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A Shock-Expansion Theory Applicable to Wings with Attached Shock Waves

By

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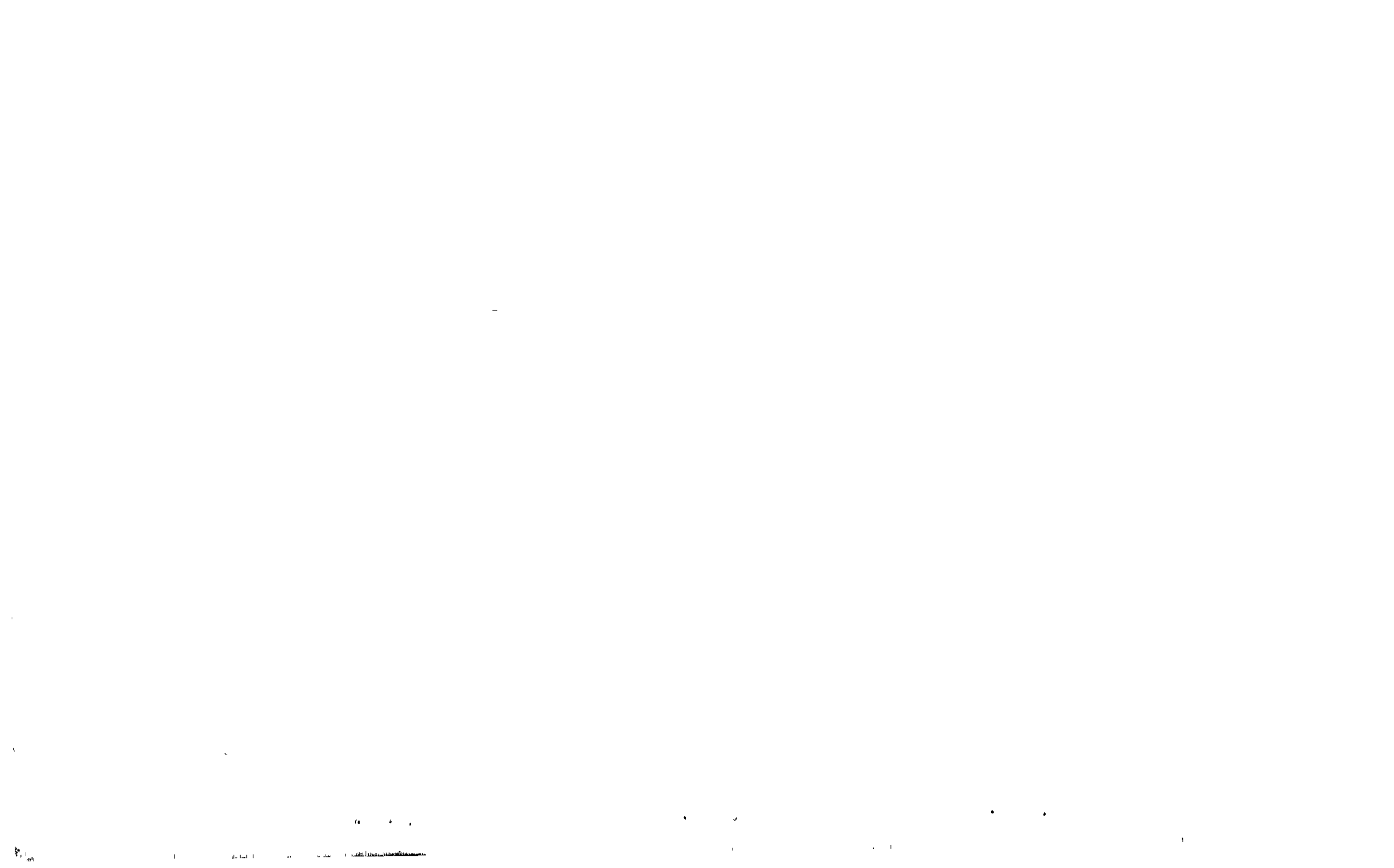
A shock-expansion theory applicable to wings
with attached shock waves

by

L.M. Sheppard
and
K.D. Thomson

SUMMARY

An extended shock-expansion theory applicable to three-dimensional wings with attached leading and trailing edge shock waves is presented in this note. Although the extended method has been derived for high Mach numbers only, preliminary comparisons of its predictions with those of linearised theory and the one experiment available indicate that it can be applied at quite low Mach numbers. For any given leading edge sweepback angle Λ there is a minimum free stream Mach number M for which the method is valid, given approximately by the condition $\frac{\tan \Lambda}{\sqrt{M^2 - 1}} < \frac{1}{3}$. Tip regions on lifting wings cannot be treated.



LIST OF CONTENTS

	<u>Page</u>
Notation	3
1 Introduction	4
2 Shock wave-Mach wave interaction	4
3 Method	5
3.1 Flow across the junction of two facets	6
3.2 Flow in facets	6
4 Treatment of zones of influence	7
4.1 Region outside the influence zones	7
4.2 The region affected by the apex influence zone only	7
4.3 The region affected by the tip influence zone only	8
4.4 The region of interaction of the apex and tip influence zones	8
5 Determination of the aerodynamic forces	8
5.1 The pressure coefficient	8
5.2 The force coefficients	9
6 Examples	9
6.1 The apex influence zone of a delta wing	10
6.2 The tip influence zone of a rectangular wing	11
7 Range of validity	11
7.1 Comparison with experiment	12
7.2 Comparison with linearised theory	12
8 Conclusions	12
References	13

LIST OF ILLUSTRATIONS

	<u>Figure</u>
Transmitted and reflected disturbances in flow field about two-dimensional body	1
Type of wing treated by method	2
Flow pattern on the plane surface of a delta wing	3
Lift curve slope of flat plate delta wings with a "supersonic" leading edge; comparison between shock-expansion theory and linearised theory	4
Comparison between theory and experiment of pressure distribution in the tip region of a rectangular wing with 9% circular arc biconvex section. $M_\infty = 1.62$	5

NOTATION

a	local speed of sound
B	$= \{M_\infty^2 - 1\}^{\frac{1}{2}}$
C_1, C_2	characteristic or Mach lines (figure 1)
C_p	pressure coefficient $(p-p_\infty)/\frac{1}{2}\rho_\infty v_\infty^2$
C_D	pressure drag coefficient, $\text{Drag}/\frac{1}{2}\rho_\infty v_\infty^2 S$
C_L	lift coefficient, $\text{Lift}/\frac{1}{2}\rho_\infty v_\infty^2 S$
ΔC_p	lower surface minus upper surface pressure coefficient
p	local static pressure
P	stagnation pressure
M	Mach number
n	$= \tan \Lambda/B$
S	wing planform area
ΔS	element of wing planform area
u,v,w	velocity components in the (x,y,z) directions
u', v'	perturbation velocities on the wing surface, respectively parallel and normal to the free stream direction
V_∞	free stream velocity
(x,y,z)	rectangular coordinate system (section 3.2)
α	local wing incidence, constant on a flat plate
γ	ratio of specific heats
δ	flow direction, relative to the undisturbed free stream
ϵ	inclination of a facet to the free stream direction
Λ	leading edge sweepback angle
ρ_∞	free stream density
∞	subscript denoting free stream conditions

1 Introduction

For wings at supersonic speeds, theoretical estimates of the aerodynamic properties are usually obtained using the linearised or small-perturbation theory. Although this theory predicts the force coefficients with fair accuracy and the surface pressure distribution with somewhat less accuracy, its predictions can be considerably in error. These errors arise when the perturbation velocities are not small compared with either the free stream velocity or the velocity of sound. At high supersonic speeds the perturbation velocities are not small compared with the velocity of sound. Hence the linearised theory is not adequate at high Mach numbers and there is a need for another method. The three-dimensional method of characteristics¹ is available; however, its application involves such a large amount of computational effort that only very simple shapes have been investigated. Fortunately, in two-dimensional supersonic flow a simple approximation to the two-dimensional method of characteristics² can be made, namely the two-dimensional shock-expansion theory, which yields results for the surface pressure distribution that are accurate enough for engineering purposes (see, for example, references 3 and 4). This approximation is applicable to infinite yawed wings and has been extended by the authors⁵ and also Vincenti and Fisher⁶ to cover certain flow regions on a class of swept, tapered wings with shock waves attached to the edges. In addition, provided that the Mach number is sufficiently high, a shock-expansion theory can be applied to bodies of revolution⁷.

In the present note, shock-expansion theory is extended to give a method for computing the surface pressure distribution on three-dimensional wings with shock waves attached to the edges. The theoretical basis for this extension, which is strictly applicable only at high Mach numbers, is described in section 2. Sections 3, 4 and 5 present the method in detail and discuss its application to the entire wing surface. Two simple illustrative examples are given in section 6 where the influence zone of the apex on a sweptback wedge and the tip region of a rectangular wing are examined. The range of validity of the present method is discussed in section 7 where comparisons with experiment and linearised theory are made. Finally, in section 8 the main conclusions of this note are given.

2 Shock wave-Mach wave interaction

Before describing the method in detail it is necessary to discuss the influence of shock waves on the flow field at the surface of a body. Consider first the flow about a two-dimensional body with attached shock waves (figure 1). Disturbances from the body are transmitted into the flow field along the family of characteristic or Mach lines C_1 . These disturbances interact with the leading edge shock wave, causing it to bend, and are partially reflected along characteristic lines such as C_2 . If the reflected disturbance is only a small proportion of the transmitted disturbance, it has negligible effect on the surface flow, which can be computed⁸ without considering flow conditions away from the surface. At a point in the flow field let the flow direction, relative to the undisturbed free stream, be δ . Eggers, Syvertson and Kraus⁴ have shown that the ratio of the gradients of δ in the C_1 and C_2 directions can be taken as a measure of the proportion of the disturbance reflected by the shock wave, i.e. the reflected disturbances are negligible when either $\frac{\partial \delta}{\partial C_1} = 0$ or $\frac{\partial \delta}{\partial C_1} / \frac{\partial \delta}{\partial C_2}$ is small just downstream of the shock wave. The general case when $\frac{\partial \delta}{\partial C_1} \neq 0$ (and $\frac{\partial \delta}{\partial C_2} \neq 0$) has been examined in reference 4; by considering disturbances incident on a plane oblique shock wave, Eggers, Syvertson and Kraus⁴ have shown that

$$\frac{\partial \delta}{\partial C_1} / \frac{\partial \delta}{\partial C_2} \ll 1$$

for all supersonic Mach numbers, provided that δ does not approach the maximum possible flow deflection angle. This gives the justification for the two-dimensional shock-expansion theory.

It will be noted that, although the body shape determines the magnitude of the disturbance transmitted to the shock wave, the proportion of this disturbance reflected by the shock wave depends directly on the shock wave. Since any shock wave pattern can be considered as built up from a number of plane oblique shock waves, it follows that the proportion of the incident disturbance reflected just downstream of the shock wave is very small, provided that the greatest flow deflection by the shock wave does not approach the maximum deflection angle. If these reflected disturbances do not coalesce they will be small throughout the entire flow field and so can be neglected. In the case of two-dimensional flow, this condition is satisfied and the whole flow field can be determined using a generalised shock-expansion method⁶. For bodies of revolution at high Mach numbers, Eggers, Savin and Syvertson^{7,8} have shown that the reflected disturbances from the nose shock wave are still small at the surface, so that the generalised shock-expansion theory is applicable in this case also.

Now, for wings with shock waves attached to the leading and trailing edges, an analysis similar to that given by Eggers and Savin⁷ for bodies of revolution can be used to show that disturbances reflected by the shock wave are unimportant at high Mach numbers. Therefore, it is assumed in this note that reflected disturbances do not coalesce to any appreciable extent at the wing surface and can be neglected. As a consequence of this, the surface flow field can be calculated without considering the shock wave pattern away from the surface.

The flow field over a wing moving at very high speeds may be investigated without considering the shock wave system explicitly since the surface flow can be determined using a strip method, in which each streamwise section is treated two-dimensionally by shock-expansion theory. (This implies immediately that reflected disturbances are negligible.) However, this method is not adequate at lower Mach numbers since each streamwise section will have a progressively increasing influence on adjacent sections as the Mach number is decreased. The extended shock-expansion theory to be described in section 3 considers the whole wing and does not treat each streamwise section two-dimensionally.

3 Method

Applications of the method are restricted to the steady supersonic flow of a perfect gas, viscosity and heat conduction being neglected. In addition, entropy changes are neglected except across shock waves, as also is any vorticity which may occur. Thus the wing can be divided into regions in which the flow is both irrotational and isentropic.

The method has been developed for the treatment of wings whose surfaces can be built up by straight generators* (figure 2), and all generators are required to be "supersonic", i.e. the local velocity component normal to each generator must be supersonic. This implies that the leading and trailing edge shock waves must be attached. Under these conditions the flow divides into four distinct regions; the zone of influence from the apex, that from the tip, a region where these two zones interact, and a region unaffected by either apex or tip.

The wing is divided into small facets bounded by generators and the aerofoil section is thus replaced by a polygon. The problem of determining the surface flow over the wing now resolves into two basic problems; finding

* Thus the wing is a ruled surface. A simple example of such a surface is illustrated in figure 2 where all the generators pass through a point.

how the flow changes in passing from one facet to the next and how it changes within each facet. The treatment of any zone of influence then becomes an application of the basic methods with appropriate boundary conditions.

3.1 Flow across the junction of two facets

The flow across the junction is similar to that over an infinite yawed wedge and, although the velocity component parallel to the junction remains constant over the ridge, the surface flow normal to the facet junction is locally two-dimensional. Hence if two facets meet to give expanding flow, the flow across the junction can be found by a Prandtl-Meyer expansion of the normal component, with the parallel velocity component remaining unaltered. When shock waves occur, as along the leading edge, a similar procedure is used.

3.2 Flow in facets

Here the flow over a plane facet will be considered. The equations of motion are referred to a rectangular coordinate system (x,y,z) in which the facet is the plane $z = 0$, i.e. the z direction is normal to the facet. Velocity components in the x,y,z directions will be denoted by u,v,w respectively. Since $w = 0$ on the facet, the three-dimensional velocity equation at the surface becomes

$$\left(1 - \frac{u^2}{a^2}\right) \frac{\partial u}{\partial x} + \left(1 - \frac{v^2}{a^2}\right) \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} - \frac{uv}{a^2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$$

where the local speed of sound a is given by the energy equation

$$\frac{a^2}{(\gamma - 1)} + \frac{u^2 + v^2}{2} = \text{constant}$$

and the irrotational flow conditions* are expressed by

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \quad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} = 0, \quad \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y} = 0.$$

It has been stressed in section 2 that disturbances reflected from the leading edge shock wave are assumed to be negligible. This implies that the flow near the facet surface remains parallel to it, i.e. $\frac{\partial w}{\partial z} = 0$. Therefore the system of equations given directly above reduces to a two-dimensional system in the plane of the facet and can be solved by the usual method of characteristics, together with the appropriate boundary conditions.

Hence if flow conditions are known along the upstream boundary of a facet and the appropriate boundary conditions on the facet sides are known also, the flow can be determined by the method of characteristics² and conditions can therefore be found on the downstream edge. The procedure outlined in section 3.1 enables the flow on the upstream edge of the next facet to be found, and so by repeating this procedure the surface flow over the whole wing can be determined.

* It is sufficient to have only the one irrotational condition $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$ i.e. the vorticity component normal to the surface vanishes. Thus zero vorticity components in the facet surface are not necessary for the application of the present method.

It now remains to examine the boundary conditions for the different zones of influence. This is done in section 4.

4 Treatment of zones of influence*

It was pointed out in section 3 that the flow pattern on a wing with "supersonic" edges can be divided into four distinct regions:-

- (a) The region away from the apex and tip influence zones.
- (b) The region affected by the apex influence zone only.
- (c) The region affected by the tip influence zone only.
- (d) The region of interaction of the apex and tip influence zones.

These four regions will now be considered in turn.

4.1 Region outside the influence zones

The region OAD in figure 2 lies outside the apex and tip influence zones and has been treated previously by the authors² using the present method, and analytically by Vincenti and Fisher⁵. Since conditions just downstream of the leading edge shock wave (or expansion wave) are constant, a characteristics mesh in the first facet will predict a constant flow region, which implies constant flow conditions in the next facet and so on over the whole region. The only flow changes are those occurring at facet junctions and these can be treated by two-dimensional techniques, as described in section 3.1. The accuracy of the method in this region is comparable with that of two-dimensional shock-expansion theory, which is sufficiently exact whether the Mach number is high or not.

4.2 The region affected by the apex influence zone only

The apex influence zone ODFC (figure 2) differs from the zone OAD in that the flow over each facet is not constant, and a characteristics mesh is required. Since OC is a line of symmetry the flow locally must be parallel to it; this constitutes one of the boundary conditions for the mesh.

Consider the segment HJKL of a facet within the apex influence zone. If the facet upstream has been examined the flow conditions just upstream of HJ are known, and hence just downstream they can be evaluated. The boundary line HL represents that portion of the apex influence zone boundary OD which crosses the facet under consideration. For a sweptback wing with leading edge shock waves occurring on both upper and lower surfaces, the boundary OD is not a shock wave but represents the upstream edge of an expansion**. Thus the flow along OD and therefore along HL is known from the flow in the region OAD.

* The techniques described in this section were developed by the authors at the Aerodynamics Division, Weapons Research Establishment (formerly the High Speed Aerodynamics Laboratory), Salisbury, during 1953-1954.

** For a sweptback wing the leading edge shock wave will deflect the flow away from the centre-line, and in the apex influence zone the streamlines will change direction so that along the centre-line they are parallel to the plane of symmetry. Such a flow can exist only if an expansion occurs within the apex influence zone. It follows that the influence zone boundary is the upstream limit of an expansion centred on O.

When the wing incidence is such that an expansion wave is attached to the leading edge on the surface being examined, it can be shown that the boundary OD of the apex influence zone is a shock wave.

Near the leading edge of the wing the calculated flow downstream of the expansion region centred on O is parallel to the centre-line. Moreover, when the boundary of the apex influence zone is a shock wave centred on O, the flow near the leading edge, but inside the apex influence zone, is parallel to the centre-line also. Thus, in this case, the initial shock wave strength and position on the first wing facet can be found. The shock wave strength and position on the next facet are determined just downstream of the facet junction from the calculated flow conditions on the two sides of the shock wave. This procedure applied to each facet in turn enables the boundary OD of the apex influence zone to be determined.

In the facet HJKL being considered the flow is now known along HJ and HL, while the stream direction along JK is known also. These conditions are sufficient for a characteristics mesh to be drawn in the facet and therefore the whole apex influence zone can be treated.

4.3 The region affected by the tip influence zone only

Tip influence zones such as GADE (figure 2) can be examined by the present method only when the wing incidence is zero and the aerofoil section is symmetrical about the plane of the wing. Under these conditions the stream sheet off the wing, dividing the upper and lower surface flows, is contained within the plane of the wing. Although the region GAB is not on the wing it will affect the flow over the wing and therefore the characteristics mesh in each facet must be extended to cover the region GAB. It is assumed that each facet can be considered to lie in the plane of the wing, so that the characteristics mesh is continuous across the wing tip AB.

The boundaries AG and AD can be either shock waves or upstream limits of expansion zones*, and their shape and strength can be found by matching the flow downstream of AG with that downstream of AD. To do this, a procedure is used whereby a number of shock wave strengths are examined for compatibility with the equations governing flow variations within a characteristics mesh. It will be noted that, since the region GAB is a plane surface, no expansions will occur across facet junctions in this region, although of course they will be required at junctions on the wing itself.

4.4 The region of interaction of the apex and tip influence zones

Having computed the flow in the apex and tip influence zones, conditions along the boundaries of the cross-over region DEF (figure 2) are known and so the flow in each facet is found by an application of the method of characteristics.

5 Determination of the aerodynamic forces

5.1 The pressure coefficient

From a knowledge of the flow pattern over a wing the pressure distribution and aerodynamic forces can be obtained. By definition, the pressure coefficient C_p is given by

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho_\infty V_\infty^2}$$

* For a sweptback wing it is found that AG is a shock wave and AD is the upstream limit of an expansion provided that the wing incidence is zero and the aerofoil section is symmetrical about the plane of the wing.

which can be rewritten as

$$C_p = \frac{2}{\gamma M_\infty^2} \left(\frac{P}{P_\infty} - 1 \right) = \frac{2}{\gamma M_\infty^2} \left(\frac{P_\infty}{P_\infty} \cdot \frac{P}{P_\infty} \cdot \frac{\rho}{\rho} - 1 \right)$$

where p , P , ρ , M and V are the static pressure, stagnation pressure, density, Mach number and flow velocity respectively, at a given point, the subscript ∞ denotes free stream conditions and γ is the ratio of specific heats. The pressure ratios $\frac{P_\infty}{P_\infty}$ and $\frac{\rho}{\rho}$ are functions of Mach number only and are determined from isentropic flow tables, while the stagnation pressure ratio $\frac{P}{P_\infty}$ is determined from plane oblique shock wave tables. This latter ratio will be determined by the shock waves upstream of the point being considered.

5.2 The force coefficients

The pressure drag coefficient at zero incidence of a wing with a section symmetrical about the plane of the wing is given by

$$C_D = \frac{1}{S} \Sigma C_p \cdot \tan \epsilon \cdot \Delta S,$$

where Σ denotes summation over one surface of half the wing,

ΔS is an element of the wing plan area,

S is the planform area

and ϵ is the inclination (measured parallel to the plane of symmetry) of the element ΔS to the free stream direction. The angle ϵ is positive near the leading edge and negative near the trailing edge.

When the wing is at incidence, the pressure drag coefficient is to be found by separate summations over the upper and lower wing surfaces. The lift coefficient is given by

$$C_L = \frac{2}{S} \Sigma \Delta C_p \cdot \cos \alpha \cdot \Delta S$$

where α is the local wing incidence, constant in the case of a flat plate,

ΔC_p denotes the lower surface pressure coefficient minus the upper surface pressure coefficient

and Σ denotes summation over one half of the wing plan area.

The centre of pressure, which lies along the wing centre-line, may be required also and can be found by applying the condition that the pitching moment about this point vanishes.

6 Examples

In order to illustrate the application of the method to a general wing, two examples are discussed in sections 6.1 and 6.2 below. The planforms considered are arranged to isolate, as far as possible, the various zones of influence. Firstly, a delta wing is considered since the only zones present are the apex influence zone and the region outside it.

Secondly, the tip influence zone of a rectangular wing with circular-arc biconvex section is examined.

6.1 The apex influence zone of a delta wing

Although the discussion of this section will be restricted to delta wings with plane upper and lower surfaces, the general qualitative picture presented is typical of the flow patterns associated with apex influence zones. Figure 3 illustrates the surface flow inside the apex influence zone when there is a shock wave, or expansion wave, attached to the leading edge. It was emphasised in section 2 that the present shock-expansion method should be used only when the leading edge shock wave is not near detachment. This condition is assumed to be fulfilled.

Consider firstly the case when there is a shock wave attached to the leading edge. Then the flow downstream of the leading edge is deflected outwards, away from the wing centre-line. Therefore an expansion wave centred on the apex will be required to ensure that the flow near the centre-line is parallel to it, i.e. inside and outside the apex influence zone there exist constant flow regions, separated by an expansion wave. For curved wing surfaces, the expansion region centred on the apex is made up of curved, not straight, lines and the flow both inside and outside the influence zone will not be constant.

The second case illustrated in figure 3 occurs when an expansion wave is attached to the leading edge. In this case the boundary of the apex influence zone is a shock wave, which separates two regions of constant flow.

As a special case of the preceding example a flat plate delta wing at incidence is examined here. The lift curve slope, at zero incidence, can be found by treating all shock and expansion waves with a small perturbation theory since their strength, in the limit, is infinitesimal. Such a procedure will be referred to as a first order shock-expansion theory and is inferior to the linearised theory, which does not neglect disturbances reflected by compression waves.

Outside the apex influence zone it is well known⁹ that, on the upper surface of the wing,

$$\frac{u'}{V_\infty} = \frac{\alpha}{B\sqrt{1-n^2}}, \quad \frac{v'}{V_\infty} = -\frac{\alpha n}{\sqrt{1-n^2}}$$

where V_∞ is the free stream velocity,

$$B = \sqrt{M_\infty^2 - 1}, \quad M_\infty \text{ being the free stream Mach number,}$$

u', v' are perturbation velocities on the wing surface, respectively parallel and normal to the free stream direction,

$$n = \tan A/B \text{ where } A \text{ is the angle of sweepback of the leading edge}$$

and α is the angle of incidence of the wing.

Now, the flow inside the apex influence zone and on the upper surface of the wing is parallel to the free stream direction; thus the flow entering the apex influence zone must undergo a compression through an angle v'/V_∞ . It can be shown quite simply that the result of first order shock-expansion theory inside the apex influence zone is

$$\frac{u'}{V_\infty} = \frac{\alpha}{B} \sqrt{\frac{1-n}{1+n}}, \quad v' = 0.$$

The pressure coefficient is defined by $C_p = -2u'/V_\infty$. Thus, inside the apex influence zone

$$C_p = -\frac{2\alpha}{B} \sqrt{\frac{1-n}{1+n}}$$

and outside the apex influence zone

$$C_p = -\frac{2\alpha}{B\sqrt{1-n^2}}.$$

On the lower surface of the wing the pressure coefficients are opposite in sign, so that the lift coefficient, given by the first order shock-expansion theory, can be shown to be

$$C_L = \frac{4\alpha}{B} \sqrt{1-n^2}.$$

The lift curve slope, at zero incidence, predicted by shock-expansion theory is therefore

$$\frac{B}{4} \left(\frac{dC_L}{d\alpha} \right)_{\alpha=0} = \sqrt{1-n^2}.$$

This result is shown in figure 4 where the exact theoretical result, which in this case is given by the linearised theory, is shown also.

6.2 The tip influence zone of a rectangular wing

The tip influence zone of a rectangular wing at zero incidence, and with a circular arc biconvex section, is considered in this section. Although the present shock-expansion theory has been deduced for "high" Mach numbers this rectangular wing will be examined at $M_\infty = 1.62$, where the experimental results of Czarnecki and Mueller¹⁰ are available. In order to apply the method presented in section 4.3 the circular arc forming the wing section was replaced by a ten-sided polygon. Thus the upper wing surface, for example, consisted of ten facets. The experience of the authors in this particular calculation indicates that wing pressure distributions can be determined by one person in from forty to eighty hours.

The flow field in the tip region displayed the features described in section 4.3. The shock wave attached to the leading edge of the wing is discontinuous at the tip leading edge where a weaker, curved shock wave extends downstream. On the wing surface, there is an expansion region centred on the tip leading edge. This region forms the boundary between the internal flow of the tip influence zone and the two-dimensional flow over the portion of the wing outside the tip zone.

The results for the pressure distribution along a particular streamwise section of the wing tip are given in figure 5. Since two-dimensional shock-expansion theory is required outside the tip influence zone the results of the extended shock-expansion theory are apparent inside the tip region only. Also shown in figure 5 are the results of experiment and linearised theory for the same streamwise section.

7 Range of validity

The range of validity of the present shock-expansion method, which has been derived for "high" Mach numbers, cannot be examined in a satisfactory manner unless adequate experimental results exist. For bodies of revolution,

Eggers, Savin and Syvertson⁸ had experimental results available and thus were able to obtain definite conclusions regarding the applicability of their generalised shock-expansion method. For the application of the present method to wings however, adequate experimental results are not available and so only a tentative range of validity can be indicated. Since two-dimensional shock-expansion theory is applicable in most cases where the shock waves are attached, thickness and incidence effects are not investigated; hence the aim of this section is to investigate what wing planform conditions must be satisfied for the present shock-expansion theory to be applicable.

7.1 Comparison with experiment

The results of the present method have been compared with experiment for the special case of a rectangular wing tip with circular arc biconvex section, the incidence being zero and the free stream Mach number 1.62. A comparison between linearised theory, experiment¹⁰ and the present method is presented in figure 5. The remarkable agreement between the experimental results and those of the shock-expansion theory is evident. This comparison shows that the one condition derived in section 2, namely that the free stream Mach number be "high", is too restrictive. It appears likely that the tip influence zone on non-lifting wings with streamwise tips can be treated by the present shock-expansion theory when the Mach number is comparatively low. However, there will be a condition restricting leading edge sweepback and this is examined in section 7.2 below using comparisons between the present method and linearised theory.

7.2 Comparison with linearised theory

In order to examine the effect of leading edge sweepback on the validity of the shock-expansion theory the lift curve slope, at zero incidence, of a flat plate delta wing is examined. Figure 4 presents the results of linearised theory, which is exact in this case, and shock-expansion theory. It will be observed that the result of the present method is within 5% of the linearised theory result for $n = \tan \Lambda/B < 0.31$. Thus a tentative range of validity for the shock-expansion theory is provided by the condition $\tan \Lambda/B < 1/3$. This may be expressed in an alternative manner by stating that the leading edge is required to be "highly supersonic".

Unfortunately, the delta wing contains no tip influence zones. If a tip influence zone on a non-lifting wing, with a streamwise tip and a single wedge section, is examined it is found that such tip influence zones can be treated with acceptable accuracy in almost all cases. Therefore the leading edge sweepback condition that $n = \tan \Lambda/B < 1/3$ provides a reasonable guide for a wing having both apex and tip influence zones. It is suggested that wings satisfying the condition $\tan \Lambda/B < 1/3$ can be investigated using the present shock-expansion theory. Of course, it is assumed that the leading and trailing edge shock waves are attached. Finally, it should be stressed that this sweepback condition is tentative and requires confirmation by experimental data.

8 Conclusions

A shock-expansion theory has been developed for treating the entire surface of a wing with attached leading and trailing edge shock waves. This method is based upon the fact that disturbances transmitted to a shock wave from a wing, or body, are not reflected significantly, provided that the free stream Mach number is high. Although the apex and tip zones of influence can be treated strictly only at high Mach numbers the region outside these influence zones is subject to no such restriction.

The range of validity of the shock-expansion theory has been investigated by comparing its predictions with the results of linearised theory and

one experiment. The essential requirement appears to be that the wing leading edge is "highly supersonic". When this requirement is fulfilled the apex and tip influence zones, as well as the region outside them, can be examined. This conclusion must be regarded as of limited quantitative value until further experimental data become available.

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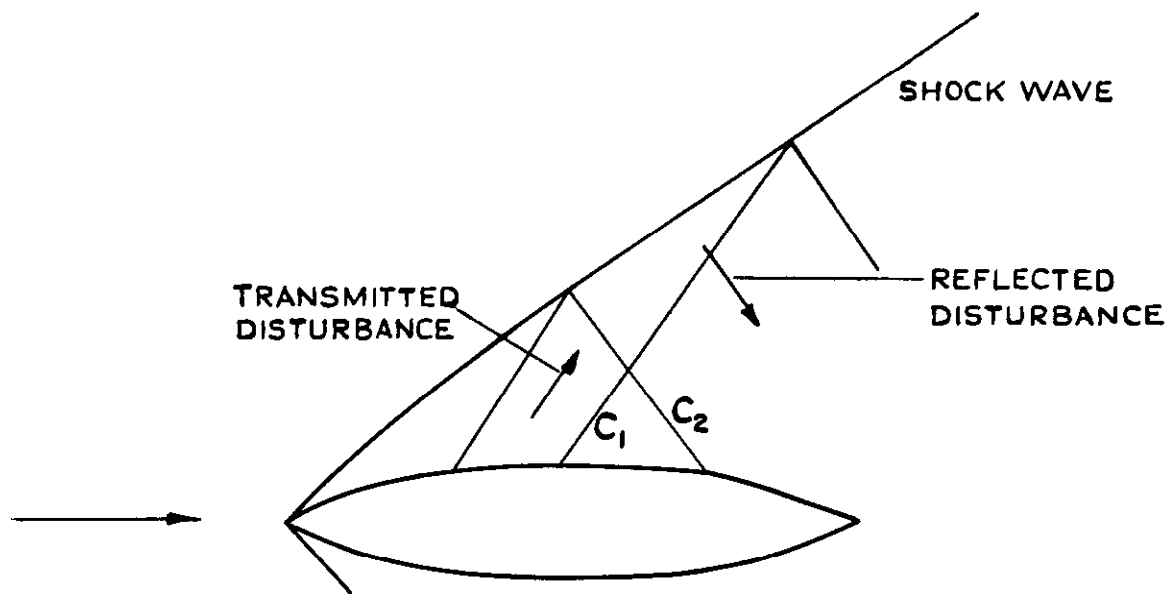


FIG.1 TRANSMITTED & REFLECTED DISTURBANCES IN FLOW FIELD ABOUT TWO-DIMENSIONAL BODY.

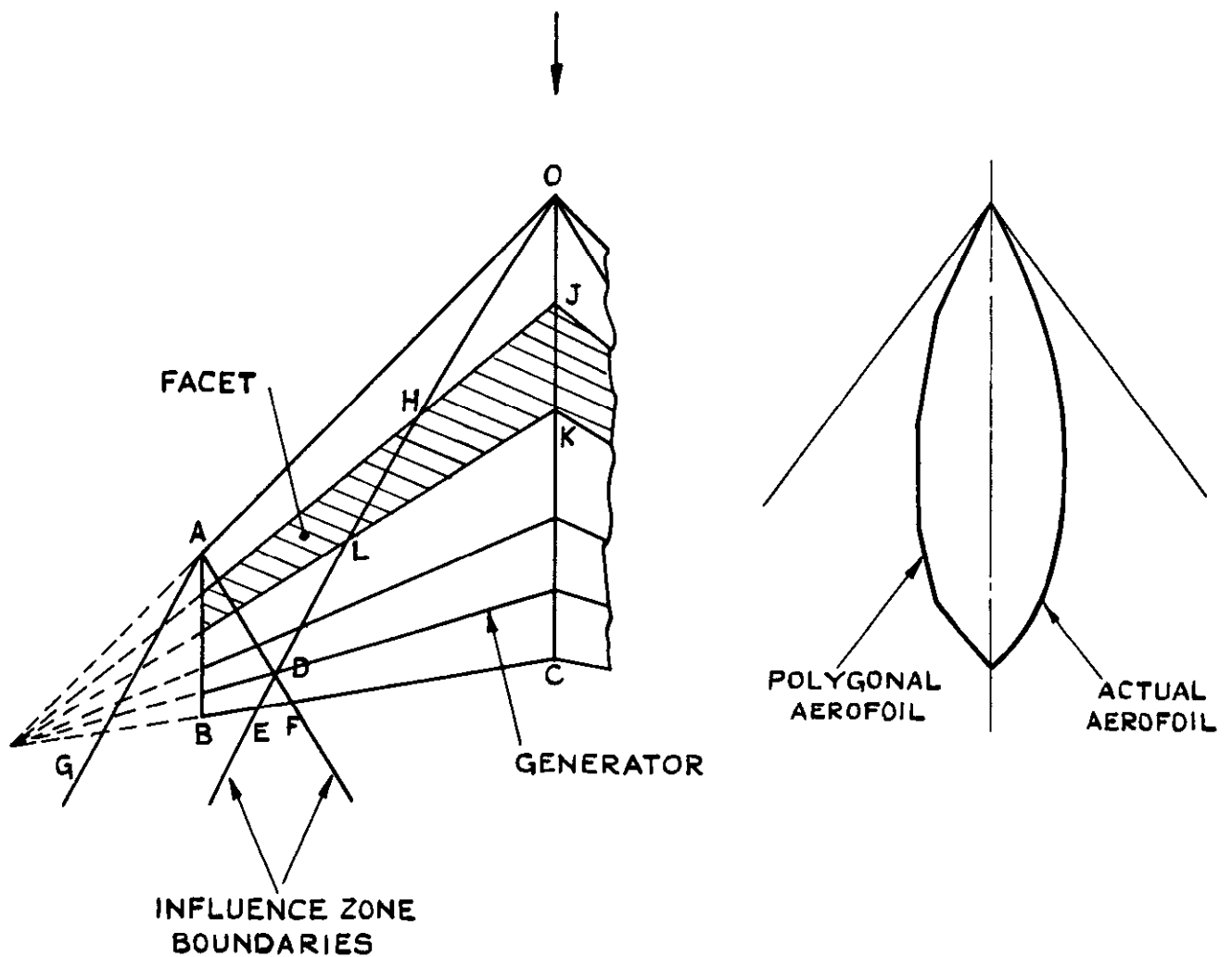
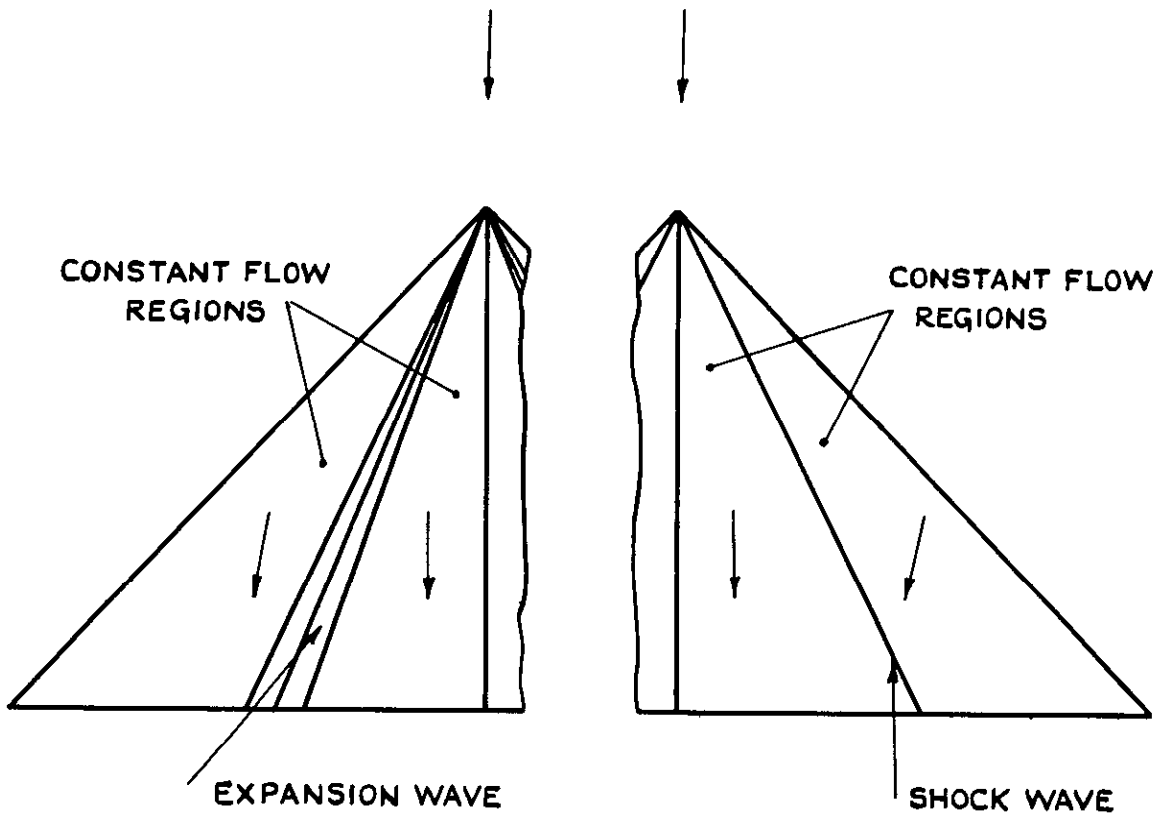


FIG.2 TYPE OF WING TREATED BY METHOD.



(a) SHOCK WAVE ATTACHED TO LEADING EDGE.

(b) EXPANSION WAVE ATTACHED TO LEADING EDGE.

FIG.3 FLOW PATTERN ON THE PLANE SURFACE OF A DELTA WING.

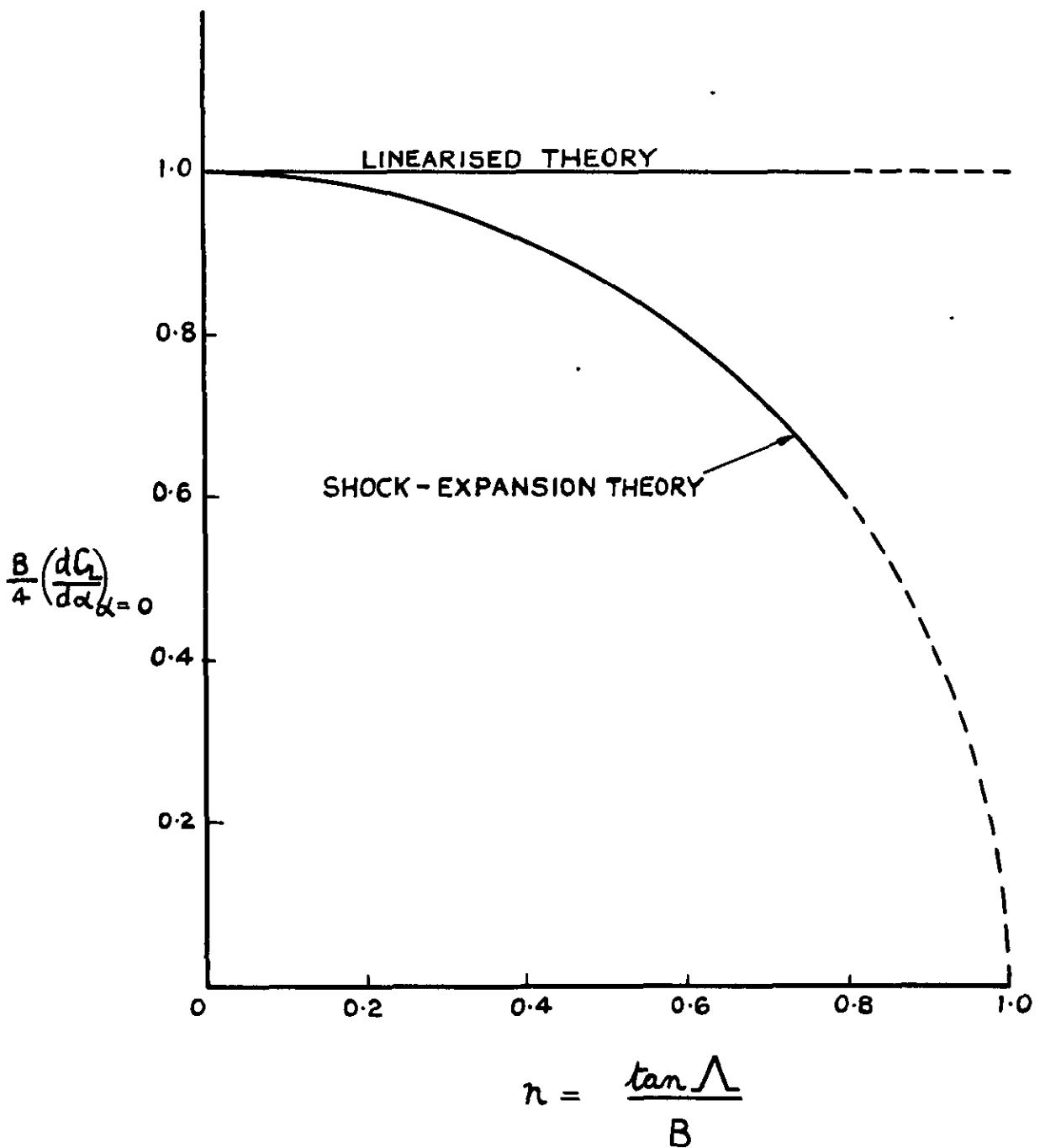


FIG.4 LIFT CURVE SLOPE OF FLAT PLATE DELTA WINGS WITH A "SUPERSONIC" LEADING EDGE; COMPARISON BETWEEN SHOCK-EXPANSION THEORY AND LINEARISED THEORY.

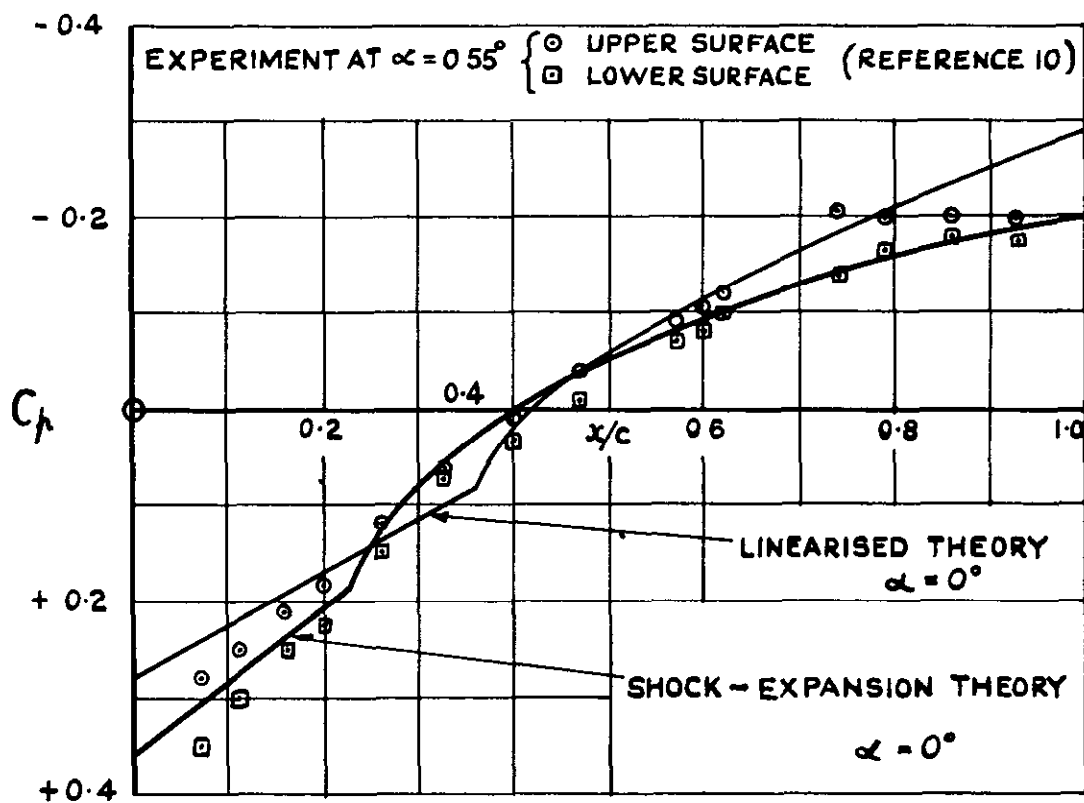
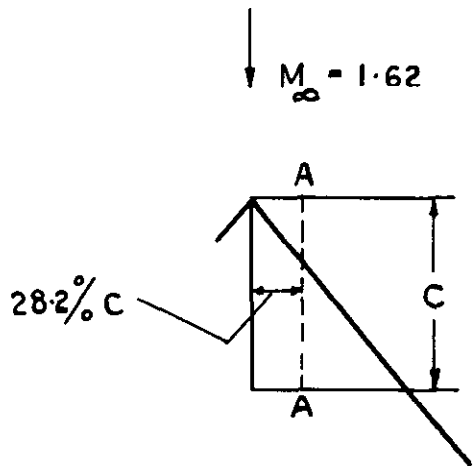


FIG.5 COMPARISON BETWEEN THEORY AND EXPERIMENT OF PRESSURE DISTRIBUTION IN THE TIP REGION OF A RECTANGULAR WING WITH 9% CIRCULAR ARC BICONVEX SECTION. $M_\infty = 1.62$
 PRESSURE DISTRIBUTION MEASURED ALONG A-A.
 $x/c = 0$ LEADING EDGE, $x/c = 1$ TRAILING EDGE.

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