at 68.7% and for the delta at 75% of the root chord. Consequently the values for x_R, the distance of the moment centre from the centre of rotation are

$$\frac{x_R}{C_o} = -0.687 + 0.483 = -0.204 \quad \text{for the gothic}$$

and $$\frac{x_R}{C_o} = -0.750 + 0.593 = -0.157 \quad \text{for the delta.}$$

The spanloading factor for the "vortex lift" has been taken as $\eta_V/\eta_A = 1.30$.

Using

$$\frac{\ell_{p_{TH}}}{a_{TH}} = -\frac{1}{16}$$

from ref.2, theoretical values were computed using the above quantities and the tunnel data of Fig.8. The values are compared with corresponding rolling balance results in Fig.9 for the three gothic wings and in Fig.10 for the delta wing. The agreement is very satisfactory in the case of the gothic planforms, for the delta there is a marked discrepancy at higher incidences, although even there the general trend of ℓ_p with α is well represented. In each case the results that would be obtained from equation 20 by ignoring the ℓ_v contribution, i.e. by correcting theory only for actual lift slope are also shown. This illustrates more clearly the importance of the ℓ_v contribution.

The failure of the data to predict more closely the low incidence damping in roll is most probably due to the fact that the basic theoretical value of $\ell_{p_{TH}}$ has been taken as that applicable to the straight delta.

Although further work is needed to extend the validity of the suggested procedure to planforms other than those considered here, the results are sufficiently encouraging to serve as a working hypothesis in the absence of more reliable data.

5 CONCLUSIONS

It is suggested that the rolling moment derivatives ℓ_r and ℓ_p of swept back and delta wings may be estimated by correcting basic theoretical estimates by experimental wind tunnel data for the lift coefficients and the derivative ℓ_v. These corrections are based on simplified concepts for the definition of so called rolling moment centres of pressure for which numerical values are given. In particular the method attempts to take into account the effects of leading edge vortex flow.

Results obtained by this procedure have been compared with data for ℓ_p from rolling balance tests on a series of narrow wings and the agreement was very satisfactory.

It was noted that for highly swept wings at incidence the values of ℓ_r and ℓ_p depend considerably on the C.G. position. The formulae given in this note are suitable for transferring such data to alternative C.G. positions.

LIST OF SYMBOLS

$a = \partial C_L/\partial\alpha$	lift slope
b	wing span
c	local chord
C_o	root chord
C_T	tip chord
C_R	distance of centre of rolling moments from apex of delta or gothic wing
$C_L = \dfrac{\text{Lift}}{\rho/2\ V^2\ S}$	lift coefficient
$C_\ell = \dfrac{\text{Rolling moment}}{\rho/2\ V^2\ S\ b}$	rolling moment coefficient

$\ell_v = \frac{\partial C_\ell}{\partial \beta}$	rolling moment due to sideslip
$\ell_p = \frac{\partial C_\ell}{\partial (pb/2V)}$	damping in roll derivative
$\ell_r = \frac{\partial C_\ell}{\partial (rb/2V)}$	rolling moment due to rate of yaw
p	rate of roll
r	rate of yaw
s	local semispan
u	incremental longitudinal velocity
V_o	flight velocity
v	incremental lateral velocity
w	incremental vertical velocity
x	chordwise co-ordinate
x_R	distance of chordwise rolling moment centre from centre of rotation (C.G. in free flight)(positive forward)
x_L	distance of chordwise rolling moment centre from wing (or lifting line) apex (positive aft)
y	spanwise co-ordinate
α	incidence
$\beta = \frac{v}{V}$	sideslip
$\eta = \frac{y}{b/2}$	non-dimensional spanwise co-ordinate
η_A	rolling moment arm of attached lift
η_V	rolling moment arm of vortex lift
ξ_R	distance of chordwise rolling moment centre from apex (or apex of lifting line)

Suffix A denotes attached lift and V vortex lift.

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APPENDIX 1

ESTIMATION OF THE ROLLING MOMENT ARM OF THE ATTACHED LIFT AND THE LEADING EDGE VORTEX LIFT

If $\eta = \frac{y}{b/2}$ is the spanwise co-ordinate of a wing and $L(\eta)$ the local lift, the rolling moment L_M acting on one half wing is given by

$$L_M = \int_0^1 L(\eta)\, \eta \, d\eta \tag{21}$$

and the moment arm of the lift acting on the semispan by

$$\eta_M = \frac{\int_0^1 L(\eta)\, \eta \, d\eta}{\int_0^1 L(\eta)\, d\eta} \quad . \tag{22}$$

If it were possible to separate the contributions of the attached flow L_A and the lift produced by the leading edge vortex into two distinct quantities, the moment arm of each could be obtained separately, i.e.

$$\eta_A = \frac{\int L_A(\eta)\, \eta \, d\eta}{\int L_A(\eta)\, d\eta} \tag{23}$$

and

$$\eta_V = \frac{\int L_V(\eta)\, \eta \, d\eta}{\int L_V(\eta)\, d\eta} \quad . \tag{24}$$

Such a separation of the total lift is not strictly possible, but for the purpose of the present investigation one may use this concept as a working hypothesis.

As a further approximation it is also assumed that the local incremental lift produced by yawing or rolling is proportional to the basic symmetric lift multiplied by the spanwise co-ordinate η, i.e.

$$L_{R_1 Y} \propto L_o(\eta)\, \eta \quad . \tag{25}$$

Now the spanwise moment arm of the incremental asymmetric lift can be computed from the basic lift distribution on the wing $L_o(\eta)$ as

$$\eta_M = \frac{\int_0^1 L_o(\eta)\,\eta^2\,d\eta}{\int_0^1 L_o(\eta)\,\eta\,d\eta} \quad . \tag{26}$$

For narrow wings, assuming conical flow it will be sufficient to compute this equation from the pressure distribution across any given chordwise location, avoiding the extreme near the apex and the trailing edge. In ref.5 it has been shown that there is in fact little variation in the pressure distribution with x. Now equation (26) has been computed for the contributions of attached and vortex lift to the rolling moments.

(i) As a working hypothesis the "attached lift" is assumed to be distributed evenly over the span. Although this assumption cannot be strictly supported by theoretical considerations, it appears to agree well with whatever experimental data there are, e.g. the pressure distributions of ref.5 for small incidences - where there is hardly any evidence of a leading edge vortex - are practically constant over the span.

Substituting L_o = const in equation (26) the rolling moment arm of this contribution is

$$\eta_A = \frac{\int_0^1 \eta^2\,d\eta}{\int_0^1 \eta\,d\eta} = \frac{2}{3} \tag{27}$$

(ii) Fig.11 shows the spanwise pressure distribution measured in ref.5 on a narrow delta of aspect ratio 1 at a station across 50% of the centre line chord. The corresponding values of total lift against incidence are given in Fig.11, where the assumed contribution of the "attached lift" is indicated by the dotted line. Deducting a constant pressure distribution corresponding to the proportion of the "attached lift" to the total lift from the values given in Fig.10 and multiplying these with η, an estimate for the vortex lift contribution due to incremental asymmetric flow is obtained as shown in Fig.13. The moment arm of this lift-distribution has been computed from equation (24) and the results are plotted in Fig.5 against incidence. It is suggested to use as a practical mean value

$$\eta_V \approx 0.865 \text{ or } \boxed{\frac{\eta_V}{\eta_A} = \frac{0.865}{0.667} = 1.30}$$

APPENDIX 2

ESTIMATION OF THE CHORDWISE ROLLING MOMENT CENTRE OF WINGS WITH SWEPT LEADING EDGES

When considering a wing in yawing motion (or in rolling at incidence) there will be a chordwise variation of local sideslip

$$\beta(x) = \frac{r\,(x - x_o)}{V} \quad . \tag{28}$$

If suitable assumptions are made for the rolling moment generated at each chordwise station it should be possible to estimate an apparent chordwise centre of pressure for this rolling moment at which the sideslip effect can be assumed to be concentrated.

(a) Narrow wings

By slender wing theory the local cross-load at any chordwise position is proportional to the local semispan, $s(x)$, and the local spanwise centre of pressure is also proportional to $s(x)$. Thus the local rolling moment is proportional to $s^2(x)$.

If we further assume that the local rolling moment is also proportional to the local sideslip $\beta(x)$ the total rolling moment generated on a slender wing with a chordwise variation of sideslip as defined in equation (28) can be calculated as

$$\frac{L}{\rho/2\; V^2} = K\,\frac{r}{V} \int_0^{c_o} (x - x_o)\; s^2(x)\; dx \tag{29}$$

where K is an aerodynamic constant.

One can define an "equivalent rolling moment" generated by a constant sideslip as appropriate to that existing at a point x_R distant from the centre of rotation as

$$\frac{L}{\rho/2\; V^2} = K\,\frac{r}{V}\, x_R \int_0^{c_o} s^2(x)\; dx \quad . \tag{30}$$

Equating equations (29) and (30) we get

$$x_R = \frac{\int_0^{c_o} (x - x_o)\; s^2(x)\; dx}{\int_0^{c_o} s^2(x)\; dx} \quad . \tag{31}$$

Introducing non-dimensional co-ordinates $\xi = \frac{x}{C_o}$ and $\eta = \frac{s}{b/2}$ we get

$$\frac{x_R}{C_o} = \frac{\int_o^1 (\xi - \xi_o)\, \eta^2 \, d\xi}{\int_o^1 \eta^2 \, d\xi} \quad . \qquad (32)$$

For the delta planform $\eta(\xi) = \xi$, therefore

$$\frac{x_R}{C_o} = \frac{\int_o^1 \xi^3 \, d\xi - \xi_o \int_o^1 \xi^2 \, d\xi}{\int_o^1 \xi^2 \, d\xi} = \frac{3}{4} - \xi_o \qquad (33)$$

i.e. the rolling moment centre is at 75% root chord.

For the gothic planform $\eta(\xi) = 1 - (1-\xi)^2$

$$\frac{x_R}{C_o} = \frac{\int_o^1 (\xi - \xi_o) \left\{1 - (1-\xi)^2\right\}^2 d\xi}{\int \left\{1 - (1-\xi)^2\right\}^2 d\xi} = \frac{11}{16} - \xi_o \qquad (34)$$

i.e. the rolling moment centre is at 68.8% root chord.

(b) For swept wings with arbitrary taper one may again apply strip theory, the strips now being chosen to lie in a streamwise direction. Let the spanwise chord-distribution and, within the limits of strip theory, the lift distribution be

$$\frac{c}{b/2}(\eta) = \frac{2}{A}\left\{1 - (2\eta - 1)\, \frac{1 - {C_T/C_o}}{1 + {C_T/C_o}}\right\} \qquad (35)$$

where $\eta = {y/s}$ is the non-dimensionalised spanwise co-ordinate and $\frac{C_T}{C_o}$ the ratio of tip chord to root chord. The spanwise centre of rolling moments y_L is then given by

$$\eta_L = \frac{y_L}{b/2} = \frac{\int_o^1 \frac{c}{b/2}(\eta)\, \eta^2 \, d\eta}{\int_o^1 \frac{c}{b/2}(\eta)\, \eta \, d\eta} \quad . \qquad (36)$$

Substituting for $C(\eta)$ from equation (35) equation (36) gives

$$\eta_L = \frac{\frac{1}{3} - \frac{1}{6}\,\frac{1 - C_T/C_o}{1 + C_T/C_o}}{\frac{1}{2} - \frac{1}{6}\,\frac{1 - C_T/C_o}{1 + C_T/C_o}} \quad . \qquad (37)$$

This function has been computed and plotted in Fig.6. The chordwise location of the centre of rolling moments measured from the apex of the lifting line (say the $\frac{1}{4}$ chord line for the subsonic regime or the $\frac{1}{2}$ chord line for supersonic flow) is finally given as

$$\xi_L = - \eta_L \tan \Lambda$$

where Λ is the sweep of the lifting line. To obtain the distance of the centre of rolling moments from the moment reference locus x_R, the distance of the apex from the moment reference (say the C.G. of the aircraft in free flight) has to be added.

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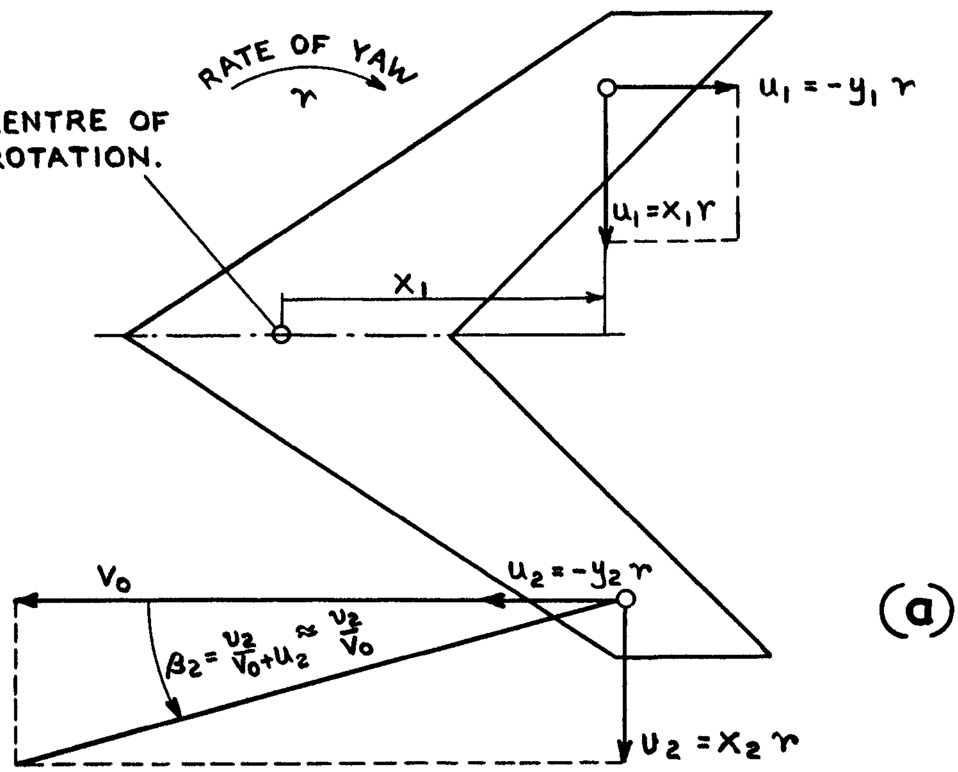

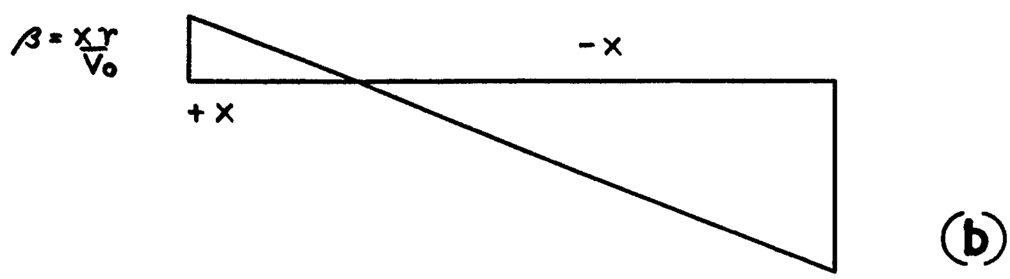

FIG. 1 (a & b) FLOW GEOMETRY & CHORDWISE DISTRIBUTION OF SIDESLIP ON A YAWING WING.

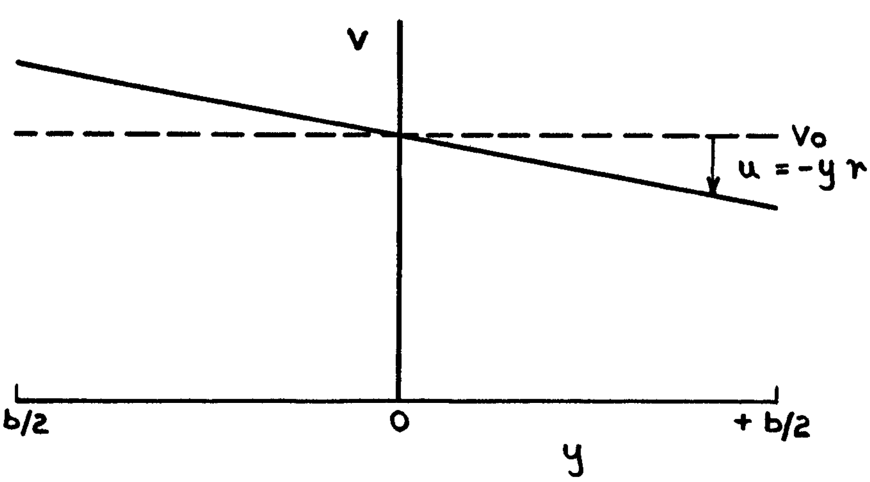

FIG. 2. SPANWISE VELOCITY DISTRIBUTION ON A YAWING WING.

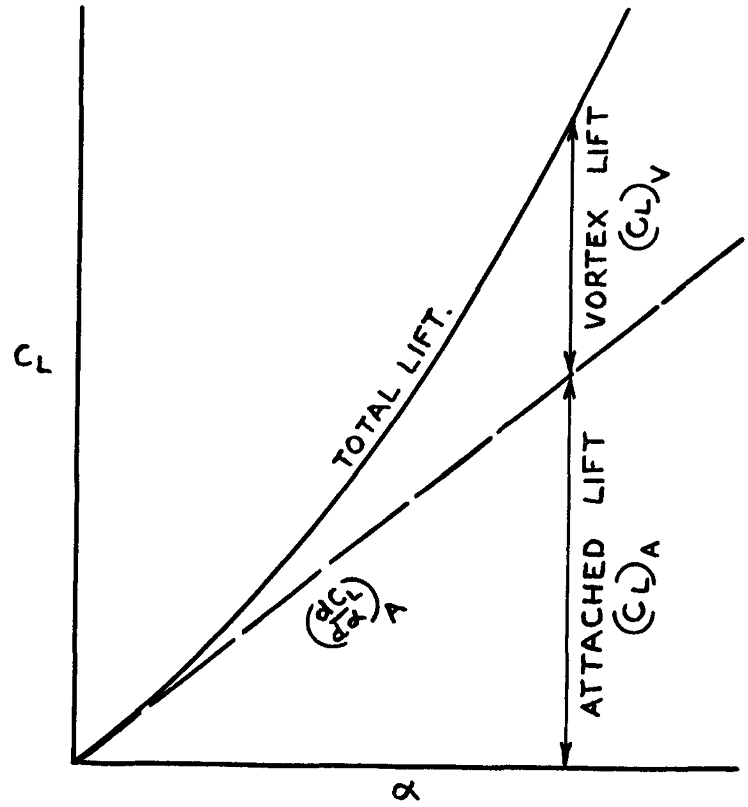

FIG. 3. DEFINITION OF "ATTACHED LIFT" & "VORTEX" LIFT FORMING THE TOTAL LIFT ON A WING WITH LEADING EDGE SEPARATION.

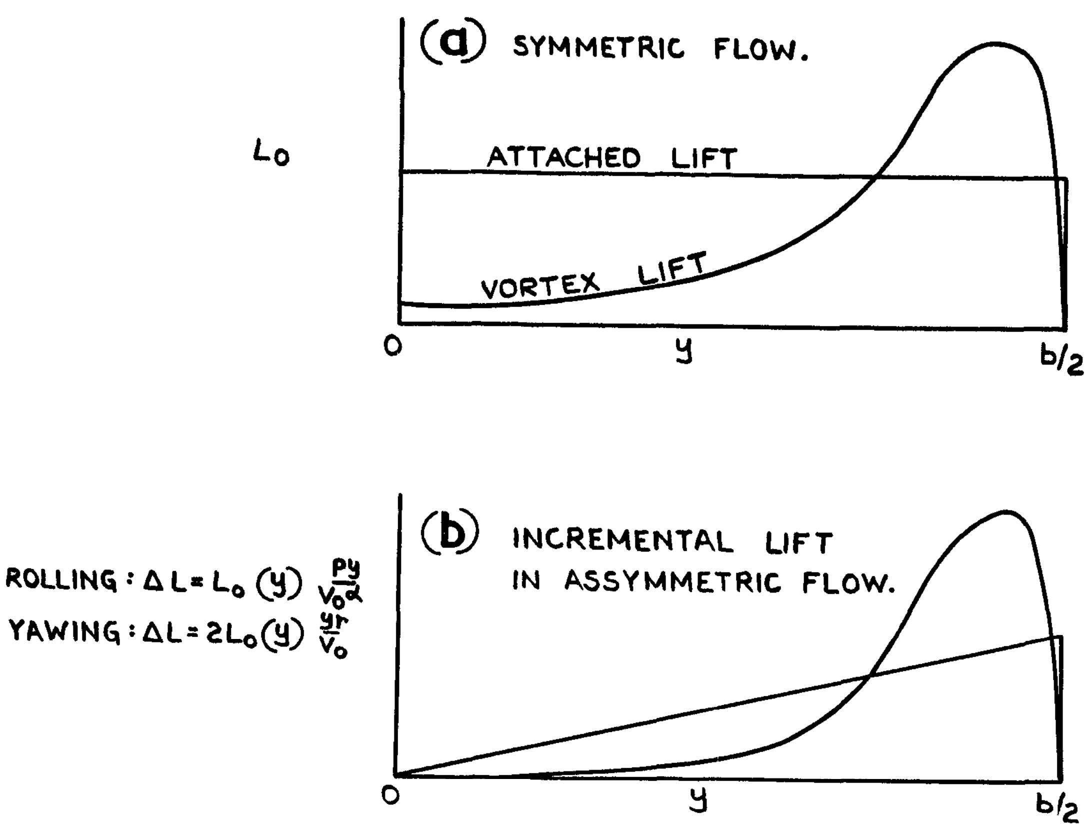

FIG. 4. (a & b) ASSUMED SPANWISE DISTRIBUTION AT A GIVEN CHORDWISE STATION OF THE ATTACHED LIFT & VORTEX LIFT IN SYMMETRIC FLOW & IN ASSYMMETRIC FLOW (YAWING OR ROLLING).

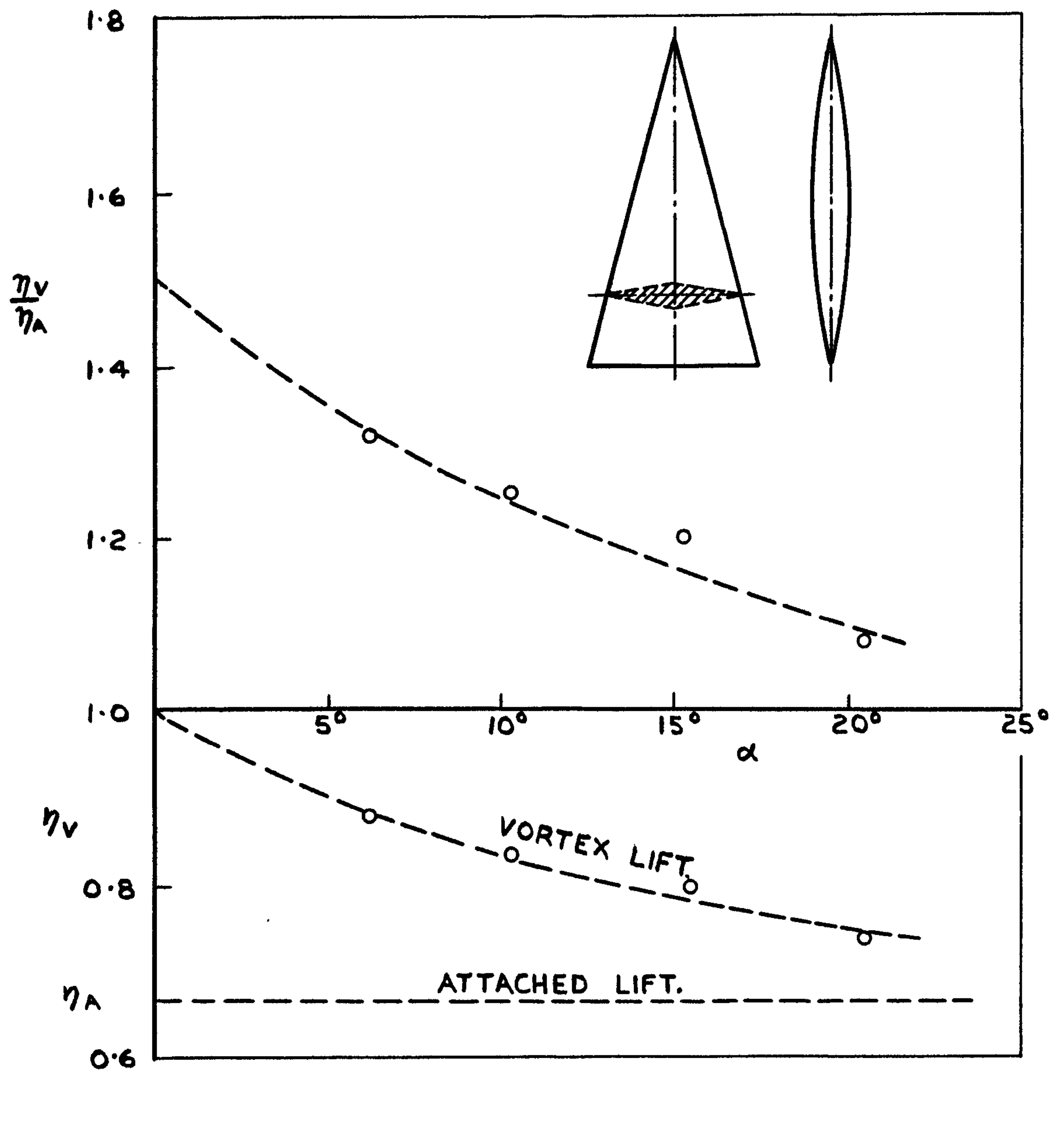

FIG. 5. ROLLING MOMENT ARM η_V OF "VORTEX LIFT" IN ASSYMMETRIC FLOW IN RELATION TO MOMENT ARM η_A OF ATTACHED LIFT. COMPUTED FROM REF. 5. ON A SLENDER DELTA WING OF ASPECT RATIO 1 WITH A 12 % THICK BICONVEX CENTRE SECTION.

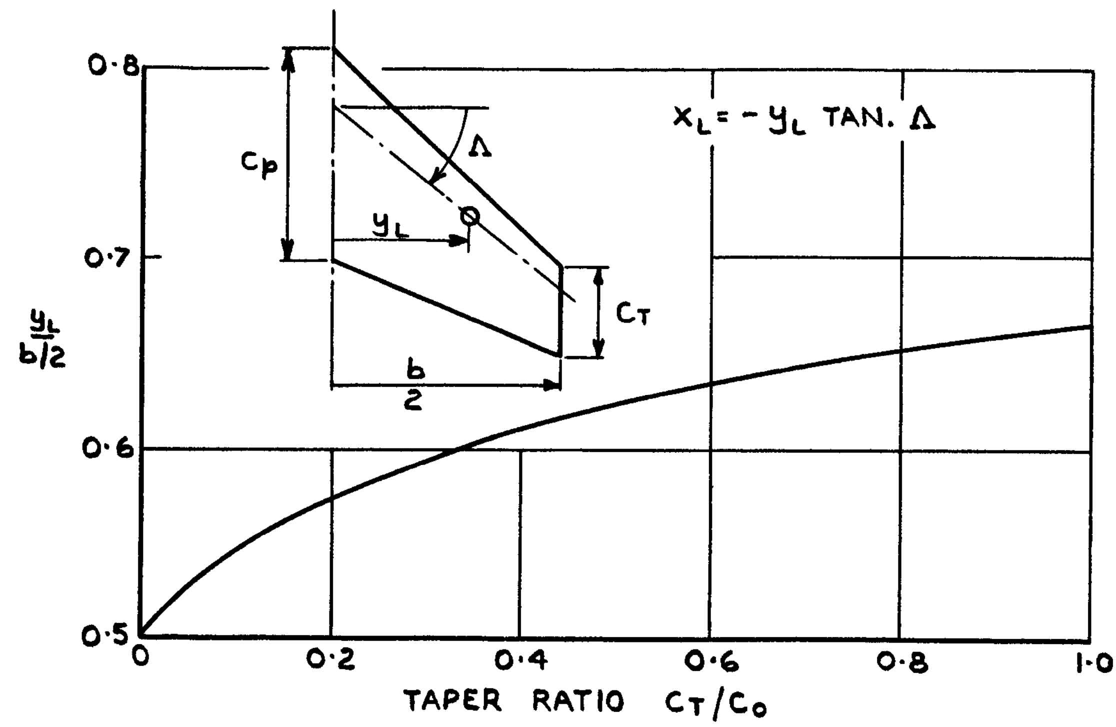

FIG. 6. THE CENTRE OF ROLLING MOMENTS FOR TAPERED SWEPT WINGS.

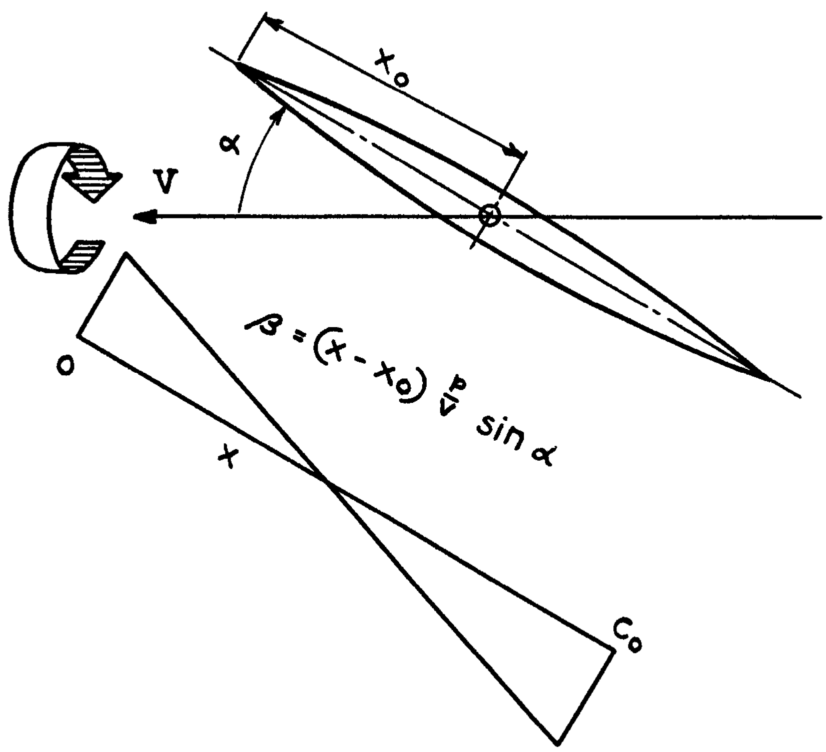

FIG. 7. CHORDWISE DISTRIBUTION OF SIDESLIP ON A SLENDER WING ROLLING AT AN INCIDENCE α.

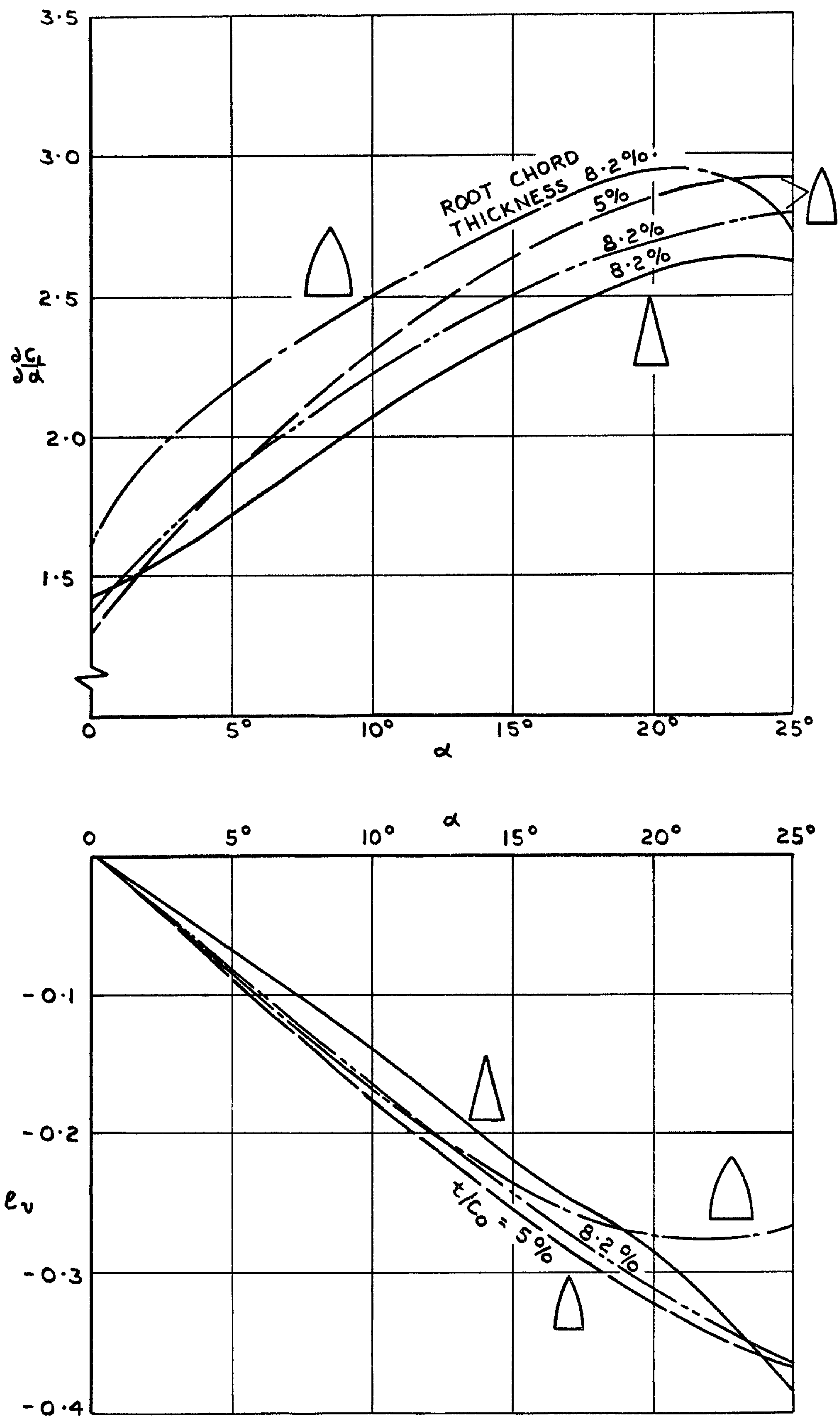

FIG. 8. WIND TUNNEL VALUES FOR $\partial C_L/\partial\alpha$ & ℓ_v OF FOUR SLENDER WINGS TESTED IN REF. 1.

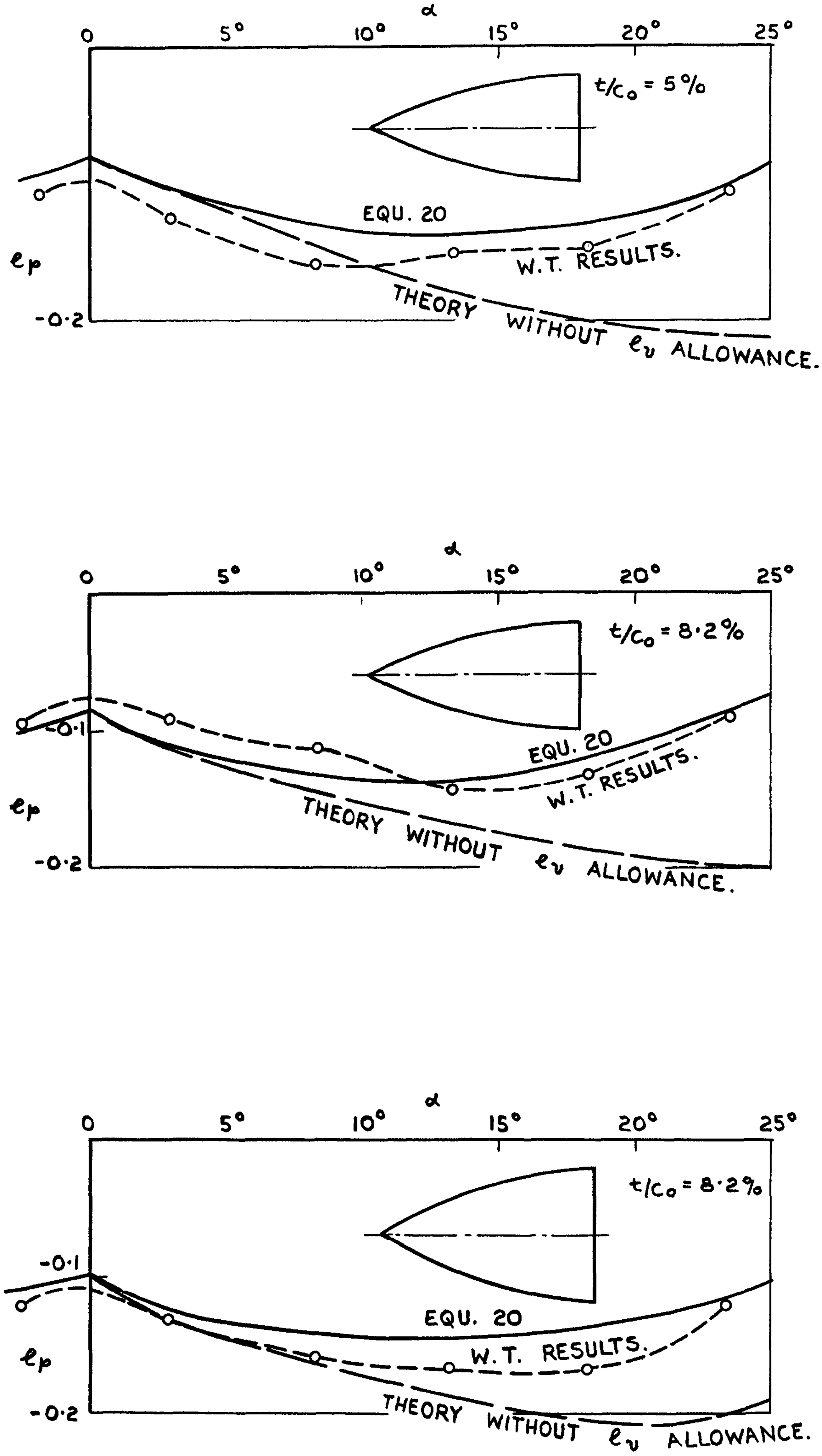

FIG. 9. DAMPING IN ROLL DATA FOR THREE GOTHIC WINGS (REF. 1) COMPARED WITH SEMIEMPERICAL THEORY EQU. 20.

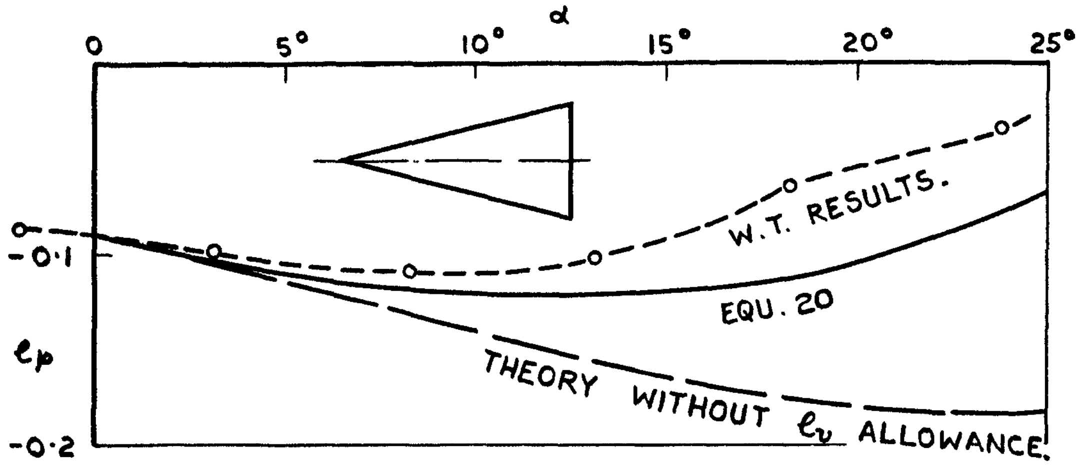

FIG. 10. DAMPING IN ROLL DATA FOR A DELTA WING (REF. 1) COMPARED WITH EQU. 20.

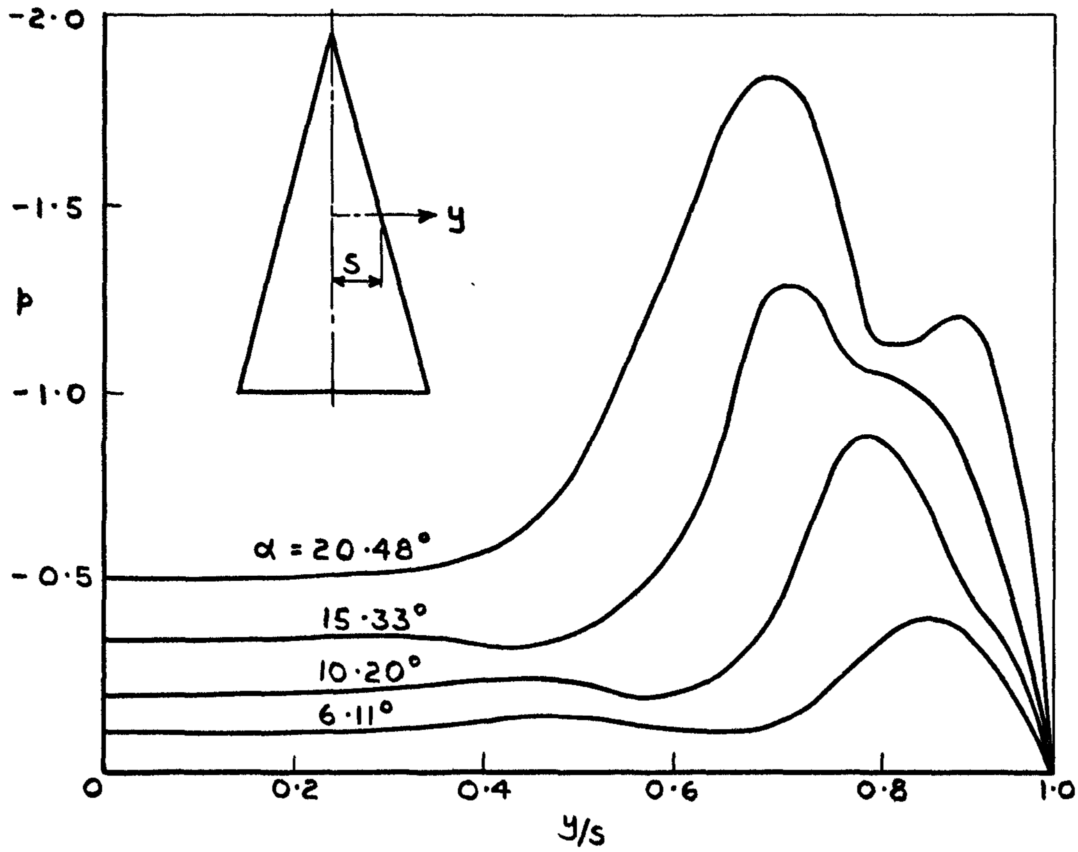

FIG. 11. SPANWISE PRESSURE DISTRIBUTION AT $X = \frac{1}{2} C_0$ OF A 12% THICK DELTA WING OF ASPECT RATIO 1 WITH BICONVEX CENTRE SECTION & DIAMOND SPANWISE CROSSECTIONS FROM REF. 5.

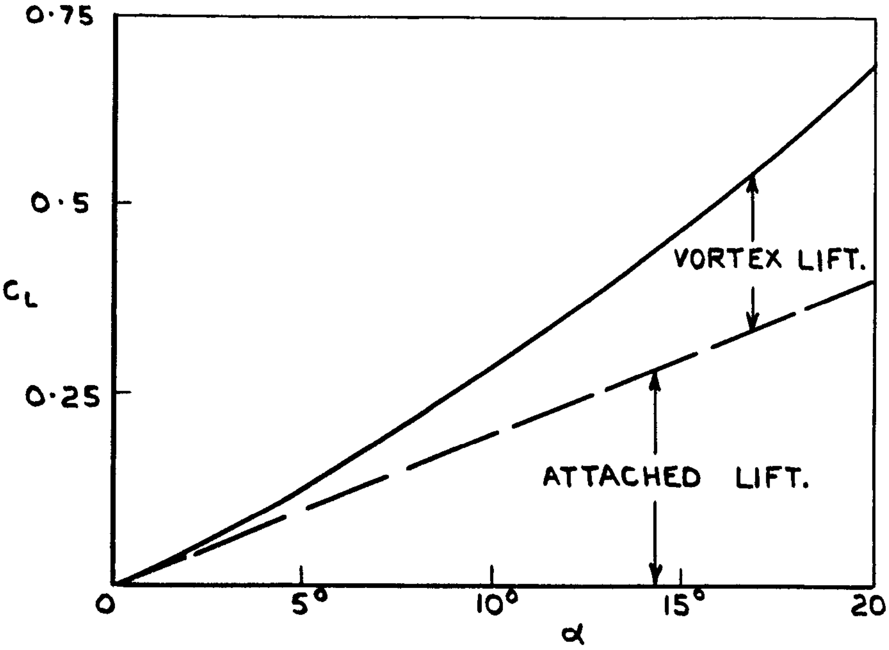

FIG. 12. WIND-TUNNEL RESULTS OF LIFT AGAINST INCIDENCE FOR THE NARROW WING OF FIG. 11.

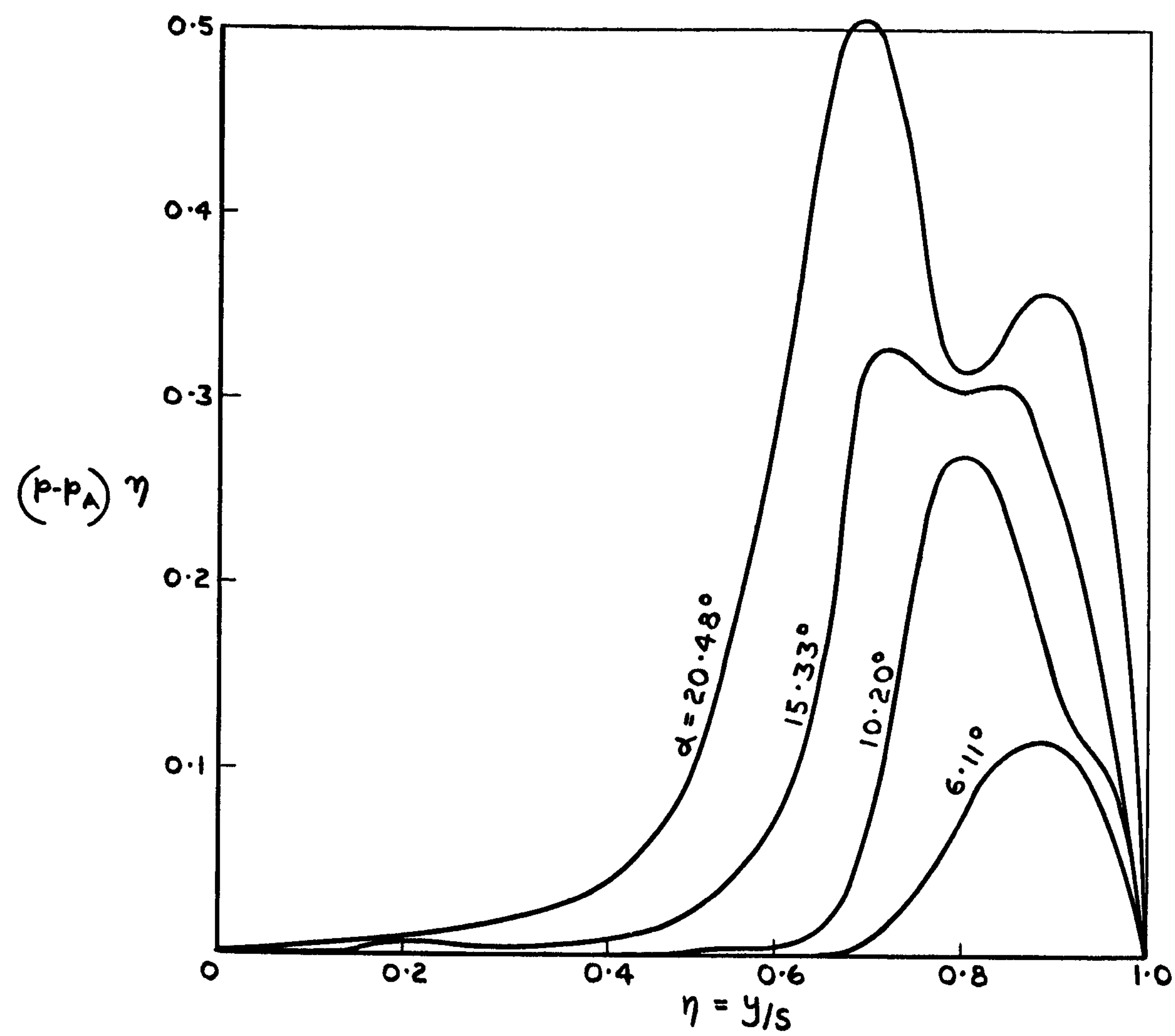

FIG. 13. ESTIMATED SPANWISE PRESSURE DISTRIBUTION OF THE VOREX LIFT INCREMENT DUE TO ASSYMETRIC FLOW OF THE WING DEFINED IN FIG. 11.

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1.8.1.2.3

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