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Strip Theory Method of Calculation for Airscrews on High-Speed Aeroplanes

By
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With an
APPENDIX

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* This report contains material from the following unpublished papers : A.R.C. Reports 5078 (April, 1941), 5233 (July, 1941), 5939 (July, 1942), 6512 (February, 1943) and 8962 (September, 1945).

Summary.—The report describes a simplified 8-point strip theory method of calculating the free-air performance of a propeller up to tip Mach numbers near the velocity of sound. It is based on the assumptions of R. & M. 1674[4] and 1849[5] together with the further simplifying assumption that the (C_L, α) curve is straight (valid below the stall) and that the (sC_L, φ) curve also is straight (valid for $J > 1 \cdot 0$). The report includes tables of parameters $(b, \tau, \zeta$ and $q)$ which are required in the calculations as functions of J, r_c and N for eight standard radii ($r_c = 0 \cdot 3, 0 \cdot 45, 0 \cdot 6, 0 \cdot 7, 0 \cdot 8, 0 \cdot 9, 0 \cdot 95, 0 \cdot 975$) for the range of values of J from $1 \cdot 0$ to $7 \cdot 0$; these are of universal application. In addition, tables of section data (C_L and C_D as functions of M and α) for various section shapes are required; these are given for Clark *Y* sections over a range of thickness in R. & M. 2036[7]; they were derived, by methods described in R. & M. 2020[8], from overall measurements of thrust and torque on full scale propellers at low values of J in the Royal Aircraft Establishment 24-ft. tunnel and are subject to revision in the light of subsequent experimental research.

The method determines values of torque grading (q_c), thrust grading (t_c) and power loss gradings (p_{C1} and p_{C2}); these are integrated arithmetically to give free air thrust, torque and efficiency (k_T, k_Q and η) by means of integrating coefficients which are also tabulated. A method is described of determining the contribution of the portion of the blades inside the radius $r_c = 0 \cdot 3$.

LIST OF SYMBOLS

a	Slope of curve of α against sC_L (assumed linear).
a_0	Value of a at low speed.
a_h	Speed of sound at height h.
A_0	Slope of low-speed lift curve (dC_{LO}/da).
b	Slope of curve of φ against sC_L (assumed linear).
B_0	Factor giving variation of C_{DO} with α_0.
c	Chord of blade element at radius r.
C_0	Value of C_{DO} when $\alpha_0 = 0$.
C_1, C_2, ...	Integrating coefficients.
C_D	Drag coefficient of blade element.
C_{DO}	Low-speed drag coefficient.
C_{DS}	Compressibility-increase in C_D over its low-speed value.
C_L	Lift coefficient of blade element.
C_{LO}	Low-speed lift coefficient.
C_{LS}	Increase in C_L over its value at the lift critical speed.
D	Airscrew diameter.
h	Operating altitude of aircraft.
J	Advance ratio (V/nD).
k_P	Total power loss coefficient (Power wastage$/2\pi\rho n^3 D^5$).
k_{PO}	Low-speed component of the profile drag power loss coefficient.
k_{P1}	Induced power loss coefficient.
Δk_{P1}	Blade root power loss coefficient.
k_{PS}	Compressibility component of the profile drag power loss coefficient.
k_Q	Torque coefficient $(Q/\rho n^2 D^5)$.
k_T	Thrust coefficient $(T/\rho n^2 D^4)$.
M	Mach number of blade element.
M_D	Critical Mach number for drag.
M_L	Critical Mach number for lift.
n	Rotational speed (r.p.s.).
N	Number of blades.
p_C	Grading coefficient of the total power loss : $dk_P/d(r_c^2)$.
p_{CO}	Grading coefficient of the low-speed component of the profile drag power loss.

List of Symbols—*contd.*

p_{c1}	Grading coefficient of the induced loss.
p_{c2}	Grading coefficient of the profile drag power loss.
p_{cs}	Grading coefficient of the compressibility component of the profile drag power loss.
q	Factor giving the power loss grading coefficient : $(\pi^3/16)r_c{}^3 \sec^3\phi_0$.
q_c	Torque grading coefficient : $dk_Q/d(r_c{}^2)$.
Q	Torque.
r	Radius at blade element.
r_c	Fractional radius at blade element (r/R).
R	Tip radius.
s	Solidity $(Nc/2\pi r)$.
t	Thickness of blade section.
t_c	Thrust grading coefficient : $dk_T/d(r_c{}^2)$.
T	Thrust.
V	Forward speed.
w_1	Inflow velocity.
W	Resultant relative air velocity at blade element.
W_0	Geometrical velocity of blade element.
α	Incidence of blade element.
α_0	Incidence referred to zero-lift datum.
β	Inflow angle $(\varphi - \varphi_0)$.
Γ	Circulation taken over blade element.
ε	Zero-lift angle.
ε_0	Zero-lift angle at low speed.
ζ	Factor giving the torque grading coefficient : $(\pi^3/16)\ r_c{}^3 \sec^2\varphi_0$.
η	Airscrew efficiency.
$\Delta\eta$	Efficiency loss due to blade roots.
θ	Blade angle.
$\varkappa$	Inflow factor corresponding to helicoid angle φ.
$\varkappa_0$	Inflow factor corresponding to helicoid angle φ_0.
ρ	Air density.
τ	Factor giving the thrust grading coefficient : $(\pi^3/8)r_c{}^2\sec^2\varphi_0$.
φ	Angle between plane of rotation and relative air velocity at blade element.
φ_0	Angle between plane of rotation and geometrical velocity of blade element.
Ω	Rotational speed (radians per second).

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1. *Introduction.*—The method of airscrew strip theory calculation described in the present report has been gradually developed in serial form[1-5]. In preparing for printing the part of the work so far unpublished, it therefore seemed worth while to recapitulate the various basic assumptions in some detail by way of introduction.

The first and most fundamental assumption is that the resultant force on an elementary strip of the airscrew blade (of length dr at radius r), is the same as it would be if the element formed part of a two-dimensional aerofoil, situated in an airstream the magnitude and direction of whose velocity is to be determined. This velocity is the resultant of the geometrical velocity of the blade element and a certain " interference " velocity due to the trailing vortex system.

Thus the problem before us resolves itself into two distinct parts—the method of dealing with the two-dimensional force coefficients and the determination of the " interference velocity ".

The recent developments with which the present report is concerned deal mainly with the first problem ; only minor changes directed to increased simplicity in computation have been made in the methods of solution of the second problem.

We shall treat the second problem in some detail, but before doing so a few remarks on the first problem are desirable.

Recent developments have been chiefly concerned with the effect of high " Mach number " on the two-dimensional section lift and drag coefficients. The ultimate aim should be to determine these coefficients from actual experiments on two-dimensional aerofoils with the addition, if necessary, of empirical corrections for any failure of the theoretical calculation of the interference velocity. The force coefficients referred to in the present paper were obtained (R. & M. 2036[7]) mainly by " back-figuring " from tests of actual propellers and would, therefore, include these empirical corrections. The data are essentially provisional, applying to a particular form of blade section, and being subject to revision whenever further experimental results become available.

2. *Determination of the Interference Velocity.*—The method is based on the Prandtl-Betz assumption of replacing the airscrew blade by a lifting line consisting of a " bound vortex " whose strength at any radius is equal to the circulation round the blade at that radius. The strength of the vortex sheet springing from the blade adjusts itself so as to satisfy the condition of continuity of vorticity ; for an airscrew it is of spiral form. The " interference velocity " at the blade element is to be calculated on the assumption that the flow is everywhere irrotational outside this vortex system. The resultant force on a bound vortex is normal to the resultant relative velocity W ; accordingly, the drag is neglected in calculating the interference velocity.

Figure 1 shows a cross-section of the air flow relative to the blade element at radius r. According to the general theorem that the vortex lines coincide with the relative streamlines, the direction of the trailing vortex at radius r, at the point where it leaves the blade, coincides with the direction of W, and the initial angle of pitch of the helical trailing vortex is equal to ϕ. It is also worth noting here that the theory for determining interference velocity is essentially a first order theory, in that the interference velocity ratio w_1/W is treated as a small quantity of which the square is to be neglected. The difference between the assumptions that the angle of pitch of the trailing vortex is equal to φ or to φ_0 is in fact of the second order, but it is worth while to prefer the former assumption in the initial stages because it leads to results which agree exactly, for the limiting case of an infinite number of blades, with results deducible from consideration of linear and angular momentum and energy.

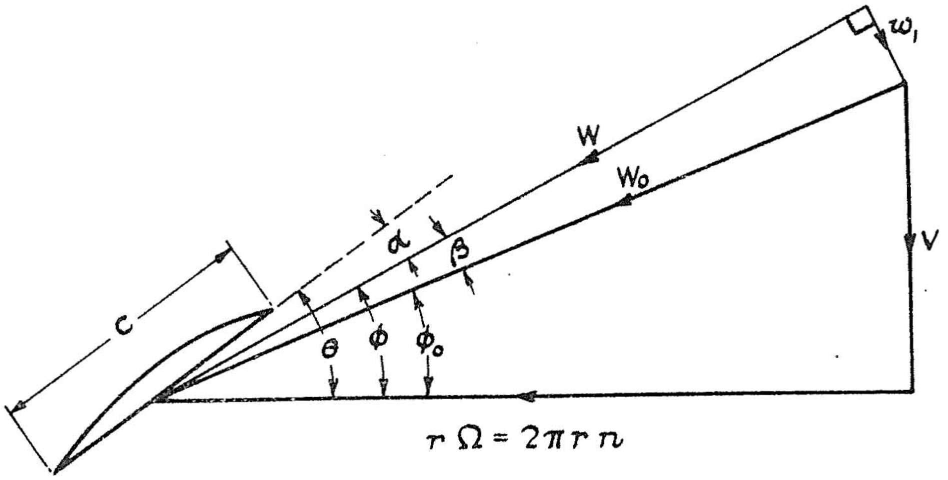

FIG. 1

It may be shown in a general manner that w_1 is normal to the vortex line and, therefore, to W as shown in Fig. 1. The essentially difficult part of the problem is to determine a relation between the magnitude of w_1 and the strength and configuration of the trailing vortices. It has so far been found necessary to ignore the variation of pitch and radius of the trailing vortices and to treat them as regular helices. It is then possible to use the standard artifice of the Prandtl theory of the lifting wing to show that the actual value of w_1 in the presence of the semi-infinite trailing vortices is one half the value that would occur for similar vortices of infinite length.

An expression for the velocity field of a single infinite helical vortex has been given by Lamb[6] in terms of Bessel functions, but the difficulty of the numerical evaluation of the principal value of an infinite integral along the blade, together with subsequent successive approximation to fit the solution to a particular working condition of a particular airscrew, makes the use of this solution prohibitive.

The method to be used here is based on Goldstein's solution for a particular case. Considering a helical vortex sheet extending to infinity in both directions, $2w_1$ is equal to the component velocity of the vortex sheet normal to itself and $2w_1 \sec\varphi$ is equal to the velocity parallel to the axis of the airscrew. For the particular case in which this quantity is independent of the radius, the trailing vortex sheet may be considered as a rigid surface moving parallel to the axis with velocity $2w_1 \sec\varphi$. This case was proved by Betz to give minimum induced energy loss.

Goldstein's solution may be considered as giving (in tabular form) the equivalent vortex strength, at any radius r, of a series of rigid infinite helicoidal surfaces (one for each blade of the airscrew) of radius R and angle of pitch φ, moving parallel to the axis with velocity $2w_1 \sec\varphi$. The circulation Γ round the blade at radius r can be immediately determined as the total strength of the vortex sheet outside that radius.

Since Γ is obviously proportional to w_1 and of the dimensions of $w_1 r$, the tables could be expressed in non-dimensional form by tabulating $\Gamma / w_1 r$ as a function of $r_c\ (= r/R)$ and φ. Actually it is convenient to use the equivalent coefficient $\varkappa$ defined by the equation

$$\varkappa = \frac{\Gamma}{w_1 r} \cdot \frac{N}{4\pi \sin\varphi} \quad \ldots \quad (1)$$

The values of $\varkappa$ are tabulated as functions of $\sin\phi$, r_c and N in Table 1a, whose derivation is discussed in Appendix 1 and Appendix 2.

The coefficient $\varkappa$ has so far been defined as determining the value of the circulation at any radius, for the particular case where the trailing vortex sheets are rigid surfaces moving with a given velocity. The artifice by which the results may be applied in the general case consists in the assumption that the relation connecting $\varkappa$ with r_c, φ and N, is *independent of the conditions at other radii.* The significance of this assumption may be made clearer by reference to Figure 2.

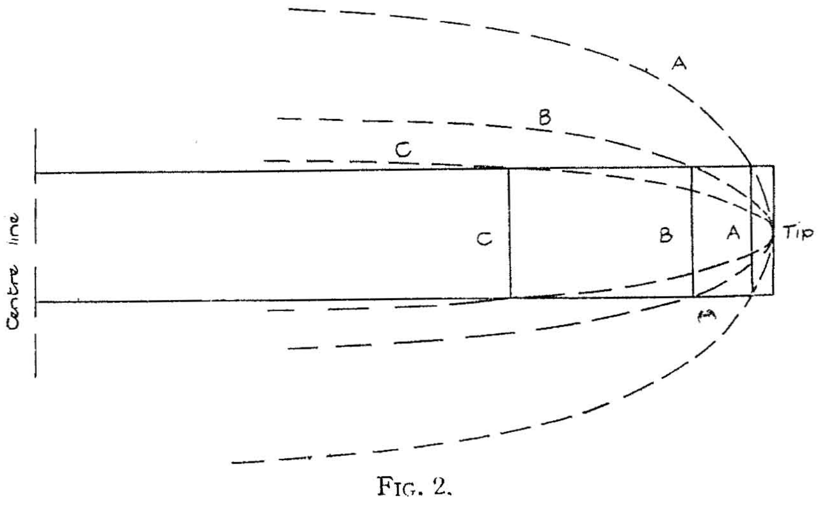

FIG. 2.

Consider the section B of a square-tipped blade and suppose that the dotted curve B represents the plan form of a blade which, with the same pitch distribution, would give rise to a wake of Betz' form for the given working condition. The proposed method calculates the interference velocity for the section at radius B on the assumption that it forms part of a blade having the dotted plan form B. For other radii the equivalent plan form is different. Thus the blade elements are treated as being effectively independent, although the value of the coefficient $\varkappa$ depends on r_c.

The following further points are worth noting.

The coefficient $\varkappa$ is defined in equation (1) in such a way that its value tends to unity as the number of blades tends to infinity. The solution for an infinite number of blades can be obtained by more elementary methods, and in this limiting case the blade elements *are* independent.

The coefficient $\varkappa$ tends to zero at the blade tip for any finite number of blades. Equation (1) then ensures that, even for a square-tipped blade, the circulation and lift coefficient tend to zero at the blade tip. This requires, according to Fig. 2, an " equivalent plan form " of infinite solidity for a square-tipped blade.

The justification of the proposed use of equation (1) is further discussed in R. & M. 1521[3]. This equation will accordingly be used as the basis of the calculation of interference velocity throughout the present report.

3. *Forces on the Blade Element : Grading Coefficients.*—The fundamental relation of equation (1) is next expressed in a form more convenient for practical use by introducing the lift coefficient at the blade element. If c denotes the chord, the lift on the element may be written—

$$\tfrac{1}{2}C_L \rho c \delta r W^2 = \delta L = \rho W \Gamma \delta r \qquad (2)$$

Hence

$$2\Gamma = cWC_L. \qquad (3)$$

Substitution for Γ in (1) then gives at once

$$\frac{Nc}{2\pi r}\, C_L = 4\varkappa\, \frac{w_1}{W} \sin \varphi \qquad (4)$$

Reference to the velocity diagram (Fig. 1) shows that $w_1/W = \tan \beta$, so that

$$sC_L = 4\, \varkappa \tan \beta \sin \varphi \qquad (5)$$

where s $(= Nc/2\pi r)$ denotes the blade solidity at the radius considered.

Also from the velocity diagram we have—

$$\theta = \alpha + \varphi \qquad (6)$$

and

$$\varphi = \varphi_0 + \beta \qquad (7)$$

Assuming a known lift curve for the section, we thus have, with equations (5), (6) and (7), four relations which serve to determine four unknowns. Usually the blade angle θ is known and the equations are to be solved for C_L, α, φ and β.

At high rates of advance a simplification is possible, due to the fact that sC_L is very nearly linear with φ. In this case equation (5) may be written, to the first order in β, as—

$$sC_L = 4\varkappa_0\, \beta \sin \varphi_0 \qquad (8)$$

or

$$\beta = bsC_L \qquad (9)$$

with

$$b = 1/4\, \varkappa_0 \sin \varphi_0 \qquad (10)^*$$

so that the three equations (6, 7 and 9) are all linear in the unknowns.

---

* These formulae are believed to be sufficiently accurate for all values of J greater than 2·0 and should give fairly accurate results down to $J = 1 \cdot 0$.

Finally, below the incidence stall the lift curve also may be assumed linear, so that we may write

$$asC_L = \alpha + \varepsilon \tag{11}$$

where

$$a = d\alpha/d(sC_L) \tag{12}$$

Equations (6), (7), (9) and (11) may then be solved in the form—

$$sC_L = (\theta - \varphi_0 + \varepsilon)/(a + b) \tag{13}$$

$$\beta = bsC_L \tag{9}$$

$$\varphi = \varphi_0 + \beta \tag{7}$$

$$\alpha = asC_L - \varepsilon \tag{11}$$

and the numerical work checked by substitution in equation (6).

The working conditions at the blade element having thus been determined, the coefficients of the torque grading, the thrust grading and the power loss gradings may now be obtained as follows :

Torque Grading.—Resolution of the forces on the blade element gives—

$$\delta Q = N\,r\,(\delta L \sin\varphi + \delta D \cos\varphi) \tag{14}$$

$$= \tfrac{1}{2} N W^2 \rho c\, r\, \delta r\, (C_L \sin\varphi + C_D \cos\varphi) \tag{15}$$

Hence

$$q_c \left(= \frac{dk_Q}{d(r_c^2)} = \frac{R}{2\rho n^2 D^5 r_c} \frac{dQ}{dr} \right) \tag{16}$$

$$= \frac{\pi}{64} \frac{W^2 r}{n^2 R^3} \frac{Nc}{2\pi r} (C_L \sin\varphi + C_D \cos\varphi). \tag{17}$$

Since $W = W_0 \cos\beta$, the difference between W and W_0 is of the second order in β and it is sufficiently accurate to replace W by W_0; equation (17) then becomes—

$$q_c = \left\{ \frac{\pi^3}{16} r_c^3 \sec^2 \varphi_0 \right\} (sC_L \sin\varphi + sC_D \cos\varphi) \tag{18}$$

in which the term within the curled brackets is a geometrical function of r_c and J only, which we shall denote by ζ.

Thrust Grading.—In an exactly similar way using

$$\delta T = N\,(\delta L \cos\varphi - \delta D \sin\varphi) \tag{19}$$

there results

$$t_c \left(= \frac{dk_T}{d(r_c^2)} \right) \tag{20}$$

$$= \tau\,(sC_L \cos\varphi - sC_D \sin\varphi) \tag{21}$$

where

$$\tau = \frac{\pi^3}{8} r_c^2 \sec^2 \varphi_0 = \frac{2}{r_c} \zeta \tag{22}$$

Values of both ζ and τ have been tabulated against J and r_c. (Tables 2c and 2d.)

Efficiency and Power Loss.—The efficiency η is defined as the ratio of the thrust power (useful power) to the torque power (power input) :—

$$\eta = TV/Q\Omega = Jk_T/2\pi k_Q \tag{23}$$

The torque power may be considered as the sum of the thrust power and the power loss P, which latter may be separated into " induced " power loss P_1 and " profile drag " power loss P_2 given for a blade element by

$$\delta P_1 = N w_1 \delta L \tag{24}$$

and

$$\delta P_2 = N W \delta D \tag{25}$$

It may be verified by direct algebra from equations (14) and (19) that

$$\Omega \delta Q = V \delta T + \delta P_1 + \delta P_2. \qquad (26)$$

We define coefficients of power loss to satisfy the following relations—

$$k_Q = J k_T / 2\pi + k_{P1} + k_{P2} \qquad (27)$$

$$p_{C1} = dk_{P1}/d(r_c^2),\ p_{C2} = dk_{P2}/d(r_c^2) \qquad (28)$$

so that

$$p_{C1} = \frac{1}{16\pi \varrho n^3 D^3 r} \frac{dP_1}{dr} \qquad (29)$$

$$p_{C2} = \frac{1}{16\pi \varrho n^3 D^3 r} \frac{dP_2}{dr}. \qquad (30)$$

Hence

$$\begin{aligned} p_{C1} &= qsC_L \sin\beta \cos^2\beta \\ &= q\beta s C_L \qquad (31) \end{aligned}$$

to the first order in β, where

$$q = \frac{1}{16} \pi^3 r_c^3 \sec^3 \varphi_0 = \zeta \sec \varphi_0 \qquad (32)$$

and

$$p_{C2} = qsC_D. \qquad (33)$$

Finally

$$\eta = 1 - (k_{P1} + k_{P2})/k_Q. \qquad (34)$$

Apart from the desirability of knowing the separate amounts of " induced " and " profile drag " losses, the use of (34) gives more accurate values of η than (23) in the neighbourhood of maximum η The thrust coefficient k_T may then be determined by (27) if desired. At low values of J it is necessary to determine the thrust from equation (21).

The only approximation involved in equations (18), (21), (31) and (33) is the replacement of cos β by unity and of sin β by β.

The calculation of q_c, p_{C1} and p_{C2} is thus now reduced to :—

(i) The tabulation of b, τ, ζ and q as functions of J, r_c and N ; values for eight standard radii, with a range of J from 1·0 to 7·0, are given in Tables 1 and 2. These tables are considered to be of universal application as they depend only on the theory of blade interference.

(ii) Knowledge of values of C_L and C_D as functions of incidence and Mach number. These section characteristics depend on the shape of blade section, and are necessarily subject to revision as further experimental data on propeller performance are obtained. The best data available on certain sections at the time of writing are discussed in general terms in § 6 below, but the appropriate tables and curves are given in a separate report (R. & M. 2036[7]). The method by which overall measurements of thrust and torque were analysed to obtain these data is described in R. & M. 2020[8]. In the range in which the assumption of a straight-line lift curve holds, C_L is represented by the values of ε and a as functions of M. Consideration of a, ε and C_D is given in § 6.

Equations (10) and (31) are correct if β is measured in radians ; in practice it is more convenient to measure all angles in degrees. The values of b given by equation (10) were therefore multiplied by $180/\pi$ for tabulation in Table 1b. Values of q from Table 2e are used as they stand in equation (33), but must be divided by $180/\pi$ for use in equation (31) if β is expressed in degrees.

4. *Integrating Coefficients.*—It is finally necessary to integrate the values of t_c, q_c, p_{C1} and p_{C2} with respect to r_c^2 between the limits 0·09 and 1·0* to give values of k_T, k_Q, k_{P1} and k_{P2}. This may be done graphically, but more consistent results may be obtained with less labour by the use of integrating coefficients C_n (given in Table 3a taken from R. & M. 2043[9]) in formulae such as—

$$k_Q = C_1 q_{c1} + C_2 q_{c2} + \ldots\ldots + C_8 q_{c8} \qquad (35)$$

where the suffices refer to the eight standard values of r_c. If separate values of k_{P1} and k_{P2} were not required, it would be sufficient to evaluate

$$k_P = k_{P1} + k_{P2} \qquad (36)$$

by integrating

$$p_C = p_{C1} + p_{C2}\,. \qquad (37)$$

Finally, the efficiency η may be obtained by using equation (34).

5. *Calculation of Mach Number.*—The Mach number of a blade section is taken to be—

$$M = W_0/a_h \qquad (38)$$

where

$$W_0 = \Omega\, r \sec \varphi_0 \qquad (39)$$

and a_h is the speed of sound at the height h at which the aircraft is flying.†

Hence $$M = (\Omega R/a_h)\, r_c \sec \varphi_0 \qquad (40)$$

where $$J = V/nD \qquad (41)$$

and $$\tan \varphi_0 = J/\pi r_c\,. \qquad (42)$$

Values of φ_0 and $r_c \sec \varphi_0$ are tabulated against J for standard values of r_c in Tables 2a and 2b.

6. *Formulae for* (C_L, α) *in Terms of Thickness Ratio and Mach Number.*—The principal object for which the method is designed is the calculation of the performance of airscrews for high-speed aeroplanes on the basis of the section lift and drag data reported in R. & M. 2036[7]. Over the range of incidence likely to be used at high speed it is sufficiently accurate to assume a linear lift curve for a given Mach number.

The lift coefficient of the section of an airscrew blade is a function of incidence, Mach number and thickness-chord ratio. It is conveniently expressed in terms of its low-speed value C_{LO}, which is assumed to be a linear function of incidence α over the range $O < C_{LO} < 0{\cdot}6$ according to the equation

$$C_{LO} = A_0 \alpha_0 \qquad (43)$$

where α_0 ($= \alpha + \varepsilon_0$) denotes the incidence referred to the zero-lift datum. The low-speed slope A_0 is taken as 0·1 per degree for values of t/c less than 17 per cent. for airscrew sections of shape similar to Clark *Y*. For larger values of t/c its value is taken provisionally from Table 1 of R. & M. 2036[7]; this table was based on experiments in the compressed air tunnel[11] on aerofoils of thickness-chord ratio up to 30 per cent.

The low-speed zero-lift angle ε_0 is taken to be that given by the theory of thin aerofoils[12], using the rapid method of evaluation given in R. & M. 1914[13].

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* The lower limit of 0·09 for r_c^2 (corresponding to 0·3 for r_c) was chosen arbitrarily; it is necessary to estimate the contribution of the portion of the blade inside this radius (blade root losses) by other methods (§ 8).

† The value of a_h as a function of height under standard conditions is given in R. & M. 1891[10].

The lift coefficient C_L, considered as a function of Mach number as well as α, is defined by distinct formulae in two separate ranges in which the values of C_L, a, ε are here distinguished by the suffices 1 and 2 respectively, the boundary between the two ranges corresponding to a critical Mach number M_L which for values of α_0 less than 3 deg. is an empirical function of thickness-chord ratio only, but depends also on incidence at higher values of α_0. Values of M_L are given in Table 3 of R. & M. 2036[7].

Range 1 includes all values of M less than M_L. In this range the lift coefficient rises with Mach number at a rate which for sections thinner than 16 per cent. follows Glauert's equation—

$$C_{L1} = (1 - M^2)^{-1/2} C_{LO} \qquad (44)$$

Comparison with equation (11) then gives the following of a and ε :—

$$a_1 = (1 - M^2)^{1/2} a_0 \qquad (45)$$

$$\varepsilon_1 = \varepsilon_0 \qquad (46)$$

where

$$a_0 = 1/sA_0 . \qquad (47)$$

Values of $(1 - M^2)^{1/2}$ and $(1 - M^2)^{-1/2}$ are given in Table 2 of R. & M. 2036[7].
For thicker sections the rate of rise with Mach number becomes progressively less than $(1 - M^2)^{-1/2}$; use is then made of Table 5 of R. & M. 2036[7].

Range 2 includes all values of M greater than M_L. In this range C_L is given by the equation—

$$C_{L2} = (1 - M_L{}^2)^{-1/2} C_{LO} + C_{LS} \qquad (48)$$

where C_{LS} is an empirical function of $(M - M_L)$ given in Table 4 of R. & M. 2036[7]. Comparison with equation (11) now gives the following values of a and ε :—

$$a_2 = (1 - M_L{}^2)^{1/2} a_0 \qquad (49)$$

$$\varepsilon_2 = \varepsilon_0 + (1 - M_L{}^2)^{1/2} C_{LS}/A_0 . \qquad (50)$$

Substitution of the appropriate values of a and ε in equations (13), (9), (7) and (11) will then determine the appropriate values of C_L, φ and α for both ranges, except that when α_0 exceeds 3 degs. the solution involves a small amount of successive approximation since M_L then depends on α as well as t/c. The principle of the successive approximation (R. & M. 2036[7]) consists in adopting the low-incidence value (M'_L) of M_L (independent of α) and the corresponding value (a') of a as a first approximation, and evaluating $\alpha_0' = a' (\theta - \varphi_0 + \varepsilon_0)/(a' + b)$. If this exceeds 3 degs. a second approximation to M_L is necessary. In practice it is found that this second approximation suffices.

Formulae for C_D *in Terms of Mach Number and* C_{LO}.—In considering the effect of high Mach number it is convenient, except at high incidence and for thick sections, to write—

$$C_D = C_{DO} + C_{DS} \qquad (51)$$

where C_{DS} is zero for low values of M.

C_{DO}.—It is assumed that the value of C_{DO} at any radius is given by the equation—

$$C_{DO} = B_0 C_0 \qquad (52)$$

where B_0 is a function of α_0 only and C_0 is a function of t/c only, given in Tables 6 and 7 of R. & M. 2036[7].

C_{DS}.—The compressibility drag C_{DS} is an empirical function of $(M - M_D)$, given in Table 9 of R. & M. 2036[7], where M_D is a critical Mach number for drag, assumed to be an empirical function of the two variables α_0 and t/c (Table 8 of R. & M. 2036[7]). When dealing with a series of calculations for all of which t/c is the same at a given radius, it is convenient to draw a family of curves of M_D against α_0, one for each standard radius.

At incidences above 8 deg. for sections thinner than 9 per cent. and above 5 deg. for sections from 9 per cent. to 16 per cent., it is more convenient to use curves of C_D plotted against M directly. These are given as Figs. 1-15 of R. & M. 2036[7].

It should be emphasised that the above formulae for C_L and C_D represent the best available evidence at the time of writing (August, 1945). If it should prove necessary to modify them when further experimental evidence becomes available, it will still be possible to use the method of § 3 with Tables 1 to 3, provided that a linear relation between C_L and α for given M still holds over a sufficient range.

7. *Specimen Calculation.*—A specimen calculation for one blade section is given in Table 4. For a complete airscrew there would be eight columns corresponding to the eight standard radii.

Values of the following quantities are supposed given—for the whole airscrew: N (number of blades), V (forward speed), n (airscrew revolutions per second), h (operating altitude, to determine the speed of sound a_h), from which are derived $\Omega R/a_h$ and J; and at each standard radius (r_c): t/c, section shape (determining ε_0), s (solidity) and θ (blade angle).

The three columns of Table 4 give examples respectively of Range 1, Range 2 at low incidence, and Range 2 at high incidence. If $M < M_L$ (Range 1), the entry for $(M - M_L)$ should be "< 0", and the value of $(1 - M^2)^{1/2}$ used instead of $(1 - M_L{}^2)^{1/2}$. If $M > M_L$, (Range 2) the value of $(M - M_L)$ is inserted so as to determine C_{LS}, and $(1 - M_L{}^2)^{1/2}$ is used instead of $(1 - M^2)^{1/2}$.

The final results for a particular working condition and blade setting of a given airscrew will be values of q_c or t_c, p_{C1}, p_{CO} and p_{CS} for the eight standard values of r_c. The integrating coefficients given in Table 3a are then used to integrate the above quantities between the limits $r_c{}^2 = 0 \cdot 09$ and $1 \cdot 0$ to give values of k_Q or k_T, k_{P1}, k_{PO} and k_{PS}. The efficiency (η) follows.

8. *Blade Root Loss.*—The lower limit of integration corresponding to the numerical coefficients of Table 3a is $r_c = 0 \cdot 3$: $(r_c)^2 = 0 \cdot 09$. The effect of the portion of the blade between this radius and the spinner is conveniently dealt with[14, 15] as a separate loss of efficiency, in the calculation of which the contribution of the root sections to the torque is in general negligible. Neglecting the induced loss, the power loss coefficient is given from equation (33) in the form—

$$\Delta k_P = \int_{\text{spinner}}^{0 \cdot 09} q s C_D \, d(r_c{}^2), \qquad (53)$$

so that the efficiency loss is simply

$$\Delta \eta = \Delta k_P / k_Q \qquad (54)$$

to the first order of small quantities.

Since

$$q = \frac{\pi^3}{16} r_c{}^3 \sec^3 \varphi_0 \qquad (32)$$

and

$$J = \pi r_c \tan \varphi_0, \qquad (42)$$

it follows that

$$\Delta k_P = \frac{J^3}{16} \int_{\text{spinner}}^{0 \cdot 09} s C_D \operatorname{cosec}^3 \varphi_0 \, d(r_c{}^2) . \qquad (55)$$

Equation (55) may be more convenient than equation (53) for rapidly estimating the root loss since the factor $\operatorname{cosec}^3 \varphi_0$ does not differ greatly from unity and may be omitted in the calculation if great accuracy is not required; the omission would not affect a comparison between different propeller designs. Alternatively, a mean value of this factor may be taken for a range of values of J.

The drag coefficient C_D for root sections has been plotted in R. & M. 2036[7] as a function of Mach number and incidence. For the evaluation of the root loss the effect of compressibility on the zero-lift angle may usually be neglected, so that equations (13) and (11) reduce to the approximate relation

$$\alpha_0 = \frac{a}{a+b}(\theta - \varphi_0 + \varepsilon_0) \quad \ldots \quad (56)$$

Values of a for thick root sections are given in R. & M. 2036[7]; values of b have been calculated from equation (10) using values of $\varkappa$ for the innermost radii obtained by the method described in Appendix 2.

The integration of equation (53) is most conveniently effected using integrating coefficients which are given in Table 3b quoted from R. & M. 2043[9]. An example of root loss calculation is included in Table 4, and the resulting power loss is included in the final value of the efficiency.

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APPENDIX 1

Calculation of $\varkappa$ from $r_c = 0{\cdot}3$ to $r_c = 0{\cdot}975$

The new method of airscrew calculation requires the tabulation of values of $\varkappa$ (the coefficient of interference velocity) for closely spaced intervals of the argument ($\sin \varphi$) for a range of values of r/R. Since the existing tables of $\varkappa$ were inadequate for this purpose, it was decided to extend these tables, making use of theoretical work[16] made available since the latter were prepared.

The aim in view was the tabulation of $\varkappa$ against $\sin \varphi$ for certain values of r/R for 2, 3 and 4-bladed airscrews. It is not always possible to compute $\varkappa$ for any arbitrary value of $\sin \varphi$ and r/R, but the approximate value $\varkappa_P$ (Ref. 4) is always calculable to any required degree of accuracy, and the ratio $\varkappa/\varkappa_P$ can be computed for certain cases, notably when $\sin \varphi$ is unity. The quantity $\varkappa_P$ was therefore calculated for all the desired values of $\sin \varphi$ and r/R.

Table 2 of R. & M. 1674[4], giving the ratio $\varkappa/\varkappa_P$ for 2 and 4-bladers, was extended in the following way. The ratios for $\sin \varphi = 1$ were computed and obtained for additional values of r/R by interpolation. The tables were then smoothed, *i.e.*, the $\varkappa/\varkappa_P$ ratios were faired against both $\sin \varphi$ and r/R by the well-known arithmetical processes for the graduation of numerical data[17]. The values of $\varkappa/\varkappa_P$ for three-bladers were then obtained by interpolation, knowing $\varkappa/\varkappa_P$ for infinity, two and four blades. The interpolation formula is quoted below; it was obtained on the assumption that $\varkappa/\varkappa_P$ is a quadratic function of $1/N$ passing through known values for $N = 2$, 4 and ∞. An additional check was obtained in this case by working out $\varkappa$, and hence the $\varkappa/\varkappa_P$ ratio, for $\sin \varphi = 1$, from Reference 16, inserting these values in the table obtained by interpolation and adjusting so as to distribute any error linearly over the range $\sin \varphi = 0$ to $\sin \varphi = 1$.

The tables were then sub-tabulated for small intervals of $\sin \varphi$. By using the appropriate values of $\varkappa_P$ the desired tables of $\varkappa$ were then obtained by simple multiplication.

The formulae used in the calculations for Table 1a are listed below :—

(*a*)
$$\varkappa_P = \frac{2}{\pi} \arccos e^{-NF/2} \quad \ldots \quad (57)$$

Where
$$F = (1 - r_c)/\sin \varphi_1 \quad \ldots \quad (58)$$

and
$$\tan \varphi_1 = r_c \tan \varphi \quad \ldots \quad (59)$$

For numerical work, we write

$$\varkappa_P = \frac{2}{\pi}\,\theta_1\,, \qquad \ldots \qquad (60)$$

where θ_1 is in radians.

Taking θ_1 in degrees and expressing $\varkappa_P$ as a function of r_c and $\sin\varphi$,

$$\log_{10}\cos\theta_1 = -\frac{N}{2}\,\frac{(1-r_c)}{\sin\varphi_1} \times 0\cdot4329 \qquad \ldots \qquad (61)$$

and

$$\sin\varphi_1 = r_c\,s/\sqrt{1-(1-r_c^2)s^2} \qquad \ldots \qquad (62)$$

where

$$s = \sin\varphi \qquad \ldots \qquad (63)$$

(*b*) For 2-bladed airscrews with $\sin\varphi = 1$,

$$\varkappa = \sqrt{(1-r_c^2)}/\pi r_c. \qquad \ldots \qquad (64)$$

(*c*) For 4-bladed airscrews with $\sin\varphi = 1$,

$$\varkappa = \frac{4}{\pi^2}\log\left\{\frac{1}{r_c^2} + \left[\frac{1}{r_c^4} - 1\right]^{1/2}\right\}. \qquad \ldots \qquad (65)$$

(*d*) Calculation of $\varkappa$ for 3-bladed airscrews with $\sin\varphi = 1$:

(i) For small values of r_c (viz., 0·3, 0·35, 0·4, 0·45)

$$\varkappa = \frac{3}{2\pi^{3/2}}\,\frac{\Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{2}{3}\right)}\,\frac{\sqrt{(1-r_c^3)}}{\sqrt{r_c}}\left\{1 + \frac{1}{5}(2r_c^3) + \frac{1\cdot4}{5\cdot11}(2r_c^3)^2 + \frac{1\cdot4\cdot7}{5\cdot11\cdot17}(2r_c^3)^3 + \ldots\right\} - \frac{3\sqrt{3}}{2\pi} \qquad (66)$$

(ii) For values of r_c approaching unity (viz. 0·975, 0·95, 0·85, 0·80)

$$\varkappa = \frac{3}{\pi^{3/2}}\,\frac{\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{2}{3}\right)}\,\frac{\sqrt{(1-r_c^3)}}{\sqrt{r_c^3}}\left[1 + \frac{1}{3}\left\{\frac{2}{3}(1-r_c^3)\right\} + \frac{1\cdot4}{3\cdot5}\left\{\frac{2}{3}(1-r_c^3)\right\}^2 + \frac{1\cdot4\cdot7}{3\cdot5\cdot7}\left\{\frac{2}{3}(1-r_c^3)\right\}^3 + \ldots\right] \qquad (67)$$

For other values of r_c

$$\varkappa = \frac{3}{\pi^{3/2}}\,\frac{\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{2}{3}\right)}\int_0^{\operatorname{arc\,cos}(r_c^3/2)}\frac{dy}{(\cos y)^{4/3}} \qquad \ldots \qquad (68)$$

$$\varkappa(r_c) - \varkappa(r_{c1}) = -\frac{9}{2\pi^{3/2}}\,\frac{\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{2}{3}\right)}\int_{r_{c1}}^{r_c}\frac{dt}{t^{3/2}(1-t^3)^{1/2}} \qquad \ldots \qquad (69)$$

where

$$(\cos y)^{2/N} = t \qquad \ldots \qquad (70)$$

(*e*) The interpolation formula for deriving the value of $\varkappa/\varkappa_P$ for 3 blades, knowing the values for infinity, 2 and 4 blades, is

$$\left(\frac{\varkappa}{\varkappa_P}\right)_3 = \frac{8}{9}\left(\frac{\varkappa}{\varkappa}\right) + \frac{2}{9}\left(\frac{\varkappa}{\varkappa_P}\right)_2 - \frac{1}{9} \qquad \ldots \qquad (71)$$

APPENDIX 2

Calculation of $\varkappa$ near the Blade Roots

By J. N. Veasey, B.A.

Since it was considered that simple extrapolation from the values of Appendix 1 to smaller radii would be unsatisfactory, and that the algebraic method used before for interpolating for other numbers of blades than 2 and 4 might be unsuitable for the function at the small radii where it attains values greater than unity, the following method was adopted. Use was made of the solution for the limiting case of infinite pitch given by Westwater and Goldstein[16].

(*a*) *For* 2 *and* 4 *Blades.*—For the limiting case of very small radii, no great simplification of the general formulae given by Goldstein[1] could be found, but for infinite pitch[16] we have equations (64) and (65) (quoted in Appendix 1) ; when $r_c \to 0$, these reduce to :—

$$\text{For 2 blades } \varkappa = 1/\pi r_c \qquad \ldots \qquad (72)$$

$$\text{For 4 blades } \varkappa = \frac{8}{\pi^2} \log \frac{1}{r_c} \cdot \qquad \ldots \qquad (73)$$

As $r_c \to 0$, the pitch angle of the helicoidal vortex sheets becomes 90 deg., and so the above relations give an approximation to $\varkappa$ when r_c is very small.

The equations suggest that $\varkappa$ becomes large towards the roots of the blades, and from equation (72) it might be expected that for conditions other than infinite pitch, $\varkappa r_c$ for two blades would remain finite when r_c approached zero, and that for four blades $\varkappa r_c$ would approach zero logarithmically.

Values of $\varkappa r_c$ for two and four blades taken from Ref. 1 and Table 7 of R. & M. 1674[4] were accordingly plotted against r_c for constant pitch ($r_c \tan \varphi$ = const.), and the curves extrapolated to $r_c = 0$ using the curves of equations (72) and (73) as a guide.

From these curves, values of $\varkappa$ at even intervals of r_c were taken and it was found that by plotting $1/\varkappa$ against $1/r_c \sec \varphi$ at constant r_c, a fairly linear and uniform set of curves was obtained. The reciprocal of $\varkappa$ was chosen as the ordinate because $\varkappa$ is usually greater than unity at the root sections, becoming very large towards the propeller axis. For abscissa $1/r_c \sec \varphi$ was used since, besides producing a smooth series of curves, it becomes zero for the condition of infinite pitch (values calculated from Ref. 16) ; moreover, the high advance ratios fall within a small range of the abscissa.

(*b*) 3, 5, 6 and 8 *Blades.*—Since the formulae in Ref. 1 for the inflow factor for these blade numbers are laborious to compute and involve Bessel Functions which have not been tabulated, only a small number of values were actually calculated for 3 and 6 blades ; the other values were interpolated graphically by plotting $1/\varkappa$ against both $1/r_c \sec \varphi$ (as for 2 and 4 blades) and $1/N$, using as a guide points obtained from the results for 2 and 4 blades and also from the solution for the limiting case of infinite pitch[16].

Since $\varkappa$ represents the ratio of the mean inflow velocity round the circumference of a circle of given radius to the inflow velocity at the vortex sheets at that radius, this ratio ($\varkappa$) will become equal to unity for an infinite number of blades, for which the vortex sheets are indefinitely close together. The inflow factor $\varkappa$ was therefore taken to be equal to unity for the case of an infinite number of blades, for all radii and advance ratios.

REFERENCES

No.	*Author.*	*Title, etc.*
1.	S. Goldstein	On the Vortex Theory of Screw Propellers. *Proc. Roy. Soc.* A, **123,** 440 (1929).
2.	C. N. H. Lock	Application of Goldstein's Airscrew Theory to Design. R. & M. 1377. November, 1930.
3.	C. N. H. Lock	An Application of Prandtl Theory to an Airscrew. R. & M. 1521. August, 1932.
4.	C. N. H. Lock and D. Yeatman	Tables for Use in an Improved Method of Airscrew Strip Theory Calculations. R. & M. 1674. October, 1934.
5.	C. N. H. Lock	A Graphical Method of Calculating the Performance of an Airscrew. R. & M. 1849. August, 1938.
6.	H. Lamb	*Proc. Camb. Phil. Soc.* **21,** 477 (1923).
7.	A. B. Haines and R. J. Monaghan	Section High-Speed Lift and Drag Data for Propeller Performance Calculations. R. & M. 2036. November, 1945.
8.	R. C. Pankhurst and A. B. Haines	Account of the Derivation of High-Speed Lift and Drag Data for Propeller Blade Sections. R. & M. 2020. August, 1945.
9.	C. N. H. Lock and A. E. Knowler	Integrating Coefficients for Airscrew Analysis. R. & M. 2043. July, 1941.
10.	R. C. Pankhurst and J. F. C. Conn	Physical Properties of the Standard Atmosphere. R. & M. 1891. January, 1941.
11.	D. H. Williams, A. H. Bell and W. G. Raymer.	Tests on a 20 per cent. Piercy Aerofoil and on N.A.C.A. 0030 with and without Flaps, in the Compressed Air Tunnel. A.R.C. No. 4511, April, 1940. (To be published.)
12.	H. Glauert	Aerofoil and Airscrew Theory, p. 91. Cambridge University Press, 1926,
13.	R. C. Pankhurst	A Method for the Rapid Evaluation of Glauert's Expressions for the Angle of Zero Lift and the Moment at Zero Lift. R. & M. 1914. March, 1944.
14.	P. A. Hufton	The Calculation of Airscrew Efficiencies at High Speeds. R.A.E. Performance Note No. 18. July, 1940. (Unpublished.)
15.	R. J. Monaghan	Estimation of the Compressibility Losses on Propeller Blade Roots. A.R.C. No. 8371. December, 1944. (Unpublished.) (The data of this report are included in Ref. 7 above.)
16.	S. Goldstein	On the Limiting Value for Infinite Pitch of a Parameter occurring in Airscrew Theory. *Proc. Camb. Phil. Soc.*, **40,** 146. (1944.)
17.	E. T. Whittaker and G. Robinson	The Calculus of Observations, Chapter XI. Blackie, 1940.

TABLE 1a

Values of $\varkappa : r_c = 0{\cdot}2$

N / Sin ϕ	2		3		4		5		6	
0·68	1·000	9	1·006	9	1·011	3	1·008	2	1·004	3
0·70	1·009	11	1·015	11	1·014	5	1·010	3	1·007	3
·72	·020	12	·026	12	·019	7	·013	5	·010	5
·74	·032	13	·038	14	·026	9	·018	7	·015	5
·76	·045	15	·052	16	·035	10	·025	9	·020	7
·78	·060	19	·068	18	·045	13	·034	10	·027	9
0·80	1·079	10	1·086	10	1·058	7	1·044	6	1·036	5
·81	·089	10	·096	11	·065	9	·050	7	·041	5
·82	·099	11	·107	12	·074	9	·057	7	·046	6
·83	·110	12	·119	14	·083	11	·064	8	·052	6
·84	·122	13	·133	14	·094	11	·072	9	·058	7
0·85	1·135	14	1·147	15	1·105	13	1·081	10	1·065	8
·86	·149	15	·162	17	·118	14	·091	10	·073	8
·87	·164	16	·179	19	·132	16	·101	13	·081	10
·88	·180	16	·198	19	·148	18	·114	15	·091	10
·89	·196	18	·217	22	·166	19	·129	15	·101	11
0·90	1·214	19	1·239	25	1·185	21	1·144	17	1·112	13
·91	·233	20	·264	28	·206	24	·161	19	·125	15
·92	·253	22	·292	30	·230	27	·180	22	·140	17
·93	·275	24	·322	33	·257	30	·202	26	·157	19
·94	·299	26	·355	36	·287	33	·228	29	·176	22
0·95	1·325	14	1·391	21	1·320	17	1·257	15	1·198	12
·955	·339	15	·412	21	·337	18	·272	16	·210	13
·96	·354	16	·433	22	·355	19	·288	17	·223	13
·965	·370	17	·455	24	·374	20	·305	18	·236	14
·97	·387	17	·479	25	·394	21	·323	19	·250	16
0·975	1·404	18	1·504	26	1·415	23	1·342	20	1·266	17
·98	·422	21	·530	28	·438	25	·362	22	·283	17
·985	·443	25	·558	29	·463	28	·384	24	·300	19
·99	·468	15	·587	15	·491	18	·408	15	·319	12
·9925	·483	16	·602	16	·509	20	·423	15	·331	12
0·995	1·499	7	1·618	5	1·529	7	1·438	7	1·343	6
·996	·506	9	·623	6	·536	8	·445	8	·349	6
·997	·515	10	·629	7	·544	9	·453	9	·355	7
·998	·525	11	·636	6	·553	10	·462	10	·362	8
·999	·536	9	·642	3	·563	9	·472	4	·370	5
0·9995	1·545	14	1·645	2	1·572	13	1·476	6	1·375	7
1·000	·559		·647		·585		·482		·382	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}25$

N Sin ϕ	2		3		4		5		6	
0·58	0·954	2	0·975	3	0·992	0	0·996	0	0·999	−1
0·60	0·956	3	0·978	4	0·992	0	0·996	1	0·998	0
·62	·959	3	·982	4	·992	1	·997	1.	998	+1
·64	·962	3	·986	4	·993	2	·998	1	·999	1
·66	·965	4	·990	6	·995	3	·999	2	1·000	1
·68	·969	4	·996	6	·998	4	1·001	3	·001	2
0·70	0·973	5	1·002	7	1·002	6	1·004	4	1·003	3
·72	·978	5	·009	8	·008	7	·008	5	·006	4
·74	·983	5	·017	10	·015	9	·013	7	·010	6
·76	·988	6	·027	11	·024	11	·020	9	·016	7
·78	·994	7	·038	14	·035	12	·029	11	·023	9
0·80	1·001	4	1·052	7	1·047	8	1·040	5	1·032	5
·81	·005	4	·059	8	·055	8	·045	7	·037	5
·82	·009	5	·067	9	·063	8	·052	7	·042	6
·83	·014	5	·076	10	·071	9	·059	8	·048	7
·84	·019	6	·086	10	·080	9	·067	8	·055	7
0·85	1·025	6	1·096	12	1·089	11	1·075	9	1·062	8
·86	·031	7	·108	12	·100	12	·084	11	·070	9
·87	·038	7	·120	14	·112	13	·095	12	·079	9
·88	·045	8	·134	15	·125	13	·107	12	·088	10
·89	·053	9	·149	15	·138	14	·119	14	·098	12
0·90	1·062	10	1·164	16	1·152	16	1·133	14	1·110	13
·91	·072	11	·180	17	·168	18	·147	16	·123	15
·92	·083	12	·197	18	·186	19	·163	18	·138	15
·93	·095	14	·215	20	·205	20	·181	19	·153	17
·94	·109	15	·235	20	·225	20	·200	20	·170	18
0·95	1·124	7	1·255	11	1·245	12	1·220	11	1·188	9
·955	·131	8	·266	11	·257	12	·231	12	·197	10
·96	·139	9	·277	11	·269	13	·243	12	·207	10
·965	·148	9	·288	12	·282	13	·255	13	·217	11
·97	·157	9	·300	13	·295	14	·268	13	·228	11
0·975	1·166	10	1·313	13	1·309	15	1·281	14	1·239	12
·98	·176	11	·326	14	·324	16	·295	15	·251	13
·985	·187	12	·340	15	·340	17	·310	16	·264	15
·99	·199	13	·355	15	·357	20	·326	18	·279	16
·995	·212	9	·370	7	·377	12	·344	10	·295	8
0·9975	1·221	12	1·377	7	1·389	15	1·354	13	1·303	10
1·000	·233		·384		·404		·367		·313	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}3$

Sin ϕ \ N	2		3		4		5		6	
0·00	1·000	0	1·000	0	1·000	0	1·000	0	1·000	0
·05	·000	− 2	·000	− 3	·000	− 1	·000	− 2	·000	0
·10	0·998	− 4	0·997	− 3	0·999	− 1	0·998	− 1	·000	− 1
·15	·994	− 3	·994	− 1	·998	0	·997	− 1	·999	− 1
·175	·991	− 2	·993	− 2	·998	− 1	·996	− 1	·998	0
0·20	0·989	− 3	0·991	− 1	0·997	− 1	0·995	0	0·998	0
·22	·986	− 3	·990	− 1	·996	0	·995	− 1	·998	− 1
·24	·983	− 3	·989	− 1	·996	− 1	·994	− 1	·997	0
·26	·980	− 2	·988	− 2	·995	− 1	·993	− 1	·997	− 1
·28	·978	− 3	·986	− 2	·994	− 1	·992	0	·996	0
0·30	0·975	− 3	0·984	− 1	0·993	− 1	0·992	− 1	0·996	0
·32	·972	− 3	·983	− 1	·992	0	·991	− 1	·996	− 1
·34	·969	− 3	·982	− 1	·992	− 1	·990	0	·995	0
·36	·966	− 3	·981	− 1	·991	− 1	·990	0	·995	− 1
·38	·963	− 3	·980	− 1	·990	0	·990	0	·994	0
0·40	0·960	− 3	0·979	0	0·990	− 1	0·990	− 1	0·994	0
·42	·957	− 4	·979	− 1	·989	− 1	·989	0	·994	− 1
·44	·953	− 3	·978	0	·988	0	·989	0	·993	0
·46	·950	− 3	·978	0	·988	− 1	·989	0	·993	− 1
·48	·947	− 3	·978	0	·987	− 1	·989	0	·992	0
0·50	0·944	− 3	0·978	0	0·986	0	0·989	0	0·992	0
·52	·941	− 3	·978	0	·986	0	·989	+ 1	·992	+ 1
·54	·938	− 3	·978	0	·986	0	·990	0	·993	0
·56	·935	− 2	·978	+ 1	·986	+ 1	·990	+ 1	·993	+ 1
·58	·933	− 3	·979	1	·987	1	·991	1	·994	1
0·60	0·930	− 2	0·980	1	0·988	1	0·992	2	0·995	1
·62	·928	− 2	·981	2	·989	2	·994	2	·996	1
·64	·926	− 2	·983	2	·991	3	·996	2	·997	2
·66	·924	− 2	·985	3	·994	3	·998	3	·999	2
·68	·922	− 2	·988	3	·997	3	1·001	3	1·001	3
0·70	0·920	− 1	0·991	4	1·000	5	1·004	4	1·004	4
·72	·919	− 1	·995	5	·005	6	·008	4	·008	4
·74	·918	− 1	1·000	5	·011	6	·012	5	·012	5
·76	·917	0	·005	7	·017	7	·017	7	·017	6
·78	·917	0	·012	7	·024	8	·024	8	·023	8
0·80	0·917	0	1·019	4	1·032	5	1·032	5	1·031	5
·81	·917	+ 1	·023	5	·037	5	·037	5	·036	5
·82	·918	1	·028	5	·042	5	·042	5	·041	5
·83	·919	1	·033	5	·047	6	·047	6	·046	6
·84	·920	2	·038	6	·053	6	·053	7	·052	6
0·85	0·922	2	1·044	6	1·059	7	1·060	7	1·058	7
·86	·924	2	·050	7	·066	8	·067	8	·065	7
·87	·926	3	·057	7	·074	9	·075	10	·072	8
·88	·929	5	·064	8	·083	10	·085	10	·080	9
·89	·934	5	·072	9	·093	11	·095	10	·089	10
0·90	0·939		1·081		1·104		1·105		1·099	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0\cdot3$—*continued*

N / Sin ϕ	2		3		4		5		6	
0·90	0·939	2	1·081	5	1·104	6	1·105	6	1·099	5
·905	·941	3	·086	5	·110	6	·111	6	·104	5
·91	·944	3	·091	5	·116	6	·117	6	·109	6
·915	·947	3	·096	5	·122	7	·123	6	·115	6
·92	·950	3	·101	6	·129	6	·129	7	·121	6
0·925	0·953	3	1·107	5	1·135	7	1·136	7	1·127	7
·93	·956	3	·112	5	·142	7	·143	7	·134	6
·935	·959	4	·117	5	·149	8	·150	8	·140	7
·94	·963	3	·122	6	·157	7	·158	7	·147	6
·945	·966	4	·128	6	·164	7	·165	8	·153	7
0·95	0·970	3	1·134	5	1·171	8	1·173	9	1·160	7
·955	·973	4	·139	6	·179	8	·182	8	·167	7
·96	·977	4	·145	7	·187	8	·190	9	·174	7
·965	·981	4	·152	6	·195	8	·199	9	·181	8
·97	·985	4	·158	6	·203	8	·208	9	·189	8
0·975	0·989	4	1·164	6	1·211	9	1·217	9	1·197	9
·98	·993	4	·170	5	·220	9	·226	9	·206	9
·985	·997	4	·175	5	·229	9	·235	9	·215	9
·99	1·001	5	·180	5	·238	9	·244	10	·224	10
·995	·006	6	·185	4	·247	9	·254	10	·234	11
1·000	1·012		1·189		1·256		1·264		1·245	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0\cdot45$

N / Sin ϕ	2		3		4		5		6	
0·00	1·000		1·000	− 1	1·000		1·000	0	1·000	0
·02	1·000		0·999	− 1	·000		·000	1	·000	1
·05	1·000		·998	0	·000		·001	0	·001	1
·07	1·000	− 1	·998	− 1	·000		·001	1	·002	1
0·10	0·999	− 1	0·997	− 1	1·000		1·002	0	1·003	1
·12	·998	0	·996	− 1	·000	− 1	·002	0	·004	0
·14	·998	− 2	·995	− 1	0·999	0	·002	1	·004	0
·16	·996	− 2	·994	− 2	0·999	− 1	·003	0	·004	1
·18	·994	− 4	·992	− 2	0·998	− 0	·003	0	·005	0
0·20	0·990	− 4	0·990	− 2	0·998	− 1	1·003	0	1·005	1
·22	·986	− 6	·988	− 2	·997	0	·003	0	·006	0
·24	·980	− 6	·986	− 3	·997	− 1	·003	0	·006	0
·26	·974	− 7	·983	− 3	·996	− 1	·003	0	·006	1
·28	·967	− 8	·980	− 3	·995	− 2	·003	0	·007	0
0·30	0·959		0·977		0·993		1·003		1·007	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0 \cdot 45$—continued

N / Sin ϕ	2		3		4		5		6	
0·30	0·959		0·977		0·993		1·003		1·007	
		−9		−4		−1		0		0
·32	·950		·973		·992		·003		·007	
		−10		−4		−1		−1		0
·34	·940		·969		·991		·002		·007	
		−11		−4		−2		0		0
·36	·929		·965		·989		·002		·007	
		−11		−5		−2		−1		0
·38	·918		·960		·987		·001		·007	
		−11		−6		−2		−1		0
0·40	0·907		0·954		0·985		1·000		1·007	
		−11		−6		−2		−2		0
·42	·896		·948		·983		·998		·007	
		−11		−6		−3		−1		−1
·44	·885		·942		·980		·997		·006	
		−12		−6		−2		−1		−1
·46	·873		·936		·978		·996		·005	
		−13		−7		−3		−1		0
·48	·860		·929		·975		·995		·005	
		−13		−6		−3		−1		−1
0·50	0·847		0·923		0·972		0·994		1·004	
		−13		−7		−3		−1		0
·52	·834		·916		·969		·993		·004	
		−13		−7		−4		−2		0
·54	·821		·909		·965		·991		·004	
		−13		−7		−3		−1		+1
·56	·808		·902		·962		·990		·005	
		−12		−7		−4		−1		0
·58	·796		·895		·958		·989		·005	
		−13		−7		−3		−1		0
0·60	0·783		0·888		0·955		0·988		1·005	
		−12		−7		−4		−1		0
·62	·771		·881		·951		·987		·005	
		−13		−7		−3		−1		1
·64	·758		·874		·948		·986		·006	
		−12		−7		−4		−1		1
·66	·746		·867		·944		·985		·007	
		−12		−7		−4		−1		0
·68	·734		·860		·940		·984		·007	
		−12		−7		−4		−1		0
0·70	0·722		0·853		0·936		0·983		1·007	
		−10		−7		−3		−1		0
·72	·712		·846		·933		·982		·007	
		−10		−7		−4		−1		0
·74	·702		·839		·929		·981		·007	
		−9		−7		−3		−1		1
·76	·693		·832		·926		·980		·008	
		−9		−6		−3		−1		0
·78	·684		·826		·923		·979		·008	
		−8		−6		−3		−1		0
0·80	0·676		0·820		0·920		0·978		1·008	
		−6		−5		−3		−1		1
·82	·670		·815		·917		·977		·009	
		−6		−5		−3		0		1
·84	·664		·810		·914		·977		·010	
		−5		−3		−1		+1		1
·86	·659		·807		·913		·978		·011	
		−5		−1		−1		0		2
·88	·654		·806		·912		·978		·013	
		−5		−1		+1		2		3
0·90	0·649		0·805		0·913		0·980		1·016	
		−4		0		1		1		4
·92	·645		·805		·914		·981		·020	
		−4		+1		1		2		4
·94	·641		·806		·915		·983		·024	
		−3		0		2		3		5
·96	·638		·806		·917		·986		·029	
		−4		1		3		4		5
·98	·634		·807		·920		·990		·034	
		−3		1		4		6		6
1·00	0·631		0·808		0·924		0·996		1·040	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}6$

N Sin ϕ	2		3		4		5		6	
0·00	1·000		1·000		1·000		1·000	0	1·000	0
·01	·000		·000	− 1	·000		·000	0	·000	1
·02	·000		·999	0	·000		·000	0	·001	0
·03	·000		·999	0	·000		·000	1	·001	0
·04	·000		·999	0	·000		·001	0	·001	1
0·05	1·000	− 1	0·999	− 1	1·000		1·001	0	1·002	0
·06	·999	0	·998	0	·000		·001	0	·002	1
·07	·999	0	·998	0	·000		·001	1	·003	0
·08	·999	− 1	·998	− 1	·000		·002	0	·003	0
·09	·998	− 1	·997	− 0	·000		·002	0	·003	1
0·10	0·997	− 1	0·997	− 1	1·000	− 1	1·002	0	1·004	0
·11	·996	− 2	·996	− 0	0·999	0	·002	0	·004	0
·12	·994	− 2	·996	− 1	·999	0	·002	0	·004	0
·13	·992	− 2	·995	− 1	·999	0	·002	0	·004	1
·14	·990	− 3	·994	− 1	·999	− 1	·002	0	·005	0
0·15	0·987	− 3	0·993	− 1	0·998	− 0	1·002	0	1·005	1
·16	·984	− 5	·992	− 2	·998	− 1	·002	0	·006	0
·17	·979	− 5	·990	− 2	·997	− 0	·002	0	·006	1
·18	·974	− 7	·988	− 2	·997	− 1	·002	1	·007	0
·19	·967	− 7	·986	− 2	·996	− 0	·003	0	·007	0
0·20	0·960	− 8	0·984	− 2	0·996	− 1	1·003	0	1·007	1
·21	·952	− 7	·982	− 3	·995	− 1	·003	0	·008	0
·22	·945	− 8	·979	− 3	·994	− 1	·003	− 1	·008	0
·23	·937	− 8	·976	− 4	·993	− 1	·002	0	·008	0
·24	·929	− 8	·972	− 3	·992	− 2	·002	0	·008	0
0·25	0·921	− 9	0·969	− 4	0·990	− 1	1·002	− 1	1·008	1
·26	·912	− 9	·965	− 4	·989	− 2	·001	0	·009	0
·27	·903	− 9	·961	− 4	·987	− 2	·001	− 1	·009	0
·28	·894	− 9	·957	− 5	·985	− 2	·000	0	·009	0
·29	·885	− 9	·952	− 5	·983	− 2	·000	− 1	·009	− 1
0·30	0·876	−18	0·947	−10	0·981	− 5	0·999	− 3	1·008	− 1
·32	·858	−19	·937	−11	·976	− 5	·996	− 3	·007	− 1
·34	·839	−19	·926	−11	·971	− 6	·993	− 3	·006	− 2
·36	·820	−19	·915	−12	·965	− 7	·990	− 4	·004	− 1
·38	·801	−19	·903	−12	·958	− 9	·986	− 4	·003	− 3
0·40	0·782	−19	0·891	−13	0·949	− 8	0·982	− 5	1·000	− 2
·42	·763	−19	·878	−14	·941	− 9	·977	− 5	0·998	− 3
·44	·744	−18	·864	−13	·932	− 9	·972	− 6	·995	− 4
·46	·726	−18	·851	−14	·923	− 9	·966	− 6	·991	− 4
·48	·708	−18	·837	−13	·914	− 9	·960	− 6	·987	− 4
0·50	0·690	−18	0·824	−14	0·905	−10	0·954	− 7	0·983	− 4
·52	·672	−17	·810	−13	·895	−10	·947	− 7	·979	− 5
·54	·655	−16	·797	−14	·885	−10	·940	− 7	·974	− 5
·56	·639	−16	·783	−13	·875	−11	·933	− 8	·969	− 5
·58	·623	−15	·770	−13	·864	−10	·925	− 7	·964	− 5
0·60	0·608		0·757		0·854		0·918		0·959	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}6$—*continued*

N / Sin ϕ	2		3		4		5		6	
0·60	0·608	-14	0·757	-13	0·854	-10	0·918	-8	0·959	-6
·62	·594	-13	·744	-13	·844	-10	·910	-8	·953	-6
·64	·581	-13	·731	-12	·834	-10	·902	-8	·947	-6
·66	·568	-12	·719	-12	·824	-10	·894	-8	·941	-6
·68	·556	-12	·707	-11	·814	-11	·886	-8	·935	-6
0·70	0·544	-10	0·696	-11	0·803	-10	0·878	-9	0·929	-7
·72	·534	-10	·685	-11	·793	-10	·869	-9	·922	-7
·74	·524	-10	·674	-11	·783	-9	·860	-8	·915	-6
·76	·514	-10	·663	-10	·774	-10	·852	-8	·909	-7
·78	·504	-8	·653	-9	·764	-9	·844	-8	·902	-6
0·80	0·496	-8	0·644	-9	0·755	-9	0·836	-8	0·896	-7
·82	·488	-8	·635	-9	·746	-9	·828	-8	·889	-7
·84	·480	-7	·626	-8	·737	-8	·820	-7	·882	-6
·86	·473	-8	·618	-8	·729	-7	·813	-7	·876	-6
·88	·465	-7	·610	-7	·722	-8	·806	-7	·870	-6
0·90	0·458	-7	0·603	-7	0·714	-7	0·799	-7	0·864	-6
·92	·451	-7	·596	-7	·707	-7	·792	-7	·858	-6
·94	·444	-7	·589	-7	·700	-6	·785	-6	·852	-6
·96	·437	-7	·582	-7	·694	-7	·779	-6	·846	-6
·98	·430	-6	·575	-6	·687	-6	·773	-6	·840	-6
1·00	0·424		0·569		0·681		0·767		0·834	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}7$

N / Sin ϕ	2		3		4		5		6	
0·00	1·000		1·000		1·000		1·000	0	1·000	0
·01	·000		·000	-1	·000		·000	0	·000	0
·02	·000		0·999	0	·000		·000	0	·000	0
·03	·000	-1	·999	0	·000		·000	0	·000	1
·04	0·999	0	·999	0	·000		·000	1	·001	0
0·05	0·999	-1	0·999	-1	1·000		1·001	0	1·001	1
·06	·998	-1	·998	0	·000		·001	0	·002	0
·07	·997	-2	·998	0	·000		·001	0	·002	0
·08	·995	-4	·998	-1	·000		·001	0	·002	0
·09	·991	-4	·997	-1	·000		·001	0	·002	1
0·10	0·987	-6	0·996	-2	1·000	-1	1·001	0	1·003	0
·11	·981	-6	·994	-1	0·999	0	·001	0	·003	0
·12	·975	-7	·993	-2	·999	-1	·001	0	·003	0
·13	·968	-7	·991	-3	·998	-1	·001	-1	·003	1
·14	·961	-8	·988	-4	·997	-1	·000	0	·004	0
0·15	0·953		0·984		0·996		1·000		1·004	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0 \cdot 7$—*continued*

N / Sin ϕ	2		3		4		5		6	
0·15	0·953	− 9	0·984	− 3	0·996	− 2	1·000	0	1·004	0
·16	·944	−10	·981	− 4	·994	− 1	·000	− 1	·004	0
·17	·934	−11	·977	− 5	·993	− 2	0·999	− 1	·004	0
·18	·923	−12	·972	− 6	·991	− 3	·998	0	·004	0
·19	·911	−12	·966	− 5	·988	− 2	·998	− 1	·004	0
0·20	0·899	−24	0·961	−13	0·986	− 6	0·997	− 4	1·004	− 1
·22	·875	−25	·948	−14	·980	− 7	·993	− 3	·003	− 2
·24	·850	−25	·934	−15	·973	− 8	·990	− 5	·001	− 1
·26	·825	−25	·919	−16	·965	− 9	·985	− 5	·000	− 3
·28	·800	−24	·903	−16	·956	−11	·980	− 6	0·997	− 3
0·30	0·776	−23	0·887	−17	0·945	−12	0·974	− 7	0·994	− 5
·32	·753	−23	·870	−17	·933	−12	·967	− 8	·989	− 5
·34	·730	−23	·853	−17	·921	−12	·959	− 9	·984	− 7
·36	·707	−22	·836	−17	·909	−13	·950	− 9	·977	− 6
·38	·685	−22	·819	−18	·896	−13	·941	− 9	·971	− 8
0·40	0·663	−21	0·801	−17	0·883	−14	0·932	−11	0·963	− 7
·42	·642	−20	·784	−18	·869	−14	·921	−11	·956	− 8
·44	·622	−20	·766	−17	·855	−14	·910	−12	·948	− 8
·46	·602	−19	·749	−17	·841	−15	·898	−11	·940	− 9
·48	·583	−18	·732	−16	·826	−14	·887	−11	·931	− 9
0·50	0·565	−16	0·716	−16	0·812	−14	0·876	−12	0·922	−10
·52	·549	−16	·700	−16	·798	−13	·864	−11	·912	−10
·54	·533	−15	·684	−15	·785	−14	·853	−12	·902	−10
·56	·518	−14	·669	−14	·771	−14	·841	−11	·892	−10
·58	·504	−12	·655	−14	·757	−13	·830	−12	·882	−10
0·60	0·492	−13	0·641	−14	0·744	−12	·0818	−11	0·872	−10
·62	·479	−11	·627	−13	·732	−13	·807	−12	·862	− 9
·64	·468	−11	·614	−13	·719	−12	·795	−11	·853	−10
·66	·457	−10	·601	−12	·707	−13	·784	−12	·843	−10
·68	·447	−10	·589	−12	·694	−12	·772	−11	·833	−10
0·70	0·437	−10	0·577	−11	0·682	−11	0·761	−10	0·823	−10
·72	·427	− 9	·566	−11	·671	−11	·751	−11	·813	−10
·74	·418	− 9	·555	−11	·660	−11	·740	−11	·803	−10
·76	·409	− 8	·544	−10	·649	−11	·729	−10	·793	− 9
·78	·401	− 8	·534	−10	·638	−10	·719	−10	·784	− 9
0·80	0·393	− 8	0·524	−10	0·628	−10	0·709	− 9	0·775	− 9
·82	·385	− 7	·514	− 9	·618	−10	·700	− 9	·766	− 9
·84	·378	− 7	·505	− 8	·608	− 9	·691	−10	·756	− 9
·86	·371	− 7	·497	− 9	·599	− 9	·681	− 9	·747	− 9
·88	·364	− 8	·488	− 8	·590	− 8	·672	− 9	·738	− 8
0·90	0·356	− 6	0·480	− 8	0·582	− 9	0·663	− 8	0·730	− 8
·92	·350	− 6	·472	− 7	·573	− 8	·655	− 8	·722	− 8
·94	·344	− 6	·465	− 7	·565	− 8	·647	− 8	·714	− 8
·96	·338	− 7	·458	− 7	·557	− 7	·639	− 8	·706	− 8
·98	·331	− 6	·451	− 7	·550	− 7	·631	− 7	·698	− 6
1·00	0·325		0·444		0·543		0·624		0·692	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}8$

N Sin ϕ	2		3		4		5		6	
0·000	1·000		1·000		1·000		1·000		1·000	
·005	·000		·000		·000		·000		·000	
·010	·000		·000		·000		·000		·000	
·015	·000	−1	·000	−1	·000		·000		·000	
·020	0·999	0	0·999	0	·000		·000		·000	
0·025	0·999	0	0·999	0	1·000		1·000		1·000	
·030	·999	−1	·999	0	·000		·000		·000	
·035	·998	−1	·999	0	·000		·000		·000	
·040	·997	−2	·999	0	·000		·000		·000	
·045	·995	−2	·999	0	·000		·000		·000	
0·050	0·993	−3	0·999	−1	1·000	−1	1·000		1·000	
·055	·990	−3	·998	−1	·999	0	·000		·000	1
·060	·987	−5	·997	−1	·999	0	·000		·001	0
·065	·982	−5	·996	−1	·999	0	·000		·001	0
·070	·977	−7	·995	−2	·999	−1	·000		·001	0
0·075	0·970	−7	0·993	−1	0·998	0	1·000		1·001	0
·080	·963	−7	·992	−2	·998	−1	·000		·001	0
·085	·956	−6	·990	−3	·997	−1	·000	−1	·001	0
·090	·950	−8	·987	−3	·996	−1	0·999	0	·001	0
·095	·942	−7	·984	−3	·995	−1	·999	0	·001	−1
0·100	0·935	−15	0·981	−7	0·994	−3	0·999	−2	1·000	0
·11	·920	−15	·974	−8	·991	−4	·997	−3	·000	−1
·12	·905	−15	·966	−9	·987	−4	·994	−2	0·999	−1
·13	·890	−15	·957	−9	·983	−5	·992	−3	·998	−1
·14	·875	−15	·948	−10	·978	−6	·989	−3	·997	−1
0·15	0·860	−16	0·938	−11	0·972	−6	0·986	−4	0·996	−2
·16	·844	−16	·927	−11	·966	−6	·982	−4	·994	−3
·17	·828	−16	·916	−11	·960	−7	·978	−4	·991	−2
·18	·812	−16	·905	−11	·953	−8	·974	−5	·989	−3
·19	·796	−16	·894	−11	·945	−8	·969	−5	·986	−3
0·20	0·780	−31	0·883	−22	0·937	−16	0·964	−11	0·983	−7
·22	·749	−31	·861	−23	·921	−17	·953	−12	·976	−9
·24	·718	−28	·838	−22	·904	−17	·941	−12	·967	−9
·26	·690	−28	·816	−23	·887	−18	·929	−14	·958	−10
·28	·662	−27	·793	−22	·869	−18	·915	−14	·948	−11
0·30	0·635	−25	0·771	−22	0·851	−19	0·901	−14	0·937	−13
·32	·610	−24	·749	−22	·832	−18	·887	−15	·924	−13
·34	·586	−22	·727	−21	·814	−18	·872	−16	·911	−13
·36	·564	−22	·706	−20	·796	−18	·856	−15	·898	−14
·38	·542	−21	·686	−19	·778	−18	·841	−16	·884	−13
0·40	0·521	−20	0·667	−19	0·760	−18	0·825	−16	·0871	−14
·42	·501	−18	·648	−19	·742	−17	·809	−16	·857	−14
·44	·483	−17	·629	−18	·725	−17	·793	−15	·843	−14
·46	·466	−17	·611	−17	·708	−17	·778	−16	·829	−14
·48	·449	−15	·594	−16	·691	−16	·762	−15	·815	−14
0·50	0·434		0·578		0·675		0·747		0·801	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}8$—*continued*

N / Sin ϕ	2		3		4		5		6	
0·50	0·434	−14	0·578	−16	0·675	−16	0·747	−15	0·801	−14
·52	·420	−13	·562	−15	·659	−16	·732	−14	·787	−15
·54	·407	−11	·547	−14	·643	−15	·718	−15	·772	−13
·56	·396	−11	·533	−13	·628	−14	·703	−14	·759	−13
·58	·385	−10	·520	−13	·614	−13	·689	−14	·746	−13
0·60	0·375	− 9	0·507	−13	0·601	−13	0·675	−13	0·733	−13
·62	·366	− 9	·494	−12	·588	−12	·662	−12	·720	−12
·64	·357	− 9	·482	−12	·576	−12	·650	−12	·708	−12
·66	·348	− 8	·470	−10	·564	−12	·638	−12	·696	−11
·68	·340	− 8	·460	−11	·552	−12	·626	−12	·685	−11
0·70	0·332	− 8	0·449	−10	0·540	−11	·614	−11	·674	−11
·72	·324	− 7	·439	− 9	·529	−10	·603	−10	·663	−11
·74	·317	− 8	·430	−10	·519	−10	·593	−10	·652	−10
·76	·309	− 7	·420	− 9	·509	−10	·583	−10	·642	−10
·78	·302	− 6	·411	− 8	·499	− 9	·573	− 9	·632	− 9
0·80	0·296	− 7	0·403	− 8	0·490	− 9	0·562	− 9	0·623	−10
·82	·289	− 6	·395	− 8	·481	− 9	·553	− 9	·613	−10
·84	·283	− 6	·387	− 8	·472	− 9	·544	− 8	·603	− 8
·86	·277	− 6	·379	− 8	·463	− 8	·536	− 8	·595	− 9
·88	·271	− 6	·371	− 7	·455	− 7	·528	− 9	·586	− 8
0·90	0·265	− 5	0·364	− 7	0·448	− 8	0·519	− 8	0·578	− 8
·92	·260	− 6	·357	− 7	·440	− 7	·511	− 8	·570	− 8
·94	·254	− 5	·350	− 6	·433	− 8	·503	− 7	·562	− 8
·96	·249	− 5	·344	− 6	·425	− 7	·496	− 8	·554	− 8
·98	·244	− 5	·338	− 6	·418	− 7	·488	− 7	·546	− 7
1·00	0·239		0·332		0·411		0·481		0·539	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}9$

N / Sin ϕ	2		3		4		5		6	
0·0000	1·000		1·000		1·000		1·000		1·000	
·0025	1·000		1·000		·000		·000		·000	
·0050	1·000	− 1	1·000		·000		·000		·000	
·0075	0·999	0	1·000	− 1	·000		·000		·000	
·0100	0·999	0	0·999	0	·000		·000		·000	
0·0125	0·999	− 1	0·999	0	1·000		1·000		1·000	
·0150	·998	− 1	·999	0	1·000	− 1	1·000	− 1	·000	
·0175	·997	− 1	·999	0	0·999	0	0·999	0	·000	
·0200	·996	− 3	·999	− 1	·999	0	·999	0	·000	
·0225	·993	− 3	·998	0	·999	0	·999	0	·000	
0·0250	0·990	− 4	0·998	− 1	0·999	0	0·999	0	1·000	
·0275	·986	− 5	·997	− 1	·999	0	·999	0	1·000	
·0300	·981	− 5	·996	− 2	·999	− 1	·999	0	·000	
·0325	·976	− 6	·994	− 1	·998	0	·999	0	·000	− 1
·0350	·970	− 7	·993	− 2	·998	− 1	·999	0	0·999	0
0·0375	0·963	− 7	0·991	− 3	0·997	− 1	0·999	0	0·999	0
·0400	·956	− 7	·988	− 3	·996	− 1	·999	− 1	·999	0
·0425	·949	− 7	·985	− 3	·995	− 1	·998	0	·999	− 1
·0450	·942	− 8	·982	− 4	·994	− 2	·998	− 1	·998	0
·0475	·934	− 8	·978	3	·992	− 1	·997	0	·998	0
0·0500	0·926	−17	0·975	− 9	0·991	− 4	0·997	− 2	0·998	0
·055	·909	−17	·966	− 9	·987	− 5	·995	− 2	·998	− 1
·060	·892	−16	·957	−10	·982	− 5	·993	− 3	·997	− 2
·065	·876	−16	·947	−10	·977	− 6	·990	− 4	·995	− 2
·070	·860	−15	·937	−11	·971	− 7	·986	− 4	·993	− 2
0·075	0·845	−15	0·926	−11	0·964	− 7	0·982	− 4	0·991	− 2
·080	·830	−15	·915	−10	·957	− 7	·978	− 5	·989	− 4
·085	·815	−15	·905	−11	·950	− 7	·973	− 5	·985	− 3
·090	·800	−14	·894	−11	·943	− 8	·968	− 6	·982	− 4
·095	·786	−14	·883	−11	·935	− 9	·962	− 5	·978	− 3
0·100	0·772	−13	0·872	−11	0·926	− 8	0·957	− 6	0·975	− 5
·105	·759	−13	·861	−11	·918	− 8	·951	− 5	·970	− 4
·110	·746	−13	·850	−10	·910	− 8	·946	− 6	·966	− 5
·115	·733	−12	·840	−10	·902	− 8	·940	− 6	·961	− 5
·120	·721	−11	·830	−10	·894	− 9	·934	− 6	·956	− 5
0·125	0·710	−11	0·820	−10	0·885	− 8	0·928	− 6	0·951	− 5
·130	·699	−11	·810	− 9	·877	− 8	·922	− 7	·946	− 5
·135	·688	−10	·801	− 9	·869	− 8	·915	− 7	·941	− 5
·140	·678	− 9	·792	− 9	·861	− 8	·908	− 7	·936	− 5
·145	·669	−10	·783	− 9	·853	− 8	·901	− 7	·931	− 5
0·150	0·659	−18	0·774	−17	0·845	−15	0·894	−13	0·926	−11
·16	·641	−18	·757	−17	·830	−15	·881	−13	·915	−11
·17	·623	−18	·740	−16	·815	−15	·868	−13	·904	−11
·18	·605	−16	·724	−15	·800	−14	·855	−13	·893	−11
·19	·589	−15	·709	−15	·786	−14	·842	−13	·882	−11
0·20	0·574		0·693		0·772		0·829		0·871	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}9$—*continued*

N / Sin ϕ	2		3		4		5		6	
0·20	0·574	-16	0·693	-15	0·772	-13	0·829	-12	0·871	-11
·21	·558	-15	·678	-14	·759	-13	·817	-12	·860	-11
·22	·543	-14	·664	-13	·746	-12	·805	-12	·849	-10
·23	·529	-14	·651	-12	·734	-13	·793	-11	·839	-11
·24	·515	-13	·639	-12	·721	-12	·782	-11	·828	-10
0·25	0·502	-12	0·627	-12	0·709	-12	0·771	-11	0·818	-11
·26	·490	-13	·615	-12	·697	-11	·760	-12	·807	-10
·27	·477	-12	·603	-12	·686	-12	·748	-11	·797	-11
·28	·465	-11	·591	-12	·674	-11	·737	-10	·786	-10
·29	·454	-11	·579	-11	·663	-11	·727	-11	·776	-10
0·30	0·443	-21	0·568	-21	0·652	-22	0·716	-21	0·766	-20
·32	·422	-19	·547	-20	·630	-20	·695	-20	·746	-20
·34	·403	-18	·527	-19	·610	-20	·675	-19	·726	-18
·36	·385	-17	·508	-19	·590	-18	·656	-19	·708	-18
·38	·368	-17	·489	-18	·572	-18	·637	-18	·690	-17
0·40	0·351	-15	0·471	-17	0·554	-17	0·619	-18	0·673	-18
·42	·336	-14	·454	-16	·537	-17	·601	-16	·655	-17
·44	·322	-13	·438	-15	·520	-15	·585	-16	·638	-16
·46	·309	-11	·423	-13	·505	-15	·569	-16	·622	-15
·48	·298	- 9	·410	-13	·490	-14	·553	-14	·607	-14
0·50	0·289	- 9	0·397	-12	0·476	-14	0·539	-14	0·593	-14
·52	·280	- 9	·385	-11	·462	-12	·525	-13	·579	-14
·54	·271	- 8	·374	-11	·450	-12	·512	-13	·565	-13
·56	·263	- 7	·363	-10	·438	-12	·499	-12	·552	-12
·58	·256	- 6	·353	-10	·426	-10	·487	-11	·540	-12
0·60	0·250	- 7	0·343	- 9	0·416	-11	0·476	-11	0·528	-12
·62	·243	- 6	·334	- 9	·405	-10	·465	-11	·516	-11
·64	·237	- 6	·325	- 9	·395	- 9	·454	-10	·505	-11
·66	·231	- 6	·316	- 7	·386	- 9	·444	-10	·494	-10
·68	·225	- 6	·309	- 8	·377	- 9	·434	- 9	·484	- 9
0·70	0·219	- 6	0·301	- 7	0·368	- 9	0·425	- 9	0·475	- 9
·72	·213	- 5	·294	- 7	·359	- 8	·416	- 9	·466	- 9
·74	·208	- 5	·287	- 7	·351	- 8	·407	- 9	·457	- 9
·76	·203	- 5	·280	- 7	·343	- 7	·398	- 8	·448	- 8
·78	·198	- 5	·273	- 6	·336	- 7	·390	- 7	·440	- 8
0·80	0·193	- 4	0·267	- 6	0·329	- 6	0·383	- 8	0·432	- 8
·82	·189	- 5	·261	- 6	·323	- 7	·375	- 7	·424	- 8
·84	·184	- 4	·255	- 6	·316	- 6	·368	- 6	·416	- 7
·86	·180	- 5	·249	- 5	·310	- 6	·362	- 6	·409	- 6
·88	·175	- 4	·244	- 5	·304	- 6	·356	- 6	·403	- 6
0·90	0·171	- 4	0·239	- 5	0·298	- 5	0·350	- 6	0·397	- 6
·92	·167	- 3	·234	- 4	·293	- 6	·344	- 6	·391	- 6
·94	·164	- 4	·230	- 5	·287	- 5	·338	- 5	·385	- 6
·96	·160	- 3	·225	- 4	·282	- 5	·333	- 6	·379	- 6
·98	·157	- 3	·221	- 4	·277	- 5	·327	- 5	·373	- 6
1·00	0·154		0·217		0·272		0·322		0·367	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}95$

N / Sin ϕ	2		3		4		5		6	
0·000	1·000		1·000		1·000		1·000		1·000	
·001	·000		·000		·000		·000		·000	
·002	·000		·000		·000		·000		·000	
·003	·000		·000		·000		·000		·000	
·004	·000		·000		·000		·000		·000	
0·005	1·000		1·000		1·000		1·000		1·000	
·006	1·000	−1	·000		·000		·000		·000	
·007	0·999	−1	·000		·000		·000		·000	
·008	·998	−1	·000		·000		·000		·000	
·009	·997	−1	·000	−1	·000	−1	·000		·000	
0·010	0·996	−2	0·999	0	0·999	0	1·000		1·000	
·011	·994	−3	·999	−1	·999	0	·000		·000	
·012	·991	−4	·998	0	·999	0	·000		·000	
·013	·987	−3	·998	−1	·999	0	·000		·000	
·014	·984	−4	·997	−1	·999	0	·000		·000	
0·015	0·980	−5	0·996	−2	0·999	−1	1·000		1·000	
·016	·975	−6	·994	−1	·998	0	1·000	−1	·000	
·017	·969	−5	·993	−2	·998	−1	0·999	0	·000	
·018	·964	−6	·991	−2	·997	−1	·999	−1	·000	
·019	·958	−6	·989	−3	·996	−1	·998	0	·000	
0·020	0·952	−13	0·986	−5	0·995	−2	0·998	−1	1·000	−1
·022	·939	−13	·981	−6	·993	−2	·997	−1	0·999	0
·024	·926	−13	·975	−8	·991	−4	·996	−1	·999	−1
·026	·913	−13	·967	−7	·987	−4	·995	−2	·998	−1
·028	·900	−14	·960	−8	·983	−4	·993	−2	·997	−1
0·030	0·886	−13	0·952	−8	0·979	−4	0·991	−3	0·996	−2
·032	·873	−13	·944	−9	·975	−5	·988	−3	·994	−2
·034	·860	−13	·935	−9	·970	−6	·985	−3	·992	−1
·036	·847	−13	·926	−9	·964	−6	·982	−4	·991	−3
·038	·834	−12	·917	−9	·958	−6	·978	−4	·988	−2
0·040	0·822	−11	0·908	−9	0·952	−7	0·974	−4	0·986	−3
·042	·811	−12	·899	−9	·945	−6	·970	−4	·983	−2
·044	·799	−11	·890	−9	·939	−7	·966	−4	·981	−3
·046	·788	−11	·881	−8	·932	−6	·962	−5	·978	−4
·048	·777	−10	·873	−9	·926	−6	·957	−5	·974	−3
0·050	0·767	−10	0·864	−8	0·920	−7	0·952	−5	0·971	−4
·052	·757	−10	·856	−9	·913	−7	·947	−5	·967	−4
·054	·747	−9	·847	−8	·906	−7	·942	−5	·963	−4
·056	·738	−9	·839	−8	·899	−7	·937	−6	·959	−4
·058	·729	−9	·831	−8	·892	−6	·931	−5	·955	−4
0·060	0·720	−9	0·823	−8	0·886	−7	0·926	−5	0·951	−4
·062	·711	−8	·815	−8	·879	−6	·921	−6	·947	−4
·064	·703	−8	·807	−8	·873	−7	·915	−5	·943	−4
·066	·695	−8	·799	−7	·866	−6	·910	−6	·939	−4
·068	·687	−8	·792	−7	·860	−6	·904	−5	·935	−4
0·070	0·679		0·785		0·854		0·899		0·931	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0\cdot95$—*continued*

Sin ϕ \ N	2		3		4		5		6	
0·070	0·679	−18	0·785	−17	0·854	−16	0·899	−13	0·931	−11
·075	·661	−17	·768	−17	·838	−15	·886	−13	·920	−13
·080	·644	−16	·751	−15	·823	−15	·873	−13	·907	−11
·085	·628	−15	·736	−15	·808	−14	·860	−13	·896	−11
·090	·613	−15	·721	−14	·794	−13	·847	−12	·885	−11
0·095	0·598	−13	0·707	−14	0·781	−13	0·835	−12	0·874	−10
·100	·585	−13	·693	−13	·768	−13	·823	−12	·864	−11
·105	·572	−12	·680	−12	·755	−12	·811	−11	·853	−10
·110	·560	−11	·668	−12	·743	−11	·800	−11	·843	−10
·115	·549	−11	·656	−11	·732	−11	·789	−11	·833	−10
0·120	0·538	−10	0·645	−11	0·721	−11	0·778	−10	0·823	−10
·125	·528	−10	·634	−10	·710	−10	·768	−10	·813	−10
·130	·518	−10	·624	−10	·700	−10	·758	−10	·803	− 9
·135	·508	− 9	·614	−10	·690	−10	·748	− 9	·794	− 9
·140	·499	− 9	·604	− 9	·680	− 9	·739	− 9	·785	− 9
0·145	0·490	− 9	0·595	− 9	0·671	− 9	0·730	− 9	0·776	− 9
·150	·481	−16	·586	−16	·662	−18	·721	−18	·767	−17
·16	·465	−15	·570	−16	·644	−16	·703	−16	·750	−16
·17	·450	−14	·554	−15	·628	−15	·687	−15	·734	−15
·18	·436	−13	·539	−14	·613	−15	·672	−15	·719	−15
·19	·423	−13	·525	−13	·598	−14	·657	−14	·704	−14
0·20	0·410	−12	0·512	−13	0·584	−13	0·643	−14	0·690	−13
·21	·398	−11	·499	−12	·571	−12	·629	−13	·677	−13
·22	·387	−12	·487	−11	·559	−12	·616	−12	·664	−12
·23	·375	−10	·476	−11	·547	−12	·604	−12	·652	−12
·24	·365	−10	·465	−11	·535	−11	·592	−11	·640	−11
0·25	0·355	−10	0·454	−10	0·524	−10	0·581	−11	0·629	−11
·26	·345	− 9	·444	−10	·514	−10	·570	−10	·618	−11
·27	·336	− 9	·434	−10	·504	−10	·560	−10	·607	−11
·28	·327	− 9	·424	− 9	·494	−10	·550	−10	·596	−10
·29	·318	− 8	·415	− 9	·484	− 9	·540	−10	·586	− 9
0·30	0·310	− 8	0·406	− 8	0·475	− 9	0·530	−10	0·577	−10
·31	·302	− 8	·398	− 9	·466	− 9	·520	− 9	·567	− 9
·32	·294	−14	·389	−16	·457	−17	·511	−17	·558	−18
·34	·280	−14	·373	−15	·440	−16	·494	−17	·540	−17
·36	·266	−13	·358	−14	·424	−15	·477	−15	·523	−17
·38	·253	−11	·344	−13	·409	−14	·462	−16	·506	−15
0·40	0·242	−11	0·331	−13	0·395	−14	0·446	−13	0·491	−15
·42	·231	− 9	·318	−11	·381	−13	·433	−14	·476	−14
·44	·222	− 9	·307	−11	·368	−12	·419	−12	·462	−13
·46	·213	− 8	·296	−10	·356	−11	·407	−13	·449	−12
·48	·205	− 7	·286	−10	·345	−11	·394	−11	·437	−12
0·50	0·198		0·276		0·334		0·383		0·425	

TABLE 1a—*continued*

Values of $\varkappa : r_c = 0 \cdot 95$—*continued*

Sin ϕ \ N	2		3		4		5		6	
0·50	0·198	− 6	0·276	− 9	0·334	− 9	0·383	−11	0·425	−12
·52	·192	− 6	·267	− 8	·325	−10	·372	−10	·413	−11
·54	·186	− 5	·259	− 7	·315	− 9	·362	−10	·402	−10
·56	·181	− 5	·252	− 7	·306	− 8	·352	− 9	·392	−10
·58	·176	− 5	·245	− 7	·298	− 8	·343	− 8	·382	− 8
0·60	0·171	− 5	0·238	− 7	0·290	− 8	0·335	− 8	0·374	− 9
·62	·166	− 4	·231	− 6	·282	− 7	·327	− 8	·365	− 8
·64	·162	− 4	·225	− 6	·275	− 7	·319	− 8	·357	− 8
·66	·158	− 4	·219	− 6	·268	− 6	·311	− 7	·349	− 8
·68	·154	− 4	·213	− 5	·262	− 7	·304	− 7	·341	− 7
0·70	0·150	− 4	0·208	− 6	0·255	− 6	0·297	− 7	0·334	− 7
·72	·146	− 4	·202	− 5	·249	− 6	·290	− 6	·327	− 7
·74	·142	− 4	·197	− 5	·243	− 5	·284	− 6	·320	− 6
·76	·138	− 3	·192	− 4	·238	− 5	·278	− 6	·314	− 6
·78	·135	− 4	·188	− 5	·233	− 5	·272	− 5	·308	− 6
0·80	0·131	− 3	0·183	− 4	0·228	− 5	0·267	− 5	0·302	− 5
·82	·128	− 3	·179	− 4	·223	− 5	·262	− 5	·297	− 6
·84	·125	− 3	·175	− 4	·218	− 5	·257	− 5	·291	− 5
·86	·122	− 3	·171	− 4	·213	− 4	·252	− 5	·286	− 5
·88	·119	− 3	·167	− 3	·209	− 4	·247	− 5	·281	− 5
0·90	0·116	− 2	0·164	− 4	0·205	− 4	0·242	− 4	0·276	− 5
·92	·114	− 3	·160	− 3	·201	− 4	·238	− 4	·271	− 4
·94	·111	− 2	·157	− 3	·197	− 3	·234	− 5	·267	− 4
·96	·109	− 3	·154	− 3	·194	− 4	·229	− 3	·263	− 5
·98	·106	− 2	·151	− 3	·190	− 3	·226	− 4	·258	− 4
1·00	0·104		0·148		0·187		0·222		0·254	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}975$

N / Sin ϕ	2		3		4		5		6	
0·0000	1·000		1·000		1·000		1·000		1·000	
·0005	·000		·000		·000		·000		·000	
·0010	·000		·000		·000		·000		·000	
·0015	·000		·000		·000		·000		·000	
·0020	·000		·000		·000		·000		·000	
0·0025	1·000	−1	1·000		1·000		1·000		1·000	
·0030	0·999	0	1·000		·000		·000		·000	
·0035	·999	0	1·000		·000		·000		·000	
·0040	·999	−2	1·000	−1	·000		·000		·000	
·0045	·997	−2	0·999	0	·000		·000		·000	
0·0050	0·995	−2	0·999	0	1·000		1·000		1·000	
·0055	·993	−3	·999	0	1·000		·000		·000	
·0060	·990	−3	·999	−1	1·000	−1	·000		·000	
·0065	·987	−4	·998	−1	0·999	0	·000		·000	
·0070	·983	−5	·997	−2	0·999	0	·000		·000	
0·0075	0·978	−5	0·995	−1	0·999	−1	1·000		1·000	
·0080	·973	−5	·994	−2	·998	0	·000		·000	
·0085	·968	−6	·992	−2	·998	−1	·000		·000	
·0090	·962	−6	·990	−3	·997	−1	·000		·000	
·0095	·956	−6	·987	−2	·996	−1	·000	−1	·000	
0·0100	0·950	−27	0·985	−12	0·995	−5	0·999	−2	1·000	−1
·012	·923	−27	·973	−15	·990	−7	·997	−4	0·999	−2
·014	·896	−27	·958	−17	·983	−10	·993	−5	·997	−3
·016	·869	−25	·941	−17	·973	−11	·988	−7	·994	−4
·018	·844	−25	·924	−19	·962	−12	·981	−8	·990	−5
0·020	0·819	−23	0·905	−18	0·950	−13	0·973	−9	0·985	−5
·022	·796	−22	·887	−18	·937	−14	·964	−9	·980	−7
·024	·774	−20	·869	−17	·923	−13	·955	−10	·973	−7
·026	·754	−19	·852	−17	·910	−14	·945	−11	·966	−8
·028	·735	−18	·835	−16	·896	−14	·934	−11	·958	−9
0·030	0·717	−17	0·819	−15	0·882	−13	0·923	−11	0·949	−9
·032	·700	−16	·804	−15	·869	−13	·912	−10	·940	−9
·034	·684	−14	·789	−14	·856	−12	·902	−11	·931	−9
·036	·670	−14	·775	−14	·844	−13	·891	−11	·922	−8
·038	·656	−13	·761	−13	·831	−12	·880	−11	·914	−9
0·040	0·643	−13	0·748	−12	0·819	−12	0·869	−11	0·905	−10
·042	·630	−12	·736	−12	·807	−12	·858	−10	·895	−9
·044	·618	−11	·724	−11	·795	−11	·848	−10	·886	−9
·046	·607	−10	·713	−12	·784	−10	·838	−10	·877	−8
·048	·597	−11	·701	−11	·774	−10	·828	−9	·869	−9
0·050	0·586	−10	0·690	−10	0·764	−10	0·819	−10	0·860	−9
·052	·576	−9	·680	−9	·754	−9	·809	−9	·851	−9
·054	·567	−9	·671	−10	·745	−10	·800	−8	·842	−8
·056	·558	−9	·661	−9	·735	−9	·792	−9	·834	−8
0·058	0·549		0·652		0·726		0·783		0·826	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0{\cdot}975$—*continued*

Sin ϕ \ N	2		3		4		5		6	
0·058	0·549	− 8	0·652	− 9	0·726	− 9	0·783	− 9	0·826	− 8
·060	·541	−19	·643	−20	·717	−20	·774	−20	·818	−19
·065	·522	−17	·623	−19	·697	−20	·754	−19	·799	−18
·070	·505	−16	·604	−17	·677	−17	·735	−18	·781	−17
0·075	0·489	−14	0·587	−16	0·660	−17	0·717	−17	0·764	−17
·080	·475	−14	·571	−14	·643	−15	·700	−16	·747	−15
·085	·461	−12	·557	−14	·628	−15	·684	−14	·732	−15
·090	·449	−12	·543	−13	·613	−13	·670	−14	·717	−14
·095	·437	−11	·530	−12	·600	−13	·656	−13	·703	−14
0·100	0·426	−10	0·518	−11	0·587	−12	0·643	−12	0·689	−12
·105	·416	− 9	·507	−11	·575	−11	·631	−12	·677	−12
·110	·407	− 9	·496	−10	·564	−11	·619	−11	·665	−12
·115	·398	− 9	·486	−10	·553	−11	·608	−12	·653	−11
·120	·389	− 8	·476	− 9	·542	−10	·596	−10	·642	−10
0·125	0·381	− 8	0·467	− 8	0·532	− 9	0·586	−10	0·632	−10
·130	·373	− 7	·459	− 8	·523	− 9	·576	− 9	·622	−10
·135	·366	− 7	·451	− 8	·514	− 9	·567	− 8	·612	− 9
·140	·359	− 6	·443	− 8	·505	− 8	·559	− 9	·603	− 9
·145	·353	− 7	·435	− 7	·497	− 7	·550	− 8	·594	− 9
0·150	0·346	− 6	0·428	− 7	0·490	− 8	0·542	− 8	0·585	− 8
·155	·340	− 6	·421	− 7	·482	− 7	·534	− 8	·577	− 8
·160	·334	− 6	·414	− 6	·475	− 7	·526	− 8	·569	− 7
·165	·328	− 6	·408	− 7	·468	− 6	·518	− 7	·562	− 8
·170	·322	− 5	·401	− 6	·462	− 7	·511	− 7	·554	− 7
0·175	0·317	− 6	0·395	− 5	0·455	− 6	0·504	− 6	0·547	− 7
·180	·311	− 5	·390	− 6	·449	− 6	·498	− 7	·540	− 6
·185	·306	− 5	·384	− 5	·443	− 6	·491	− 6	·534	− 7
·190	·301	− 5	·379	− 6	·437	− 6	·485	− 6	·527	− 6
·195	·296	− 4	·373	− 5	·431	− 5	·479	− 6	·521	− 6
0·200	0·292	− 9	0·368	− 9	0·426	−11	0·473	−10	0·515	−12
·21	·283	− 8	·359	− 9	·415	−10	·463	−11	·503	−11
·22	·275	− 9	·350	− 9	·405	− 9	·452	−10	·492	−10
·23	·266	− 8	·341	− 8	·396	− 9	·442	− 9	·482	−10
·24	·258	− 7	·333	− 9	·387	− 9	·433	− 9	·472	−10
0·25	0·251	− 7	0·324	− 7	0·378	− 8	0·424	− 9	0·462	− 9
·26	·244	− 7	·317	− 8	·370	− 8	·415	− 9	·453	− 9
·27	·237	− 7	·309	− 7	·362	− 8	·406	− 8	·444	− 8
·28	·230	− 6	·302	− 7	·354	− 7	·398	− 8	·436	− 9
·29	·224	− 6	·295	− 7	·347	− 7	·390	− 7	·427	− 8
0·30	0·2174	−58	0·288	− 6	0·340	− 7	0·383	− 8	0·419	− 8
·31	·2116	−57	·282	− 6	·333	− 7	·375	− 7	·411	− 7
·32	·2059	−54	·276	− 6	·326	− 6	·368	− 7	·404	− 7
·33	·2005	−53	·270	− 6	·320	− 7	·361	− 7	·397	− 7
·34	·1952	−50	·264	− 6	·313	− 6	·354	− 6	·390	− 7
0·35	0·1902		0·258		0·307		0·348		0·383	

TABLE 1a—*continued*

Values of $\varkappa$: $r_c = 0\cdot975$—*continued*

Sin ϕ \ N	2		3		4		5		6	
0·35	0·1902	−47	0·258	− 5	0·307	− 6	0·348	− 7	0·383	− 7
·36	·1855	−46	·253	− 5	·301	− 6	·341	− 5	·376	− 6
·37	·1809	−43	·248	− 5	·295	− 5	·336	− 6	·370	− 6
·38	·1766	−41	·243	− 5	·290	− 6	·330	− 5	·364	− 6
·39	·1725	−39	·238	− 5	·284	− 5	·325	− 6	·358	− 6
0·40	0·1686	−38	0·233	− 5	0·279	− 5	0·319	− 6	0·352	− 6
·41	·1648	−36	·228	− 4	·274	− 5	·313	− 5	·346	− 6
·42	·1612	−35	·224	− 4	·269	− 5	·308	− 5	·340	− 5
·43	·1577	−33	·220	− 4	·264	− 4	·303	− 5	·335	− 6
·44	·1544	−32	·216	− 5	·260	− 5	·298	− 5	·329	− 5
0·45	0·1512	−30	0·211	− 3	0·255	− 4	0·293	− 5	0·324	− 5
·46	·1482	−28	·208	− 4	·251	− 5	·288	− 4	·319	− 4
·47	·1454	−27	·204	− 4	·246	− 4	·284	− 5	·315	− 5
·48	·1427	−25	·200	− 3	·242	− 4	·279	− 4	·310	− 4
·49	·1402	−23	·197	− 4	·238	− 3	·275	− 4	·306	− 5
0·50	0·1379	−22	0·1935	−30	0·235	− 3	0·271	− 4	0·301	− 4
·51	·1357	−21	·1905	−29	·231	− 3	·267	− 3	297	− 4
·52	·1336	−20	·1876	−29	·228	− 4	·264	− 4	·293	− 4
·53	·1316	−19	·1847	−29	·224	− 3	·260	− 4	·289	− 4
·54	·1297	−18	·1818	−28	·221	− 3	·256	− 3	·285	− 4
0·55	0·1279	−18	0·1790	−28	0·218	− 3	0·253	− 4	0·281	− 3
·56	·1261	−17	·1762	−26	·215	− 3	·249	− 3	·278	− 4
·57	·1244	−17	·1736	−26	·212	− 3	·246	− 3	·274	− 3
·58	·1227	−16	·1710	−24	·209	− 3	·2429	−33	·271	− 4
·59	·1211	−16	·1686	−23	·206	− 2	·2396	−31	·267	− 3
0·60	0·1195	−31	0·1663	−46	0·2036	−54	0·2365	−60	0·264	−62
·62	·1164	−31	·1617	−44	·1982	−51	·2305	−56	·2583	−60
·64	·1133	−30	·1573	−42	·1931	−49	·2249	−54	·2523	−59
·66	·1103	−29	·1531	−40	·1882	−47	·2195	−51	·2464	−55
·68	·1074	−28	·1491	−38	·1835	−45	·2144	−48	·2409	−52
0·70	0·1046	−27	0·1453	−37	0·1790	−42	0·2096	−47	0·2357	−51
·72	·1019	−27	·1416	−36	·1748	−41	·2049	−45	·2306	−48
·74	·0992	−27	·1380	−35	·1707	−39	·2004	−43	·2258	−46
·76	·0965	−26	·1345	−33	·1668	−37	·1961	−42	·2212	−43
·78	·0939	−25	·1312	−32	·1631	−35	·1919	−40	·2169	−43
0·80	0·0914	−24	0·1280	−30	0·1596	−35	0·1879	−38	0·2126	−41
·82	·0890	−23	·1250	−29	·1561	−33	·1841	−37	·2085	−39
·84	·0867	−22	·1221	−27	·1528	−33	·1804	−35	·2046	−38
·86	·0845	−21	·1194	−27	·1495	−31	·1769	−34	·2008	−36
·88	·0824	−20	·1167	−26	·1464	−30	·1735	−33	·1972	−35
0·90	0·0804	−18	0·1141	−24	0·1434	−29	0·1702	−32	0·1937	−34
·92	·0786	−17	·1117	−23	·1405	−27	·1670	−32	·1903	−32
·94	·0769	−16	·1094	−22	·1378	−26	·1638	−30	·1871	−32
·96	·0753	−15	·1072	−22	·1352	−26	·1608	−30	·1839	−32
·98	·0738	−13	·1050	−21	·1326	−25	·1578	−30	·1807	−30
1·00	0·0725		0·1029		0·1301		0·1548		0·1777	

TABLE 1b

Values of $b°$: $N = 2$

J \ r_c	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	14·95	−76	18·27	−79	21·38	−82	31·08	−99	42·57	−117	53·32	−130	70·0	−14	104·9	−15	152·1	−20	218·3	−24
·1	14·19	−61	17·48	−66	20·56	−69	30·09	−76	41·40	−86	52·02	−90	68·6	−10	103·4	−10	150·1	−13	215·9	−18
·2	13·58	−49	16·82	−55	19·87	−58	29·33	−60	40·54	−68	51·12	−76	67·6	−8	102·4	−6	148·8	−9	214·1	−13
·3	13·09	−40	16·27	−47	19·29	−50	28·73	−50	39·86	−56	50·36	−66	66·9	−7	101·8	−7	147·9	−6	212·8	−11
·4	12·69	−34	15·80	−40	18·79	−43	28·23	−43	39·30	−47	49·70	−59	66·2	−7	101·1	−9	147·3	−6	211·7	−7
1·5	12·35	−29	15·40	−34	18·36	−37	27·80	−40	38·83	−41	49·11	−49	65·5	−6	100·2	−7	146·7	−9	211·0	−10
·6	12·06	−25	15·06	−30	17·99	−33	27·40	−36	38·41	−35	48·62	−41	64·9	−5	99·5	−9	145·8	−11	210·0	−17
·7	11·81	−21	14·76	−26	17·66	−30	27·04	−33	38·06	−33	48·21	−40	64·4	−5	98·6	−8	144·7	−10	208·3	−17
·8	11·60	−19	14·50	−23	17·36	−27	26·71	−30	37·73	−33	47·81	−38	63·9	−5	97·8	−7	143·7	−10	206·6	−16
·9	11·41	−17	14·27	−21	17·09	−25	26·41	−28	37·40	−28	47·43	−27	63·4	−4	97·1	−7	142·7	−11	205·0	−14
2·0	11·24	−15	14·06	−19	16·84	−22	26·13	−26	37·12	−25	47·16	−23	63·0	−4	96·4	−7	141·6	−11	203·6	−14
·1	11·09	−13	13·87	−17	16·62	−20	25·87	−23	36·87	−21	46·93	−20	62·6	−3	95·7	−7	140·5	−9	202·2	−14
·2	10·96	−11	13·70	−15	16·42	−18	25·64	−20	36·66	−19	46·73	−19	62·3	−3	95·0	−6	139·6	−7	200·8	−12
·3	10·85	−10	13·55	−13	16·24	−17	25·44	−19	36·47	−17	46·54	−18	62·0	−2	94·4	−4	138·9	−6	199·6	−9
·4	10·75	−9	13·42	−12	16·07	−15	25·25	−18	36·30	−17	46·36	−17	61·8	−2	94·0	−2	138·3	−5	198·7	−8
2·5	10·66	−8	13·30	−11	15·92	−14	25·07	−16	36·13	−19	46·19	−17	61·6	−2	93·8	−2	137·8	−4	197·9	−7
·6	10·58	−7	13·19	−10	15·78	−13	24·91	−15	35·94	−18	46·02	−17	61·4	−2	93·6	−2	137·4	−4	197·2	−5
·7	10·51	−7	13·09	−9	15·65	−12	24·76	−13	35·76	−15	45·85	−16	61·2	−2	93·4	−2	137·0	−3	196·7	−5
·8	10·44	−6	13·00	−8	15·53	−11	24·63	−12	35·61	−13	45·69	−15	61·0	−1	93·2	−1	136·7	−3	196·2	−3
·9	10·38	−6	12·92	−7	15·42	−9	24·51	−11	35·48	−12	45·54	−12	60·9	−1	93·1	−2	136·4	−1	195·9	−3
3·0	10·32	−5	12·85	−6	15·33	−8	24·40	−10	35·36	−10	45·42	−10	60·8	−1	92·9	−1	136·3	−1	195·6	−2
·1	10·27	−5	12·79	−6	15·25	−8	24·30	−10	35·26	−9	45·32	−7	60·7	−1	92·8	−1	136·2	−1	195·4	−1
·2	10·22	−4	12·73	−5	15·17	−7	24·20	−9	35·17	−7	45·25	−7	60·6	−1	92·7	−1	136·1	−1	195·3	−1
·3	10·18	−4	12·68	−5	15·10	−6	24·11	−8	35·10	−7	45·18	−7	60·5	0	92·6	0	136·0	0	195·2	0
·4	10·14	−4	12·63	−5	15·04	−5	24·03	−7	35·03	−8	45·11	−6	60·5	−1	92·6	0	136·0	0	195·2	0
3·5	10·10	−4	12·58	−4	14·99	−5	23·96	−7	34·95	−6	45·05	−6	60·4	0	92·6	0	136·0	0	195·2	1
·6	10·06	−3	12·54	−5	14·94	−5	23·89	−6	34·89	−6	44·99	−6	60·4	−1	92·6	0	136·0	0	195·3	1
·7	10·03	−3	12·49	−4	14·89	−5	23·83	−5	34·83	−5	44·93	−6	60·3	0	92·6	0	136·0	1	195·4	0
·8	10·00	−3	12·45	−3	14·84	−4	23·78	−5	34·78	−5	44·87	−6	60·3	−1	92·6	0	136·1	0	195·4	1
·9	9·97	−3	12·42	−4	14·80	−4	23·73	−5	34·73	−4	44·81	−4	60·2	0	92·6	0	136·1	0	195·5	1
4·0	9·94	−3	12·38	−3	14·76	−3	23·68	−4	34·69	−3	44·77	−4	60·2	0	92·6	0	136·1	1	195·6	1
·1	9·91	−2	12·35	−2	14·73	−3	23·64	−4	34·66	−3	44·73	−3	60·2	0	92·6	0	136·2	0	195·7	1
·2	9·89	−2	12·33	−3	14·70	−3	23·60	−4	34·63	−3	44·70	−3	60·2	−1	92·6	0	136·2	1	195·8	1
·3	9·87	−2	12·30	−2	14·67	−3	23·56	−4	34·60	−3	44·67	−2	60·1	0	92·6	0	136·3	1	195·9	2
·4	9·85	−2	12·28	−3	14·64	−3	23·52	−4	34·57	−3	44·65	−2	60·1	0	92·6	0	136·4	1	196·1	1
4·5	9·83		12·25		14·61		23·48		34·54		44·63		60·1		92·6		136·5		196·2	

TABLE 1b—*continued*

Values of b° : $N = 2$—continued

r_0 / J	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
4·5	9·83	−1	12·25	−2	14·61	−3	23·48	−2	34·54	−3	44·63	−3	60·1	−1	92·6	0	136·5	0	196·2	1
·6	9·82	−2	12·23	−2	14·58	−2	23·46	−3	34·51	−3	44·60	−2	60·0	0	92·6	0	136·5	1	196·3	1
·7	9·80	−2	12·21	−2	14·56	−2	23·43	−3	34·48	−3	44·58	−3	60·0	0	92·6	0	136·6	1	196·4	0
·8	9·78	−2	12·19	−2	14·54	−2	23·40	−3	34·45	−3	44·55	−2	60·0	0	92·6	1	136·7	0	196·4	1
·9	9·76	−2	12·17	−2	14·52	−2	23·37	−3	34·42	−3	44·53	−2	60·0	−1	92·7	0	136·7	1	196·5	1
5·0	9·74	−1	12·15	−1	14·50	−1	23·34	−3	34·39	−2	44·51	−3	59·9	0	92·7	0	136·8	0	196·6	1
·1	9·73	−1	12·14	−2	14·49	−1	23·31	−2	34·37	−2	44·48	−2	59·9	0	92·7	0	136·8	1	196·7	0
·2	9·72	−1	12·12	−1	14·48	−2	23·29	−2	34·35	−3	44·46	−2	59·9	0	92·7	0	136·9	0	196·7	1
·3	9·71	−1	12·11	−2	14·46	−1	23·27	−2	34·32	−3	44·44	−2	59·9	0	92·7	0	136·9	0	196·8	0
·4	9·70	−1	12·09	−1	14·45	−1	23·25	−2	34·29	−3	44·42	−2	59·9	0	92·7	0	136·9	1	196·8	1
5·5	9·69	−1	12·08	−1	14·44	−1	23·23	−2	34·26	−2	44·40	−2	59·9	0	92·7	0	137·0	0	196·9	0
·6	9·68	−1	12·07	−1	14·43	−1	23·21	−2	34·24	−1	44·38	−2	59·9	0	92·7	0	137·0	0	196·9	0
·7	9·67	−1	12·06	−2	14·42	−1	23·19	−1	34·23	−1	44·36	−2	59·9	0	92·7	0	137·0	0	196·9	1
·8	9·66	−1	12·04	−1	14·41	−1	23·18	−1	34·22	−1	44·34	−2	59·9	0	92·7	0	137·0	0	197·0	0
·9	9·65	−1	12·03	−1	14·40	−1	23·17	−1	34·21	−1	44·32	−2	59·9	0	92·7	0	137·0	0	197·0	0
6·0	9·64		12·02		14·39		23·16		34·20		44·30		59·9		92·7		137·0		197·0	
6·5	9·59		11·97		14·35		23·09		34·13		44·21		59·9		92·7		137·0		197·2	
7·0	9·55		11·93		14·31		23·03		34·10		44·16		59·9		92·7		137·0		197·2	

TABLE 1b—*continued*

Values of $b°: N = 3$

J \ r_0	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	14·82	−80	17·49	−91	19·74	−97	27·69	−127	36·15	−155	43·83	−170	55·59	−195	80·5	−23	115·2	−29	163·5	−40
·1	14·02	−65	16·58	−76	18·77	−81	26·42	−103	34·60	−122	42·13	−136	53·64	−153	78·2	−18	112·3	−25	159·5	−28
·2	13·37	−54	15·82	−65	17·96	−68	25·39	−84	33·38	−97	40·77	−107	52·11	−118	76·4	−13	109·8	−17	156·7	−24
·3	12·83	−46	15·17	−55	17·28	−57	24·55	−67	32·41	−78	39·70	−88	50·93	−98	75·1	−10	108·1	−13	154·3	−20
·4	12·37	−39	14·62	−47	16·71	−48	23·88	−56	31·63	−65	38·82	−71	49·95	−82	74·1	−9	106·8	−12	152·3	−16
1·5	11·98	−33	14·15	−40	16·23	−42	23·32	−48	30·98	−53	38·11	−59	49·13	−68	73·2	−8	105·6	−9	150·7	−12
·6	11·65	−29	13·75	−34	15·81	−37	22·84	−41	30·45	−46	37·52	−52	48·45	−56	72·4	−8	104·7	−8	149·5	−11
·7	11·36	−25	13·41	−29	15·44	−32	22·43	−35	29·99	−41	37·00	−46	47·89	−46	71·6	−7	103·9	−8	148·4	−10
·8	11·11	−23	13·12	−25	15·12	−28	22·08	−31	29·58	−36	36·54	−38	47·43	−38	70·9	−6	103·1	−7	147·4	−9
·9	10·88	−20	12·87	−22	14·84	−25	21·77	−30	29·22	−31	36·16	−32	47·05	−32	70·3	−5	102·4	−7	146·5	−9
2·0	10·68	−18	12·65	−19	14·59	−22	21·47	−26	28·91	−26	35·84	−26	46·73	−28	69·8	−4	101·7	−7	145·6	−8
·1	10·50	−16	12·46	−17	14·37	−20	21·21	−24	28·65	−22	35·58	−24	46·45	−25	69·4	−3	101·0	−6	144·8	−7
·2	10·34	−14	12·29	−15	14·17	−18	20·97	−22	28·43	−22	35·34	−22	46·20	−23	69·1	−3	100·4	−5	144·1	−6
·3	10·20	−12	12·14	−13	13·99	−16	20·75	−20	28·21	−21	35·12	−20	45·97	−21	68·8	−2	99·9	−4	143·5	−5
·4	10·08	−10	12·01	−12	13·83	−14	20·55	−18	28·00	−19	34·92	−18	45·76	−20	68·6	−2	99·5	−3	143·0	−5
2·5	9·98	−9	11·89	−11	13·69	−12	20·37	−16	27·81	−19	34·74	−14	45·56	−18	68·4	−2	99·2	−2	142·5	−5
·6	9·89	−8	11·78	−10	13·57	−11	20·21	−16	27·64	−15	34·60	−14	45·38	−16	68·2	−2	99·0	−2	142·0	−4
·7	9·81	−7	11·68	−9	13·46	−10	20·05	−14	27·49	−12	34·46	−14	45·22	−14	68·0	−2	98·8	−2	141·6	−3
·8	9·74	−7	11·59	−7	13·36	−9	19·91	−13	27·37	−11	34·32	−12	45·08	−12	67·8	−1	98·6	−2	141·3	−3
·9	9·67	−6	11·52	−7	13·27	−9	19·78	−12	27·26	−11	34·20	−12	44·96	−10	67·7	−1	98·4	−1	141·0	−2
3·0	9·61	−6	11·45	−7	13·18	−8	19·66	−12	27·15	−11	34·08	−9	44·86	−10	67·6	−1	98·3	−1	140·8	−2
·1	9·55	−5	11·38	−6	13·10	−7	19·54	−11	27·04	−10	33·99	−9	44·76	−10	67·5	−1	98·2	−1	140·6	−2
·2	9·50	−5	11·32	−6	13·03	−6	19·43	−10	26·94	−10	33·90	−8	44·66	−10	67·4	−1	98·1	0	140·4	−1
·3	9·45	−4	11·26	−5	12·97	−5	19·33	−8	26·84	−9	33·82	−8	44·56	−10	67·3	−1	98·1	−1	140·3	−1
·4	9·41	−4	11·21	−4	12·92	−5	19·25	−8	26·75	−9	33·74	−7	44·46	−9	67·2	0	98·0	0	140·2	−1
3·5	9·37	−4	11·17	−5	12·87	−5	19·17	−7	26·66	−9	33·67	−7	44·37	−9	67·2	−1	98·0	−1	140·1	0
·6	9·33	−3	11·12	−4	12·82	−4	19·10	−7	26·57	−6	33·60	−7	44·28	−8	67·1	−1	97·9	0	140·1	−1
·7	9·30	−3	11·08	−4	12·78	−4	19·03	−7	26·51	−6	33·53	−6	44·20	−8	67·0	0	97·9	−1	140·0	0
·8	9·27	−3	11·04	−4	12·74	−4	18·96	−6	26·45	−6	33·47	−6	44·12	−7	67·0	0	97·8	−1	140·0	−1
·9	9·24	−3	11·00	−3	12·70	−3	18·90	−5	26·39	−6	33·41	−5	44·05	−5	67·0	0	97·7	−1	139·9	0
4·0	9·21	−2	10·97	−3	12·67	−3	18·85	−5	26·33	−6	33·36	−4	44·00	−4	67·0	−1	97·6	−1	139·9	−1
·1	9·19	−2	10·94	−3	12·64	−3	18·80	−5	26·27	−4	33·32	−4	43·96	−3	66·9	0	97·5	0	139·8	0
·2	9·17	−2	10·91	−2	12·61	−3	18·75	−5	26·23	−4	33·28	−4	43·93	−3	66·9	0	97·5	−1	139·8	−1
·3	9·15	−3	10·89	−2	12·58	−3	18·70	−4	26·19	−4	33·24	−4	43·90	−3	66·9	−1	97·4	−0	139·7	0
·4	9·12	−2	10·87	−2	12·55	−2	18·66	−4	26·15	−4	33·20	−4	43·87	−3	66·8	0	97·4	−1	139·7	0
4·5	9·10		10·85		12·53		18·62		26·11		33·16		43·84		66·8		97·3		139·7	

TABLE 1b—*continued*

Values of b° : $N = 3$—*continued*

J \ r_c	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
4·5	9·10	−2	10·85	−2	12·53	−3	18·62	−4	26·11	−4	33·16	−4	43·84	−3	66·8	0	97·3	0	139·7	−1
·6	9·08	−1	10·83	−3	12·50	−2	18·58	−4	26·07	−4	33·12	−4	43·81	−3	66·8	−1	97·3	0	139·6	0
·7	9·07	−1	10·80	−2	12·48	−2	18·54	−3	26·03	−4	33·08	−4	43·78	−2	66·7	0	97·3	0	139·6	0
·8	9·06	−2	10·78	−2	12·46	−2	18·51	−3	25·99	−4	33·04	−3	43·76	−2	66·7	0	97·3	−1	139·6	−1
·9	9·04	−2	10·76	−2	12·44	−1	18·48	−3	25·95	−3	33·01	−2	43·74	−2	66·7	−1	97·2	0	139·5	0
5·0	9·02	−1	10·74	−1	12·43	−1	18·45	−2	25·92	−3	32·99	−2	43·72	−2	66·6	0	97·2	0	139·5	0
·1	9·01	−1	10·73	−1	12·42	−2	18·43	−3	25·89	−2	32·97	−2	43·70	−2	66·6	0	97·2	0	139·5	0
·2	9·00	−1	10·72	−1	12·40	−1	18·40	−2	25·87	−3	32·95	−2	43·68	−2	66·6	0	97·2	−1	139·5	0
·3	8·99	−1	10·71	−1	12·39	−2	18·38	−2	25·84	−2	32·93	−2	43·66	−2	66·6	−1	97·1	0	139·5	0
·4	8·98	−1	10·70	−1	12·37	−1	18·36	−2	25·82	−2	32·91	−2	43·64	−1	66·5	0	97·1	0	139·5	−1
5·5	8·97	−1	10·69	−1	12·36	−1	18·34	−2	25·80	−2	32·89	−2	43·63	−2	66·5	0	97·1	0	139·4	0
·6	8·96	−1	10·68	−1	12·35	−1	18·32	−2	25·78	−2	32·87	−2	43·61	−1	66·5	0	97·1	−1	139·4	0
·7	8·95	0	10·67	−1	12·34	−2	18·30	−3	25·76	−2	32·85	−2	43·60	−1	66·5	0	97·0	0	139·4	0
·8	8·95	−1	10·66	−1	12·32	−1	18·27	−2	25·74	−2	32·83	−2	43·59	−2	66·5	−1	97·0	0	139·4	0
·9	8·94	−1	10·65	−1	12·31	−1	18·25	−2	25·72	−1	32·81	−2	43·57	−1	66·4	0	97·0	0	139·4	0
6·0	8·93		10·64		12·30		18·23		25·71		32·79		43·56		66·4		97·0		139·4	
6·5	8·89		10·59		12·26		18·16		25·64		32·70		43·52		66·4		97·0		139·4	
7·0	8·86		10·55		12·23		18·10		25·58		32·64		43·49		66·4		97·0		139·3	

TABLE 1b—*continued*

Values of $b°$: $N = 4$

J \ r_c	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	15·39	−77	17·54	−89	19·53	−100	25·87	−138	33·24	−177	39·64	−197	49·20	−213	69·7	−25	98·3	−32	138·6	−45
·1	14·62	−65	16·65	−75	18·53	−82	24·49	−109	31·47	−138	37·67	−153	47·07	−172	67·2	−20	95·1	−24	134·1	−27
·2	13·97	−55	15·90	−63	17·71	−68	23·40	−90	30·09	−112	36·14	−124	45·35	−134	65·2	−16	92·7	−20	131·4	−25
·3	13·42	−47	15·27	−53	17·03	−56	22·50	−75	28·97	−92	34·90	−101	44·01	−110	63·6	−12	90·7	−16	128·9	−20
·4	12·95	−40	14·74	−45	16·47	−47	21·75	−63	28·05	−76	33·89	−81	42·91	−88	62·4	−11	89·1	−13	126·9	−16
1·5	12·55	−34	14·29	−39	16·00	−42	21·12	−52	27·29	−64	33·08	−70	42·03	−74	61·3	−8	87·8	−10	125·3	−14
·6	12·21	−29	13·90	−34	15·58	−38	20·60	−46	26·65	−53	32·38	−60	41·29	−61	60·5	−8	86·8	−10	123·9	−13
·7	11·92	−25	13·56	−30	15·20	−34	20·14	−38	26·12	−46	31·78	−52	40·68	−55	59·7	−7	85·8	−9	122·6	−12
·8	11·67	−22	13·26	−26	14·86	−31	19·76	−34	25·66	−39	31·26	−44	40·13	−51	59·0	−5	84·9	−8	121·4	−11
·9	11·45	−19	13·00	−23	14·55	−28	19·42	−29	25·27	−34	30·82	−36	39·62	−43	58·5	−5	84·1	−6	120·3	−11
2·0	11·26	−17	12·77	−20	14·27	−25	19·13	−26	24·93	−30	30·46	−31	39·19	−37	58·0	−4	83·5	−5	119·2	−8
·1	11·09	−15	12·57	−18	14·02	−23	18·87	−24	24·63	−26	30·15	−27	38·82	−33	57·6	−4	83·0	−5	118·4	−7
·2	10·94	−13	12·39	−16	13·79	−20	18·63	−21	24·37	−24	29·88	−25	38·49	−28	57·2	−4	82·5	−4	117·7	−6
·3	10·81	−12	12·23	−14	13·59	−18	18·42	−20	24·13	−21	29·63	−24	38·21	−24	56·8	−3	82·1	−4	117·1	−5
·4	10·69	−10	12·09	−13	13·41	−16	18·22	−18	23·92	−19	29·39	−22	37·97	−21	56·5	−3	81·7	−4	116·6	−5
2·5	10·59	−10	11·96	−12	13·25	−15	18·04	−17	23·73	−17	29·17	−17	37·76	−19	56·2	−3	81·3	−4	116·1	−4
·6	10·49	−8	11·84	−10	13·10	−13	17·87	−15	23·56	−16	29·00	−15	37·57	−18	55·9	−2	80·9	−4	115·7	−4
·7	10·41	−8	11·74	−9	12·97	−12	17·72	−14	23·40	−14	28·85	−14	37·39	−17	55·7	−2	80·5	−3	115·3	−4
·8	10·33	−7	11·65	−8	12·85	−11	17·58	−13	23·26	−12	28·71	−14	37·22	−15	55·5	−1	80·2	−2	114·9	−3
·9	10·26	−6	11·57	−7	12·74	−10	17·45	−11	23·14	−12	28·57	−13	37·07	−13	55·4	−1	80·0	−1	114·6	−3
3·0	10·20	−6	11·50	−7	12·64	−9	17·34	−10	23·02	−12	28·44	−13	36·94	−12	55·3	−1	79·9	−2	114·3	−3
·1	10·14	−6	11·43	−7	12·55	−8	17·24	−10	22·90	−10	28·31	−12	36·82	−11	55·2	−2	79·7	−1	114·0	−3
·2	10·08	−5	11·36	−6	12·47	−7	17·14	−9	22·80	−9	28·19	−9	36·71	−10	55·0	−1	79·6	−1	113·7	−2
·3	10·03	−4	11·30	−5	12·40	−7	17·05	−8	22·71	−9	28·10	−9	36·61	−9	54·9	−1	79·5	−1	113·5	−2
·4	9·99	−4	11·25	−5	12·33	−6	16·97	−8	22·62	−7	28·01	−8	36·52	−9	54·8	−1	79·4	−1	113·3	−2
3·5	9·95	−4	11·20	−4	12·27	−6	16·89	−8	22·55	−7	27·93	−8	36·43	−8	54·7	−1	79·3	−1	113·1	−2
·6	9·91	−4	11·16	−5	12·21	−5	16·82	−7	22·48	−7	27·85	−8	36·35	−7	54·6	−2	79·2	−1	112·9	−2
·7	9·87	−4	11·11	−4	12·16	−4	16·75	−6	22·41	−6	27·77	−6	36·28	−6	54·4	−1	79·1	−1	112·7	−2
·8	9·83	−3	11·07	−4	12·12	−4	16·69	−5	22·35	−7	27·71	−6	36·22	−6	54·3	−1	79·0	−1	112·5	−1
·9	9·80	−3	11·03	−4	12·08	−4	16·64	−5	22·28	−6	27·65	−6	36·16	−6	54·2	−1	78·9	−1	112·4	−1
4·0	9·77	−2	10·99	−3	12·04	−4	16·59	−5	22·22	−5	27·59	−4	36·10	−6	54·1	−1	78·8	−2	112·3	−1
·1	9·75	−2	10·96	−3	12·00	−3	16·54	−5	22·17	−5	27·55	−5	36·04	−6	54·0	−1	78·6	−1	112·2	−1
·2	9·73	−2	10·93	−3	11·97	−3	16·49	−4	22·12	−5	27·50	−5	35·98	−6	53·9	0	78·5	−1	112·1	−1
·3	9·71	−3	10·90	−2	11·94	−3	16·45	−4	22·07	−4	27·45	−5	35·92	−6	53·9	−1	78·4	−1	112·0	−1
·4	9·68	−2	10·88	−3	11·91	−2	16·41	−3	22·03	−4	27·40	−5	35·86	−5	53·8	0	78·3	−1	111·9	−1
4·5	9·66		10·85		11·89		16·38		21·99		27·35		35·81		53·8		78·2		111·8	

TABLE 1b—*continued*

Values of b° : $N = 4$—*continued*

J \ r_c	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
4·5	9·66	− 2	10·85	− 2	11·89	− 3	16·38	− 4	21·99	− 4	27·35	− 5	35·81	− 6	53·8	0	78·2	0	111·8	− 1
·6	9·64	− 2	10·83	− 3	11·86	− 2	16·34	− 3	21·95	− 4	27·30	− 3	35·75	− 5	53·8	− 1	78·2	− 1	111·7	− 1
·7	9·62	− 1	10·80	− 2	11·84	− 2	16·31	− 3	21·91	− 4	27·27	− 3	35·70	− 4	53·7	0	78·1	0	111·6	− 1
·8	9·61	− 2	10·78	− 2	11·82	− 2	16·28	− 3	21·87	− 3	27·24	− 3	35·66	− 4	53·7	− 1	78·1	− 1	111·5	0
·9	9·59	− 2	10·76	− 2	10·80	− 2	16·25	− 3	21·84	− 3	27·21	− 3	35·62	− 4	53·6	0	78·0	0	111·5	0
5·0	9·57	− 1	10·74	− 2	11·78	− 1	16·22	− 3	21·81	− 3	27·18	− 3	35·58	− 4	53·6	− 1	78·0	− 1	111·5	− 1
·1	9·56	− 2	10·72	− 1	11·77	− 2	16·19	− 3	21·78	− 2	27·15	− 3	35·54	− 3	53·5	0	77·9	0	111·4	0
·2	9·54	− 1	10·71	− 2	11·75	− 1	16·16	− 2	21·76	− 2	27·12	− 2	35·51	− 3	53·5	− 1	77·9	0	111·4	− 1
·3	9·53	− 2	10·69	− 1	11·74	− 2	16·14	− 2	21·74	− 2	27·10	− 1	35·48	− 3	53·4	0	77·9	− 1	111·3	0
·4	9·51	− 1	10·68	− 2	11·72	− 1	16·12	− 2	21·72	− 3	27·09	− 1	35·45	− 2	53·4	0	77·8	0	111·3	0
5·5	9·50	− 1	10·66	− 1	11·71	− 1	16·10	− 2	21·69	− 3	27·08	− 2	35·43	− 3	53·4	0	77·8	− 1	111·3	− 1
·6	9·49	− 1	10·65	− 1	11·70	− 1	16·08	− 2	21·66	− 2	27·06	− 1	35·40	− 2	53·4	0	77·7	0	111·2	0
·7	9·48	− 1	10·64	− 2	11·69	− 1	16·06	− 2	21·64	− 2	27·05	− 1	35·38	− 2	53·4	− 1	77·7	− 1	111·2	0
·8	9·47	− 1	10·62	− 1	11·68	− 1	16·04	− 1	21·62	− 2	27·03	− 1	35·36	− 2	53·3	0	77·6	0	111·2	− 1
·9	9·46	− 1	10·61	− 1	11·67	− 1	16·03	− 1	21·60	− 1	27·02	− 1	35·34	− 2	53·3	0	77·6	− 1	111·1	0
6·0	9·45		10·60		11·66		16·02		21·59		27·01		35·32		53·3		77·5		111·1	
6·5	9·40		10·55		11·62		15·94		21·51		26·86		35·23		53·2		77·3		111·0	
7·0	9·36		10·51		11·59		15·89		21·44		26·81		35·19		53·1		77·3		110·9	

TABLE 1b—*continued*

Values of b° : N = 5

r_e / J	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	15·7	−7	17·6	−8	19·5	−10	25·1	−15	31·7	−19	37·5	−21	45·6	−22	62·9	−24	88·0	−32	123·0	−41
·1	15·0	−6	16·8	−7	18·5	− 8	23·6	−12	29·8	−15	35·4	−17	43·4	−19	60·5	−20	84·8	−26	118·9	−32
·2	14·4	−5	16·1	−6	17·7	− 6	22·4	− 9	28·3	−12	33·7	−14	41·5	−15	58·5	−17	82·2	−21	115·7	−26
·3	13·9	−4	15·5	−5	17·1	− 6	21·5	− 8	27·1	−10	32·3	−11	40·0	−12	56·8	−13	80·1	−16	113·1	−21
·4	13·5	−4	15·0	−5	16·5	− 5	20·7	− 7	26·1	− 8	31·2	− 9	38·8	−10	55·5	−11	78·5	−14	111·0	−17
1·5	13·1		14·5	−4	16·0	− 4	20·0	− 5	25·3	− 7	30·3	− 8	37·8	− 8	54·4	−10	77·1	−12	109·3	−15
·6	12·8		14·1		15·6	− 4	19·5		24·6	− 6	29·5	− 6	37·0	− 6	53·4	− 8	75·9	−10	107·8	−13
·7	12·5		13·8		15·2	− 4	19·0		24·0	− 5	28·9	− 6	36·4	− 6	52·6	− 7	74·9	− 8	106·5	−11
·8	12·2		13·5		14·8		18·6		23·5	− 4	28·3	− 5	35·8	− 5	51·9	− 6	74·1	− 8	105·4	−11
·9	12·0		13·3		14·5		18·3		23·1	− 4	27·8	− 4	35·3	− 5	51·3	− 6	73·3	− 7	104·3	−10
2·0	11·8		13·0		14·2		18·0		22·7		27·4		34·8		50·7	− 5	72·6	− 6	103·3	− 9
·1	11·7		12·8		14·0		17·7		22·4		27·0		34·4		50·2		72·0	− 5	102·4	− 8
·2	11·5		12·6		13·8		17·4		22·1		26·7		34·0		49·8		71·5	− 5	101·6	− 8
·3	11·4		12·5		13·6		17·2		21·9		26·4		33·7		49·4		71·0	− 4	100·8	− 6
·4	11·3		12·3		13·4		17·0		21·6		26·2		33·5		49·1		70·6	− 4	100·2	− 6
2·5	11·2		12·2		13·3		16·9		21·4		26·0		33·2		48·8		70·2		99·6	− 5
·6	11·1		12·1		13·1		16·7		21·3		25·8		33·0		48·5		69·8		99·1	
·7	11·0		12·0		13·0		16·6		21·1		25·6		32·8		48·3		69·5		98·7	
·8	10·9		11·9		12·9		16·4		20·9		25·4		32·6		48·1		69·2		98·3	
·9	10·8		11·8		12·8		16·3		20·8		25·3		32·4		47·9		68·9		98·0	
3·0	10·8		11·7		12·7		16·2		20·7		25·4		32·3		47·7		68·7		97·6	
·1	10·7		11·7		12·6		16·1		20·6		25·0		32·1		47·6		68·5		97·3	
·2	10·7		11·6		12·6		16·0		20·5		24·9		32·0		47·4		68·3		97·0	
·3	10·6		11·6		12·5		15·9		20·4		24·8		31·9		47·3		68·1		96·8	
·4	10·6		11·5		12·4		15·8		20·3		24·7		31·7		47·2		67·9		96·5	
3·5	10·5		11·4		12·4		15·7		20·2		24·6		31·6		47·0		67·7		96·3	
·6	10·5		11·4		12·3		15·7		20·1		24·5		31·5		46·9		67·6		96·1	
·7	10·5		11·4		12·3		15·6		20·0		24·4		31·4		46·8		67·4		95·9	
·8	10·4		11·3		12·2		15·6		20·0		24·4		31·4		46·7		67·3		95·7	
·9	10·4		11·3		12·2		15·5		19·9		24·3		31·3		46·6		67·2		95·5	
4·0	10·4		11·2		12·1		15·4		19·9		24·3		31·2		46·5		67·0		95·4	
·1	10·3		11·2		12·1		15·4		19·8		24·2		31·1		46·5		66·9		95·2	
·2	10·3		11·2		12·0		15·4		19·8		24·2		31·1		46·4		66·8		95·1	
·3	10·3		11·2		12·0		15·3		19·7		24·1		31·0		46·3		66·7		95·0	
·4	10·3		11·1		12·0		15·3		19·7		24·1		31·0		46·2		66·6		94·8	
4·5	10·2		11·1		12·0		15·2		19·6		24·0		30·9		46·2		66·5		94·7	

TABLE 1b—*continued*

Values of b° : $N = 5$—*continued*

J \ r_c	0·2	0·25	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975
4·5	10·2	11·1	12·0	15·2	19·6	24·0	30·9	46·2	66·5	94·7
·6	10·2	11·1	12·0	15·2	19·6	24·0	30·9	46·1	66·4	94·6
·7	10·2	11·0	11·9	15·2	19·6	23·9	30·8	46·0	66·4	94·5
·8	10·2	11·0	11·9	15·1	19·5	23·9	30·8	46·0	66·3	94·5
·9	10·2	11·0	11·9	15·1	19·5	23·9	30·7	45·9	66·2	94·4
5·0	10·2	11·0	11·8	15·1	19·4	23·8	30·7	45·9	66·2	94·3
·1	10·1	11·0	11·8	15·0	19·4	23·8	30·7	45·8	66·1	94·2
·2	10·1	11·0	11·8	15·0	19·4	23·8	30·6	45·8	66·0	94·1
·3	10·1	10·9	11·8	15·0	19·4	23·7	30·6	45·7	66·0	94·1
·4	10·1	10·9	11·8	15·0	19·3	23·7	30·6	45·7	65·9	94·0
5·5	10·1	10·9	11·8	15·0	19·3	23·7	30·6	45·6	65·9	94·0
·6	10·1	10·9	11·8	14·9	19·3	23·6	30·5	45·6	65·8	93·9
·7	10·1	10·9	11·7	14·9	19·3	23·6	30·5	45·6	65·8	93·8
·8	10·0	10·9	11·7	14·9	19·3	23·6	30·5	45·5	65·8	93·8
·9	10·0	10·9	11·7	14·9	19·2	23·6	30·5	45·5	65·7	93·7
6·0	10·0	10·9	11·7	14·9	19·2	23·6	30·4	45·4	65·7	93·7
6·5	9·97	10·8	11·6	14·8	19·1	23·5	30·4	45·3	65·5	93·5
7·0	9·94	10·8	11·6	14·7	19·1	23·4	30·3	45·2	65·4	93·3

TABLE 1b—*continued*

Values of $b°$: $N = 6$

r_c / J	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	15·9	−6	17·8	−9	19·5	−10	24·7	−15	30·9	−20	36·1	−22	43·5	−25	58·6	−26	80·5	−31	112·2	−40
·1	15·3	−5	16·9	−6	18·5	−8	23·2	−12	28·9	−15	33·9	−18	41·0	−19	56·0	−21	77·4	−25	108·2	−30
·2	14·8	−5	16·3	−6	17·7	−6	22·0	−10	27·4	−13	32·1	−14	39·1	−15	53·9	−17	74·9	−20	105·2	−25
·3	14·3	−4	15·7	−5	17·1	−6	21·0	−8	26·1	−11	30·7	−12	37·6	−13	52·2	−14	72·9	−17	102·7	−21
·4	13·9		15·2	−4	16·5	−5	20·2	−7	25·0	−8	29·5	−10	36·3	−10	50·8	−12	71·2	−14	100·6	−18
1·5	13·6		14·8	−4	16·0	−4	19·5	−5	24·2	−8	28·5	−8	35·3	−8	49·6	−10	69·8	−12	98·8	−15
·6	13·3		14·4		15·6	−4	19·0		23·4	−6	27·7	−7	34·5	−8	48·6	−9	68·6	−10	97·3	−14
·7	13·0		14·1		15·2		18·5		22·8	−5	27·0	−6	33·7	−7	47·7	−7	67·6	−9	95·9	−12
·8	12·8		13·8		14·9		18·1		22·3	−5	26·4	−5	33·0	−5	47·0	−6	66·7	−8	94·7	−11
·9	12·6		13·6		14·6		17·7		21·8	−4	25·9	−4	32·5	−5	46·4	−6	65·9	−7	93·6	−9
2·0	12·4		13·3		14·4		17·4		21·4		25·5		32·0		45·8	−5	65·2	−6	92·7	−9
·1	12·3		13·1		14·1		17·1		21·1		25·1		31·6		45·3		64·6	−6	91·8	−8
·2	12·1		13·0		13·9		16·9		20·8		24·7		31·2		44·9		64·0	−5	91·0	−7
·3	12·0		12·8		13·7		16·6		20·5		24·4		30·8		44·5		63·5	−4	90·3	−7
·4	11·9		12·7		13·6		16·4		20·2		24·2		30·5		44·1		63·1	−4	89·6	−6
2·5	11·8		12·6		13·4		16·3		20·0		23·9		30·3		43·8		62·7		89·0	−5
·6	11·7		12·4		13·3		16·1		19·8		23·7		30·0		43·5		62·3		88·5	
·7	11·6		12·4		13·2		15·9		19·6		23·5		29·8		43·2		61·9		88·0	
·8	11·6		12·3		13·1		15·8		19·5		23·3		29·6		43·0		61·6		87·6	
·9	11·5		12·2		13·0		15·7		19·4		23·1		29·4		42·8		61·3		87·2	
3·0	11·4		12·1		12·9		15·6		19·2		23·0		29·3		42·6		61·0		86·8	
·1	11·4		12·0		12·8		15·5		19·1		22·9		29·1		42·4		60·8		86·5	
·2	11·3		12·0		12·7		15·4		19·0		22·8		29·0		42·2		60·6		86·2	
·3	11·3		11·9		12·7		15·3		18·9		22·6		28·8		42·0		60·4		85·9	
·4	11·2		11·9		12·6		15·2		18·8		22·5		28·7		41·9		60·2		85·6	
3·5	11·2		11·8		12·6		15·1		18·7		22·4		28·6		41·8		60·0		85·4	
·6	11·2		11·8		12·5		15·1		18·6		22·4		28·5		41·7		59·8		85·1	
·7	11·1		11·7		12·4		15·0		18·6		22·3		28·4		41·5		59·6		84·9	
·8	11·1		11·7		12·4		14·9		18·5		22·2		28·3		41·4		59·5		84·7	
·9	11·1		11·7		12·4		14·9		18·4		22·2		28·2		41·3		59·3		84·5	
4·0	11·0		11·6		12·3		14·8		18·4		22·1		28·2		41·2		59·2		84·3	
·1	11·0		11·6		12·3		14·8		18·3		22·0		28·1		41·1		59·1		84·2	
·2	11·0		11·6		12·2		14·7		18·3		22·0		28·0		41·1		59·0		84·0	
·3	11·0		11·6		12·2		14·7		18·2		21·9		27·9		41·0		58·8		83·9	
·4	11·0		11·5		12·2		14·6		18·2		21·9		27·9		40·9		58·7		83·7	
4·5	10·9		11·5		12·2		14·6		18·1		21·8		27·8		40·8		58·6		83·6	

TABLE 1b—*continued*

Values of b° : $N = 6$—*continued*

r_c / J	0·2	0·25	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975
4·5	10·9	11·5	12·2	14·6	18·1	21·8	27·8	40·8	58·6	83·6
·6	10·9	11·5	12·1	14·6	18·1	21·8	27·8	40·8	58·6	83·5
·7	10·9	11·4	12·1	14·5	18·0	21·7	27·7	40·7	58·5	83·4
·8	10·9	11·4	12·1	14·5	18·0	21·7	27·7	40·6	58·4	83·3
·9	10·9	11·4	12·1	14·5	18·0	21·7	27·7	40·6	58·3	83·2
5·0	10·8	11·4	12·0	14·5	17·9	21·6	27·6	40·5	58·2	83·1
·1	10·8	11·4	12·0	14·4	17·9	21·6	27·6	40·4	58·2	83·0
·2	10·8	11·4	12·0	14·4	17·9	21·6	27·5	40·4	58·1	82·9
·3	10·8	11·4	12·0	14·4	17·8	21·5	27·5	40·3	58·1	82·8
·4	10·8	11·4	12·0	14·4	17·8	21·5	27·5	40·3	58·0	82·7
5·5	10·8	11·3	12·0	14·3	17·8	21·5	27·4	40·2	57·9	82·7
·6	10·8	11·3	11·9	14·3	17·8	21·4	27·4	40·2	57·9	82·6
·7	10·8	11·3	11·9	14·3	17·7	21·4	27·4	40·2	57·8	82·5
·8	10·8	11·3	11·9	14·3	17·7	21·4	27·4	40·1	57·8	82·5
·9	10·7	11·3	11·9	14·3	17·7	21·4	27·3	40·1	57·7	82·4
6·0	10·7	11·3	11·9	14·2	17·7	21·4	27·3	40·0	57·7	82·4
6·5	10·7	11·2	11·8	14·2	17·6	21·3	27·2	39·9	57·5	82·1
7·0	10·7	11·2	11·8	14·1	17·6	21·2	27·1	39·7	57·3	81·9

TABLE 2a

Values of φ_o°

r_o J	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	57·86	240	51·85	262	46·70	271	35·27	263	27·95	232	24·45	212	21·70	194	19·48	178	18·52	171	18·08	167
·1	60·26	210	54·47	233	49·41	244	37·90	243	30·27	221	26·57	205	23·64	188	21·26	174	20·23	167	19·75	164
·2	60·36	184	56·80	206	51·85	221	40·33	227	32·48	211	28·62	197	25·52	183	23·00	169	21·90	164	21·39	161
·3	64·20	163	58·86	185	54·06	199	42·60	212	34·59	201	30·59	189	27·35	177	24·69	165	23·54	159	23·00	156
·4	65·83	144	60·71	165	56·05	181	44·72	198	36·60	191	32·48	182	29·12	171	26·34	161	25·13	155	24·56	153
1·5	67·27	129	62·36	149	57·86	164	46·70	184	38·51	182	34·30	174	30·83	165	27·95	156	26·68	152	26·09	149
·6	68·56	116	63·85	135	59·50	150	48·54	171	40·33	172	36·04	167	32·48	159	29·51	151	28·20	147	27·58	145
·7	69·72	104	65·20	123	61·00	136	50·25	160	42·05	163	37·71	159	34·07	154	31·02	146	29·67	143	29·03	141
·8	70·76	94	66·43	111	62·36	126	51·85	151	43·68	155	39·30	153	35·61	148	32·48	142	31·10	138	30·44	137
·9	71·70	86	67·44	102	63·62	115	53·36	139	45·23	147	40·83	146	37·09	142	33·90	137	32·48	135	31·81	133
2·0	72·56	78	68·46	93	64·77	106	54·75	130	46·70	139	42·29	139	38·51	137	35·27	133	33·83	130	33·14	129
·1	73·34	72	69·49	86	65·83	98	56·05	122	48·09	132	43·68	132	39·88	132	36·60	129	35·13	127	34·43	126
·2	74·06	66	70·35	80	66·81	91	57·27	115	49·41	125	45·00	128	41·20	126	37·89	124	36·40	122	35·69	121
·3	74·72	61	71·15	73	67·72	84	58·42	108	50·66	119	46·28	122	42·46	122	39·13	120	37·62	118	36·90	118
·4	75·33	56	71·88	68	68·56	78	59·50	101	51·85	113	47·50	116	43·68	117	40·33	115	38·80	115	38·08	114
2·5	75·89	52	72·56	63	69·34	73	60·51	95	52·98	108	48·66	111	44·85	112	41·48	112	39·95	111	39·22	111
·6	76·41	49	73·19	59	70·07	69	61·46	90	54·06	102	49·77	107	45·97	108	42·60	108	41·06	107	40·33	107
·7	76·90	45	73·78	55	70·76	64	62·36	85	55·08	97	50·84	101	47·05	104	43·68	104	42·13	104	41·40	103
·8	77·35	42	74·33	52	71·40	60	63·21	81	56·05	93	51·85	98	48·09	100	44·72	101	43·17	101	42·43	100
·9	77·77	40	74·85	48	72·00	56	64·02	75	56·98	88	52·83	94	49·09	96	45·73	97	44·18	97	43·43	97
3·0	78·17	37	75·33	45	72·56	53	64·77	72	57·86	84	53·77	88	50·05	92	46·70	93	45·15	94	44·40	94
·1	78·54	35	75·78	43	73·09	50	65·49	68	58·70	80	54·65	85	50·97	88	47·63	91	46·09	91	45·34	91
·2	78·89	33	76·21	40	73·59	47	66·17	64	59·50	76	55·50	81	51·85	86	48·54	87	47·00	87	46·25	88
·3	79·22	31	76·61	38	74·06	45	66·81	61	60·26	74	56·31	79	52·71	82	49·41	84	47·87	85	47·13	85
·4	79·53	29	76·99	36	74·51	42	67·42	58	61·00	70	57·10	76	53·53	79	50·25	82	48·72	82	47·98	83
3·5	79·82	28	77·35	34	74·93	40	68·00	56	61·70	66	57·86	73	54·32	76	51·07	78	49·54	80	48·81	80
·6	80·10	26	77·69	33	75·33	38	68·56	53	62·36	64	58·59	69	55·08	73	51·85	76	50·34	76	49·61	77
·7	80·36	25	78·02	30	75·71	36	69·09	50	63·00	62	59·28	66	55·81	71	52·61	74	51·11	74	50·38	75
·8	80·61	24	78·32	29	76·07	34	69·59	48	63·62	59	59·94	64	56·52	68	53·35	71	51·85	72	51·13	72
·9	80·85	22	78·61	28	76·41	33	70·07	46	64·21	56	60·58	62	57·20	66	54·06	68	52·57	70	51·85	71
4·0	81·07	22	78·89	27	76·74	31	70·53	45	64·77	54	61·20	59	57·86	63	54·74	67	53·27	68	52·56	68
·1	81·29	20	79·16	25	77·05	30	70·98	42	65·31	52	61·79	57	58·49	61	55·41	64	53·95	65	53·24	66
·2	81·49	20	79·41	24	77·35	29	71·40	40	65·83	50	62·36	55	59·10	59	56·05	62	54·60	64	53·90	64
·3	81·69	18	79·65	23	77·64	27	71·80	39	66·33	48	62·91	53	59·69	58	56·67	61	55·24	61	54·54	62
·4	81·87	18	79·88	22	77·91	26	72·19	37	66·81	46	63·44	52	60·27	55	57·28	58	55·85	60	55·16	60
4·5	82·05		80·10		78·17		72·56		67·27		63·96		60·82		57·86		56·45		55·76	

TABLE 2a—*continued*

Values of φ_o°—*continued*

r_c J	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
4·5	82·05	17	80·10	21	78·17	25	72·56	36	67·27	45	63·96	49	60·82	53	57·86	56	56·45	57	55·76	58
·6	82·22	17	80·31	20	78·42	24	72·92	34	67·72	43	64·45	48	61·35	51	58·42	55	57·02	56	56·34	57
·7	82·39	15	80·51	20	78·66	23	73·26	33	68·15	41	64·93	46	61·86	50	58·97	53	57·58	55	56·91	55
·8	82·54	15	80·71	18	78·89	22	73·59	32	68·56	40	65·39	44	62·36	49	59·50	51	58·13	52	57·46	53
·9	82·69	15	80·89	18	79·11	22	73·91	30	68·96	38	65·83	43	62·85	46	60·01	50	58·65	52	57·99	52
5·0	82·84	14	81·07	18	79·33	20	74·21	30	69·34	38	66·26	41	63·31	46	60·51	49	59·17	49	58·51	50
·1	82·98	13	81·25	16	79·53	20	74·51	28	69·72	36	66·67	41	63·77	43	61·00	47	59·66	49	59·01	49
·2	83·11	13	81·41	16	79·73	19	74·79	28	70·08	34	67·08	39	64·20	43	61·47	45	60·15	46	59·50	47
·3	83·24	12	81·57	15	79·92	18	75·07	26	70·42	34	67·47	37	64·63	41	61·92	44	60·61	46	59·97	47
·4	83·36	12	81·72	15	80·10	18	75·33	26	70·76	32	67·84	37	65·04	40	62·36	43	61·07	44	60·44	44
5·5	83·48	11	81·87	15	80·28	17	75·59	24	71·08	32	68·21	35	65·44	39	62·79	42	61·51	43	60·88	44
·6	83·59	12	82·02	13	80·45	16	75·83	24	71·40	30	68·56	34	65·83	38	63·21	41	61·94	42	61·32	43
·7	83·71	11	82·15	14	80·61	16	76·07	23	71·70	30	68·90	34	66·21	36	63·62	39	62·36	41	61·75	41
·8	83·82	10	82·29	13	80·77	15	76·30	23	72·00	28	69·24	32	66·57	36	64·01	38	62·77	40	62·16	40
·9	83·92	10	82·42	12	80·92	15	76·53	21	72·28	28	69·56	31	66·93	34	64·39	38	63·17	38	62·56	39
6·0	84·02		82·54		81·07		76·74		72·56		69·87		67·27		64·77		63·55		62·95	
6·5	84·48		83·11		81·75		77·73		73·83		71·31		68·86		66·49		65·34		64·77	
7·0	84·87		83·60		82·33		78·58		74·93		72·56		70·25		68·01		66·91		66·37	

TABLE 2b

Values of r_c sec φ_o

J \ r_c	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975		1·0	
1·0	0·376	27	0·405	25	0·437	24	0·551	19	0·679	16	0·769	14	0·861	12	0·955	11	1·002	11	1·026	10	1·049	10
·1	·403	28	·430	27	·461	25	·570	20	·695	16	·783	14	·873	13	·966	12	1·013	11	1·036	11	1·059	11
·2	·431	29	·457	26	·486	25	·590	21	·711	18	·797	16	·886	15	·978	13	1·024	12	1·047	12	1·070	12
·3	·460	28	·483	28	·511	26	·611	22	·729	18	·813	17	·901	15	·991	13	1·036	13	1·059	13	1·082	13
·4	·488	30	·511	28	·537	27	·633	23	·747	20	·830	17	·916	16	1·004	15	1·049	14	1·072	14	1·095	13
1·5	0·518	29	0·539	28	0·564	27	0·656	24	0·767	20	0·847	19	0·932	16	1·019	15	1·063	15	1·086	14	1·108	14
·6	·547	30	·567	29	·591	28	·680	24	·787	21	·866	19	·948	18	1·034	16	1·078	15	1·100	15	1·122	15
·7	·577	30	·596	29	·619	28	·704	25	·808	22	·885	20	·966	18	1·050	17	1·093	16	1·115	16	1·137	16
·8	·607	30	·625	29	·647	28	·729	25	·830	22	·905	20	·984	19	1·067	17	1·109	17	1·131	16	1·153	16
·9	·637	30	·654	30	·675	29	·754	26	·852	23	·925	21	1·003	19	1·084	18	1·126	18	1·147	17	1·169	16
2·0	0·667	31	0·684	30	0·704	29	0·780	26	0·875	23	0·946	22	1·022	20	1·102	19	1·144	18	1·164	18	1·185	18
·1	·698	30	·714	30	·733	29	·806	26	·898	24	·968	22	1·042	21	1·121	19	1·162	18	1·182	18	1·203	18
·2	·728	31	·744	30	·762	29	·832	27	·922	25	·990	23	1·063	21	1·140	20	1·180	19	1·200	19	1·221	18
·3	·759	31	·774	30	·791	30	·859	28	·947	24	1·013	23	1·084	22	1·160	20	1·199	20	1·219	20	1·239	19
·4	·790	31	·804	30	·821	29	·887	27	·971	26	1·036	24	1·106	22	1·180	21	1·219	20	1·239	20	1·258	20
2·5	0·821	30	0·834	31	0·850	30	0·914	28	0·997	25	1·060	24	1·128	23	1·201	22	1·239	21	1·259	20	1·278	20
·6	·851	31	·865	30	·880	30	·942	28	1·022	26	1·084	24	1·151	23	1·223	21	1·260	21	1·279	21	1·298	21
·7	·882	31	·895	31	·910	30	·970	28	1·048	26	1·108	25	1·174	24	1·244	23	1·281	22	1·300	21	1·319	21
·8	·913	32	·926	30	·940	31	·998	29	1·074	27	1·133	25	1·198	24	1·267	22	1·303	22	1·321	22	1·340	21
·9	·945	31	·956	31	·971	30	1·027	29	1·101	27	1·158	26	1·222	24	1·289	23	1·325	22	1·343	22	1·361	22
3·0	0·976	31	0·987	31	1·001	30	1·056	29	1·128	27	1·184	26	1·246	24	1·312	24	1·347	23	1·365	22	1·383	22
·1	1·007	31	1·018	31	1·031	31	1·085	29	1·155	27	1·210	26	1·270	25	1·336	23	1·370	23	1·387	23	1·405	22
·2	1·038	31	1·049	31	1·062	30	1·114	29	1·182	28	1·236	26	1·295	25	1·359	24	1·393	23	1·410	23	1·427	23
·3	1·069	32	1·080	31	1·092	31	1·143	29	1·210	27	1·262	27	1·320	26	1·383	25	1·416	24	1·433	24	1·450	23
·4	1·101	31	1·111	31	1·123	31	1·172	29	1·237	28	1·289	27	1·346	26	1·408	24	1·440	24	1·457	24	1·473	24
3·5	1·132	31	1·142	31	1·154	30	1·201	30	1·265	28	1·316	27	1·372	26	1·432	25	1·464	24	1·481	24	1·497	24
·6	1·163	32	1·173	31	1·184	31	1·231	30	1·293	29	1·343	27	1·398	26	1·457	25	1·488	25	1·505	24	1·521	24
·7	1·195	31	1·204	31	1·215	31	1·261	30	1·322	28	1·370	27	1·424	26	1·482	26	1·513	25	1·529	25	1·545	24
·8	1·226	31	1·235	31	1·246	31	1·291	29	1·350	29	1·397	28	1·450	27	1·508	25	1·538	25	1·554	25	1·569	25
·9	1·257	32	1·266	32	1·277	31	1·320	30	1·379	29	1·425	28	1·477	27	1·533	26	1·563	26	1·579	25	1·594	25
4·0	1·289	31	1·298	31	1·308	31	1·350	30	1·408	29	1·453	28	1·504	27	1·559	26	1·589	25	1·604	25	1·619	25
·1	1·320	32	1·329	31	1·339	31	1·380	31	1·437	29	1·481	28	1·531	27	1·585	27	1·614	26	1·629	26	1·644	25
·2	1·352	31	1·360	31	1·370	31	1·411	30	1·465	29	1·509	28	1·558	27	1·612	26	1·640	26	1·655	26	1·669	26
·3	1·383	32	1·391	32	1·401	31	1·441	30	1·494	30	1·537	29	1·585	28	1·638	27	1·666	26	1·681	26	1·695	26
·4	1·415	31	1·423	31	1·432	31	1·471	30	1·524	29	1·566	28	1·613	28	1·665	27	1·692	27	1·707	26	1·721	26
4·5	1·446		1·454		1·463		1·501		1·553		1·594		1·641		1·692		1·719		1·733		1·747	

TABLE 2b—*continued*

Values of r_c sec φ_o—continued

r_c / J	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975		1·0	
4·5	1·446	32	1·454	31	1·463	32	1·501	31	1·553	29	1·594	29	1·64	28	1·692	27	1·719	26	1·733	26	1·747	26
·6	1·478	31	1·485	32	1·495	31	1·532	30	1·582	30	1·623	29	1·669	28	1·719	27	1·745	27	1·759	27	1·773	26
·7	1·509	32	1·517	31	1·526	31	1·562	31	1·612	29	1·652	29	1·697	28	1·746	27	1·772	27	1·786	27	1·799	27
·8	1·541	32	1·548	32	1·557	31	1·593	30	1·641	30	1·681	29	1·725	28	1·773	28	1·799	27	1·813	26	1·826	27
·9	1·573	31	1·580	31	1·588	32	1·623	31	1·671	30	1·710	29	1·753	28	1·801	27	1·826	27	1·839	27	1·853	27
5·0	1·604	32	1·611	32	1·620	31	1·654	31	1·701	30	1·739	29	1·781	29	1·828	28	1·853	28	1·866	28	1·880	27
·1	1·636	31	1·643	31	1·651	31	1·685	30	1·731	30	1·768	29	1·810	28	1·856	28	1·881	28	1·894	27	1·907	27
·2	1·667	32	1·674	31	1·682	31	1·715	31	1·761	30	1·797	29	1·838	29	1·884	28	1·909	27	1·921	27	1·934	27
·3	1·699	31	1·705	32	1·713	32	1·746	31	1·791	30	1·826	30	1·867	29	1·912	28	1·936	28	1·948	28	1·961	28
·4	1·730	32	1·737	31	1·745	31	1·777	31	1·821	30	1·856	29	1·896	29	1·940	28	1·964	28	1·976	28	1·989	27
5·5	1·762	32	1·768	32	1·776	32	1·808	31	1·851	30	1·885	30	1·925	29	1·968	29	1·992	28	2·004	28	2·016	28
·6	1·7 4	32	1·800	32	1·808	31	1·839	30	1·881	30	1·915	30	1·954	29	1·997	28	2·020	28	2·032	28	2·044	28
·7	1·826	31	1·832	31	1·839	31	1·869	31	1·911	30	1·945	29	1·983	29	2·025	29	2·048	28	2·060	28	2·072	28
·8	1·857	32	1·863	32	1·870	32	1·900	31	1·941	31	1·974	30	2·012	29	2·054	29	2·076	29	2·088	28	2·100	28
·9	1·889	31	1·895	31	1·902	31	1·931	31	1·972	30	2·004	30	2·041	30	2·083	28	2·105	28	2·116	28	2·128	28
6·0	1·920		1·926		1·933		1·962		2·002		2·034		2·071		2·111		2·133		2·144		2·156	
6·5	2·079		2·084		2·091		2·117		2·154		2·184		2·218		2·256		2·277		2·287		2·298	
7·0	2·237		2·242		2·248		2·273		2·307		2·335		2·367		2·403		2·422		2·432		2·442	

TABLE 2c *Values of* τ

r_c J	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975	△
1·0	0·742	1·178	1·788	2·292	2·873	3·532	3·891	4·077	82
·1	0·824	1·260	1·870	2·374	2·956	3·615	3·973	4·160	90
·2	0·914	1·350	1·961	2·465	3·046	3·705	4·063	4·250	98
·3	1·012	1·449	2·059	2·563	3·144	3·803	4·162	4·348	106
·4	1·118	1·555	2·165	2·669	3·250	3·909	4·268	4·454	114
1·5	1·232	1·668	2·279	2·783	3·364	4·023	4·382	4·568	122
·6	1·354	1·790	2·401	2·904	3·486	4·145	4·503	4·690	130
·7	1·484	1·920	2·530	3·034	3·615	4·274	4·633	4·819	138
·8	1·621	2·057	2·668	3·172	3·753	4·412	4·770	4·957	145
·9	1·767	2·203	2·813	3·317	3·898	4·557	4·916	5·102	153
2·0	1·920	2·356	2·966	3·470	4·051	4·710	5·069	5·255	161
·1	2·081	2·517	3·127	3·631	4·212	4·871	5·230	5·416	169
·2	2·250	2·686	3·296	3·800	4·381	5·040	5·399	5·585	177
·3	2·426	2·862	3·473	3·977	4·558	5·217	5·575	5·762	185
·4	2·611	3·047	3·657	4·161	4·743	5·401	5·760	5·946	192
2·5	2·803	3·239	3·850	4·354	4·935	5·594	5·952	6·139	200
·6	3·004	3·440	4·050	4·554	5·135	5·794	6·153	6·339	208
·7	3·212	3·648	4·258	4·762	5·343	6·002	6·361	6·547	216
·8	3·428	3·864	4·474	4·978	5·559	6·218	6·577	6·763	224
·9	3·651	4·088	4·698	5·202	5·783	6·442	6·801	6·987	232
3·0	3·883	4·319	4·930	5·433	6·015	6·674	7·032	7·219	240
·1	4·123	4·559	5·169	5·673	6·254	6·913	7·272	7·458	247
·2	4·370	4·806	5·417	5·920	6·502	7·161	7·519	7·706	255
·3	4·625	5·061	5·672	6·176	6·757	7·416	7·774	7·961	263
·4	4·888	5·324	5·935	6·439	7·020	7·679	8·038	8·224	271
3·5	5·159	5·595	6·206	6·710	7·291	7·950	8·309	8·495	279
·6	5·438	5·874	6·485	6·989	7·570	8·229	8·587	8·774	287
·7	5·725	6·161	6·771	7·275	7·857	8·516	8·874	9·061	295
·8	6·019	6·455	7·066	7·570	8·151	8·810	9·169	9·355	302
·9	6·322	6·758	7·368	7·872	8·454	9·112	9·471	9·657	310
4·0	6·632	7·068	7·678	8·182	8·764	9·423	9·781	9·968	318
·1	6·950	7·386	7·997	8·500	9·082	9·741	10·099	10·286	326
·2	7·276	7·712	8·322	8·826	9·408	10·067	10·425	10·612	334
·3	7·610	8·046	8·656	9·160	9·741	10·400	10·759	10·945	342
·4	7·951	8·387	8·998	9·502	10·083	10·742	11·100	11·287	350
4·5	8·301	8·737	9·347	9·851	10·433	11·092	11·450	11·637	357
·6	8·658	9·094	9·705	10·209	10·790	11·449	11·807	11·994	365
·7	9·024	9·460	10·070	10·574	11·155	11·814	12·173	12·359	373
·8	9·397	9·833	10·443	10·947	11·528	12·187	12·546	12·732	381
·9	9·778	10·214	10·824	11·328	11·909	12·568	12·927	13·113	389
5·0	10·166	10·602	11·213	11·717	12·298	12·957	13·315	13·502	397
·1	10·563	10·999	11·609	12·113	12·695	13·353	13·712	13·899	404
·2	10·967	11·403	12·014	12·518	13·099	13·758	14·116	14·303	412
·3	11·380	11·816	12·426	12·930	13·511	14·170	14·529	14·715	420
·4	11·800	12·236	12·846	13·350	13·932	14·590	14·949	15·135	428
5·5	12·228	12·664	13·274	13·778	14·360	15·018	15·377	15·564	436
·6	12·664	13·100	13·710	14·214	14·796	15·454	15·813	16·000	444
·7	13·108	13·544	14·154	14·658	15·239	15·898	16·257	16·444	452
·8	13·559	13·995	14·606	15·110	15·691	16·350	16·708	16·895	460
·9	14·019	14·455	15·065	15·569	16·150	16·809	17·168	17·355	467
6·0	14·486	14·922	15·532	16·036	16·618	17·276	17·635	17·822	
6·5	16·940	17·376	17·987	18·491	19·072	19·731	20·090	20·276	
7·0	19·591	20·027	20·638	21·141	21·723	22·382	22·740	22·927	

TABLE 2d *Values of* ζ

r_e / J	0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	0·111	13	0·265	18	0·536	25	0·802	29	1·149	33	1·589	38	1·848	39	1·988	40
·1	·124	13	·283	21	·561	27	·831	32	1·182	36	1·627	40	1·887	43	2·028	44
·2	·137	15	·304	22	·588	30	·863	34	1·218	40	1·667	44	1·930	47	2·072	48
·3	·152	16	·326	24	·618	31	·897	37	1·258	42	1·711	48	1·977	50	2·120	51
·4	·168	17	·350	25	·649	35	·934	40	1·300	46	1·759	51	2·027	54	2·171	56
1·5	0·185	18	0·375	28	0·684	36	0·974	43	1·346	48	1·810	55	2·081	58	2·227	59
·6	·203	20	·403	29	·720	39	1·017	45	1·394	52	1·865	58	2·139	62	2·286	63
·7	·223	20	·432	31	·759	41	1·062	48	1·446	55	1·923	62	2·201	65	2·349	67
·8	·243	22	·463	33	·800	44	1·110	51	1·501	58	1·985	66	2·266	69	2·416	71
·9	·265	23	·496	34	·844	46	1·161	53	1·559	62	2·051	69	2·335	73	2·487	75
2·0	0·288	24	0·530	36	0·890	48	1·214	57	1·621	64	2·120	72	2·408	76	2·562	78
·1	·312	25	·566	38	·938	51	1·271	59	1·685	67	2·192	76	2·484	80	2·640	83
·2	·337	27	·604	40	·989	53	1·330	62	1·752	71	2·268	80	2·564	84	2·723	86
·3	·364	28	·644	42	1·042	55	1·392	64	1·823	74	2·348	83	2·648	88	2·809	90
·4	·392	28	·686	43	1·097	58	1·456	68	1·897	77	2·431	86	2·736	91	2·899	94
2·5	0·420	31	0·729	45	1·155	60	1·524	70	1·974	80	2·517	90	2·827	95	2·993	97
·6	·451	31	·774	47	1·215	62	1·594	73	2·054	83	2·607	94	2·922	99	3·090	102
·7	·482	32	·821	48	1·277	65	1·667	75	2·137	87	2·701	97	3·021	103	3·192	105
·8	·514	34	·869	51	1·342	67	1·742	79	2·224	89	2·798	101	3·124	106	3·297	109
·9	·548	34	·920	52	1·409	70	1·821	81	2·313	93	2·899	104	3·230	110	3·406	113
3·0	0·582	36	0·972	54	1·479	72	1·902	84	2·406	96	3·003	108	3·340	114	3·519	117
·1	·618	38	1·026	55	1·551	74	1·986	86	2·502	99	3·111	111	3·454	118	3·636	121
·2	·656	38	1·081	58	1·625	77	2·072	89	2·601	102	3·222	115	3·572	121	3·757	124
·3	·694	39	1·139	59	1·702	78	2·161	93	2·703	105	3·337	119	3·693	125	3·881	128
·4	·733	41	1·198	61	1·780	82	2·254	94	2·808	108	3·456	121	3·818	129	4·009	132
3·5	0·774	42	1·259	63	1·862	83	2·348	98	2·916	112	3·577	126	3·947	132	4·141	136
·6	·816	43	1·322	64	1·945	86	2·446	100	3·028	115	3·703	129	4·079	136	4·277	140
·7	·859	44	1·386	66	2·031	89	2·546	103	3·143	117	3·832	132	4·215	140	4·417	144
·8	·903	45	1·452	68	2·120	90	2·649	106	3·260	121	3·964	137	4·355	144	4·561	147
·9	·948	47	1·520	70	2·210	94	2·755	109	3·381	124	4·101	139	4·499	147	4·708	151
4·0	0·995	48	1·590	72	2·304	95	2·864	111	3·505	128	4·240	143	4·646	151	4·859	155
·1	1·043	48	1·662	73	2·399	98	2·975	114	3·633	130	4·383	147	4·797	155	5·014	159
·2	1·091	50	1·735	75	2·497	100	3·089	117	3·763	134	4·530	150	4·952	158	5·173	163
·3	1·141	52	1·810	77	2·597	102	3·206	120	3·897	136	4·680	154	5·110	163	5·336	166
·4	1·193	52	1·887	79	2·699	105	3·326	122	4·033	140	4·834	157	5·273	166	5·502	171
4·5	1·245	54	1·966	80	2·804	107	3·448	125	4·173	143	4·991	161	5·439	170	5·673	174
·6	1·299	55	2·046	82	2·911	110	3·573	128	4·316	146	5·152	164	5·609	173	5·847	178
·7	1·354	55	2·128	84	3·021	112	3·701	130	4·462	149	5·316	168	5·782	177	6·025	182
·8	1·409	58	2·212	86	3·133	114	3·831	134	4·611	153	5·484	172	5·959	181	6·207	186
·9	1·467	58	2·298	88	3·247	117	3·965	136	4·764	155	5·656	175	6·140	185	6·393	189
5·0	1·525	59	2·386	89	3·364	119	4·101	139	4·919	159	5·831	178	6·325	188	6·582	194
·1	1·584	61	2·475	91	3·483	121	4·240	141	5·078	162	6·009	182	6·513	192	6·776	197
·2	1·645	62	2·566	93	3·604	124	4·381	145	5·240	165	6·191	186	6·705	196	6·973	201
·3	1·707	63	2·659	94	3·728	126	4·526	147	5·405	168	6·377	189	6·901	200	7·174	205
·4	1·770	64	2·753	96	3·854	128	4·673	149	5·573	171	6·566	192	7·101	203	7·379	208
5·5	1·834	66	2·849	98	3·982	131	4·822	153	5·744	174	6·758	196	7·304	207	7·587	213
·6	1·900	66	2·947	100	4·113	133	4·975	155	5·918	178	6·954	200	7·511	211	7·800	216
·7	1·966	68	3·047	102	4·246	136	5·130	158	6·096	180	7·154	203	7·722	214	8·016	220
·8	2·034	69	3·149	103	4·382	138	5·288	161	6·276	184	7·357	207	7·936	219	8·236	224
·9	2·103	70	3·252	105	4·520	140	5·449	164	6·460	187	7·564	210	8·155	222	8·460	228
6·0	2·173		3·357		4·660		5·613		6·647		7·774		8·377		8·688	
6·5	2·541		3·910		5·396		6·472		7·629		8·879		9·543		9·885	
7·0	2·939		4·506		6·191		7·399		8·689		10·072		10·802		11·177	

TABLE 2e *Values of q*

These values are used as they stand in Eq. 33 but they must be divided by $180/\pi$ for use in Eq. 31 if β is expressed in degrees

J \ r_c	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
1·0	0·103	24	0·128	26	0·162	28	0·325	34	0·607	43	0·881	48	1·237	54	1·686	59	1·949	62	2·091	64
·1	0·127	28	0·154	30	0·190	32	0·359	40	0·650	47	0·929	54	1·291	59	1·745	66	2·011	69	2·155	70
·2	0·155	33	0·184	35	0·222	37	0·399	44	0·697	53	0·983	59	1·350	66	1·811	73	2·080	76	2·225	78
·3	0·188	38	0·219	40	0·259	41	0·443	49	0·750	59	1·042	65	1·416	72	1·884	79	2·156	83	2·303	84
·4	0·226	43	0·259	45	0·300	47	0·492	55	0·809	65	1·107	72	1·488	79	1·963	86	2·239	90	2·387	93
1·5	0·269	48	0·304	50	0·347	53	0·547	61	0·874	71	1·179	78	1·567	86	2·049	94	2·329	98	2·480	99
·6	0·317	55	0·354	56	0·400	59	0·608	68	0·945	77	1·257	85	1·653	93	2·143	101	2·427	106	2·579	108
·7	0·372	61	0·410	63	0·459	65	0·676	73	1·022	85	1·342	92	1·746	100	2·244	109	2·533	113	2·687	116
·8	0·433	68	0·473	70	0·524	72	0·749	81	1·107	91	1·434	100	1·846	109	2·353	118	2·646	122	2·803	124
·9	0·501	75	0·543	77	0·596	79	0·830	88	1·198	99	1·534	108	1·955	116	2·471	125	2·768	130	2·927	133
2·0	0·576	82	0·620	84	0·675	87	0·918	96	1·297	107	1·642	115	2·071	125	2·596	134	2·898	139	3·060	141
·1	0·658	91	0·704	93	0·762	95	1·014	104	1·404	116	1·757	124	2·196	133	2·730	144	3·037	149	3·201	151
·2	0·749	98	0·797	100	0·857	103	1·118	112	1·520	124	1·881	133	2·329	142	2·874	152	3·186	157	3·352	161
·3	0·847	107	0·897	109	0·960	111	1·230	121	1·644	132	2·014	142	2·471	152	3·026	162	3·343	168	3·513	170
·4	0·954	117	1·006	119	1·071	121	1·351	130	1·776	142	2·156	151	2·623	161	3·188	172	3·511	177	3·683	180
2·5	1·071	125	1·125	127	1·192	130	1·481	139	1·918	152	2·307	161	2·784	171	3·360	182	3·688	188	3·863	190
·6	1·196	135	1·252	138	1·322	140	1·620	149	2·070	162	2·468	171	2·955	182	3·542	193	3·876	198	4·053	202
·7	1·331	146	1·390	147	1·462	150	1·769	160	2·232	171	2·639	182	3·137	192	3·735	203	4·074	210	4·255	212
·8	1·477	156	1·537	158	1·612	160	1·929	170	2·403	183	2·821	192	3·329	203	3·938	215	4·284	220	4·467	224
·9	1·633	167	1·695	169	1·772	171	2·099	181	2·586	194	3·013	204	3·532	214	4·153	226	4·504	232	4·691	235
3·0	1·800	18	1·864	18	1·943	183	2·280	192	2·780	205	3·217	215	3·746	226	4·379	238	4·736	244	4·926	247
·1	1·98	19	2·04	20	2·126	194	2·472	204	2·985	217	3·432	227	3·972	238	4·617	249	4·980	256	5·173	260
·2	2·17	20	2·24	20	2·320	206	2·676	216	3·202	229	3·659	239	4·210	251	4·866	263	5·236	269	5·433	272
·3	2·37	21	2·44	22	2·526	219	2·892	228	3·431	241	3·898	252	4·461	263	5·129	275	5·505	282	5·705	285
·4	2·58	23	2·66	22	2·745	231	3·120	242	3·672	254	4·150	264	4·724	276	5·404	289	5·787	295	5·990	298
3·5	2·81	24	2·88	25	2·976	245	3·362	254	3·926	268	4·414	278	5·000	290	5·693	302	6·082	309	6·288	312
·6	3·05	25	3·13	25	3·221	258	3·616	268	4·194	281	4·692	292	5·290	303	5·995	316	6·391	323	6·600	326
·7	3·30	27	3·38	27	3·479	272	3·884	282	4·475	295	4·984	305	5·593	317	6·311	330	6·714	337	6·926	341
·8	3·57	28	3·65	28	3·751	286	4·166	296	4·770	310	5·289	321	5·910	332	6·641	345	7·051	351	7·267	355
·9	3·85	30	3·93	30	4·037	301	4·462	310	5·080	324	5·610	334	6·242	347	6·986	360	7·402	367	7·622	372
4·0	4·15	31	4·23	32	4·338	315	4·772	326	5·404	340	5·944	350	6·589	362	7·346	375	7·769	382	7·994	384
·1	4·46	33	4·55	33	4·653	332	5·098	341	5·744	354	6·294	365	6·951	378	7·721	391	8·151	398	8·378	401
·2	4·79	34	4·88	34	4·985	346	5·439	357	6·098	370	6·659	382	7·329	393	8·112	406	8·549	414	8·779	418
·3	5·13	36	5·22	36	5·331	364	5·796	373	6·468	387	7·041	397	7·722	410	8·518	424	8·963	430	9·197	434
·4	5·49	37	5·58	38	5·695	379	6·169	390	6·855	403	7·438	414	8·132	426	8·942	440	9·393	447	9·631	451
4·5	5·86		5·96		6·074		6·559		7·258		7·852		8·558		9·382		9·840		10·082	

TABLE 2e—*continued* *Values of q—continued*

These values are used as they stand in Eq. 33 but they must be divided by 180/π for use in Eq. 31 if β is expressed in degrees

J \ r_e	0·2		0·25		0·3		0·45		0·6		0·7		0·8		0·9		0·95		0·975	
4·5	5·86	39	5·96	39	6·074	396	6·559	406	7·258	420	7·852	432	8·558	444	9·382	457	9·840	464	10·082	468
·6	6·25	41	6·35	41	6·470	414	6·965	424	7·678	438	8·284	448	9·002	460	9·839	474	10·304	482	10·550	485
·7	6·66	43	6·76	43	6·884	431	7·389	441	8·116	455	8·732	467	9·462	479	10·313	492	10·786	499	11·035	504
·8	7·09	45	7·19	45	7·315	450	7·830	460	8·571	473	9·199	484	9·941	497	10·805	511	11·285	519	11·539	521
·9	7·54	46	7·64	46	7·765	467	8·290	478	9·044	492	9·683	503	10·438	515	11·316	529	11·804	536	12·060	541
5·0	8·00	47	8·10	49	8·232	487	8·768	496	9·536	511	10·186	521	10·953	534	11·845	548	12·340	555	12·601	559
·1	8·47	51	8·59	50	8·719	506	9·264	516	10·047	529	10·707	541	11·487	554	12·393	568	12·895	576	13·160	678
·2	8·98	52	9·09	52	9·225	525	9·780	536	10·576	549	11·248	560	12·041	573	12·961	586	13·471	594	13·738	598
·3	9·50	54	9·61	55	9·750	544	10·316	554	11·125	568	11·808	580	12·614	592	13·547	607	14·065	614	14·336	618
·4	10·04	56	10·16	56	10·294	566	10·870	576	11·693	590	12·388	601	13·206	614	14·154	628	14·679	635	14·954	640
5·5	10·60	58	10·72	58	10·860	586	11·446	596	12·283	610	12·989	621	13·820	634	14·782	648	15·314	656	15·594	660
·6	11·18	61	11·30	61	11·446	606	12·042	617	12·893	631	13·610	643	14·454	655	15·430	669	15·970	677	16·254	681
·7	11·79	62	11·91	62	12·052	629	12·659	638	13·524	653	14·253	664	15·109	677	16·099	691	16·647	699	16·935	702
·8	12·41	64	12·53	65	12·681	649	13·297	660	14·177	674	14·917	685	15·786	698	16·790	713	17·346	720	17·637	724
·9	13·05	68	13·18	67	13·330	672	13·957	682	14·851	695	15·602	707	16·484	720	17·503	735	18·066	743	18·361	747
6·0	13·73		13·85		14·002		14·639		15·546		16·309		17·204		18·238		18·809		19·108	
6·5	17·40		17·54		17·708		18·396		19·374		20·194		21·154		22·259		22·869		23·188	
7·0	21·70		21·85		22·024		22·762		23·811		24·688		25·714		26·892		27·541		27·881	

TABLE 3

Integrating coefficients

TABLE 3a

(*Eight points, square root,* $\alpha = \frac{1}{2}$, *see* R. & M. 2043[9])

r_c	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975
r_c^2	0·09	0·2025	0·36	0·49	0·64	0·81	0·9025	0·951
Integrating coefficient	0·03307	0·16668	0·13147	0·14282	0·16079	0·14466	0·05481	0·06745

TABLE 3b

Integrating coefficients for root loss

Spinner Radius r_{c0}	Coefficient at 0·2 radius	Coefficient at 0·25 radius	Coefficient at 0·3 radius
0·10	0·06548	−0·00269	0·01721
0·11	0·05946	+0·00222	0·01622
0·12	0·05320	0·00719	0·01521
0·13	0·04678	0·01211	0·01421
0·14	0·04029	0·01686	0·01325
0·15	0·03385	0·02130	0·01235
0·16	0·02756	0·02529	0·01155
0·17	0·02155	0·02869	0·01086
0·18	0·01594	0·03134	0·01032
0·19	0·01087	0·03306	0·00997
0·20	0·00648	0·03367	0·00985
0·21	0·00290	0·03303	0·00997
0·22	+0·00018	0·03114	0·01028
0·23	−0·00169	0·02810	0·01069
0·24	−0·00275	0·02409	0·01106
0·25	−0·00308	0·01935	0·01123
0·26	−0·00280	0·01422	0·01098
0·27	−0·00209	0·00912	0·01007
0·28	−0·00118	0·00460	0·00819
0·29	−0·00037	0·00130	0·00497
0·30	0	0	0

TABLE 4

Specimen Calculations

The following specimen calculations refer to a 5-bladed airscrew operating at $J = 2{\cdot}65$

r_e	0·95	0·95	0·95	
t/c	0·062	0·062	0·062	From blade details.
$r_e \sec \varphi_0$	1·270	1·270	1·270	Table 2b.
M	0·758	0·882	0·882	Equation (40).
θ°	45·0	45·0	49·0	Blade details.
ε_0°	2·94	2·94	2·94	Blade details.
φ_0°	41·59	41·59	41·59	Table 2a.
s	0·064	0·064	0·064	Blade details.
A_0	0·1	0·1	0·1	Table 1 of R. & M. 2036[7].
a_0	156·2	156·2	156·2	Equation (47).
M_L'	0·784	0·784	0·784	Table 3 of R. & M. 2036[7].
$(1 - M_L'^2)^{1/2}$	0·621	0·621	0·621	Table 2 of R. & M. 2036[7].
$(a')^\circ$	97·0	97·0	97·0	Equation (49).
b°	69·6	69·6	69·6	Table 1*b*.
$(\theta - \varphi_0 + \varepsilon_0)^\circ$	6·35	6·35	10·35	
$(a' + b)^\circ$	166·6	166·6	166·6	
$(\alpha_0')^\circ$	3·70	3·70	6·02	$a'(\theta - \varphi_0 + \varepsilon_0)/(a' + b)$.
M_L	0·782	0·782	0·750	Table 3 of R. & M. 2036[7]. $M_L = M_L'$ when $\alpha_0' \leqslant 3^\circ$.
$M - M_L$	<0	0·100	0·132	
C_{LS}	—	−0·002	−0·043	Table 4 of R. & M. 2036[7].
$(1 - M^2)^{1/2}$	0·652	—	—	Table 2 of R. & M. 2036[7].
$(1 - M_L^2)^{1/2}$	—	0·623	0·661	Table 2 of R. & M. 2036[7].
$(1 - M_L^2)^{1/2} C_{LS}/A_0$	—	−0·01	−0·28	
ε°	2·94	2·93	2·66	§ 6.
a°	101·8	97·3	103·3	§ 6.
$(\theta - \varphi_0 + \varepsilon)^\circ$	6·35	6·34	10·07	
$(a + b)^\circ$	171·4	166·9	172·9	
sC_L	0·0371	0·0380	0·0583	Equation (13).
C_L	0·58	0·59	0·91	
$(asC_L)^\circ$	3·78	3·70	6·02	
α_0°	3·78	3·71	6·30	Equal to asC_L in Range 1 and $asC_L - (1 - M_L^2)^{1/2} C_{LS}/A_0$ in Range 2.
β°	2·58	2·64	4·06	Equation (9).
φ°	44·17	44·23	45·65	Equation (7).
$(\alpha_0 + \varphi)^\circ$	47·95	47·94	51·95	Check $= \theta + \varepsilon_0$.
M_D	0·742	0·746	0·592	Table 8 of R. & M. 2036[7].
$M - M_D$	0·016	0·136	0·290	
B_0	0·998	0·997	1·124	Table 6 of R. & M. 2036[7].
C_0	0·00804	0·00804	0·00804	Table 7 of R. & M. 2036[7].
C_{D0}	0·0080	0·0080	0·0090	Equation (52)
C_{DS}	0·0006	0·0342	0·0860	Table 9 of R. & M. 2036[7].
C_D	0·0086	0·0422	0·0950	Equation (51).
sC_D	0·00055	0·00270	0·00608	
$\cos \varphi$	0·717	0·717	0·699	
$\sin \varphi$	0·697	0·698	0·715	
$sC_L \sin \varphi$	0·0258	0·0265	0·0417	
$sC_D \cos \varphi$	0·0004	0·0019	0·0042	
Sum	0·0262	0·0284	0·0459	$(sC_L \sin \varphi + sC_D \cos \varphi)$.
ζ	2·972	2·972	2·972	Table 2d.
q_e	0·0778	0·0844	0·1365	Equation (18).
q	3·975	3·975	3·975	Table 2e.
p_{C1}	0·0066	0·0070	0·0164	Equation (31).
p_{C0}	0·0020	0·0020	0·0023	Equation (33).
p_{CS}	0·0002	0·0087	0·0219	Equation (33).

Specimen Column for Blade Root Loss

r_e	0·25	
t/c	0·23	Blade details.
$\theta°$	74·60	Blade details.
$\varphi_0°$	73·48	Table 2a.
$\varepsilon_0°$	6·20	Blade details.
M	0·610	Equation (40) with Table 2b.
s	0·51	Blade details
$a°$	21·4	§ 6.
$b°$	12·0	Table 1b.
$(\theta - \varphi_0 + \varepsilon_0)°$	7·32	
$(a + b)°$	33·4	
$\alpha_0°$	4·68	Equation (56).
C_D	0·028	Fig. 12 of R. & M. 2036[7].
q	1·319	Table 2e.
$q\,sC_D$	0·0189	

Example of Integrations

r_e	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975	Integrated results
q_e	0·1354	0·1740	0·1784	0·1561	0·1340	0·1030	0·0844	0·0662	k_Q = 0·1248
p_{C1}	0·0208	0·0236	0·0203	0·0150	0·0115	0·0081	0·0070	0·0058	k_{P1} = 0·0132
p_{CO}	0·0058	0·0045	0·0038	0·0033	0·0029	0·0024	0·0020	0·0018	k_{PO} = 0·0030
p_{CS}	0	0	0·0012	0·0021	0·0049	0·0081	0·0087	0·0074	k_{PS} = 0·0034

Root Loss

r_e	0·20	0·25	0·30	Integral
$q\,sC_D$	0·1090	0·0189	0·0028	$\Delta\, k_P$ = 0·00137

$\Delta\eta$ = 0·011

k_{P1}/k_Q = 0·106
k_{PO}/k_Q = 0·024
k_{PS}/k_Q = 0·027
k_P/k_Q = 0·157
η = 0·843
Root loss $\Delta\eta$ = 0·011
Final η = 0·832

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