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# Stress distribution in Pressurized Cabins: An Experimental Study by means of Xylonite Models.

*by*

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# Stress Distribution in Pressurized Cabins: An Experimental Study by means of Xylonite Models

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## SUMMARY

This report is concerned with the effect of openings on the stresses in plain, pressurized circular cylindrical shells.

A review of relevant theory is presented and the design, construction and testing of Xylonite models is described.

Results are summarised and discussed, and conclusions are drawn together with proposals for future work.

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## CONTENTS



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## 1. Introduction

The shells of aircraft pressure cabins have certain discontinuities and are subjected to various forms of constraint. The skin stresses, inferred by the straightforward application of membrane theory to analyses related to pressure loads, are thus not truly representative of the actual conditions obtaining in the shell, since local bending and stress concentration effects are neglected. To arrive at the exact stress distribution, or at least a close approximation thereto, some refinement of this theory is required.

The general problem of stress determination in pressure cabins is highly complex[1,2,3], so that to achieve some degree of success it is expedient to reduce the main problem to its elements and progressively refine theory, or empirical rules, to take into account the various factors involved. According to Williams[1], the principal sources of discontinuity in the main cabin are the windows and doors. The present investigation is therefore directed towards establishing the influence of openings on the stresses in a plain cylindrical shell, not constrained by stringers or hoops, when it is subjected to internal pressure.

The presence of windows and doors in the cabin wall represents portions of stress bearing sheet being replaced by non-stress transmitting elements. Local distortions in the flow of the membrane stresses thus occur, resulting in what are commonly referred to as "concentrations of stress". Experience has shown that the fatigue strength of pressurized cabins with respect to pressure cycles most certainly is significant[4,5], so that the magnitudes of these stress concentrations, which form the focal points for fatigue failures, must be brought to a minimum.

The stress intensification due to the presence of the hole may be relieved by applying suitable reinforcement to its edge, and ideally, this would reduce the concentration to zero. It has been shown that, in fact, it is always possible - theoretically at least - to design a hole and reinforcing such that the stress distribution in the remainder of the sheet is undisturbed. Such a hole is said to be neutral[6].

To investigate experimentally the stress field around various openings, having various types of reinforcing, it is convenient to apply strain measurement tests to model cylinders rather than to full scale structures. Models are frequently used in experimental engineering science; nevertheless, it is usually necessary to develop certain specialised techniques for a given type of problem. The choice of the model material itself may be the major consideration when deciding procedure; this is the case in the present investigation. Xylonite was used in this work, and a discussion on its suitability and influence on experimental technique is presented in Section 3.

Strain gauges provide a means of point by point analysis. For this problem however, it is useful to have information regarding the overall stress field in the neighbourhood of the hole. The photoelastic method of stress analysis provides such information, so that the application of the technique to tests on flat plates, supplementing the strain gauge work, should provide useful data. Considerable preparatory work has been done in this direction, but shortage of time has prevented the exploitation of this aspect of the project.

This investigation, then, is concerned with the effect of various openings on the stress in a plain shell. It represents the early stages of a project which it is hoped will ultimately encompass a complex scale model, representative of an actual aircraft main cabin, complete in all its details. Regarded in this light, the present investigation may therefore be regarded as a vehicle for the establishment of a suitable experimental technique which will be common to all the model work related to this type of problem.

## 2. Theoretical Basis of the Problem

### 2.1 Dimensional analysis of the stresses in a pressurized cylindrical shell having a reinforced opening

Unless there is axial symmetry, the solution of shell problems is extremely difficult[7]. When problems are mathematically intractable, a high degree of success is often attained by applying dimensional reasoning to the problem. Such a treatment is attempted here.

Let the stress $\sigma$ in the shell at the hole boundary depend on the pressure, $p$; hole diameter, $d$; cylinder diameter, $D$; shell thickness, $h$; cross-sectional area of cutout reinforcement, $A$; Poisson's Ratio, $\nu$; Young's Modulus, $E$. Then

$$\sigma = \phi[p, D, d, h, A, \nu, E] \qquad \ldots(1)$$

where $\phi$ is some function. Applying dimensional reasoning, this function takes the form

$$\sigma = \frac{p.D}{h}.\phi_1\left[\left(\frac{d}{D}\right), \left(\frac{A}{D^2}\right), \left(\frac{p}{E}\right), \nu\right] \qquad \ldots(2)$$

The terms $p/E$ and $\nu$ infer the influence of strain, and if this is constant, the terms may be ignored. Gurney[8], states that if $D/d > A$ the problem may be regarded as one of plane stress; then, since it is known that the size of the hole does not affect the stress distribution in a very large plate, the equation for stress reduces to the form

$\sigma$/

$$\sigma = \frac{p.D}{h}.\phi_2\left\{\left(\frac{A}{D^2}\right)\right\} \qquad \ldots(3)$$

That is, for a given size of shell, the value of the stress is the nominal increased by a factor which depends on the amount of reinforcing around the cutout. From the neutral hole theory, $\phi_2$ may be chosen such that no stress concentration occurs. In predicting full scale performance from the model results, one may then write

$$\frac{\left(\frac{\sigma.h}{p.D}\right)_m}{\left(\frac{\sigma.h}{p\ D}\right)_f} = \frac{\phi_2\left\{\left(\frac{A}{D^2}\right)\right\}_m}{\phi_2\left\{\left(\frac{A}{D^2}\right)\right\}_f}$$

or

$$\sigma_f = \sigma_m.\left(\frac{pf}{pm}\right).\left(\frac{Df}{Dm}\right).\left(\frac{hm}{hf}\right).\frac{\phi_2\left\{\left(\frac{A}{D^2}\right)\right\}_f}{\phi_2\left\{\left(\frac{A}{D^2}\right)\right\}_m} \qquad \ldots(4)$$

### 2.2 Cutouts in stress bearing sheet

Mansfield[2] has discussed the design of neutral holes in pressurized shells, and he has shown that for a developable surface, the shape of a neutral hole corresponds to that in the developed sheet. For the case of a spherical shell also, the neutral hole corresponds to that in the flat plate provided the hole diameter is small in comparison with the shell diameter. Gurney[8] also states that provided the ratio of shell diameter to hole diameter is not less than about 4:1, the problem may be regarded as one of plane stress.

A number of investigators have studied the problem of reinforcing holes in flat plates and, in view of the remarks of the preceding paragraph, it seems reasonable to utilize their general conclusions in designs related to thin shells. It should be borne in mind, however, that this theory does not take into account the conditions for radial equilibrium of an element of shell subjected to a pressure load, so that for completeness, local bending effects must be incorporated by means of the principle of super position. With this proviso in mind, some benefit will accrue from a discussion of flat plates.

In the neighbourhood of holes in a stress bearing sheet, the values of stress are considerably above the average. The value of the stress concentration factor (S.C.F.), defined as the ratio of the maximum direct stress to the larger principal stress in regions remote from the hole, is dependent upon the shape of the hole and the nature of the loading.

These areas of very high stress intensity are local in character, the magnitude of the S.C.F., decaying rapidly with the distance from the edge of the hole. As an illustration; for a sheet containing a circular hole and sustaining equal principal stresses $\sigma_0$, the value of the circumferential stress is:-

$$\sigma_\theta = \sigma_0 \left(1 + \frac{a^2}{r^2}\right) \qquad \ldots(5)$$

where a = radius of hole and r = distance from hole centre.

It is thus evident that a design which utilizes sheet of a thickness appropriate to the maximum existing stress would be grossly inefficient in terms of structural weight, and that edge reinforcement is the only reasonable way of bringing these local stresses down to safe levels. Ideally, a hole would be reinforced in such a manner that no stress concentration would occur; it is interesting to note, however, that in attempting to achieve this end, it is possible to over-stiffen the hole region, introducing what Williams[1] refers to as a "hard spot" which "attracts" load to itself by magnifying the radial pull in the surrounding sheet.

The application of edge reinforcing to holes has commanded attention for some time. As early as 1924, Timoshenko[9] obtained an approximate solution of the problem using the theory of curved bars. By first applying the theory to an unreinforced opening, he was able to justify that method by comparison with the known exact solution; then, for a reinforced opening, the only changes involved are in the properties of the modified cross section of the "apparent curved beam". The method, however, does not take into account the stress concentrations which arise at the junction of plate and the ring due to sudden changes in section.

Gurney[8], using the principles of theory of elasticity, attempted to find the optimum reinforcement of constant section which would render a circular hole neutral under specified conditions. Two important conclusions may be drawn from this work. First, neutrality can only be obtained under the special conditions of uniform stress in all directions. Secondly, the bending stiffness of the reinforcing, in the plane of the sheet is not as important as the cross sectional area in governing the stress values.

Benskin[10], in 1944, extended Gurney's work to allow for a thin ring along the inside edge of a doubler plate.

Levy and others[11] have found that the optimum reinforcement is that which has the material arranged to give a very compact ring.

Reissner and Morduchow[12] treated a slightly more general problem. Like Gurney, they used the theory of elasticity and restricted themselves to circular cutouts but they permitted variation of the section area and moment of inertia along the circumference of the reinforcing ring. The properties of the reinforcing were then expressed as rather cumbersome functions of the appropriate stress function for the plate. They concluded that only for the case of equal principal stresses could neutrality be obtained and that the bending stiffness of the reinforcing was not as important as the cross sectional area. In fact, for the case of equal principal stresses, the moment of inertia was found to be arbitrary.

The work so far discussed was directed towards obtaining an optimum reinforcement for a circular cutout, bringing the stress concentration to zero, or at least a minimum value. Mansfield[6] based his work on an entirely new concept; that is, specifying zero stress concentration and allowing the hole and reinforcing to take whatever form theory predicted.

If one is given a free choice as to the shape of the hole and the stiffness of the reinforcement, then it can be shown that it is always possible to design a hole whose presence does not affect the stress

distribution/

distribution in the remainder of the sheet. Such a hole, which is elastically equivalent to the uncut sheet, is then neutral.

By considering the equilibrium of an element of sheet bounding on the hole, neglecting the bending strength of the reinforcement, and specifying compatibility of strains at the boundary of the sheet and reinforcement, then the shape of the neutral hole is given by:-

$$\phi = 0 \qquad \text{...(6)}$$

where $\phi$ is the Airy stress function defining the stresses in the sheet and which satisfies the biharmonic equation

$$\nabla^4 \phi = 0 . \qquad \text{...(7)}$$

The stress components, in terms of Cartesian co-ordinates, are related to the stress function by the equations:-

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} : \quad \sigma_y = \frac{\partial^2 \phi}{\partial x^2} : \quad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} \qquad \text{...(8)}$$

It is seen, therefore, that the shape of a neutral hole depends on the stress distribution in the sheet. In general, the principal stresses are unequal, but if they are of the same sign, the true shape of the neutral hole is elliptical with the major axis in the direction of the larger principal stress. Furthermore, the cross sectional area of the reinforcement is not constant, but it is a maximum and minimum at the ends of the major and minor axes respectively. If the principal stresses are equal and of the same sign, the ellipse degenerates into a circle and the reinforcement is of constant section. For the particular case of a thin wall cylinder having no hoops or stringers and subjected to internal pressure, the hoop stress is twice the longitudinal stress and the ellipse then has axes in the ratio $\sqrt{2}:1$.

For a circular hole, the reinforcement cross sectional area is:-

$$A_r = \frac{r.t}{1 - \nu} \qquad \text{...(9)}$$

and for the $\sqrt{2}:1$ ellipse:-

$$A_r = \frac{\sqrt{2}.r.t \left(1 + \frac{x^2}{r^2}\right)^{3/2}}{1 - 2\nu + 3\frac{x^2}{r^2}} \qquad \text{...(10)}$$

where $2r$ = minor axis of ellipse and diameter of circular neutral hole.

$\nu$ = Poisson's Ratio

$\sigma_y = 2\sigma_x$ = principal stresses

$x$ = distance along minor axis, with origin at the intersection of the axes.

More/

More recently, Mansfield[13] has restudied the reinforcing of circular holes. Since, in general, this shape of cutout cannot be neutral, he has determined the optimum shapes and the corresponding S.C.F.'s for various conditions. These are expressed in terms of perturbation stress coefficients and like the elliptical neutral hole, the reinforcements are not very attractive from a manufacturing point of view.

3. The Model

3.1 Selection of a suitable material

In problems of applied elasticity, the material concerned is assumed to be homogeneous, isotropic and to obey Hooke's law. Wood, which from a number of considerations is highly suited for model work, does not satisfy these requirements so that the choice remains between a metal and a plastic. If a metal were used, then:-

(a) Fabrication would not be easy.

(b) To obtain similarity between model and full scale would require rather high pressures. Since air under pressure has a very high energy content, the danger from possible failures of an explosive character, render this feature highly undesirable.

(c) The use of a liqud for pressurization involves the use of a tank similar to those at the Royal Aircraft Establishment but on a model scale. For strain gauge work, this is to be avoided if possible since otherwise, difficulties in the strain gauging techniques would be encountered.

From a consideration of such factors as these, one is guided to the selection of a material of low modulus such as a plastic. Xylonite has been used successfully on previous occasions[14] and it has been chosen as the material for the present work. A detailed discussion of the plastic is presented by Redshaw and Palmer in the paper cited.

3.2 Design of the models

The models, which were to be pressurized by air, were plain cylinders, each having two openings of different type.

Each cylinder was nominally 10" diameter and 24" long. With these proportions, the bending stresses resulting from the constraining influence of the bulkheads, being of a damped oscillatory character, were negligible in the neighbourhood of the holes. The shells, of 0.030" thick Xylonite sheet, had one longitudinal joint and the two openings were diametrally opposed to each other and 90 degrees away from the seam. To permit access to the interior of the cylinder for mounting strain gauges, end caps common to all cylinders were used. These were held in position by eighteen 4 BA screws and located by means of a central spigot fitting into an accurately bored hole in the bulkheads. To provide an effective seal against leakage of air at the windows and end caps, a smear of vaseline was found to be adequate.

In the testing position, the seam was underneath so that a line joining the centres of the holes was horizontal. Each cylinder was simply supported on short lengths of tubing projecting from the end caps, then no constraint was offered to deflection under pressure.

3.2.1 Design of openings

It was decided that four openings would be examined.

(a) A plain circular hole with no reinforcement.

(b) A circular hole with a nominal reinforcement. The reinforcing ring was made equal in weight to the disc of material removed - for this purpose, the outside diameter of the ring is the limit of the hole as seen by the sheet. The reinforcing was placed entirely on the inside of the shell, and to minimise the local bending effect due to eccentricity, the thickness of the ring was only twice that of the sheet.

(c) A circular hole, designed to be neutral for the case of equal principal stresses. The reinforcement was compact and equally disposed about the middle surface of the shell.

(d) An elliptical neutral hole designed according to the theory provided by Mansfield. Here again, the limit of the hole as seen by the sheet was the ellipse defining the outer edge of the reinforcing ring.

Details of the openings are given in Figs. 1A, 1B, 1C, 1D.

The "window panes" were small pieces of Xylonite placed inside the shell, the air pressure holding them in position whilst the test was in progress. On release of pressure, there was a tendency for the panes to slip out of position, so that to overcome this inconvenience, string was cemented on. (See Fig. 8).

3.3 Manufacture of the model

The first manufacturing operation was preparation of the shell. Openings were first cut in the developed sheet. Where a reinforcing ring was used, these openings corresponded to the inner dimensions of the complete cutout. To assist in the location of the reinforcement and strain gauges, very faint lines corresponding the hole axes were scribed on the sheet. The sheet was then wrapped around and placed in a fixture, a run of Bexol SX1 solution applied to the longitudinal joint and the subassembly allowed to set overnight. The bulkheads, previously bored, could then be finish turned on the outside diameter to suit the shell, cemented in position and the end caps made. Finally, the reinforcing rings were formed to the correct curvature in a warmed metal tool, and then cemented in position. In this way homogeneity of the ring is obtained. Fig. 2 shows a completed cylinder having in position a template for strain gauges.

A tensile test peice was cut from the same sheet as the shell and used for control tests.

4. The Pneumatic System

In the Section dealing with strain recording, the significance of mechanical creep in the model material is discussed. Here it will be sufficient to state that once the cylinder has been admitted to a certain pressure, it is essential that this pressure be maintained within close limits. This implies that since the volume of the cylinder is continually increasing due to creep, and since also a small amount of leakage is

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almost inevitable, air must be continuously supplied at the correct rate. Preliminary trials showed that manual control was inadequate, so that the pneumatic system shown schematically in Fig. 3 was devised. This incorporates a precision regulator valve by means of which an extremely close control of the model pressure was possible.

The characteristics of the valve over different ranges of upstream pressure and having valve setting as parameter were found. Fig. 4 shows a typical set of curves from which it will be seen that the output is not sensitive to upstream pressure fluctuations when the nominal pressure is about 25 p.s.i. This could be maintained manually to within 0.3 p.s.i. with no difficulty and was therefore adopted as standard for test work.

The numbers 1.81, etc., in Fig. 4 refer to the valve output, in inches of mecury, at the initial setting.

## 5. Automatic Strain Measuring Set

Xylonite exhibits pronounced mechanical creep so that if consistent results are to be obtained, a strict programme of events must be observed during the testing period. Redshaw and Palmer[14] have obtained a strain-time curve which shows that very little creep occurs two minutes after the application of a load. The testing technique employed by these investigators was therefore adopted. For both control and model tests, records of strain were taken two minutes after the application of a load increment; after a further minute, the next increment was applied and the procedure repeated.

From the above paragraph, it is evident that if there are more than one or two gauge stations, some form of automatic recording is necessary. For this purpose, an automatic strain measuring set was designed and built to be used in conjunction with the New Electronic Products "series 1000" multi-channel galvanometer recorder. The design of the set was based on the familiar D.C. Wheatstone bridge and was such that ten channels were available for simultaneous recording.

The set, Figs. 5 and 6, incorporated the use of two eight bank, twenty-five way uniselector switches, electrically coupled, impulses being applied to the uniselector relay by means of a master contactor clock. The switch positions were connected to the centre tappings of 5 ohm potentiometers which constituted the trimming resistors by means of which an initial balance of the bridges was obtained. The common terminals of the uniselector were connected to the mid points of the ratio arms via the recorder galvanometer and impedance matching resistors. The latter were necessary since the effective resistance of the bridge alone was not equal to that required for optimum damping of the galvanometers. Ten signals could be recorded simultaneously, so that to avoid interaction between the galvometers, ten pairs of ratio arms were used.

The strain gauge leads were fed into junction boxes mounted on a panel, and energised by a ring main, Fig. 7. The boxes were connected to the set proper by two, 12 core cables per box, so that only two leads per station entered the set, thus simplifying its design. To avoid differences in "effective" gauge factor from station to station, to obtain temperature compensation and for matching purposes generally, all leads which performed the same function were of the same gauge and length.

The effect of changes in switch contact resistance, which could be comparable to the changes in the gauges under load, were minimized in the design employed. As will be seen in Fig. 5, all the switching which actually took place was in the galvanometer line, and

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since the resistance was 290 ohms, changes of as much as 0.1 ohm would affect galvanometer current to a negligible extent only. The signal was recorded as a trace on photographic paper, and tests showed that scrupulous cleanliness with regard to switch contacts was imperative, since otherwise, fuzzy, meaningless traces could result.

All bridges were fed from constant output batteries which had a high ampere-hour capacity. Using the set, 120 gauge stations could be recorded in $12\frac{1}{2}$ seconds in one cycle.

### 5.1 Calibration of the set

To calibrate the set, it was necessary that a signal corresponding to a known value of strain should be fed into the recorder. For this purpose, electric resistance strain gauges mounted on an aluminium beam in pure bending were employed.

The gauge factors of these gauges were first checked using a null method in the usual Wheatstone bridge arrangement. They were then connected to the junction boxes and for a series of values of the central deflection of the beam, measured with a 0.0001" dial gauge, traces were obtained. In this way, the electrical system as a whole was calibrated eliminating the need to make any subsequent correction for lead resistance, etc. A statistical analysis was then applied to the results and a calibration factor obtained in terms of units of strain per inch deflection on the trace.

## 6. Electrical Resistance Strain Gauges and Gauging Technique

The building of models in materials of low modulus and small section, introduces a problem which is not met in ordinary strain gauge work on structures of steel, for example. The local stiffening effect of the gauge now becomes significant. There are two results consequent on this. First, the recorded strain is less than that which would have occurred had the gauge not been present, and second, the strain distribution in the neighbourhood of the gauge is disturbed. It is possible to make an allowance for the first effect after the method discussed by Dove[15]; however, because of the latter much more complex influence, it is highly desirable that the gauge stiffness should be very small.

The present investigation was concerned with stress concentrations, so that the overall dimensions of the gauge had to be small, thus imposing an additional requirement upon it. In view of this, some time was devoted to the development of a miniature strain gauge. Due to shortage of time, however, this work had to be discontinued and it was decided to use the Tinsley type 6H gauge, as it was the smallest gauge available at an economical price.

Most of the gauge stations were along axes of symmetry so that two gauges along the known principal directions were sufficient for stress determination from strain measurements. Since the gauges were comparatively large and close pitching was desired, use was made of the symmetrical conditions at the hole, and complementary gauges were mounted in homologous positions, Fig. 8. At regions not on axes of symmetry, "equivalent rosettes" were arranged.

Gauges were mounted inside and outside the shell using Bexol solution. The leads from the inner gauges were brought out through small holes at distances remote from the gauges, leakage of air being prevented by cementing with Bexol.

To estimate the stiffening effect of the gauges, pairs were mounted on tensile test specimens and calibrated. It was found that the apparent Young's Modulus was increased by a factor of 1.39 for the type of gauge employed on sheet 0.030 inches thick.

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## 7. Experimental Set-up and Method of Testing

The experimental set-up is seen in Figs. 6 and 7. All the controls for the pneumatic system were located together in such a manner that the operator could observe the upstream pressure gauge, mercury manometer and manipulate the controls whilst seated. The recorder and recording set were adjacent to each other so that a second operator could operate both. All the apparatus, including the rig for determining Young's Modulus, were in one laboratory which was free from draughts, and where temperature and humidity could be maintained approximately constant.

The testing procedure was as follows:-

(a) Bridge volts measured using a potentiometer.

(b) Pressure applied to the model and the stop watch started.

(c) After two minutes, the pressure was recorded and two records of strain taken. Taking two records of strain and subsequently using an average value for signal, eliminated the danger of misreading the trace deflection and troubles due to possible erratic traces.

(d) After a further minute, the next pressure increment was applied, and the procedure repeated.

After the tests, the bridge volts were checked.

Immediately on completion of the main test, Young's Modulus was determined using a tensile test piece. By carrying out a control test at the same time as the main experiment, changes in the elastic properties with temperature and humidity were allowed for. Fig. 9 shows a test piece in position in the loading rig. A typical load deflection curve for Xylonite is shown in Fig. 10.

A statistical analysis of the results was carried out and this value of the modulus used in stress calculations related to the corresponding main test.

Poisson's Ratio was found using two methods. The first, discussed by Searle[16], making use of the relation between the main and anticlastic curvature for a beam in pure bending, gave good consistent results in the neighbourhood of $\nu = 0.39$, Fig. 11, the optical interference method of Cornu, see Ref. 17, was not very satisfactory due to difficulties with the Xylonite as a test material.

## 8. Photoelastic Investigation

### 8.1 Photoelastic rig

A loading rig was designed and built to apply a biaxial stress sytem to sheet; the construction was such that the stresses in the two directions were applied simultaneously. The magnitudes of the applied loads were measured by means of tie rods which carried gauges connected up to give a four active arm Wheatstone bridge, the output of which was measured with a potentiometer.

The proportions of sheet were such that the ratio of width to diameter of hole was over 6.25:1. Quoting from a paper by A. J. Durelli and W. M. Murray[18], this enabled a good approximation to stresses in an infinite sheet to be obtained. To achieve rapid diffusion of stress into the sheet, a wiffle tree arrangement was used, Fig. 12.

8.2 Photoelastic material

Araldite B, which has only comparatively recently been introduced as a photoelastic material, was selected for the present work. This material is a thermosetting plastic in which a resin is used in conjunction with a hardener.

The resin is supplied in the form of a yellow solid and the hardener as a white powder. To mix, both are molten and then stirred together. Care is necessary in mould preparation and casting technique if good results are to be obtained and some time was devoted to the establishment of suitable methods.

8.3 Photoelastic tests

In the time available, only a few exploratory tests were carried out.

9. Results

9.1 General

The results relevant to the following topics were presented in the appropriate Sections in the main body of the report.

(a) Characteristics of the pneumatic system.

(b) Stiffening effect of strain gauges.

(c) Calibration of the strain measuring set.

The elastic constants, obtained as discussed in the text were:-

(a) Young's Modulus, (Section 7) for the cylinder containing openings 1 and 2, E was sensibly constant at $0.35 \times 10^6$ p.s.i. For openings 3 and 4, the value was $0.36 \times 10^6$ p.s.i. During all tests, the temperature was constant to within about 3 degrees Centigrade, and although there was some variation in the relative humidity, nevertheless, in the dozen or so tests, the above values for E were recurrent. A typical load deflection curve for Xylonite was shown in Fig. 10.

(b) Poisson's Ratio, (Section 7). The values obtained were consistently close to 0.39, and this was used in the stress calculations. The experimental curve relating the principal curvatures was shown in Fig. 11.

9.2 Strain records and calculations

Values of strain were recorded in the neighbourhood of four cutouts. Strain records were the deflections of a trace on photographic paper, and as outlined in the appropriate Section, the strain was recorded for a series of pressure increments. Then, improved accuracy was obtained by plotting deflection as a function of cylinder pressure, and using the slope of the resulting straight line in calculating the strain per unit pressure drop across the cylinder wall. The stresses are therefore in terms of 1 p.s.i. of cylinder pressure.

The membrane stresses were calculated and in the diagrams, the theoretical hoop and longitudinal stresses are represented by the chain-dot lines. In Figs. 13 to 31 the stresses are plotted as functions of distance from the cutout centre, and the curves relevant to a given opening are as follows:-

Opening No./

| Opening No. | Figure No.(s) |
|---|---|
| 1 | 13 - 16 |
| 2 | 17 - 21 |
| 3 | 22 - 26 |
| 4 | 27 - 31 |

## 10. Discussion of Results

### 10.1 General observations

The neutral hole is an extremely useful concept, but in view of the nature of the hole and the complex shape of the reinforcing further discussion of its applicability to pressure vessels is desirable. A few general observations will therefore be made before the results are considered.

As was pointed out in Section 2.2, the shape of the neutral hole and the cross sectional area of the reinforcing depends on the stress system obtaining in the sheet. This means that for a structure subjected to live loads which do not hold a fixed relation to each other, or if the nature of the loading envisaged at the design stage is not exactly realised, then the neutral hole cannot be achieved in practice. An aircraft pressure cabin experiences various aerodynamic loads, superimposed upon the pressure load, which results in random bending and torsion of the cabin as a whole. It therefore follows that, for this case, a true neutral hole is not a practical possibility.

Other factors also militate against a neutral hole in a pressure vessel.

In developing the neutral hole theory, it was assumed that the reinforcing had only tensile stiffness, in practice, there must also be some flexural rigidity. It has been adequately proved that the bending stiffness in the plane of the sheet is of no consequence, however, in the case of a curved pressure containing wall, the stiffness of the reinforcing in a direction normal to the shell offers constraint to radial change of curvature, with consequent introduction of local bending influences.

The pressure load on the window pane must be supported by bending of the shell, so that some stress concentration is inevitable on this account.

In an aircraft cabin, any reinforcing applied to a hole must be principally inside the shell otherwise undesirable aerodynamic drag is experienced. This eccentricity will introduce further local bending.

All the previous reasoning points to the fact that the theory at present available is inadequate to define a neutral hole in the wall of a pressure vessel, so that the complex shape of the hole and reinforcing are not justified in practice.

From equation 10, it is clear that from a design and manufacturing point of view, the absolutely neutral hole is not a very attractive proposition, so that provided a sufficiently low value of S.C.F. can be attained, the notion of an approximately neutral hole is worthy of consideration. For example, a contour made up of circular arcs is quite easy to manufacture to close limits, and to check dimensionally. This

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is an important consideration, since if the notion is accepted, it is known that the S.C.F. predicted at the design stage will not be vitiated by manufacturing difficulties, and furthermore, any deviations resulting from faulty manufacture can be detected by comparatively straightforward inspection methods. This will not be the case for the absolutely neutral hole.

In spite of the foregoing remarks, the value of the concept of neutrality applied to a pressure vessel is not underestimated. The form which the Mansfield theory for flat plates predicts can be regarded as a starting point for determining the optimum shape of the approximately neutral hole. For such a complex problem, an empirical approach would probably lead to a satisfactory solution.

### 10.2 Strain gauge tests

#### 10.2.1 General

Before proceeding to a discussion of the main results, it is important to realise the significance of the very large, local stiffening effect which the strain gauges employed have imposed on 0.030" thick Xylonite sheet. In effect, the gauges, which were mounted on either side of the sheet, introduce a discontinuity which must inevitably distort the strain pattern in its neighbourhood. Due to the close pitching of the gauges and to the stress gradients prevalent, strain gauge readings were almost certain to be mutually affected, so that it is rather unrealistic to quote stress concentration factors based on the results of the limited number of tests carried out. The urgent need for a good miniature gauge of small stiffness, is at once evident.

Regarding the size of models, it is considered that whereas the dimensions employed were convenient from the point of view of manufacture, and mounting of internal strain gauges, the possibility of a larger size should be considered for future work and so imposing less exacting demands on strain gauges. In this connection, there is, up to the present, a restriction imposed by the size of Xylonite sheet available which is compatible with models having only one longitudinal seam.

#### 10.2.2 Opening No. 1

The results relevant to the unreinforced opening No. 1, take the form which one would expect, the stress concentration due to the presence of the hole being aggravated by the local bending which results from the pressure load on the window pane. The stresses $\sigma_{\ell i}$ and $\sigma_{\ell o}$ in Fig. 13, and $\sigma_{hi}$ and $\sigma_{ho}$ in Fig. 14, would reduce to zero at the hole boundary, being radial with respect to the hole. The curves are thus typical of those obtained in shell theory with bending, being of a damped oscillatory character.

The membrane stress curves of Figs. 15 and 16, have a general form comparable to those obtained theoretically.

#### 10.2.3 Opening No. 2

As seen from Figs. 17 and 19, the introduction of a reinforcing disc has considerably reduced the stress concentration. Even though the eccentricity of the reinforcing is very small, there is still much bending effect in evidence. Fig. 21 shows that in proceeding along the circumference of a circle of 1.4" radius*, the maximum stresses do not

occur/

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*This is the closest one could get to the hole with the gauges.

occur at intermediate positions between the hoop and longitudinal axes. The inflexions in the curves must follow from the fact that only a quadrant is represented, and that there is symmetry at the hole.

10.2.4 Opening No. 3

Figs. 22 and 24 show that a useful reduction in stress concentration factor, as compared with opening No. 1, is obtained by using a circular hole designed to be neutral under equal principal stresses. Again, bending influences are much in evidence and the critical section is not intermediate between the hoop and longitudinal axes.

10.2.5 Opening No. 4

The deviation from the field of uniform stress is rather more than can be accounted for by experimental and model constructional error, so that on the evidence available, this opening cannot be regarded as neutral. Nevertheless, a good reduction in stress concentration is observed, Figs. 27 to 28.

The similarity between the results for this opening and No. 3 is interesting in as much as the performance of a straightforward hole is comparable to that of the much more complex one.

Fig. 31 shows the principal stresses along a quadrant of an ellipse whose axes are 0.3" larger than those of the cutout. Again, this is the closest that one could approach the boundary with the gauges available.

In most cases, at regions remote from the hole, there is good agreement between the theoretical and experimental values of the membrane stresses.

## 11. Conclusions and Proposals for Future Work

In view of the limited nature of the test series, only qualitative conclusions are drawn.

The results show that the direct application of flat plate theory alone to problems related to cutouts in pressurized shells is inadequate for a complete definition of the stress system; significant bending effects must be taken into account.

The application of reinforcing to the edges of the cutouts has considerably reduced stress concentration, but on the limited evidence available, the neutral hole theory for flat plates does not appear to be applicable to pressure vessels. In fact, the performance of a straightforward reinforced circular hole has been found to be comparable with that of the much more complex elliptical hole.

A satisfactory technique has been developed for the testing of Xylonite cylinders, but the value of such tests is considerably reduced by the lack of a suitable miniature strain gauge of low stiffness. The need for such a gauge is urgent.

It is felt that efforts should be directed towards the determination of an optimum approximately neutral hole suitable for pressure vessels. In view of the complexity of the problem, probably the most beneficial approach would be the application of an empirical technique, first, treating of a vessel whose walls are not constrained by hoops or stringers and then proceeding to a vessel so constrained.

This investigation has represented the early stages of a project which it is hoped will ultimately encompass a complex model representative of an actual aircraft main cabin.

It is emphasised that the test series was very limited, the investigation as described was of a preliminary character and the report is therefore an interim one.

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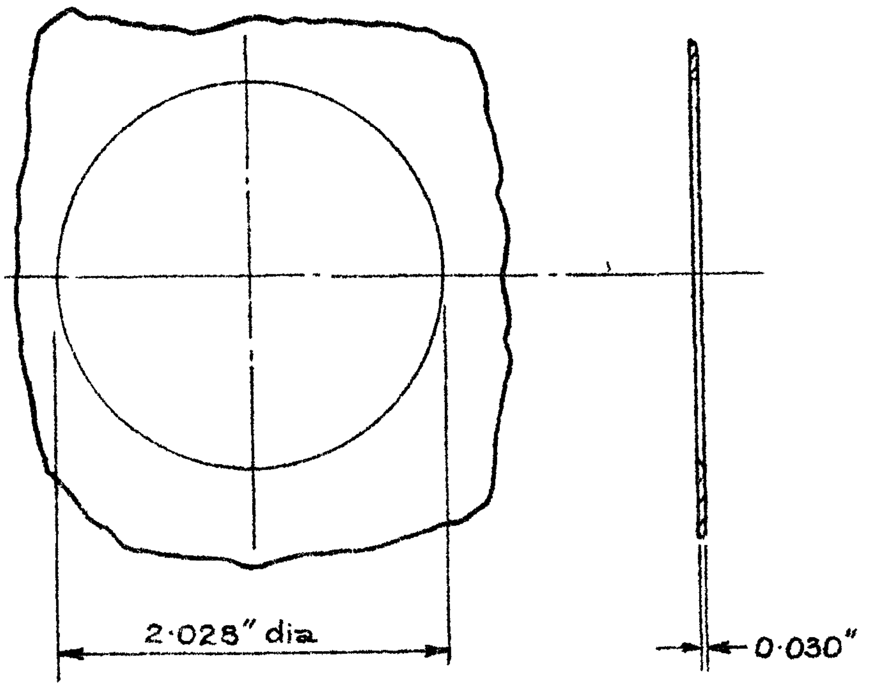
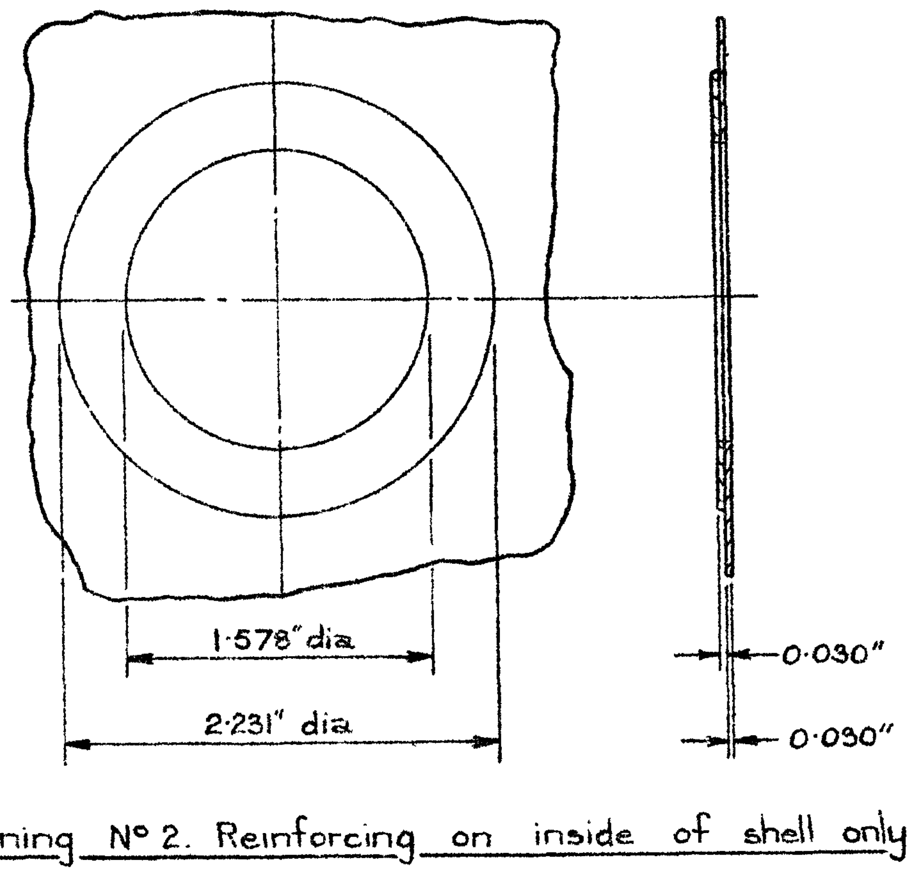
Fig. 1a & b.



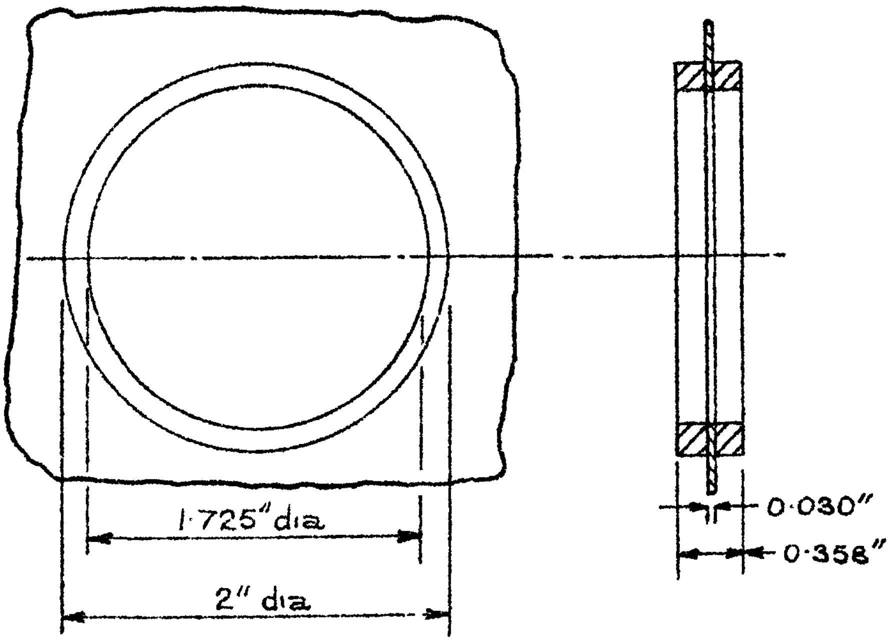

(a) Opening No 1. No reinforcing.



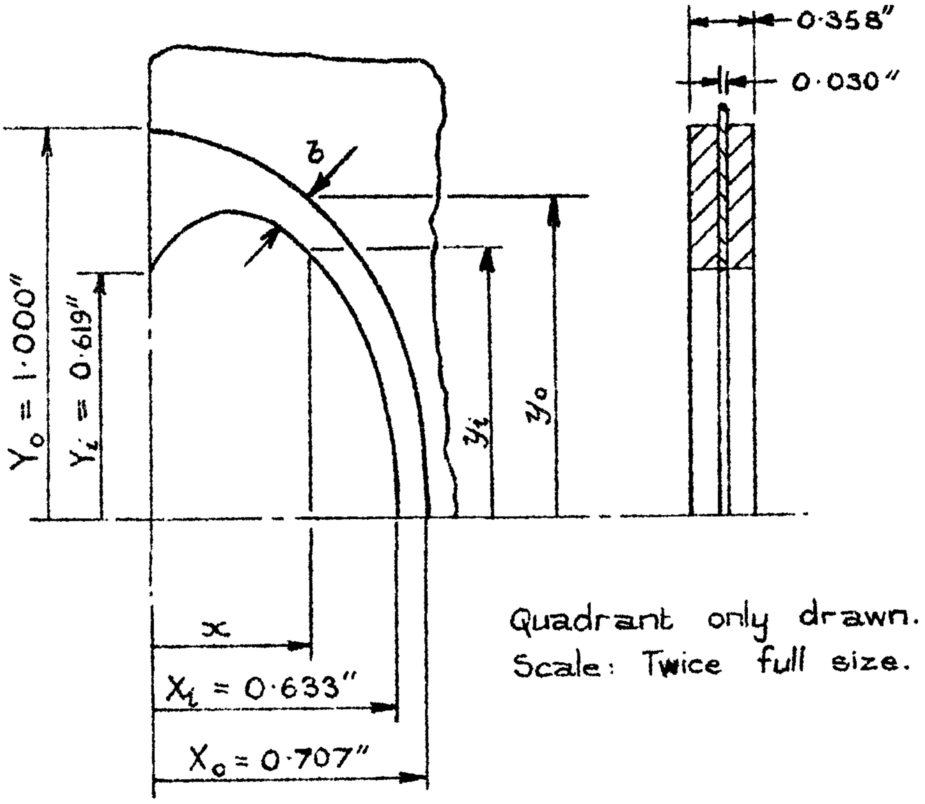

(b) Opening No 2. Reinforcing on inside of shell only.

D.C.

FIG. 1c&d.



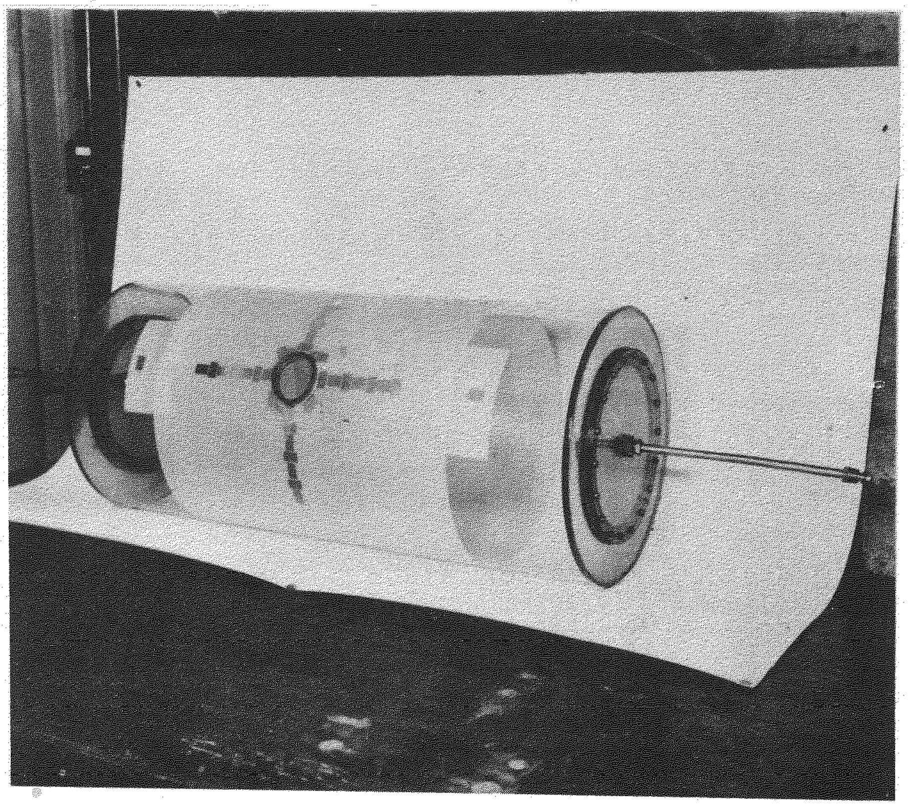

(c) Opening N° 3. Equal reinforcing on both sides of middle surface.




(d) Opening N° 4. Equal reinforcing on both sides of middle surface. x's and y's given in tabular form. b proportional to section area.

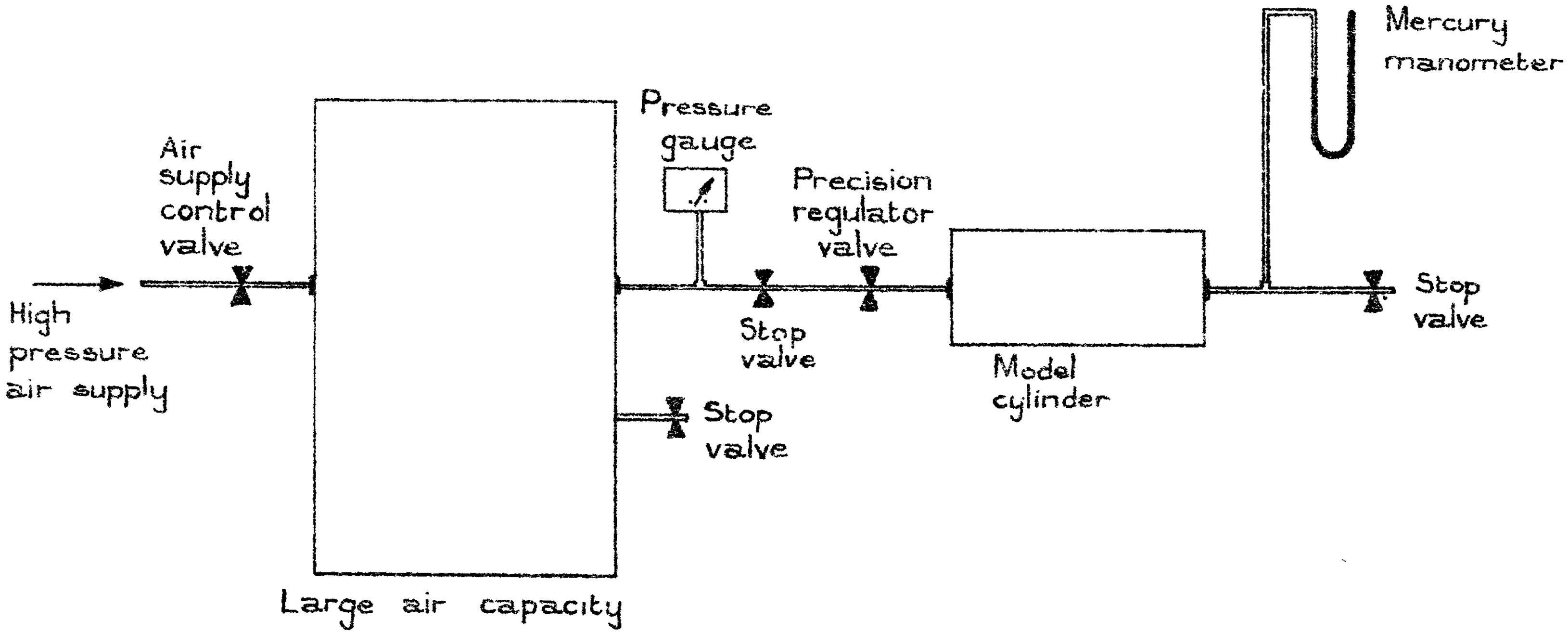
a.c.

FIG.2.



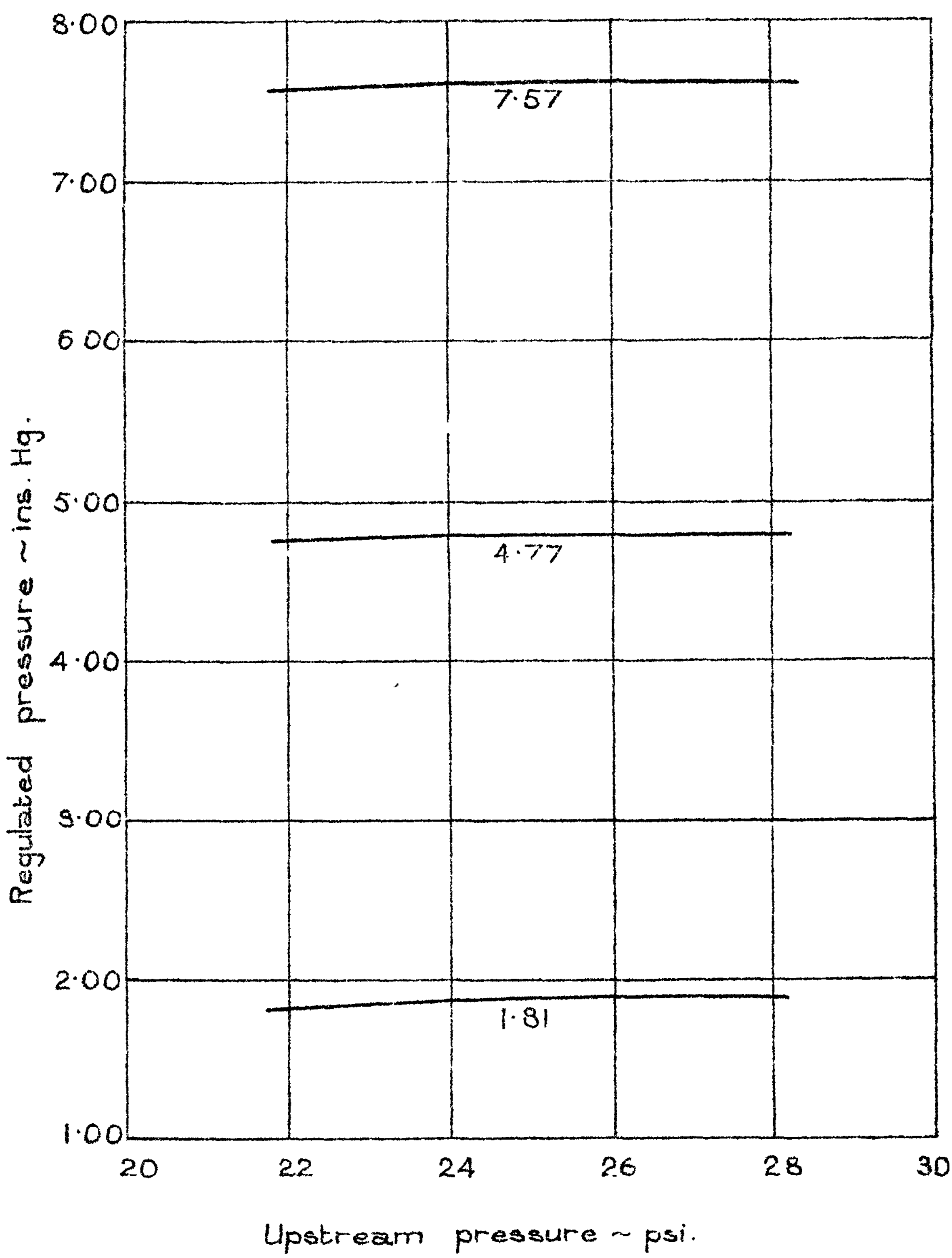
COMPLETED CYLINDER SHOWING TEMPLATE FOR LOCATION OF STRAIN GAUGES.

Fig. 3.




Schematic arrangement. Pneumatic system.

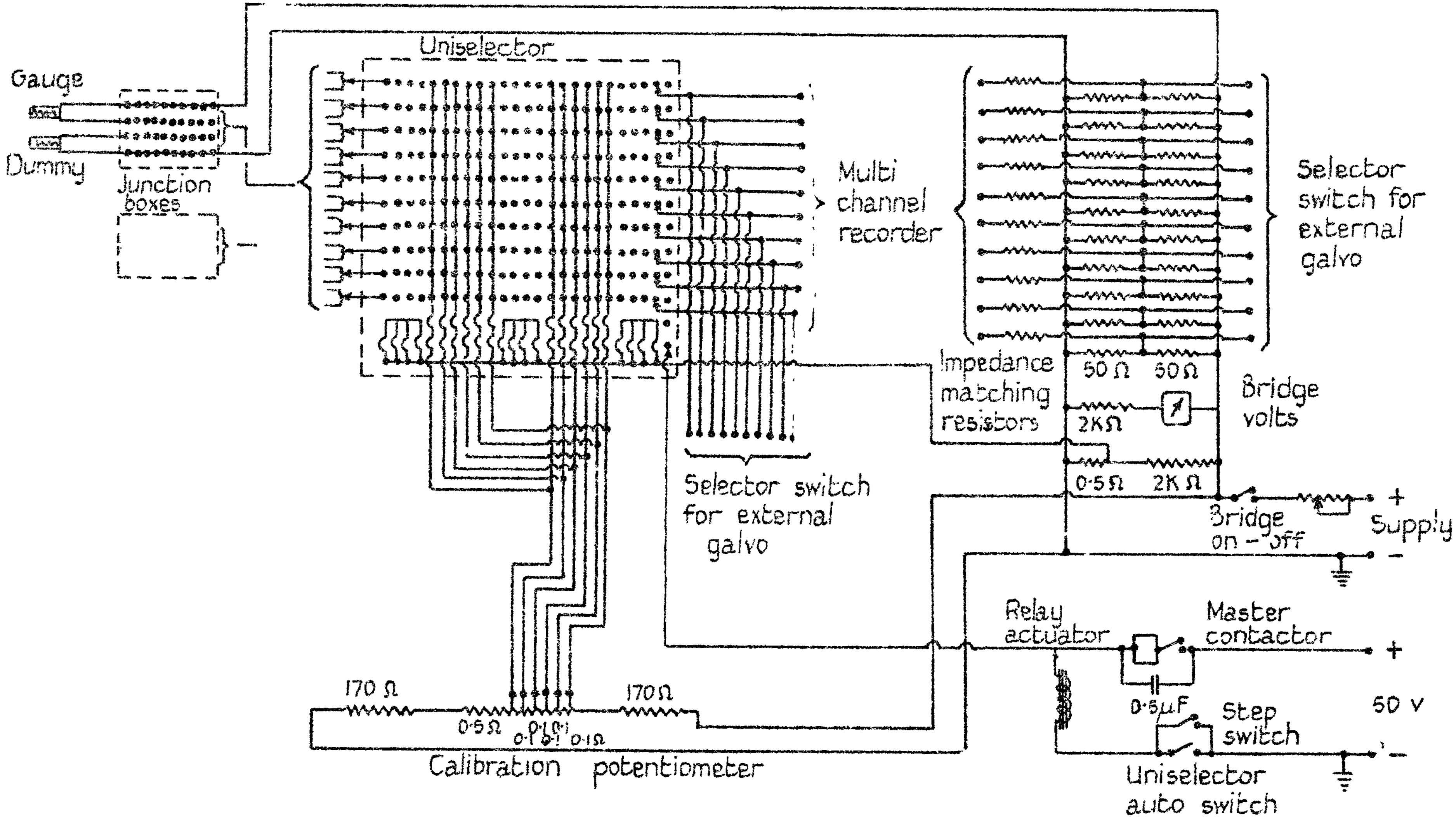
Fig. 4.




Characteristics of pressure regulator valve.

D.C.

FIG. 5.




Schematic arrangement. Automatic strain measuring set.

O.T.

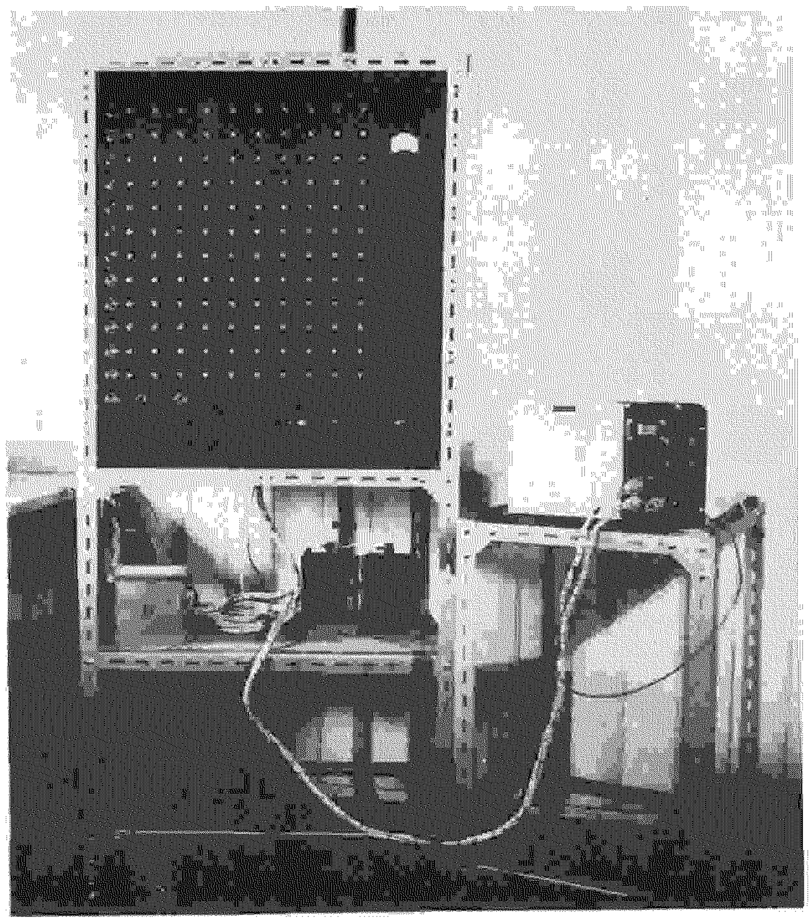


FIG. 6.

AUTOMATIC STRAIN RECORDING SET AND GALVANOMETER RECORDER.

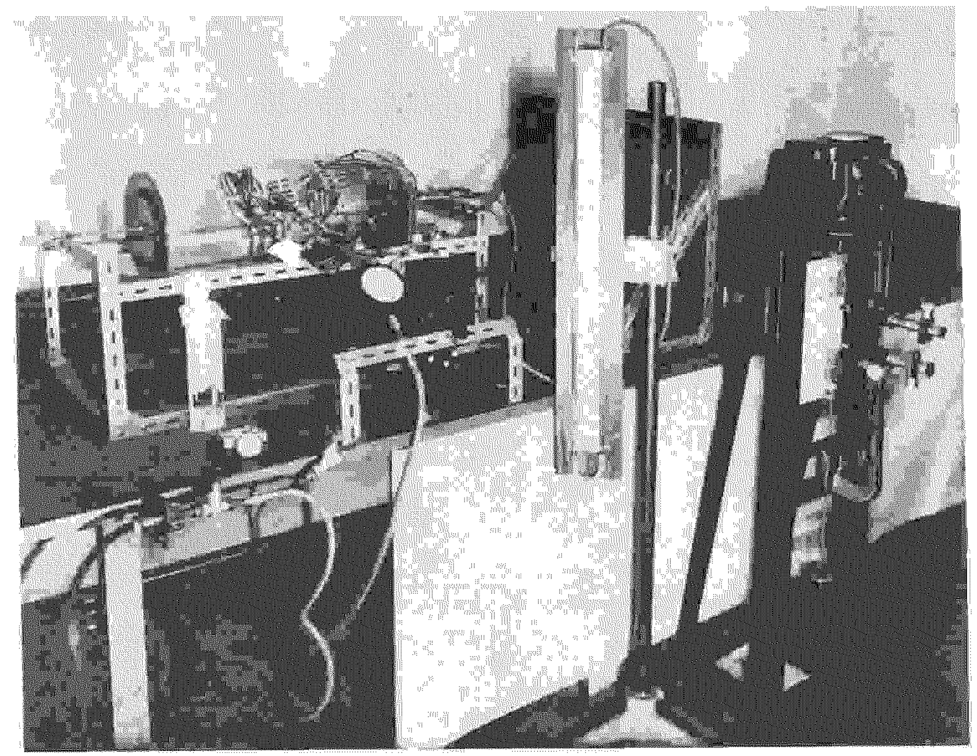


FIG. 7.

THE EXPERIMENTAL SET UP.

THE RECORDER IS JUST OUT OF THE CAMERA RANGE TO THE RIGHT HAND SIDE.

FIGS. 8 & 9.

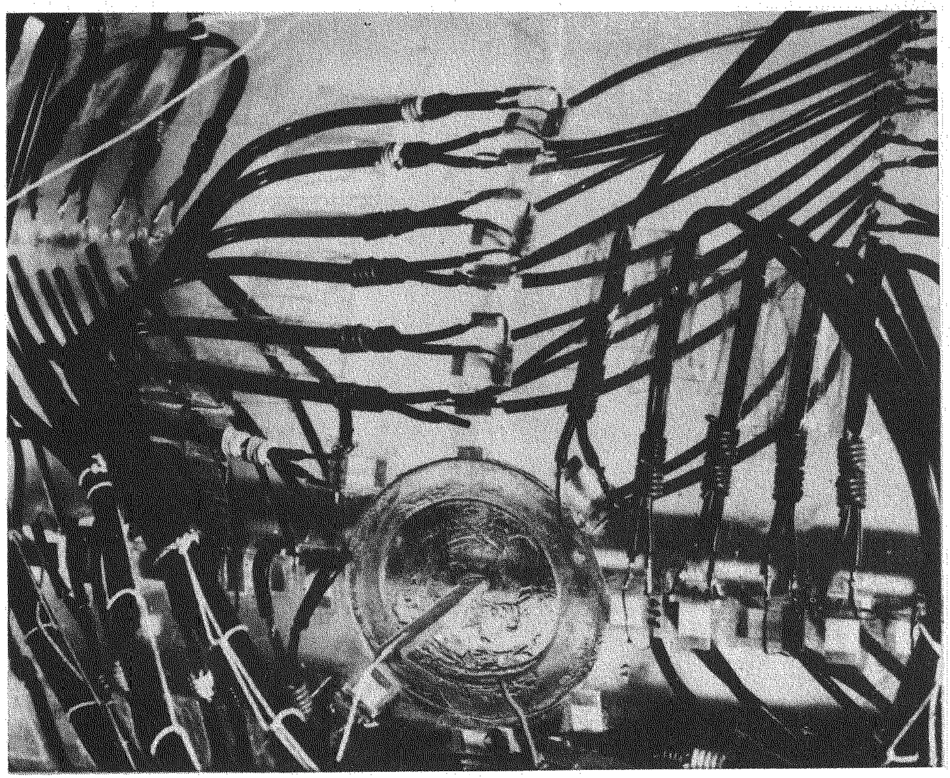


FIG. 8.

CLOSE UP VIEW SHOWING COMPLEMENTARY LOCATION OF GAUGES.

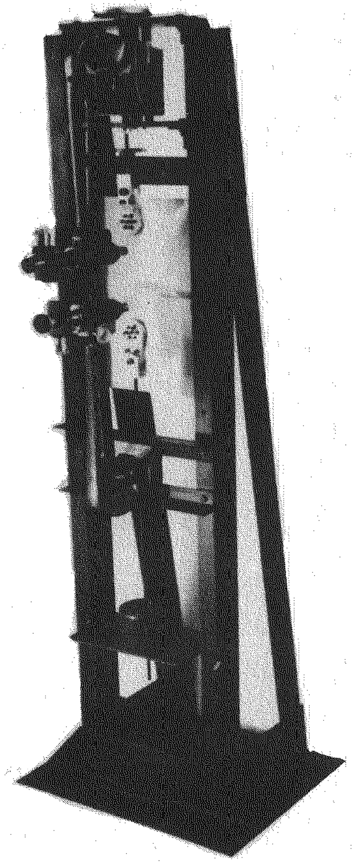
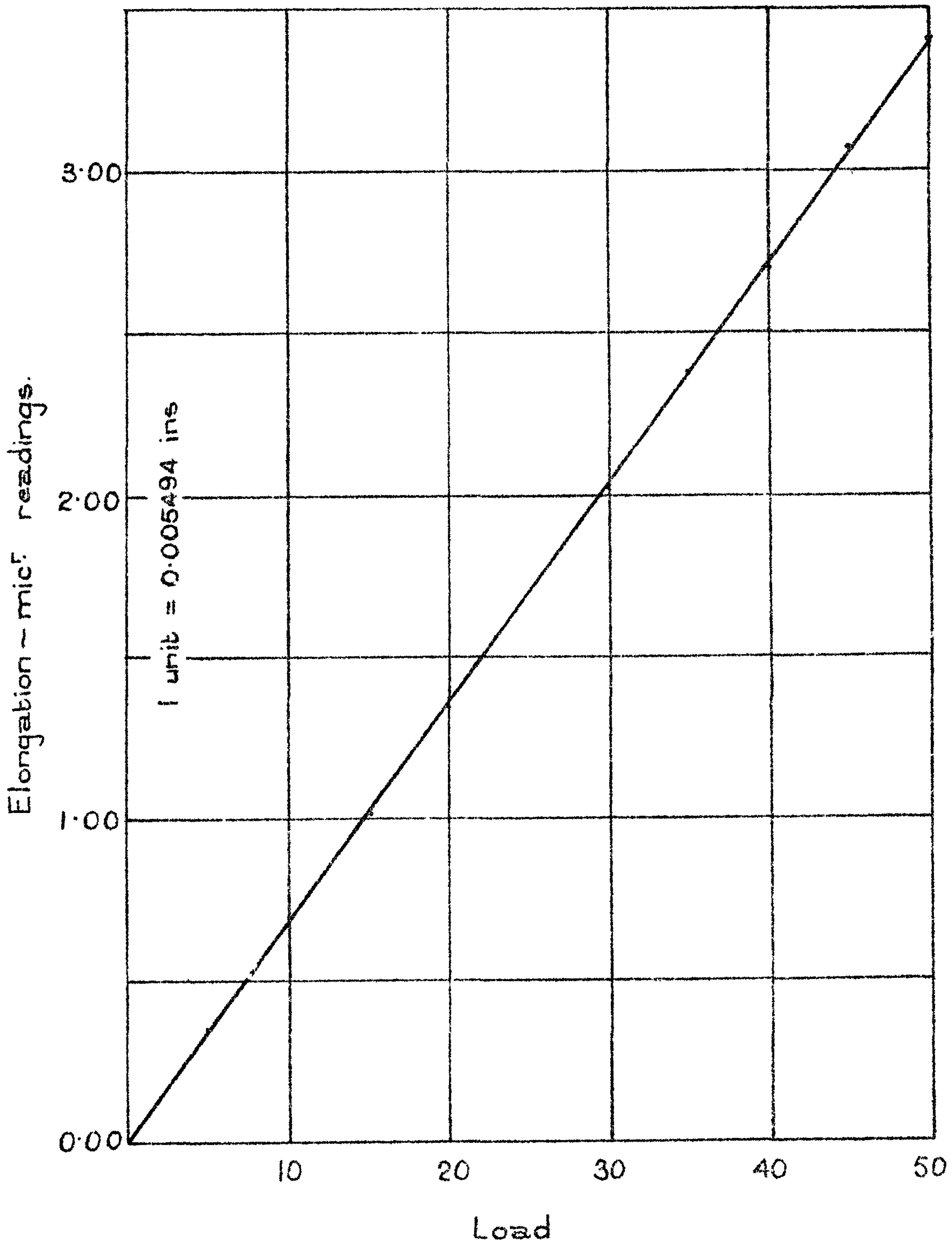


FIG. 9

RIG TO FIND YOUNG'S MODULUS FOR XYLONITE. THE TELESCOPES ARE USED TO MEASURE THE RELATIVE DISPLACEMENTS OF TWO LINES ON THE TEST PIECE.

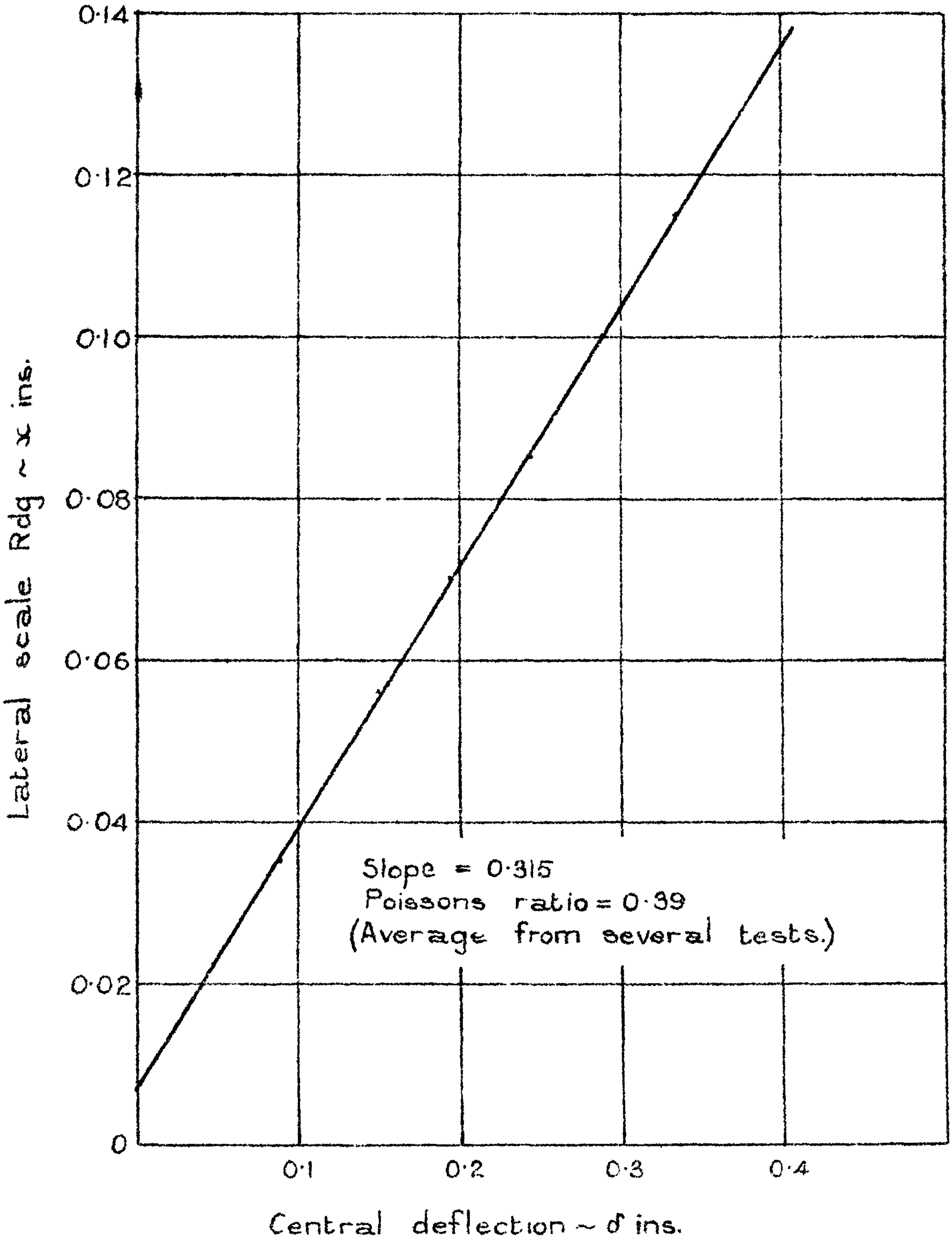
Fig. 10.




Typical load ~ deflection curve for xylonite.

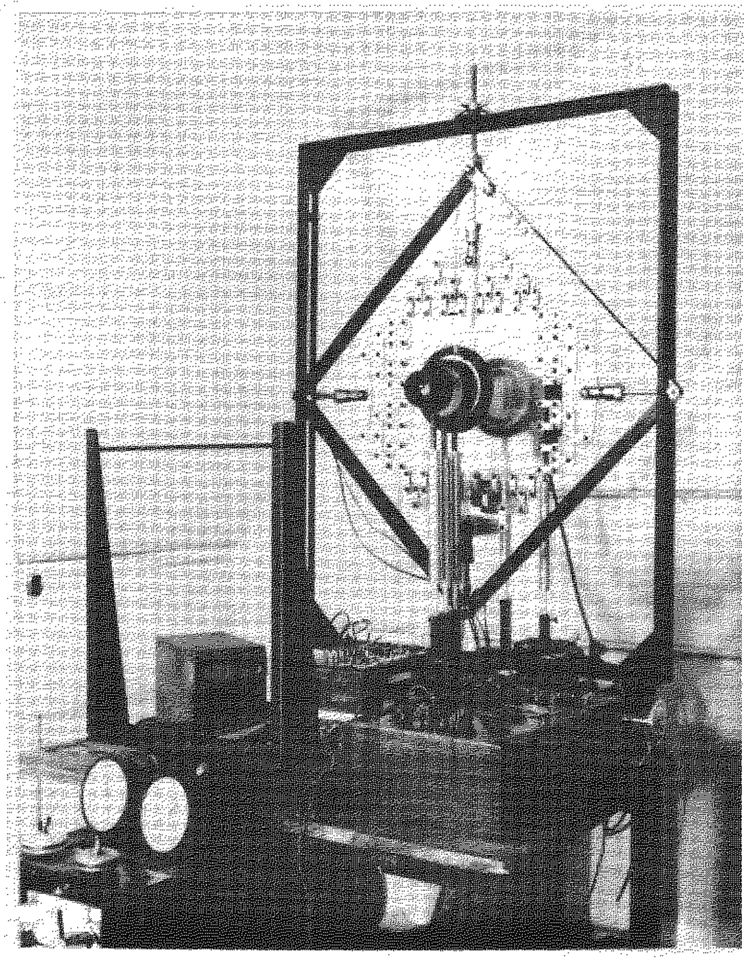
D.C.

FIG. 11.



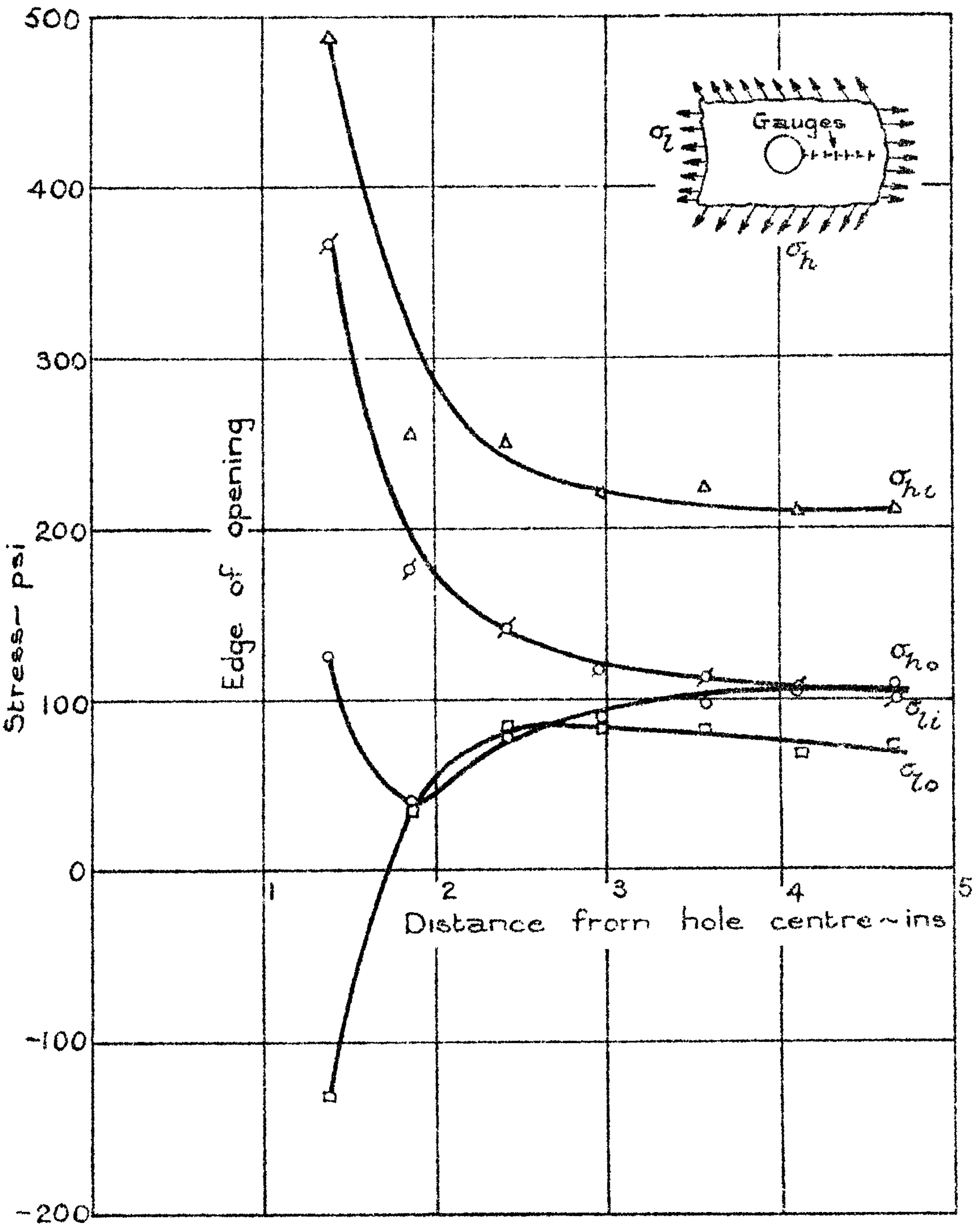

Typical curve relating curvature to anticlastic curvature for a beam in pure bending.

FIG. 12



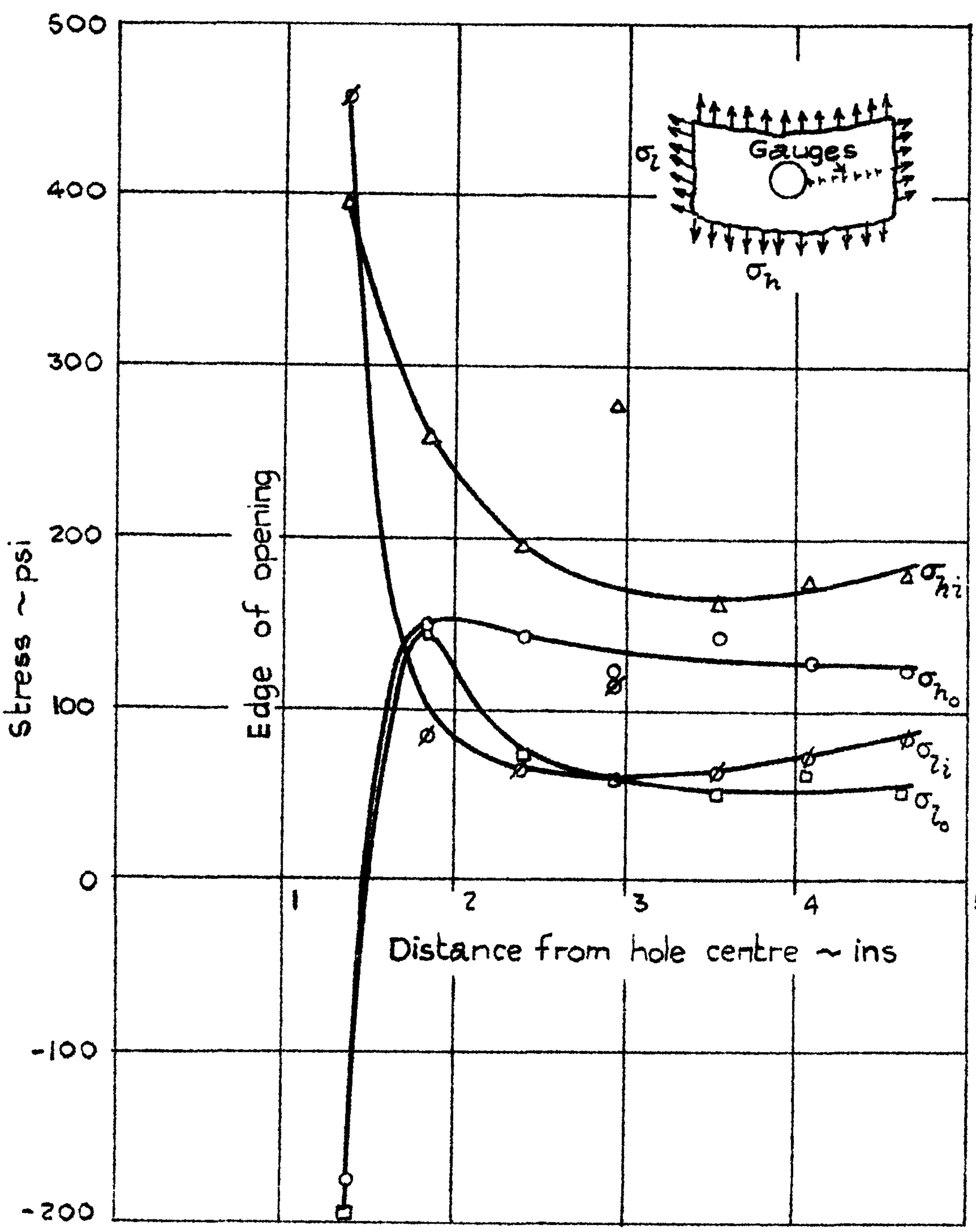
RIG TO APPLY A BIAXIAL STRESS FIELD TO A FLAT PLATE

Fig. 13.




Total stresses inside and outside shell longitudinal axis.
Opening N° 1.

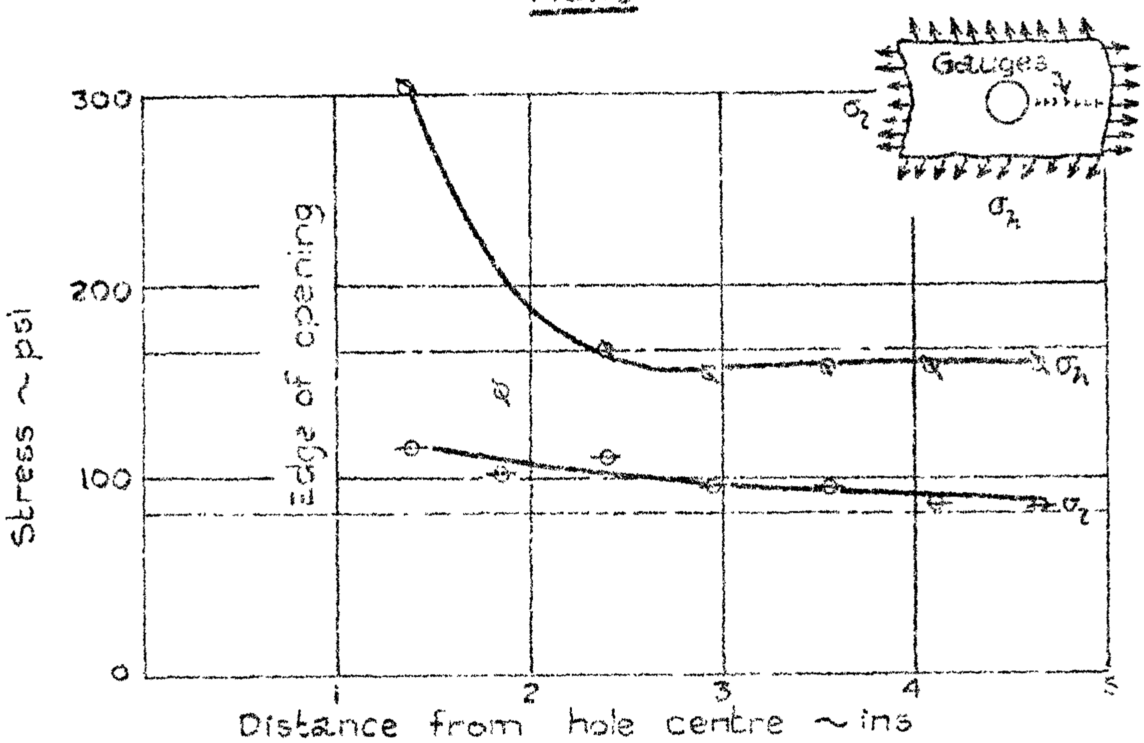
FIG. 14.




Total stresses inside and outside shell hoop axis. Opening N° 1.

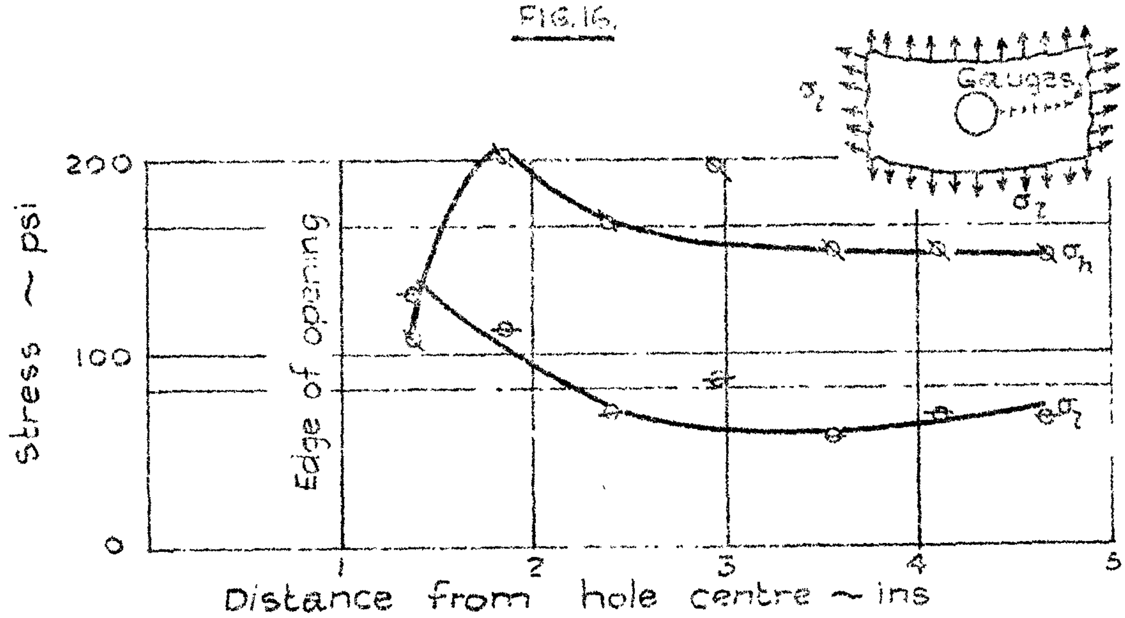
FIG.15 & 16.

FIG.15.




Membrane stresses longitudinal axis. Opening No 1.

FIG.16.

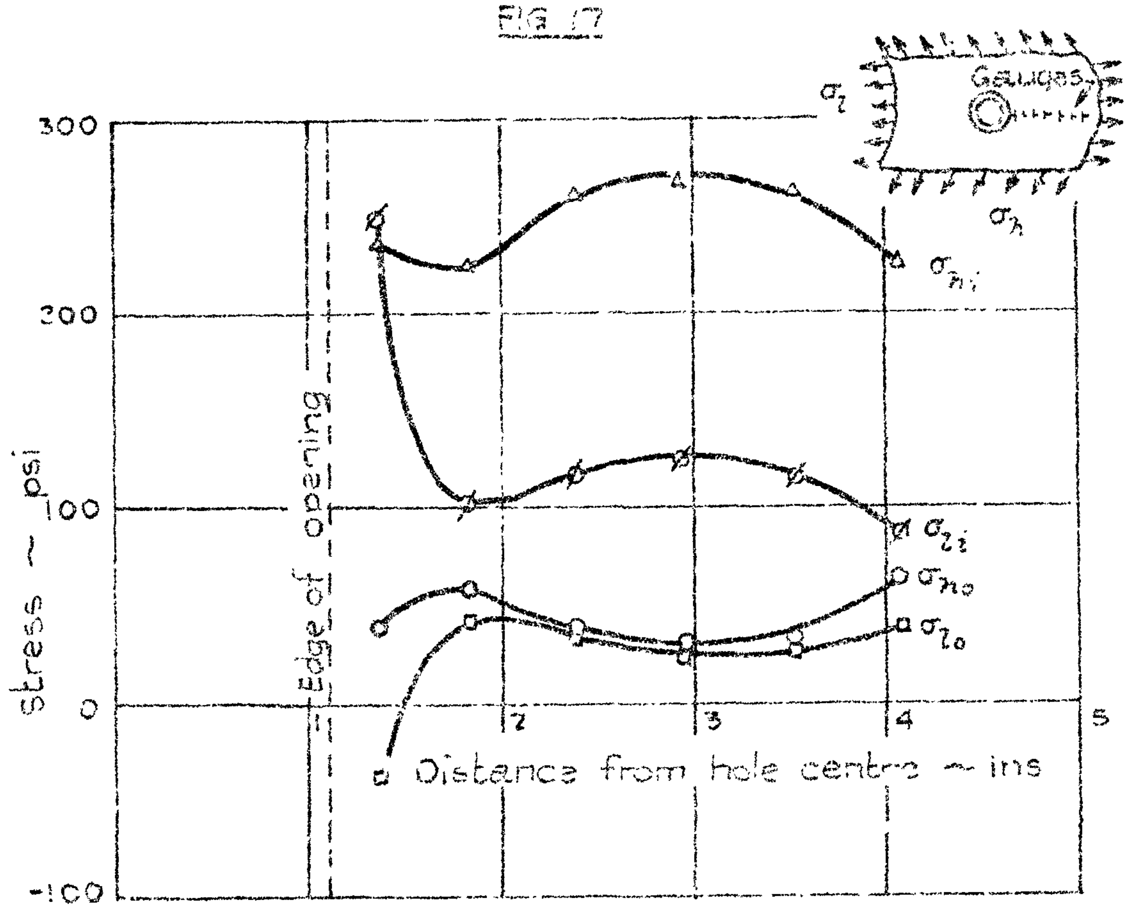



Membrane stresses hoop axis. Opening No 1

M.C.

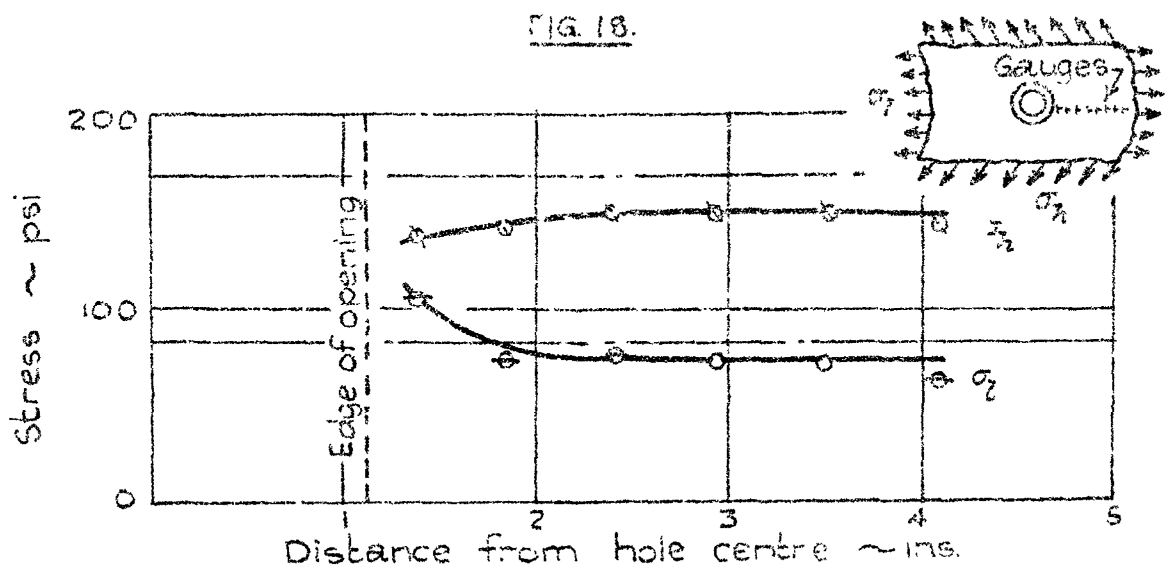
FIG. 17 & 18

FIG. 17




Total stresses inside and outside shell longitudinal axis. Opening N° 2.

FIG. 18.

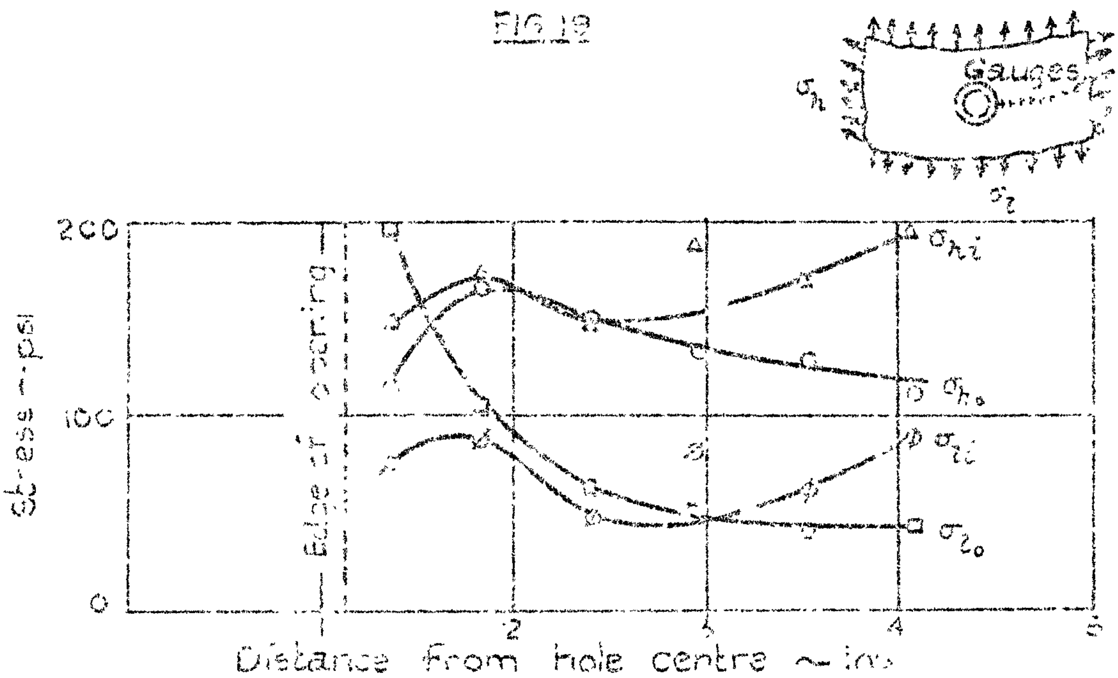



Membrane stresses longitudinal axis. Opening N° 2.

M.C.

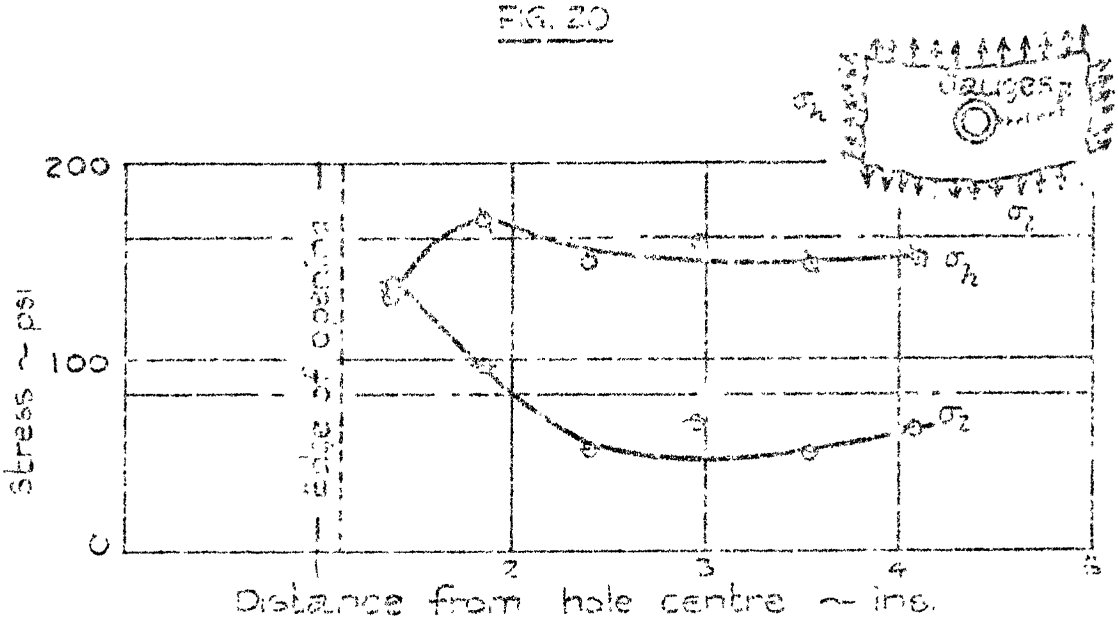
FIG 19.

FIG 19




Total stresses inside and outside shell hoop axis. Opening N° 2

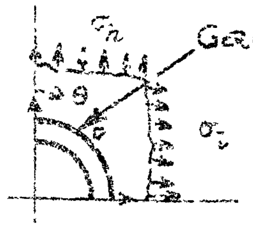
FIG. 20

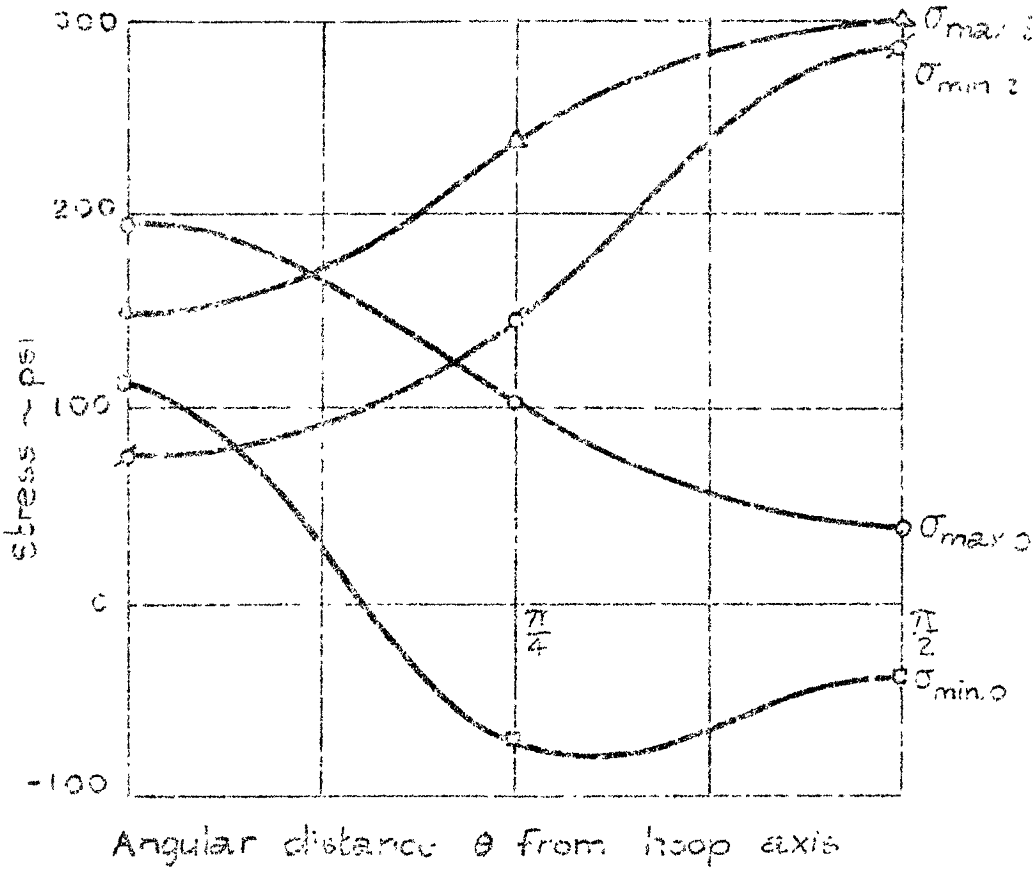



Membrane stresses hoop axis. Opening N° 2.

M.C.

Fig. 21



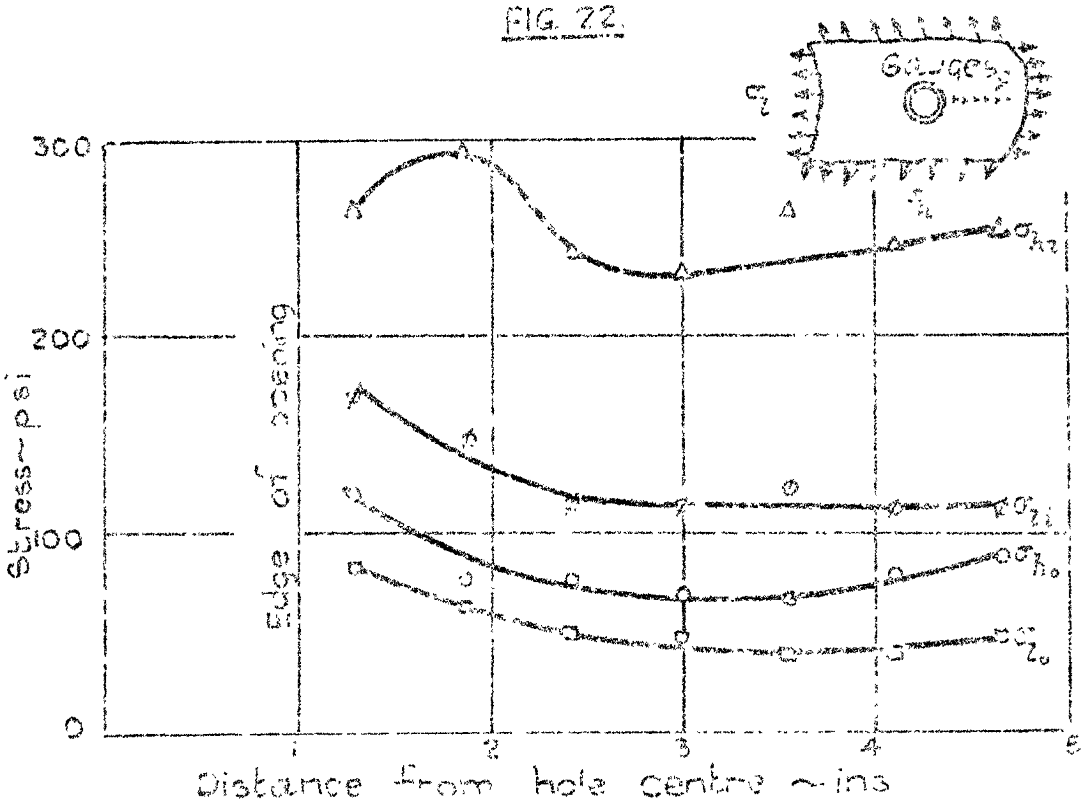




Principal stresses along hole circumference inside and outside shell. Opening N° 2.

M.C.

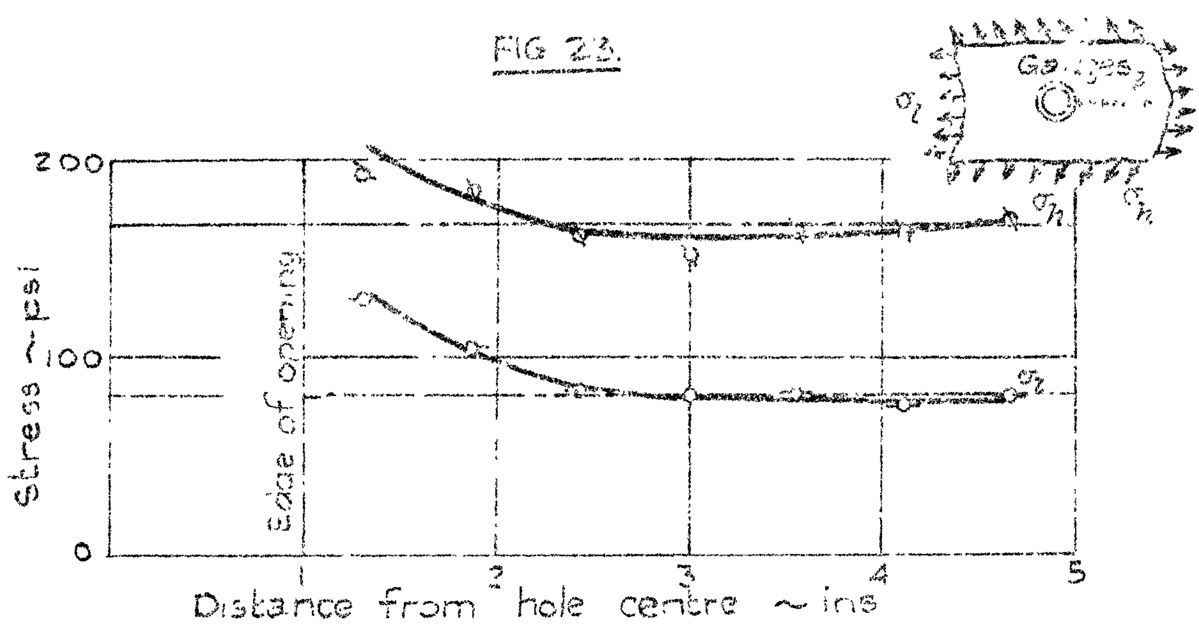
FIG. 22 & 23.

FIG. 22.




Total stresses inside and outside shell Longitudinal axis.
Opening No 3.

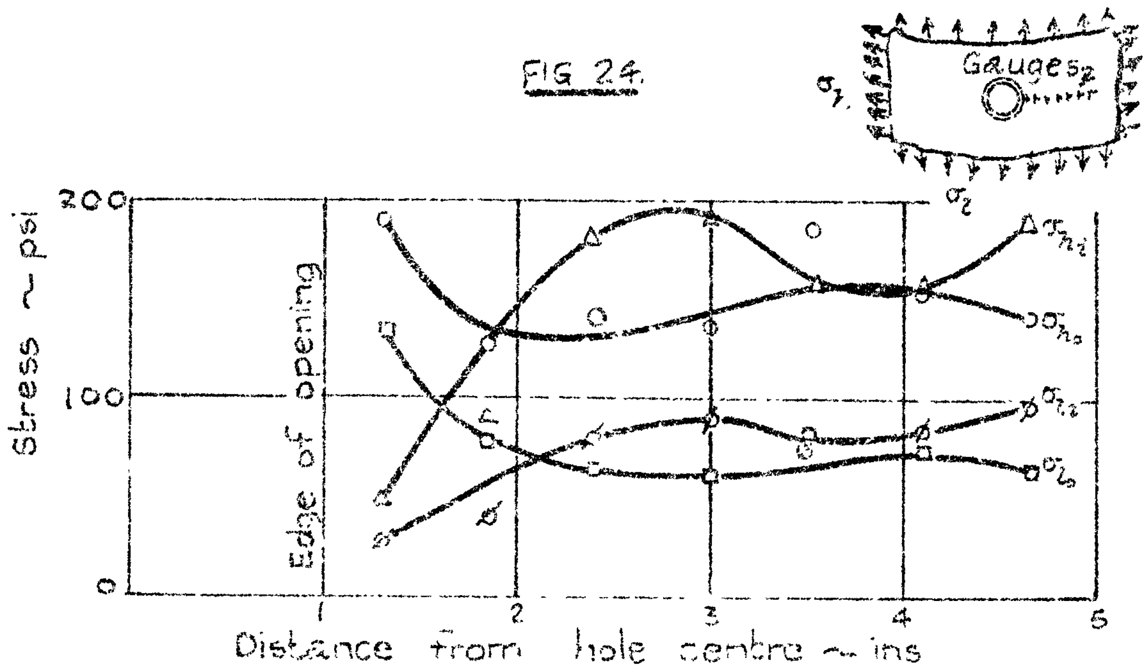
FIG 23.




Membrane stresses longitudinal axis Opening No 3

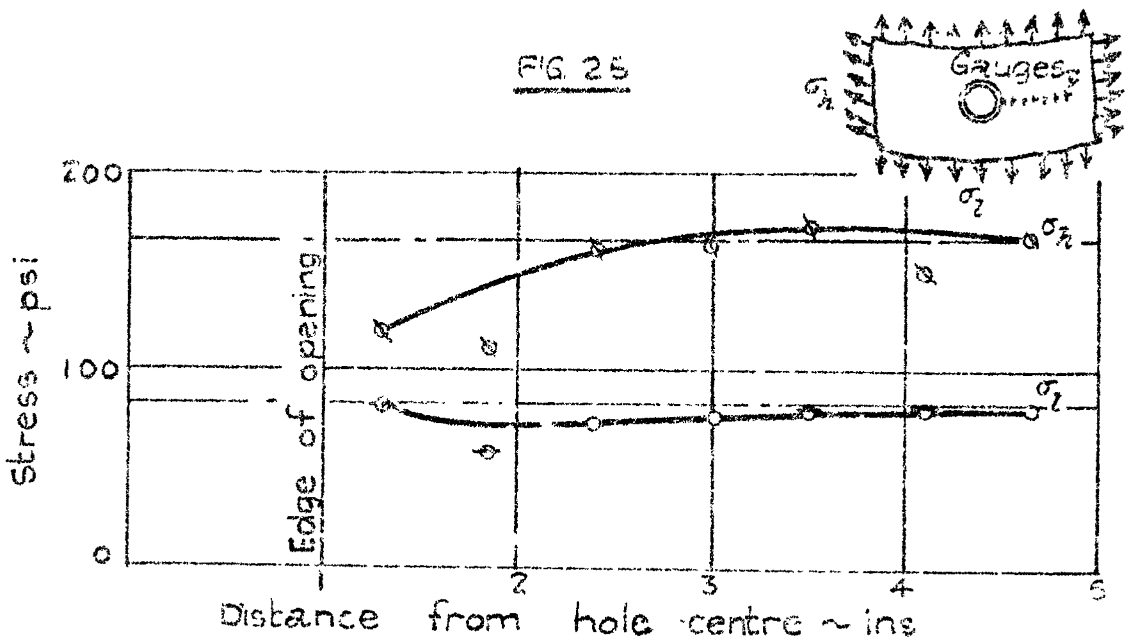
FIG. 24 & 25.

FIG 24.



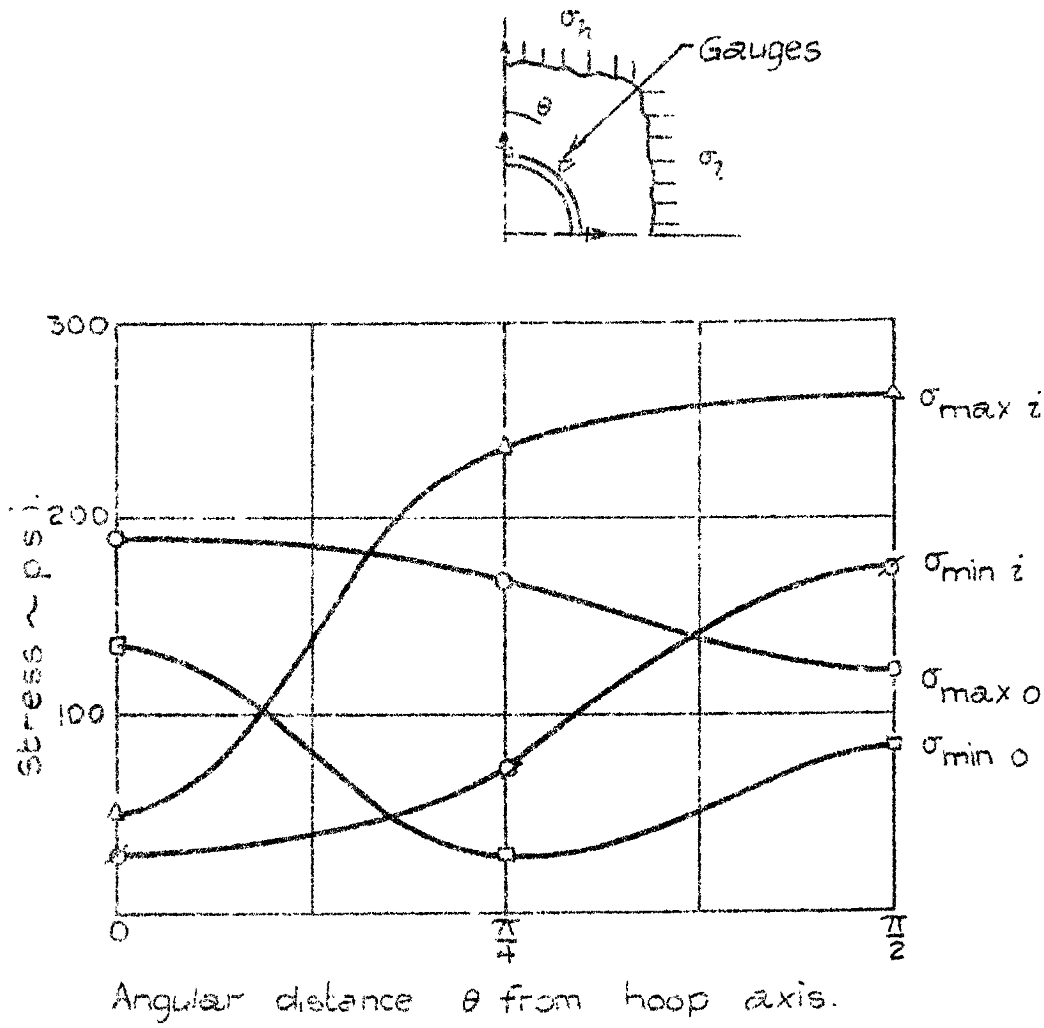

Total stresses inside and outside shell Hoop axis Opening N° 3.

FIG 25




Membrance stresses hoop axis. Opening N° 3.

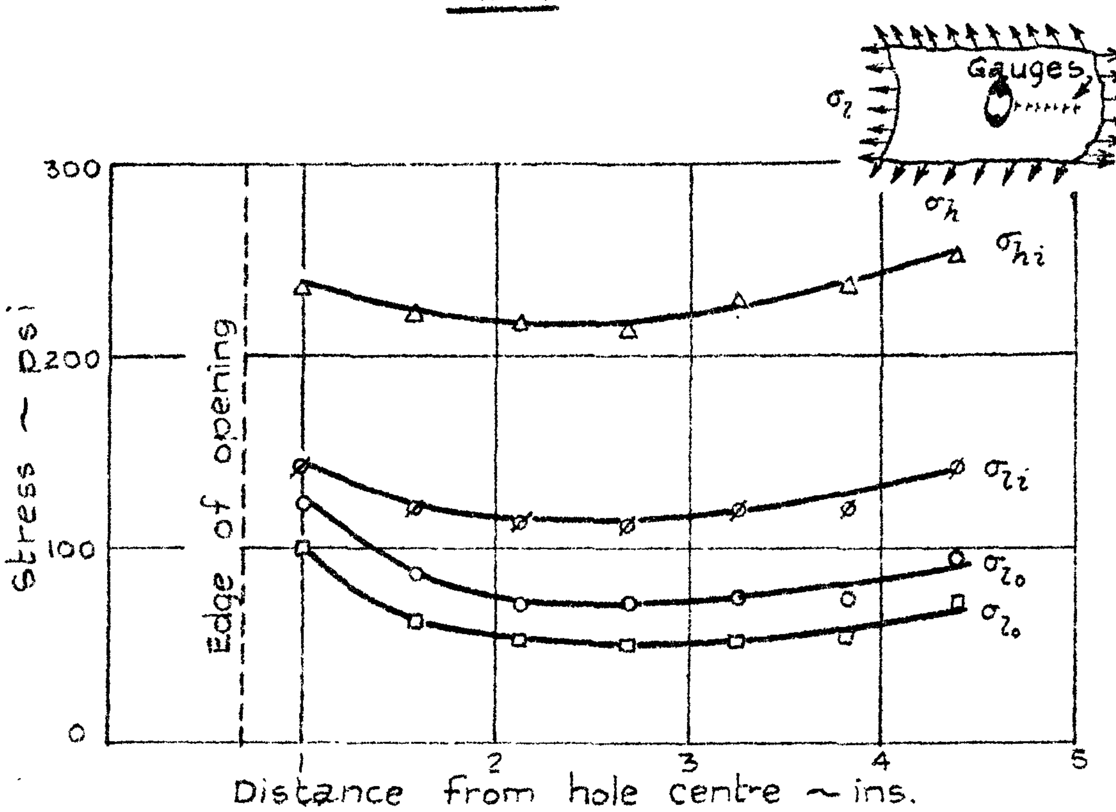
FIG. 26.




Principal stresses along hole circumference. Inside and outside shell. Opening N°3.

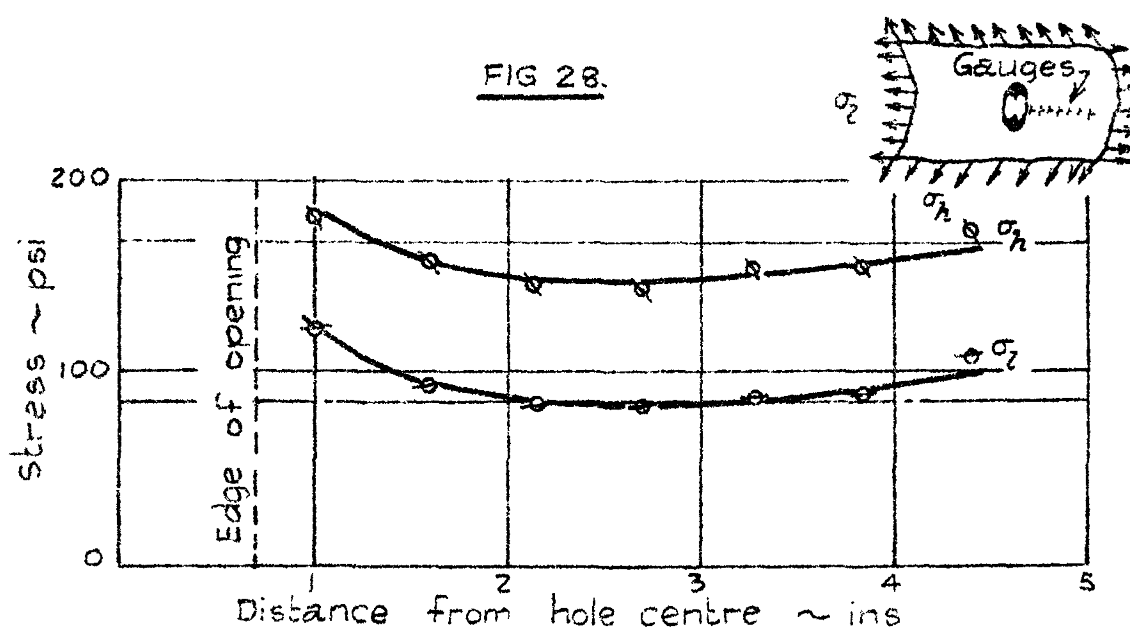
FIG 27 & 28.

FIG. 27.




Total stresses inside and outside shell longitudinal axis. Opening N° 4.

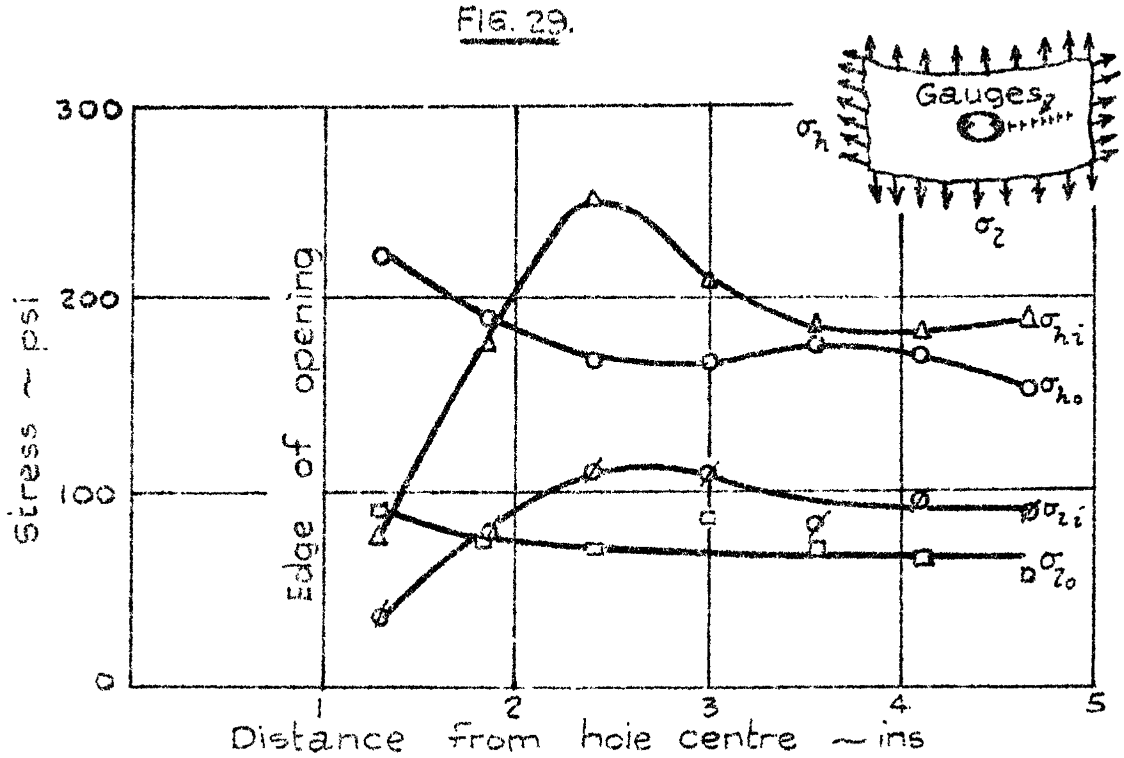
FIG 28.




Membrane stresses longitudinal axis Opening N° 4.

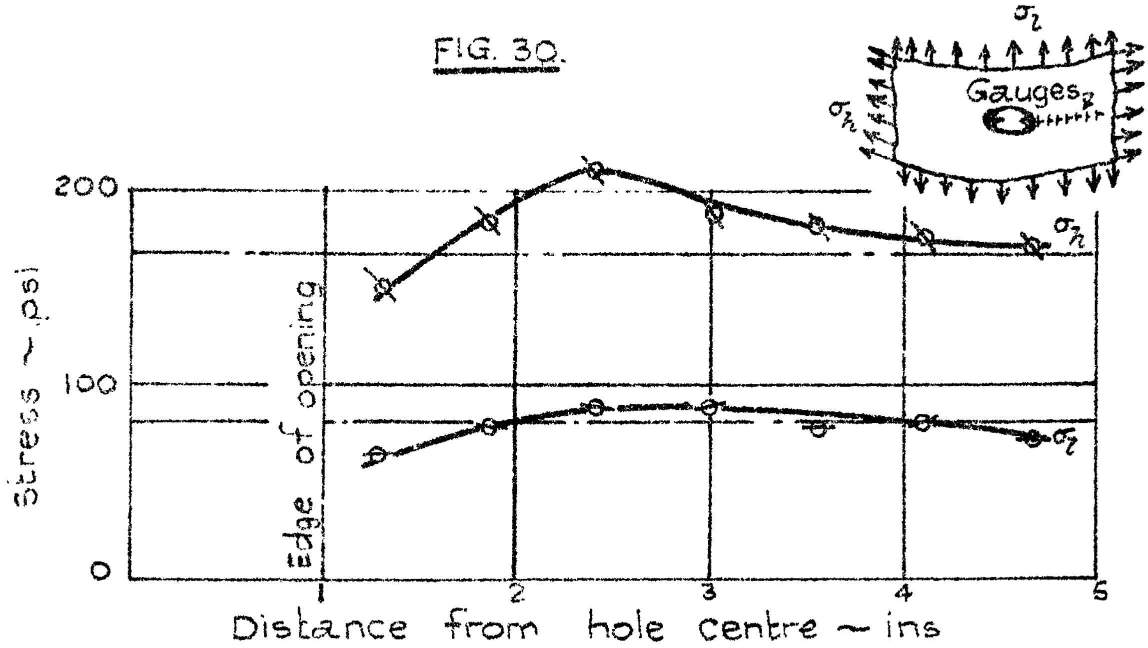
FIG. 29.&30.

FIG. 29.




Total stresses inside and outside shell hoop axis. Opening N°4

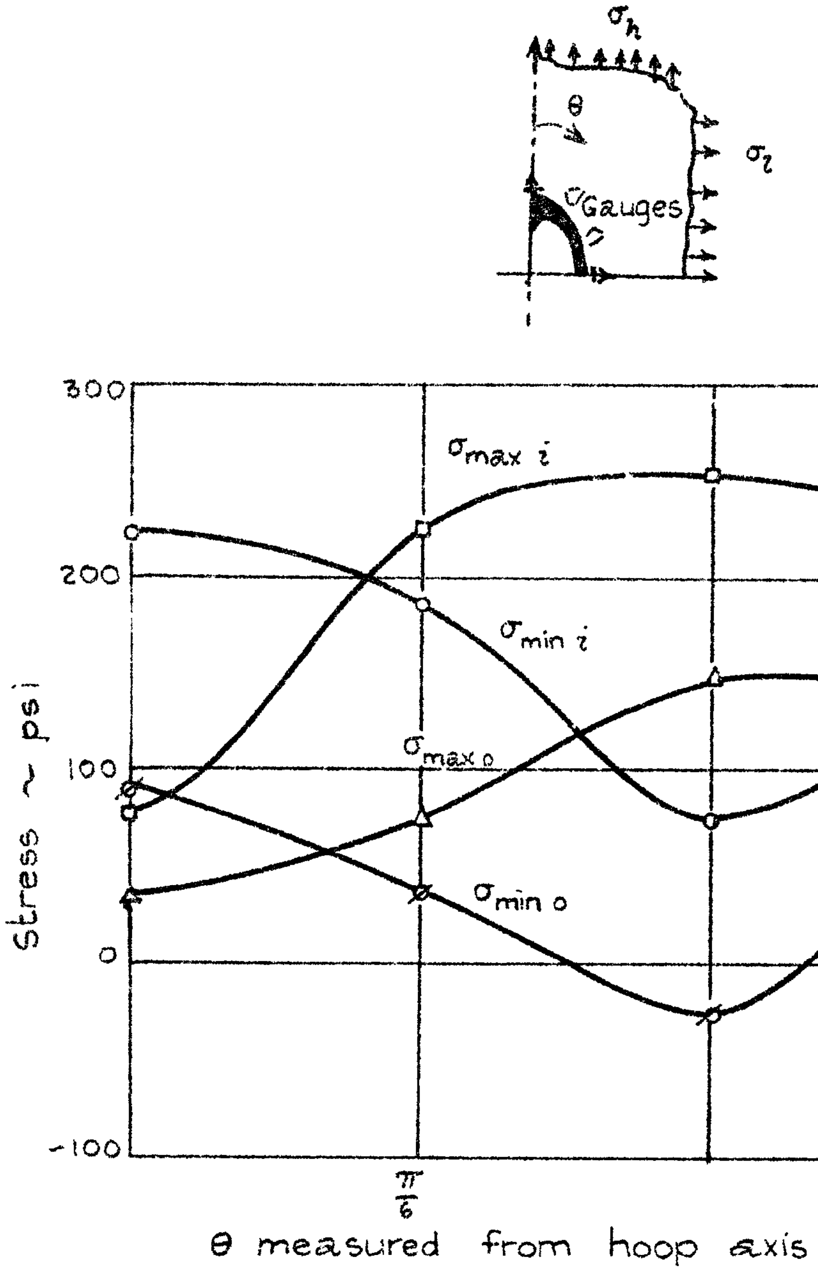
FIG. 30.




Membrane stresses hoop axis. Opening N°4.

N.C

FIG. 31




Maximum and minimum principal stresses along a quadrant of the hole circumference. Inside and outside shell. Opening N°4.

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C.P. No. 503

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