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The Longitudinal Response of an Aircraft with Auto-Pilot, including an Incidence Term in the Height Control Equation

By

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SUMMARY

This Technical Note gives the results of a theoretical investigation into the dynamic stability of an aircraft under automatic height control. Inaccuracies in the barometric height information due to incidence changes are shown to be destabilising. Either the short period or long period motion may lose damping depending on the sign of the static pressure error.

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1 Introduction

Aircraft height information is often obtained from a static pressure head or vent. If a static pressure signal is fed to an auto-pilot for the purpose of controlling height, errors in this signal are likely to result in poor height keeping. For instance an American auto-pilot firm has mentioned the occurrence of a 10-15 seconds period pitching oscillation, which they say was due to a positive incidence change producing the equivalent of a decrease in height. 50 ft height error per degree incidence was enough to produce an oscillation and after much flight testing the firm concluded that the signal error should not be allowed to exceed 25 ft per degree.

Also the performance of a British bomber when flying under automatic height control was improved considerably by moving the static vent. There is reason to assume that the improvement was due to a reduction in some spurious content of the static pressure signal.

Some of the reasons for faulty static pressure are: (1) the actual position of the static vent, (2) the position, size, number and condition of the orifices in the static head[1,2], (3) the angle of sideslip[1], (4) the angle of incidence[1,3,4]. This Note deals with the effect of changes of incidence on the static pressure, and hence on the longitudinal stability of a height-controlled aircraft. Analogue computer responses representing pitch and height errors are shown, demonstrating the effect of this spurious height signal being fed into the auto-pilot of a medium bomber. (see Table I). A positive change in incidence may change the static pressure signal in a sense equivalent to either a positive or negative height excursion, and both possibilities have been considered.

Records are shown for an idealised auto-pilot and also for the more practical case with phase advance in the elevator channel and lag on a pitch gyro platform.

All responses are to a horizontal step gust.

2 Control equations

The equations are for a typical modern auto-pilot (Fore-and-aft pendulum and rate gyro on pitch platform). The meaning of symbols not defined can be found in R & M 1801[5].

$$D\eta = \frac{1 + NT_1 D}{1 + T_1 D} \cdot G\left(D\theta - D\,\theta_p + \frac{1}{T_m} x\right) \qquad (1)$$

$$\theta_p = -\frac{1}{1 + T_2 D} \cdot P\left[(h + I\alpha) + \frac{1}{T_h}\int (h + I\alpha)\, dt\right] \qquad (2)$$

where

D = d/dt

x = pendulum signal = $\theta - \theta_p + D(u-u_g)/g$

θ_p = auto-pilot pitch platform angle

u_g = a head-on gust disturbance, u being the aircraft speed disturbance measured relative to the air

N is a phase advance circuit parameter

T_1, T_2 are network time constants

T_m is the pendulum monitor time constant

T_h is the height integrator time constant

G and P are control gearings.

The height deviation h is related to θ and the incidence α by the equation

$$Dh = V_e (\theta - \alpha) = V_e \theta - w ,$$

where V_e = true speed in equilibrium, and small deviations only are assumed. $(h + I\alpha)$ is the contaminated height signal, and I defines the strength of the incidence effect.

The control equations in their non-dimensional form are:

$$\hat{D}\eta = \frac{1 + N \tau_1 \hat{D}}{1 + \tau_1 \hat{D}} \cdot G \left(\hat{D}\theta - \hat{D}\, \theta_p + \frac{1}{\hat{T}_m} x \right)$$

and

$$\theta_p = - \frac{1}{1 + \tau_2 \hat{D}} \cdot \hat{P} \left[(\hat{h} + \hat{I}\alpha) + Q \int (\hat{h} + \hat{I}\alpha)\, d\tau \right]$$

$$\hat{D}\hat{h} = \theta - \hat{w} = \theta - \alpha, (\hat{w} = \alpha)$$

where;

$$\hat{D} \equiv \frac{d}{d\tau} , \qquad \tau = \frac{t}{\hat{t}} , \qquad \hat{t} = \frac{m}{\rho\, SV_e} ,$$

$$\tau_1 = \frac{T_1}{\hat{t}} , \qquad \tau_2 = \frac{T_2}{\hat{t}} ,$$

$$\hat{T}_m = \frac{\hat{t}}{T_m} , \qquad Q = \frac{\hat{t}}{T_h} ,$$

$$\hat{P} = P\, V_e\, \hat{t} , \quad \hat{I} = \frac{I}{V_e \hat{t}} ,$$

$$x = \theta - \theta_p + \frac{2}{C_L} \hat{D} (\hat{u} - \hat{u}_g) .$$

The aircraft non-dimensional equations are given in Appendix I.

$\hat{P} = 0.6$ (equivalent to 1° of platform for 65 ft change in height) and G = 1.0 are normal settings of the height control and elevator gearings for this type of aircraft.

2.1 Magnitude of incidence signal

The incidence contaminated height signal is $(h + I\alpha)$;

Thus 1 degree of incidence is equivalent to $\frac{I}{57.3}$ ft.

Since $I = \hat{I}\,V_e\,\hat{t}$

1 degree of incidence $\equiv \frac{\hat{I}\,V_e\,\hat{t}}{57.3}$ which, for the aircraft of Table I, becomes $39.15\,\hat{I}$ ft. It is convenient to refer to the magnitude of the incidence effect by quoting this figure of "$39.15\,\hat{I}$ ft per degree."

3 Discussion of analogue computer results

Responses in pitch and height to a sustained horizontal gust are shown for both positive and negative incidence effect as defined in 3.1 and 3.2. The overall height gearing normally used in an aircraft of this type is one degree of pitch (or pitch gyro platform) for 65 ft height change and records are shown for this and two other gearings, one weaker and one stronger. The time constant of the height integral term is 4.5 seconds throughout. The phase advance network in the elevator channel of the auto-pilot, and some phase lag on the pitch platform, have been included for some of the cases in an attempt to simulate the practical auto-pilot more closely.

The response of the uncontrolled aircraft is shown in Fig.1(a), and in Fig.1(b) is the response with the auto-pilot without lags and assuming a perfect height signal. The normal pitching motion of the aircraft is a combination of two oscillatory modes, one very heavily damped and having a short period (9 seconds), and the other poorly damped having a relatively long period (98 seconds). The latter oscillation is apparent in Fig.1(a).

3.1 "Positive" incidence effect

For the purpose of this Note, if the nature of the error in the static pressure is such that a positive change in incidence results in a decrease in pressure (i.e. an apparent increase in height), this is defined as a "positive" effect.

Fig.2(a) shows that as the incidence error increases (positively) the damping of the long period motion is decreased and the initial responses in pitch and height are increased. Figs.2(b) and 2(c) show the same effect for different height gearings. Moderate amounts of lag make little difference to the stability (Figs.3(a) and 3(b)), and Fig.4 shows how the damping is reduced by the integral terms i.e. integral of height and the pitch pendulum.

3.2 "Negative" incidence effect.

As stated in para.1 a positive change in incidence may in some circumstances result in an increase in static pressure or an apparent decrease in height. This we have called a "negative" effect.

Appendix II shows that the negative effect is equivalent to reducing the derivative m_w or the stiffness of the short period motion. Damping indices and period of oscillation for the controlled aircraft are plotted against varying $\omega \left(= -\frac{\mu_1\, m_w}{i_B}\right)$ in Figs.10 and 11 and the stability quartic is seen to factorise into a complex pair and two real roots. The value of the smaller root depends on the ratio of the last two coefficients of the stability quartic which for a simple pitch control is approximately

$$\frac{-\omega k\ z_u + \delta\ G_\theta\ P_1}{\delta\ G_\theta\ N_1} \qquad \text{(see Appendix II)}$$

This is very small and changes very little for all values of ω. The frequency of the oscillation decreases as ω is reduced, the value of the other real root increases and since the total damping remains constant there is therefore a reduction in the damping of the oscillation.

The analogue computer results show responses for three height control gearings with varying incidence error (Figs. 5(a)(b) and (c)). As the period of the oscillation in the controlled case is only three seconds, auto-pilot lag has a greater effect and it appears to be advantageous to have a certain amount of it in the control as may be seen by comparing Fig. 5(a) with Figs. 6(a)(b) and (c).

4 Roots of the stability equation

The roots of the 7th order stability equation have been calculated for a height signal strength of 1 degree equivalent pitch change for 65 ft and an incidence strength I varying between -80 ft per degree and +400 ft per degree. Figs. 8 and 9 show the results in terms of damping indices and oscillation periods. (If a mode of motion has a damping index k, the amplitude of the mode is proportional to exp(-kt)). With positive incidence effect the short period motion remains heavily damped and the frequency increases slightly (Fig. 8), while the long period slowly loses damping (Fig. 9). With negative effect, the short period motion rapidly loses damping, becoming unstable at approximately $\hat{I} = -1.5$ (-60 ft per degree). The long period damping is practically unaffected but the period lengthens.

5 Conclusions

In a controlled aircraft the effect of an incidence-contaminated height signal is detrimental to the stability irrespective of the sign of the error.

If a positive incidence change results in a positive height error signal the main effect is on the long period motion, whereas if a positive incidence change causes a negative height error signal the short period motion is affected. The positive incidence effect does not seem to be serious in that a strength of several hundreds of feet per degree incidence is required before the long period damping becomes very low.

The negative incidence effect on the other hand can make the short period oscillation unstable when the error strength is of the order of 60 ft per degree. The critical negative incidence strength will vary with the aircraft aerodynamics and also with auto-pilot gearings. An approximate formula quoted in Appendix II indicates that the negative incidence effect becomes less when the aircraft has a large manoeuvre margin and when the auto-pilot θ gearing is large. Since the incidence signal originates as a component of the "height signal", an increase in the height gearing will amplify the destabilising influence.

The evidence of the practical cases mentioned in the introduction supports the conclusions drawn from the theoretical work. The period quoted by the American firm is longer than that obtained in this investigation, but nothing is known of the characteristics of the aeroplane, or of the flight conditions of their tests. Fig. 10 illustrates that with certain values of pitch gearing and the aircraft derivative m_w the period of the oscillation may be increased to 12 seconds.

LIST OF SYMBOLS

a_o, a_1 etc	coefficients of stability equation.
C_L	lift coefficient.
D	differential operator, d/dt.
$\hat{D}$	differential operator, $d/d\tau$.
G	auto-pilot elevator gearing.
g	acceleration due to gravity.
h	height error.
$\hat{h}$	non-dimensional height error = $\theta - \hat{w}$.
I	strength of incidence error term in control equation.
$\hat{I}$	non-dimensional form of I; $\hat{I} = \frac{I}{V_e \hat{t}}$.
i_B	aircraft inertia coefficient about y-axis.
k	damping index (see Section 4).
$m_u, m_w, m_{\dot{w}}, m_q, m_\eta$	pitching moment derivatives.
N	constant in phase advance network.
P	auto-pilot height lock gearing.
$\hat{P}$	non-dimensional form of P; $\hat{P} = P V_e \hat{t}$.
Q	strength of height integral term in non-dimensional equation; $Q = \frac{\hat{t}}{T_h}$.
T_1, T_2	time constants of phase advance network and platform lag.
T_m, T_h	time constants of pitch pendulum and height integrator.
$\hat{t}$	unit of time in non-dimensional equations, $\frac{m}{\rho S V_e}$
u_g	horizontal gust disturbance.
$\hat{u}_g$	non-dimensional form of u_g; $\hat{u}_g = \frac{u_g}{V_e}$.
u	speed error along x-axis (relative to gust).
$\hat{u}$	non-dimensional form of u; $\hat{u} = \frac{u}{V}$.
V_e	forward speed of aircraft in equilibrium.
w	speed error along z-axis.
$\hat{w}$	non-dimensional form of w; $\hat{w} = \frac{w}{V}$
x	quantity measured by pitch pendulum. $x = \theta - \theta_p + D(u-u_g)/g = \theta - \theta_p + 2\hat{D}(\hat{u}-\hat{u}_g)/C_L$

LIST OF SYMBOLS (CONTD.)

x_u, x_w, z_u, z_w	longitudinal force derivatives.
α	angle of incidence.
δ	elevator effect coefficient, $-\dfrac{\mu_1\, m_\eta}{i_B}$.
κ	$-\dfrac{\mu_1\, m_u}{i_B}$.
λ	root of stability equation.
μ_1	aircraft relative density, $\dfrac{m}{\rho S \ell}$.
ν	$-\dfrac{m_q}{i_B}$.
τ	time in airsecs, $\dfrac{t}{\hat{t}}$.
$\tau_1,\ \tau_2$	non-dimensional form of T_1, T_2; $\tau_1 = \dfrac{T_1}{\hat{t}}$.
θ	angle of pitch.
θ_p	angle of pitch gyro platform.
ω	$-\dfrac{\mu_1\, m_w}{i_B}$.
χ	$-\dfrac{\mu_1\, m_{\dot{w}}}{i_B}$
η	elevator angle
C_1, D_1, N_1, P_1, R_1	portmanteau functions defined in Appendix II.

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4	J. E. Nethaway	Low speed wind tunnel calibration of a Mk.9A pitot-static head. C.P. 244 March, 1955.
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APPENDIX I

Aircraft non-dimensional equations

The aircraft and control equations in their non-dimensional form are:-

$$\hat{D}(\hat{u} - \hat{u}_g) = x_u \hat{u} + x_w \hat{w} - \frac{C_L}{2} \theta$$

$$\hat{D}(\hat{w} - \theta) = z_u \hat{u} + z_w \hat{w}$$

$$\hat{D}^2 \theta = - \kappa \hat{u} - \chi \hat{D} \hat{w} - \omega \hat{w} - \nu \hat{D} \theta - \delta \eta$$

$$\hat{D} \hat{h} = \theta - \hat{w}$$

$$\hat{D} \eta = \left(\frac{1 + N \tau_1 \hat{D}}{1 + \tau_1 \hat{D}} \right) G . \left(\hat{D} \theta - \hat{D} \theta_p + E . \frac{C_L}{2} \hat{x} \right)$$

$$\theta_p = - \frac{1}{1 + \tau_2 \hat{D}} . \hat{P} \left[(\hat{h} + \hat{I} \hat{w}) + Q \int (\hat{h} + \hat{I} \hat{w}) \, d\tau \right]$$

$$\hat{x} = \theta - \theta_p + \frac{2}{C_L} \hat{D}(\hat{u} - \hat{u}_g).$$

where,

$$\kappa = - \frac{\mu_1 m_u}{i_B} \qquad \chi = - \frac{\mu_1 m_{\dot{w}}}{i_B}$$

$$\omega = - \frac{\mu_1 m_w}{i_B} \qquad \nu = - \frac{m_q}{i_B}$$

$$\delta = - \frac{\mu_1 m_\eta}{i_B}$$

APPENDIX II

Coefficients of stability equation

The stability equation for the uncontrolled aircraft is

$$a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_o = 0$$

where

$$a_4 = 1$$

$$a_3 = \nu + \chi - x_u - z_w$$

$$a_2 = (x_u z_w - x_w z_u) - \nu (x_u + z_w) - x_u \chi + \omega$$

$$a_1 = \nu (x_u z_w - x_w z_u) - \frac{C_L}{2} \cdot z_u \chi - x_u \omega - \kappa \left(\frac{C_L}{2} - x_w\right)$$

$$a_o = - \frac{C_L}{2} \cdot z_u \omega + \frac{C_L}{2} z_w \kappa .$$

With the basic control equation,

$$\eta = G_\theta \theta + \hat{G}_h \hat{h} + \hat{G}_{\bar{h}} \int \hat{h} \, d\tau$$

i.e. elevator proportional to angle of pitch, height error and integral height error, the additional terms to the above coefficients are:

$$\begin{array}{llll}
a_4 & 0 & & \\
a_3 & 0 & & \\
a_2 & \delta G_\theta & & \\
a_1 & \delta G_\theta N_1 & & \\
a_o & \delta G_\theta P_1 & + \delta \hat{G}_h C_1 & \\
a_{-1} & & + \delta \hat{G}_h (P_1 - R_1) & + \delta \hat{G}_{\bar{h}} C_1 \\
a_{-2} & & & + \delta \hat{G}_{\bar{h}} (P_1 - R_1)
\end{array}$$

where;

$$C_1 = - z_w$$

$$N_1 = -(x_u + z_w)$$

$$P_1 = (x_u z_w - x_w z_u)$$

$$R_1 = - \frac{C_L}{2} z_u .$$

This is the normal long period control law with height lock. Since there is no addition to coefficient a_3, the sum damping of the system is unaltered, but merely redistributed. The G_θ gearing has the effect of increasing the short period frequency, and decreasing the phugoid, while damping is transferred from the short period motion to the long.

The effect of incidence on the static pressure can be represented in the elevator control equation by an equivalent incidence or $\hat{w}$ term ($\alpha = \hat{w}$).

i.e $$\eta = G_\theta\,\theta + \hat{G}_h\,(\hat{h} + \hat{I}\,\hat{w}) + G_{\bar{h}}\left(\int \hat{h}\,d\tau + \hat{I}\int \hat{w}\,d\tau\right)$$

This gives the following additional terms to the coefficients;

a_4	0	
a_3	0	
a_2	$\delta\,\hat{G}_h\,\hat{I}$	
a_1	$\delta\,\hat{G}_h\,\hat{I}\,Q_1$	$+\,\delta\,\hat{G}_{\bar{h}}\,\hat{I}$
a_o	$\delta\,\hat{G}_h\,\hat{I}\,R_1$	$+\,\delta\,\hat{G}_{\bar{h}}\,\hat{I}\,Q_1$
a_{-1}		$+\,\delta\,\hat{G}_{\bar{h}}\,\hat{I}\,R_1$

where $Q_1 = -x_u$.

For the aircraft of Table I the coefficients of the stability equation have the following values:-

a_4	1.0
a_3	10.23
a_2	$31.228 + 165.6\,G_\theta + 165.6\,\hat{G}_h\,\hat{I}$
a_1	$0.893 + 427.25\,G_\theta + 3.312\,\hat{G}_h\,\hat{I} + 165.6\,\hat{G}_{\bar{h}}\,\hat{I}$
a_o	$1.226 + 9.141\,G_\theta + 423.936\,\hat{G}_h + 7.982\,\hat{G}_h\,\hat{I} + 3.312\,\hat{G}_{\bar{h}}\,\hat{I}$
a_{-1}	$1.159\,\hat{G}_h + 423.936\,\hat{G}_{\bar{h}} + 7.982\,\hat{G}_{\bar{h}}\,\hat{I}$
a_{-2}	$1.159\,\hat{G}_{\bar{h}}$

For $G_\theta = 1.0$, $\hat{G}_h = 0.6$, $\hat{G}_{\bar{h}} = 0.0252$ the relative contributions to each of the coefficients are indicated in the following Table.

Contribution From	Basic Aircraft	G_θ Term	$\hat{G}_h$ Term	$\hat{G}_{\bar{h}}$ Term	$\hat{G}_h\,\hat{I}$ Term	$\hat{G}_{\bar{h}}\,\hat{I}$ Term
a_4	1.0					
a_3	10.23					
a_2	31.228	165.6			99.36 $\hat{I}$	
a_1	0.893	427.25			1.987$\hat{I}$	4.173 $\hat{I}$
a_o	1.226	9.14	254.362		4.739 $\hat{I}$	0.083 $\hat{I}$
a_{-1}			0.696	10.683		0.201 $\hat{I}$
a_{-2}				0.029		

The last two coefficients a_{-1}, a_{-2} are very small compared with the rest. Thus it appears reasonable to drop these terms, reducing the stability sextic to a quartic. This is equivalent to extracting two small real roots approximately equal to $\frac{a_{-2}}{a_{-1}}$ and $\frac{a_{-1}}{a_o}$. Taking the main contributions to a_o, a_{-1}, a_{-2} the two roots become (for $\hat{I} = 1$):

$$\frac{a_{-2}}{a_{-1}} \simeq \frac{\delta\, \hat{G}_{\bar{h}}\, (P_1 - R_1)}{\delta\, \hat{G}_{\bar{h}}\, C_1} = \frac{P_1 - R_1}{-z_w} = 0.00273$$

$$\frac{a_{-1}}{a_o} \simeq \frac{\delta\, \hat{G}_{\bar{h}}\, C_1}{\delta\, \hat{G}_h\, C_1} = \frac{\hat{G}_{\bar{h}}}{\hat{G}_h} = \frac{1}{\hat{T}_h} \qquad \left(T_h = \text{height integrator time constant}\right)$$

The real root $\frac{P_1 - R_1}{-z_w}$ will be a subsidence if $P_1 - R_1 > 0$.

Both roots are effectively independent of $\hat{I}$ for the range of values considered. In the remaining quartic the only large contribution from the incidence effect is to coefficient a_2; the quartic now becoming

a_4	1.0	
a_3	10.23	
a_2	196.828	$+ 99.36\, \hat{I}$
a_1	428.143	
a_o	264.728	

The main effect of changes in a_2 will be in the frequencies of the oscillating modes. (cf G_θ gearing). If I is positive the coefficient a_2 will increase and the effect on the long period motion will be comparatively gradual. With negative values of I the coefficient a_2 will diminish rapidly and eventually become negative. Thus with only small magnitudes of the "negative" effect there is an instability in one of the modes. In this case, the short period motion rapidly becomes unstable and increases in period. Thus with "negative effect" the stability change is rapid, while with "positive effect" the change is comparatively slow. Using Routh's Discriminant for the quartic, for positive stability,

$$a_1\, a_2\, a_3 - a_o\, a_3^2 - a_1^2 > 0$$

or

$$a_2 > \frac{a_1}{a_3} + \frac{a_o\, a_3}{a_1}$$

In terms of aeroplane and auto-pilot parameters the above expression becomes approximately equal to;

$$\omega - \frac{\hat{G}_h}{G_\theta}\,(N_1 + \nu + \chi) + \frac{\delta\, G_0^{\prime}(\nu + \chi)}{(N_1 + \nu + \chi)} + \delta\, \hat{G}_h\, \hat{I} > 0\,.$$

i.e. for aircraft of Table I,

$$148.651 + 99.36\, \hat{I} > 0.$$

Thus for positive values of I, both modes of the quartic are always stable, but with negative I, one mode will become unstable when

$$148.651 + 99.36\,\hat{I} = 0,$$

i.e

$$\hat{I} = -1.496.$$

Thus the negative effect is the more serious, since with positive I, one of the modes may lose damping gradually, but will never become unstable, while with negative I the short period motion rapidly becomes unstable.

TABLE I

Aerodynamic and auto-pilot data

Height	40,000 ft	S	960 sq ft
Mach number	0.75	ℓ	32 ft
Weight W	40,620 lb	V_i	214 kts
W/S	42.4	V	726 ft/sec
Aspect Ratio	4.26	C_L	0.274
μ_1	69	$\hat{t}$	3.09
i_B	0.1	$V\hat{t}$	2243.3
x_u	-0.02	z_u	-0.365
x_w	0.011	z_w	-2.56
m_u	0.00123	κ	-0.849
m_w	-0.0282	ω	19.5
$m_{\dot{w}}$	-0.00457	χ	3.15
m_q	-0.45	ν	4.5
m_η	-0.24	δ	165.6
G	1.0	N	3.0
T_m	11.5 sec	T_n	45 sec

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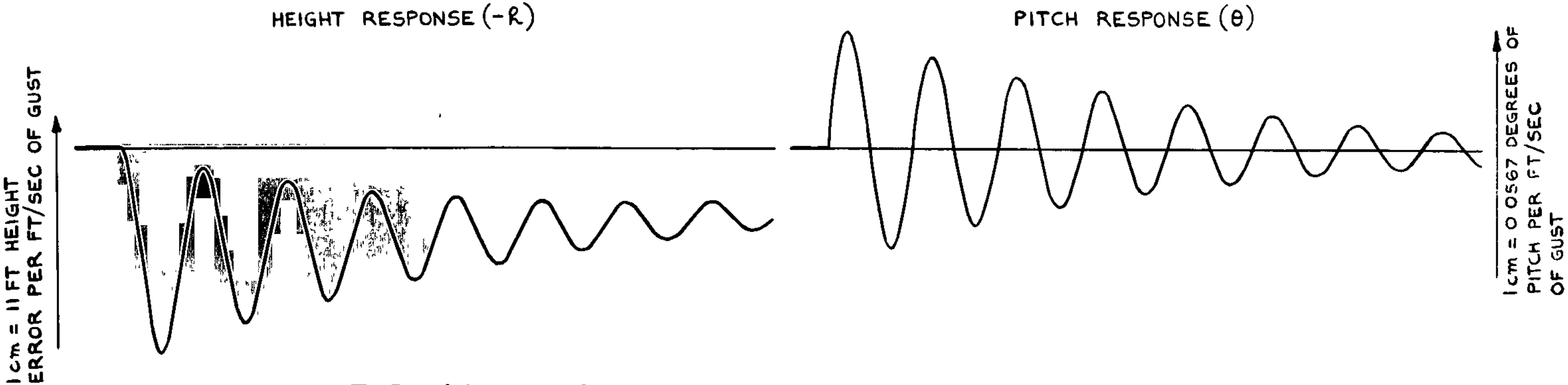
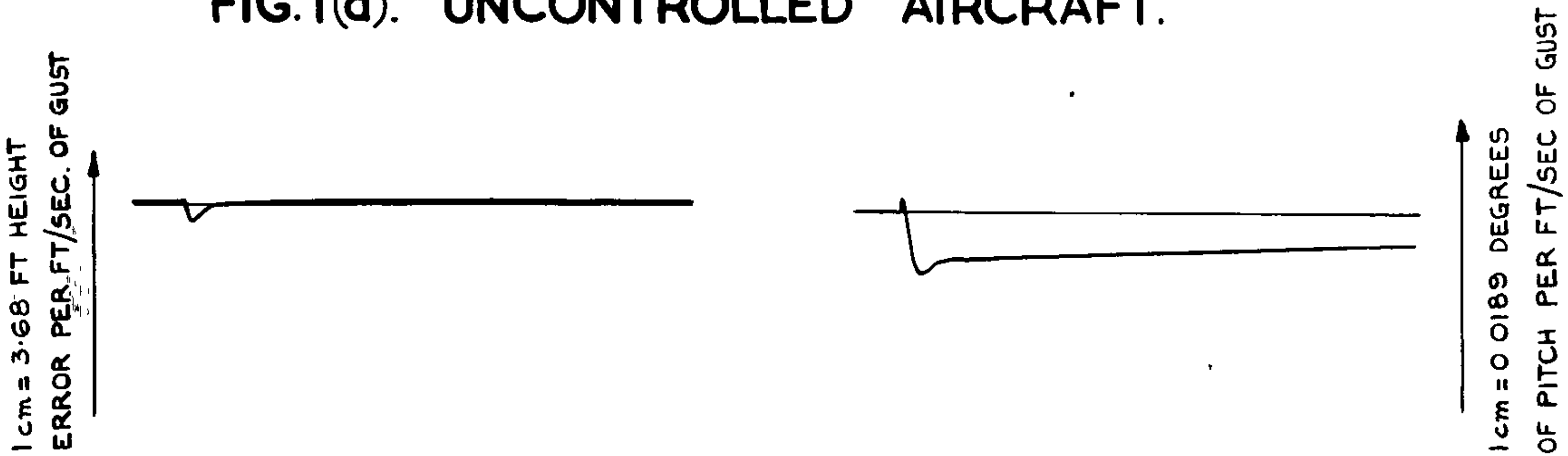


FIG. 1(a). UNCONTROLLED AIRCRAFT.

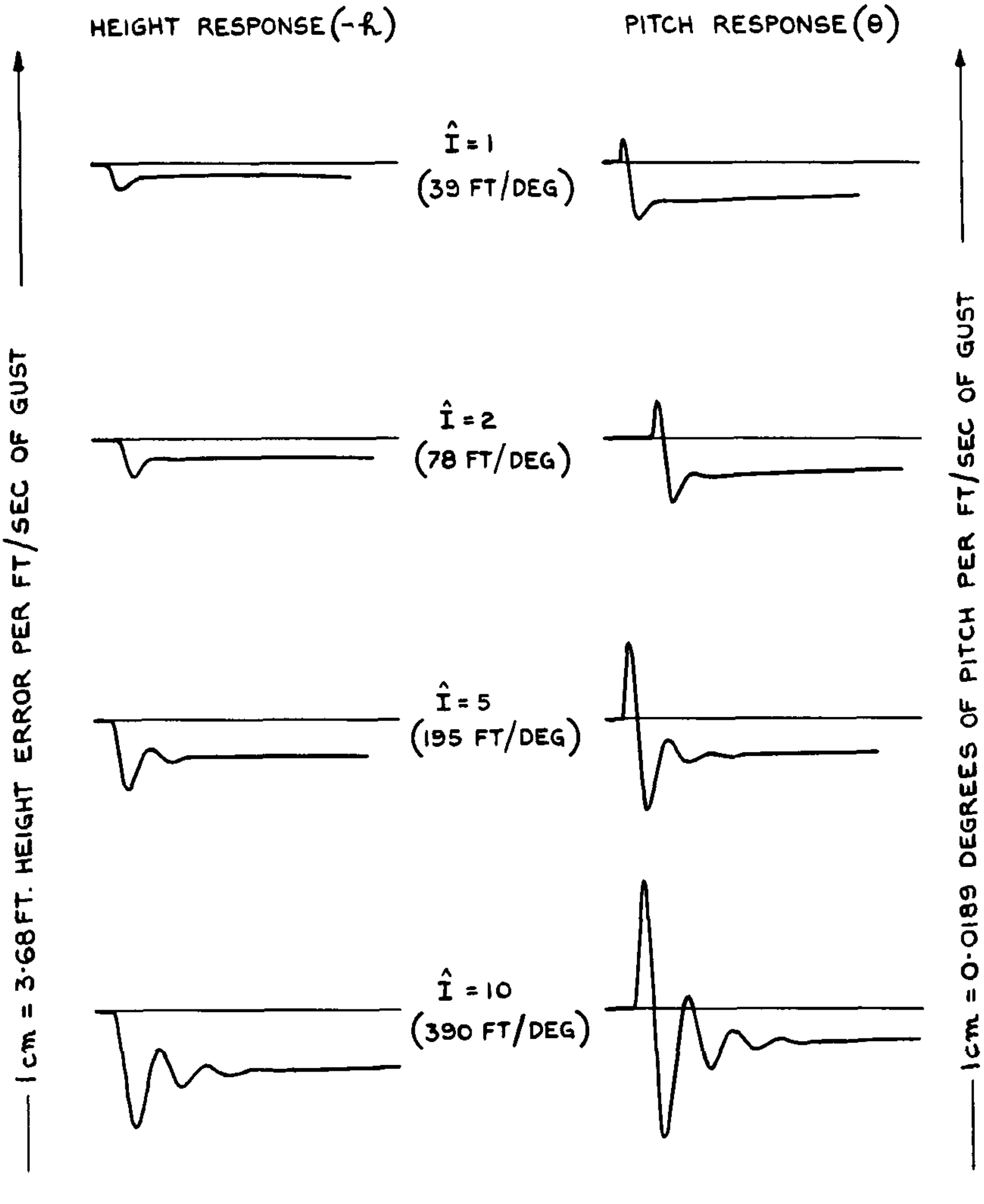


TIME SCALE : 1 cm = 61·8 SECS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT

HEIGHT INTEGRATOR TIME CONSTANT = 4·5 SECS

FIG. 1(b). AIRCRAFT WITH AUTOPILOT – NO PLATFORM LAG OR INCIDENCE EFFECT.



TIME SCALE : 1 cm = 61·8 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT

HEIGHT INTEGRATOR TIME CONSTANT = 45 SECS

FIG. 2 (a). POSITIVE INCIDENCE EFFECT
– NO PLATFORM LAG.

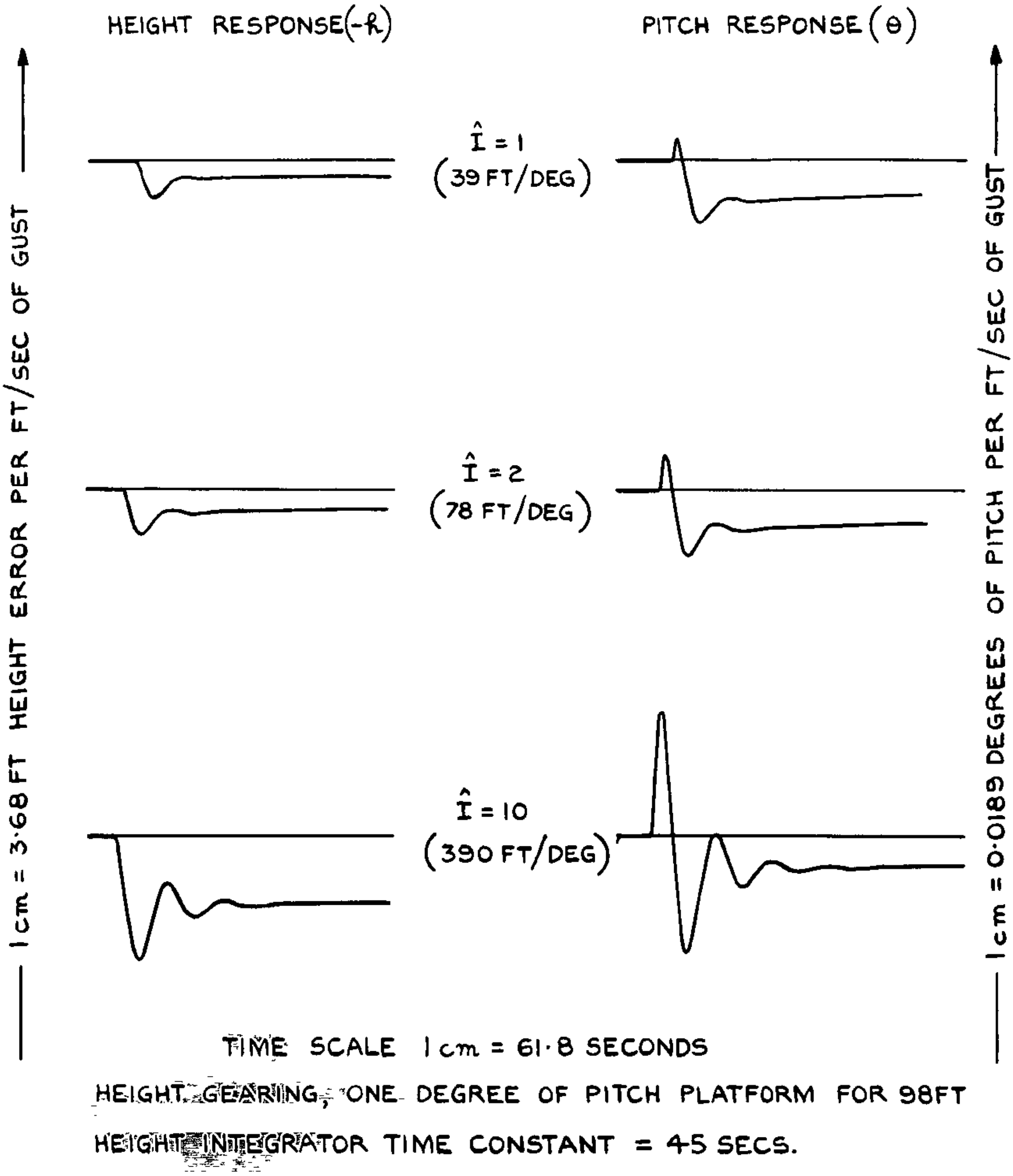
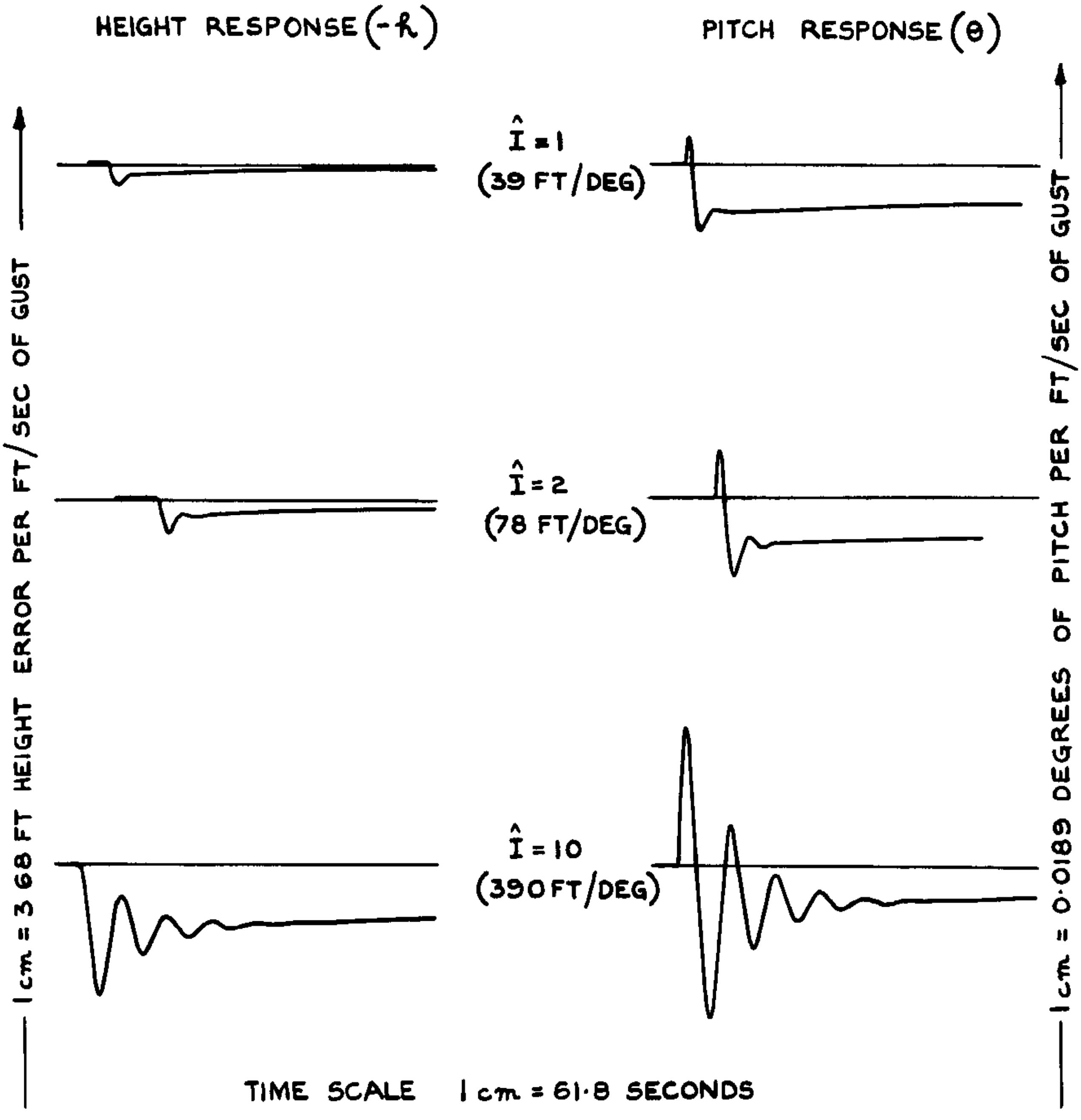


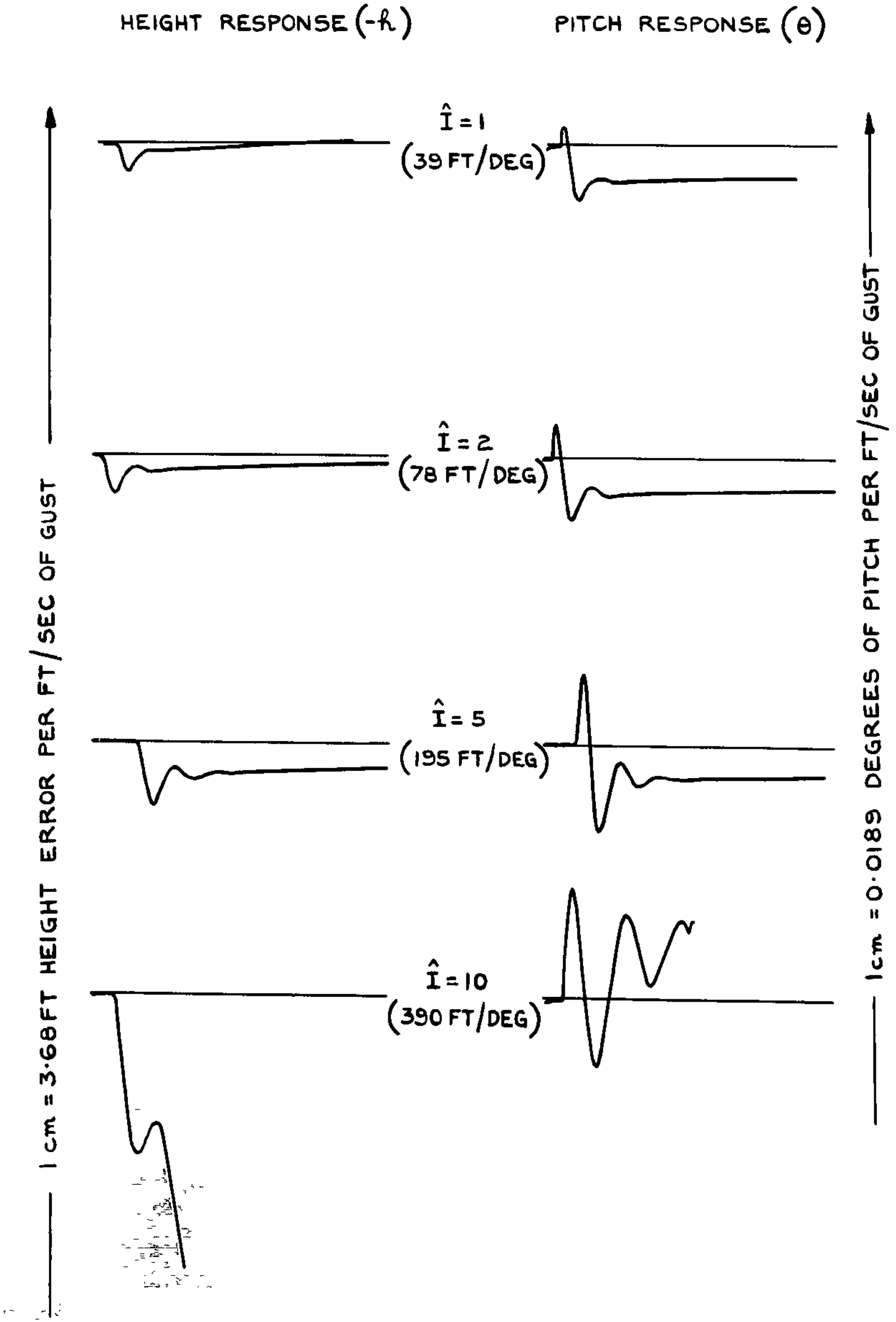
FIG. 2(b). POSITIVE INCIDENCE EFFECT
– NO PLATFORM LAG.



HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 39 FT.

HEIGHT INTEGRATOR TIME CONSTANT = 45 SECS

FIG. 2 (c). POSITIVE INCIDENCE EFFECT
— NO PLATFORM LAG.

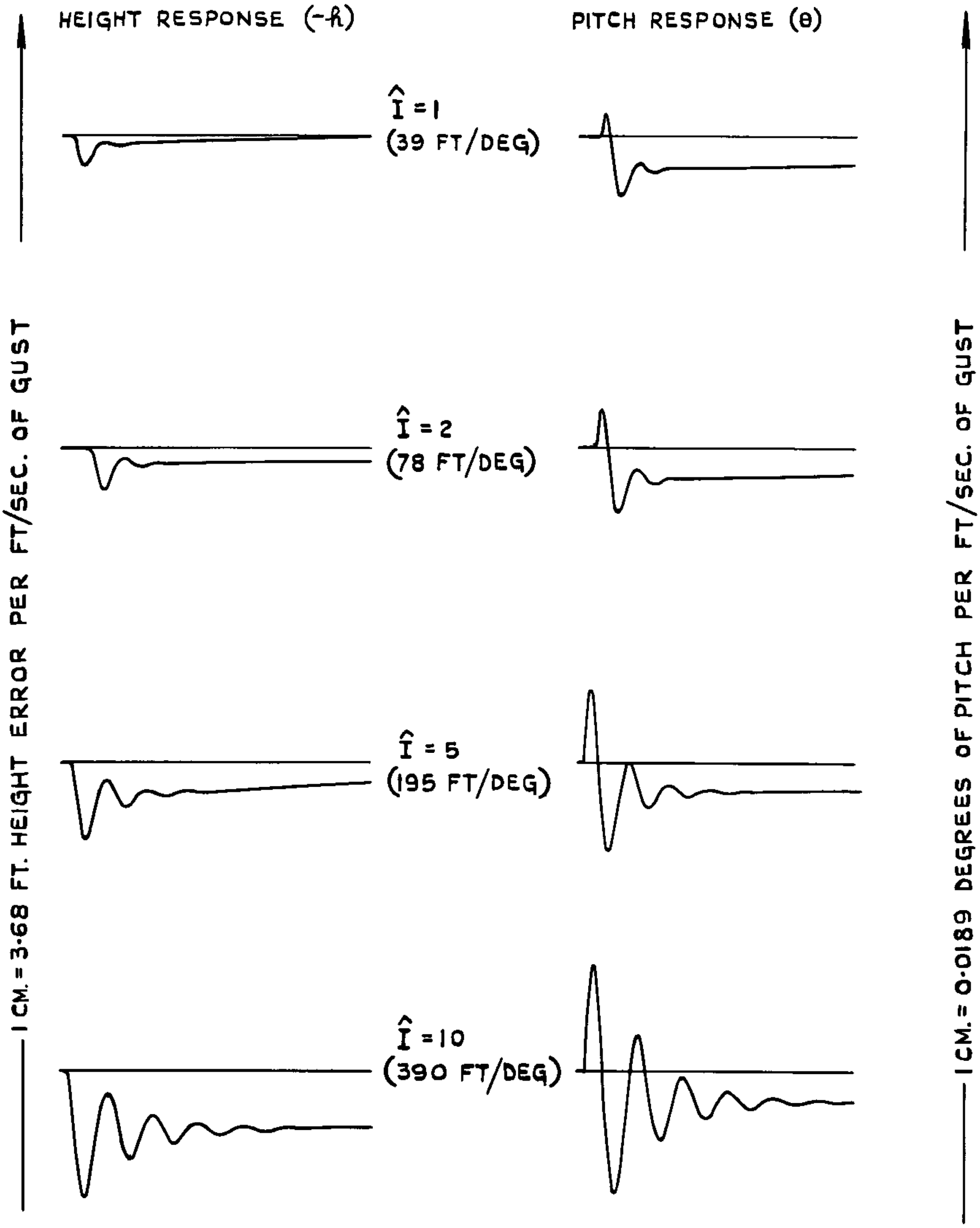


TIME SCALE: 1 cm = 61·8 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT.

HEIGHT INTEGRATOR TIME CONSTANT, 45 SECS

FIG. 3 (a). POSITIVE INCIDENCE EFFECT— PLATFORM LAG (T_2 = 0·309 SECONDS) PHASE ADVANCE ON ELEVATOR.

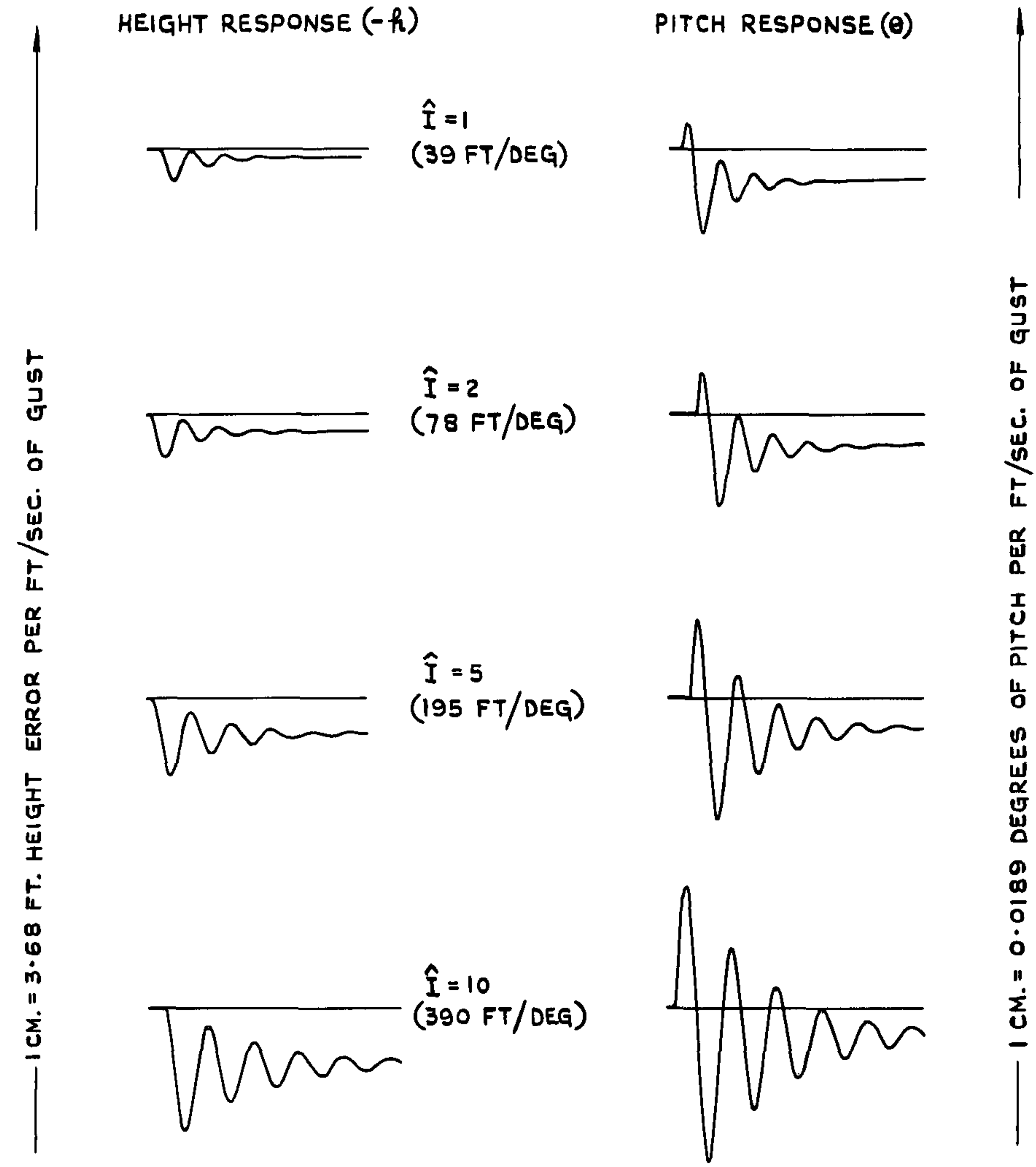


TIME SCALE : 1 CM. = 61·8 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT.

HEIGHT INTEGRATOR TIME CONSTANT, 45 SECS.

FIG.3(b). POSITIVE INCIDENCE EFFECT — PLATFORM LAG (T_2 = 0·927 SECONDS) PHASE ADVANCE ON ELEVATOR.

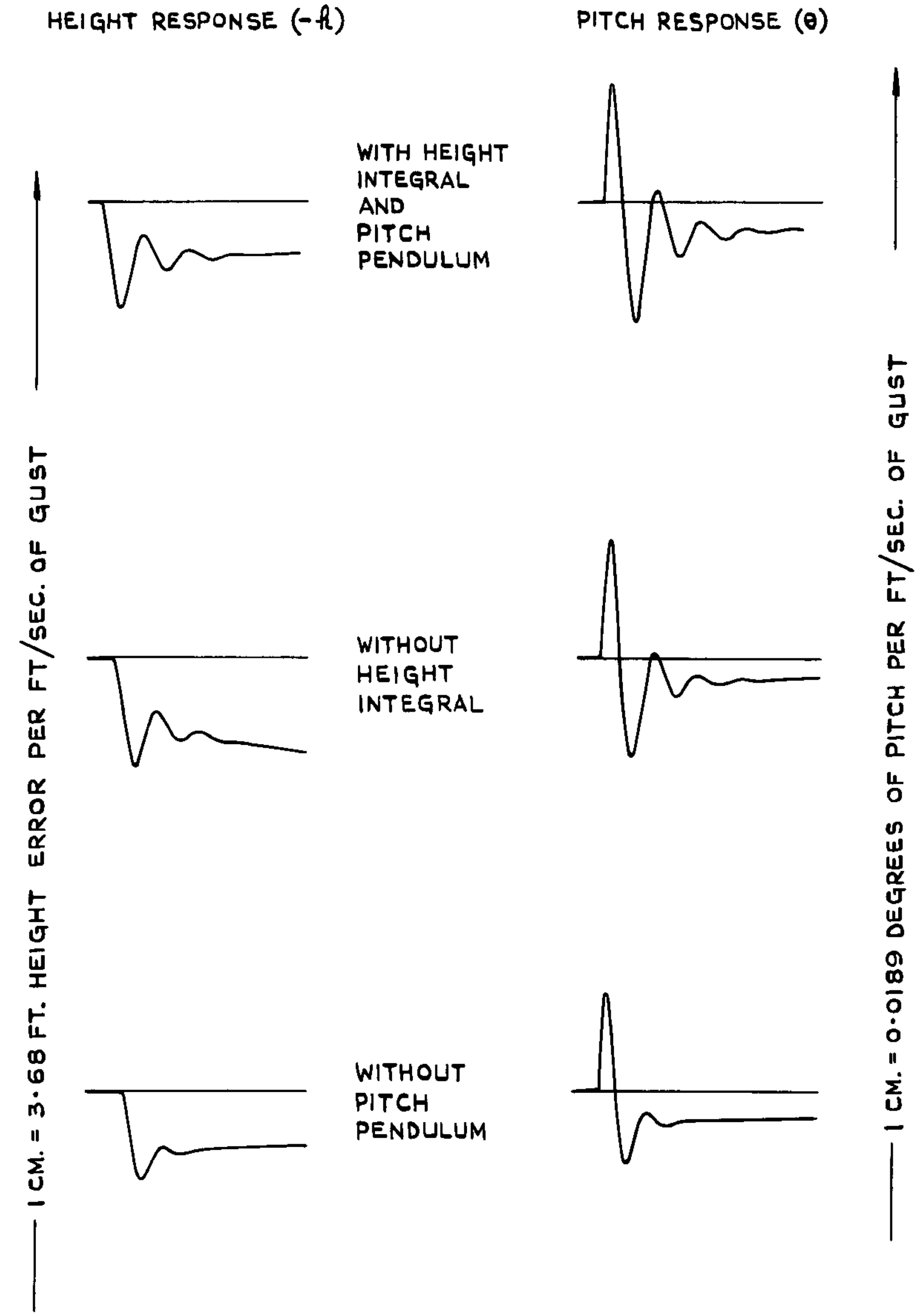


TIME SCALE : 1 CM. = 61·8 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT.

HEIGHT INTEGRATOR TIME CONSTANT, 45 SECS.

FIG.3(c). POSITIVE INCIDENCE EFFECT – PLATFORM LAG (T_2 = 3·09 SECONDS) PHASE ADVANCE ON ELEVATOR.

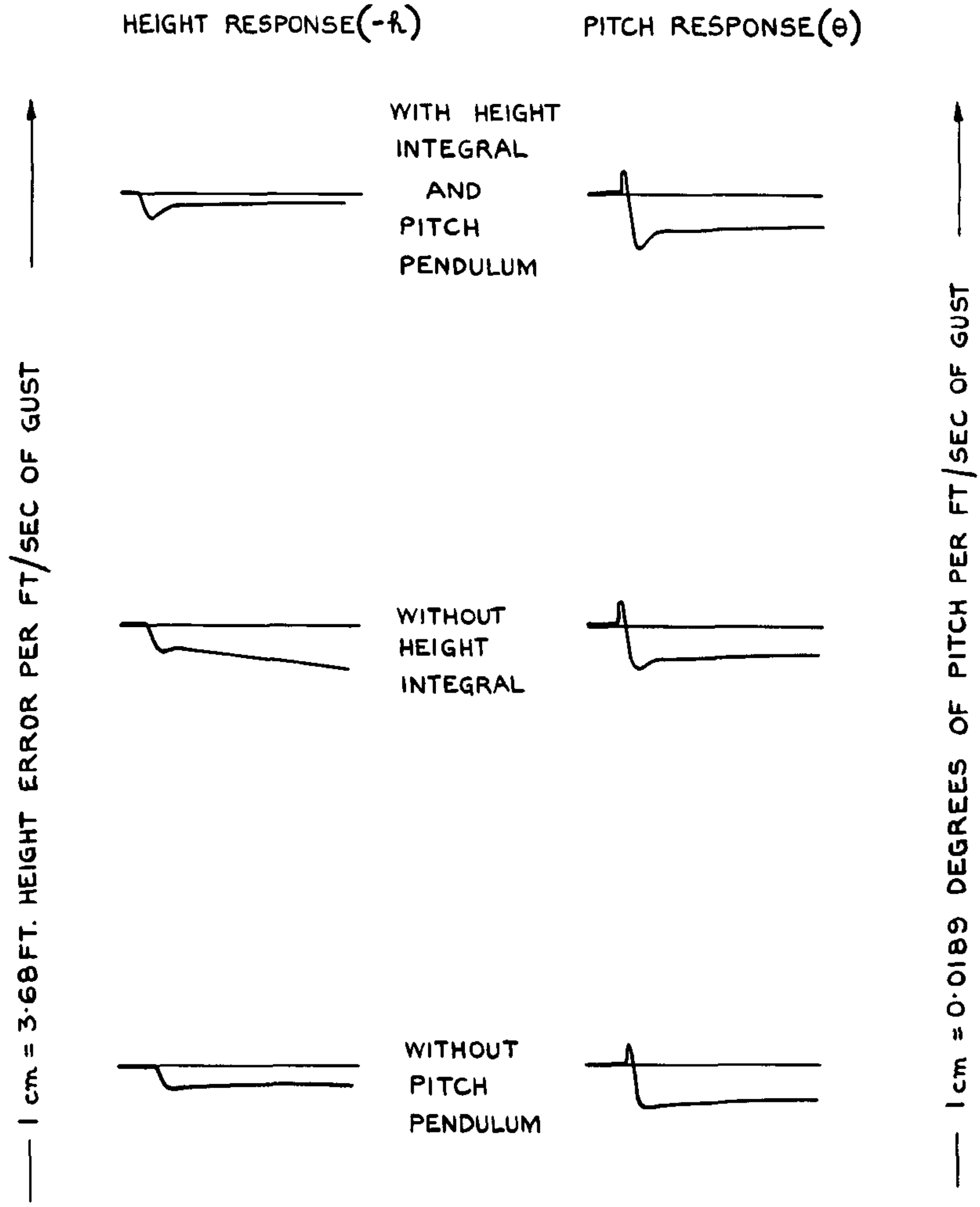


TIME SCALE : 1 CM. = 61·8 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT.

STRENGTH OF INCIDENCE ERROR TERM, 390 FT/DEG.

FIG.4(a). POSITIVE INCIDENCE EFFECT WITHOUT HEIGHT INTEGRAL AND PITCH PENDULUM.

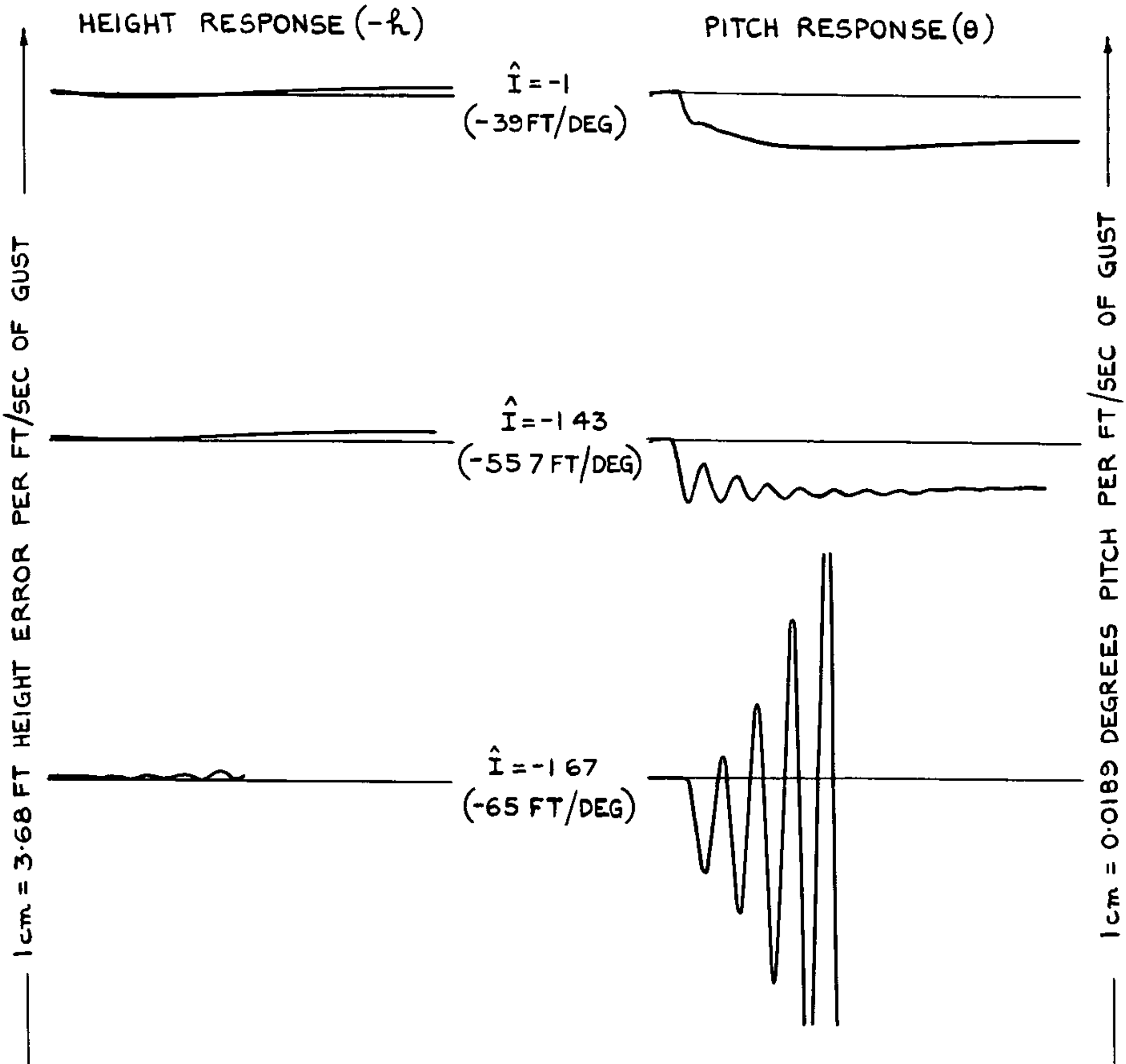


TIME SCALE: 1 cm = 61·8 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT

STRENGTH OF INCIDENCE ERROR TERM, 39 FT/DEG

FIG. 4(b). POSITIVE INCIDENCE EFFECT WITHOUT HEIGHT INTEGRAL AND PITCH PENDULUM.

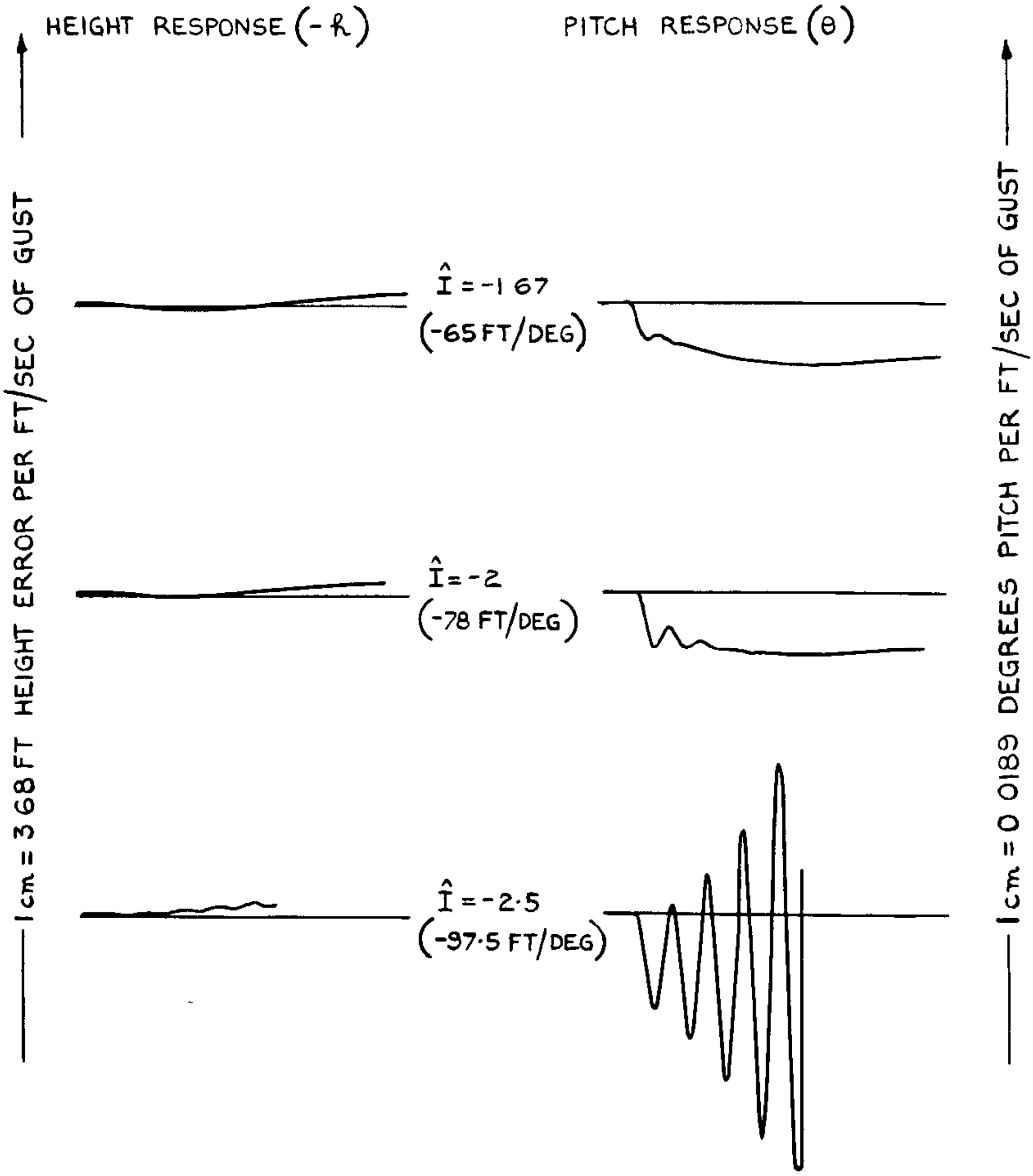


TIME SCALE : 1cm = 6·18 SECS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65FT

HEIGHT INTEGRATOR TIME CONSTANT = 45 SECS

FIG.5 (a). NEGATIVE INCIDENCE EFFECT – NO PLATFORM LAG.



TIME SCALE: 1cm = 6·18 SECS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 98 FT

HEIGHT INTEGRATOR TIME CONSTANT = 45 SECS

FIG. 5 (b). NEGATIVE INCIDENCE EFFECT — NO PLATFORM LAG

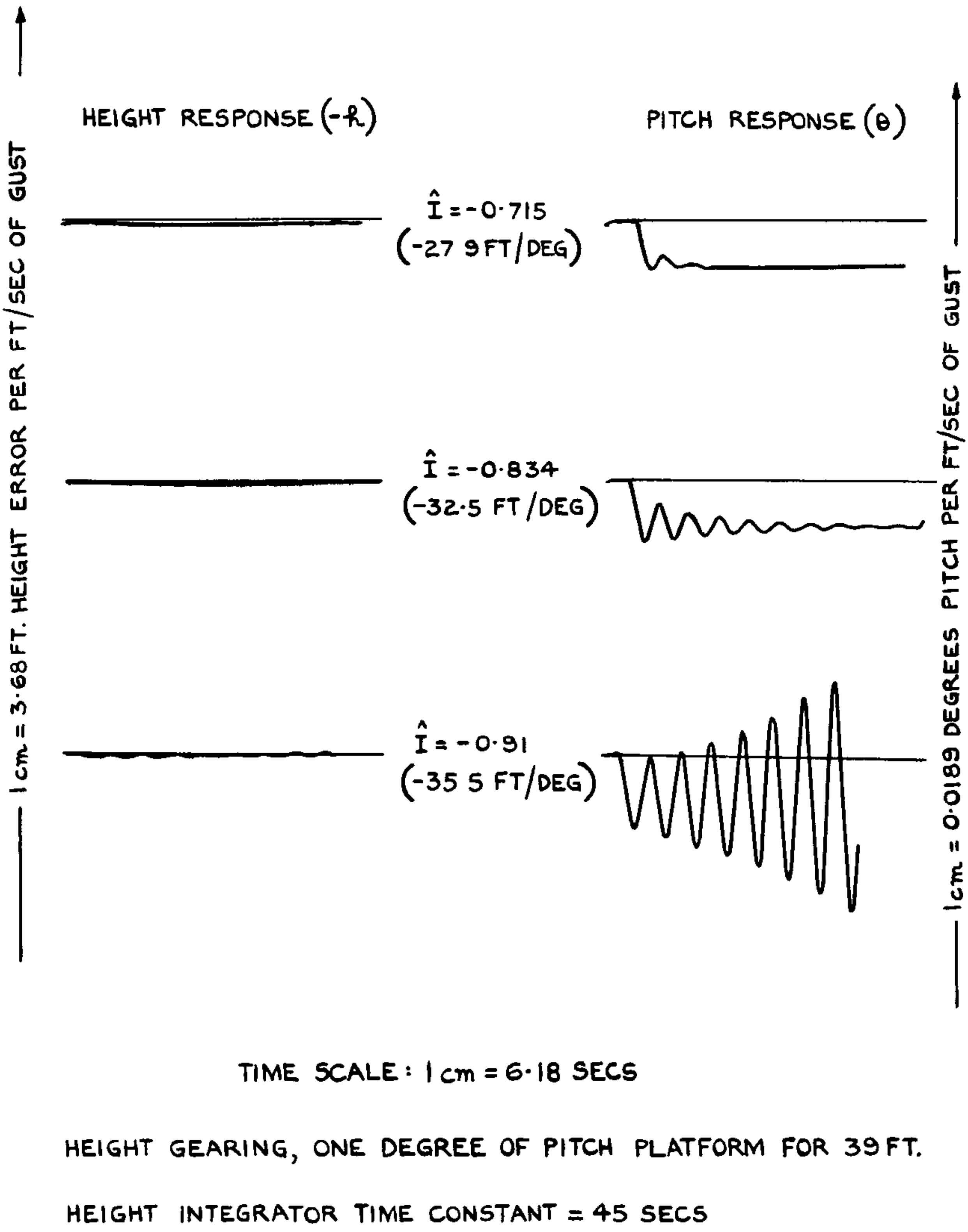
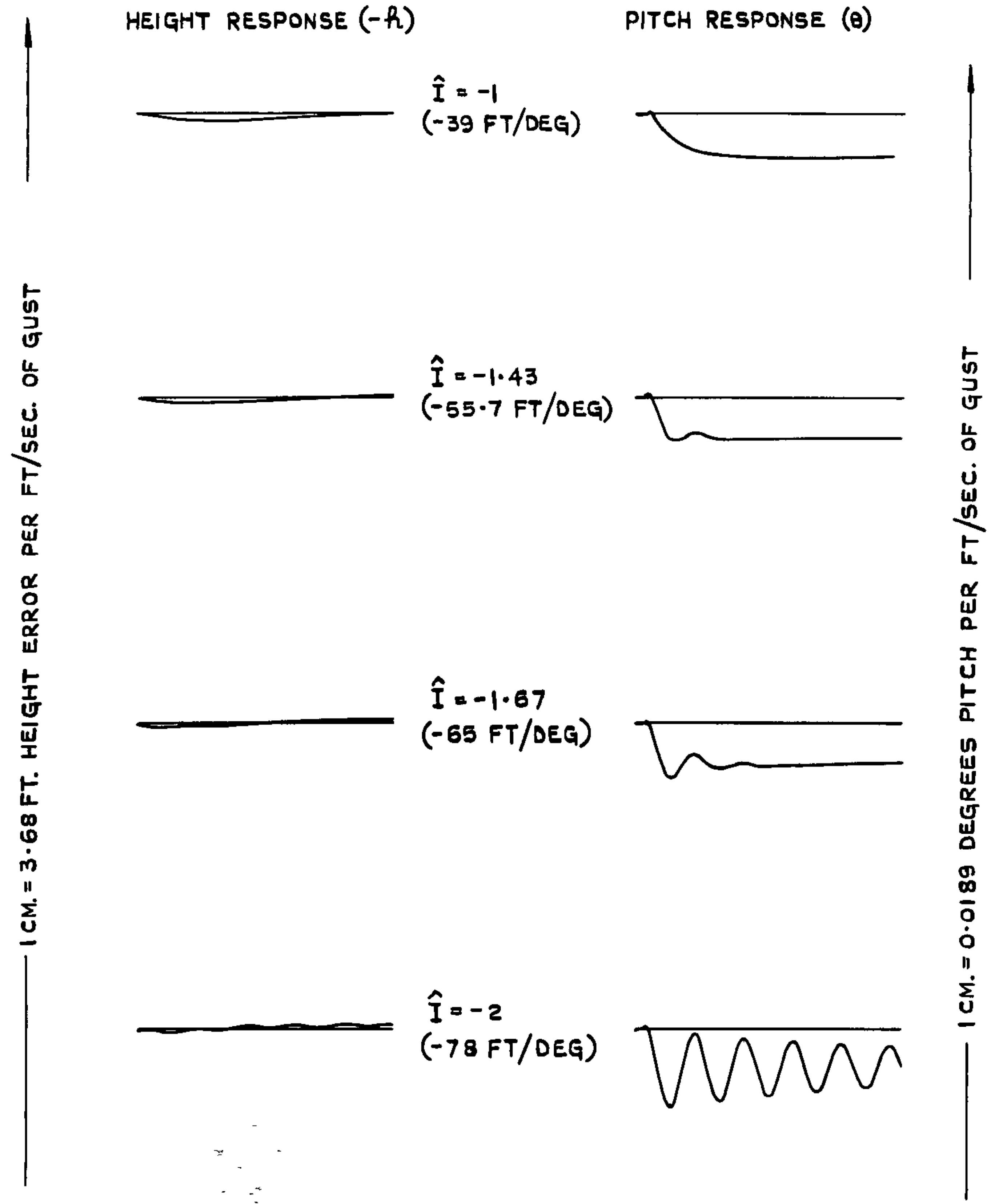


FIG. 5(c). NEGATIVE INCIDENCE EFFECT
-NO PLATFORM LAG.

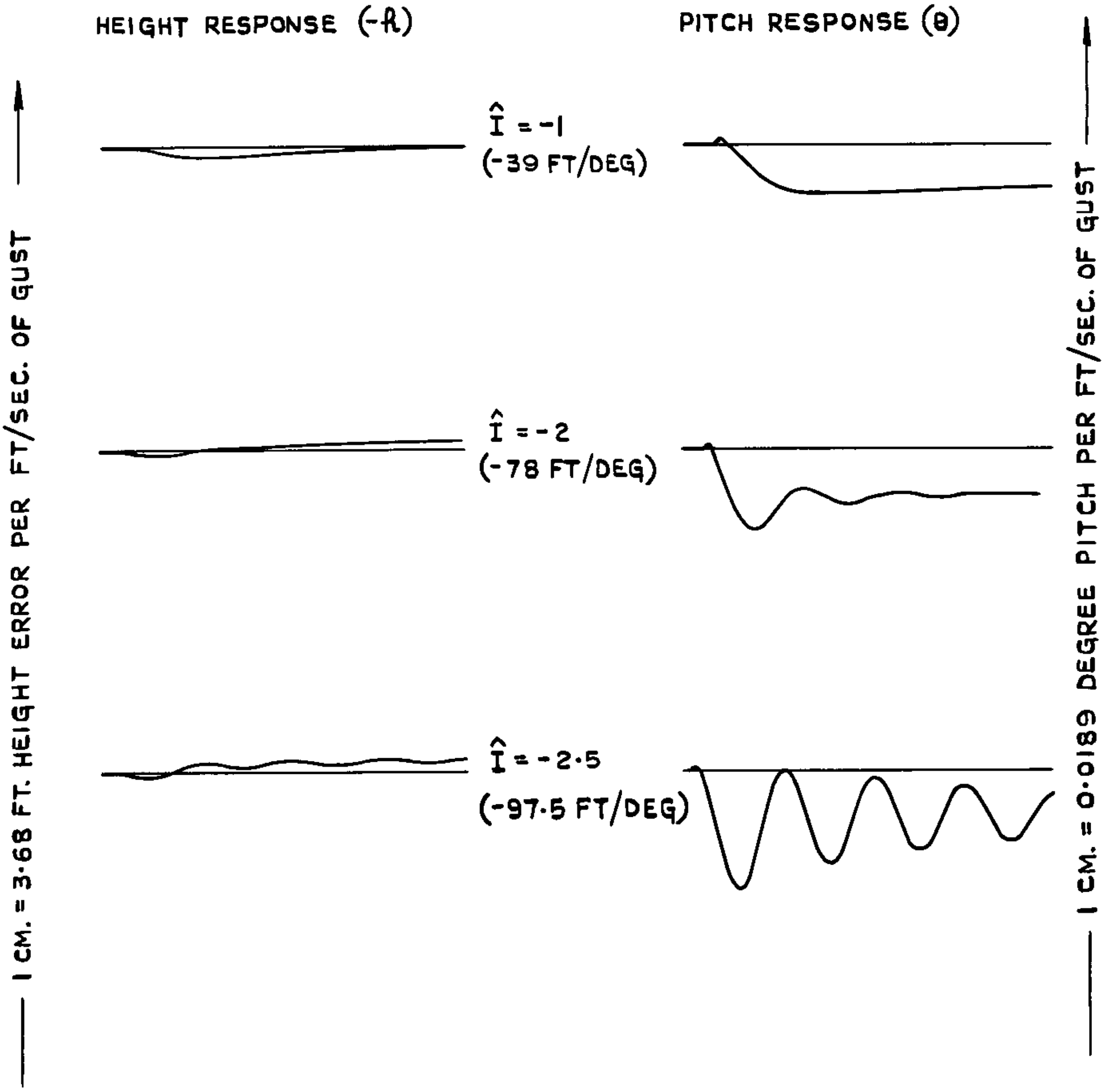


TIME SCALE : 1 CM. = 6·18 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT.

HEIGHT INTEGRATOR TIME CONSTANT, 45 SECS.

FIG. 6(a). NEGATIVE INCIDENCE EFFECT — PLATFORM LAG (T_2 = 0·309 SECONDS) PHASE ADVANCE ON ELEVATOR.

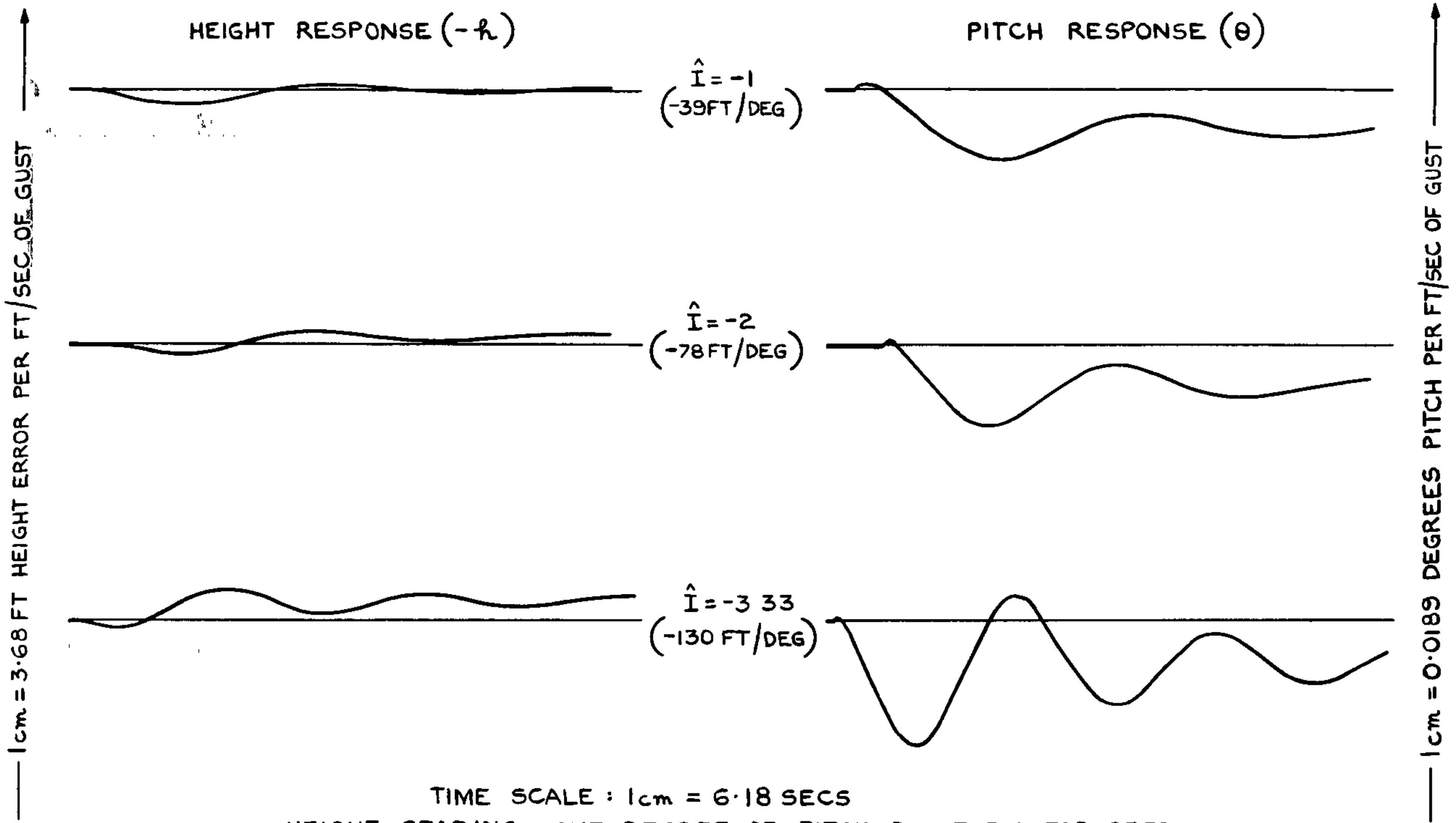


TIME SCALE : 1 CM. = 6·18 SECONDS

HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65 FT.

HEIGHT INTEGRATOR TIME CONSTANT, 45 SECS.

FIG. 6(b). NEGATIVE INCIDENCE EFFECT – PLATFORM LAG (T_2 = 0·927 SECONDS) PHASE ADVANCE ON ELEVATOR.



TIME SCALE : 1cm = 6·18 SECS
HEIGHT GEARING, ONE DEGREE OF PITCH PLATFORM FOR 65FT
HEIGHT INTEGRATOR TIME CONSTANT = 45 SECS

FIG. 6(c). NEGATIVE INCIDENCE EFFECT – PLATFORM LAG (T_2 = 3·09 SECS) – PHASE ADVANCE ON ELEVATOR.

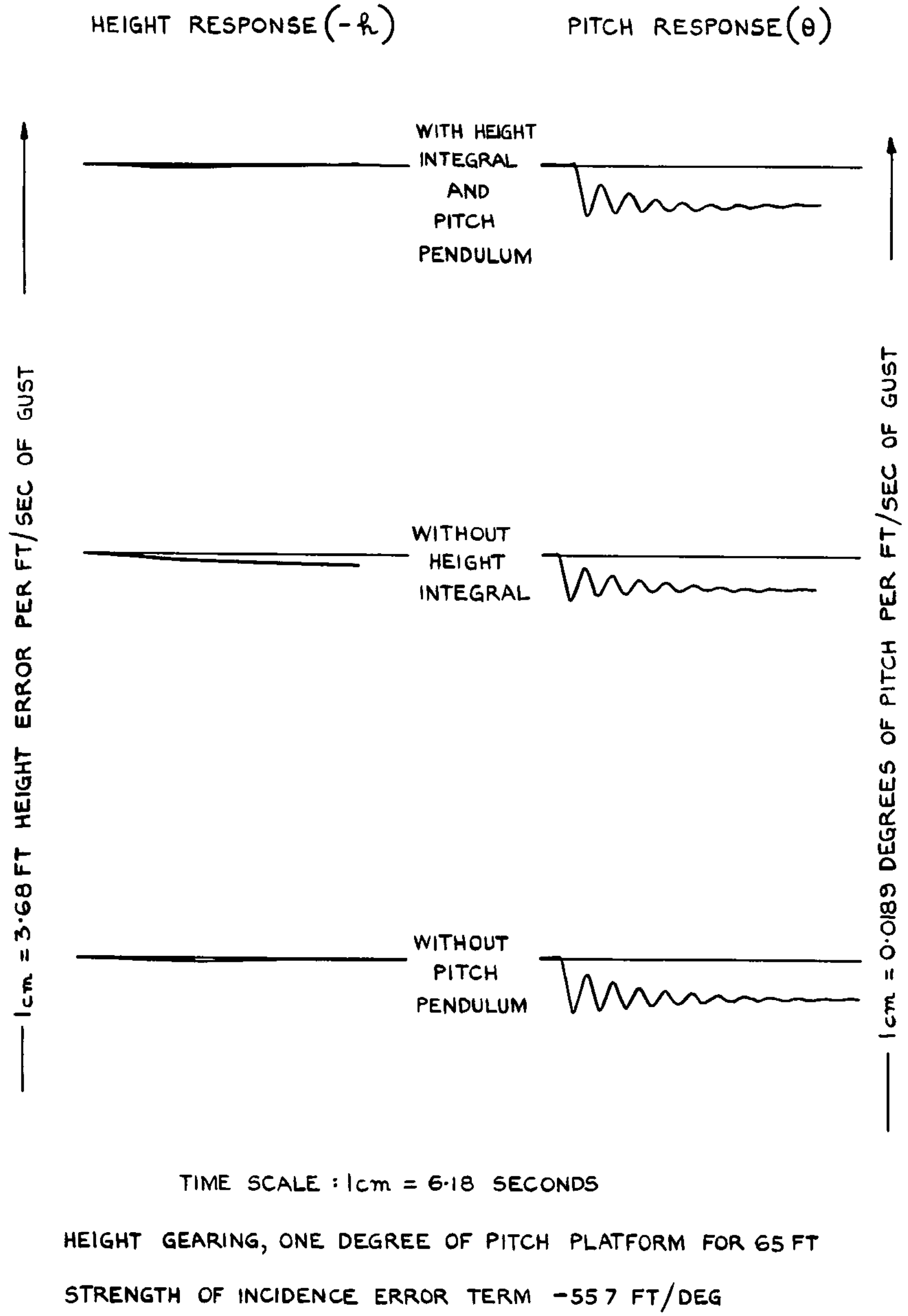


FIG.7. NEGATIVE INCIDENCE EFFECT WITHOUT HEIGHT INTEGRAL AND PITCH PENDULUM.

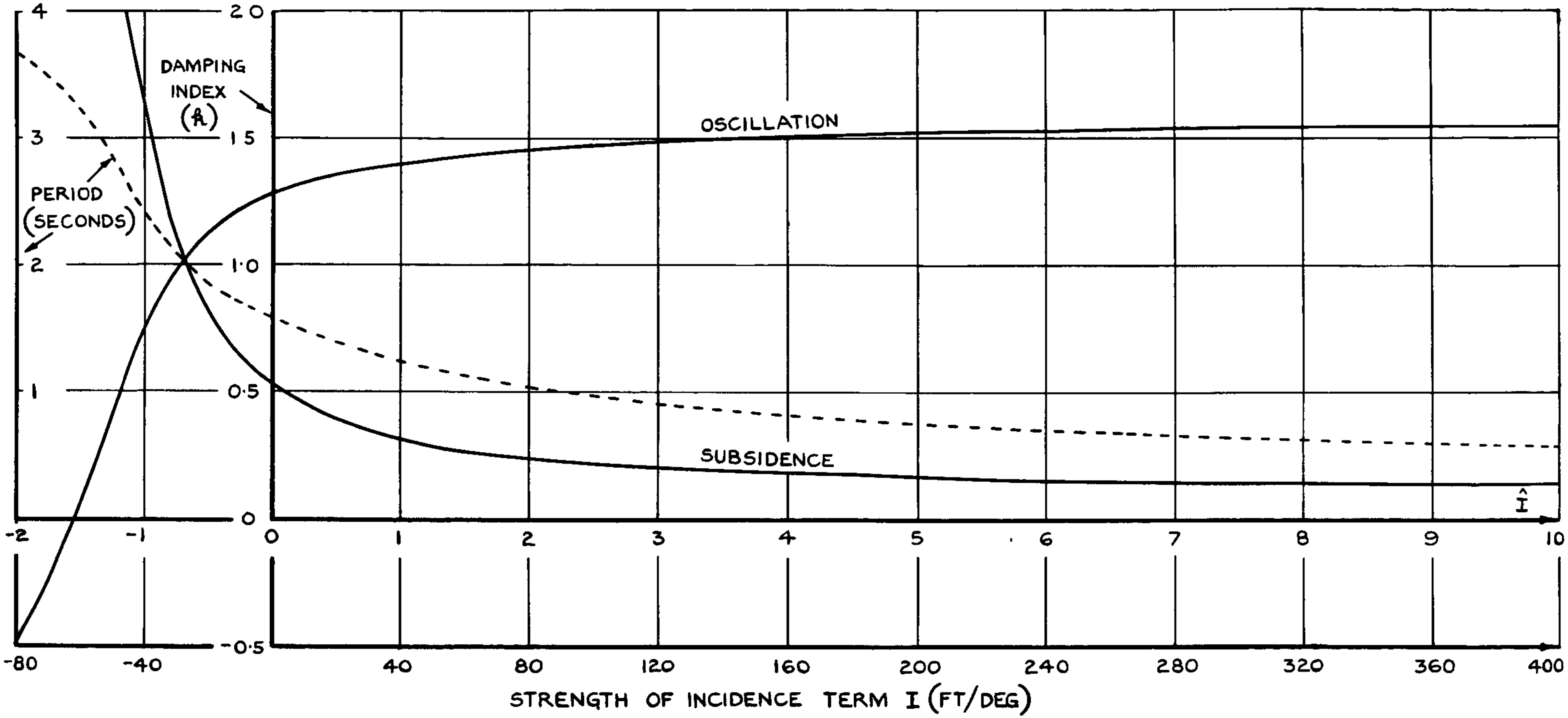


FIG. 8. EFFECT OF I ON SHORT PERIOD STABILITY.

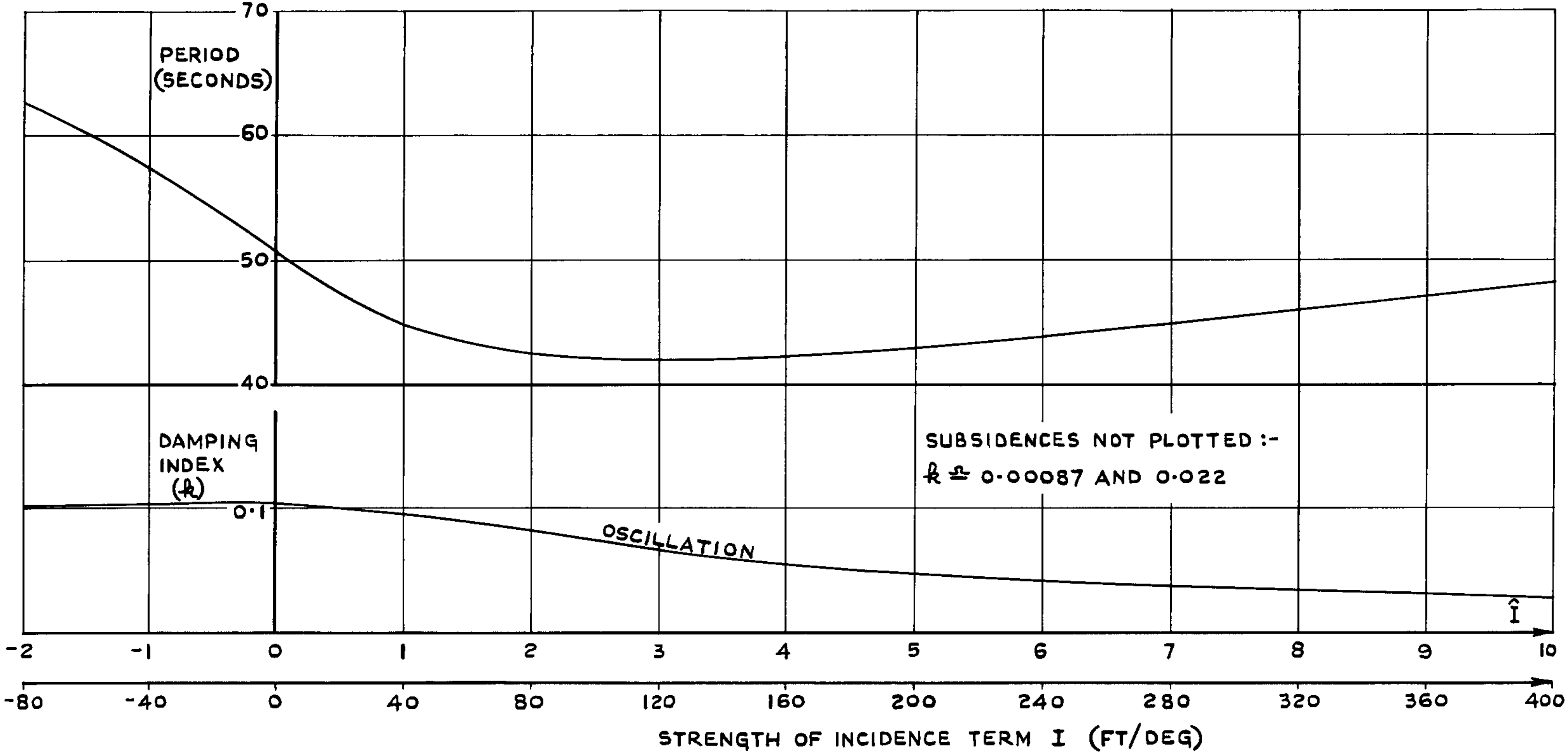


FIG.9. EFFECT OF I ON LONG PERIOD STABILITY.

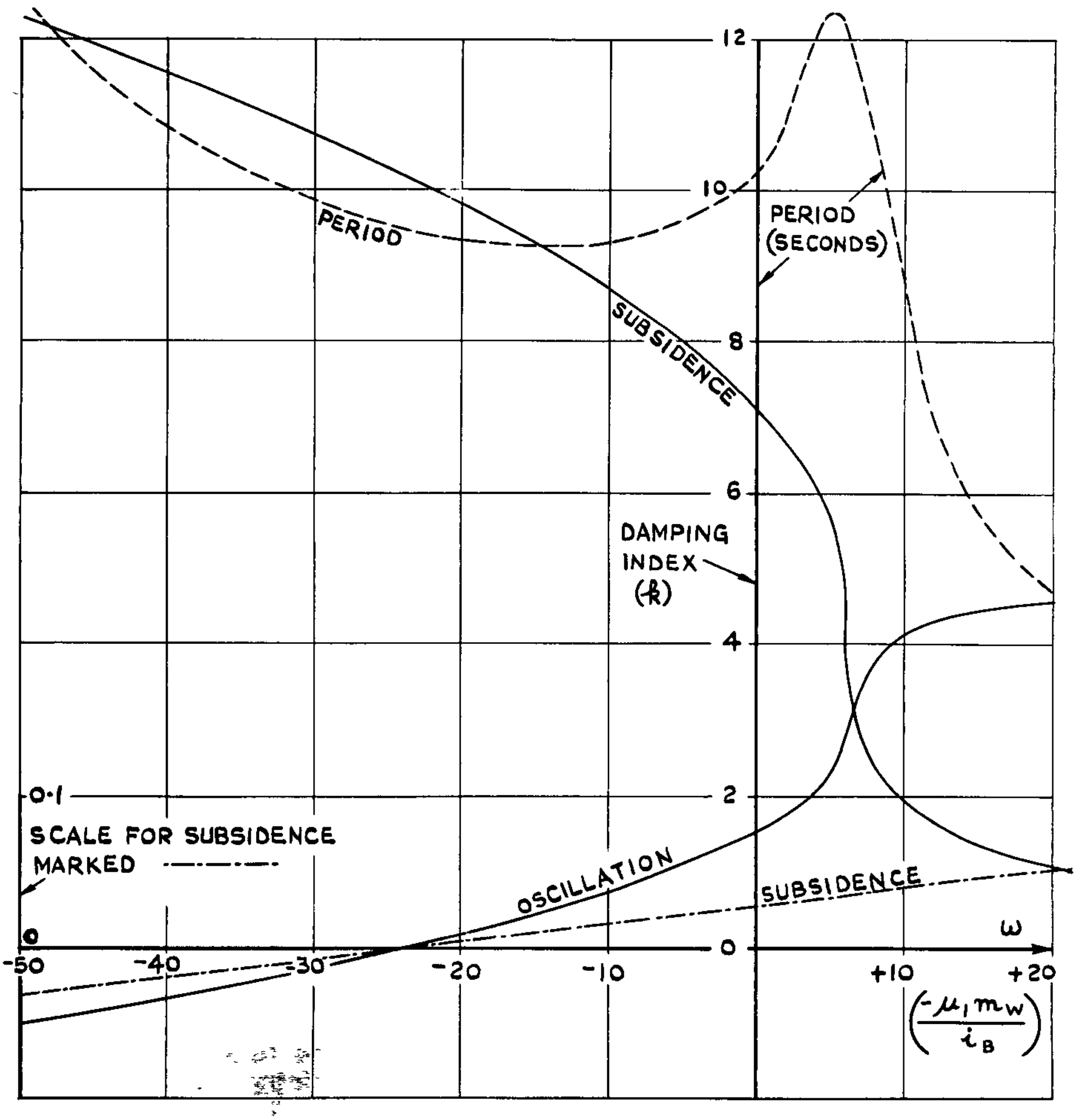


FIG.10. EFFECT OF ω ON STABILITY OF AIRCRAFT WITH PITCH CONTROL ($\eta = 0{\cdot}1\ \theta$).

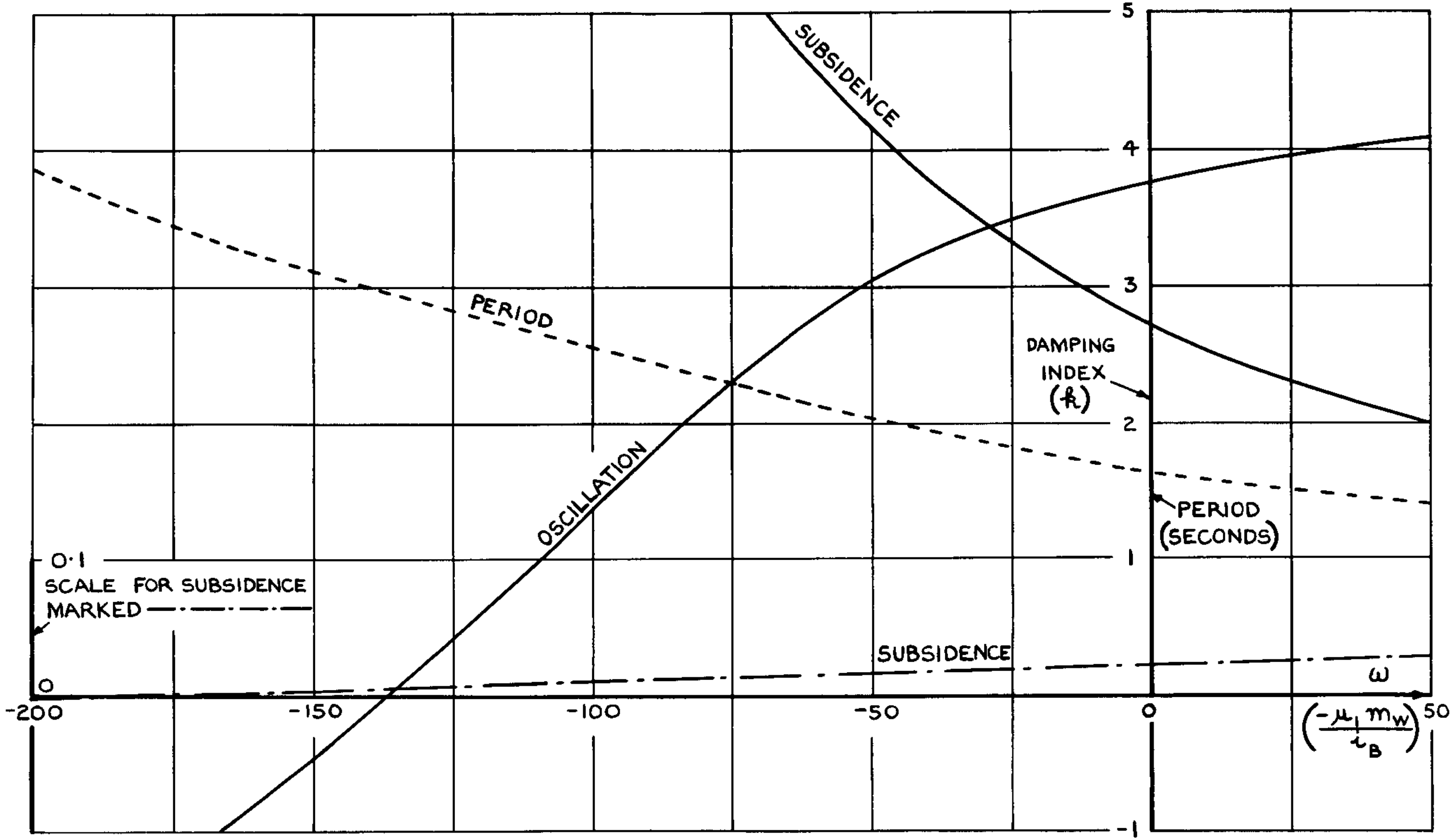


FIG.11. EFFECT OF ω ON STABILITY OF AIRCRAFT WITH PITCH CONTROL ($\eta = \theta$)

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