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Some Improvements in the Design of Thick Suction Aerofoils

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Some Improvements in the Design of Thick Suction Aerofoils

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8th March, 1950

Summary

An attempt has been made to design more satisfactory thick suction aerofoils than have hitherto been obtained with the exact method. Methods have been developed for representing more closely the physical conditions corresponding to an abrupt velocity fall with suction at a slot, and less severe types of velocity fall have also been considered. Experiments are needed to decide which is the best method of improvement and to clarify the difficulties encountered with the original exact theory shapes.

1. Introduction

In wind-tunnel tests on GLAS II and GLAT III shapes designed by exact theory, the suction quantities required to prevent separation were considerably larger than estimated theoretical values[1, 2, *] and with the GLAS II aerofoil unstable conditions were found in which separation occurred intermittently. On the other hand, the 30% symmetrical Griffith aerofoil designed by approximate theory and the Australian experimental modification of the GLAS II shape both worked reasonably well.[3, 4, 5].

The exact theory sections were designed to have a large discontinuous fall in velocity at the intended position for the suction slot,[6] and the surface curvatures in that vicinity are large since the theoretical shape approximates to a logarithmic spiral. The need for large suction quantities and the instability encountered with GLAS II suggest that the flow has difficulty in surmounting a large abrupt pressure rise (even with boundary layer suction) without separating from the highly curved /

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* The results from the recent R.A.E. tests on GLAT III have not yet been reported.

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curved surface./ For a complete understanding of this difficulty, fundamental research on the behaviour of the boundary layer under such conditions is required. As a preliminary, an attempt has been made to design more satisfactory exact theory shapes, by seeking to represent more closely the physical conditions corresponding to an abrupt velocity fall with suction at a slot, or by specifying instead a type of velocity fall which experience suggests should be easier to achieve in practice.

2. Outline of Theoretical Methods

Two distinct methods for representing more closely the physical conditions at the slot have been investigated. In the first method, the sink effect due to suction is taken into account and the slot shape worked out with the rest of the aerofoil by following a suggestion by Glauert[6] (1947). Essentially, a sink of the required strength is assumed to lie at the point $\theta = \beta$ on the unit circle from which the aerofoil is transformed, so as to correspond to a suction slot on the aerofoil; but the design velocity distribution is specified to be finite and continuous everywhere in order to provide a slot of finite width on the aerofoil. A second method, suggested by Griffith[7] (1948) and already applied to diffusers by Howell[8] (1948), is to design for the streamline which has a rapid continuous fall in velocity (over a distance comparable with the slot width) to the stagnation point at the intended position for the rear lip of the slot. The aerofoil ordinates upstream of the slot are then assumed to be obtained by removing the boundary-layer displacement thickness along the inwardly drawn normal of the calculated shape.*

The alternative approach, which from practical experience seems beneficial, is to make the velocity decrease continuously over a small extent of surface, and thereby both spread the pressure rise and reduce the surface curvatures in that vicinity. In this connection, it has been found instructive to consider the characteristics of the 30% Griffith section[3, 4] and the Australian modification of the GLAS II[5] in the general discussion of § 5.

The mathematical treatment for the first method (inclusion of sink effect) is given in §3, and that for the remaining methods (which ignore sink effect) in §4.

3. /

/ The surface curvatures and the severity of the velocity fall increase markedly with increasing thickness-chord ratio and camber; the radii of curvature of the surface may become comparable with the boundary-layer thickness.

* This does not completely fix the slot entry shape which must be chosen so that the stagnation point on the rear lip remains in the correct position.

3. Theoretical Treatment for the Inclusion of Sink Effect and Slot Shape

The symbol q_{am} will be used to denote the velocity over the aerofoil surface, for unit stream velocity, at incidence α with a sink of strength $2\pi m$ at the point corresponding to $\theta = \beta$ on the unit circle*. By considering the corresponding flow past the unit circle, it can quickly be shown that

$$q_{am}/q_{oo} = \left| \frac{\cos(\frac{1}{2}\theta - \alpha)}{\cos\frac{1}{2}\theta} + \frac{m}{4} \operatorname{cosec}\tfrac{1}{2}\beta \operatorname{cosec}\tfrac{1}{2}(\theta-\beta)\sec\tfrac{1}{2}\theta \right| .$$

If the stagnation point behind the sink is at $\theta = \beta - \gamma$, we then have

$$m = 4\cos\tfrac{1}{2}(\beta-\gamma-2\alpha)\sin\tfrac{1}{2}\beta\sin\tfrac{1}{2}\gamma , \qquad \ldots(3.1)$$

and

$$q_{am}/q_{oo} = \left| \frac{\sin\frac{1}{2}(\theta-\beta+\gamma)\cos\frac{1}{2}(\theta-\gamma-2\alpha)}{\sin\frac{1}{2}(\theta-\beta)\cos\frac{1}{2}\theta} \right| . \qquad \ldots(3.2)$$

The problem is considerably simplified if the sink strength is assumed to be adjusted so that γ remains constant as α is varied, in which case

$$q_{am}/q_{om} = \left| \cos\tfrac{1}{2}(\theta-\gamma-2\alpha)/\cos\tfrac{1}{2}(\theta-\gamma) \right| . \qquad \ldots(3.3)$$

The design velocity distribution can then be conveniently specified in the form

$$\log q_{om} = \begin{cases} \log\left|\cos\frac{1}{2}(\theta-\gamma)/\cos\frac{1}{2}(\theta-\gamma-2\alpha_1)\right| , & \alpha_1+\alpha_2+\gamma < \theta < \pi+\alpha_1+\alpha_2+\gamma \\ +\log\left|\cos\frac{1}{2}(\theta-\gamma)/\cos\frac{1}{2}(\theta-\gamma-2\alpha_2)\right| , & \pi+\alpha_1+\alpha_2+\gamma < \theta < 2\pi+\alpha_1+\alpha_2+\gamma \\ +S & 0 < \theta < 2\pi \end{cases} \qquad \ldots(3.4)$$

where S takes the values $\log q_{\alpha_1 m}$, $\log q_{\alpha_2 m}$ over the upper design surface $\alpha_1+\alpha_2+\gamma < \theta < \pi+\alpha_1+\alpha_2+\gamma$ and the lower design surface $\pi+\alpha_1+\alpha_2+\gamma < \theta < 2\pi+\alpha_1+\alpha_2+\gamma$ respectively. Equation (3.2) with $\alpha = 0$ gives the further relation

$$\log q_{oo} = \log q_{om} + \log\left|\{\sin\tfrac{1}{2}(\theta-\beta)\cos\tfrac{1}{2}\theta\}/\{\sin\tfrac{1}{2}(\theta-\beta+\gamma)\cos\tfrac{1}{2}(\theta-\gamma)\}\right| \qquad \ldots(3.5)$$

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* Measured from the no-lift attitude corresponding to zero sink strength.

A typical thick cambered section of the GLAS II type is then given by

$$S = \begin{cases} l & 0 < \theta < 2\pi \\ + (\pi+\alpha_1+\alpha_2+\gamma-\theta) \cot \frac{1}{2} (\alpha_1-\alpha_2), & \pi+\alpha_1+\alpha_2+\gamma-\varepsilon<\theta<\pi+\alpha_1+\alpha_2+\gamma \\ + b , & \beta<\theta<\pi+\alpha_1+\alpha_2+\gamma-\varepsilon \\ + c(\sin\theta - \sin \overline{\beta-\gamma}) , & \beta-\gamma<\theta<\beta \\ - j (\cos\theta - \cos\delta) , & 2\pi-\delta<\theta<2\pi \\ - j (1-\cos\delta) - k , & 0<\theta<\beta \\ + \frac{1}{2} k P_6 (\theta) , & [\text{Trailing edge term}] \end{cases} \qquad \ldots(3.6)$$

For continuity of $\log q_{om}$ at $\theta = \pi+\alpha_1+\alpha_2+\gamma-\varepsilon$ and $\theta = \beta$, we have

$$b = \varepsilon \cot \tfrac{1}{2} (\alpha_1-\alpha_2) = c (\sin\beta - \sin \overline{\beta-\gamma}) - j (1-\cos\delta) - k, \quad \ldots(3.7)$$

and then the velocity distribution is continuous everywhere. It will be observed that the term $c(\sin\theta - \sin \overline{\beta-\gamma})$ produces a velocity rising continuously from the rear lip of the slot entry at $\theta = \beta-\gamma$ into the slot at $\theta = \beta$. With this simple expression the rear lip takes the form of a cusp as there is a non-zero velocity there on the aerofoil contour. It would not be difficult to modify (3.6) in order to obtain a stagnation point at $\theta = \beta-\gamma$ on the aerofoil and a rear lip of small radius, but this would probably not differ much from the cusped shape. A well-rounded rear lip could probably be obtained by the introduction of further terms into (3.6), but the numerical computation of the aerofoil shape would become extremely involved and lengthy.

The formulae for the aerofoil coordinates (x,y) are

$$x = \int (2 \sin\theta / q_{oo}) \cos \chi \, d\theta, \quad y = \int (2 \sin\theta / q_{oo}) \sin \chi \, d\theta, \quad \ldots(3.8)$$

where $\chi(\theta) = (1/2\pi) \, P \int_{-\pi}^{\pi} \log q_{oo}(t) \cot \frac{1}{2} (\theta - t) \, dt$ is the Fourier conjugate of $\log q_{oo}(\theta)$. The function $\log q_{oo}$ must satisfy the relations

$$\int_{-\pi}^{\pi} \log q_{oo} \, d\theta = 0, \quad \int_{-\pi}^{\pi} \log q_{oo} \cos\theta \, d\theta = 0, \quad \int_{-\pi}^{\pi} \log q_{oo} \sin\theta d\theta = 0 \quad \ldots(3.9)$$

and the pitching moment coefficient at zero-lift without suction is

$$C_{Mo} = (-4/c_d^2) \int_0^{2\pi} \log q_{oo} \sin 2\theta \, d\theta \quad \ldots(3.10)$$

where c_d is the design chord which takes a value between 3 and 4. The contributions of the separate terms of $\log q_{oo}$ to the conjugate χ and to the integrals (3.9) and (3.10), can mostly be written down immediately from the formulae listed in Appendix I of Ref. 6, but a few additional formulae are listed in the Appendix to the present paper. When α_1, α_2, β, γ, δ and ε are assigned, the constants l, b, c, j and k are given

by /

by the relations (3.7) and (3.9). The value of γ is chosen to yield the required value of $C_Q = 2\pi n/e_d$ at some assigned incidence, and ε is chosen to give a reasonable value for C_{M0} - usually of negligible magnitude.

The design can then be completed following the usual procedure described in Ref.6. In the integration to obtain the aerofoil shape, it is necessary to check, not that the contour closes up, but that there is a slot entry of a width equal to C_Q/q_{cm} $(\theta=\beta)$ of the chord.

4. Theoretical Treatment without Sink effect

When sink effect is ignored the design velocity distribution can be specified in the more usual form

$$\log q_0 = \begin{cases} \log \left| \cos \tfrac{1}{2}\theta / \cos (\tfrac{1}{2}\theta - \alpha_1) \right| , & \alpha_1+\alpha_2 < \theta < \pi+\alpha_1+\alpha_2 \\ +\log \left| \cos \tfrac{1}{2}\theta / \cos (\tfrac{1}{2}\theta - \alpha_2) \right| , & \pi+\alpha_1+\alpha_2 < \theta < 2\pi+\alpha_1+\alpha_2 \\ + S , & 0 < \theta < 2\pi \end{cases} \quad \ldots(4.1)$$

where S takes the values $\log q_{\alpha_1}$, $\log q_{\alpha_2}$ over the upper surface

$\alpha_1+\alpha_2 < \theta < \pi+\alpha_1+\alpha_2$ and the lower surface $\pi+\alpha_1+\alpha_2 < \theta < 2\pi+\alpha_1+\alpha_2$ respectively. For the purpose of illustration, we shall consider velocity distributions for symmetrical suction aerofoils of the GLAT III type.

The original GLAT III shape has simply $\alpha_1 = -\alpha_2 = \alpha$ and

$$S = \begin{cases} l , & 0 < \theta < \pi \\ +K_\zeta , & \pi - \dfrac{\pi}{12} < \theta < \pi \\ - k , & \theta < \theta < \beta , \end{cases} \quad \ldots(4.2)$$

continued as an even function of θ ; $\theta = \beta$ is the position of the discontinuity, l and k are constants, and K_ζ is a term introduced merely to increase the nose-radius of the section (see §4 of Ref.6.).
The replacement of the k term in (4.2) by the term

$$\begin{cases} - c\,(\cos\theta - \cos\beta) , & 0 < \theta < \beta \\ + c\,(\cos\theta - \cos \overline{\beta-\gamma}) , & 0 < \theta < \beta-\gamma \end{cases} \quad \ldots(4.3)$$

spreads the velocity fall evenly over the extent of surface $\beta-\gamma < \theta < \beta$.
The inclusion of a further term

$$h \log \left| \tan \tfrac{1}{2} (\theta-\beta+\gamma) \right| - h \log \tan \tfrac{1}{2}\gamma , \qquad \beta-2\gamma < \theta < \beta \quad \ldots(4.4)$$

provides a design velocity distribution falling rapidly (γ small) from $\theta = \beta$ to a stagnation point at $\theta = \beta-\gamma$ and then rising again as far as $\theta = \overline{\beta-2\gamma}$.

It /

It can readily be shown that h must in general lie between 0 and 1,* and that the section shape will then have a discontinuity of $h\pi$ in the slope of its tangent at the stagnation point, i.e. there will be an angled-bend with exterior angle $h\pi$.

The usual Fourier relations - equation (3.9) with $\log q_o$ replacing $\log q_{oo}$ - give the constants l and e when the values of α, β, γ, and h (if included) are assigned. The design can then be completed by the procedure recommended in Ref.6. When the term involving h is included, the integrands in the relations of the type (3.8) for x and y have an infinity at $\theta = \beta$ due to the zero in q_o. Nevertheless, provided $0 < h < 1$,* the integrals are convergent and can be evaluated by representing the integrands within a narrow range about the infinity by simple integrable functions to which they are asymptotic as $\theta \to \beta$.

5 Applications and General Discussion

GLAS II with sink effect and slot shape

The original cambered GLAS II section was design by Glauert[6] (1947) and has been tested both at the N.P.L. and in Australia. A modification designed to include sink effect and a slot is shown in Fig. 1a, and the corresponding velocity distribution is given in Fig. 1b.+ For simplicity in the theoretical design, the rear lip has been given the form of a cusp, and a rather large C_Q of 0.009 has been assumed. In practice, the cusp would have to be rounded off. A well-rounded rear lip could probably be designed for, but the numerical work would become extremely lengthy and involved. It is seen in Fig.1a that the sides of the slot approach asymptotically to the same direction at a constant width apart. The severe curvatures of the original section are somewhat reduced owing to a paring away of the front shoulder and a filling-in of the concavity behind; but the relative magnitudes of the paring away and filling-in are not obvious because the slot position is slightly altered. The magnitude of the modifications would of course have been less if a smaller value of C_Q (corresponding to transition at the slot) had been used.

It should be mentioned that the modification of GLAS II obtained theoretically by Hurley and Hurst[9] (1948), namely a slight paring away of the shoulder, was obtained by approximate calculations using a velocity distribution with a discontinuity instead of a rapid but continuous fall as prescribed here; this led to a slot whose sides were asymptotic to a pair of concentric circles. The mathematical treatment of the problem was also somewhat different from that developed at the N.P.L.

Australian/

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* If β is an integral multiple of π, h need only lie between 0 and 2. For example, with $\beta = 0$, we obtain a trailing-edge angle of $(2 - h)\pi$.

\+ A thicker and more highly cambered section, designed with high-lift applications in mind, is shown in Fig.2.

Australian experimental modification of GLAS II

The Australians pared away extensively the shoulder of the original GLAS II profile ahead of the position of the velocity discontinuity in order to remove the severe curvatures in that vicinity (see Fig. 1a), and as a result the velocity fall became evenly spread over a region beginning about 0.05c ahead of the original discontinuity. With suction at three slots in this region, the modified section worked satisfactorily[5]. Section shapes with a velocity fall of this type can readily be designed (ignoring sink effect) by the exact method (see §4) or by Goldstein's approximate theory.

Experimental results for 30% Griffith section[3, 4]

This symmetrical section was designed by Goldstein's approximate theory to have an abrupt velocity fall at 0.8c from the leading edge, and has slight adverse velocity gradients beginning about 0.1c ahead of this position. The surface curvatures in the vicinity of the slot are consequently rather less severe than would have been obtained if the section had been designed to have a single discontinuous fall in velocity by exact theory. In tests at a Reynolds number of 3×10^6, transition occurred just in front of the slot because of the adverse gradients there, and the experimental C_Q values agreed well with the corresponding theoretical estimates. However, at a lower Reynolds number of 1×10^6, considerably higher suction quantities were needed because of the occurrence of laminar separation instead of transition.

GLAT III shapes

The original GLAT III symmetrical shape was designed by Glauert[6] (1947) and has recently been tested at the R.A.E. Two modified shapes are shown in Fig. 3a, one with the velocity decreasing rapidly to a stagnation point at the intended position for the rear lip of the slot (as suggested by Griffith[7]), and the other with the velocity falling steadily over a small extent of surface (instead of discontinuously as in the original shape), where in both cases the velocity fall is assumed to take place over a distance comparable with the slot width.* The first modification is marked because of the angled bend which corresponds to the required stagnation point. The second modification exhibits a slight reduction in surface curvatures but this would probably be insignificant in practice; a much more widely spread fall would be necessary to effect an appreciable change in shape. It should be pointed out that the general thinning and thickening obtained with the first and second modifications respectively are partly associated with the changes in the chordwise location of the velocity fall.

6./

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* In order to simplify the computations, the slot width was assumed to be somewhat larger than would be required for removal of a laminar boundary-layer.

6. Conclusions

The Australian tests have shown that shapes with the velocity fall spread evenly over an appreciable extent of surface (in which case large surface curvatures are simultaneously avoided) are likely to work satisfactorily, provided several suction slots* are employed over the region of adverse velocity gradient. The possibility of estimating theoretically the number and spacing of the suction slots required is being examined by the present writer.

It is important, however, to ascertain whether the need for several slots can be avoided. For example, the 30% symmetrical Griffith aerofoil, with a slight adverse velocity gradient promoting transition ahead of the slot and with the remaining fall in velocity effectively spread across the slot width, has worked reasonably well. Moreover, it is possible that an abrupt pressure rise can be more easily surmounted with the aid of suction if the boundary layer becomes turbulent just before reaching the slot than if it remains laminar the whole way.

The inclusion of sink effect and slot shape in the design should lead to some improvement, though the change in shape may not be significant if the assumed C_Q is very small. An alternative method of representing more closely the physical conditions at the slot, namely by designing for the stagnation streamline on the rear lip, also affords interesting possibilities.

Experiments alone can decide the relative merits of these various methods of design.

Acknowledgement. The writer is indebted to Miss E. M. Love, Miss C. M. Tracy and Miss L. M. Esson for their assistance with the computational work and preparation of diagrams for this paper.

APPENDIX /

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* Or possibly distributed suction through porous areas.

APPENDIX

List of Additional Conjugates

The Fourier conjugate function of a function $f(\theta)$ with periodicity 2π in θ is given by Poisson's integral as

$$G(\theta) = (1/2\pi) \int_{-\pi}^{\pi} f(\phi) \cot \tfrac{1}{2} (\theta-\phi)\, d\phi ,$$

The following integrals, which correspond to the first five constants in the Fourier series for $f(\theta)$, are also required in aerofoil design.

$$\{A, B, C, D, E\} = \int_{-\pi}^{\pi} f(\theta) \quad \{1,\ \cos\theta,\ \sin\theta,\ \cos 2\theta,\ \sin 2\theta\}\ d\theta$$

A few additions to the list of conjugates and constants given in Appendix I of Ref. 6 will now be tabulated. The terms of G enclosed in square brackets are only of interest for a determination of the no-lift angle.

1. $\log \left| \cos \tfrac{1}{2} (\theta-\gamma)/\cos \tfrac{1}{2} (\theta-\gamma-2\alpha_1)\right|$ for $\alpha_1+\alpha_2+\gamma < \theta < \pi + \alpha_1+\alpha_2+\gamma$

$+ \log \left| \cos \tfrac{1}{2} (\theta-\gamma)/\cos \tfrac{1}{2} (\theta-\gamma-2\alpha_2)\right|$ for $\pi+\alpha_1+\alpha_2+\gamma < \theta < 2\pi+\alpha_1+\alpha_2+\gamma$

$$G = F\{\tan \tfrac{1}{2}(\alpha_1-\alpha_2) \tan \tfrac{1}{2} (\theta-\alpha_1-\alpha_2-\gamma)\} + [\tfrac{1}{2}(\alpha_1+\alpha_2)]$$

$$A = -2\pi\{X(\alpha_1-\alpha_2) + X(\pi - \alpha_1-\alpha_2) - X(\pi)\}$$

$$B = \{2 \sin \alpha_1 \cos(\alpha_1+\gamma) - 2 \sin \alpha_2 \cos(\alpha_2+\gamma)\} \log \cot \tfrac{1}{2} (\alpha_1-\alpha_2)$$
$$+ \pi\{\sin \alpha_1 \sin(\alpha_1+\gamma) + \sin \alpha_2 \sin(\alpha_2+\gamma)\}$$

$$C = \{2 \sin \alpha_1 \sin(\alpha_1+\gamma) - 2 \sin \alpha_2 \sin(\alpha_2+\gamma)\} \log \cot \tfrac{1}{2} (\alpha_1-\alpha_2)$$
$$- \pi\{\sin \alpha_1 \cos(\alpha_1+\gamma) + \sin \alpha_2 \cos(\alpha_2+\gamma)\}$$

$$D = 2 \sin(\alpha_1-\alpha_2) \cos 2(\alpha_1+\alpha_2+\gamma) - \tfrac{1}{2}\pi\{\sin 2\alpha_1 \sin 2(\alpha_1+\gamma)$$
$$+ \sin 2\alpha_2 \sin 2(\alpha_2+\gamma)\} - \{\sin 2\alpha_1 \cos 2(\alpha_1+\gamma)$$
$$- \sin 2\alpha_2 \cos 2(\alpha_2+\gamma)\} \log \cot \tfrac{1}{2} (\alpha_1-\alpha_2)$$

$$E = 2 \sin(\alpha_1-\alpha_2) \sin 2(\alpha_1+\alpha_2+\gamma) + \tfrac{1}{2}\pi\{\sin 2\alpha_1 \cos 2(\alpha_1+\gamma)$$
$$+ \sin 2\alpha_2 \cos 2(\alpha_2+\gamma)\} - \{\sin 2\alpha_1 \sin 2(\alpha_1+\gamma)$$
$$- \sin 2\alpha_2 \sin 2(\alpha_2+\gamma)\} \log \cot \tfrac{1}{2} (\alpha_1-\alpha_2)$$

2. $\log \left| \left\{ \sin \frac{1}{2} (\Theta-\beta) \cos \frac{1}{2}\Theta \right\} / \sin \frac{1}{2} (\Theta-\beta+\gamma) \cos \frac{1}{2} (\Theta-\gamma) \right|$ for $0 < \Theta < 2\pi$

$$G = 0$$

$$A = 0$$

$$B = 4\pi \sin \tfrac{1}{2} \gamma \cos \tfrac{1}{2} (\beta-\gamma) \sin \tfrac{1}{2} \beta$$

$$C = -4\pi \sin \tfrac{1}{2} \gamma \cos \tfrac{1}{2} (\beta-\gamma) \cos \tfrac{1}{2} \beta$$

$$D = 2\pi \sin \gamma \sin (\beta-\gamma) \cos \beta$$

$$E = 2\pi \sin \gamma \sin (\beta-\gamma) \sin \beta$$

3. $\cos (\Theta-\lambda)$ for $\lambda < \Theta < \mu$

$$G = \frac{\mu-\lambda}{2\pi} \sin (\Theta-\lambda) + \frac{\cos (\Theta-\lambda)}{\pi} \log \left| \frac{\sin \frac{1}{2} (\Theta-\lambda)}{\sin \frac{1}{2} (\Theta-\mu)} \right| + \left[\frac{1}{2\pi} \left\{ 1 - \cos (\mu-\lambda) \right\} \right]$$

$$A = \sin (\mu-\lambda)$$

$$B = \frac{1}{4} \left\{ 2 (\mu-\lambda) \cos \lambda + \sin (2\mu-\lambda) - \sin \lambda \right\}$$

$$C = \frac{1}{4} \left\{ 2(\mu-\lambda) \sin \lambda - \cos (2\mu-\lambda) + \cos \lambda \right\}$$

$$D = \frac{1}{6} \left\{ \sin (3\mu-\lambda) + 3 \sin (\mu+\lambda) - 4 \sin 2 \lambda \right\}$$

$$E = \frac{1}{6} \left\{ - \cos (3\mu-\lambda) - 3 \cos (\mu+\lambda) + 4 \cos 2\lambda \right\}$$

4. $\cos (\Theta-\mu)$ for $\lambda < \Theta < \mu$

$$G = \frac{\mu-\lambda}{2\pi} \sin (\Theta-\mu) + \frac{\cos(\Theta-\mu)}{\pi} \log \left| \frac{\sin \frac{1}{2} (\Theta-\lambda)}{\sin \frac{1}{2} (\Theta-\mu)} \right|$$

$$\left[1 \right.$$

$$A = \sin(\mu-\lambda)$$

$$B = \frac{1}{4}\{2(\mu-\lambda)\cos\mu + \sin(\mu-2\lambda) + \sin\mu\}$$

$$C = \frac{1}{4}\{2(\mu-\lambda)\sin\mu + \cos(\mu-2\lambda) - \cos\mu\}$$

$$D = \frac{1}{6}\{\sin(\mu-3\lambda) - 3\sin(\mu+\lambda) + \sin 2\mu + 3\sin 2\mu\}$$

$$E = \frac{1}{6}\{\cos(\mu-3\lambda) + 3\cos(\mu+\lambda) - \cos 2\mu - 3\cos 2\mu\}$$

5. $\underline{\log \left| \tan \tfrac{1}{2}(\Theta-\xi) \right| \text{ for } \xi-\Upsilon<\Theta<\xi+\Upsilon}$

$$G = -F\{\tan \tfrac{1}{2}\Upsilon \cot \tfrac{1}{2}(\Theta-\xi)\} + \frac{\log\tan\tfrac{1}{2}(\Theta-\xi)}{\pi} \log \left| \frac{\sin\tfrac{1}{2}(\Theta-\xi+\Upsilon)}{\sin\tfrac{1}{2}(\Theta-\xi-\Upsilon)} \right|$$

$$A = -2\pi\{X(\Upsilon) + X(\pi-\Upsilon) - X(\pi)\}$$

$$B = 2\cos\xi\ (\sin\Upsilon \log\tan\tfrac{1}{2}\Upsilon - \Upsilon)$$

$$C = 2\sin\xi\ (\sin\Upsilon \log\tan\tfrac{1}{2}\Upsilon - \Upsilon)$$

$$D = \cos 2\xi\ (\sin 2\Upsilon \log\tan\tfrac{1}{2}\Upsilon - 2\sin\Upsilon)$$

$$E = \sin 2\xi\ (\sin 2\Upsilon \log\tan\tfrac{1}{2}\Upsilon - 2\sin\Upsilon)$$

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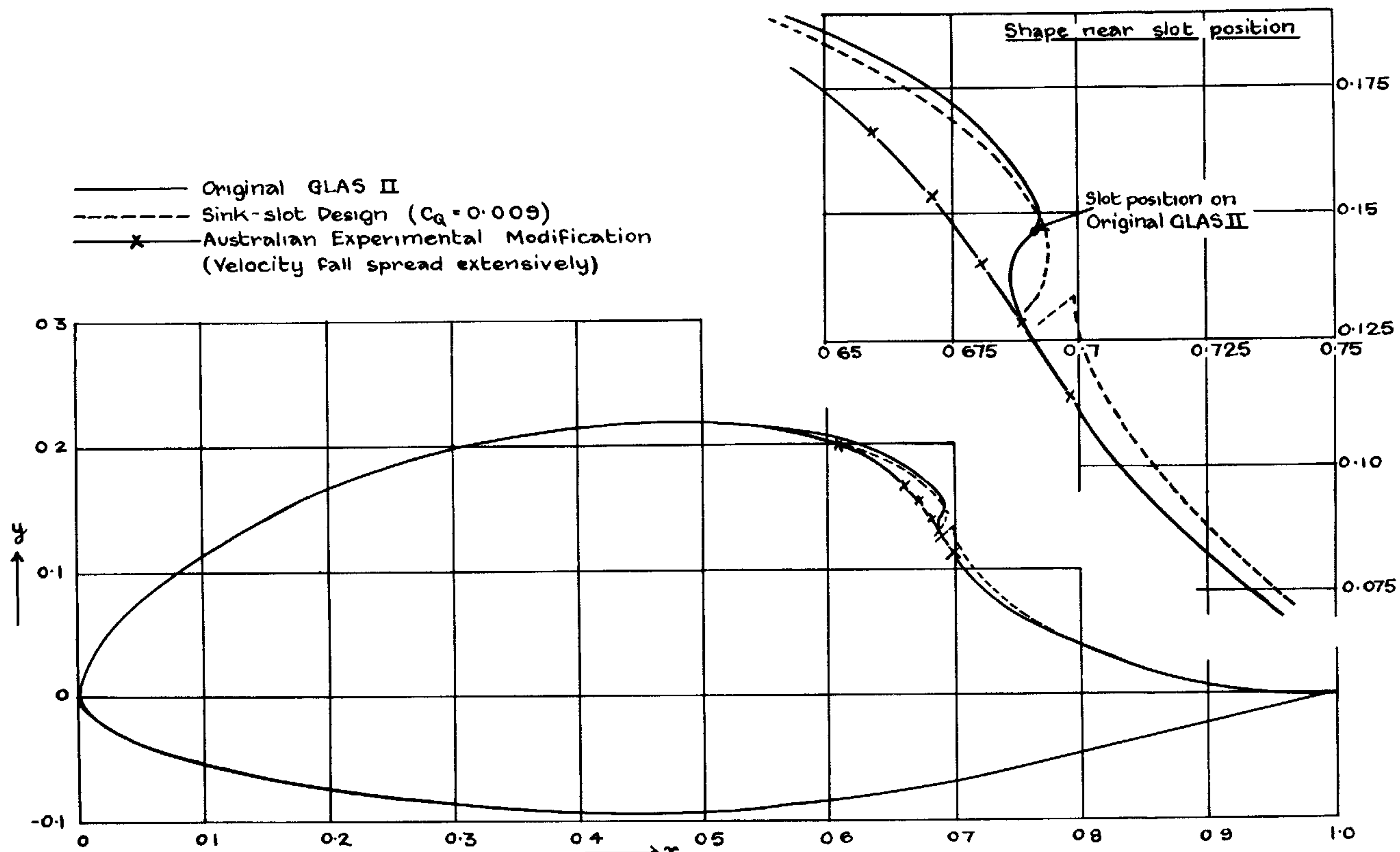

Modified GLAS II Shapes

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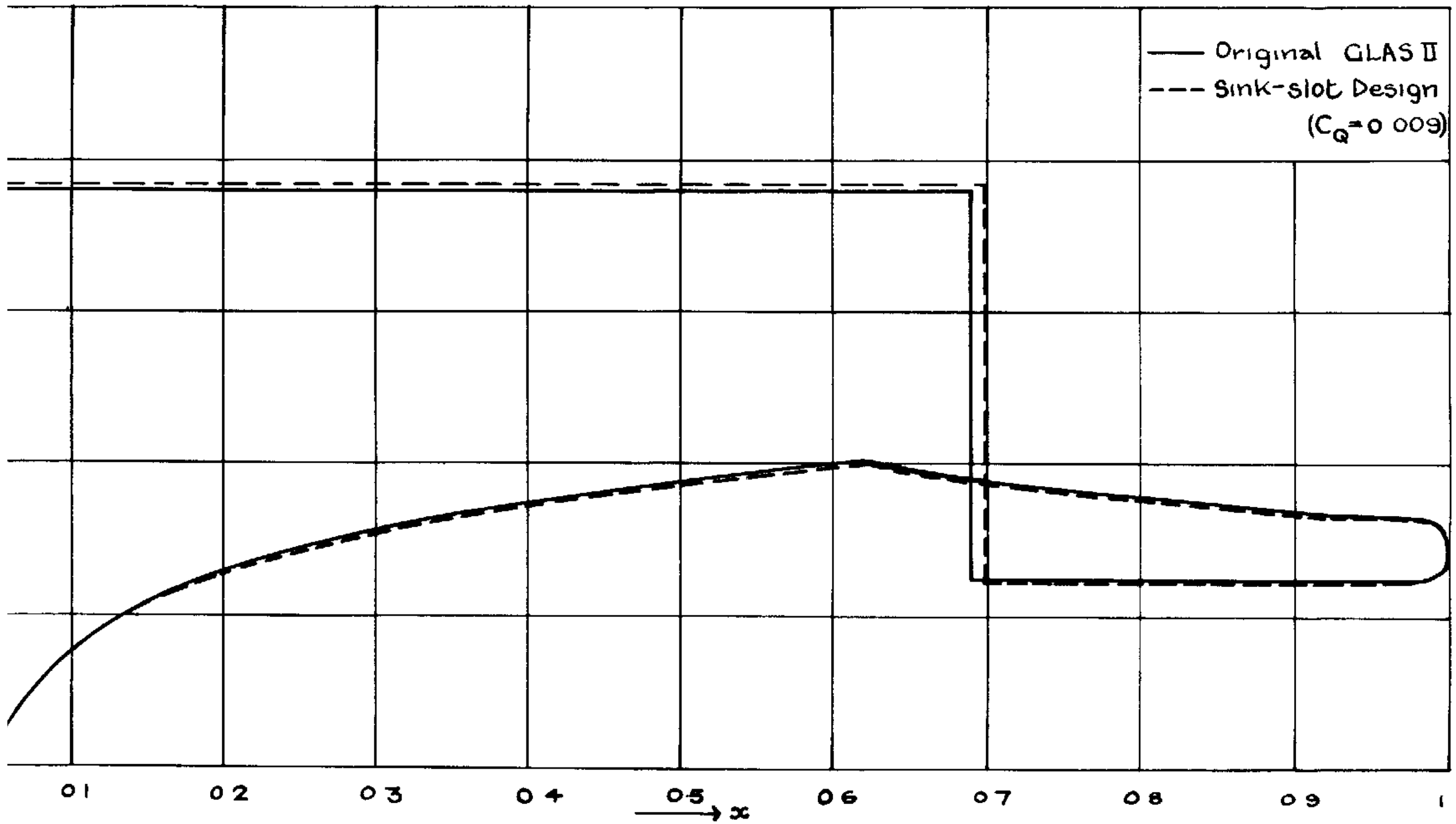
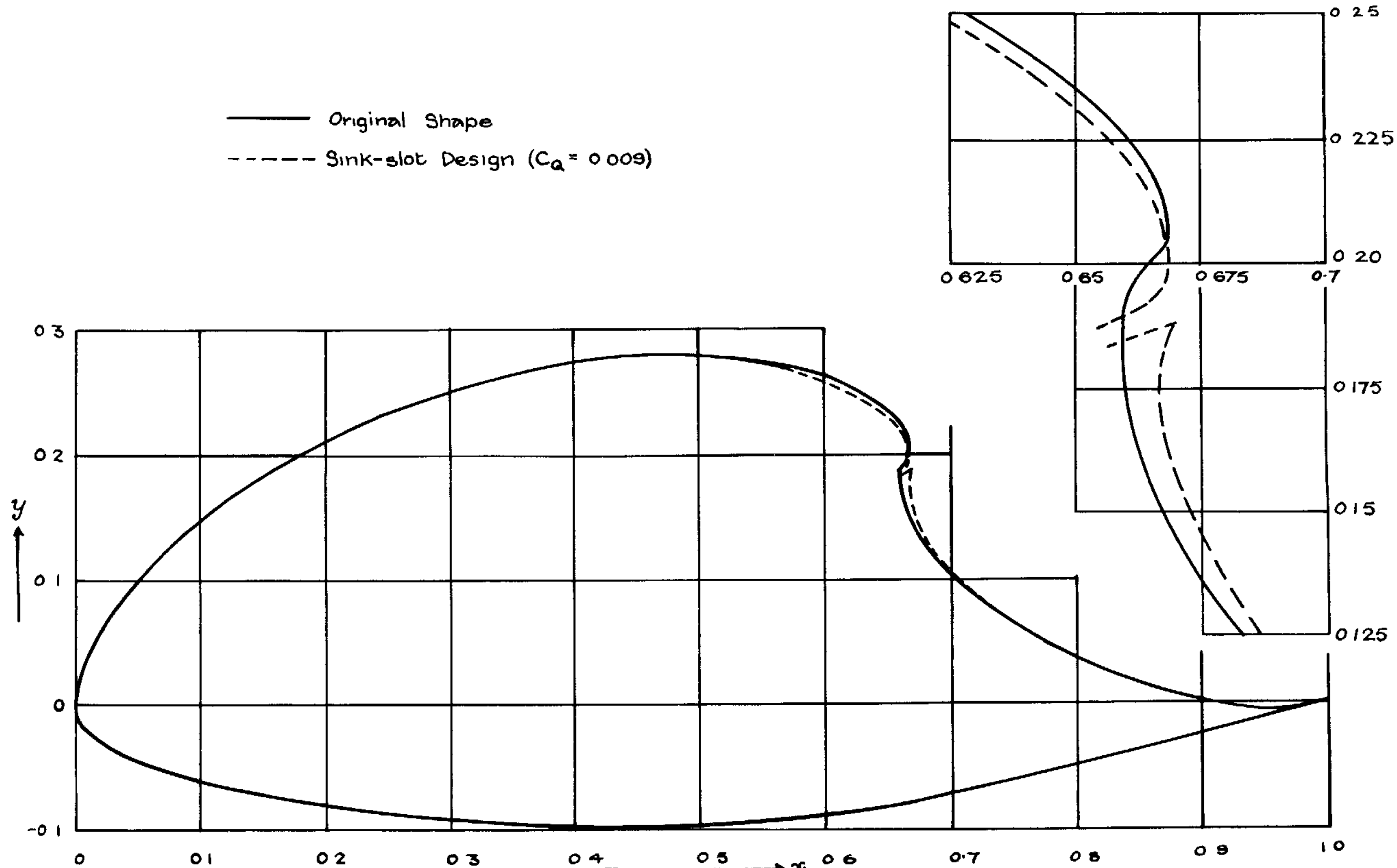
Fig 1a



Velocity Distribution for Modified GLAS II Shape (C_L = 2 00)



Modified 38% Thick Cambered Section

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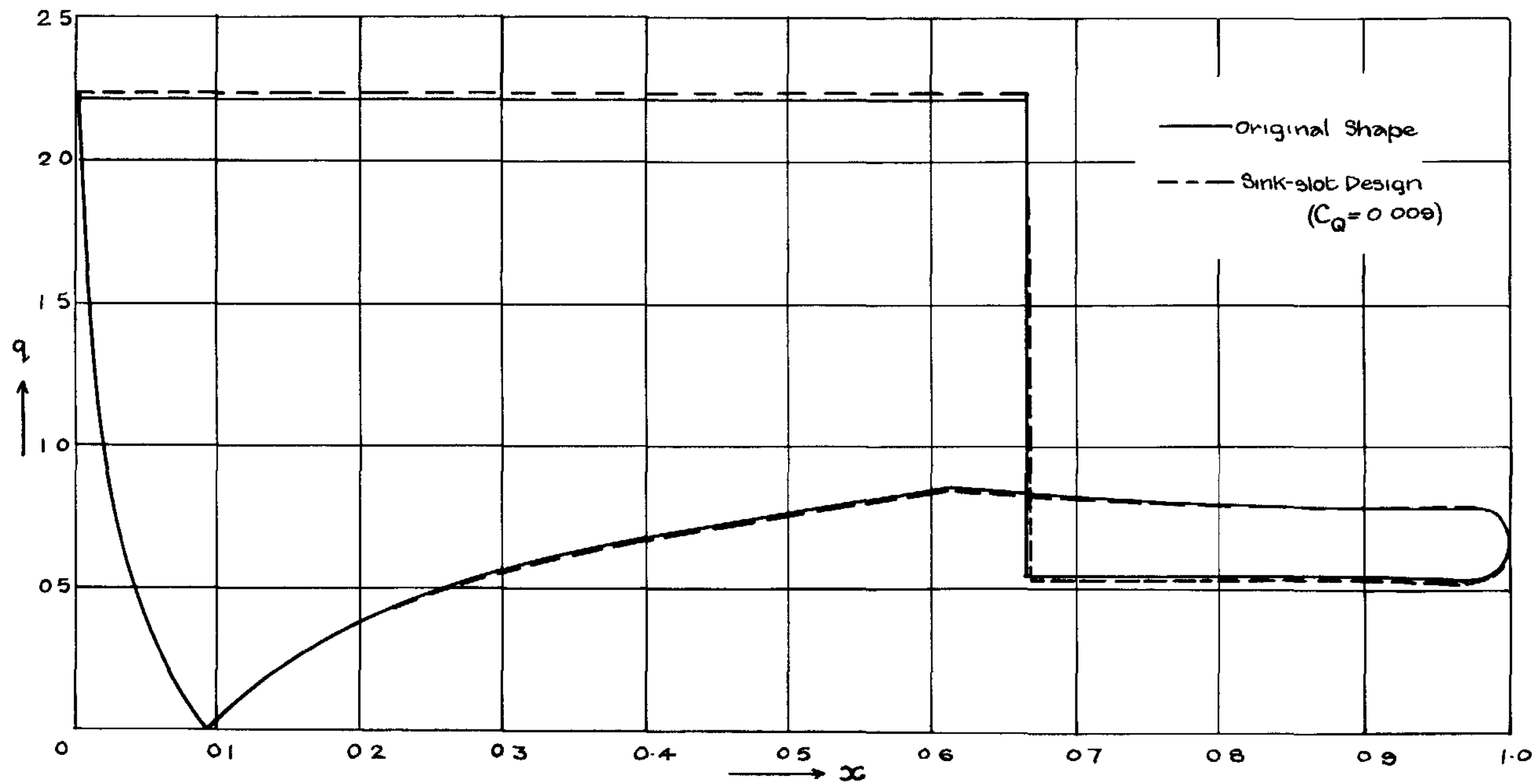
FIG 2b



Velocity Distribution for Modified 38% Thick Cambered Section (C_L = 3 03)

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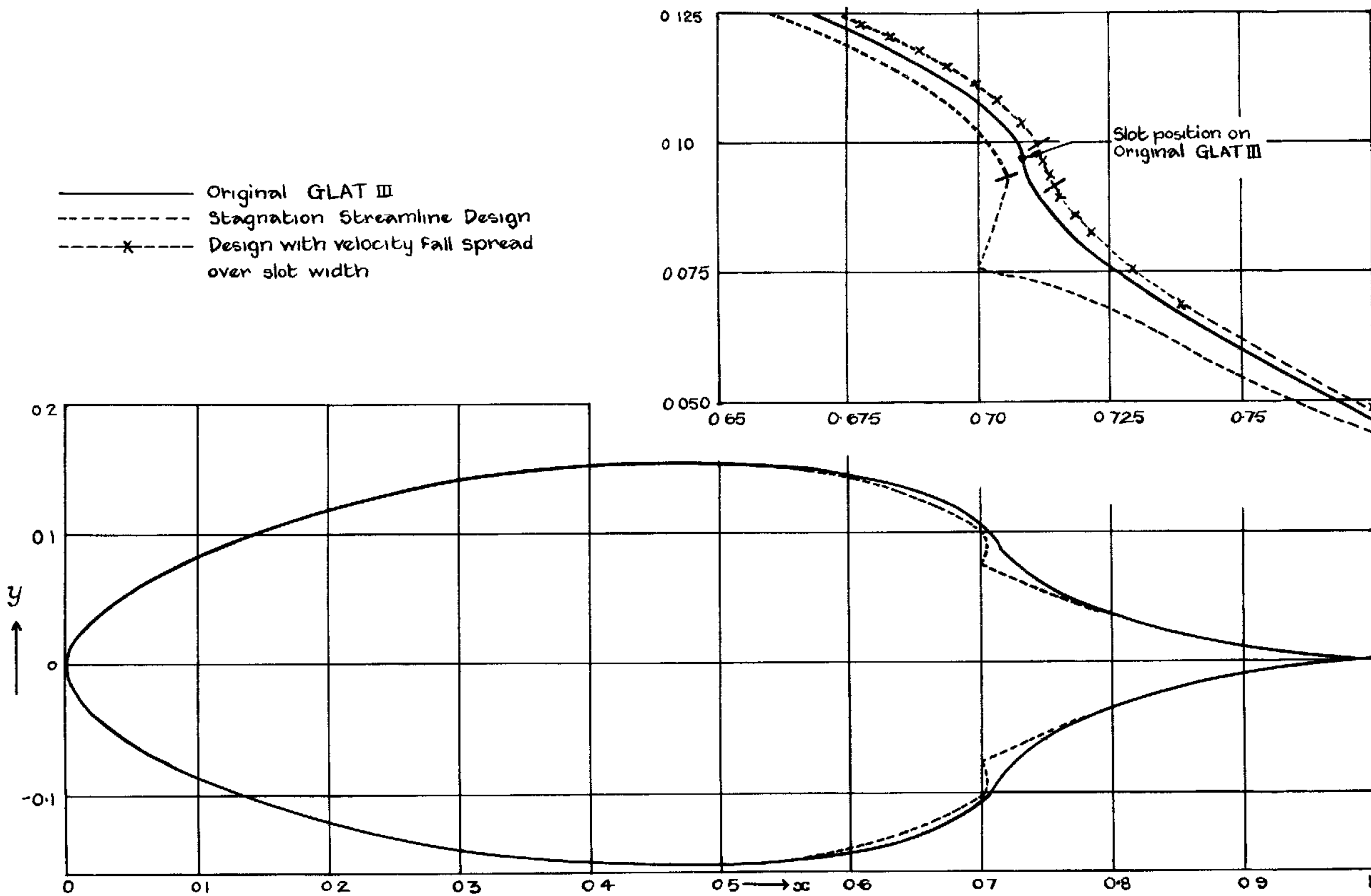
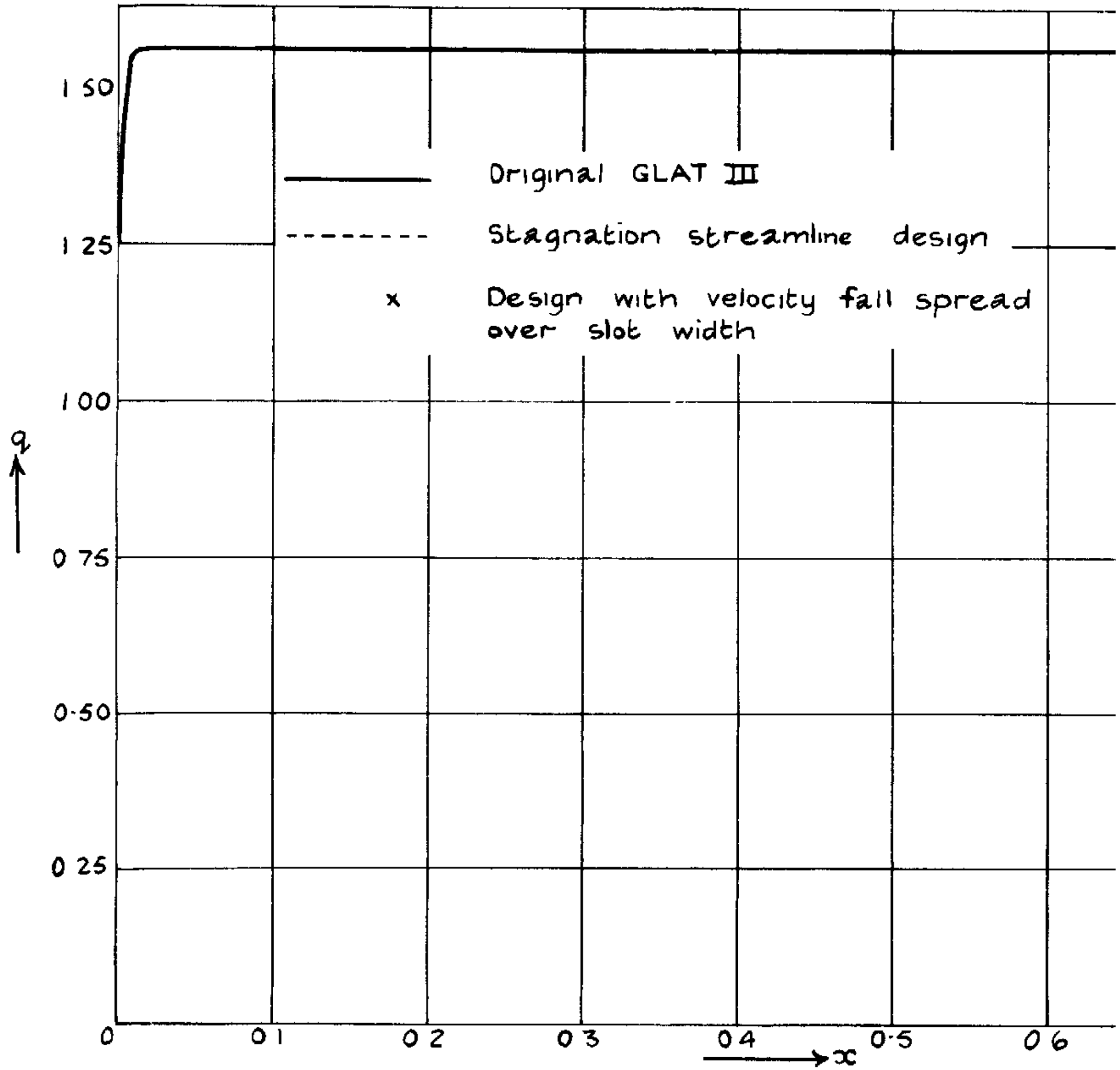
Fig 3.a



Modified GLAT III Shapes

B.B



Velocity distributions for Modified GLAT III

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