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The Aerodynamic Drag of Near Earth Satellites

by

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ADDENDA

Since this paper was written, early in 1959, several papers dealing with molecular reflection at the surface of a solid have appeared. None of them add anything, however, to our knowledge of accommodation coefficients except for the suggestion that the values obtained by Wiedmann may be inaccurate due to experimental errors. If this is so, the accommodation coefficient would be less than the value assumed here, in which case the drag coefficients would be slightly higher than those quoted.

Page 11. The reference area for the drag coefficient is the projected area in the direction of motion for all bodies except the inclined flat plate, for which it is the area of the plate.

To determine the density of the upper atmosphere from the orbital motion of a near earth satellite a knowledge of the satellite's drag is required. In this note, the drag of a body in free molecule flow, the flow regime appropriate to a satellite in orbit, is discussed, and the molecular speed ratio is related to the properties of the upper atmosphere. The mechanism of molecular reflection at the satellite's surface and the surface temperature are considered. The drag coefficients of some simple shapes are quoted for appropriate molecular speed ratios, and the drag coefficients are obtained for cylinders and a cone with axes inclined at any angle to the direction of motion.

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1 INTRODUCTION

Estimates of the density of the upper atmosphere have been made recently from the perturbations of the orbits of near earth satellites, the results of various investigations being summarized in Ref.1. For this purpose a knowledge of the satellite's attitude and drag coefficient are required. So far, various constant values have been assumed for the drag coefficient, usually 2, and various assumptions have been made for the mode of rotation so that an average frontal area can be obtained for use as the reference area. The fact that the drag coefficient will depend on the shape of the individual satellites has tended to be neglected. More reliable values for the frontal area are being obtained now that the mode of rotation of the satellites is becoming better understood. More accurate values for the drag coefficient are therefore also required if the density is to be determined accurately.

The purpose of this note is to review present knowledge of the aerodynamics of bodies moving in the upper atmosphere and suggest values for the drag coefficients which can be used for the determination of density at altitudes between 100 and 200 nautical miles, where free molecule flow is the appropriate flow regime.

2 ASSUMPTIONS OF FREE MOLECULE FLOW THEORY

Free molecule flow is defined as occurring where the mean free path of the molecules is larger than a typical linear dimension of the body. The ratio of the length of the mean free path to a typical linear dimension is the Knudsen number K, so that free molecule flow occurs for large values of K.

In recent years several theoretical and a few experimental investigations have been made to determine the heat transfer to a body and the aerodynamic forces acting on it in free molecule flow, and recently Schaaf and Chambré[2] have given a summary of this work.

The usual assumptions, when calculating the forces acting on a body in free molecule flow, are:

(i) Collisions between impinging and re-emitted molecules are negligible.

(ii) The gas molecules have a Maxwellian distribution of thermal velocity superimposed upon the uniform mass velocity.

(iii) The molecules are reflected specularly or diffusely from the body surface.

The mechanism of molecular reflection is discussed in Section 4. On the basis of assumption (i) the total force acting on the body can be split into two parts: one due to the impinging molecules and another due to the re-emission of the molecules from the surface.

Measurements of the heat transfer to circular cylinders[3] indicate that fully developed free molecule flow exists for Knudsen numbers greater than 2. In a recent paper[4], Liu has given the theoretical solution for the drag of a flat plate at zero incidence in an almost free molecule flow making the following assumptions:

(i) The rate of collision between the molecules incident on and reflected from the surface is small compared with that between the incident molecules and the surface element under consideration.

(ii) The probability of a reflected molecule colliding twice with the incident molecules is negligible compared with the probability of its colliding once.

(iii) The molecules are reflected diffusely.

The results of this paper indicate a value of at least 3 for K if the drag of a flat plate is not to be more than 2 per cent less than the value for fully developed free molecule flow with values of $\sqrt{\text{Reynolds No.}}/(\text{Mach No.})$ appropriate to satellites. The variation of mean free path with altitude, which is determined in Section 3 and illustrated in Fig.6, indicates that K will exceed 3 for a satellite in orbit and the correct drag will be that predicted by free molecule flow theory.

3 MOLECULAR SPEED RATIO FOR A SATELLITE IN ORBIT

The most important parameter governing the forces and heating of a body in free molecule flow is the molecular speed ratio s defined by

$$s = \frac{\text{speed of satellite}}{\text{most probable molecular speed}} .$$

It is well known from the kinetic theory of gases[5] that, if a Maxwellian velocity distribution is assumed, the most probable molecular speed v_m is related to the root mean square speed C by the relation

$$v_m^2 = \frac{2}{3} C^2 \qquad (1)$$

and that C is given by

$$C = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3RT}{M}} \qquad (2)$$

where p is the pressure, ρ the density, T the absolute temperature, R the gas constant and M the molecular weight.

At present, knowledge of the properties of the upper atmosphere is in a state which can only be termed "fluid", as neither the temperature nor the composition are accurately known, so that the molecular weight is also unknown. It might appear that to determine the most probable molecular speed would be a difficult task. Fortunately, this is not so if the atmosphere is assumed to be isothermal in the region under consideration, giving an exponential variation of density with altitude

$$\rho = \rho_o \exp\left\{-\frac{h}{H}\right\} \qquad (3)$$

where h is the altitude above some reference level where the density is ρ_o, and H is the local scale height which is related to the local atmospheric properties by the relation

$$H = \frac{RT}{Mg} \qquad (4)$$

g being the local value of the acceleration due to gravity.

Combining equations (1), (2) and (4) gives a simple relationship between the most probable molecular speed and the scale height

$$v_m = \sqrt{\frac{2RT}{M}} = \sqrt{2Hg} \,. \tag{5}$$

Values of the scale height can be determined from a theoretical relationship for the decrease of the perigee radius of a satellite[6] or from the rate of change of density with height[1], obtained by plotting values of density derived from satellite observations. Neither method is very accurate, but it is probable that H lies between 25 and 50 nautical miles for altitudes between 100 and 200 nautical miles. Values of v_m are shown in Fig.1 for scale heights of 20, 30, 40 and 50 nautical miles.

For a satellite moving in an elliptic orbit, the components of the velocity are given by

$$\left.\begin{aligned} v_r &= \sqrt{\frac{\mu}{a\,(1-e^2)}}\; e \sin\psi \\ v_n &= \sqrt{\frac{\mu}{a\,(1-e^2)}}\; (1 + e\cos\psi) \end{aligned}\right\} \tag{6}$$

where v_r is the velocity component along the radius vector, v_n is the velocity component perpendicular to the radius vector, e is the eccentricity of the satellite's orbit, a is the semi-major axis, μ the constant of the earth's gravitational field (assumed spherically symmetric) and ψ is the true anomaly. Most of the action of air drag takes place over a small arc near perigee, and since the velocity will not vary greatly as the satellite moves through this arc, it is convenient to take the velocity v_n at perigee as the speed of the satellite. This velocity is shown in Fig.2 for the altitude range of interest for eccentricities of 0, 0.1 and 0.2. The molecular speed ratio appropriate to a particular height depends on the values of the scale height and on the eccentricity of the orbit and is given in Figs.3 to 5 for perigee altitudes of 100, 150 and 200 nautical miles. Examination of these figures indicates that the range of interest is from s = 6 to s = 9.

The length of the mean free path λ of a gas molecule is given by[5]

$$\lambda = \frac{0.707}{\pi N \sigma^2} \tag{7}$$

where σ is the diameter of the molecules and N is the number of molecules per unit volume which is given by

$$\rho = m N \tag{8}$$

m being the mass of one molecule. Krassovsky[7] has reported preliminary results obtained from instruments aboard satellite 1958 δ2 (Sputnik 3) which indicate that above about 135 nautical miles (250 km) the atmosphere is in the main of atomic composition, many of the atoms being ionized. The main constituents above 250 km are oxygen and nitrogen, and to simplify the calculation of λ the atmosphere is taken as a monatomic gas of molecular weight 15 (this figure may, of course, have to be modified as more experimental results become available). The variation of the length of the mean free path with altitude is given in Fig.6 for the density variation of Ref.1, assuming a value of 1.8×10^{-8} cm for the radius of the molecules. The value of molecular weight used is the lowest that is likely to occur and the length of the mean free path will be underestimated at the lower altitudes.

4 MECHANISM OF MOLECULAR REFLECTION

The mechanism of re-emission can be divided into several types; for the purpose of theoretical analysis the two which are usually considered possible are specular and diffuse. Specular reflection occurs when the angle of reflection of a molecule is equal to the angle of incidence, i.e. the tangential velocity component remains unchanged and the normal component is reversed. With diffuse reflection re-emission takes place according to the Knudsen cosine law which states that the direction of motion of the molecules after impact is independent of the direction of motion before impact, and that the number of molecules re-emitted between the angles θ and $\theta + \delta\theta$ with the normal to the surface is proportional to $\cos\theta\,\delta\theta$. The velocity distribution is assumed to be Maxwellian and its magnitude is determined by the temperature of re-emission, which is related to the surface temperature. The most probable velocity of re-emission v_{m_r} in terms of the temperature of the re-emitted gas T_r is given by

$$v_{m_r}^2 = \frac{2RT_r}{M}, \tag{9}$$

assuming the number of molecules dissociated by impact is negligible. The temperature of re-emission is usually related to the surface temperature by introducing the accommodation coefficient α which is the ratio of the energy change of the molecules that strike the surface to the energy change that would occur if they were emitted at the surface temperature T_W. It is defined by

$$\alpha = \frac{E_i - E_r}{E_i - E_W} \tag{10}$$

where E_i is the energy flux of the incident molecules, E_r that of the re-emitted molecules and E_W that of molecules at the surface temperature. Strictly, different accommodation coefficients should be defined for the three types of molecular energies: translational, rotational and vibrational. In practice only one value of α is used.

In general, reflection will not be entirely diffuse or entirely specular, but will be a combination of both; a fraction f will be reflected diffusely and a fraction (1-f) specularly. For entirely specular reflection $\alpha = f = 0$ and for entirely diffuse reflection with complete accommodation $\alpha = f = 1$. There are only meagre experimental values of α and f available. Measurements by Millikan[8] indicate that f is nearly unity* and measurements

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* A recently published paper by Hurlbut[15] also indicates that f is probably between 0.95 and 1.

by Wiedmann[9] indicate that α is approximately unity, although these results are not for high speed flow. For cylinders in free molecule flow it has been found[3] that the heat transfer data are well correlated with theory for the accommodation coefficient equal to 0.9.

The surface of a material used for a satellite, even if highly polished, would not be smooth compared with molecular dimensions so that uniform reflection in any particular direction could hardly be expected. For this reason and in view of the small amount of experimental evidence available it is believed that the assumption of entirely diffuse reflection should be made to obtain numerical values of drag. In the absence of measured values of α, for large molecular speed ratios, the best approximation is $\alpha = 1$.

5 TEMPERATURE OF A SATELLITE

For bodies moving high in the atmosphere, i.e. above about 130 nautical miles, it is well known that aerodynamic heating is negligible and that solar radiation is the predominating influence determining the temperature, which varies as the body moves into and out of the earth's shadow. The range through which the temperature varies will be governed by several factors, including the absorptivity and the emissivity of the surface and the heat capacity of the shell. If the absorptivity is high the temperature variation of the satellite could be as great as 100°C, or more, during one revolution round the earth, although for a small absorptivity it would be much less. Below an altitude of 130 nautical miles aerodynamic heating begins to have an appreciable effect. The satellites so far launched have only flown below 130 nautical miles for a small fraction of their orbits and aerodynamic heating would be small compared with the effect mentioned above.

The temperature of satellite 1958 α (Explorer I), which uses a passive technique for temperature control, has been investigated by Buwalda and Hibbs[10] and compared with measured values of the shell temperature. Resistance thermometers were placed at the tip of the nose cone, on the nose cone and aft on the cylindrical section. Shell temperatures ranging from -25°C to 75°C were observed, although most of the observations were in a narrow band near 0°C which was about 25°C wide. During this period its perigee was at about 200 nautical miles.

For the purpose of calculations in this note the satellites have been assumed to have a shell temperature of 0°C, which appears reasonable in the light of the experimental evidence from Explorer I.

6 FORCES ON A BODY IN FREE MOLECULE FLOW

6.1 Force due to impinging molecules

The equations used in this note are those given by Stalder and Zurick in Ref.11. The co-ordinate system for an element of area is shown in Fig.7. The x-axis is normal to the surface and directed into it. The satellite's velocity vector $\underline{U}$ has components U_x, U_y and U_z in the directions of the local axes.

Stalder and Zurick give the differential component of the force, in the direction defined by ℓ_x, ℓ_y and ℓ_z, due to the impinging molecules acting on the element of area dA, as

/Equ.(11)

$$dG_i = \frac{\beta^3 m N}{\pi^{3/2}} \Bigg[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} c_x \left(\ell_x c_x + \ell_y c_y + \ell_z c_z\right) \exp\left\{-\beta^2\left[\left(c_x - U_x\right)^2 + \left(c_y - U_y\right)^2 + \left(c_z - U_z\right)^2\right]\right\} dc_x\, dc_y\, dc_z$$

$$- \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{0} c_x \left(\ell_x c_x + \ell_y c_y + \ell_z c_z\right) \exp\left\{-\beta^2\left[\left(c_x - U_x\right)^2 + \left(c_y - U_y\right)^2 + \left(c_z - U_z\right)^2\right]\right\} dc_x\, dc_y\, dc_z \Bigg] dA$$

$$= \frac{\rho U^2}{2} \frac{1}{\sqrt{\pi}\, s^2} \Bigg\{ \Bigg[\left(\ell_x \beta U_x + \ell_y \beta U_y + \ell_z \beta U_z\right) \left\{ \sqrt{\pi}\, \beta U_x \left[1 + \operatorname{erf}\left(\beta U_x\right)\right] + \exp\left(-\beta^2 U_x^2\right)\right\} + \frac{\sqrt{\pi}}{2} \ell_x \left[1 + \operatorname{erf}\left(\beta U_x\right)\right]\Bigg]$$

$$- \Bigg[\left(\ell_x \beta U_x + \ell_y \beta U_y + \ell_z \beta U_z\right) \left\{ \sqrt{\pi}\, \beta U_x \left[1 - \operatorname{erf}\left(\beta U_x\right)\right] - \exp\left(-\beta^2 U_x^2\right)\right\} + \frac{\sqrt{\pi}}{2} \ell_x \left[1 - \operatorname{erf}\left(\beta U_x\right)\right]\Bigg] \Bigg\} dA \qquad (11)$$

$$= \frac{\rho U^2}{2} \frac{2}{\sqrt{\pi}\, s^2} \left\{ \left[\ell_x \beta U_x + \ell_y \beta U_y + \ell_z \beta U_z\right] \left[\sqrt{\pi}\, \beta U_x \operatorname{erf}\left(\beta U_x\right) + \exp\left(-\beta^2 U_x^2\right)\right] + \frac{\sqrt{\pi}}{2} \ell_x \operatorname{erf}\left(\beta U_x\right)\right\} dA \qquad (12)$$

where $\beta = \frac{1}{V_m} = \frac{s}{U}$ = reciprocal of most probable molecular speed

c_x, c_y, c_z = components of the total velocity of a molecule in the direction of the x, y and z axes respectively.

The total velocity of a molecule is the vector sum of the satellite's velocity and the thermal velocity of the molecule. Equation (11), though unwieldy, is retained because the first part represents the force on the front side of the element of area and the second part that on the rear side, and these forces are required separately when considering a conical body in Section 7.5. It should be noted that the direction cosines appropriate to the forward facing side of the element are always used in equation (11), as in Ref.11.

6.2 Force produced by specular reflection

For specular reflection the momentum change is perpendicular to the surface, as the momenta of the impinging and re-emitted molecules are equal. The force is found from equation (12) using

$$\left.\begin{aligned} \ell_x &= 1 \\ \ell_y &= 0 \\ \ell_z &= 0\,. \end{aligned}\right\} \tag{13}$$

It is given by

$$dG_s = \frac{\rho U^2}{2} \frac{4}{\sqrt{\pi}\, s^2} \left\{ \sqrt{\pi} \left(\beta^2 U_x^2 + \tfrac{1}{2}\right) \operatorname{erf}\left(\beta U_x\right) + \beta U_x \exp\left(-\beta^2 U_x^2\right) \right\} dA\,. \tag{14}$$

6.3 Force produced by diffuse re-emission

The normal re-emission force on the front side of a differential area inclined to the flow is given by Stalder and Zurick as

$$dG_r = \frac{\rho U^2}{2} \frac{\chi}{2\, s\, s_r} dA \tag{15}$$

where $\chi = \exp\left(-s^2 \ell_{xd}^2\right) + \sqrt{\pi}\, s\, \ell_{xd} \left[1 + \operatorname{erf}\left(s\, \ell_{xd}\right)\right]$ (16)

ℓ_{xd} = cosine of the angle between the normal to the element of area and the direction of the drag

and $s_r = s\sqrt{\dfrac{T}{T_r}}$ = molecular speed ratio of re-emission.

On the rear side, distinguished by a dash, the force is

$$dG'_r = \frac{\rho U^2}{2} \frac{\chi'}{2\, s\, s_r} dA \tag{17}$$

where $\chi' = \exp\left(-s^2 \ell_{xd}'^2\right) - \sqrt{\pi}\, s\, \ell'_{xd} \left[1 - \operatorname{erf}\left(s\, \ell'_{xd}\right)\right]$. (18)

7 DRAG OF SOME SIMPLE SHAPES IN FREE MOLECULE FLOW

Several authors have investigated the drag of simple shapes in free molecule flow by integrating equations (12), (15) and (17). The results quoted in 7.1 and 7.2 are from Ref.11.

7.1 Drag with specular reflection

(i) Flat plate normal to the stream

$$C_{D_s} = 4\left(1+\frac{1}{2s^2}\right) \operatorname{erf}(s) + \frac{4}{\sqrt{\pi}\, s} \exp(-s^2) . \tag{19}$$

The second term is negligible for molecular speed ratios appropriate to satellites.

(ii) Circular cylinder perpendicular to the stream

$$C_{D_s} = \frac{4\sqrt{\pi}}{3s} \exp\left(-\frac{s^2}{2}\right)\left\{I_o\left(\frac{s^2}{2}\right) + \left(\frac{1+2s^2}{2}\right)\left[I_o\left(\frac{s^2}{2}\right) + I_1\left(\frac{s^2}{2}\right)\right]\right\} \tag{20}$$

where I_o and I_1 are Bessel functions of the first kind and imaginary argument.

For large values of s the Bessel functions can be expanded asymptotically to give

$$C_{D_s} = \frac{8}{3}\left[1 + \frac{1}{s^2}\right] . \tag{21}$$

(iii) Sphere

$$C_{D_s} = 2\left(1 + \frac{1}{s^2} - \frac{1}{4s^4}\right) \operatorname{erf}(s) + \frac{2s^2+1}{\sqrt{\pi}\, s^3} \exp(-s^2) . \tag{22}$$

The second term is negligible for molecular speed ratios appropriate to satellites.

7.2 Drag with diffuse re-emission

(i) Flat plate normal to the stream

$$C_D = 2\left(1+\frac{1}{2s^2}\right) \operatorname{erf}(s) + \frac{2}{\sqrt{\pi}\, s} \exp(-s^2) + \frac{\sqrt{\pi}}{s_r} . \tag{23}$$

(ii) Flat plate inclined to the stream at angle of attack γ

$$C_D = \frac{2}{\sqrt{\pi}\, s} \exp(-s^2 \sin^2\gamma) + 2\sin\gamma\left(1+\frac{1}{2s^2}\right) \operatorname{erf}(s \sin\gamma) + \frac{\sqrt{\pi}\sin^2\gamma}{s_r} . \tag{24}$$

(iii) Circular cylinder perpendicular to the stream

$$C_D = \frac{\sqrt{\pi}}{s} \exp\left(-\frac{s^2}{2}\right)\left\{I_o\left(\frac{s^2}{2}\right) + \left(\frac{1+2s^2}{2}\right)\left[I_o\left(\frac{s^2}{2}\right) + I_1\left(\frac{s^2}{2}\right)\right]\right\} + \frac{\pi^{3/2}}{4s_r} \quad (25)$$

$$= 2\left(1 + \frac{1}{s^2}\right) + \frac{\pi^{3/2}}{4\,s_r} \text{ for large values of s.} \quad (26)$$

(iv) Sphere

$$C_D = 2\left(1 + \frac{1}{s^2} - \frac{1}{4s^4}\right) \operatorname{erf}(s) + \frac{2s^2+1}{\sqrt{\pi}\,s^3} \exp(-s^2) + \frac{2\sqrt{\pi}}{3s_r} . \quad (27)$$

7.3 Drag of an inclined circular cylinder of infinite length

The drag of a circular cylinder with its axis inclined to the direction of motion at an angle γ, as shown in Fig.8, and with diffuse reflection has been obtained from equations (12), (15) and (17). The calculation is given in the appendix. The result for an infinite cylinder, using the area parallel to the cylinder axis as the reference area, is

$$C_D = \frac{\pi^{\frac{1}{2}}}{s} \exp\left(-\frac{s^2\sin^2\gamma}{2}\right)\left\{I_o\left(\frac{s^2\sin^2\gamma}{2}\right) + \left(\frac{1+2s^2}{2}\right)\sin^2\gamma\left[I_o\left(\frac{s^2\sin^2\gamma}{2}\right) + I_1\left(\frac{s^2\sin^2\gamma}{2}\right)\right]\right\} + \frac{\pi^{3/2}}{4s_r}\sin^2\gamma . \quad (28)$$

7.4 Drag of an inclined cylinder of finite length

To obtain the drag of a circular cylinder of finite length and radius r, the drag of the two ends is considered separately from that of the curved surface. The total drag is the sum of two parts: the drag of an inclined cylinder of length L and the drag of a flat plate of area πr^2. For a cylinder inclined at an angle γ as shown in Fig.8 the drag is given by

$$C_D S = \frac{D}{\frac{1}{2}\rho U^2}$$

$$= 2rL\left\{\frac{\pi^{\frac{1}{2}}}{s}\exp\left(-\frac{s^2\sin^2\gamma}{2}\right)\left[I_o\left(\frac{s^2\sin^2\gamma}{2}\right) + \left(\frac{1+2s^2}{2}\right)\sin^2\gamma\left(I_o\left(\frac{s^2\sin^2\gamma}{2}\right) + I_1\left(\frac{s^2\sin^2\gamma}{2}\right)\right)\right] + \frac{\pi^{3/2}}{4s_r}\sin^2\gamma\right\} + \pi r^2 \frac{2}{\sqrt{\pi}\,s}\left[\exp(-s^2\cos^2\gamma) + \sqrt{\pi}\,s\cos\gamma\left(1+\frac{1}{2s^2}\right)\operatorname{erf}(s\cos\gamma) + \frac{\pi s}{2s_r}\cos^2\gamma\right] \quad (29)$$

where S is the area normal to the direction of motion.

7.5 Drag of a conical body

For a conical body there are four cases to consider, depending on whether the apex or base is leading (see Fig.9) and on the angle of attack (γ or η). If the apex is leading and the angle of attack is less than the semi-vertex angle of the cone δ, the curved surface is entirely exposed to the main flow. When the angle of attack exceeds the semi-vertex angle some of the curved surface is partially shielded from the main flow. The direction cosine ℓ between the stream velocity and the normal to the surface of the cone is given by

$$\ell = \cos\gamma \sin\delta - \sin\gamma \cos\delta \cos\phi \tag{30}$$

where ϕ is an angle measured around the base as shown in Fig.9. The partially shielded area is bounded by lines along which the stream velocity is parallel to the surface, i.e. by lines defined by $\ell = 0$, which occur at values of ϕ given by

$$\phi_1 = \cos^{-1} \frac{\tan\delta}{\tan\gamma} . \tag{31}$$

Similarly, if the base is leading, the curved surface is entirely shielded from the main flow if the angle of attack is less than the semi-vertex angle, and is partially exposed if the angle of attack exceeds the semi-vertex angle.

The element of area considered is shown in Fig.9 and is

$$dA = \frac{r^2}{2\sin\delta} d\phi \tag{32}$$

where r is the radius of the base. If $\delta < \gamma < \pi/2$, the total cross-sectional area perpendicular to the direction of motion is

$$S = \frac{r^2}{\sin\delta} \int_{\phi_1}^{\pi} (\cos\gamma \sin\delta - \sin\gamma \cos\delta \cos\phi)\, d\phi$$

$$= \frac{r^2}{\sin\delta} \left[\cos\gamma \sin\delta\,(\pi - \phi_1) + \sin\gamma \cos\delta \sin\phi_1\right] . \tag{33}$$

If $0 \leqslant \gamma \leqslant \delta$, the area is

$$S = \pi r^2 \cos\gamma . \tag{34}$$

The total drag is found by adding the drag of the curved surface to that of the base. For $0 \leqslant \gamma \leqslant \pi/2$, the drag of the base is that of the shielded side of a flat plate. Using equations (11) and (17), it is given by

$$S\,C_{D_{base}} = \frac{\sqrt{\pi}\,r^2}{s}\left\{\exp(-s^2\cos^2\gamma) - \sqrt{\pi}\,s\cos\gamma\left(1+\frac{1}{2s^2}\right)\left[1-\operatorname{erf}(s\cos\gamma)\right]\right\}$$

$$-\frac{\pi r^2\cos\gamma}{2\,s\,s_r}\left\{\exp(-s^2\cos^2\gamma) - \sqrt{\pi}\,s\cos\gamma\left[1-\operatorname{erf}(s\cos\gamma)\right]\right\}. \quad (35)$$

For $0 \leqslant \eta \leqslant \pi/2$ the drag of the base is that of the front side of a flat plate and is given by

$$S\,C_{D_{base}} = \frac{\sqrt{\pi}\,r^2}{s}\left\{\exp(-s^2\cos^2\eta) + \sqrt{\pi}\,s\cos\eta\left(1+\frac{1}{2s^2}\right)\left[1+\operatorname{erf}(s\cos\eta)\right]\right\}$$

$$+\frac{\pi r^2\cos\eta}{2\,s\,s_r}\left\{\exp(-s^2\cos^2\eta) + \sqrt{\pi}\,s\cos\eta\left[1+\operatorname{erf}(s\cos\eta)\right]\right\}. \quad (36)$$

The drag of the curved surface (c.s.) will now be written down making use of the work of Stalder and Zurick.

(a) Apex leading, $0 \leqslant \gamma \leqslant \delta$.

$$S\,C_{D_{cs}} = \frac{r^2}{\sqrt{\pi}\,s^2\sin\delta}\int_0^{\pi}\Omega\,d\phi + \frac{r^2}{2\,s\,s_r\sin\delta}\int_0^{\pi}\ell\,\chi\,d\phi \quad (37)$$

where

$$\Omega = s\left\{\exp(-s^2\ell^2) + \sqrt{\pi}\,s\ell\,[1+\operatorname{erf}(s\ell)]\right\} + \frac{\sqrt{\pi}}{2}\ell\,[1+\operatorname{erf}(s\ell)]$$

$$\chi = \exp(-s^2\ell^2) + \sqrt{\pi}\,s\,\ell\,[1+\operatorname{erf}(s\ell)].$$

For $\gamma = 0$, this reduces to

$$S\,C_{D_{cs}} = \frac{\sqrt{\pi}\,r^2}{s\sin\delta}\left\{\exp(-s^2\sin^2\delta) + \sqrt{\pi}\,s\sin\delta\left(1+\frac{1}{2s^2}\right)\left[1+\operatorname{erf}(s\sin\delta)\right]\right\}$$

$$+\frac{\pi r^2}{2\,s\,s_r}\left\{\exp(-s^2\sin^2\delta) + \sqrt{\pi}\,s\sin\delta\left[1+\operatorname{erf}(s\sin\delta)\right]\right\}. \quad (38)$$

(b) Apex leading, $\delta \leqslant \gamma \leqslant \pi/2$.

$$S\,C_{D_{cs}} = \frac{r^2}{\sqrt{\pi}\, s^2 \sin\delta}\left\{\int_0^{\phi_1} \Omega'\, d\phi + \int_{\phi_1}^{\pi} \Omega\, d\phi\right\}$$

$$+ \frac{r^2}{2\, s\, s_r \sin\delta}\left\{-\int_0^{\phi_1} \ell'\, \chi'\, d\phi + \int_{\phi_1}^{\pi} \ell\, \chi\, d\phi\right\} \quad (39)$$

where

$$\Omega' = s\left\{\exp\,(-s^2\, \ell'^2) - \sqrt{\pi}\, s\, \ell'\, [1 - \mathrm{erf}\,(s\ell')]\right\} - \frac{\sqrt{\pi}}{2}\, \ell'\, [1 - \mathrm{erf}\,(s\,\ell')]$$

$$\chi' = \exp\,(-s^2 \ell'^2) - \sqrt{\pi}\; s\; \ell'\; [1 - \mathrm{erf}\,(s\ell')]$$

and $\ell' = -\ell$.

When $\gamma = \pi/2$ the integrals can be evaluated to give

$$S\,C_{D_{cs}} = \frac{\sqrt{\pi}\, r^2}{s \sin\delta} \exp\left(-\frac{s^2 \cos^2\delta}{2}\right)\left\{I_0\left(\frac{s^2 \cos^2\delta}{2}\right)\right.$$

$$\left. + s^2\left(1 + \frac{1}{2s^2}\right) \cos^2\delta \left[I_0\left(\frac{s^2 \cos^2\delta}{2}\right) + I_1\left(\frac{s^2 \cos^2\delta}{2}\right)\right]\right\}$$

$$+ \frac{\pi^{3/2}\, r^2 \cos^2\delta}{4\, s_r \sin\delta}\,. \quad (40)$$

For high molecular speed ratios, the Bessel functions can be expanded asymptotically to give

$$S\,C_{D_{cs}} = \frac{r^2}{s^2 \sin\delta \cos\delta}\left\{\left(1 - \frac{1}{4\, s^2 \cos^2\delta}\right) + (s^2 + \tfrac{1}{2}) \cos^2\delta \left(2 + \frac{1}{2\, s^2 \cos^2\delta}\right)\right\}$$

$$+ \frac{\pi^{3/2}\, r^2 \cos^2\delta}{4\, s_r \sin\delta}\,. \quad (41)$$

(c) Base leading, $0 \leqslant \eta \leqslant \delta$.

In this case the entire curved surface is shielded from the main flow. Using equation (11), the drag is given by

$$S\,C_{D_{cs}} = \frac{r^2}{\sqrt{\pi}\, s^2 \sin\delta}\int_0^{\pi} \Omega_1\, d\phi - \frac{r^2}{2\, s\, s_r \sin\delta}\int_0^{\pi} \ell\, \chi_1\, d\phi \quad (42)$$

where

$$\Omega_1 = s\left\{\exp\left(-s^2 \ell^2\right) - \sqrt{\pi}\, s\, \ell\, [1 - \mathrm{erf}\,(s\ell)]\right\} - \frac{\sqrt{\pi}}{2}\, \ell\, [1 - \mathrm{erf}\,(s\,\ell)]$$

$$\chi_1 = \exp\left(-s^2 \ell^2\right) - \sqrt{\pi}\, s\, \ell\, [1 - \mathrm{erf}\,(s\,\ell)]$$

and $\ell = \cos\eta \sin\delta - \sin\eta \cos\delta \cos\phi$.

For $\eta = 0$, the integrals can be evaluated to give

$$S\, C_{D_{os}} = \frac{\sqrt{\pi}\, r^2}{s \sin\delta}\left\{\exp\left(-s^2 \sin^2\delta\right) - \sqrt{\pi}\, s \sin\delta\left(1 + \frac{1}{2s^2}\right)[1 - \mathrm{erf}\,(s \sin\delta)]\right\}$$

$$- \frac{\pi r^2}{2\, s\, s_r}\left\{\exp\left(-s^2 \sin^2\delta\right) - \sqrt{\pi}\, s \sin\delta\, [1 - \mathrm{erf}\,(s \sin\delta)]\right\}. \qquad (43)$$

(d) Base leading, $\delta \leqslant \gamma \leqslant \pi/2$.

An area bounded by the lines $\phi_1 = \cos^{-1} \dfrac{\tan\delta}{\tan\eta}$ is exposed to the main flow and the drag is now given by

$$S\, C_{D_{os}} = \frac{r^2}{\sqrt{\pi}\, s^2 \sin\delta}\left\{\int_0^{\phi_1} \Omega_2\, d\phi + \int_{\phi_1}^{\pi} \Omega_1\, d\phi\right\}$$

$$+ \frac{r^2}{2\, s\, s_r \sin\delta}\left\{\int_0^{\phi_1} \chi_2\, d\phi - \int_{\phi_1}^{\pi} \chi_1\, d\phi\right\} \qquad (44)$$

where

$$\Omega_2 = s\left\{\exp\left(-s^2 \ell'^2\right) + \sqrt{\pi}\, s\, \ell'\, [1 + \mathrm{erf}\,(s\ell')]\right\} + \frac{\sqrt{\pi}}{2}\, \ell'\, [1 + \mathrm{erf}\,(s\ell')]$$

$$\chi_2 = \exp\left(-s^2 \ell'^2\right) + \sqrt{\pi}\, s\, \ell'\, [1 + \mathrm{erf}\,(s\ell')]$$

$$\ell' = -\ell$$

and Ω_1, χ_1 and ℓ have the same definitions as in (c).

8 RESULTS

At the altitudes considered in this note the atmosphere is sufficiently tenuous for free molecule flow to occur, the Knudsen number being greater than 3, even at 100 nautical miles. Taking the scale height from the model atmosphere of Ref.1, equation (4) indicates a temperature of about 1200°K at an altitude of 200 nautical miles. If a surface temperature of 0°C is assumed for the satellite (see Section 5), together with an accommodation coefficient of unity, the molecular speed ratio of re-emission will be about twice the incident molecular speed ratio. The drag coefficients of some simple shapes have been calculated for molecular speed ratios from 5 to 10 and for values of $\sqrt{\frac{T}{T_r}}$ of 1 and 2 which are two likely values for altitudes between 100 and 200 nautical miles. A value of unity for $\sqrt{\frac{T}{T_r}}$ corresponds approximately to $\alpha = 0.95$. The results are shown in Figs.10 to 12 for a flat plate normal to the direction of motion, a sphere and an infinite circular cylinder respectively. There is a difference of 5 per cent or less in the drag coefficient for the two values of $\sqrt{\frac{T}{T_r}}$. For comparison, results for specular reflection are also given, when the drag can be either greater than or less than that for diffuse re-emission depending on the shape of the body. The greatest error would be for a flat plate normal to the direction of motion when $C_{D_s} \sim 2C_D$, but no satellite is likely to have this shape. For a sphere C_{D_s} is 4 per cent less than C_D if $\sqrt{\frac{T}{T_r}} = 2$ and 8 per cent less if $\sqrt{\frac{T}{T_r}} = 1$. For an infinite cylinder C_{D_s} is 27 per cent greater than C_D if $\sqrt{\frac{T}{T_r}} = 2$ and 21 per cent greater if $\sqrt{\frac{T}{T_r}} = 1$.

If reflection is neither entirely diffuse or entirely specular (probably the true state of affairs) the drag coefficient would be

$$f\ C_D + (1-f)\ C_{D_s} .$$

As it is extremely unlikely that f would be below 0.8, the errors involved in assuming f = 1 would not exceed 5 per cent for a cylinder and would be considerably less for a sphere.

For the purpose of calculating drag, the American Explorer satellites can be taken as circular cylinders, 80 inches long and 6 inches diameter. Using equation (29) values of $S\ C_D$ have been obtained for all possible inclinations to the direction of motion and are given in Fig.13 for s = 7.1, $\sqrt{\frac{T}{T_r}} = 1$ and for s = 7.1, $\sqrt{\frac{T}{T_r}} = 2.1$. These values are for Explorer I with its perigee at an altitude of nearly 200 nautical miles. Explorer I began life spinning about the longitudinal axis, but it is believed[12] that there was a fairly rapid change to rotation about the axis of maximum moment of inertia. The value of the drag coefficient to be used will depend on the mode of rotation. The two extreme modes of rotation are (a) spinning propeller-wise and (b) tumbling end-over-end: for (a) the satellite is spinning like an aeroplane propeller with the axis of spin parallel to the direction of motion; and for (b) the axis of spin is perpendicular to the direction of motion. For a cylinder with the dimensions given above, rotating about an axis transverse to its length, the drag coefficient $\left(\frac{\text{mean value of } S C_D}{\text{mean value of } S}\right)$ is plotted in Fig.14 against the angle ε between the axis of rotation and the satellite's velocity vector. The mean value of the area perpendicular to the direction of motion is

$$2\,r^2 \sin\varepsilon + \frac{4\,r\,L}{\pi} E(\varepsilon)$$

where $E(\varepsilon)$ is the complete elliptic integral of the second kind. The drag coefficient lies between 2.12 and 2.25 for $\sqrt{\frac{T}{T_r}} = 2$ and it would be about 0.10 greater for $\sqrt{\frac{T}{T_r}} = 1$.

The equation determining density at perigee ρ_p from the motion of a satellite in an elliptic orbit whose period is changing at a rate $\frac{dT}{dt}$ is[13]

$$\rho_p = -\frac{dT}{dt}\frac{m}{3\,S\,C_D}\sqrt{\frac{2e}{\pi a H}}\left\{1 - 2e - \frac{H}{8\,a\,e} + O\left(e^2, \frac{H^2}{a^2 e^2}\right)\right\} \qquad (45)$$

where m is the mass of the satellite and the scale height H is assumed constant over the small region near perigee where air drag is important. It can be seen from this equation that the quantity required is really $S\,C_D$ and not C_D. For spinning about the axis of maximum moment of inertia, the value of $S\,C_D$ for $\sqrt{\frac{T}{T_r}} = 2.1$ lies between 7.07 ft^2 for mode (a) and 4.83 ft^2 for mode (b), and in the absence of further information the best estimate of $S\,C_D$ is the mean of these two values, i.e. $S\,C_D = 5.95\ ft^2$. The value would be 6.20 for $\sqrt{\frac{T}{T_r}} = 1$. The effect of varying the length-diameter ratio is shown in Fig.15.

The satellite 1958 δ2 (Sputnik 3) is approximately conical in shape and to obtain its drag a cone of similar shape has been considered. This cone had a base diameter of 68 inches and a semi-vertex angle of 17.6°. Values of $S\,C_D$ were obtained by numerical integration of equation (35) to (44) for $s = 8.8$ and $s_r = 17$ which are appropriate to a perigee altitude of 120 nautical miles and an eccentricity of 0.1. The results are shown in Fig.16. In this case the extreme values of $S\,C_D$ occur for mode (a) and an intermediate mode of rotation. For mode (a) $S\,C_D$ is 55.07 ft^2 and for mode (b) the average value is 55.69 ft^2. The area perpendicular to the direction of motion is shown in Fig.17 for all possible inclinations to the direction of motion and the corresponding drag coefficients are shown in Fig.18. Assuming the satellite is spinning about an axis transverse to the axis of the cone the mean value of $S\,C_D$ and S are plotted in Fig.19 as functions of the inclination of the axis of rotation to the direction of motion.

9 CONCLUSIONS

Reliable estimates of the drag of a satellite can be obtained even though errors arise due to lack of knowledge of (1) the molecular speed ratio; (2) the mechanism of molecular reflection, including the fraction of molecules reflected specularly, the accommodation coefficient and the temperature of the satellite; and (3) its mode of rotation.

For the reasons given in Section 4, it is believed that the molecules are diffusely re-emitted from the surface. If as many as 20 per cent of the molecules were reflected specularly, the error in the drag would not exceed 5 per cent for a cylinder and would be negligible for a sphere. The error introduced by assuming complete accommodation, is probably less than the error introduced by assuming an incorrect mode of rotation if the satellite is spinning. For the cylinder considered in Section 8, ($d \sim 0.075$ L), the use of a mean value of S C_D of 1.78 Ld for density determination would produce a maximum error of 19 per cent if either spinning propellerwise or tumbling end-over-end was the sole mode of rotation. Values of S C_D for spinning circular cylinders of different length-diameter ratio are shown in Fig.15. Better accuracy can be obtained for spherical satellites, the drag of the sphere being combined with an average drag of the antennae. The drag of the antennae is less accurately known than that of the sphere, but their area is usually less and their contribution to the total drag is smaller. If the conical Sputnik 3 is assumed to be spinning about an axis transverse to its length, the mean value of S C_D is 1.486 Ld, which will be in error by less than 4 per cent, and which corresponds to a mean drag coefficient of 2.12 if S is taken as the mean of the values in Fig.19.

The scale height can only be obtained approximately from a density-altitude curve derived from satellite observations. Figs.3 to 5 indicate a large variation of molecular speed ratio with scale height at a particular altitude. Fortunately, the drag is not very sensitive to the molecular speed ratio, an error of 1 in s affecting the drag by less than 1.5 per cent.

No account is taken of the drag due to the accumulation of electric charge[14] or the consequences of dissociation of molecules on impact, since these effects are believed to be negligible at altitudes between 100 and 200 nautical miles.

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LIST OF SYMBOLS

Symbol	Definition
a	semi-major axis of satellite's orbit
dA	area of an element of surface
c_i	component of total velocity of a molecule in the direction i
C	root mean square speed of molecules
C_D	drag coefficient for diffuse reflection
C_{D_s}	drag coefficient for specular reflection
D	total drag
D_i	drag due to incident molecules
D_r	drag due to reflected molecules

LIST OF SYMBOLS (Contd.)

e	eccentricity of satellite's orbit
E_i	energy flux of incident molecules
E_r	energy flux of re-emitted molecules
E_W	energy flux of molecules re-emitted at the wall temperature
f	fraction of molecules reflected diffusely
g	local acceleration due to gravity
G_i	momentum force of impinging molecules
G_r	normal diffuse re-emission momentum force
G_s	total normal specular momentum force
h	altitude above reference level
H	local scale height
K	Knudsen number
ℓ_i	direction cosine between the component of force required and the i-axis
L	length of cylinder or axis of cone
m	mass of one molecule
M	molecular weight of the atmosphere
N	number of molecules per unit volume
p	atmospheric pressure
r	radius of cylinder or base of cone
R	universal gas constant
s	molecular speed ratio
s_r	molecular speed ratio of re-emission
S	frontal area of body
T	temperature of the atmosphere
T_r	temperature of gas re-emitted from the surface of the satellite
U	mass velocity
U_i	component of mass velocity in direction of i-axis
v_m	most probable molecular speed

LIST OF SYMBOLS (Contd.)

v_{m_r}	most probable molecular speed of re-emission
v_n	component of satellite's velocity perpendicular to the radius vector
v_r	component of satellite's velocity along the radius vector
x,y,z	local Cartesian coordinates
α	accommodation coefficient
β	reciprocal of most probable molecular speed
γ	angle of incidence of axis of cylinder or cone to stream
δ	semivertex angle of cone
ε	angle between axis of rotation of cylindrical satellite and its velocity vector
η	angle of incidence of axis of cone to stream (base leading)
θ	angular variable defined by Fig.4
λ	mean free path
μ	constant of earth's gravitational field
ρ	density
ρ_o	density at reference altitude
σ	diameter of a molecule
ϕ	angle measured around base of cone as shown in Fig.9
χ	dimensionless quantity defined by $\chi = \exp(-s^2 \ell_x^2) + \sqrt{\pi} . s \ell_x [1 + \text{erf}(s \ell_x)]$
χ'	dimensionless quantity defined by $\chi' = \exp(-s^2 \ell_x'^2) - \sqrt{\pi} . s \ell_x' [1 - \text{erf}(s \ell_x')]$
ψ	the true anomaly
erf(a)	error function $\frac{2}{\sqrt{\pi}} \int_0^a \exp(-t^2)\, dt$

Superscript refers to shielded surface only

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APPENDIX

THE DRAG OF AN INCLINED CIRCULAR CYLINDER

The drag acts in the direction of $\underline{U}$ so that in the notation of section 6 the stream velocity has components

$$\left.\begin{aligned} U_x &= U \sin\gamma \sin\theta \\ U_y &= -U \sin\gamma \cos\theta \\ U_z &= U \cos\gamma \end{aligned}\right\} \qquad (46)$$

with the direction cosines

$$\left.\begin{aligned} \ell_x &= \sin\gamma \sin\theta \\ \ell_y &= -\sin\gamma \cos\theta \\ \ell_z &= \cos\gamma \end{aligned}\right\} \qquad (47)$$

where θ is the angle defining the element of area as shown in Fig.7. The force due to the impinging molecules acting on the differential element of area illustrated in Fig.8, obtained by substituting equations (46) and (47) into equation (12), is

$$dG_i = \frac{\rho U^2}{2} \frac{2}{\sqrt{\pi}\, s^2} \Big\{ s \Big[\sqrt{\pi}\, s \sin\gamma \sin\theta \operatorname{erf}(s \sin\gamma \sin\theta) + \exp(- s^2 \sin^2\gamma \sin^2\theta) \Big] + \frac{\sqrt{\pi}}{2} \sin\gamma \sin\theta \operatorname{erf}(s \sin\gamma \sin\theta) \Big\} dA$$

where

$$dA = r\, L\, d\theta \, .$$

Therefore the total drag on the cylinder due to the momentum of the impinging molecules is

$$D_i = 2 r L \frac{\rho U^2}{2} \int_0^{\frac{\pi}{2}} \left[\frac{2}{\pi^{\frac{1}{2}} s} \exp\left(- s^2 \sin^2 \gamma \sin^2 \theta\right) + \right.$$

$$\left. + 2 \left(1 + \frac{1}{2 s^2}\right) \sin \gamma \sin \theta \operatorname{erf}\left(s \sin \gamma \sin \theta\right) \right] d\theta$$

$$= 2 r L \frac{\rho U^2}{2} \left[\pi^{\frac{1}{2}} \frac{\exp\left(- \frac{s^2 \sin^2 \gamma}{2}\right)}{s} I_o \left(\frac{s^2 \sin^2 \gamma}{2}\right) + \right.$$

$$+ \left(1 + \frac{1}{2 s^2}\right) \pi^{\frac{1}{2}} s \sin^2 \gamma \exp\left(- \frac{s^2 \sin^2 \gamma}{2}\right) \left\{ I_o \left(\frac{s^2 \sin^2 \gamma}{2}\right) \right.$$

$$\left. \left. + I_1 \left(\frac{s^2 \sin^2 \gamma}{2}\right) \right\} \right]$$

$$= 2r L \frac{\rho U^2}{2} \pi^{\frac{1}{2}} \frac{\exp\left(- \frac{s^2 \sin^2 \gamma}{2}\right)}{s} \left[I_o \left(\frac{s^2 \sin^2 \gamma}{2}\right) + \right.$$

$$+ \left(\frac{1 + 2 s^2}{2}\right) \sin^2 \gamma \left\{ I_o \left(\frac{s^2 \sin^2 \gamma}{2}\right) + \right.$$

$$\left. \left. + I_1 \left(\frac{s^2 \sin^2 \gamma}{2}\right) \right\} \right] .$$

The drag on a differential element of area due to re-emission, obtained from equations (15) to (18), is

$$\frac{\rho U^2}{2} \cdot \frac{\sin\theta \sin\gamma}{s\, s_r} (\chi - \chi') \, dA$$

$$= \frac{\rho U^2}{2} \cdot \frac{\sqrt{\pi} \sin^2\theta \sin^2\gamma}{s_r} \, dA \, .$$

Therefore the total drag due to re-emission is

$$D_r = \frac{\rho U^2}{2} \frac{2\sqrt{\pi} \sin^2\gamma}{s_r} \int_0^{\frac{\pi}{2}} r\, L \sin^2\theta \, d\theta$$

$$= \frac{\rho U^2}{2} \, 2\, rL \, \frac{\sqrt{\pi} \sin^2\gamma}{s_r} \, \frac{\pi}{4} \, .$$

The total drag acting on the cylinder is given by

$$D = D_i + D_r$$

$$= \frac{\rho U^2}{2} \, 2\, r\, L \left[\pi^{\frac{1}{2}} \frac{\exp\left(-\frac{s^2 \sin^2\gamma}{2}\right)}{s} \left\{ I_o \left(s^2 \frac{\sin^2\gamma}{2} \right) + \right. \right.$$

$$\left. + \sin^2\gamma \left(\frac{1 + 2\, s^2}{2} \right) \left[I_o \left(\frac{s^2 \sin^2\gamma}{2} \right) + I_1 \left(\frac{s^2 \sin^2\gamma}{2} \right) \right] \right\}$$

$$\left. + \frac{\pi^{\frac{3}{2}}}{4\, s_r} \sin^2\gamma \right] .$$

The drag coefficient based on the area parallel to the cylinder axis is

$$C_D = \frac{D}{\frac{\rho U^2}{2} \cdot 2\,r\,L} = \pi^{\frac{1}{2}} \frac{\exp\left(-\frac{s^2 \sin^2 \gamma}{2}\right)}{s} \left\{ I_0\left(\frac{s^2 \sin^2 \gamma}{2}\right) + \right.$$

$$+ \sin^2 \gamma \left(\frac{1 + 2\,s^2}{2}\right) \left[I_0\left(\frac{s^2 \sin^2 \gamma}{2}\right) + \right.$$

$$\left. \left. + I_1\left(\frac{s^2 \sin^2 \gamma}{2}\right) \right] \right\} + \frac{\pi^{\frac{3}{2}}}{4\,s_r} \sin^2 \gamma \, .$$

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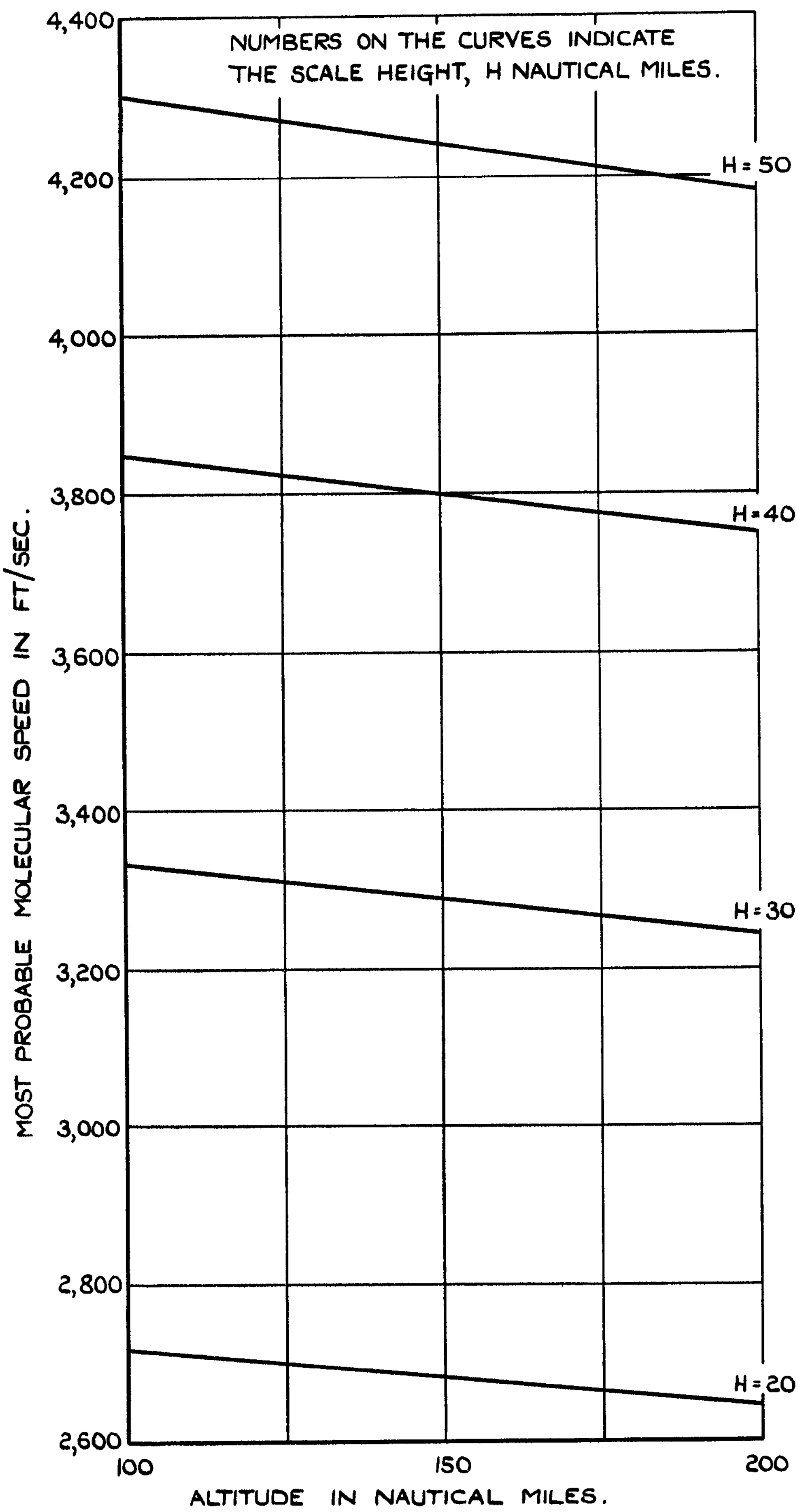


FIG. 1. VARIATION OF MOST PROBABLE MOLECULAR SPEED WITH ALTITUDE.

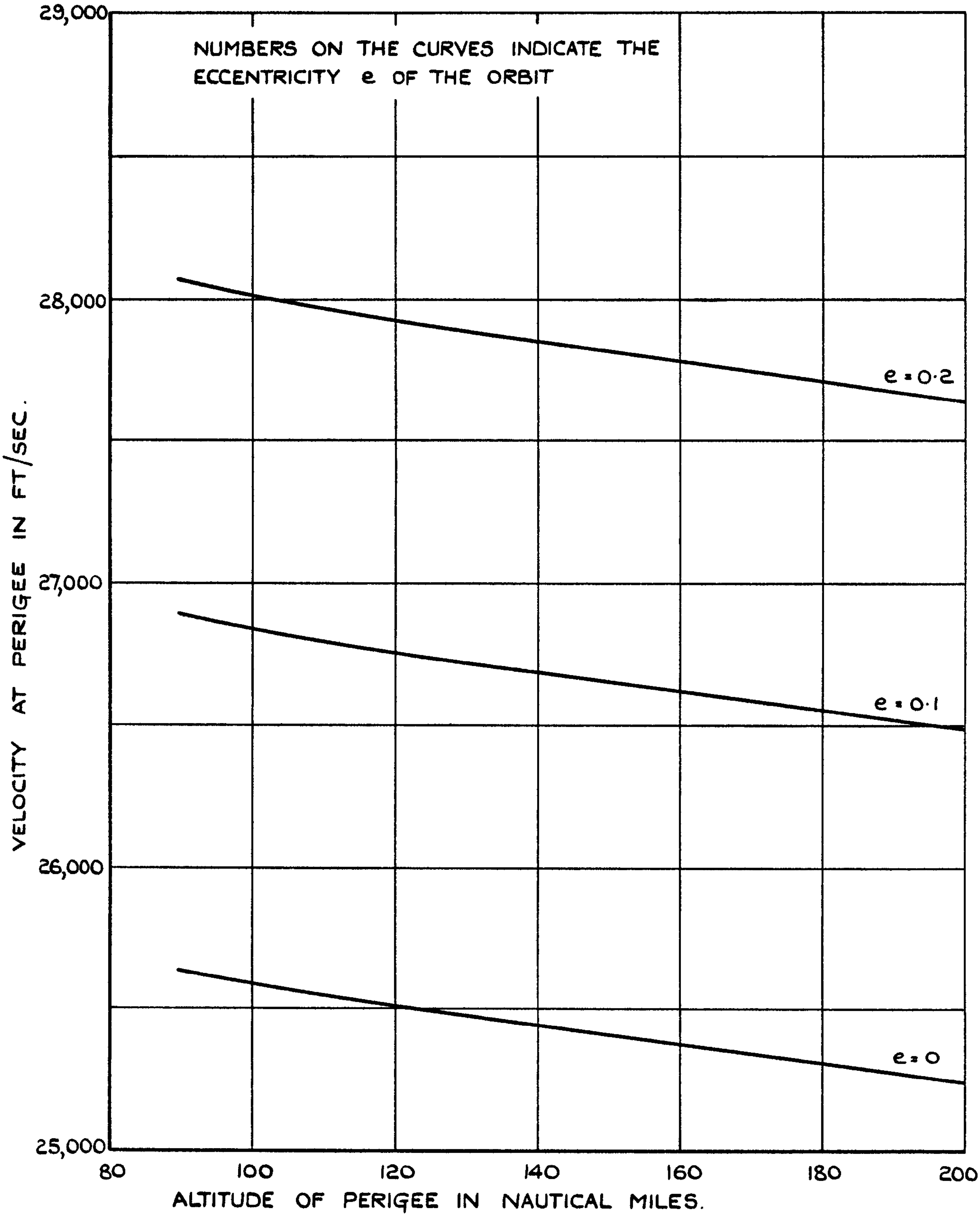


FIG. 2. VARIATION OF VELOCITY AT PERIGEE WITH ALTITUDE.

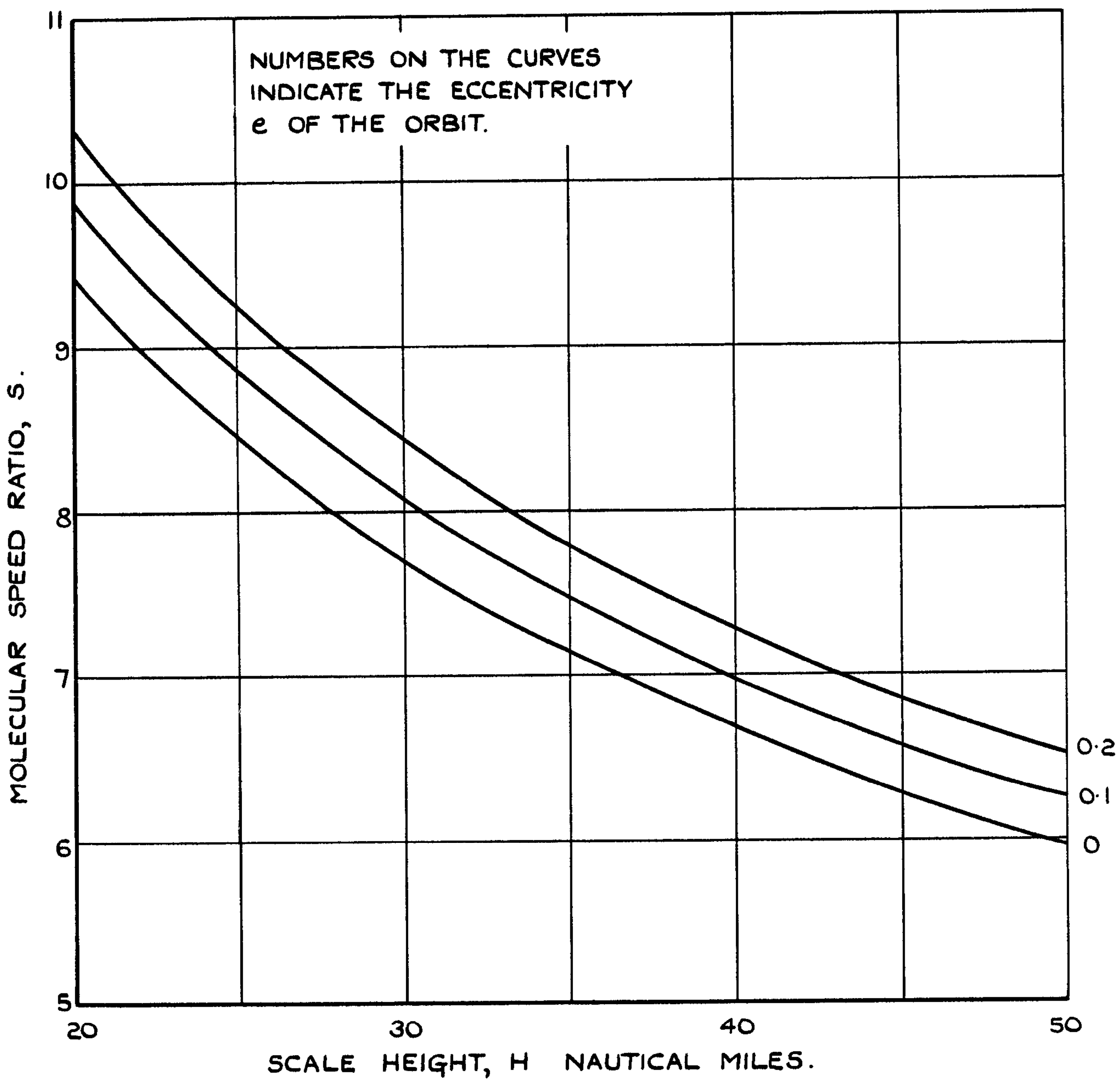


FIG. 3. MOLECULAR SPEED RATIO AT A PERIGEE ALTITUDE OF 100 N.M. FOR VARIOUS SCALE HEIGHTS.

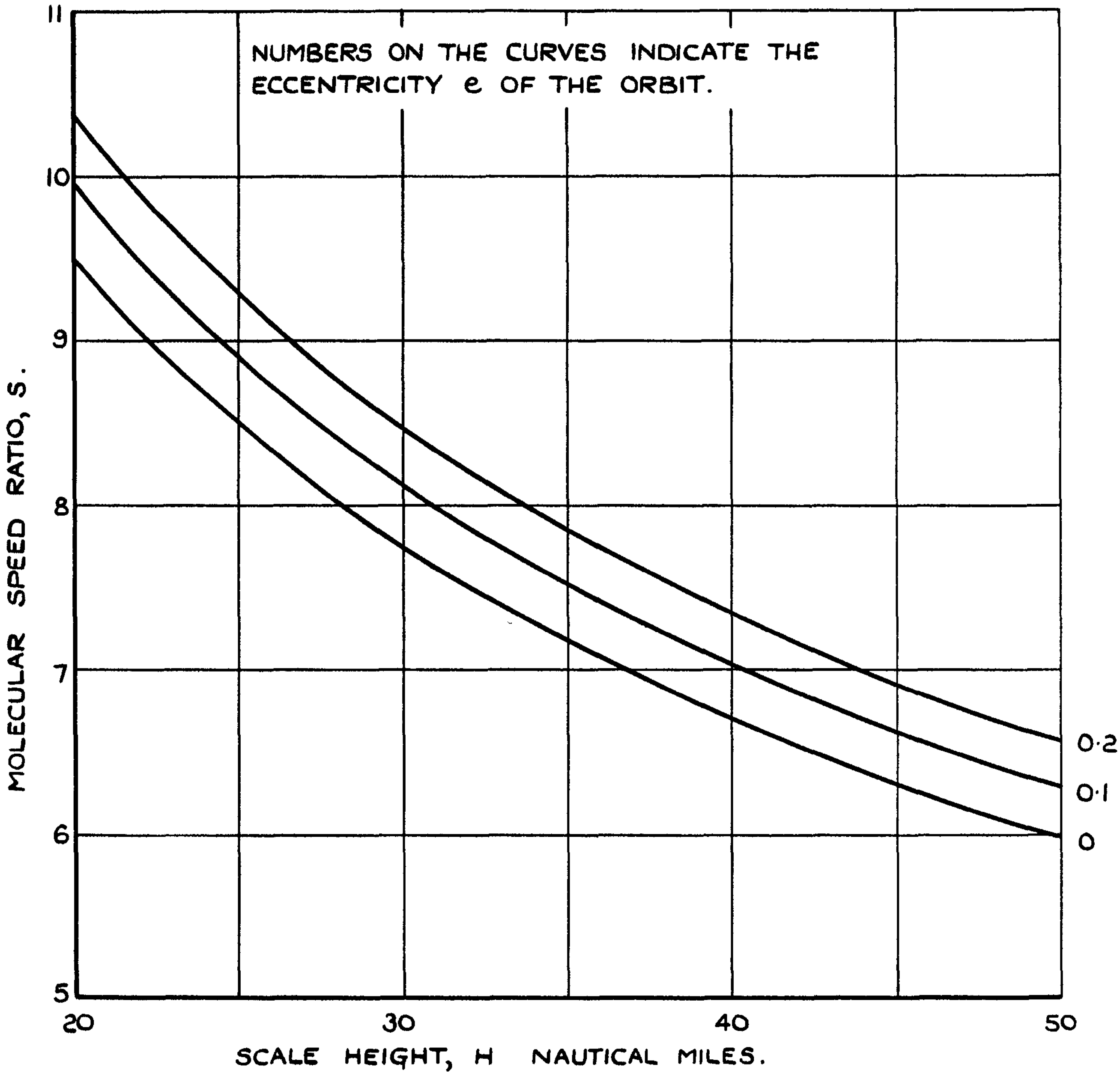


FIG. 4. MOLECULAR SPEED RATIO AT A PERIGEE ALTITUDE OF 150 N.M. FOR VARIOUS SCALE HEIGHTS.

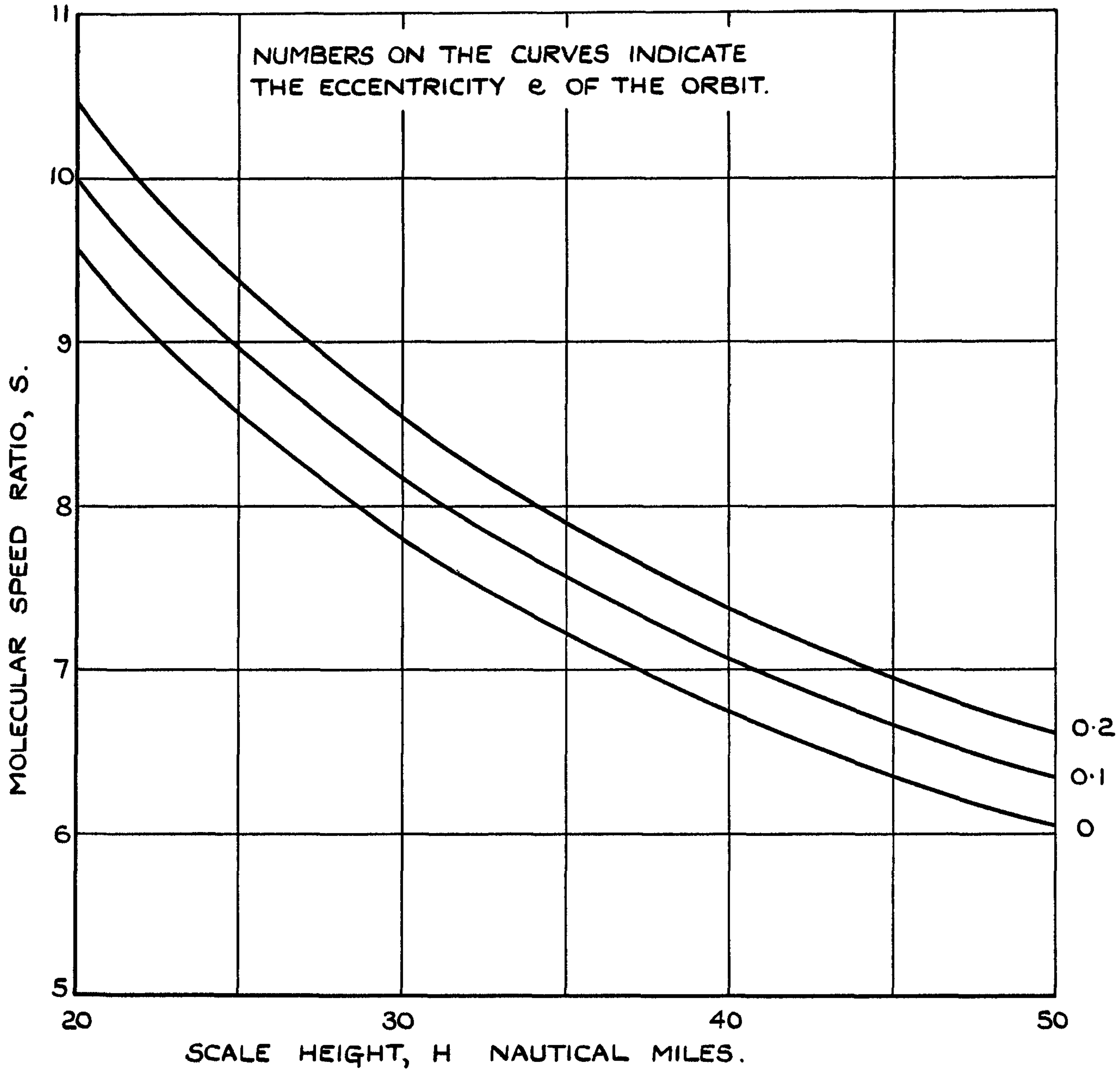


FIG. 5. MOLECULAR SPEED RATIO AT A PERIGEE ALTITUDE OF 200 N.M. FOR VARIOUS SCALE HEIGHTS.

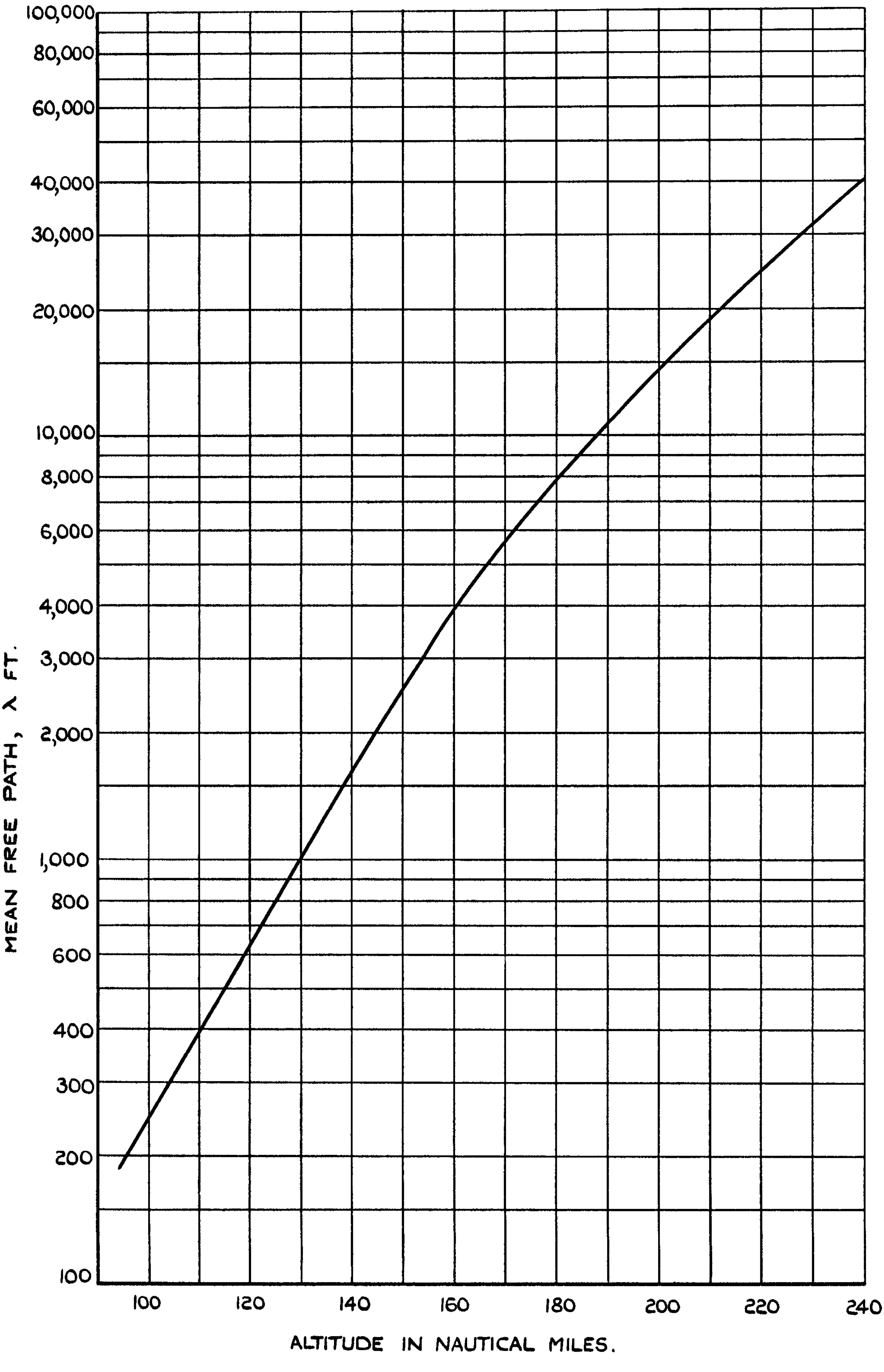


FIG. 6. MEAN FREE PATH AS A FUNCTION OF ALTITUDE ASSUMING M=15.

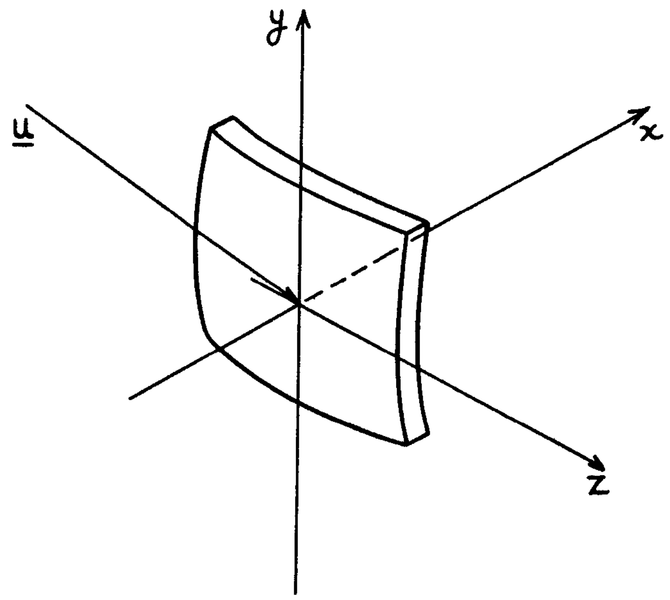


FIG. 7. COORDINATE SYSTEM FOR ELEMENT OF AREA, dA.

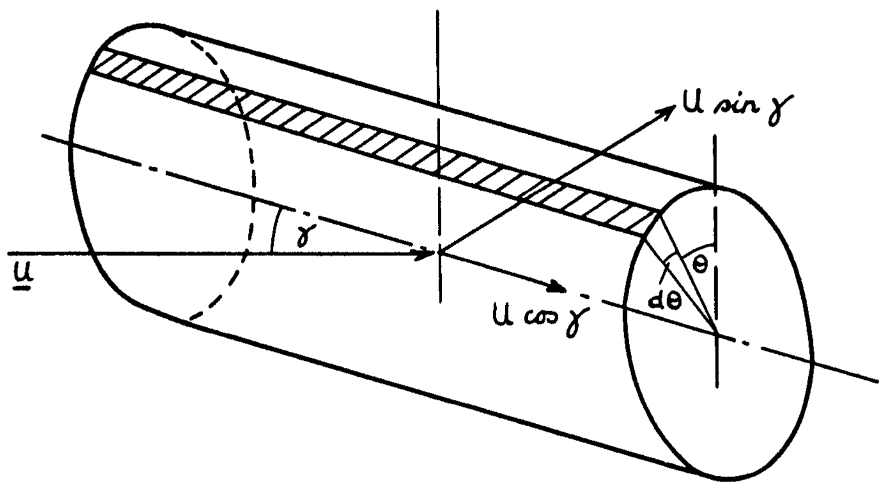
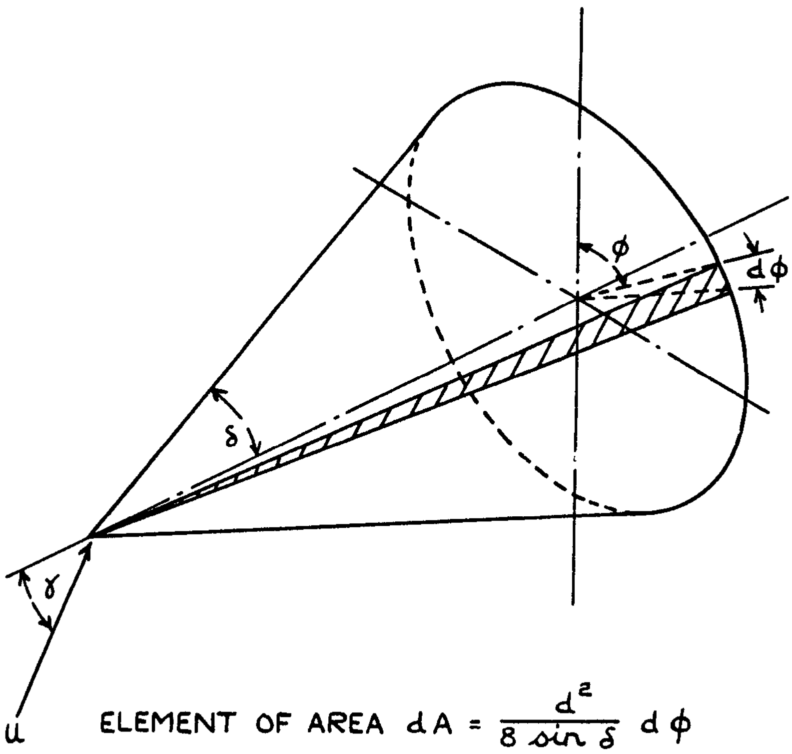
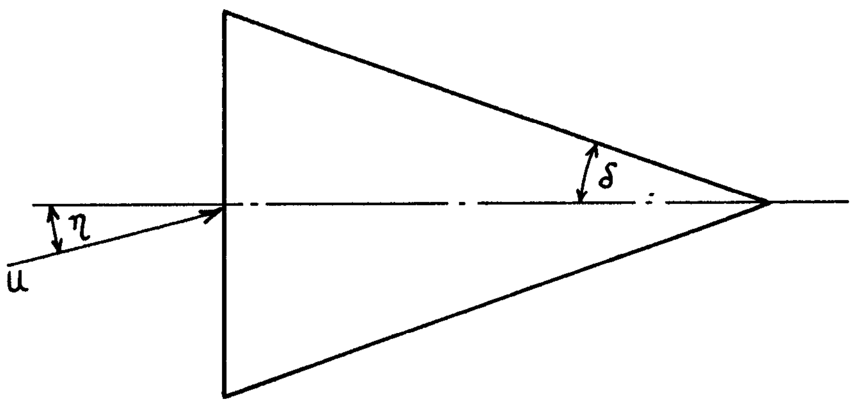


FIG. 8. ELEMENTAL AREA FOR INCLINED CYLINDER, OF LENGTH L AND RADIUS r, $dA = rL\,d\theta$.



(a) APEX LEADING, ANGLE OF ATTACK = γ



(b) BASE LEADING, ANGLE OF ATTACK = η

FIG.9 (a & b) NOTATION FOR A CONICAL BODY.

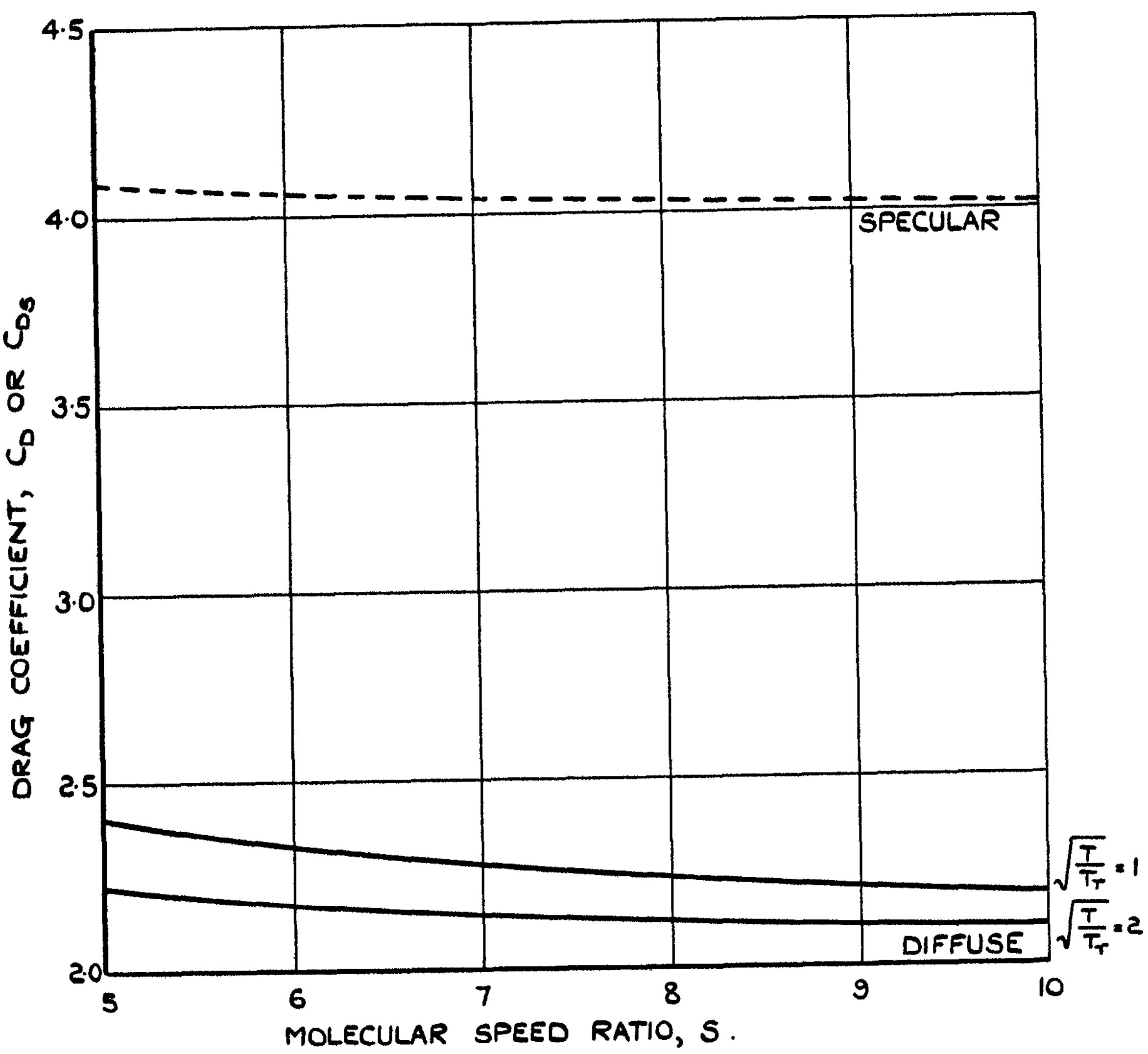


FIG. 10. DRAG OF A FLAT PLATE NORMAL TO THE DIRECTION OF MOTION.

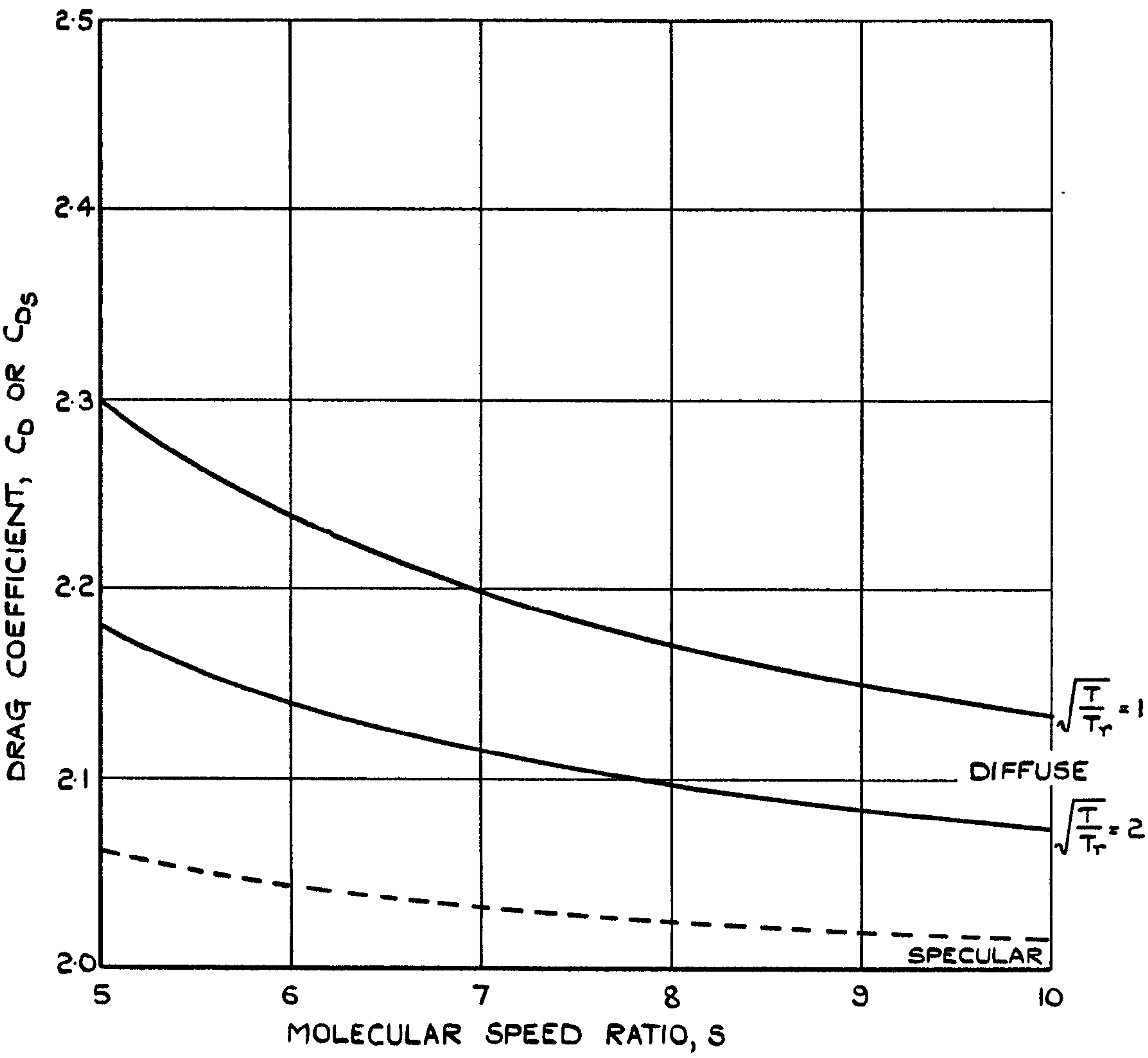


FIG. 11. DRAG OF A SPHERE.

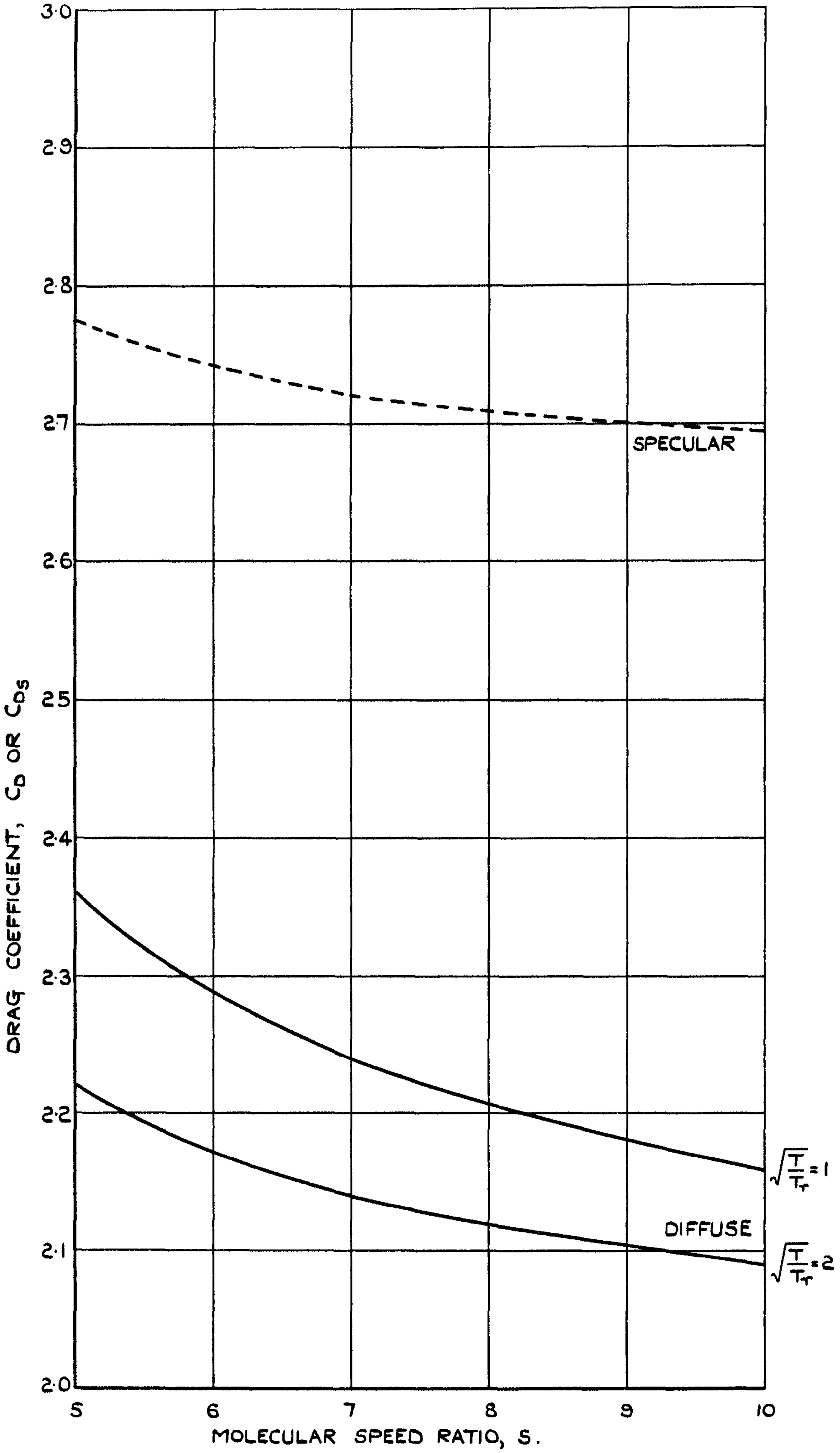


FIG. 12. DRAG OF AN INFINITE CIRCULAR CYLINDER WITH AXIS PERPENDICULAR TO THE DIRECTION OF MOTION.

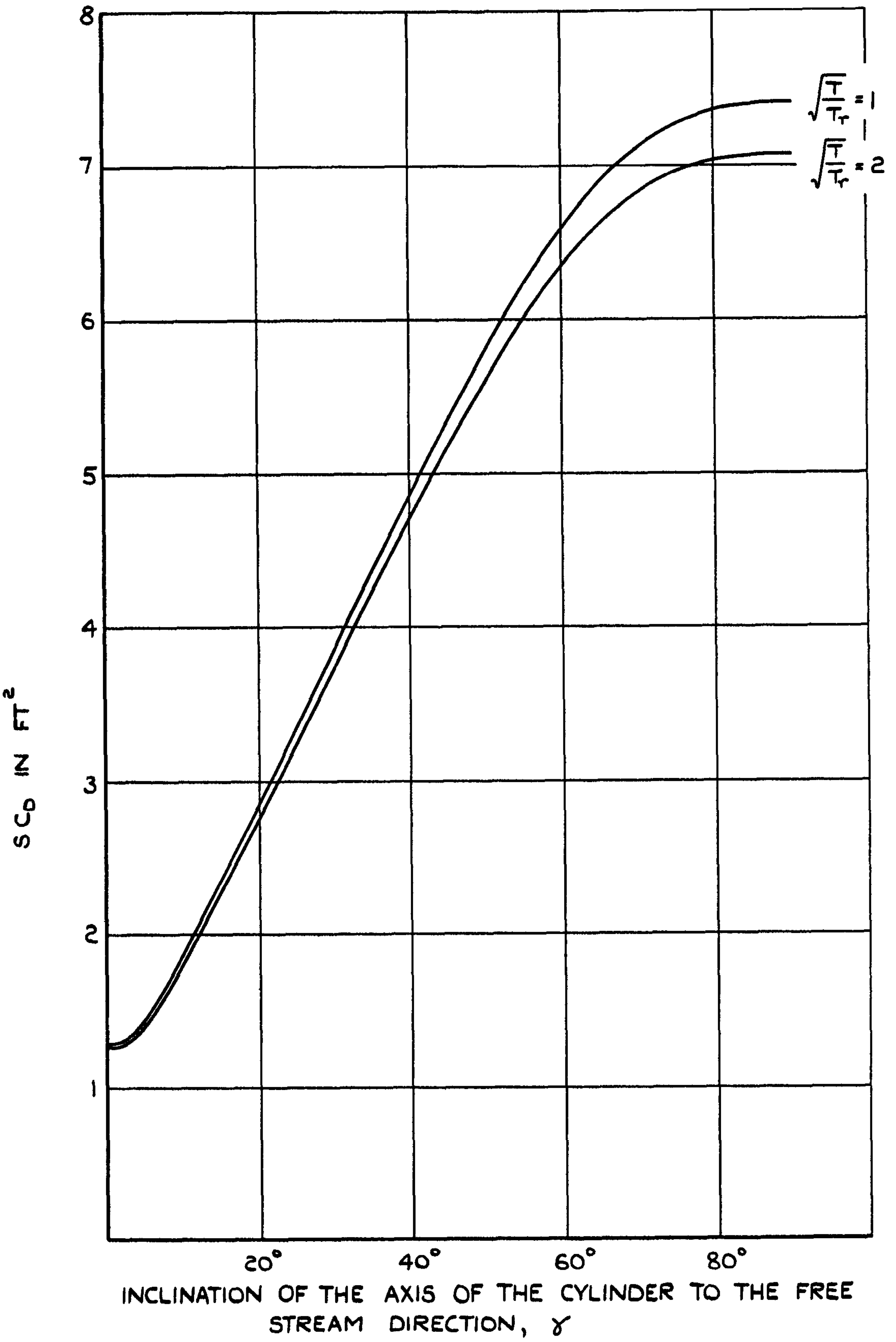


FIG. 13. SC_D FOR AN INCLINED CYLINDER WITH THE DIMENSIONS GIVEN IN SECTION 8 (S=7·1)

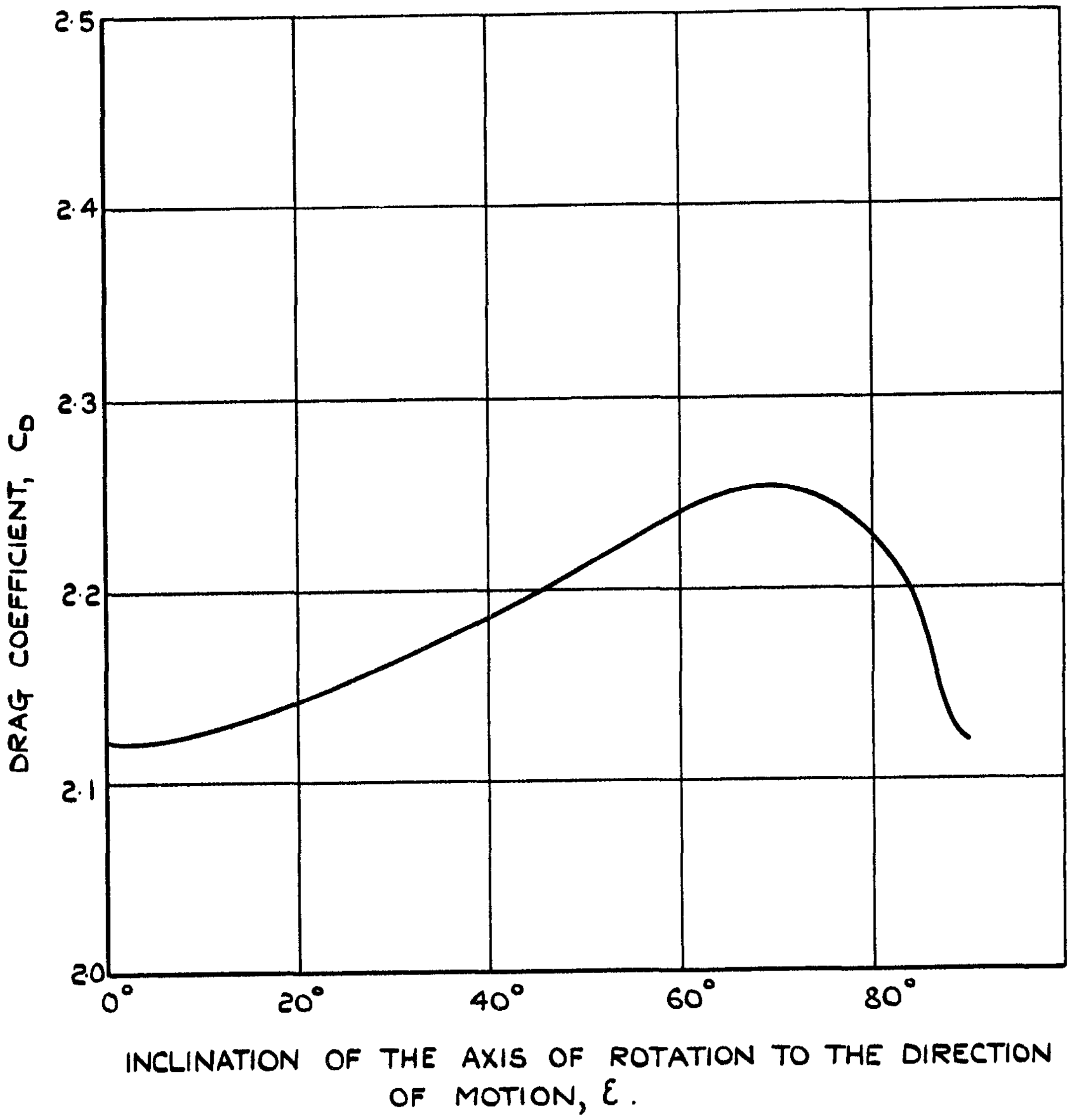


FIG. 14. VARIATION OF DRAG COEFFICIENT WITH INCLINATION OF AXIS OF ROTATION TO DIRECTION OF MOTION, FOR A CYLINDER SPINNING ABOUT AN AXIS TRANSVERSE TO ITS LENGTH $\left(S = 7 \cdot 1, \sqrt{\frac{T}{T_r}} = 2 \cdot 1\right)$

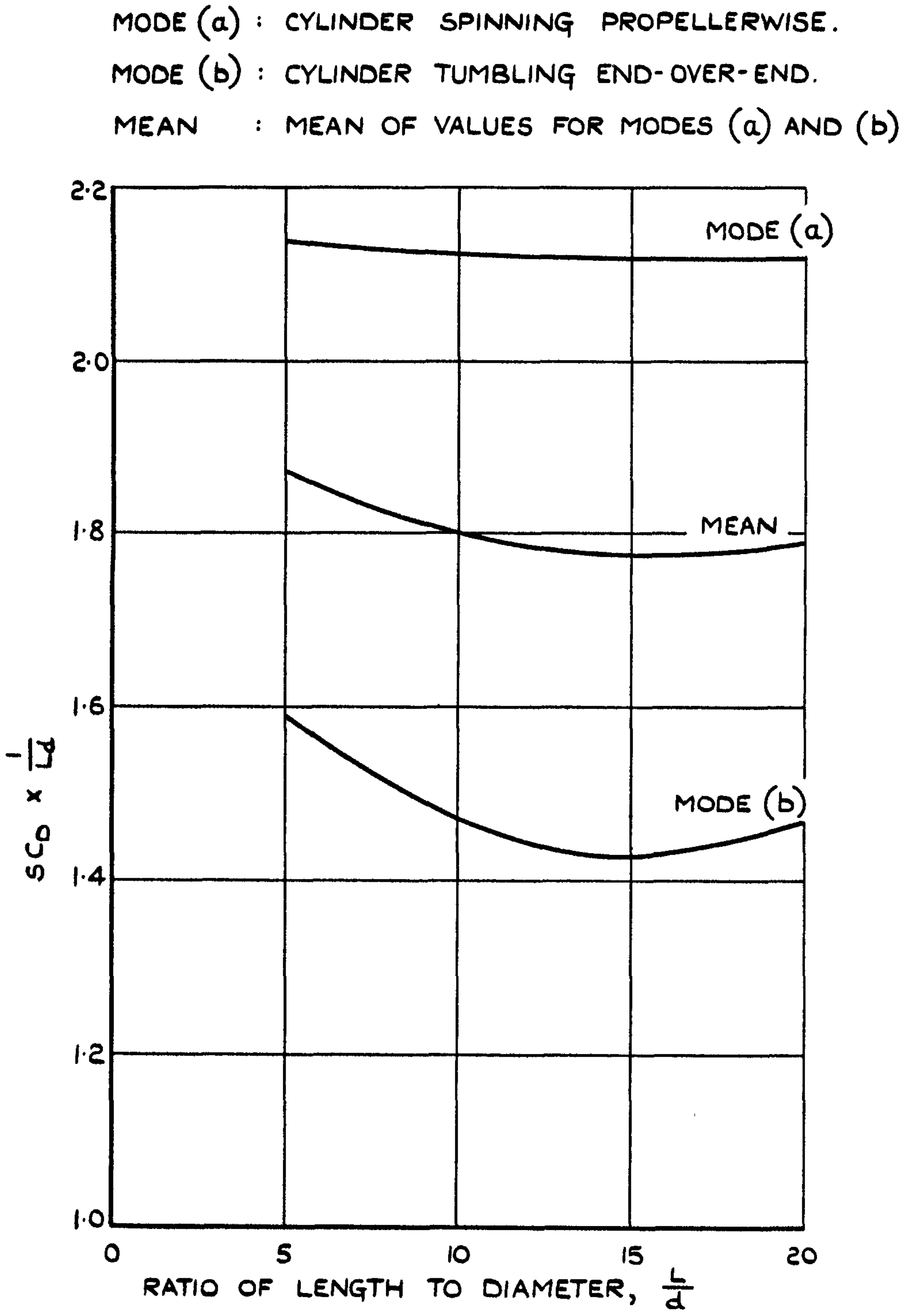


FIG.15. SC_D FOR SPINNING CIRCULAR CYLINDERS OF DIFFERENT LENGTH - DIAMETER RATIOS $\left(S = 7{\cdot}1,\ \sqrt{\frac{T}{T_r}} = 2{\cdot}1\right)$

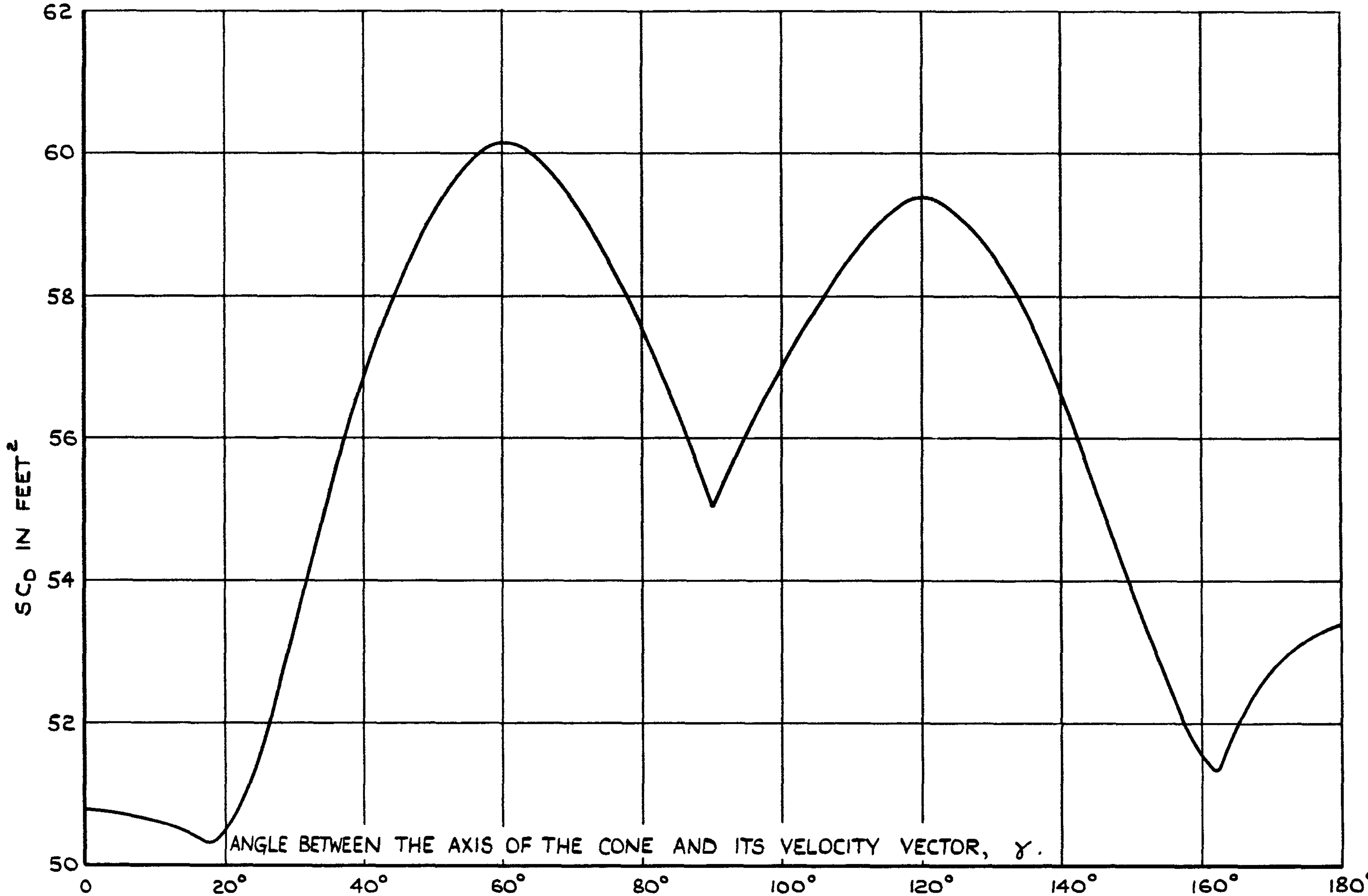


FIG. 16. SC_D FOR AN INCLINED CONE WITH THE DIMENSIONS GIVEN IN SECTION 8 (S=8·8)

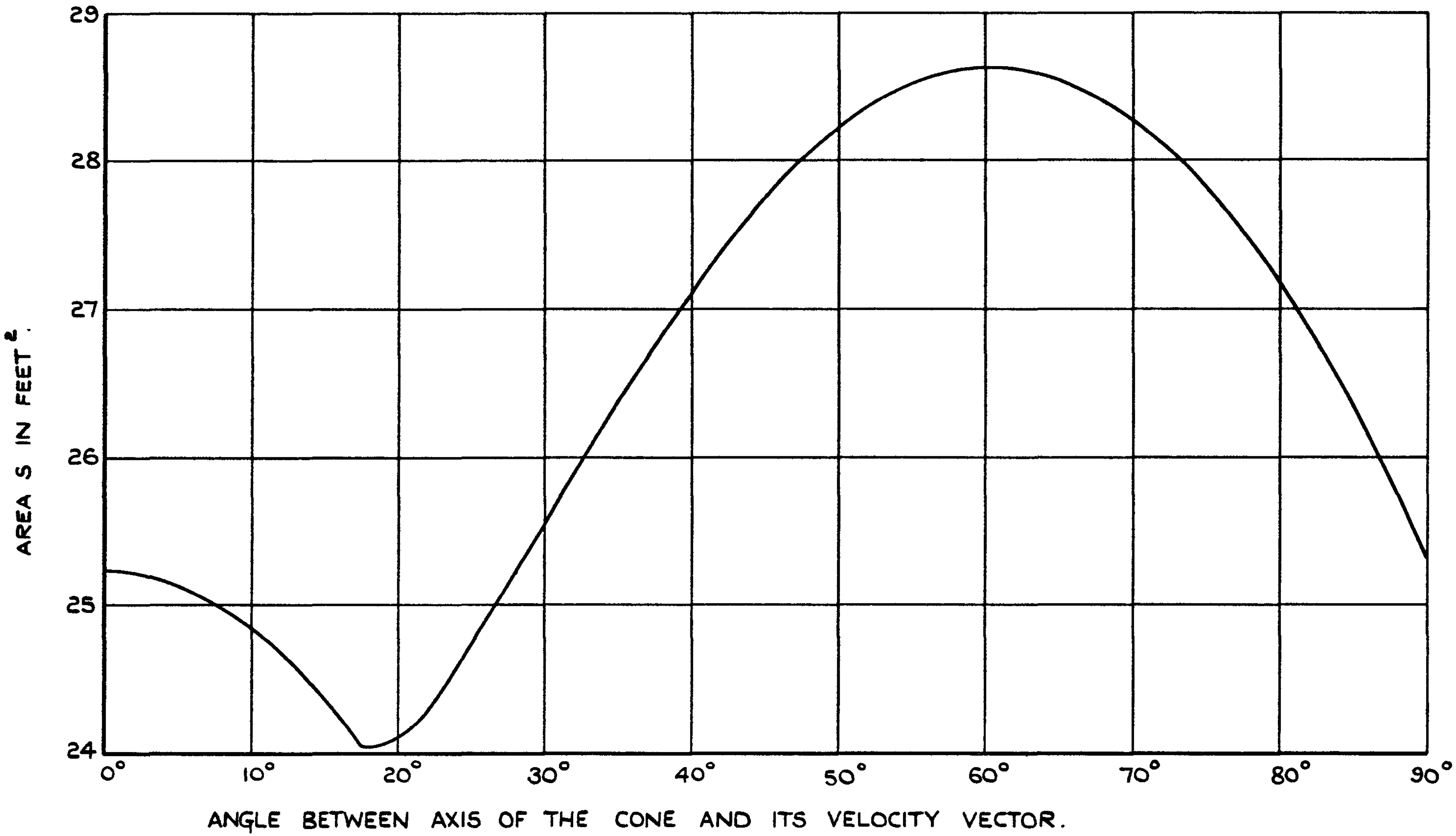


FIG.17. CROSS-SECTIONAL AREA PERPENDICULAR TO THE DIRECTION OF MOTION FOR AN INCLINED CONE WITH THE DIMENSIONS GIVEN IN SECTION 8.

FIG. 18. DRAG COEFFICIENT BASED ON THE CROSS-SECTIONAL AREA PERPENDICULAR TO THE DIRECTION OF MOTION FOR AN INCLINED CONE HAVING THE DIMENSIONS GIVEN IN SECTION 8.

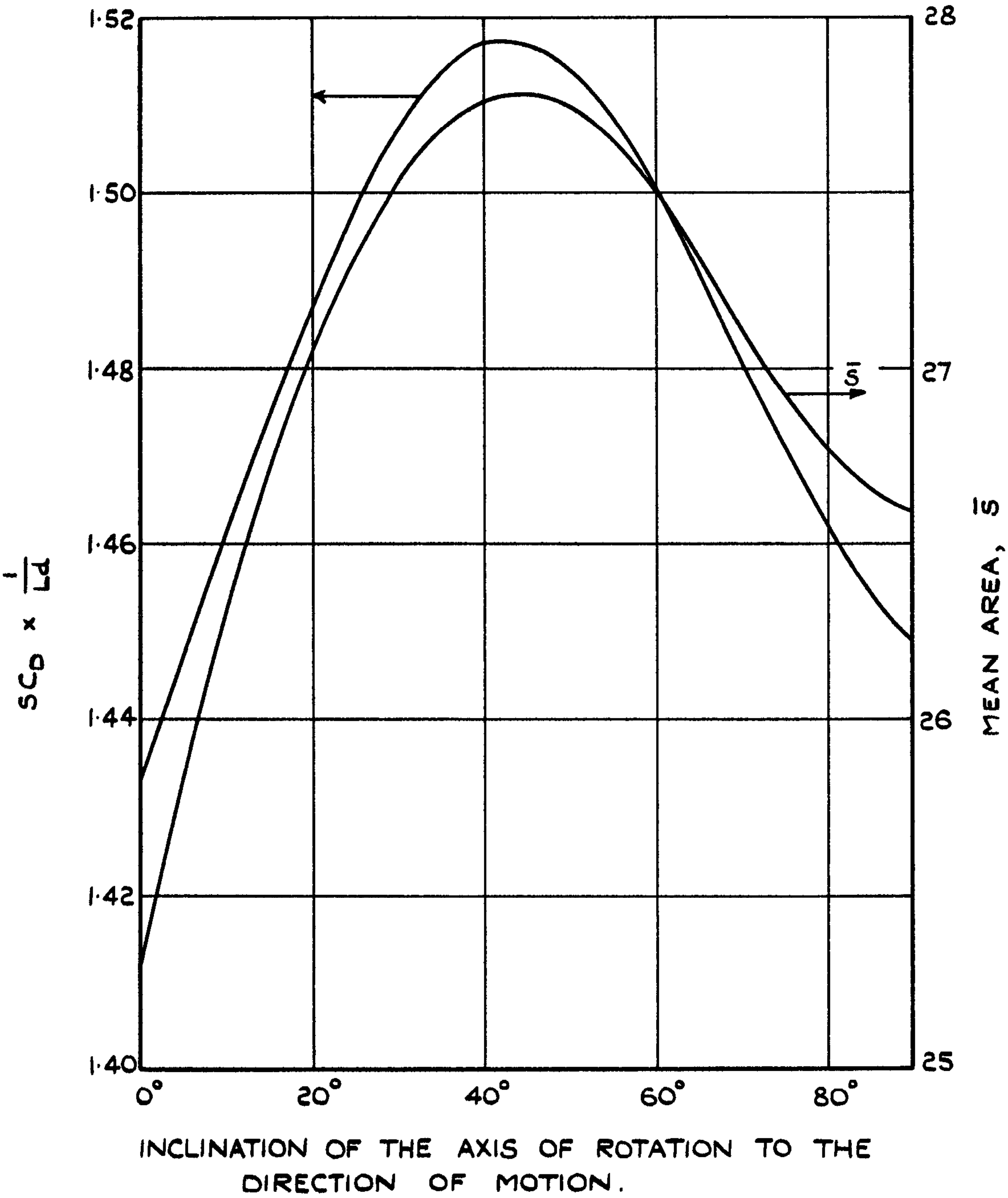


FIG. 19. VARIATION OF SC_D AND MEAN AREA $\bar{S}$ WITH INCLINATION OF AXIS OF ROTATION TO DIRECTION OF MOTION, FOR A CONE WITH THE DIMENSIONS GIVEN IN SECTION 8 SPINNING ABOUT AN AXIS TRANSVERSE TO ITS LENGTH (S = 8·8)

C.P. No.523 533.6.013.12: 525.7: 523.4

THE AERODYNAMIC DRAG OF NEAR EARTH SATELLITES. Cook, G.E. Sept. 1959.

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