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An Approximate Method for the Estimation of the Design Point Efficiency of Axial Flow Turbines

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An approximate method for the estimation of the design point efficiency of axial flow turbines

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SUMMARY

A method is outlined for making rapid approximate estimates of the design point efficiency of conventional axial flow turbines. For a number of existing turbines a comparison has been made between the predicted efficiencies and the efficiencies actually recorded on test. As a result of this comparison a final empirical correction to the calculated efficiencies is suggested to give final predictions which should compare favourably with actual values. It appears from the analysis that the turbine stage temperature drop coefficient, $\frac{2\ Kp\ \Delta T}{U_m^2}$, and the stage reaction are of major significance when considering stage efficiency. Contemporary units show relatively poor efficiency when designed for a high temperature drop coefficient and the reflection of this upon the choice of an optimum turbine desain for certain applications such as long range aircraft, industrial, and marine plant is briefly discussed.

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1.0 Introduction

During past years the efficiency to be expected from a project turbine design has been largely a matter of conjecture and no rational method has existed whereby a fairly reliable estimate might be made. The reason for this has been primarily due to the absence of reliable data regarding pressure losses and deflections through a row of turbine blades for specified conditions of blade form, gas incidence angle, Mach number, Reynolds number, etc. Data of this type is now being gradually accumulated, however, and when complete information is finally available the task of estimating turbine performance will reduce to that of suitable computation. Nevertheless, accurate computation will be comparatively lengthy and there will always be a need for a rapid approximate method for making estimates of turbine efficiency just at the design point. The creation of such a method may ultimately be expected to define critical parameters controlling maximum attainable turbine efficiencies as well as to offer a quick estimate of the efficiency of any specific unit and thus may be doubly useful to designers during the initial conjectural stage of a new engine design. The purpose of this note is to suggest a possible way of achieving this objective.

The method adopted has been to calculate the possible efficiencies of a wide range of hypothetical "50% reaction" and "impulse" turbine stages and to indicate a way of deriving the efficiency of any arbitrary unit by a relatively simple process of interpolation between these hypothetical units. A final empirical correction is suggested which has been derived by direct comparison of uncorrected estimates of efficiency for a number of turbines with values obtained by experiment.

The hypothetical reaction and impulse turbines are of the simplest possible conception and the important assumptions made with regard to their performance may be listed as follows:-

(1) A flow of negligible compressibility (or Mach number)

(2) The performance of a stage is determined by reference only to the mean diameter blade sections and it is assumed that the performance will be unaffected by the value of the hub ratio.

(3) The incidence on each row of every hypothetical turbine is 0°, or very near, at the design point.

(4) The blade pitch/chord ratio in each row is chosen to make the blade lift coefficient based on outlet velocity ($C_{L_{V_2}}$) equal to 0.7 at the design (zero) incidence.

These assumptions may appear justifiable, from the point of view of simplifying the computation, if it is wished primarily to predict the design point efficiency. In the calculation of efficiency it is, of course, the assumptions regarding blade pressure losses which predominate. Section 2.0 is devoted to a discussion of available information regarding pressure loss coefficients relevant to turbine blading. Data on conventional turbine blades from both cascade and full scale turbine tests has indicated that variation of flow Mach number influences pressure losses only to a secondary degree so that the assumption of negligible compressibility is unlikely to affect appreciably the calculated efficiency of a stage, even if in practice it operates at high Mach numbers, providing that the incidences and Reynolds numbers in the calculation correspond to the true working condition of the actual turbine. The assumption that the performance of a stage may be deduced by reference only to the mean diameter blade section is, perhaps, open to more doubt at the present time, particularly on stages in which the hub ratio is small and the blade roots operate close to impulse conditions. On high reaction turbines no evidence has yet been noted to suggest the necessity for any "work done factors" and the calculated performance, based on mean diameter blade geometry, has shown reasonable agreement with available test results. On low reaction turbines, however, the problem is at present more obscure and the assumption in question may be contributory to the over-estimates of efficiency on some of these latter types of turbine that are discussed in section 6.0.

Nevertheless, the final empirical correction to the estimated efficiencies which is derived by direct comparison with experiment may be expected to allow to some extent for the non-validity of the simplifying assumptions outlined above and to correct for the relatively inadequate nature of the blade pressure loss data available.

2.0 Total pressure losses through rows of turbine blades

As outlined in reference 1 the losses are sub-divided into (1) profile loss, (2) secondary loss, and (3) annulus loss.

The notation and sign convention used in this note is stated in Appendix I and illustrated in figure 1.

2.1 Profile losses at zero blade incidence

These are assessed approximately from references 2 to 5. In figures 2 and 3 the values of the profile loss coefficient Y_p (defined as $Y_p = \dfrac{\text{profile loss}}{P_{tot_{outlet}} - p_{stat_{outlet}}}$), for zero incidence are plotted against pitch/chord ratio for blades giving various gas outlet angles both for a range of nozzle type blades, defined here by $\beta_1 = 0°$, and a range of impulse type blades, defined here by $\beta_1 = -\alpha_2$. To find a value of Y_p for an arbitrary blade section it is necessary to determine the value of the quantities β_1/α_2, α_2, and s/c for the blade considered and use the expression:-

$$Y_p = Y_{p(\beta_1 = 0°)} + (\beta_1/\alpha_2)^2 \left[Y_{p(\beta_1 = -\alpha_2)} - Y_{p(\beta_1 = 0°)}\right] \quad .....(1)$$

when $\beta_1/\alpha_2 < 0$,

or $$Y_p = Y_{p(\beta_1 = 0°)} \quad ....(2)$$

when $\beta_1/\alpha_2 \geq 0$.

The values of $Y_{p(\beta_1 = 0°)}$ and $Y_{p(\beta_1 = -\alpha_2)}$ in equation (1) or (2) may be obtained from figures 2 and 3 for the values of s/c and α_2 appropriate to the blade under consideration.

It should be appreciated that these curves are obtained from a number of test results via a process of interpolation and extrapolation and are necessarily approximate. The losses at low values of s/c are estimated roughly on the assumption that when the blades are so close together as to touch one another the blade loss coefficient must be infinite. The losses measured on various reaction blades in different tunnels agreed well but the losses measured on impulse blades have apparently varied widely in different tunnels (cf. refs. 3 and 4). It is thought that this latter variation is partially attributable to differences in tunnel gas stream turbulence, amount of laminar flow on the blades and point of flow break away near trailing edge. For the purposes of turbine performance estimation the results from tunnels showing the higher blade losses (references 4 and 5) have been selected from which to determine the curves in figure 3 since these results are believed to be more appropriate to the order of turbulence and amount of laminar flow that is likely to exist in an actual turbine.

2.2 Secondary losses

These are calculated from the empirical formula suggested in reference 1:-

$$\frac{\text{Secondary loss}}{P_{tot_{outlet}} - P_{stat_{outlet}}} = Y_s = 0.04 \left[1 - (\beta_1/\alpha_2)\right] C_{L_{V_2}}^{2} \qquad \ldots\ldots.(3)$$

when $\beta_1/\alpha_2 < 0$

$$\text{or,} \qquad Y_s = 0.04\, C_{L_{V_2}}^{2} \qquad \ldots\ldots\ldots.(4)$$

when $\beta_1/\alpha_2 > 0$

This loss is assumed to allow for tip clearance, or shroud, loss in addition to other losses associated with three dimensional flows. The tip clearance loss in practice, however, is an obscure quantity although some unpublished test data suggests that the influence of tip clearance may be severe, causing as much as 2 to 2½% fall in efficiency for an increase of tip clearance of 1% of the blade height. Efficiencies calculated using equation (3) for secondary loss have shown good agreement with test results on high reaction turbines in which the tip clearances were roughly 1½ to 2% of the blade height and, since the profile losses on high reaction blades can be fairly reliably estimated from cascade tests, it might be assumed that equation (3) is approximately valid for clearances of about this value. Turbines with considerably larger clearances might be expected to operate at rather lower efficiencies than those calculated, but the magnitude of the reduction is at present indeterminate.

2.3 Annulus loss

This is assumed as:-

$$\frac{\text{Annulus loss}}{P_{tot_{outlet}} - P_{stat_{outlet}}} = Y_a = \frac{0.02}{h/c} \qquad \ldots\ldots\ldots.(5)$$

When applying this rule to high reaction nozzle guide vanes the loss may be halved since equation (5) is based on a high friction coefficient and may, therefore, over estimate the loss, particularly when the aspect ratio is small (as is often the case on a first stage nozzle row).

2.4 Total loss

The total loss through a row of blades is the sum of the component losses

$$\frac{\text{Total loss}}{P_{tot_{outlet}} - P_{stat_{outlet}}} = Y_t = Y_p + Y_s + Y_a \qquad \ldots\ldots\ldots.(6)$$

3.0 Calculation of the efficiencies of a series of hypothetical impulse and reaction turbines

Assuming that the Mach numbers of the flows in a turbine stage are sufficiently low to treat the gas density as a constant then it is shown in Appendix II that:-

$$\eta = \frac{1}{1 - \frac{1}{2}\left(\frac{V_{a_1}}{U}\right)\left\{\frac{Y_{t_s}\sec^2\alpha_0 + \lambda^2 Y_{t_r}\sec^2\alpha_2}{\frac{U}{V_{a_1}} - (\tan\alpha_0 + \lambda\tan\alpha_2)}\right\}} \quad \text{........(7)}$$

Where, η = isentropic total head stage efficiency

Y_{t_s} = total loss coefficient in stator row

Y_{t_r} = total loss coefficient in rotor row

λ = V_{a_2}/V_{a_1}

It is further assumed that,

(1) The incidence angle on each row is zero (i.e. $\beta_1 = \alpha_1$, $\beta_3 = \alpha_3$).

(2) The value of h/c on each row is 2.0.

(3) The value of s/c on each row is such as to give a value of $C_{L_{V2}}$ on each row equal to 0.7 when the incidence is zero.

(4) Losses in each row are assessed as shown in section 2.0.

(5) $V_{a_1} = V_{a_2}$, (i.e. $\lambda = 1.0$).

It then becomes possible to calculate from equation (7) the stage efficiencies of a wide range of hypothetical multistage 50% reaction turbines (in which $\alpha_2 = \alpha_0$) and multistage impulse turbines (in which $\alpha_1 = \beta_1 = -\alpha_2$). Results from such a computation are illustrated in figures 4 and 5, where the calculated stage efficiencies of multistage impulse and reaction turbines are plotted against corresponding values of the stage temperature drop coefficient for a number of values of stator gas outlet angle, α_0. Stage efficiencies of single stage turbines are also indicated.

When the outlet swirl from a rotor row is negative (i.e. in the same direction as the direction of rotation) then the assumptions regarding blade loss dictate an identical stage efficiency for both multistage and single stage turbines. When the outlet swirl is positive then the single stage efficiency for a given value of temperature drop coefficient and α_0 is greater than the corresponding multistage stage efficiency due to the fact that the inlet nozzle row to a single stage turbine will have less deflection and less loss than an interstage nozzle in a multistage turbine. In practice, however, it should be remembered that the outlet swirl from the more efficient single stage turbine may lead to high exhaust duct losses, or losses in an extra row of diffusing vanes (if such a stator row is inserted downstream of the rotor row to remove the swirl velocity), which may more than compensate the higher stage efficiency of the turbine itself.

The curves in figures 4 and 5 illustrate that:-

(1) If the outlet swirl from the rotor is negative or slightly positive the turbine efficiency for a given value of the temperature drop coefficient increases as the nozzle outlet angle, α_0, is increased.

(2) For a fixed value of nozzle outlet angle there is an optimum value of the temperature drop coefficient, at which the stage efficiency has a maximum value.

(3) Higher maximum stage efficiencies are obtainable from reaction stages than from impulse stages.

With regard to (1), calculation suggests that little improvement of efficiency is to be derived by exceeding stator outlet angles of 70°, although it is not possible to be very specific on this point due to the absence of appropriate turbine or cascade tunnel data.

4.0 Derivation of the efficiency of an actual turbine

For interpolating from the curves to find the efficiencies of arbitrary turbines in which the reaction lies between 0% and 50% the following procedure is suggested:-

(1) For the turbine under consideration find:-

(a) the design value of $\frac{2.Kp.\Delta T}{U_m^2}$ [mean value per stage]

(b) the value of (β_1/α_2) at mean diameter (or the design value of (α_1/α_2) at mean diameter if the blade angles are unknown or undetermined).

(c) the value of α_o at mean diameter.

(d) the design value of $C_{L_{V_2}}$ on the rotor at mean diameter.

(2) From figure 5, for the above value of $\frac{2.Kp.\Delta T}{U_m^2}$ and α_o, find η_I.

(3) From figure 4, for the same value of $\frac{2.Kp.\Delta T}{U_m^2}$ and α_o, find η_R and $(\beta_1/\alpha_2)_R$.

Then a first approximation to the stage efficiency may be deduced as:-

$$\eta = \eta_R - \left[\frac{(\beta_1/\alpha_2)_R - (\beta_1/\alpha_2)}{1 + (\beta_1/\alpha_2)_R} \right]^2 (\eta_R - \eta_I) \qquad \text{.........(8)}$$

This is an approximation based on the assumption that when interpolating for efficiency between a 50% reaction and an impulse turbine stage having a given value of $\frac{2.Kp.\Delta T}{U_m^2}$ the efficiency varies parabolically with β_1/α_2. This law is empirical and is derived from a few sample calculations of the efficiency of turbines varying reaction.

If the value of $C_{L_{V_2}}$ on the rotor is greater than 0.7 then the efficiency found from (8) should be corrected to:-

$$\eta = 1 - (1 - \eta_{(8)}) \left(0.875 + 0.125 \left[\frac{C_{L_{V_2}}}{0.7} \right]^2 \right) \qquad \text{........(9)}$$

This correction is again empirical and is approximately derived from a number of calculations of the efficiencies of turbines having various values for blade pitch/chord ratio.

The above procedure has been outlined for a single stage turbine. For a multistage turbine mean values for the parameters $\frac{2.Kp.\Delta T}{U_m^2}$ (per stage), β_1/α_2, α_o, and $C_{L_{V_2}}$ should be selected. The corresponding stage efficiency may be assumed to represent the turbine overall efficiency.

This latter assumption saves a further step in the computation and also may tend to reduce errors in the prediction since it will be seen later in section 6.0 that the predictions in most instances are higher than corresponding experimental values. The assumption that overall efficiency equals stage efficiency may reduce this discrepancy if the overall pressure ratio and number of stages is large.

A typical example of the application of this method is given in Appendix III.

4.1 Additional corrections for Reynolds number, blade size, and blade aspect ratio

The turbine efficiencies as determined by the foregoing analysis are intended to apply to stages in which the mean Reynolds number = 2×10^5, mean rotor aspect ratio = 2.0, and mean blade chord > 0.8". If the actual values of these parameters in a turbine under consideration differ appreciably from these then it is desirable to correct the predicted efficiency accordingly.

A suggested correction for Reynolds number is given in fig. 6, this correction being based primarily upon results given in reference 6. Suggested corrections for aspect ratio and blade chord are illustrated in fig. 7. The blade chord correction is intended to allow for increased losses that may be caused by poor profile shapes and relatively large trailing edge thicknesses on blade chord lengths less than 0.8". The corrections for aspect ratio and blade chord are relatively small, the first being derived very approximately from the expression assumed for annulus loss, given in section 2.3, and the last being quite arbitrary.

5.0 Comparison of predicted and experimental efficiencies

In figure 8 a comparison is made of the predicted efficiencies of a number of turbines with experimental results that are available. It is important to note that the experimental efficiency values are relevant to the design point of the units. The experimental efficiency at the design point of a turbine seldom coincides with the maximum efficiency developed; the latter usually occurs at an appreciably lower power output than the design figure and corresponds to a lower operating incidence on the blade (or lower numerical value of $\frac{2.K_p.\Delta T}{U_m^2}$) than the design value. It will be noted that the predictions agree fairly well with experiment for those turbines having low numerical values of $\frac{2.K_p.\Delta T}{U_m^2}$ per stage (generally high reaction turbines) but for those having high numerical values of $\frac{2.K_p.\Delta T}{U_m^2}$ the predicted efficiencies tend to be considerably greater than the experimental values. This may, perhaps, be primarily attributable to either or both of the following causes (1) the formula for secondary loss under-estimates the general level of the basic secondary loss in high deflection, low reaction, blade rows working with a typical flow distribution at inlet to a turbine rotor row, or (2) the variation of percentage error with $\frac{2.K_p.\Delta T}{U_m^2}$ shown in fig. 8 is largely coincidental and represents an extra loss of efficiency caused by large tip clearances, annulus flare, effect of hub ratio, or other reasons not accounted for in the predictions. Possible causes of efficiency losses mentioned in (2) may be exampled as follows: turbine No. 12 had a step in the outer annulus wall in front of each rotor row; turbines Nos. 7 and 9 had a relatively large flare on the outer diameter of the annulus wall across the rotor row; turbine No. 8 had blade profiles with a relatively high degree of curvature (on both camber line and blade form) over the leading half of the blade sections and tip clearances of the order of $2\frac{1}{2}\%$ of the blade height; the large errors on turbines Nos. 4 and 6 are probably due to a particularly low value of hub ratio on these turbines which led to a large numerical value of $\frac{2.Kp.\Delta T}{U^2}$ at the blade root, with appreciable "recompression", even though the mean diameter sections, on which the performance is estimated, had relatively high reaction. It is very improbable that these causes may wholly explain away the discrepancies between the calculated and experimental results and some part of the error must be accountable to the simplifying assumptions made in sections 2.0 and 3.0. It is possible, for example, that an improvement might be made to the calculation by altering the form of the secondary loss formula given in section 2.2 to give substantially higher secondary losses on impulse blades, but the doubt attached to the true source of the error makes it desirable to await further tests on cascade tunnels and turbines before any specific alterations are considered in detail. However, since the errors vary fairly consistently with stage temperature drop coefficient a simple empirical correction to the estimated efficiencies is advocated. The suggested correction is shown by the broken line in figure 8 and is intended to give final efficiencies which should be attainable in practice providing care is taken to provide smooth annulus walls, without a large flare, together with relatively small blade tip or shroud clearances.

6.0 Note on the choice of an optimum turbine design

Figure 8 is particularly interesting in that it shows that the experimental efficiencies of high work capacity turbines (i.e. those having a large numerical value of $\frac{2.Kp.\Delta T}{U_m^2}$) have been consistently low. In particular it is fair to state that in most instances these test efficiencies are lower than those anticipated when the units were designed. Very broadly, the design point efficiencies show a tendency to fall from about 90% or slightly more for high reaction designs (in which $\frac{2.Kp.\Delta T}{U_m^2}$ Ω -2.0 to -2.5) to about 80% for high gas deflection, low reaction turbines (in which $\frac{2.Kp.\Delta T}{U_m^2}$ Ω -5.0). It is well known that the thermal efficiency of a simple gas turbine plant is particularly susceptible to the expansion efficiency. A change in expansion efficiency on a simple gas turbine plant without heat exchanger from 80% to 90% may increase the thermal efficiency of an engine from 15% to as much as 25% (as illustrated by Crowe, ref. 8). On an aircraft jet engine, without heat exchanger, of 6:1 design pressure ratio (sea level static) flying at 500 m.p.h. the reduction of specific fuel consumption for the same increase in expansion efficiency may be of the order of 20% (cf. Reeman, Gray, and Morris; reference 9).

The analysis in sections 5.0 and 6.0 suggests that turbine efficiency may, for a first approximation, be regarded as dependent upon the design value of $\frac{2.Kp.\Delta T}{U_m^2}$ and that the highest efficiencies are obtained with nearly 50% reaction, high nozzle and blade outlet angles, low outlet swirl, and $\frac{2.Kp.\Delta T}{U_m^2}$ equal to -2.0 to -2.5. On many contemporary aero engines the value of $\frac{2.Kp.\Delta T}{U_m^2}$ ranges between -3.5 and -5.0, these numerically high values generally being adopted to keep engine weight and size as small as possible. Figure 8 indicates, however, that the penalty imposed upon turbine efficiency for reducing turbine size to a minimum may be severe. Although a low engine specific weight may outweigh specific consumption in importance on a short endurance interceptor fighter aircraft the specific consumption becomes very much more important on long range aircraft; it seems probable that in many instances turbines having a stage value of $\frac{2.Kp.\Delta T}{U_m^2}$ of about -2.5 may improve the engine specific consumption

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more than enough to compensate for the higher weight as compared with one having $\frac{2.K_p.\Delta T}{U_m^2}$ of -4.0 to -4.5 and thus provide a better engine for such long range aircraft.

The use of low numerical values of $\frac{2.K_p.\Delta T}{U_m^2}$ on many contemporary units would imply, speaking broadly, either (a) maintaining the same number of stages and increasing the blade speeds by 30 to 40% or (b) maintaining approximately the same blade speeds and increasing the number of stages, in many instances doubling the original number. Method (a) is obviously impracticable if present maximum permissible disc stresses are not to be greatly exceeded, so that method (b) may be more acceptable in practice. Doubling the number of turbine stages (keeping the same mean blade speeds) might be expected to nearly double the weight of the turbine component (including rotor, stator, and bearing support members) on a single stage unit. If doubling the number of stages necessitated replacing a single bearing overhung construction by a two bearing assembly the component weight may be rather more than doubled. Thus the increase in aero jet engine weight caused by doubling the number of stages may, very approximately, be 15% - 25%, depending upon whether the original turbine were a single or two stage unit.

It is important to note that the reduction of the numerical value of $\frac{2.K_p.\Delta T}{U_m^2}$ and use of high stage reaction will result in a reduction of axial velocity at outlet from the turbine and permit small outlet swirl. This may reduce the exhaust cone losses in a jet engine appreciably and this in itself may contribute a valuable increase in the total expansion efficiency which would be additional to that obtained by the higher turbine component efficiency.

On land-bound industrial gas turbine plant and on marine plant low specific consumption is again important, together with plant capital cost. In general the inherently higher efficiencies of high reaction turbines may be expected to reduce running costs more than enough to compensate for higher initial capital costs when compared with low reaction turbine units.

7.0 Conclusions

(1) An empirical method of predicting the efficiency of any conventional type of axial flow turbine is suggested. The values of efficiency so deduced are estimated in accordance with measurements of pressure loss coefficients in turbine blades obtained from cascade tunnels and a few turbine experimental results.

(2) Predicted values of efficiency on high reaction turbines are in good agreement with available experimental results. Substantial errors are noted, however, on low reaction turbines. These errors appear to be dependant, to a first approximation, on the mean value of the parameter $\frac{2.K_p.\Delta T}{U_m^2}$ (per stage) and an empirical correction to the calculated efficiency becomes possible which will give final predictions having real significance.

(3) The calculations indicate clearly the influence of nozzle outlet angle and the parameter $\frac{2.K_p.\Delta T}{U_m^2}$ (per stage) on turbine efficiency. The highest calculated efficiencies occur on 50% reaction turbines having high gas outlet angles from the blading and $\frac{2.K_p.\Delta T}{U_m^2}$ of -2.0 to -2.5.

(4) On many applications of the gas turbine plant it is very probable that the use of a relatively large turbine having high reaction and low numerical value of $\frac{2.K_p.\Delta T}{U_m^2}$ (about -2.0 to -2.5) may result in a better engine than

one having a turbine component of minimum size and weight with a high numerical value of $\frac{2.K_p.\Delta T}{U_m^2}$ (stage). This applies essentially to jet engines for long range aircraft, and to plant for industrial and marine use.

REFERENCES

No.	Author	Title
1	D.G. Ainley	Performance of axial flow turbines. Proc. of Inst. of Mech. Engineers (1948) Vol.159.
2	J. Reeman	Note on positive stagger cascades. R.A.E. Tech. Note No. Eng.202 (1943).
3	J. Reeman	Performance of cascades of aerofoils at positive stagger. A.R.C 10, 829. 1946. (unpublished)
4	E. Bridle	Some high speed tests on turbine cascades. R.& M. 2697. February, 1949.
5	K. Todd	Practical aspects of cascade wind tunnel research. Proc. I. Mech. E. Vol.157 (1947).
6	D.G. Ainley, S.E. Petersen & R.A. Jeffs	Overall performance characteristics of a four stage reaction turbine. R.& M. 2416. September, 1946.
7	K. Bammert & G. Korbacher	The characteristic diagram of single stage reaction gas turbines. M.O.S. (A) Volkenrode Rep. & Trans. No.639 (1946).
8	T.A. Crowe	The gas turbine as applied to marine propulsion. Proc. I. Mech. E. Vol. 158 (1948).
9	J. Reeman, P.G. Gray & C. Morris	Some introductory notes on the simple jet engine. A.R.C. 9271. (1946). (unpublished)
10	A. R. Howell	Fluid dynamics of axial compressors. Proc. I. Mech. E. Vol.153 (1945).

APPENDIX I

Notation and convention of signs

(a) Notation

α_0	stator outlet gas angle.
α_1	rotor inlet gas angle.
α_2	rotor outlet gas angle.
α_3	stator inlet gas angle.
β_0	stator outlet blade angle.
β_1	rotor inlet blade angle.
β_2	rotor outlet blade angle.
β_3	stator inlet blade angle.
ζ_S	stator stagger angle.
ζ_R	rotor stagger angle.
U_1	rotor blade velocity at inlet to row.
U_2	rotor blade velocity at outlet from row.
V_0	gas velocity at outlet from stator row.
V_1	gas velocity relative to rotor inlet.
V_2	gas velocity relative to rotor outlet.
V_3	gas velocity at inlet to stator row.
V_{a_1}	axial velocity at inlet to rotor row.
V_{a_2}	axial velocity at outlet from rotor row.
V_{w_0}	gas whirl velocity at outlet from stator.
V_{w_1}	gas whirl velocity relative to rotor inlet.
V_{w_2}	gas whirl velocity relative to rotor outlet.
V_{w_3}	gas whirl velocity at inlet to stator.
P_1	total pressure upstream of stator row.
P_2	total pressure downstream of stator row.
P_{stat_2}	static pressure between stator and rotor row.
P_{2_r}	total pressure relative to rotor row at inlet to row.
P_{3_r}	total pressure relative to rotor row at outlet from row.
P_{stat_3}	static pressure downstream of rotor row.
P_3	total pressure after rotor row.
λ	V_{a_2}/V_{a_1}.

γ ratio of specific heats.

ρ gas density.

Kp specific heat at constant pressure.

R gas constant.

Y_{ts} total loss coefficient for a stator row.

Y_{tr} total loss coefficient for a rotor row.

c blade chord.

s blade pitching.

h blade height.

C_{LV_2} blade lift coefficient based on outlet velocity.

$\overline{\omega}_r$ loss of total pressure through a rotor row (relative to blades).

$\overline{\omega}_s$ loss of total pressure through a stator row.

(b) Convention of signs

This has been chosen to follow directly from that already well established in axial compressor theory in this country (ref.10). The extension of the compressor system to a typical turbine stage is illustrated in fig. 1.

It should be noted, with the arrangement drawn, that the sign of the outlet gas angles, blade angles, and swirl velocities relative to each row become negative. Furthermore the blade velocity must also be regarded as negative in order that the stage temperature rise (or work done on the gas by the blading) shall be negative (i.e. a temperature drop across the stage for a turbine).

The sign to be attributed to a gas swirl velocity is always the same as that of the associated vector swirl angle.

Gas axial velocity is always positive.

It is important to appreciate that the signs to be attributed to the gas swirl angles and velocities must always be determined by reference to the blade row with which these velocity components are directly associated. In other words, the sign attributed to absolute velocities and angles (i.e. velocities and angles which would be registered by instruments which are stationary relative to the machine casing) must be determined by reference to the immediately adjacent stationary blade row. The signs attributed to relative velocities and angles must be determined by reference to the immediately adjacent moving row (relative to which the velocities and angles are supposed to refer).

NOTE: The swirl velocity at outlet from a turbine is always negative when it swirls in the same direction as the direction of rotation of the rotor.

APPENDIX II

Derivation of expressions for turbine stage efficiency and temperature drop coefficient

It is assumed that the Mach numbers of the flows through the stage are sufficiently low for the density to be treated as constant without appreciable error and that the stage pressure ratio, P_3/P_1, is sufficiently small to make the approximation that

$$P_3/P_1 = \left[1 + \frac{\Delta T'}{T_1}\right]^{\frac{\gamma}{\gamma-1}} \simeq 1 + \left(\frac{\gamma}{\gamma-1}\right) \cdot \frac{\Delta T'}{T_1} \qquad \ldots\ldots\ldots(10)$$

where $\Delta T'$ = isentropic total temperature rise across the stage.

It is also assumed that the mean loss of total pressure in any blade row may be represented by a coefficient, Y_t, where:-

$$\text{Total head loss} = Y_t . \tfrac{1}{2} . \rho . V_{outlet}^{\;2}$$

Thus the total head loss in the stator row becomes

$$\bar{\omega}_s = Y_{ts} . \tfrac{1}{2} . \rho . \left[\frac{V_{a_1}}{\cos \alpha_0}\right]^2 \qquad \ldots\ldots\ldots\ldots(11)$$

and the total head loss in the rotor row (relative to the blade row) becomes

$$\bar{\omega}_r = Y_{tr} . \tfrac{1}{2} . \rho . \left[\frac{V_{a_2}}{\cos \alpha_2}\right]^2 \qquad \ldots\ldots\ldots\ldots\ldots(12)$$

Only an axial flow stage of constant mean diameter will be considered, so that $U_1 = U_2$.

$$\text{Work done on gas/lb. of gas} = K_p . \Delta T = U(V_{a_2} \tan \alpha_3 - V_{a_1} \tan \alpha_0) \ldots(13)$$

(Note that this work is negative on a turbine, with the convention of signs adopted).

From the velocity triangles at inlet to and outlet from the rotor row:-

$$V_{a_1} \tan \alpha_0 = U - V_{a_1} \tan \alpha_1 \qquad \ldots\ldots\ldots\ldots\ldots(14)$$

$$V_{a_2} \tan \alpha_3 = U - V_{a_2} \tan \alpha_2 \qquad \ldots\ldots\ldots\ldots\ldots(15)$$

Thus, combining (13) and (15) and assuming that the "work done factor" (Ω) = 1.0 for a turbine,

$$\frac{2 . K_p . \Delta T}{U^2} = 2 - 2 \frac{V_{a_1}}{U} (\tan \alpha_0 + \lambda \tan \alpha_2) \qquad \ldots\ldots\ldots\ldots(16)$$

From equations (11) and (12)

$$P_2 = P_1 - Y_{ts} . \tfrac{1}{2} . \rho . \left[\frac{V_{a_1}}{\cos \alpha_0}\right]^2 \qquad \ldots\ldots\ldots\ldots\ldots(17)$$

$$P_{3_r} = P_{2_r} - Y_{t_r}.\tfrac{1}{2}.\rho. \left[\frac{V_{a_2}}{\cos\alpha_2}\right]^2 \qquad ....(18)$$

From the Bernoulli equation:-

$$P_{2_r} = P_{stat_2} + \tfrac{1}{2}.\rho. \left[\frac{V_{a_1}}{\cos\alpha_1}\right]^2$$

$$P_{3_r} = P_{stat_3} + \tfrac{1}{2}.\rho. \left[\frac{V_{a_2}}{\cos\alpha_2}\right]^2$$

and
$$P_3 = P_{stat_3} + \tfrac{1}{2}.\rho. \left[\frac{V_{a_2}}{\cos\alpha_3}\right]^2$$

Combining these with (17) and (18) we may obtain

$$\frac{P_3 - P_1}{\rho} = \tfrac{1}{2} V_{a_1}{}^2 (\lambda^2 \sec^2\alpha_3 - \lambda^2 \sec^2\alpha_2 + \sec^2\alpha_1 - \sec^2\alpha_0)$$

$$- \tfrac{1}{2} V_{a_1}{}^2 (Y_{t_s}.\sec^2\alpha_0 + \lambda^2.Y_{tr}.\sec^2\alpha_2) \qquad ....(19)$$

which may be shown to reduce to:-

$$\frac{P_3 - P_1}{\rho} = Kp.\Delta T - \tfrac{1}{2} V_{a_1}{}^2 (Y_{ts} \sec^2\alpha_0 + \lambda^2 Y_{tr}.\sec^2\alpha_2) \qquad ....(20)$$

Now from equation (10):-

$$(P_3 - P_1) . \frac{T_1}{P_1} . \left(\frac{\gamma-1}{\gamma}\right) = \Delta T' \qquad ....(21)$$

But the equation for a perfect gas gives:-

$$\frac{P_1}{T_1} = R.\rho. = Kp.\left(\frac{\gamma-1}{\gamma}\right).\rho$$

Therefore, from (21),:-

$$\frac{P_3 - P_1}{\rho} = Kp.\Delta T' \qquad ....(22)$$

For a turbine:- $\eta_{stage} = \dfrac{\Delta T}{\Delta T'}$ ....(23)

Therefore, from equations (23), (22) and (20),:-

$$\eta_{stage} = \frac{Kp.\Delta T}{Kp.\Delta T - \tfrac{1}{2}.V_{a_1}{}^2.(Y_{ts} \sec^2\alpha_0 + \lambda^2.Y_{tr} \sec^2\alpha_2)} \qquad ....(24)$$

Combining (23) and (16), we may obtain for a turbine stage

$$\eta_{stage} = \frac{1}{1 - \tfrac{1}{2}.\left(\frac{V_{a_1}}{U_1}\right) \left\{\frac{Y_{ts} \sec^2\alpha_0 + \lambda^2.Y_{tr} \sec^2\alpha_2}{(U/V_{a_1}) - (\tan\alpha_0 + \lambda.\tan\alpha_2)}\right\}}$$

APPENDIX III

Example estimate of the efficiency of a single stage turbine

The following illustrates the application of the method to predict the efficiency of turbine No. 2, figure 8.

The relevant data at m/d for this turbine may be listed as follows.

$$\beta_o = 8^\circ$$
$$\beta_1 = 16^\circ$$
$$\alpha_o = -63.8^\circ$$
$$\alpha_2 = -58.4^\circ$$
$$\text{Rotor } s/c = 0.64$$
$$\text{Rotor } h/c = 1.71$$
$$V_{a_1} = V_{a_2}, \text{ i.e. } \lambda = 1.0$$

For zero incidence on rotor $\alpha_1 = \beta_1 = 16^\circ$

Value of C_{LV_2} on rotor $= 2.s/c\ (\tan\alpha_1 - \tan\alpha_2)\ \dfrac{\cos^2\alpha_2}{\cos\alpha_m}$

where $\tan\alpha_m = \frac{1}{2}(\tan\alpha_1 + \tan\alpha_2)$

Substituting above values gives:-

$$C_{LV_2} = 0.812$$

From equations (14) and (16), when $\lambda = 1.0$

$$\frac{2.Kp.\Delta T}{U_m^{\ 2}} = 2 - 2\ \frac{\tan\alpha_o + \tan\alpha_2}{\tan\alpha_o + \tan\alpha_1} = -2.19$$

$$\beta_1/\alpha_2 = -\frac{16}{58.4} = -0.274$$

From fig. 5, when $\dfrac{2.Kp.\Delta T}{U_m^{\ 2}} = -2.19$ and $\alpha_o = -63.8^\circ$; $\eta_I = 87.5\%$

From fig. 4, when $\dfrac{2.Kp.\Delta T}{U_m^{\ 2}} = -2.19$ and $\alpha_o = -63.8^\circ$; $\eta_R = 92.5\%$ and $(\beta_1/\alpha_2)_R = -0.09$

Then from equation (8), the first approximation to turbine efficiency:-

$$\eta = 92.5 - \left\{\frac{0.274 - 0.09}{1 - 0.09}\right\}^2 (92.5 - 87.5) = 92.3\%$$

Since C_{LV_2} on rotor is greater than 0.7 then a second approximation, from equation (9) becomes:-

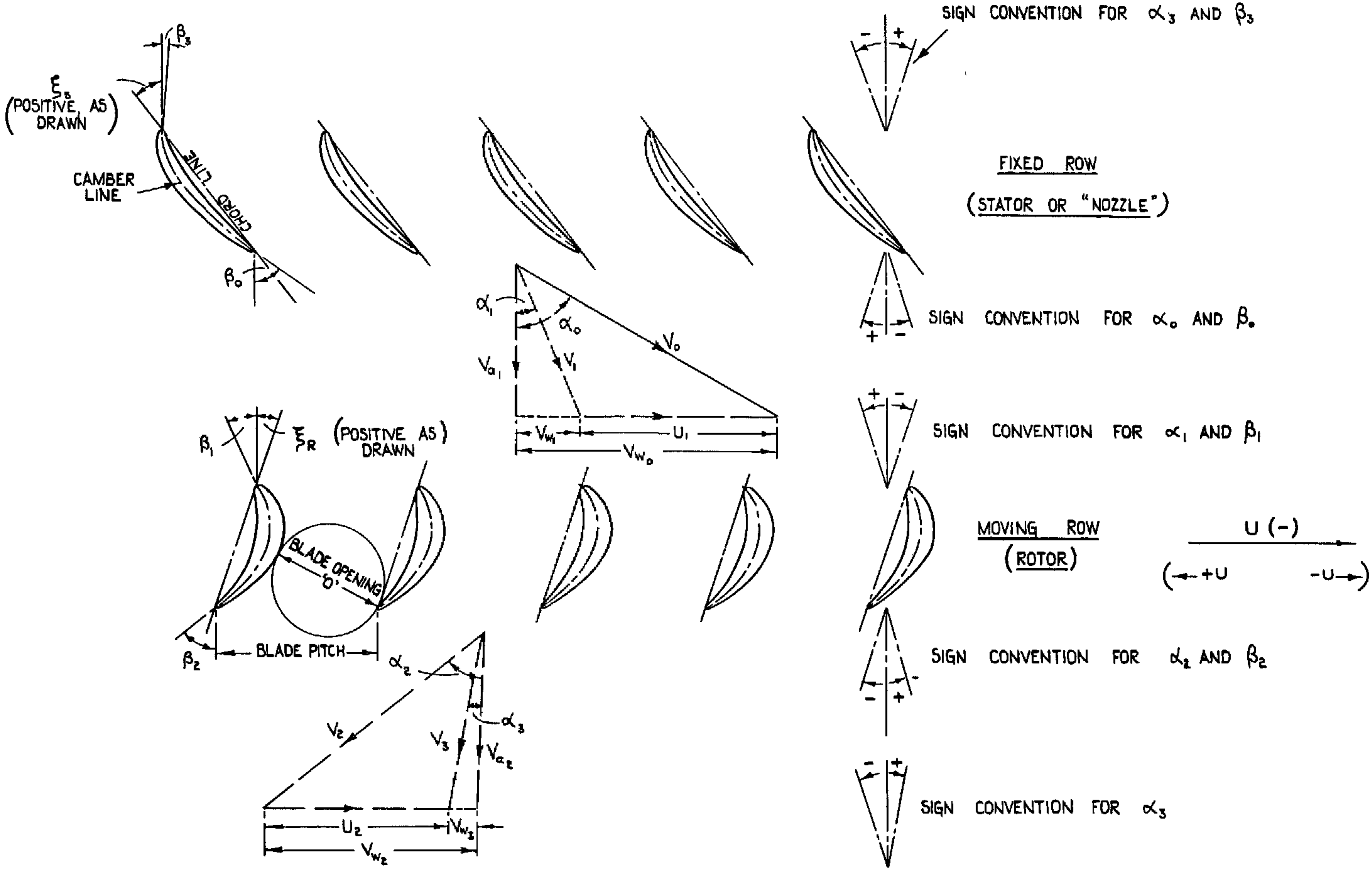
$$\eta = 1 - (1 - .923)\ (0.875 + 0.125 \times (\frac{0.812}{0.7})^2) = 91.95\%$$

Finally, correcting for rotor aspect ratio from fig. 7.

$$\eta = 91.95 - 0.25 = 91.7\%, \text{ for } R_N = 2 \times 10^5$$

FIG 1.

NOTATION & SIGN CONVENTION FOR VELOCITY TRIANGLES ON AN AXIAL FLOW TURBINE STAGE. (POSITIVE STAGGER ON BLADES.)



NOTE - WHEN THE STAGGER ANGLE IS POSITIVE IN SIGN THEN THE ANGLE DEFINED BY $\cos^{-1} o/s$ MUST BE ASSIGNED NEGATIVE.

FIG. 2.

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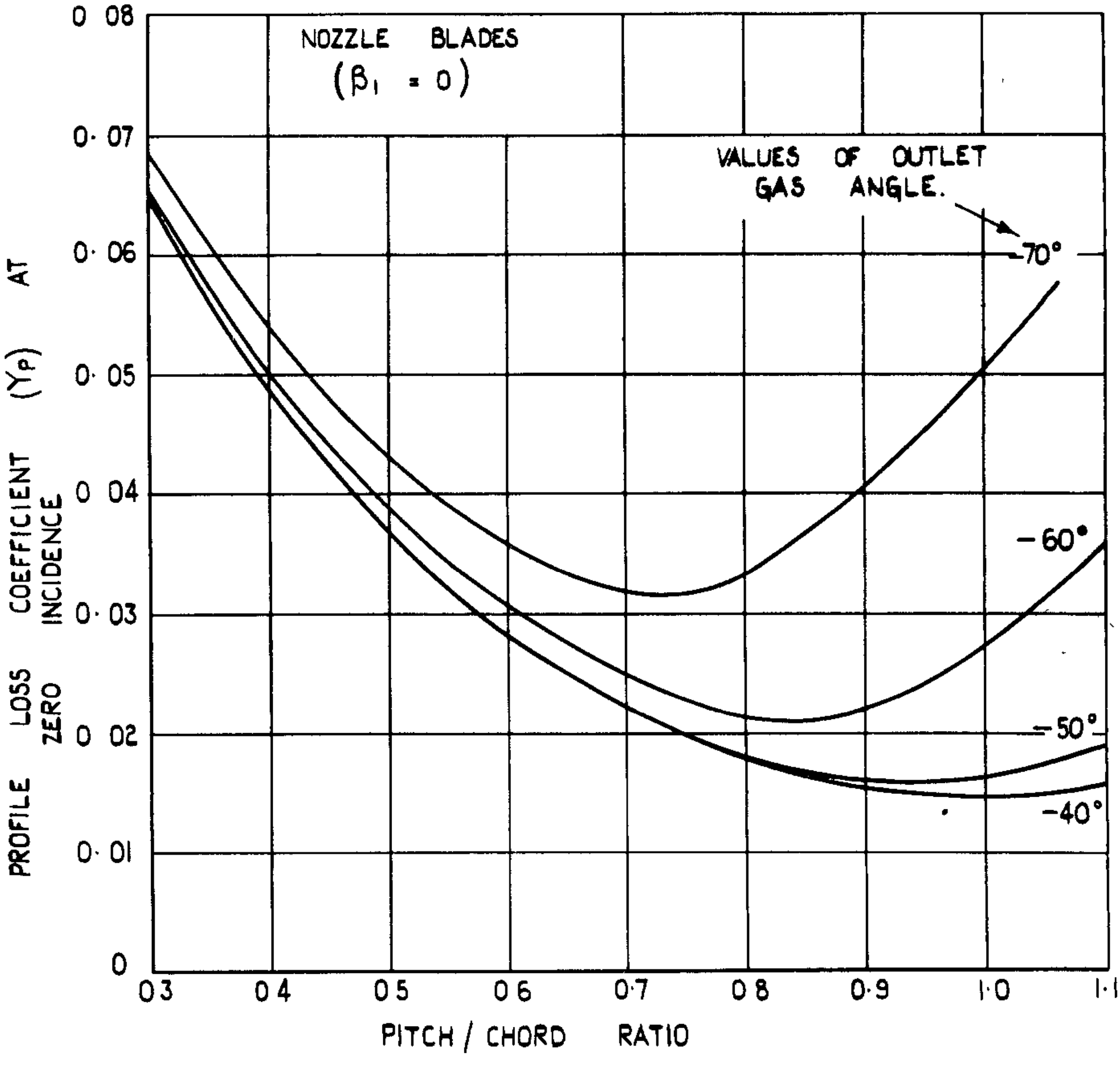
PROFILE LOSS COEFFICIENTS FOR TURBINE NOZZLE BLADES.



PROFILE LOSS COEFFICIENTS FOR TURBINE IMPULSE BLADES.

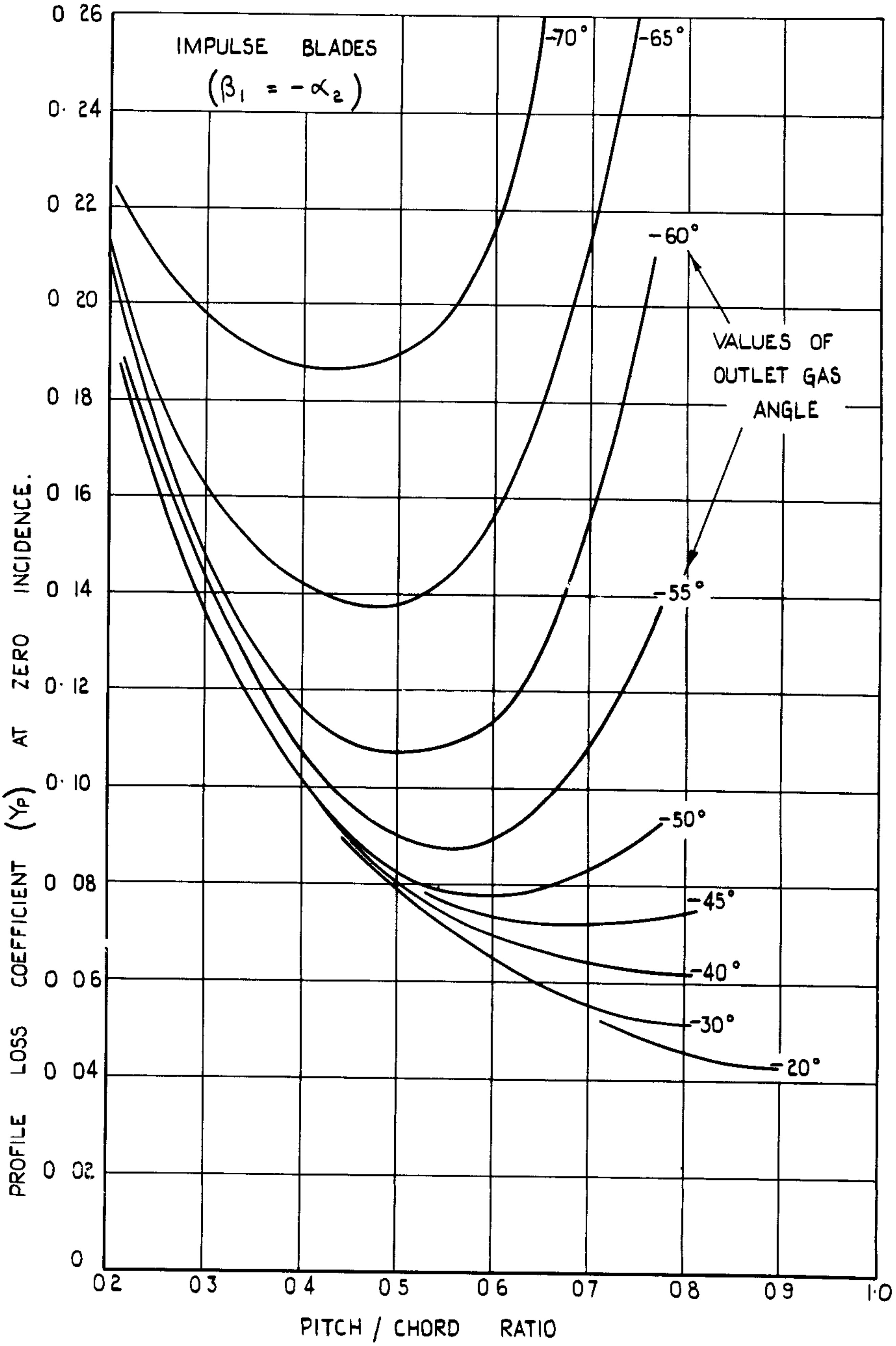


FIG. 4.

STAGE EFFICIENCIES OF '50% REACTION' TURBINES.

($\alpha_2 = \alpha_0$)

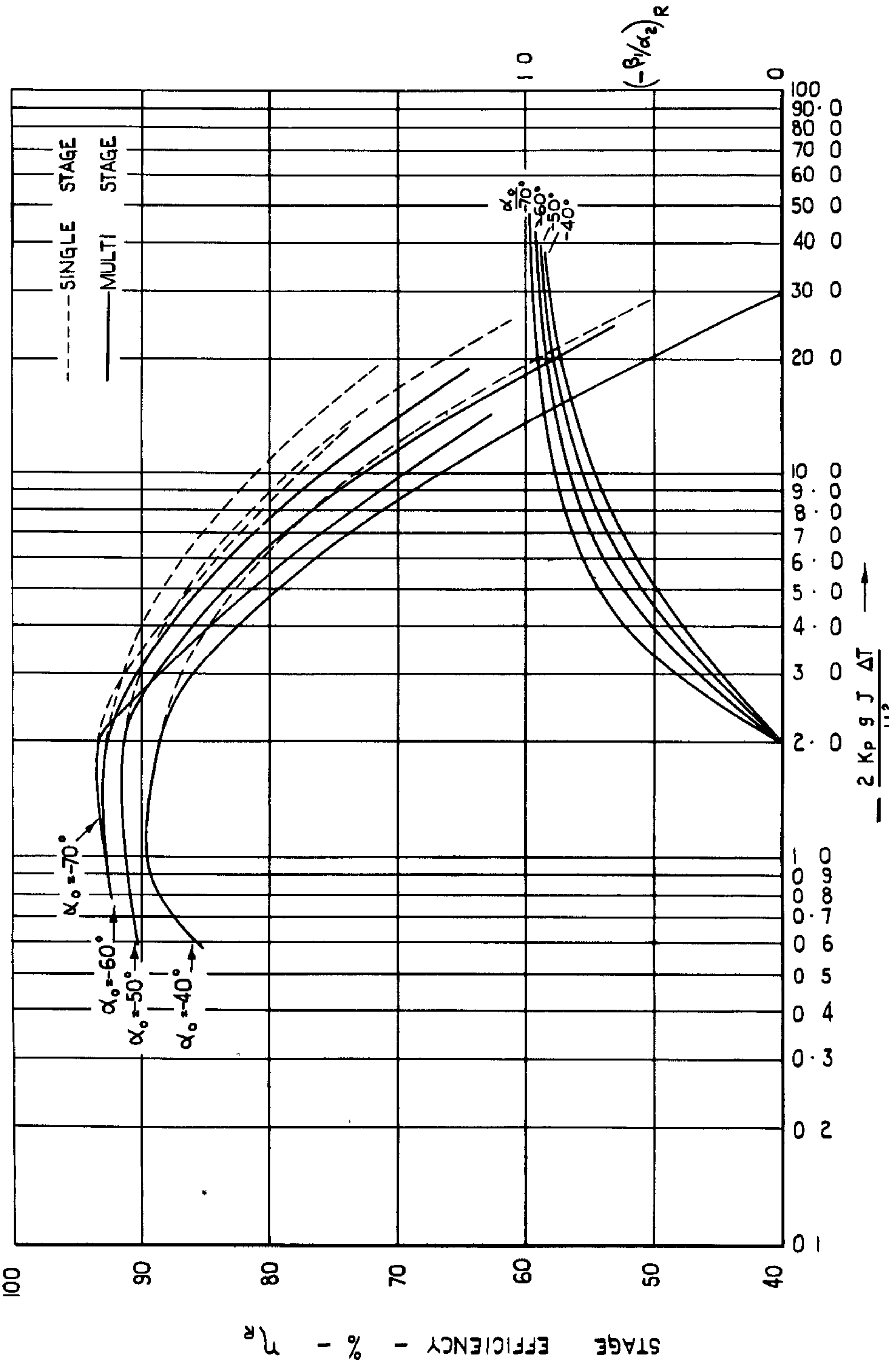


FIG. 5.

STAGE EFFICIENCIES OF 'IMPULSE' TURBINES $(\beta_1/\alpha_2 = -1\cdot0)$

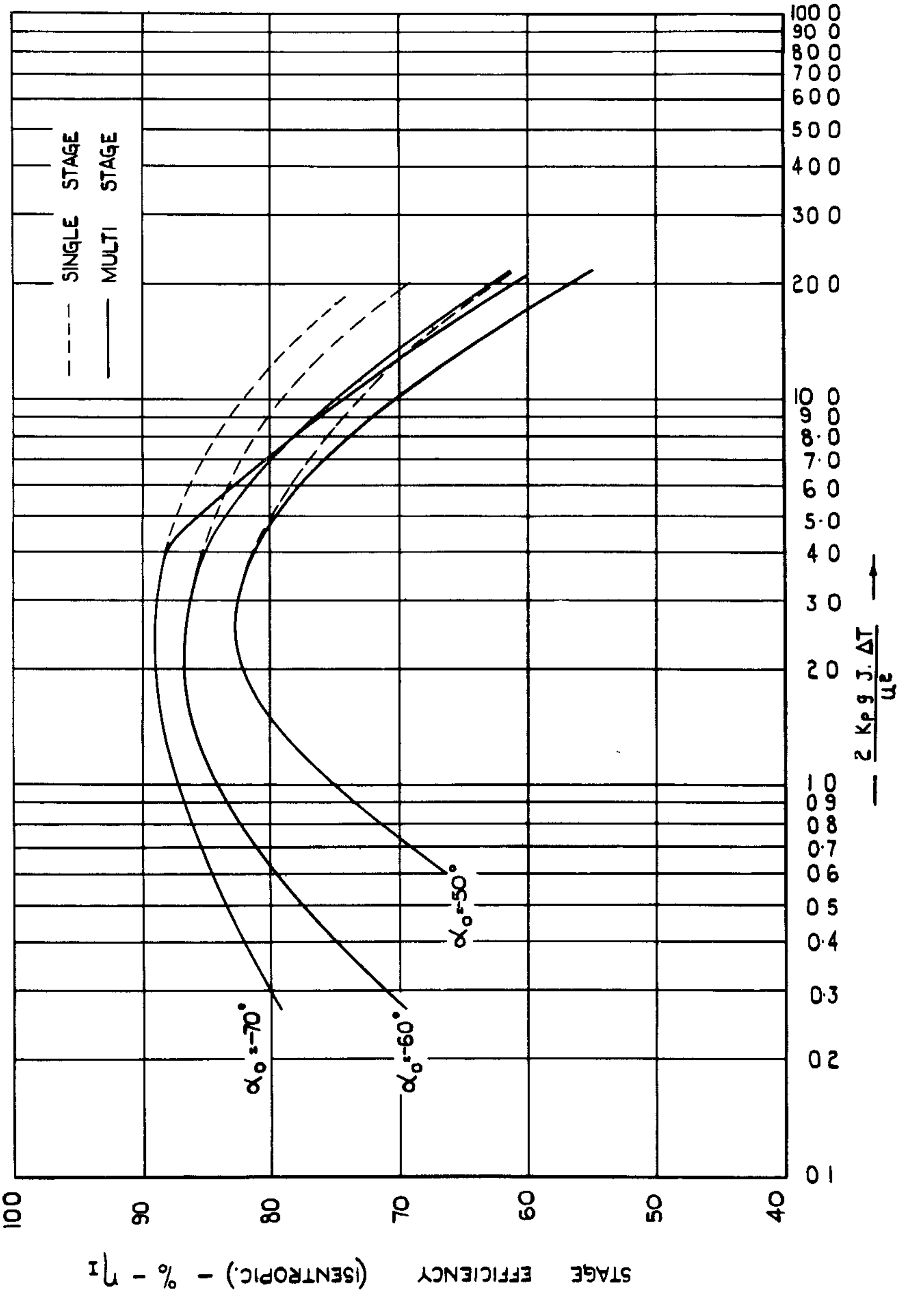


FIG. 6.

REYNOLDS No CORRECTION.

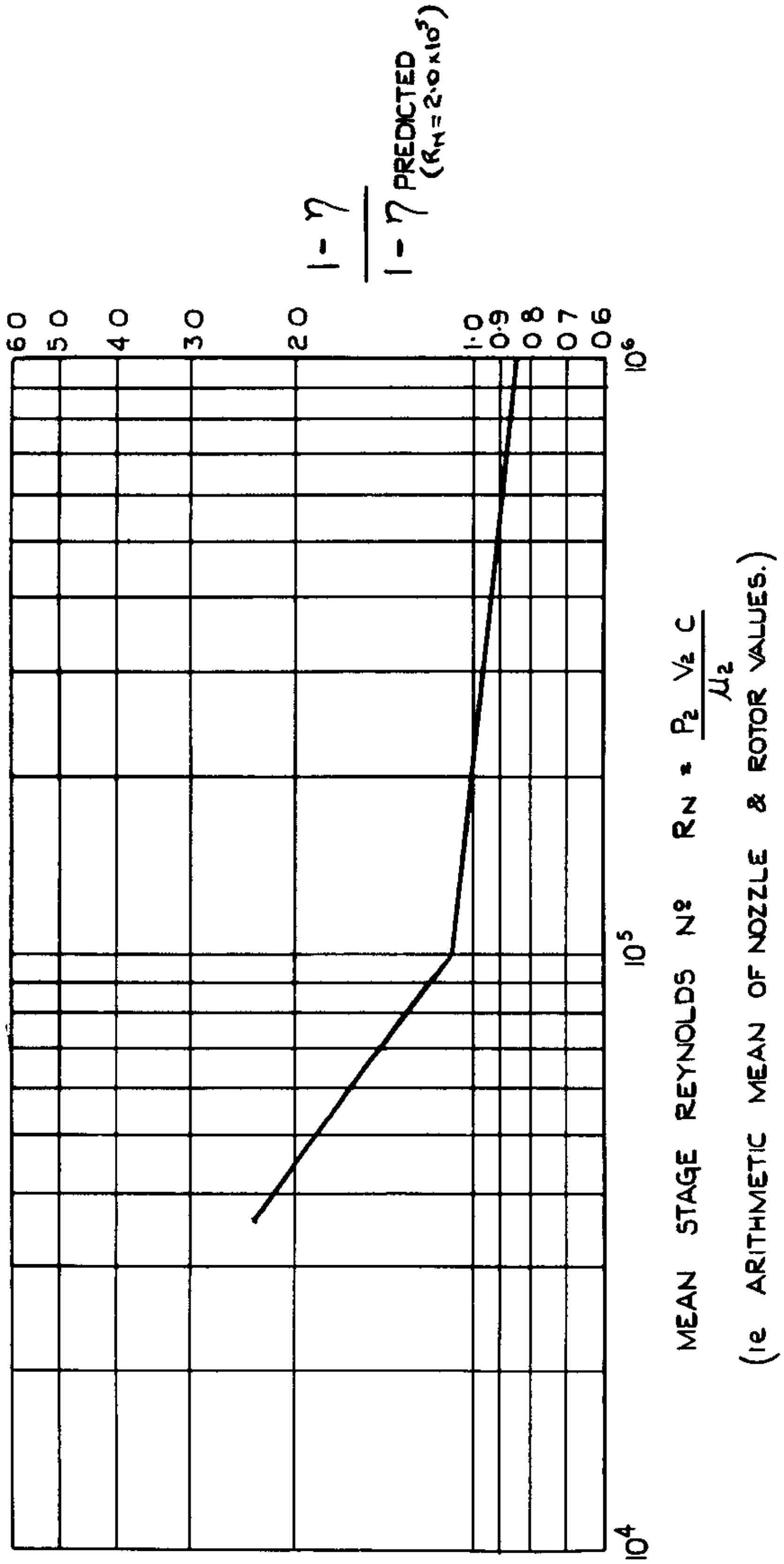
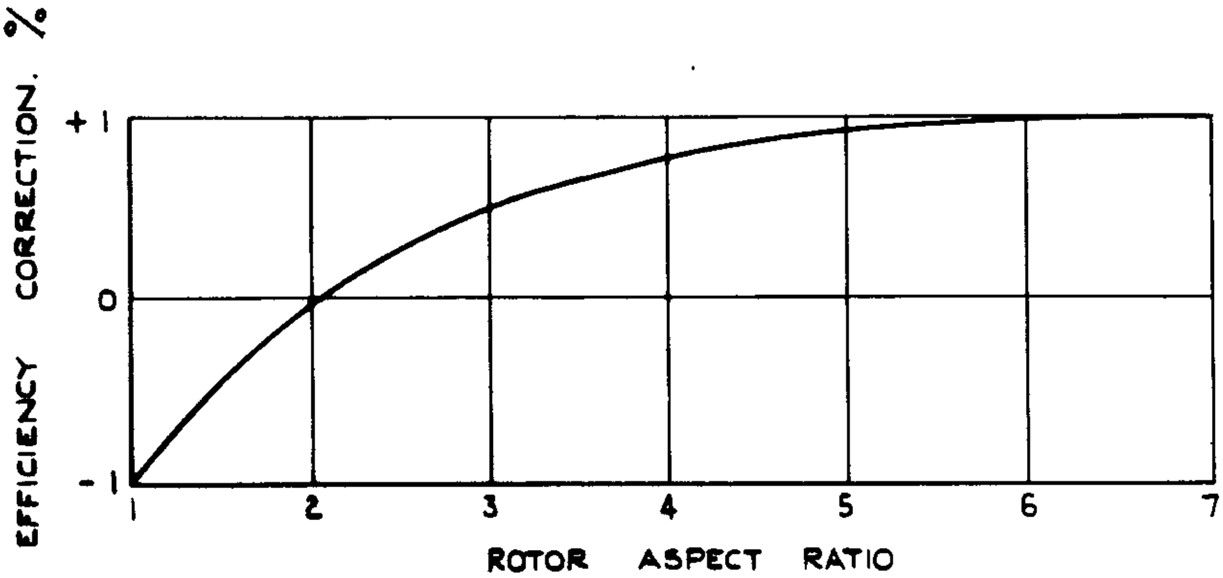


FIG. 7.

ROTOR ASPECT RATIO CORRECTION.



BLADE 'SCALE' CORRECTION.

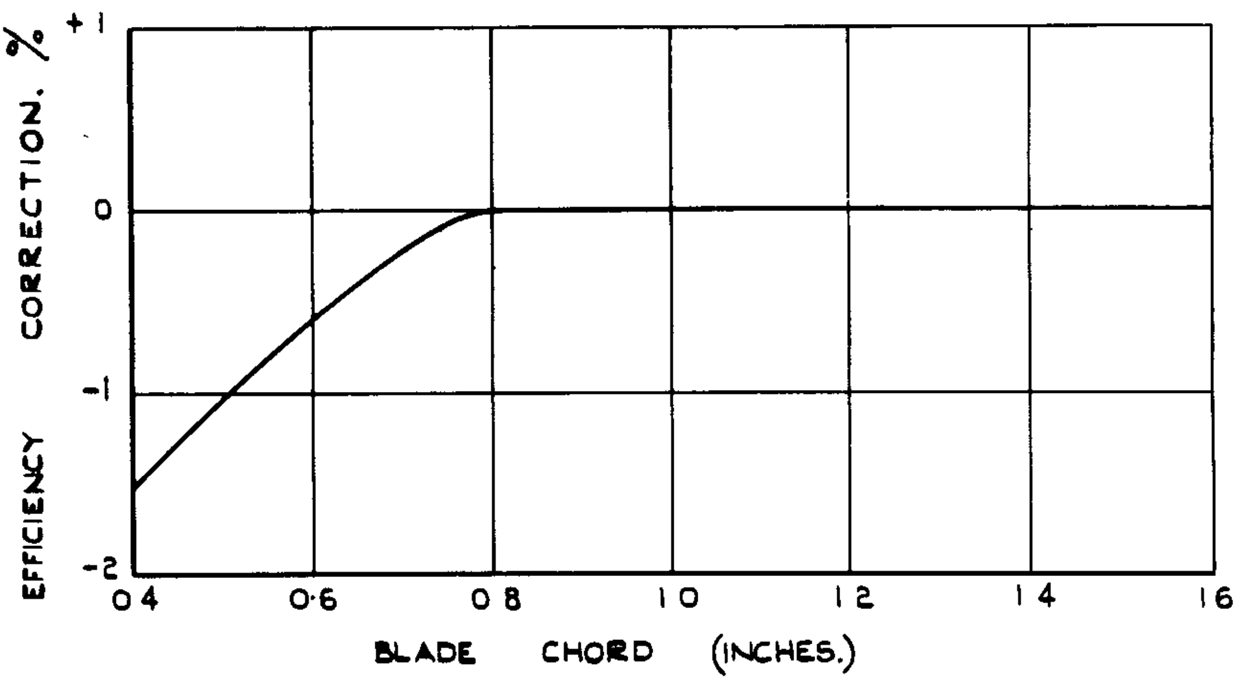


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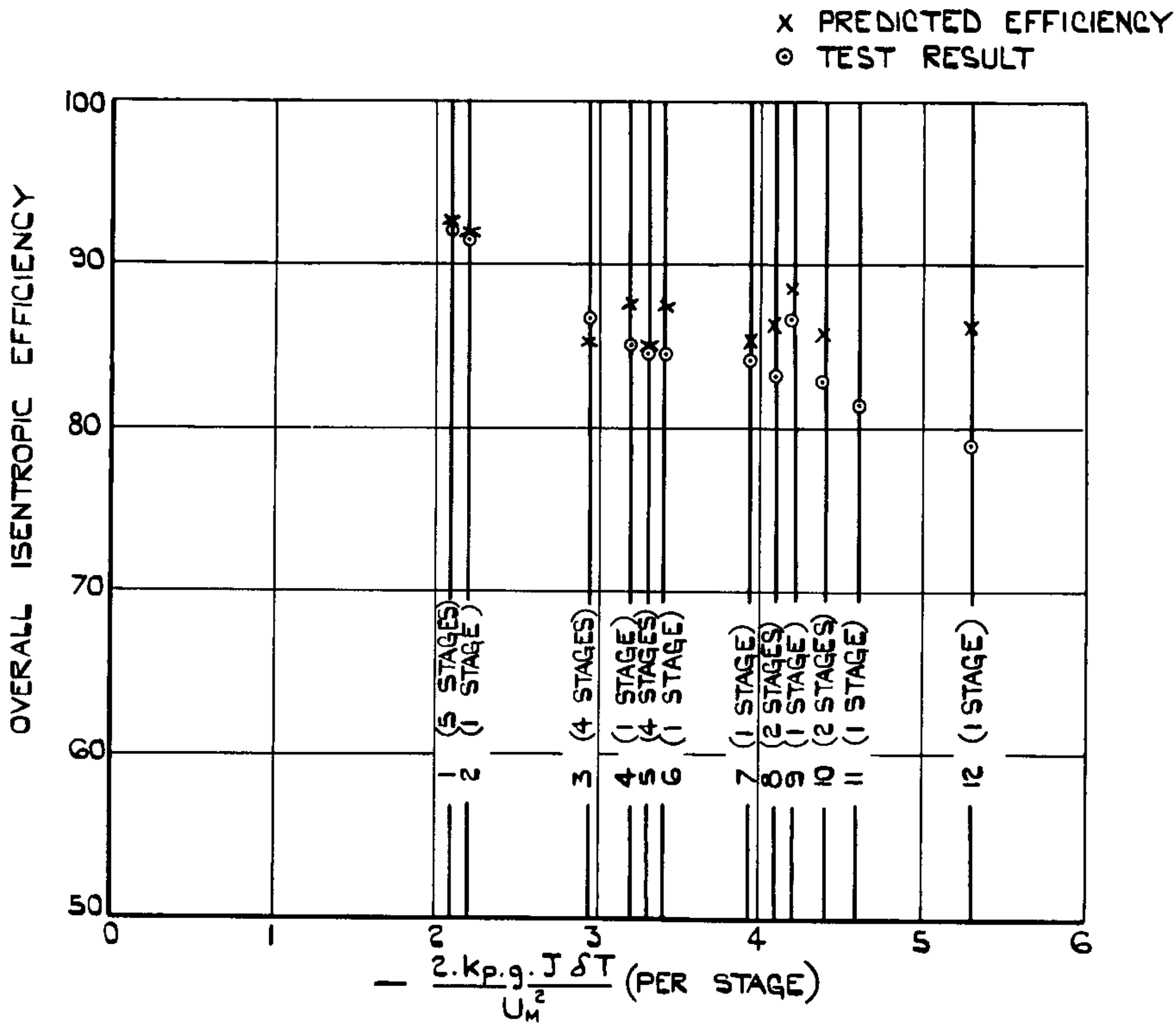
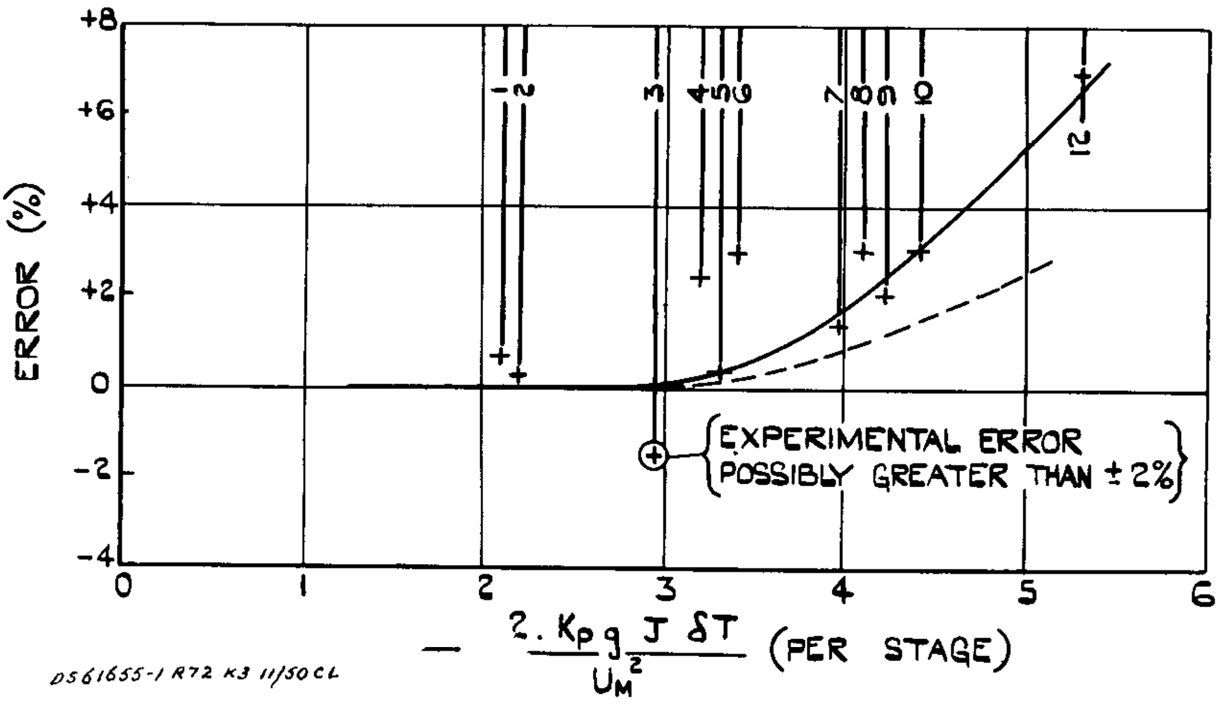
FIG. 8.

COMPARISON OF PREDICTED AND EXPERIMENTAL EFFICIENCIES.





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