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A Discussion of Two-Dimensional Turbulent Base Flows

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Summary.

The paper presents a review of recent experimental and theoretical research on two-dimensional base flows. The main object is to illustrate the extent to which the basic properties of the flow are understood in each of the different flow régimes: subsonic periodic, subsonic non-periodic, and supersonic.

The main conclusion from this study is that further work is needed throughout the whole field, but attention is drawn to those areas where the need is most urgent.

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1. *Introduction.*

Since the last review of research on base flows was written[1] considerable advances have been made both in the experimental and the theoretical fields. Accordingly, it was felt to be an opportune time to review progress once more. The present paper is fairly limited in scope, attention being confined to the case of two-dimensional turbulent flows. The reason for this restriction is two-fold. In the first place, even the work in this limited field is of direct importance in, and relevant to, a wide range of problems not only in aeronautics but in other branches of engineering.

The more important reason, however, is that our understanding of two-dimensional turbulent base flows is at a different stage from that of either laminar separated flows, on the one hand, or three-dimensional base flows, on the other. The laminar problem now stands on a different category because a complete theoretical solution is possible in principle and may, indeed, not be far off. Already advanced integral methods for the treatment of supersonic laminar base flows are being developed[2,3], and even in the subsonic case where the flow is unsteady it has been found possible to compute the development of the vortex street using numerical solutions of the Navier-Stokes equations (*see*, for example, Ref. 4). Further advances in the laminar problem are likely to be governed only by the size of the available computers.

In contrast, for turbulent flow, important details of the flow model are still not fully understood – even in the relatively simple supersonic case. The theories remain semi-empirical and will almost certainly remain so for a long time to come. In the subsonic and transonic cases the basic problems are only just beginning to be appreciated and a great deal more research is needed. The present review is concerned with a discussion of our understanding of the basic fluid dynamics of turbulent base flows, and in this context it would clearly be premature to complicate the matter further by considering three-dimensional effects.

The advances which have been made in the field of base-flow research during the last few years are of a different nature according to the particular flow régime concerned. In the case of the subsonic periodic flow – which exists behind an isolated section, such as an aerofoil with a blunt trailing edge – we are still at the stage of trying to understand the fundamental properties of the flow and the formation of the vortex street. A number of important experiments have recently been carried out with this aim (Section 2.2). These experiments are interesting, in the first instance, from a fluid-dynamic standpoint, but they have also a practical importance. From the experiments more is known about methods for reducing the strength of the vortex street, and hence for reducing the drag of sections with thick trailing edges.

Besides the periodic flow, a steady subsonic base flow can also exist. This type of flow occurs naturally downstream of a rearward-facing step in an otherwise continuous surface. Also, the steady subsonic base flow can be considered to be the limiting flow configuration which would be attained if the strength of the vortex street behind an isolated section could be reduced to zero. In this sense it is sometimes convenient to refer to an 'underlying' steady flow in the case of the periodic wake; but it must be made clear that it is not intended to convey the impression that the periodicity is superimposed on an underlying steady flow in any fluid-dynamic sense.

The problem of the non-periodic subsonic base flow is more amenable to theoretical treatment than the flow involving the vortex street. A start in this direction has already been made, and a brief statement of the present author's work in this field is given in Section 2.3. This work is aimed at the prediction of the base pressures on steps, and the prediction of the characteristics of aerofoil sections with thick trailing edges under conditions where the vortex street has been successfully eliminated.

Whereas for the subsonic case interest in base flows has been fairly limited, a great deal of effort has been put into research on supersonic base flows over the last fifteen to twenty years. A large volume of experimental data has been acquired, and the understanding of the nature of the flow, which is essentially steady, is already quite advanced. The development of theories for predicting the base pressure has played a considerable role in research in this field, but in many respects the theories are still inadequate as regards the detailed mechanics of the flow, or can be regarded as little more than correlations of data. The various theories are discussed in some detail in Sections 3.3, 3.4 and 3.5.

It has not been possible to include a section devoted to a detailed discussion of base flows at transonic speeds because very little relevant research has been done beyond that referred to in Ref. 1. On the other hand a few words will not be out of place on the general relationship between phenomena in the transonic range and those in the subsonic and supersonic ranges both of which are treated in detail in the body of the paper. As noted in Ref. 1, an abrupt fall in base pressure occurs over a small interval in Mach number near or just below unity, at least for sections of relatively small base height (expressed as a fraction of the chord). There is evidence[6] to suggest that this fall in base pressure can be interpreted simply as an adjustment in the level of base pressure accompanying the progression from a subsonic- to a supersonic-type base flow. Further research is needed before this simple picture can be accepted with confidence, but it is a convenient working hypothesis for use at the moment. Accordingly it will be assumed in the present paper that the supersonic-type base flow is established at $M = 1$ and that measurements at this condition can be correlated with data obtained at higher supersonic speeds. It is now, of course, well-known that the base-pressure coefficient reaches an absolute minimum at or near sonic velocity.

For further information on transonic base flows the reader is referred to Refs. 1 and 6.

2. *Base Pressures and Drag at Subsonic Speeds.*

2.1. *Profile Drag and Trailing-edge Thickness.*

As far as the aeronautical context of subsonic base flows is concerned, this work can perhaps be put into its proper perspective by first considering conventional sharp-trailing-edge aerofoil sections. Little work has been devoted to determining the limits of 'aerodynamic sharpness', that is to say the maximum thickness that a trailing edge may have before the characteristics of the aerofoil – and in particular the drag – differ significantly from those of an ideally sharp section. This matter is of some practical importance because it may be that aircraft designers have been over-cautious in this respect in the past; on the other hand perhaps the aim should be to make wings sharper than they have generally been.

A few tests have been made to measure the changes in profile drag which occur when the trailing edge of an aerofoil is either thickened or cut back*, and some data were shown in Ref. 5 where a plot was presented of C_D against the trailing-edge thickness, h, expressed as a fraction of the chord c. A similar diagram is shown in Fig. 1, and further data have been added. Most of the data were obtained at a fairly low chord Reynolds number, and the general conditions were such that the drag level appropriate to $h = 0$ is about 0·0095. Apart from one set of data, where this is less clear, all the results indicate that there is some small range of h/c, near zero, over which the value of C_D is more or less the same as for $h/c = 0$. In general this range seems to extend up to a value of h/c of between 0·5 and 1 per cent.

Now one would think intuitively that this figure would probably decrease as the Reynolds number increases – perhaps such that the ratio of the 'critical' trailing-edge thickness to the thickness of the boundary layer remains constant. It may be a useful exercise to use a criterion of this sort to provide a rule-of-thumb estimate as to whether a certain trailing edge can be regarded as 'aerodynamically sharp' at any given Reynolds number.

One might argue as follows. Let it be assumed that the value of C_D is related to the total momentum thickness at the trailing edge, θ_T, (i.e. the sum of the values on each surface), and the velocity outside the boundary layer at separation, u_T, by the formula given by Squire and Young

$$C_D = \frac{2\theta_T}{c}\left(\frac{u_T}{u_\infty}\right)^{3\cdot 2}.$$

The value of u_T/u_∞ will vary from one section to another (depending largely on trailing-edge angle) and is also dependent on Reynolds number. However, for typical wind-tunnel conditions it is frequently of order 0·9. Using this value, and taking C_D to be 0·0095, we find that $\theta_T/c = 0{\cdot}007$.

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*No discussion is made here of the way in which the increased thickness over the rear of the section is applied. Particularly at high subsonic speeds there is evidence that intelligent thickening of the aerofoil near the trailing edge can improve the flow over the section itself. Thus there is not always an overall debit due to adding trailing-edge thickness.

Comparing this value with the critical value of h/c mentioned above, it would seem that for a section to be 'aerodynamically sharp' the trailing-edge thickness must be of the same order or less than the total boundary-layer momentum thickness at separation.

Clearly this criterion can only provide a rough guide and it is probably valid only over a limited range of conditions. In particular one would expect that the shape of the boundary-layer velocity profile would be important and that, if the value of H (the ratio of displacement to momentum thickness) were large, a trailing edge of given thickness compared with θ_T would be less significant than if H were nearer the flat-plate value. This objection would be met by the use of a criterion involving the displacement thickness instead of θ_T; it would have been more difficult to infer a representative value from the data of Fig. 1.

The increase in profile drag which occurs when the trailing-edge thickness is increased above the 'critical' value arises from the low pressures acting on the base and, usually, on the rearward part of the aerofoil also. If a section is truncated the pressures acting over the rear are generally lower than those on the original sharp section on account of the upstream influence from conditions on the base.

Care must be exercised in connection with definitions of 'base drag' at subsonic speeds. Firstly, as indicated in the previous paragraph, not all the extra drag can be related to the base pressure itself (as is the case at supersonic speeds). Secondly, there are instances where the drag of a truncated section is higher than that of the original sharp section even though the base pressure coefficient,

$$C_{p_b} = \frac{p_b - p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2},$$

is positive. Thus if the 'base drag' is defined as equal to $-C_{p_b} \cdot h$ (the usual convention) it may appear odd at first sight that this quantity would, in such cases, be negative. However this difficulty is only one of definition. Further consideration shows that the increased drag due to truncation arises not because the pressures over the rear of the section are below ambient, but because they are lower than on the original aerofoil.

A large proportion of the additional drag associated with a thick trailing edge is due to the formation of a vortex street in the wake, the concentrations of vorticity representing a momentum 'loss'. On the other hand Hoerner[5] showed that part of the additional drag would be present even if the flow in the wake were completely steady. In this sense one can think of part of the additional drag as due to the steady effects and the remainder due to the underlying steady base flow, but this concept should not be misinterpreted (*see* Section 1, above).

2.2. *The Periodic Base Flow.*

2.2.1. (a) *The nature of the flow.* It is now well-known that at subsonic speeds the wake behind an isolated two-dimensional section, such as an aerofoil section with a blunt trailing edge, breaks up into a vortex street, even when the boundary layers are turbulent at separation. An attempt was made in Ref. 6 to define what is meant by a vortex street in turbulent flow, and it was suggested that in order to ascertain whether a street is present it is necessary to observe the flow for a time which is long compared with the periodic time of the shedding process. For this reason spark photographs do not necessarily show a well-defined street where it is in fact present. On the other hand, the analysis of hot-wire signals from in or near the wake[7,8] or the observation of the wake using a stroboscope[6], can amply demonstrate its existence.

The chief result of the existence of a vortex street, from a practical point of view, is the high drag as compared with what is expected in an equivalent steady flow[5]. For instance, for the same ratio of base height to the thickness of the boundary layer, the base drag on an aerofoil section may be three times what is measured on a rearward-facing step[6]. In the former case a vortex street is present, in the latter the flow is steady – or at least aperiodic.

In Ref. 1 a selection of data was shown illustrating the variation of base pressure with Mach number for a range of different sections. The main conclusions were that the base-pressure coefficient is largely insensitive to compressibility effects up to the critical Mach number – in general the value of $(-C_{p_b})$ increases slightly with Mach number in this range – and that the value of $(-C_{p_b})$ increases with increase of the ratio of trailing-edge thickness to chord, h/c. It was found possible to correlate C_{p_b} with h/c for a limited range of section shapes. As an indication of the general levels of base pressure involved, the value of C_{p_b} for a section with $h/c = 0{\cdot}05$ is of order $-0{\cdot}6$. However, data have since been accumulated which do not fit the correlation, the measured values of $(-C_{p_b})$ being too small for the given value of h/c.

There seems little hope of establishing a reliable correlation of base-pressure data at subsonic speeds. It is probable that the base pressure depends on the geometry of the section, as well as on the ratio h/c, and on the boundary-layer thickness. As regards the section shape, the experience we have on the sensitivity of the base pressure to the detailed geometry of the base (see below, Section 2.2.2) should serve to illustrate the difficulties involved. Nor is it likely, for a considerable time to come, that a method will emerge for predicting base pressures (and drag) in the unsteady-flow régime, except perhaps for very small trailing-edge thicknesses. In this latter case it may be possible to extend the currently available methods for conventional sections, but even then it will probably be necessary to ignore the periodicity in the wake. For larger degrees of bluntness there is not even an adequate flow model.

Various models of the flow have been constructed, the best known being that originally proposed by Heisenberg[9] and put on a firmer basis more recently by Roshko[10]. This model considers that the flow past the section itself is essentially steady and can be analysed using classical 'free streamline' theory. Some distance downstream in the wake a 'coupling region' is assumed to exist where the free streamlines are joined to a Kármán vortex street. An important feature of this model is that only a fraction of the vorticity shed continuously from each separation point finds its way into the discrete eddies in the street. It appears that under normal conditions the fraction of vorticity is about a half, but that it is reduced by devices such as splitter plates which interfere with the formation of the street.

For the type of sections which are considering (that is, sections of reasonably small thickness-chord ratio with sharp corners at the trailing edge) the Heisenberg/Roshko model is plausible in so far as it has been observed that the vortex street appears to form a short distance downstream of the section, not at the base itself[6,7,8]. On the other hand the assumption that the unsteady effects are confined to the wake may be a serious one (although not so serious as in the case of the circular cylinder, for example). Indeed the passage of pressure disturbances forward over the section can be seen in flow photographs[6].

In the context of this model we may mention the theoretical work which has been done on the general problem of the rolling-up of vortex sheets[11,12]. This work might be regarded as aimed at an elucidation of the mechanics of the 'coupling region'.

An alternative model[13] considers that the unsteady effects are not merely confined to the wake but play an essential role round the section as well. The basis of this model is a consideration of the nature of the displacement surface presented to the external flow by the section and the region of separated flow immediately downstream of the base. As far as the external flow is concerned the section is attached to a deformable rearward extension corresponding to the separated region. It is postulated that the separated region is unstable to disturbances orientated in a direction normal to the chordline and, in fact, suffers oscillatory deformations. Accordingly, the external flow behaves rather like that past an aerofoil with an oscillating flap; an oscillatory circulation is set up round the aerofoil and vorticity is shed into the wake in sympathy forming the vortex street.

This model explains why the provision of a constraint in the wake against asymmetric disturbances is effective in weakening or even suppressing the street. Such a constraint is provided by a splitter plate*; a rearward-facing step – past which the flow is steady – can be regarded as equivalent to a section with a very long splitter plate. It will be seen later that the effect of a trailing-edge cavity can also be explained in terms of this model.

It has sometimes been suggested that the mechanism for the vortex formation lies in the instability of free stagnation points at subsonic speeds. If the flow were steady a free stagnation point – the 'point of confluence' as it is usually termed in the literature on base flows – would exist at the downstream end of the

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*Although the effect of a splitter plate can also be explained on the basis of the Heisenberg/Roshko model.

region of reversed flow. The concept of the instability of the point of confluence can be reconciled with either of the two flow models described above. On the one hand it could be argued that this is an interpretation of what happens in the coupling region of the Heisenberg/Roshko model. On the other, it could be argued that the instability of the confluence point is directly related to the stability of the separated region as a whole.

Mention must also be made of the flow model proposed by Shaw[14] in which the essential mechanism responsible for the vortex shedding is thought to be the acoustic resonance associated with the passage of sound waves round the section. There is some common ground between Shaw's model and the oscillatory-circulation model.

Thus, in broad terms, there are two alternative flow models. The one assumes that the vortex street is formed as a result of the instability of the two shear layers springing from the separation points, but that the influence of the street on the section is limited to its being a 'closure condition' at the end of the shear layers. The other assumes that there is a lack of uniqueness in the external flow field as a whole due to the existence of the separated region, and that the vortex street is only a manifestation of the periodicity occurring in the flow past the section itself. The main conclusion, on this matter, from the discussions in the present paper is that neither of the two models is adequate to explain all the observed effects but that each is useful as a basis for explaining certain results. It may therefore emerge that the two models are not, in fact, alternatives but merely represent different aspects of the unsteady-flow phenomenon.

2.2.2. *Some recent experiments.* The most interesting experiments conducted recently have been those concerned with devices which interfere with the formation of the vortex street. These tests are clearly of practical importance in that they may lead to the development of practical drag-reduction methods. But at the same time they have helped to elucidate the fluid-dynamic properties of the flow.

The experiments in this category can usually be divided into two groups: the first are concerned with the use of mechanical devices (splitter plates, etc.), the second with suction or injection (base bleed). In some instances, however, aspects of both are involved.

The most common mechanical device for interfering with the formation of the street is the splitter plate[6,7] (Fig. 2). This is a thin, two-dimensional flat plate extending into the wake from the centre of the base. Attention was drawn in Ref. 6 to the fact that the increase of base pressure was not monotonic with increase of the length of the splitter plate, but that the base pressure reached a local maximum for a plate of order one base-height in length. This effect has now been repeated in other tests and three sets of results are shown in Fig. 3 for comparison. There is evidence that the local maximum occurs for a shorter splitter plate when the section is boat-tailed. This is to be expected since the streamlines are then converging at the trailing edge and the wake is presumably thinner. There is also some evidence that the local maximum occurs for a shorter splitter plate as the Reynolds number is reduced, but the reasons for this are less clear.

To judge by the results in Fig. 3 the use of a splitter plate of the appropriate length to achieve the local maximum in base pressure leads to a reduction in base drag of about 50 per cent. It also appears that at this condition the base drag is only about 25 per cent above the level to which it is tending for a very long splitter plate. This final level presumably corresponds to that appropriate to the steady base flow. To achieve the complete elimination of the street clearly demands a splitter plate several times as long as the trailing-edge thickness and this may present considerable structural – and control – difficulties in a practical case. However, the use of a splitter plate of length about equal to the trailing-edge thickness appears to be a somewhat better practical proposition and still gives a worthwhile decrease in base drag.

Bearman[7] has conducted some very interesting tests to determine the nearest point, downstream of the trailing edge, where the vortex street can be regarded as fully developed. This corresponds to a peak in the local velocity fluctuations and also to the region of low static pressure in the wake observed in Ref. 6. Bearman's work demonstrated that for short splitter plates the 'point of vortex formation' is displaced downstream by the length of the plate. This process appears to be the most effective in increasing the base pressure, and it leads to the local maximum when l/h is equal to about unity. If the length of the splitter plate is increased further the shear layers themselves become unstable, as was also noted in Ref. 6.

In this condition the vortex street forms nearer to the end of the plate than before and the device appears less efficient as a means of increasing the base pressure. Bearman shows that the increase in base pressure can be related directly to the amount by which the vortex-formation point is displaced downstream.

For very long splitter plates the flow reattaches to the plate and the vortex shedding is more or less suppressed. In these cases, Bearman shows that the length of the separation bubble (i.e. the distance from the trailing edge to the reattachment point) is approximately inversely proportional to the base pressure coefficient. This is to be expected on the basis of linearised inviscid theory applied to the external flow, and is confirmed by tests carried out at the N.P.L. (*see* Ref. 13). What is more surprising, however, is that when, with shorter splitter plates, there is vortex shedding the vortex-formation point occupies the position where the reattachment point would be expected to lie if the same relationship with the base pressure held. This is a most important result. In some respects it supports the view that the flow upstream of the 'coupling region' (the vortex-formation point?) is essentially steady and that the periodic effects are confined to the downstream part of the wake. On the other hand it affords evidence that it is the instability of the point of confluence (which would become the reattachment point if there were a wall present) which is the key to the periodic behaviour.

Generally speaking, the results for splitter plates can be interpreted as evidence for either of the two flow models. In the context of the Heisenberg/Roskho model it would be argued that the effect of the splitter plate is to displace the 'coupling region' downstream until the shear layers have diffused further, thereby reducing the proportion of the vorticity which becomes concentrated into the street. On the other other hand it could be argued that the presence of the splitter plate provides a symmetrical constraint on the separated region and thus reduces the possible amplitude of the oscillatory deformation. However, an important criticism of this model arises because of the difficulty of explaining the kink in the curve of base pressure against splitter-plate (Fig. 3).

Another device which tends to reduce the strength of the vortex street and increase the base pressure is a trailing-edge cavity (Fig. 2). In one form this is a simple downstream-facing cavity formed in the base[6,15]; a more sophisticated form[16] is a carefully profiled shape designed into an aerofoil section, with suction to assist the formation of a stable eddy in the cavity (this has been termed the 'snow-cornice' aerofoil). In the latter case, on which detailed theoretical and experimental work has been done, the aim is to eliminate the street entirely. The experience of Hazen *et al*[17] is, however, that this may be too uneconomical a goal to aim at in view of the large suction quantities involved.

Little is known about the effects of cavity geometry in the simple case where suction is not applied and the device is just intended to weaken the vortex street. The test reported in Ref. 6 showed a reduction in base drag of about 20 per cent for a 'square' cavity (i.e. of streamwise depth equal to the trailing-edge thickness). In another test carried out at the N.P.L.[15] (using a different section) a reduction of 30 per cent was obtained using a cavity of depth equal to 1·7 times the base height.

Some tests have also been conducted at the N.P.L. to examine the effects of combining a cavity and a splitter plate[18]. However, no configuration was found which was superior to a splitter plate mounted on a solid base. Moreover some odd results were obtained in which the base pressure was found to be sensitive to the length of splitter plate extending *upstream* into the cavity as well as downstream into the wake. The general conclusion of this work was that combinations of a cavity and a splitter plate are not promising from a practical point of view and should probably be avoided. On the other hand, from a fluid-dynamic standpoint, there are some interesting problems to be examined.

The stabilising effect which a rear cavity has on the wake lends considerable support to the flow model which involves deformations of the separated region. The walls of the cavity can be visualised as constraining the upstream part of the standing eddies which, while not forming in a stable way as would in a steady wake, probably exist in some partly formed state. The same argument can be used to explain why a rounded trailing edge, of appreciable radius, can in some cases lead to a significant *increase* of drag compared with a rectangular base[6,19]. In this instance the eddies would be required to form on a convex surface which, intuitively, seems more difficult. It is not easy to see how either of these facts could be accommodated in the Heisenberg/Roshko flow model.

An important variation of the trailing-edge cavity is the ventilated cavity. A range of tests has been carried out at the N.P.L.[15] on an aerofoil fitted with a cavity which is ventilated in different ways. Two basic schemes were tried, using either perforations or streamwise slots in the walls of the cavity. The former is essentially an automatic base-bleed system in which the bleed air is accepted from the boundary layer approaching the trailing edge. The latter contains some of the same elements but it is possible that, in addition, the edges of the slots act vortex generators, increasing the mixing in the shear layers and giving them increased stability. As far as the base pressure is concerned both types of ventilation give much the same order of increase but the slotted cavity is found to be superior as regards the overall decrease in drag. It would appear that in the case of the perforations the streamwise force acting on the walls of the cavity (due presumably to the pressure differential across the holes) partly offsets the decrease in base drag. No significant drag increment appears to arise in this way in the case of the slotted cavity.

A few exploratory tests were also carried out[20] with a slotted-wall cavity fitted to the model described in Ref. 6. It was found that the base pressure was fairly sensitive to the ratio of the length of the slots to the streamwise depth of the cavity. Optimum results appeared to be obtained when the upstream part of the cavity was solid (about 15 to 20 per cent of the depth) and the slots covered the remainder, running out at the downstream end of the cavity walls. With this configuration the increase in base pressure was as good as, or better than, that obtained with a splitter plate of length equal to the base height. It is not easy to see why the extent of the slots in the cavity walls should be important. Some further work would be welcome to investigate this point.

The relative effectiveness of splitter plates and trailing-edge cavities (solid and ventilated) as devices for reducing base drag is compared in Table 1.

The other method of reducing base drag at subsonic speeds which has been examined in some detail is base bleed, i.e. the continuous injection of low-velocity air through the base. It is not to be confused with the injection of a high-velocity jet through a spanwise slot in the base[21]; the action of this latter is probably different and resembles more the effect of a splitter plate. Some early exploratory work on base bleed was done by Holder[22] and this was followed up by some tests on the same model by Moulden[23]. This work established that significant reductions in drag could be achieved over a small range of bleed quantity. More recently, systematic work has been done by Wood[8].

Wood's tests[8] have shown that, as at supersonic speeds, optimum results are obtained by bleeding the air through a vent nearly filling the base so that for a given mass flow the injection velocities are minimised. The results indicate that there is a progressive increase of base pressure with bleed quantity until the injection velocity becomes excessive. For a value of the bleed coefficient C_q* of about 0·12 the drag appears to have fallen to the level appropriate to a steady wake – as indicated by the results obtained using a long splitter plate – and there is little further decrease in drag for larger values of C_q. In order to achieve the value of $C_q = 0{\cdot}12$ without the injection velocity becoming excessive, it seems that the height of the vent must be at least about 0·6 of the trailing-edge thickness. When allowance is made for the equivalent drag of the bleed quantity it seems doubtful whether the net reduction in drag is any greater than could be obtained using a short splitter plate.

Wood finds that as the bleed quantity increases the point of vortex formation is displaced downstream. This observation is interesting and is consistent with the observations using splitter plates[7] but the general question of the mechanism behind the effect of bleed on the subsonic periodic base flow has not yet been elucidated.

Whilst on the subject of methods for reducing the strength of the vortex street, mention may be made of the work of Brown and Goddard[97] who have investigated the effect of external sound waves on a vortex street. In a recent paper Quinn[98] has commented on the potentialities of such a scheme for reducing drag. On the whole this does not seem a very attractive scheme from a practical point of view even if significant reductions of drag could be achieved on an experimental basis.

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$$C_q = \frac{u_j h'}{u_\infty h}$$

where u_j is the injection velocity and h' the height of the vent.

2.3. *Steady Subsonic Base Flow.*

The available experimental data relating to the steady subsonic base flow is not extensive. A few tests have been carried out at the N.P.L. on a rectangular step[6] and some further results were referred to in Ref. 25. Apart from these measurements a very limited amount of data exists from tests on aerofoils where a splitter plate, or some such device, has been used to reduce the strength of the vortex street to an acceptable minimum.

On the other hand the steady flow régime is more amenable to a theoretical treatment and a start has been made in this direction[25]. This work will be discussed later in the present Section.

In the case of rearward-facing steps, as such, the base pressure itself is probably of most importance; in the case of aerofoil sections, however, the profile drag is a more meaningful quantity. For this reason a convenient starting point in this discussion will be the procedure for predicting the profile drag of conventional sharp-trailing-edge sections.

The most commonly-used methods for predicting profile drag in subsonic flow make use of the fact that far downstream of the section the momentum thickness of the wake must be proportional to the profile drag. This result forms the basis of the classical method of Squire and Young[26]. The procedure is to use the potential-flow pressure distribution over the section to calculate the development of the boundary layer up to the trailing edge and then to use a simplified analysis of the wake to calculate the growth of the latter from the trailing edge to infinity. Subsequently the method has been modified in certain details and also extended to compressible subsonic flow[27,28] but for our present purposes the original incompressible form will suffice.

The wake relation was obtained by an approximate integration of the momentum-integral equation along the wake, using an empirical expression for the shape factor, H. The relation could be written

$$C_D = \frac{2\theta_T}{c}\left(\frac{u_T}{u_\infty}\right)^{\frac{H_T+5}{2}},$$

where, as in Section 2.1 above, θ_T and u_T denote the total momentum thickness and the velocity outside the wake at the trailing edge, respectively. H_T denotes the shape factor (the ratio of displacement to momentum thickness) of the wake at the separation point.

Now, so long as the trailing-edge thickness is small – i.e. compared with any measure of the boundary-layer thickness – the pressure distribution will still approximate to that for the case of the sharp trailing edge; so will the momentum thickness of each boundary layer. The only effect the slight blunting will have is to increase slightly the value of H_T; the displacement thickness of the wake will be increased by the thickness of the trailing edge. The effect of this on C_D will, however, be small since H_T enters the expression for C_D only as H_T+5.

This argument in general terms lends support to the observation which was made in Section 2.1 above to the effect that the profile-drag coefficient is insensitive to trailing-edge blunting until the thickness of the trailing edge becomes appreciable compred with the boundary-layer thickness.

On the other hand if the degree of blunting becomes significant the expression for C_D loses its validity, since the value of H_T will exceed the order of values for which the empirical relation can be expected to hold. Moreover, the pressure distribution becomes distorted from its form on the sharp aerofoil. In general, of course, periodic effects will also become apparent but in this Section we are considering the case where the vortex street is suppressed by some means.

The present author has suggested a method[25] for tackling the case where the trailing-edge thickness is large compared with the boundary-layer thickness. [So far, however, there is no analysis which is valid for the intermediate range of trailing-edge thicknesses.] For the case where the trailing-edge thickness is large, the analysis can be conveniently divided into two parts: an analysis of the external flow past a displacement surface presented by the section, its attached boundary layer and the wake behind it, and an analysis of the flow in the wake itself.

The analysis of the external flow is essentially a mixed boundary-value problem since, while the section geometry is given and the growth of the boundary layer known at least approximately, the shape of the downstream part of the displacement surface – representing the separated region and wake – is not known at the outset. Instead, in the method of Ref. 25, certain restraints are imposed on the pressure distribution in the initial part of the wake so that it should resemble that through a typical region of separated flow. The region downstream of a step provided experimental data which could be used to establish the type of restraints necessary. This approach enables the pressure distribution round the section to be determined as a function of the base pressure – which as far as this part of the analysis is concerned is a free parameter as in the usual 'free streamline' theory. The analysis was performed using linear inviscid theory. It is recognised that this imposes limitations on the generality of the work but it was thought justified from reasons of simplicity at least until further experience was gained.

The analysis of the turbulent flow in the wake is aimed at matching the development of the wake to that of the external flow. This is not done at each point along the wake but appeal is made, as in the case studied by Squire and Young, to compatibility at infinity downstream. Since from the analysis of the external flow the pressure distribution is known as a function of the base pressure it is a straightforward matter to estimate the profile drag as a function of the base pressure simply by evaluating the pressure drag and skin-friction drag directly. (The restriction to appreciable trailing-edge thickness enables this to be done more easily than would be the case for a sharp section.) Knowledge of the profile drag as a function of the base pressure, then defines the asymptotic momentum thickness of the wake in terms of the base pressure. It remains to integrate the momentum-integral equation along the wake to provide a further relation between conditions at separation and those at infinity.

In the analysis of Ref. 25 the momentum-integral equation is applied, not to the complete flow in the wake, but to the part outside the mean dividing streamline through the separated region; the recirculating flow immediately behind the base is excluded from the integration. A few simplifying approximations are made in order to integrate the equation but these are thought to be satisfactory at the present state of the art. The approximations are little more serious than those contained in the analysis of Squire and Young, and the treatment of the wake in the method of Ref. 25 can be regarded as a logical extension of their work to cover the case of non-zero trailing-edge thickness.

The validity of the wake analysis was confirmed to an encouraging degree by measurements of the base pressure on steps, although the comparison was only meaningful within the framework of the assumption that the effect of the downstream wall on the pressure field is small. The fact that data from the same experiments was used in the analysis does not invalidate the comparison. In the case of the flow past a rectangular step the analysis of the external flow (which involved empirical information concerning the pressure distribution downstream of separation) plays little part in the determination of the base pressure. It only arises at all, in this case, as far as the length of the separated region is concerned and this information could have been provided by other means (*see* Ref. 13).

The predicted variation of base pressure with Mach number for a long, parallel-sided section, or for a rectangular step, is shown in Fig. 4. It is noted that with increase of Mach number the base-pressure coefficient falls slightly. The rate of decrease is however only about a half of that which would occur in the case of the pressure coefficients on the surface of an aerofoil. The variation of C_p with Mach number indicated by the Prandtl-Glauert law is shown in Fig. 4 for comparison. The more sophisticated compressibility laws indicate an even larger rate of decrease of the pressure coefficients.

The main application of the method is to the prediction of the effects of section geometry, Reynolds number, etc., on base pressures. In such exercises the validity of the analysis of the external flow must be assumed but this is likely to be as good as that of the flow model as a whole.

In Ref. 25 some calculations were presented showing the effect of boat-tailing on base pressure. The general effect, that is a rapid increase of base pressure even for moderate degrees of boat-tailing, has now been confirmed by tests carried out at Cambridge University[24]. Calculations were also presented showing the effect on an aerofoil pressure distribution of trailing-edge truncation using an aerofoil of RAE.103 section for an example. Two cases, with different degrees of blunting, were considered and these were compared with the basic RAE.103 section. To make the comparison more meaningful, it was assumed that the sections were stretched such that they all had a final thickness-chord ratio of 10 per cent. The calculations were carried out, as it were, for infinite Reynolds number; i.e. the boundary layer was assumed to be of zero thickness at separation. Under such conditions it appeared that the profile drag increased from zero (i.e. the potential-flow value for zero trailing-edge thickness) by about 0·0014 for each 1 per cent increase of trailing-edge thickness expressed as a fraction of the chord.

This result is to be expected since there is no range of h/c in which the trailing-edge thickness is small compared with the thickness of the boundary layer. To examine the effect of blunting for the case of finite Reynolds number, some rough calculations have subsequently been performed assuming, for simplicity, a linear growth of the boundary layer with chordwise position. These results are mainly of qualitative interest but they can be considered important in demonstrating the expected trend. Fig. 5 shows the predicted variation of C_D with h/c for chord Reynolds numbers of 10^6 and 10^7, and it is seen that at these Reynolds numbers there is the same insensitivity to small increases of h/c, near zero, as is evident from the experimental data of Fig. 1.

Although not included in the original analysis of Ref. 25, the method can also be extended to include the effects of base bleed. This is achieved simply by rewriting the formulation of the wake analysis to take account of a continuous flux of low-momentum air through the base. Some calculations have been done for the case of a long, parallel-sided section, or rectangular step, and the results are shown in Fig. 6. In these calculations the effects of the approaching boundary layer have been ignored, and incompressible flow has been assumed. The experimental results of Wood[8] are shown for comparison. It will be recalled that for small bleed rates, values of C_q less than about 0·12, the strength of the vortex street was still significant in Wood's experiment. For larger values of C_q, however, the slow increase of C_{p_b} with increasing bleed rate is predicted satisfactorily by the theory; the slight difference in level between theory and experiment in this range is, of course, consistent with the boundary-layer effects.

Fig. 6 demonstrates that while bleed may be an effective means of eliminating the periodicity, it cannot be considered an economic method for decreasing the base drag once the unsteady effects have been eliminated.

3. *Base Pressures at Supersonic Speeds.*

3.1. *Experimental Data.*

Measurements of base pressure at supersonic speeds have been reported by a large number of investigators over the last twenty years. The experimental data referring to two-dimensional turbulent flow are fairly comprehensive over the Mach number range 1·5 to 3 (*see* Fig. 7) but rather sparse outside this range, particularly at low supersonic speeds. Although flight data exist, including the important results for the X–15 aircraft (Ref. 29), most of the measurements have been made in wind tunnels using either aerofoil models or step models. Step models have tended to be favoured in fundamental investigations, chiefly because it is easier to vary the height of the base and also easier to measure the pressures downstream of the base. No systematic tests have been reported in which the results obtained on step models have been compared with those on aerofoil models but there do not seem to be large discrepancies between them. If this is true it is largely fortuitous and there are theoretical reasons for doubting that they should agree. This will be mentioned later in the paper (Section 3.4, below).

A large number of measurements of base pressure at supersonic speeds are plotted in Fig. 7. It is seen that the value of C_{p_b} is strongly dependent on Mach number although other factors clearly have some effect. At a particular Mach number the base pressure is chiefly a function of the base height as compared with the boundary-layer thickness at separation; this is been known for a long time. There has recently arisen a controversy, however, as to whether the base pressure depends explicitly on Reynolds number (in addition to the implicit dependence by way of the boundary-layer thickness). It will be seen later in the Report (Sections 3.4 and 3.5, below) that there are theoretical grounds for suspecting this but there is little support from experimental data. For the present it will be assumed that this effect is negligible. Nor does it appear that the base pressure is influenced much by the geometry of the section ahead of the base – at least for sections of a reasonably small thickness-chord ratio. Until there is evidence to the contrary, therefore, we conclude that the base pressure depends only on Mach number and on the ratio of the base height, h, to the thickness of the approaching boundary layer.

As the base height becomes large compared to the boundary-layer thickness the base pressure seems to approach an effective limit. It is of no practical importance to discuss what happens in the limit of $h/\theta \to \infty$ but the concept of a 'limiting base pressure', for which

$$C_{p_b} = (C_{p_b})_l,$$

is a useful hypothetical datum from which to measure the effect of decreasing h/θ to finite values. It must be stressed, however, that the precise value assigned to $(C_{p_b})_l$ at a particular Mach number depends on how the extrapolation to $h/\theta = \infty$ is carried out.

It has been recognised for many years that the base pressure rises from its limiting value as the base height decreases compared with the boundary-layer thickness. Chiefly on theoretical grounds, it is usually assumed that the correct measure of the boundary-layer thickness to use is the momentum thickness, θ. Accordingly it will be assumed that C_{p_b} is a function of Mach number and h/θ. Given this assumption one can look for some empirical correlation of the data. The value of this exercise, is two-fold; if successful, it both provides a check on the coherence of the measurements and provides a means of interpolating between the data for the purposes of predicting base pressures in other cases. The limitations of such an empirical correlation are, of course, obvious and will not be repeated here.

One possible form of correlation is shown in Fig. 8, where $\{(C_{p_b})_l - C_{p_b}\}$ is plotted as a function of h/θ. The ordinate represents the difference between the measured base-pressure coefficient and the assumed value of $(C_{p_b})_l$ for the same Mach number. In fact the variation of $(C_{p_b})_l$ with Mach number is chosen to give the best correlation. Allowing for a degree of scatter in the data a collapse of the points is obtained. The measurements plotted in Fig. 8 relate mainly to tests where θ has actually been measured (notably the very comprehensive measurements of Hastings[30]) although some data have been included where this quantity could be fairly reliably estimated* from the information provided. The trend of the points in Fig. 8 suggests a relation of the form

$$(C_{p_b})_l - C_{p_b} = 0{\cdot}175 \left(\frac{\theta}{h}\right)^{\frac{1}{2}}.$$

$(C_{p_b})_l$ is plotted against Mach number in Fig. 7 and is tabulated in Table 2. Fig. 7 indicates that the curve of $(C_{p_b})_l$ against Mach number is an effective upper limit to the spread of the points.

The data presented in Fig. 8 cover a Mach number range of 1 to 6. It must be made clear, however, that only within the range 1·5 to 3 are the data sufficiently comprehensive to warrant very much confidence in the correlation. Outside this range the correlation must be used only as a guide. Nor does the relation shown above hold down to arbitrarily small values of h/θ, an obvious limit being the value for which the expression predicts $C_{p_b} = 0$. This restriction becomes more severe at the higher Mach numbers. The correlation evidently breaks down earlier than this, in fact, for it seems likely that C_{p_b} should approach zero as $h/\theta \to 0$.

Fig. 8 shows that the base-pressure coefficient is a strong function of h/θ only for small values of this ratio – less than about 50. For values of h/θ above 50, C_{p_b} is relatively insensitive to changes in base height and, in this range, is effectively a function of Mach number only. This observation lends support to correlations of base pressure, such as that of Staylor and Goldberg[31], in which C_{p_b} is taken to be simply a function of Mach number. But in order to use such correlations successfully it is necessary to know their range of validity and the work discussed in the previous paragraphs of this Section can be regarded as providing this information.

On the other hand, the fact that C_{p_b} decreases rapidly with increase of h/θ for very small values of the latter and that, especially at the lower Mach numbers, appreciable values of C_{p_b} are associated with very small values of h/θ, suggests that roughness drag (that is, roughnesses in the form of small rearward-facing steps) may reach a maximum at Mach numbers of order unity.

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*Some of these computations were done by Mr. N. Thompson of the Royal Aeronautical Society and grateful acknowledgements are due to him.

3.2. *The Nature of the Flow.*

The nature of the supersonic base flow is now reasonably well understood and there is little dispute among the theoreticians as to the gross features of the flow model which forms the basis of analytical treatments of the problem. Indeed, schematic diagrams similar to Fig. 9 are to be found in scores of published papers.

The boundary layer which has developed on the surface of the body approaches the trailing edge and separates there to form a free-mixing layer. Since the base pressure is usually lower than the pressure on the afterbody an expansion fan springs from the corner and the shear layer is deflected away from the external flow. Some distance downstream of the base the free shear layer either reattaches to a solid wall or coalesces with a second shear layer and continues downstream. The reattachment of confluence occurs in a region of abrupt pressure rise, in contrast to the region closer to the base in which the pressure gradients are small. In the case of the flow past a step the position of the reattachment line can easily be found by the use of a surface oil technique or by some other means which senses the surface shear (the reattachment point is a point of zero surface shear). The reattachment point is found to lie part of the way up the abrupt pressure rise. Presumably the equivalent is true of the flow in the wake of an isolated aerofoil section.

The mixing layer induces a recirculating flow immediately downstream of the base but (unless there is bleed into the cavity) the mass flux of the recirculating flow must be conserved, and the theoretical flow models posit the existence of a mean dividing streamline joining the separation point of the point of confluence (or reattachment point). If there is bleed the streamline springing from the separation point is taken to be distinct from that which passes through the point of confluence and the mass flux between them to be equal to the bleed mass flux.

These considerations concerning the streamline pattern in the separated region have a meaning only in the context of the assumption that the flow is steady (although turbulent flow is accommodated). That the flow in the wake is steady is an assumption and must be weighed against the evidence that exists pointing to periodic effects (in the downstream part of the wake) even at high supersonic speeds[32]. Nevertheless the assumption is a sensible one and there has been no suggestion that the periodic effects play any significant part in the determination of the pressure field as, of course, they do at subsonic speeds. For the case of the flow past a step there is even more likelihood that a steady-flow model will be physically realistic, and the fact that a well-defined reattachment line can be observed offers strong support to this view (*see*, for example, Ref. 30).

3.3. *General Features of the Theories.*

A common feature of the theoretical methods for predicting base pressure is the assumption that the base pressure depends on the pressure recovery which can be sustained by the wake, or by the reattaching boundary layer. The final recovery pressure far downstream is assumed known; frequently it is either equal to, or can be approximated by, free-stream static pressure. The base pressure is then found by subtraction from this known pressure the computed pressure rise occurring between the base and infinity downstream.

The pressure recovery downstream of the base is estimated by an analysis of the development of the wake, or the reattaching boundary layer, and its interaction with the external supersonic stream. In their method of analysis the theories can be divided into two groups: those which depend on the dissection of the base flow into a number of discrete parts and those which employ integral techniques. The two groups will now be discussed in turn.

3.4. *The 'analytic' approach.*

This class of theories owes its inspiration to the original thoughts of Chapman[34], Korst[35] and Kirk[36] but the degree of sophistication has increased considerably since the early days. The theories assume that the flow can be dissected into a number of parts which can be analysed separately:

(a) The flow approaching the separation point.

(b) The interaction between the boundary layer and the expansion fan (or shock) which springs from the separation point.

(c) The development of the separated shear layer.

(d) The recompression associated with the confluence of the two shear layers (or the reattachment of the shear layer to the downstream wall, in the case of the flow past a step).

The various regions may be identified in Fig. 9.

(a) *The flow approaching separation.* In the simplest cases, where there is an expansion at separation, the pressure on the afterbody remains independent of conditions on the base up to a point a few boundary-layer thicknesses from the corner. The development of the boundary layer can thus be estimated from the known pressures on the body without reference to the precise value of the base pressure. As a result, at least up to a short distance from separation, the boundary-layer velocity profile will reflect only the upstream conditions – if the afterbody has flat surfaces the velocity profile will approximate to the flat-plate type. On the other hand cases do occur where there is a shock instead of an expansion at separation. This arises when the base pressure is higher than the pressures on the afterbody. Such conditions can occur even on a parallel-sided section, or a rectangular step, over a small range of Mach numbers at transonic speeds[6]. It is more usual, however, in complex base flows involving embedded jets where the jet total pressure considerably exceeds that of the free-stream. In these cases the boundary layer may thicken or separate upstream of the corner, and the conditions at separation depend more critically on the level of the base pressure. This problem is similar to the flow in a compression corner or the flow near a shock-impingement point on a boundary layer and must be treated using the techniques of shock-wave/boundary-layer interaction.

(b) *The interaction at the separation point.* Here we shall consider only the case where the interaction is with an expansion fan. It is assumed in the theories that since the expansion occurs over a short distance, in terms of the boundary-layer thickness, the effect of the expansion on the velocity profile is essentially inviscid. Several attempts have been made to calculate the change in momentum thickness resulting from different expansion ratios[36–40]. The results are broadly similar, the small discrepancies which exist being due largely to different approximations of a computational nature. Roberts[40] has compared his own method with those of Carrière and Sirieix[38] and the present author[39] for a Mach number of 2.

(c) *The development of the shear layer.* The theories all involve the assumption that the development of the shear layer approximates to that of the constant-pressure mixing layer between a uniform stream and a quarter-infinite 'dead-air' region, appropriate initial conditions being related to the boundary layer at separation. Three assumptions are implicit:

(i) The shear layer in fact corresponds to that developing between a uniform stream and a fluid at rest.

(ii) The growth of this idealised layer can be calculated, at least for the simple case of zero initial thickness (i.e. the asymptotic mixing layer).

(iii) The effect of the initial boundary layer on the development of the shear layer is understood.

Little work has been done to assess the validity of (i), but over a range of conditions it is probably not seriously in error. This assumption may become questionable for high Mach numbers (above about 3, say) because the uniformity of the external stream deteriorates. Also, if the boundary layer is thick compared with the base height, the assumption that the region immediately downstream of the nose approximates to a 'dead-air' region would appear to lose its validity. In this case not only may the finite velocities in the separated region affect the mixing process but the condition of constant pressure may be violated.

As regards (ii), the information which is required is the velocity on the mean dividing streamline and, in the latest theories, the thickness of the shear layer at a given distance from separation, also. For low-speed flow the velocity on the dividing streamline (which is a function of the velocity profile) and the rate of spread of the layer are known fairly accurately; the effects of compressibility are, however, less certain.

Two assumptions are being currently made concerning the effect of compressibility on the velocity profile. In some work it is assumed that the profile expressed in terms of a parameter $\sigma y/x$ is independent of Mach number (Ref. 41, e.g.) and compressibility only affects the densities. (Here, σ is a factor describing the rate of spread of the shear layer.) In other work[42] the velocity profile is considered to be independent of Mach number when expressed in terms of $\sigma\psi/x$, where ψ is some stream function. The latter alternative is consistent with considerations of a compressibility transformation (such as that of Ref. 43) in which invariance of the stream function is a basic assumption by analogy with the laminar case. The difference between the two assumptions is not a trivial one. In the one case the velocity on the dividing streamline is a constant fraction (about 0·58) of that at the edge of the layer. In the other, this fraction increases with Mach number. Roberts' calculations[40], using Crane's velocity profile[44], show that the value of this fraction increases to about 0·64 at $M = 3$. Since the solution for the base pressure involves the square of this fraction a difference of 20 per cent arises at this Mach number.

It is not easy to resolve this question definitively, but Maydew and Reed's measurements [45], which extend up to a Mach number of nearly 2, lend support to the view that the velocity profile in terms of $\sigma y/x$ is independent of compressibility effects. Furthermore, Coles' work[46] on turbulent attached boundary layers has known that a compressibility transformation based on the assumption of invariance of the stream function is difficult to substantiate experimentally.

The effect of compressibility on the rate of spread of the shear layer is still less certain. The experimental data relating to σ show a scatter of about 4 to 1 at a Mach number of 2 (*see* Ref. 47, e.g.). The reasons for this are not very clear but it would appear that there are at least two contributory factors. Firstly, Bradshaw[49] has suggested that part of the measured 'compressibility' effect may, in fact, have been a spurious effect of Reynolds number, the importance of which has only been recognised recently. More will be said about this phenomenon later. Secondly, Johannesen[48] has noted that the rate of growth of the turbulent shear can be affected significantly by disturbances in the external flow.

Nor do the available theories of turbulent mixing (Ref. 50, e.g.) shed much light on the true state of affairs. It would seen that the question can only be solved by further detailed experimental work. In the meantime it is probably safest to rely on the measurements of Maydew and Reed[45], for Mach numbers up to 2.

Until recently it was thought that the effect of the initial boundary layer on the development of the mixing layer was satisfactorily understood. Using an extension of Görtler's first approximation[51] the present author indicated a method[52] for calculating the development of the velocity profiles from the initial form (that of the initial boundary layer) to the asymptotic form far downstream. Detailed calculations using this method were later performed by McDonald and Acklam[53]. One conclusion of this work was that, some distance downstream of separation the initial perturbations had decayed except for an *increase* of scale of the velocity profiles. This meant that the shear layer could be replaced by one growing, from zero initial thickness, from a virtual origin *upstream* of the real separation point, as had formerly been suggested by Kirk[36].

The possibility that, as a general result, this might be a fallacy was suggested by Bradshaw's experiments[49]. The conclusion of these was that, except at high Reynolds numbers, the virtual origin may be *downstream* of separation – at least so long as the initial boundary layer is turbulent. The explanation of this is that the level of shear stresses in the initial boundary layer is lower than that in the asymptotic free-mixing layer, and that the shear stresses do not immediately adjust themselves to the new level. In the initial region, whose length may depend, say, on the inverse of the Reynolds number, the shear stresses are lower than those in the asymptotic shear layer and the rate of spread of the layer correspondingly less. At low Reynolds numbers, in particular, the reduced rate of spread may offset the effect of the finite initial thickness.

So far no attempt has been made to rectify what appears to be a flaw in the theories, but clearly the matter must be settled if the physics of the flow are to be correctly represented. An important consequence of Bradshaw's observations is that a mechanism may be introduced by which the base pressure can depend explicitly on Reynolds number, aside from the implicit dependence by way of the boundary-layer thickness. Alternatively, it has been suggested that this effect may largely offset another Reynolds number dependence which has recently been suspected. This question will be referred to again later.

(d) *The recompression region.* The recompression region extends from the point at which the pressure starts to rise, to a point well downstream of the point of confluence (or reattachment, in the case of the flow past a step).

In the original theories of Chapman[34], Korst[35] and Kirk[36] the assumption was made that the final pressure, to which conditions recovered at the end of the recompression region, could be equated to the total pressure of fluid on the mean dividing streamline through the free shear layer. This was one of the central assumptions of the theories, and it is capable of two interpretations:

(i) The compression of the fluid on the dividing streamline is quasi-isentropic and the point of confluence (or reattachment point) lies at or near the top of the pressure rise; i.e. the part of the pressure recovery occurring downstream of this point can be ignored even if it exists.

(ii) Neither of these assertions is true but, by a fortunate coincidence, the overall pressure rise is more or less correctly given by assuming that they are. This would mean that, even though the point of confluence, or reattachment, is not at the top of the pressure rise, the effects of this are balanced by the 'loss' of total pressure occurring during recompression.

It is not clear which of these two interpretations the original authors intended. Their discussion of the 'escape criterion' suggests the former, whereas the comments of Chapman and Korst in Ref. 54 indicate otherwise. Even quite recently attempts have been made to substantiate the details of the original recompression criterion [55,58,104], although the authors of one of these papers have since found that this could not be done[101]. The criterion is still widely used, however, throughout the U.S., and even in Russia[56,57], as a basis for calculations of base pressure.

Chapman and Korst (but not Kirk) confined their published work to the case of zero boundary-layer thickness at separation. In this case it seems that the initial part of the pressure rise (i.e. upstream of the point of confluence) may be a fairly large fraction of the overall pressure rise, possibly exceeding a half. However, as the boundary-layer thickness increases, this fraction becomes less[30,59,60] until, with very thick boundary layers it may be no more than one tenth.

Several attempts have been made to improve on the original recompression criterion on an empirical basis. The assumption of isentropic recompression of fluid on the dividing streamline is usually retained (indeed it is difficult to argue convincingly how significant 'losses' of total pressure could arise), but it is recognised that the pressure continues to rise downstream of the point of confluence or reattachment. The present author[39] and Cooke[61] made the assumption that the initial pressure rise was a constant fraction, N, of the total. The value of N was assessed from measurements of the actual position of the reattachment point in experiments on reattaching flows. It was difficult to obtain a precise value and N was taken equal to 0·35 by the present author and 0·5 by Cooke. It must be stressed that these values were assessed from experiments on reattaching flows; their relevance to the flow in the wake of an isolated base could not be checked but the assumption was made that they applied to that case also. The modification of the theory along these lines produced a marked improvement between predicted and measured base pressures. Moreover, it was shown in Ref. 39 that the excursions in base pressure on a step at transonic speeds could be related directly to the corresponding measured variation of N.

As soon as it becomes clear that the value of N varied significantly with the thickness of the boundary layer, the validity of this approach was doubted, and alternative forms of the recompression criterion have been proposed. Carrière and Sirieix[62] have suggested one based on the angle of declination of the external streamlines, at the reattachment point, relative to the wall on to which the flow is reattaching their work being related specifically to the reattachment case. If it is assumed that the wall can be extended downstream in the same plane, their criterion relates the pressure at the reattachment point to the final recovery pressure. But as it stands the criterion makes no explicit assumption to this effect. The advantages of the definition of their criterion has been demonstrated by some tests carried out by Carrière and Sirieix[62]. In these tests it was shown that, even if the downstream wall does not extend indefinitely in the same direction, so long as it extends a short distance beyond the actual reattachment point the base pressure remains unaffected. A few experiments of a similar nature have also been reported in Ref. 101.

The assumption that the pressure rise downstream of the point of confluence or reattachment is independent of the boundary-layer thickness at separation is made explicitly by Roberts[40]. In a recent paper Roberts has analysed measured *base-pressure* data to infer the magnitude of the downstream part of the pressure rise as a function of Mach number. The validity of the remainder of the analysis had to be assumed in this exercise and it is not possible to ascertain from Roberts' paper whether the values obtained really represent the downstream pressure rise or are just the values of a disposable parameter necessary to make theory and experiment agree. All the same a good measure of correlation was obtained and the work obviously merits attention.

Of more fundamental significance is the work of McDonald which has formed the basis of a number of papers[42,63,64]. McDonald has made a number of important advances in the theory particularly as regards the treatment of the recompression region. Again the analysis applies explicitly to the base of the flow past a step but the usual assumption is made that the work has a more general validity. In one paper[63] an attempt is made to break away from the assumption that the recompression of fluid on the dividing streamline is isentropic. In its place an analysis of the initial part of the recompression is made on the lines of the model of the separating boundary layer proposed by Stratford[65] and Townsend[66]. Only in the outer layer is conservation of total pressure along streamlines assumed; near the wall at the reattachment point the velocities are taken to increase as $y^{\frac{1}{2}}$ as in Stratford's work. Suitable conditions are taken at the join. This approach represents a considerable advance over the earlier methods but it involves an added complication in that the pressure gradient at the reattachment point must be specified by some other means. In the absence of any reliable information on this point, rather gross assumptions have had to be made to assess the order of magnitude of the pressure gradient. The computed results differ little from those obtained using the simple approximation of isentropic recompression, and the comparison affords a degree of confirmation to the validity of the earlier work in this respect.

McDonald[42] has also suggested a method for dealing with the part of the pressure rise occurring downstream of the reattachment point. The central assumption in this analysis is that the pressure rise is related to the change in shape factor (i.e. the ratio of displacement to momentum thickness) of the reattaching boundary layer between the reattachment point and far downstream – where the boundary layer has recovered to the flat-plate type. [It will be noted that in this method the conditions are downstream need to be defined explicitly. Some modifications might therefore be required to enable the theory to cover the type of flows considered in the experiments conducted by Carrière and Sirieix[62].]

A similar approach was used by Reshotko and Tucker[67] for the separating boundary layer. However, an important difference exists between the two types of flow. Whereas in both cases the pressure rises in the downstream direction, the shape factor increases in the separating boundary layer but decreases in the reattaching layer. In McDonald's earlier papers it was assumed, nevertheless, that a certain pressure rise was associated with a given numerical change in the shape factor, irrespective of the direction of the change. Although this argument did not seem entirely satisfactory it received some confirmation from the work of Curle[68] for separating and reattaching laminar boundary layers. Curle's method was based on a small-perturbation analysis and involved the assumption that the momentum thickness was effectively constant through the separation and reattachment processes. For this case the pressure rise at separation was found to be exactly equal to that at reattachment.

In a very recent paper[64] McDonald has attempted to put his argument on a firmer basis.

By relating the downstream pressure rise to the shape factor appropriate to the boundary layer on a flat plate, McDonald's method predicts a dependence of the base pressure on Reynolds number in addition to the accepted dependence on the boundary-layer thickness. The computed results presented in Ref. 64 indicate that the predicted variation with Reynolds number is quite significant; however no effect of Reynolds number – at least of this order – can be detected in the experimental data. McDonald[64] suggests that there is some other factor, with a Reynolds number dependence of opposite sign, which cancels that due to the effect on the boundary-layer conditions far downstream in the real flow, but which has not been taken into account in the flow model. As to where this is arising, McDonald points to the possible Reynolds-number effect on the initial development of the free shear layer suggested by Bradshaw's experiments[49].

This is clearly an important point which demands further study. The question will be raised again in the next Section in relation to Rom's work[70,71].

Aside from this question another is of some importance, concerning the degree of similarity between the flow past isolated bases and the flow past steps. Little effort has gone into examining the validity of the assumption that the two types of flow are similar, or that data and results relating to one could be applied generally. The assumption appears to have originated with the older theories like that of Korst in which the conditions downstream of the point of confluence or reattachment (where the presence of a wall could be expected to have some influence) were thought to be unimportant, and recieved experimental confirmation in that the base pressures measured in the two cases were not greatly dissimilar. On the other hand the experimental evidence is far from conclusive, and developments in the theories have drawn attention to the possibility that essential differences between the two flows may exist. It is now recognised that the conditions in the flow downstream of reattachment or confluence play an important role in the determination of the pressure field as a whole. Furthermore the interaction of the pressures and the shear stresses in the region close to a reattachment point is a complicated process as was shown by the experiments of Mueller and Robertson[103]. The same is likely to be true in the region of confluence of two shear layers but it is difficult to believe that the difference of the shear stresses, due to there being no solid boundary present in this case, is unimportant. If the two flows are otherwise similar this is a largely fortuitous result and must be demonstrated by detailed research, either experimental or theoretical.

3.5. *Integral Methods.*

While integral methods are being used increasingly for laminar base flows[2,3,72,73], important work on turbulent base flows along these lines is largely represented by the research of Rom and de Krasinsky. Both have employed the notation and techniques originally proposed by Crocco and Lees[69].

De Krasinsky's theoretical work[58] was confined to a straightforward extension of the method of Crocco and Lees to the axisymmetric case. But in addition de Krasinsky has carried out some important experiments to try to confirm some of the basic assumptions of the method. It will be his conclusions from these experiments which will be of most interest in the present discussion.

Rom's theoretical work[70,71] is of interest in its own right and may be taken to represent the most significant attempt to construct a successful base-flow theory using the Crocco-Lees mixing concepts. The basic tool in the method is the momentum – integral equation, written in the characteristic notation,

$$\frac{d\kappa}{dx} = \frac{1-\kappa}{m}\frac{dm}{dx} + \frac{\kappa F}{M_e}\frac{dM_e}{dx},$$

in which κ is a kind of viscous layer shape factor, m is the local mass flux in the layer and F is some function of κ. M_e is the local Mach number at the edge of the viscous/turbulent layer. The assumption that $F(\kappa)$ is a unique function, takes the place of the usual assumption in integral methods that the velocity profiles form a simple family in terms of one or two parameters. In its original form[69] the momentum equation contained a further term involving the shear stress at the wall (if a wall is present). This term is omitted throughout Rom's work, both for the flow past an isolated base and for that past a step.

The essential feature of the method is the integration of the momentum equation from the separation point, where the initial conditions are provided by the approaching boundary layer, to infinity downstream. The integration is done in two stages. The first stage corresponds to the separated region and here it is assumed, fairly, that the pressure is constant. Accordingly, the second term on the right-hand side of the equation is omitted, and the remaining terms are integrated by making an assumption about the rate of entrainment into the shear layer from the external flow. This assumption implies that the rate of entrainment decreases as the distance from separation increases. This is in contrast to the usual assumption (which is otherwise universally accepted) that the rate of entrainment is independent of the distance from the separation point, at least outside the initial region of development. Rom's defence of his assumption is based on the interpretation of his own measurements. It is difficult to establish whether any significant differences in predicted base pressures are involved since this part of the analysis contains a free parameter which is evaluated empirically.

It is in his treatment of the recompression region that Rom's method may be most open to criticism. In contrast to the assumption which has so far been made in all the 'analytic' methods (Section 3.4, above), Rom draws a distinction between the flow past an aerofoil section with a blunt trailing edge and the flow past a step. As far as this is concerned he may well be right; but his arguments as to the ways in which the two flows differ do not seem altogether convincing. The arguments were developed in a paper on laminar base flows[72] but taken over qualitatively to the turbulent case. Rom asserts that 'The recompression in the case of the wake is found to be associated with mass exchange. This is different from the case of reattaching flows where the reattachment and the recompression associated with it were found to be essentially inviscid.' Little evidence is produced to support the assertion although the conclusion of Chapman *et al*[34] (whose flow model is similar to Korst's) are quoted in support of the remarks about the reattachment case.

In the subsequent analysis Rom[71] neglects the left-hand side of the momentum equation for the case of the flow past an isolated base, and the first term on the right-hand side in the case of the flow past a step. Furthermore, in the former case, information has to be specified as to the mass flux in the wake at infinity downstream, this being required as a boundary condition for the integration of the equation. In Rom's theory an expression for the mass flux at infinity is proposed without a sufficiently plausible justification. In our view Rom's general procedure has too great a degree of arbitrariness about it to be accepted without supporting evidence (particularly as regards the analysis of the isolated base), and that this detracts from the importance of the potentially valuable work.

The analysis of the flow past a step is rather more satisfactory and it is interesting to note that Rom's final expression for the pressure rise along the downstream wall is of a similar form to that derived by McDonald[42] using an adaption of the work of Reshotko and Tucker[67]. As was the case with McDonald's analysis, the method of Rom for the flow past a step predicts a dependence of the base pressure on Reynolds number (in addition to that arising *via* the sensitivity to the boundary-layer thickness). However the predicted effect of Reynolds number is of opposite sign in the two theories. Rom claims that there is some experimental evidence to support his assertion that the base pressure on steps *increases* with increasing Reynolds number, other factors being equal; although he adds that tests at higher Reynolds numbers are required to settle the matter definitively. It would seem that this experimental work should be undertaken with some urgency. Furthermore we should wish to press for some tests to be carried out to investigate the possible differences between the flow past steps and that past isolated sections.

More theoretical work on these various points is also required to follow up the interesting leads suggested by Rom's work and, perhaps, to put some of the assumptions on a firmer basis.

The method of Rom described above was based on the Crocco-Lees concepts but only in a fairly general way and there was no quantitative reliance on any of the correlations of empirical data which were indicated in the original paper of Crocco and Lees. In contrast to this, a considerable part of the work of de Krasinsky[58] was devoted to examining the validity of these correlations and attempting to define them quantitatively. De Krasinsky concluded however that within the accuracy of his measurements the existance of the correlations could not be established. This is a serious point because it seems to indicate that either the correlations do not exist in which case the validity of the method as a whole becomes questionable, or experiments of a much higher degree of precision than is currently possible are needed to establish them. The feeling seems to be that while the integral approach has obvious possibilities, as has been demonstrated by Rom's work, the difficulties are likely to be as great as those of the 'analytic' theories discussed in the previous Section.

Whilst on the subject of integral methods it seems appropriate to mention some work which does not relate directly to the approach used by Rom but can be grouped with it in a wider context. This is the interesting analysis carried out by Hastings[30] on his own measurements. Hastings related a suitably-defined base-pressure coefficient to the local wall shear stress in the boundary layer just ahead of separation. The form of the relation was suggested by work on forward-facing steps and on Stanton tubes. This is a novel approach which should receive more attention in future studies of the subject. The particular attraction of Hastings' correlation is that it applies to the case of very thick boundary layers. Such conditions are outside the range of the 'analytic' and possibly also the integral methods discussed so far.

3.6. *Comparison of the Various Methods.*

From the discussion in the previous two Sections it should be clear that there are many uncertainties in all the current methods for the prediction of base pressures in supersonic turbulent flow. Consequently little purpose will be served by extensive comparisons with each other and with experimental results. The fact that a particular method gives results in agreement with experiment does not necessarily confirm the various assumptions made in it because the probability of some cancellation of errors is not inconsiderable. Moreover, some of the methods (that of Roberts[40], for example) involve a free parameter which is itself evaluated by direct comparison with experiment.

However a few comparisons are shown in Figs. 10, 11 and 12, for Mach numbers of 1·5, 2·0 and 3·0. The principal aim here is to illustrate the effect which the uncertainties in the different methods have on the predicted base pressures. In some of the methods a choice of results is possible in that a value of the Reynolds number has to be assumed. This is the case with the methods of McDonald[64], Rom[71] and Hastings[30]. The calculated results shown in Figs. 10, 11 and 12 relate to the condition where the Reynolds number based on the momentum thickness of the initial boundary layer, θ, is equal to 5,000. In the case of Rom's work a further choice is possible between the theory relating to the flow past a step and that past an isolated base. The calculations presented refer to the case of the flow past a step.

In some instances the momentum thickness of the boundary layer at separation was not given explicitly in the results shown in the original papers, but had to be inferred from stated values of the Reynolds number, etc. Where this was necessary the momentum thickness was calculated using the following expression:—

$$\theta = k\,L\,Re_L^{-1/5},$$

where Re_L is the Reynolds number based on the effective run of turbulent boundary layer upstream of the base, L, and the values of k assumed were as follows:

M	k
1·5	0·0312
2·0	0·0286
3·0	0·0228

Rather than plot actual experimental data points, for comparison, in Figs. 10, 11 and 12 it was decided to show instead the values of C_{p_b} derived from the empirical correlation described in Section 3.1, above. It is probably fair to say that this procedure provides a more coherent reference, by which the accuracy of the various methods can be judged, then would be provided by the showing data points themselves.

The general impression which is gained from the comparisons in Figs. 10 to 12 is that the agreement between theory and experiment is better than might have been hoped for – Although it should be remembered that some of the methods essentially have the status of data correlations and have therefore been forced to agree with a body of experimental results. At a Mach number of 2, for example, (Fig. 11), all the methods give results which agree to within about 15 per cent over a range of h/θ from 50 to 400. The trend of C_{p_b} with h/θ is not predicted so well, however. In most of the cases too large a variation of C_{p_b} with h/θ is predicted (except, of course, by the empirical correlation of Staylor and Godberg[31] which expressly ignores the dependence on h/θ), although the methods of McDonald[64] and Rom[71] are least at fault in this respect. It is encouraging to see that the latest methods are giving more accurate results than the earlier ones, and that there seems to be some return for the effort which is put into the analysis. The corresponding disadvantage is that both McDonald's and Rom's analyses predict a significant dependence on Reynolds which does not seem to be real.

Over the range of h/θ from about 100 to 400 there is little variation of C_{p_b} and the latter appears to be effectively a function of Mach number alone (unless subsequent work confirms the existence of a Reynolds number effect). This observation lends support to the validity of the correlation proposed by Staylor and Goldberg[31]. Except at the higher Mach numbers, where their curve seems too low (Fig. 12), Staylor and Goldberg's values are in good agreement with the present correlation. However, the comparisons in Figs. 10 to 12 indicate that this approximation is only valid for values of h/θ greater than about 100.

In the past some significance has often been attached to the value to which C_{p_b} tends as the ratio h/θ tends to infinity. The present feeling, however, is that this is a rather academic point. The correlation established in Section 3.1 indicates a decrease in C_p of only about 0·01 as h/θ increases from a value of 400 to infinity. But this depends solely on the method of extrapolation. Therefore the values of $(C_{p_b})_l$ shown in Fig. 7 and Table 2 should be regarded as strictly meaningful only in the context of the correlation for finite values of h/θ. The difficulties involved in extrapolating to $h/\theta = \infty$ have recently been noted by Roshko and Thomke[101].

3.6. *Methods for Increasing Base Pressure.*

Bleed – that is the continuous injection of low-velocity air through the base – is a well-tried method of increasing base pressure at supersonic speeds. The effects and limitations of base bleed have been understood for a long time (*see* e.g. Ref. 1) and it is not intended to discuss them again in the present Report. But it does seem appropriate to mention some experiments which have been performed at the NPL[75] on an automatic bleed device similar to the ventilated cavity mentioned in relation to subsonic base flows (Section 2.2.2, above).

This consists of a cavity ventilated by perforations (i.e. holes) similar to that sketched in Fig. 2c. In contrast to the case of subsonic speeds, the tests reported in Ref. 75 indicated that a slotted cavity is of very limited effectiveness in supersonic flow. This can probably be explained in that the flow turns abruptly into the cavity through each slot without losing much of its kinetic energy in the process. On the other hand, for base bleed to be effective the bleed air must have had its dynamic pressure destroyed before being introduced into the base flow. This retardation of the air is achieved by the passage of the air through the holes in the case of the perforated cavity walls. The idea is not new (*see*, for example, Ref. 74) but the tests recently carried out were aimed at examining the effect of different hole sizes, hole density, cavity-wall thickness and so on. The main conclusion of the work was that none of these factors was particularly important but that the increase in base pressure depended chiefly on the total open area of the holes (Fig. 13). The results are in substantial agreement with those of Jones[74] which were obtained at a slightly lower Mach number.

The increase in base pressure achieved must nevertheless be at the expense of a drag increment due to the streamwise force on the cavity walls. This is no doubt associated with the destruction of the streamwise momentum of the air passing through the holes. Jones found that under suitable conditions a net reduction in drag could be realised. Further tests are planned at the N.P.L. to examine this aspect.

Aside from the use of automatic bleed devices as a means of reducing base drag there seems to be considerable scope for modifying the pressure distribution in the reattachment region. The main applications of this may well be in the non-aeronautical field. Automatic bleed leads to a more gradual recompression in the reattachment region and may offer the prospect of reducing the high heat-transfer rates occurring there.

In the context of reducing base drag as such, little attention seems to have been devoted to the favourable exploitation of interference from other parts of the aircraft or vehicle. The possibilities seem very considerable particularly in three-dimensional configurations. Even in two-dimensional or axi-symmetric configurations the possibilities exist as shown by de Krasinsky[58].

4. *General Discussion and Concluding Remarks.*

The impression which is gained fairly consistently from the detailed discussion in the preceding Sections is that, while the achievements of recent years have been considerable, there are few grounds for complacency and a great deal remains to be done throughout the whole field of two-dimensional turbulent base flows.

In the supersonic régime on which research has been in progress for many years, there is as yet no theory which takes proper account of the detailed mechanics of the flow. Even the best available methods for predicting base pressure are little more than sophisticated data correlations, and there has been little progress at all towards the prediction of other information – for instance the pressure distribution downstream of the base. Nor are the available measurements sufficiently comprehensive throughout the

Mach number range to merit very much confidence in the various methods viewed as correlations of data. In the subsonic speed range the nature of the flow is only beginning to be understood, particularly in the unsteady-flow régime, and in this area research for some time to come must concentrate largely on the construction of an adequate flow model. As far as base flows in the neighbourhood of $M = 1$ are concerned it is significant that there has been virtually no research to review in the present Report beyond that already referred to in Ref. 1.

Nevertheless against this gloomy background it is also evident that very important research has been undertaken in this area over the last few years. In some cases the importance of the work has lain in exposing the nature of the problems involved whereas, previously, a falsely optimistic picture of the state of the art had been held. A case in point is the work on the theory of supersonic base flow. After the period of stagnation following the publication of the work of Chapman and Korst, the revival of interest in this problem and the determination of a number of investigators to reach a satisfactory solution are most encouraging. In the subsonic speed range, on the other hand, little research had been done at all until three or four years ago (except on bluff bodies). Since then several important series of experiments have been carried out to elucidate the properties of the flow and to provide a guide to further studies in this field. The study of the vortex-shedding phenomenon has proved to be of considerable fluid-dynamic interest besides its importance from a practical point of view. In the context of the steady subsonic base flow theoretical work has already made an impact.

It will now be profitable to summarise the main achievments of the research reviewed and to draw attention to a number of specific points which could form the basis for work over the next few years.

(a) *Subsonic speeds.*

In an attempt to put work on blunt-trailing-edge aerofoil sections into perspective, an examination was made in Section 2.1 of data showing the effect on profile drag of thickening or cutting back the trailing edge of conventional sections. As a rough guide it was found that the profile drag is noticeably increased above its level for a sharp section when the trailing-edge thickness exceeds the total momentum thickness of the boundary layers on the two surfaces of the aerofoil. The additional drag which results from increasing the trailing-edge thickness above this 'critical' value is partly due to the formation of a vortex street in the wake. But it is recognised that even if the periodicity were entirely suppressed the drag would still be higher than that of a sharp section. The base flow behind a blunt trailing edge under such conditions is assumed to resemble the steady flow past a rearward-facing step.

Several experiments have been conducted to explore the properties of the unsteady flow past a blunt-trailing-edge aerofoil section (Section 2.2). A good deal is now known about the effects of splitter plates, trailing-edge cavities (both solid and ventilated), and base bleed on the vortex formation and the base pressure. Splitter plates and ventilated cavities, in particular, are effective drag-reducing devices. A significant result of this work has been the discovery of a relation between the base pressure and the point downstream of the trailing edge, at which the vortex street appears to form. On the other hand there is not yet an adequate flow model in terms of which all the observed effects can be explained. There is a need to determine to what extent the flow about the section itself can be regarded as steady – as is assumed in the Heisenberg/Roshko model, or to what extent a periodic circulation round the section is the key to the whole phenomenon. Measurements of the instantaneous pressures and forces might help in this respect.

Research is also urgently needed on the question of the dependence of the strength of the vortex street on the section geometry (in particular on the degree of boat-trailing). The effects of incidence and of Reynolds number have not been fully explored either. Nor can it be assumed that the present data on splitter plates, cavities and so on, are adequate or that the possibilities for devices of this nature have been exhausted.

An important start has been made on the problem of analysing the steady subsonic base flow, and predictions can be made of the effects of Mach number, boundary-layer thickness, section geometry and bleed on the base pressure, (Section 2.3). However a large number of assumptions had to be made in the analysis and, although comparisons with the very limited available experimental data have been most encouraging, much more work must be done to confirm or improve the various aspects of the theory.

There is scope for a detailed research programme involving both experimental and theoretical work. It would seem worthwhile concentrating more attention on the case where the base height is comparable with the thickness of the boundary layer. Particularly in aeronautical applications, this case is likely to be of greater practical interest than cases where the base height is large compared with the boundary-layer thickness (as is assumed in the existing analysis).

As far as the theoretical work is concerned, it must be emphasised that what has been done so far is only a first step. Our experience of the fortunes of calculation methods for supersonic flow should warn against excessive confidence in the adequacy of the theory. For instance it is not unlikely that the question of Reynolds number dependence – which is currently a matter of argument in the supersonic case – may emerge as a difficulty in the subsonic analysis also.

(b) *Supersonic speeds.*

A large volume of experimental data has been amassed particularly between Mach numbers of 1·5 and 3. Within this range base pressures can be predicted with reasonable confidence simply by interpolation, and a formula is suggested in Section 3.1 for doing this. Outside this range the data are less comprehensive and further measurements would be welcome. Experimental work is also required on two fundamental points: first, whether any significant differences exist between the base flows behind steps and isolated bases and, second, whether the dependence of base pressure on Reynolds number is in fact more complicated than has usually been assumed. The assumption that Reynolds number influences the flow only *via* the thickness of the initial boundary layer has recently been called in question as the result of theoretical work.

A few remarks are made in Section 3.7 about methods of increasing base pressure at supersonic speeds. Particular reference is made to some recent tests on an automatic-bleed system.

There has been little controversy about the gross features of the supersonic flow model for a number of years and the theories predict the base pressure reasonably accurately for Mach numbers between 1·5 and 3, (Section 3.6). But in several cases this is at least partly attributable to the fact that they contain disposable parameters which are adjusted to ensure agreement with some of the data. As regards the details of the analysis and the underlying assumptions the methods remain largely inadequate and the increase in sophistication has often scarcely kept pace with the discovery of unforeseen difficulties.

In the case of the theories based on the Chapman/Korst/Kirk model (Section 3.4) there is not a single feature of the flow which can be analysed with anything approaching quantitative confidence. Some of the uncertainties are of a quite fundamental nature like the rate of spread of a turbulent free shear layer, or the effect of initial conditions on the development of a shear layer. It is highly probable that information on such questions is required in a whole range of applications beside the present one. Other difficulties are more strictly limited to the study of base flows such as the treatment of the recompression region. In the most recent theories there have been significant advances in the treatment of the reattachment region for the case of the flow past a step. But the relevance of the results to the recompression region downstream of an isolated base has not been examined closely. There is also the general question concerning the range of conditions over which it is valid to dissect the flow field into supposedly separate components.

The integral methods (Section 3.5) may possibly have a greater potential range of validity, but the work which has so far been done in this direction has shown that the difficulties to be overcome are no less serious than in the methods discussed in Section 3.4. There seems to be sufficient justification for advocating more attention to the integral methods. However further work need not be along the lines of the Crocco-Lees method; this is only one possible approach and is not necessarily the best one.

As in the subsonic case, there is a need for work on base flows where the base height is not large compared with the boundary-layer thickness. The suggestions of Hastings (Section 3.5) are interesting in this respect, and it is commented in Ref. 64 that an approach based on perturbations to the turbulent boundary layer could be profitable.

Lastly it would perhaps be useful to put the problem of the analysis of turbulent base flows into its correct perspective. It was believed at one time that the supersonic, turbulent base flow could be analysed independently of, and more readily than, the turbulent attached boundary layer. This depended chiefly on one or two assumptions such as the validity of the Chapman/Korst/Kirk reattachment criterion.

However now that these assumptions have been shown to be invalid, attempts to improve the theories are leading to the demand for more information about attached boundary layers. Moreover, this process is exposing serious shortcomings in our understanding of attached turbulent boundary layers – even in incompressible flow. For example, attempts to analyse the boundary layer downstream of a reattachment point have drawn attention to the differences in behaviour between a boundary layer in an adverse pressure gradient of *decreasing* severity and one in a gradient which is becoming 'more adverse' with distance downstream. Another example which can be cited is the question of the effect of the initial boundary layer on the base flow. It appears that the validity of all the theories for predicting base pressure may now be in doubt (whatever their other limitations) because they have not taken proper account of the level of the shear stresses in the boundary layer at separation[49]. But it is precisely this question of the influence of the initial shear stress levels which is at present being examined in relation to attached turbulent boundary layers[99,100,102].

Thus the conclusion from this seems to be that further progress in the theory of turbulent base flows must depend to a large extent on necessary improvements in the theory of turbulent layers. On the other hand the work on base flows has stimulated research on boundary layers and may continue to do so.

TABLE 1

Comparison of Effectiveness of Various Devices for Increasing Base Pressure at Subsonic Speeds.

References		Original level of C_{p_b}	Splitter plate (optimum)		Cavity		Ventilated cavity (slots)	
			C_{p_b}	Percentage reduction in base drag	C_{p_b}	Percentage reduction in base drag	C_{p_b}	Percentage reduction in base drag
Bearman[7]	($M = 0$)	−0·58	−0·29	50%				
Nash *et al* (Ref. 6 and unpublished work)	($M = 0{\cdot}4$)	−0·62	−0·30	52%	−0·49	21%	−0·30	52%
	($M = 0{\cdot}8$)	−0·75	−0·42	44%	−0·58	23%	−0·30	60%
Osborne and Pearcey[15]	($M = 0{\cdot}4$)	−0·54			−0·37	31%	−0·25	54%
	($M = 0{\cdot}8$)	−0·62			−0·42	32%	−0·25	60%
Nash *et al*[18]	($M = 0{\cdot}4$)	−0·47	−0·23	51%				
	($M = 0{\cdot}8$)	−0·54	−0·28	48%				

TABLE 2

Limiting Base Pressure for Thin Boundary Layers.

M	$(C_{p_b})_l$
1·0	−0·800
1·2	−0·520
1·4	−0·390
1·6	−0·326
1·8	−0·285
2·0	−0·252
2·2	−0·224
2·4	−0·200
2·6	−0·178
2·8	−0·159
3·0	−0·142
3·2	−0·127
3·4	−0·114
3·6	−0·103
3·8	−0·093
4·0	−0·085
4·2	−0·077
4·4	−0·070
4·6	−0·065
4·8	−0·060
5·0	−0·056

As a rough guide, for higher Mach numbers $(C_{p_b})_l$ can probably be taken equal to $\frac{2}{\gamma M^2}$.

LIST OF SYMBOLS

M	Mach number
u	Velocity
p	Pressure
ρ	Density
c	Chord (of aerofoil)
h	Base height (of aerofoil section; the equivalent height of a step would be $h/2$)
θ	Boundary-layer momentum thickness
C_p	Pressure coefficient $= \dfrac{p - p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2}$
C_D	Profile-drag coefficient $= \dfrac{\text{Drag}}{\frac{1}{2}\rho_\infty u_\infty^2}$
C_q	Bleed coefficient $= \dfrac{\text{Mass flow}}{\rho_\infty u_\infty h}$
Re	Chord Reynolds number

Subscripts

∞	Free-stream conditions
b	Condition on base
l	Hypothetical limit for large base height compared with boundary-layer thickness
T	Value at trailing edge of aerofoil

Other symbols, occurring infrequently, are defined in the text.

REFERENCES

No.	Author(s)	Title, etc.
1	J. F. Nash	A review of research on two-dimensional base flow. A.R.C. R. & M. 3323; March, 1962.
2	L. Lees and B. L. Reeves ..	Supersonic separated and reattaching laminar flows: General theory and application to adiabatic boundary layer-shock wave interactions. GALCIT Separated flows research project. Tech. Report 3; October 1963.
3	J. N. Nielsen, L. L. Lynes, .. F. K. Goodwin and M. Holt	Calculation of laminar separation with free interaction by the method of integral relations. AIAA 2nd Aerospace Sciences Meeting, New York; January 1965.
4	F. H. Harlow and J. E. Fromm	Computer experiments in fluid dynamics. *Scientific American*, Vol. 212, pp. 104–110, 1965.
5	S. F. Hoerner	Fluid-dynamic drag. (Published by the author); 1958.
6	J. F. Nash, V. G. Quincey and J. Callinan	Experiments on two-dimensional base flow at subsonic and transonic speeds. N.P.L. Aero Report 1070; A.R.C. R. & M. No. 3427. January 1963.
7	P. W. Bearman	Investigation of the flow behind a two-dimensional model with a blunt trailing edge and fitted with splitter plates. *J. Fluid Mech.*, Vol. 21, Part 2, pp. 241–255. 1965.
8	C. J. Wood	The effect of base bleed on a periodic wake. *J. R. Ae. Soc.*, Vol. 68, pp. 477–482. 1964.
9	W. Heisenberg	Die absoluten der Karmanschen Wirbelbewegung. *Phys. Z*, Vol. 23, p. 363; 1922.
10	A. Roshko	On the wake and drag of bluff bodies. *J. Ae. Sci.*, Vol. 22, p. 124. 1955.
11	F. R. Hama and E. R. Burke	On the rolling up of a vortex street. Univ. of Maryland, Tech. Note BN–220, AFOSR TN 60–1069; September 1960.
12	F. H. Abernathy and .. R. E. Kronauer	The formation of vortex streets. *J. Fluid Mech.*, Vol. 13, Part 1, pp. 1–20. May 1962.
13	J. F. Nash	Factors influencing the base pressure on blunt-trailing-edge aerofoils in high-speed flight. Ph.D. Thesis, London University; May 1964.

No.	*Author(s)*	*Title, etc.*
14	R. A. Shaw	An explanation of vortex shedding on the basis of pulses travelling at the speed of sound. A.R.C. 18 455. May 1956.
15	J. Osborne and H. H. Pearcey	(Unpublished N.P.L. work). 1960.
16	F. O. Ringleb	Separation control by trapped vortices. (Chapter in *Boundary Layer and Flow Control.* Vol. 1 ed. G. V. Lachmann. Pergamon Press; 1961).
17	D. C. Hazen, R. F. Lehnert, T. E. Sweeney and F. O. Ringleb	Preliminary report on circulation control by means of trailing-edge suction and the cusp effect. Princeton University, Aero. Eng. Report 234; June 1953.
18	J. F. Nash, V. G. Quincey and and R. J. Wood.	(Unpublished N.P.L. work). 1964.
19	R. F. Sargent	Tests on NACA 0012 with various thick trailing edges at high speeds. A.R.C. R. & M. 2209; November 1942.
20	J. F. Nash, V. G. Quincey and J. Callinan	(Unpublished N.P.L. work). 1962.
21	P. L. Poisson-Quinton and .. P. Jousserandot	Influence du soufflage au voisinege du bord de finite sur les caracteristiques aerodynamique d'une aile aux grandes vitesses. *La Recherche Aéronautique*, No. 56, pp. 21–32. February 1957.
22	D. W. Holder	(Unpublished N.P.L. work). 1960.
23	T. H. Moulden	(Unpublished N.P.L. work). 1962.
24	D. J. Maull	(Unpublished work, Cambridge University). 1965.
25	J. F. Nash	An analysis of the subsonic flow past symmetrical blunt-trailing-edge aerofoil sections at zero incidence, in the absence of a vortex street. A.R.C. R. & M. 3436. August 1964.
26	H. B. Squire and A. D. Young	The calculation of the profile drag of aerofoils. A.R.C. R. & M. 1838; November 1937.
27	A. D. Young and .. .. N. E. Winterbottom	Note on the effect of compressibility on the profile drag of aerofoils at subsonic Mach numbers in the absence of shock waves. A.R.C. R. & M. 2400; May 1940.
28	J. F. Nash, T. H. Moulden and J. Osborne	On the variation of profile drag coefficient below the critical Mach number. A.R.C. C.P.758; November 1963.

No.	*Author(s)*	*Title, etc.*
29	E. J. Saltzman	Base pressure coefficients obtained from the X–15 airplane for Mach numbers up to 6. NASA, TN D–2420; August 1964.
30	R. C. Hastings	Turbulent flow past to two-dimensional bases in supersonic streams. A.R.C. R. & M. 3401. December 1963.
31	W. F. Staylor and T. J. Goldberg	Afterbody pressures on two-dimensional boattailed bodies having turbulent boundary layers at Mach 5·98. NASA TN D–2350; July 1964.
32	G. E. Gadd, D. W. Holder and J. D. Regan	Base pressures in supersonic flow. A.R.C. C.P.271; March 1955.
33	D. R. Chapman	An analysis of base pressure at supersonic velocities and comparison with experiment. NACA TN 2137; July 1950.
34	D. R. Chapman, D. M. Kuehn and H. K. Larson	Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transition. NACA Report 1356, 1958.
35	H. H. Korst	A theory for base pressures in transonic and supersonic flow. *J. App. Mech.*, Vol. 23, No. 4, pp. 593–600. December 1956.
36	F. N. Kirk	An approximate theory of base pressure in two-dimensional flow at supersonic speeds. R.A.E. TN Aero 2377; December 1959.
37	A. D. Young and S. Kirkby ..	The profile drag of biconvex and double wedge wing sections at supersonic speeds. *Symposium on boundary-layer effects in aerodynamics.* N.P.L. 31st March–2nd April, 1955, Paper No. 7 H.M.S.O.
38	P. Carrière and M. Sirieix ..	Facteurs d' influence du recollement d'u ecoulement supersonique. *Proc. 10th. Congress of App. Mech.*, Stresa, September 1960. ONERA MT 20, 1961.
39	J. F. Nash	An analysis of two-dimensional turbulent base flow, including the effect of the approaching boundary layer. A.R.C. R. & M. 3344; July 1962.
40	J. B. Roberts	On the prediction of base pressure in two-dimensional supersonic turbulent flow. A.R.C. R. & M. 3434. November 1964.

No.	*Author(s)*	*Title, etc.*
41	H. H. Korst, R. H. Page and M. E. Childs	Compressible two-dimensional jet mixing at constant pressure. University of Illinois, ME–TN 392.1. 1954.
42	H. McDonald	Turbulent shear layer re-attachment with special emphasis on the base pressure problem. *Aero. Quart.*, Vol. 15, pp. 247–280. 1964.
43	A. Mager	Transformation of the compressible turbulent boundary layer. *J. Aero. Sci.*, Vol. 25, pp. 305–311. 1958.
44	L. J. Crane	The laminar and turbulent mixing of jets of compressible fluids. Pt. II. The mixing of two emi-infinite streams. *J. Fluid Mech.*, Vol. 3, Part 1, pp. 81–92. 1957.
45	R. C. Maydew and J. F. Reed	Turbulent mixing of axisymmetric jets (in the half-jet region) with quiescent air Sandia Corp. Res. Rep. SC–4764 (RR); March 1963. (Published in a shortened form in *AIAA Journal*, June 1963).
46	D. Coles	The turbulent boundary layer in a compressible fluid. *Phys. of Fluids*, Vol. 7, pp. 1403–1423. September 1964.
47	R. S. Channapragada ..	Compressible jet spread parameter for mixing zone analyses. *AIAA Journal*, Vol. 1, pp. 2188–2190, 1963.
48	N. H. Johannesen	Further results on the mixing of free axially-symmetrical jets of Mach number 1·40. A.R.C. R. & M. 3292. May 1959.
49	P. Bradshaw	The effect of initial conditions on the development of a free shear layer. Published in *J. Fluid Mech.* Vol. 26, 1966, pp. 225–236.
50	J. A. Hill and J. E. Nicholson	Compressibility effects on fluid entrainment by turbulent mixing layers. NASA CR–131; November 1964.
51	H. Görtler	Berechnung von Aufgaben der freien Turbulenz auf Grund eines neuen Näherungsansatzes. *Z. angew. Math. Mech.* Vol. 22 pp. 244–254. 1942.
52	J. F. Nash	The effect of an initial boundary layer on the development of a turbulent free shear layer. A.R.C. CP 682; June 1962.
53	H. McDonald and I. A. Acklam	Some numerical results of the turbulent free shear layer with a finite initial boundary layer. B.A.C. (Warton) Rep. Ae. 212; June 1964.

No.	*Author(s)*	*Title, etc.*
54	D. R. Chapman and .. .. H. H. Korst	Discussion of 'theory for base pressures in transonic and supersonic flow'. *J. App. Mech.*, Vol. 24, pp. 484–485. 1957.
55	A. Roshko and G. J. Thomke	Flow separation and re-attachment behind a downstream-facing step. Douglas Rep. SM–43056–1; January 1964.
56	R. K. Tagirov	Determination of the base pressure and the base temperature following the sudden expansion of a sonic or supersonic stream. Mekanika i Mashinostroenie, Izvestia Akademii Nauk, U.S.S.R., Otdelenie Tekhnicheskikh Nauk, No. 5. p. 30. September/October 1961.
57	R. K. Tagirov	Calculation of heat transfer in the base flow of two different streams Mekhanika i Mashinostroenie, Izvestia Akademii Nauk U.S.S.R., No. 6, p. 55. November/December 1963.
58	J. E. de Krasinsky	A study of the separated base flow behind bodies of revolution in supersonic streams. Ph.D. Thesis, London University. November 1964.
59	R. A. White	Turbulent boundary layer separation from smooth-convex surfaces in supersonic two-dimensional flow. Ph.D. Thesis, Iniv. of Illinois; 1963.
60	E. Eminton	Simple theoretical and experimental studies of the flow through a three-shock system in a corner. A.R.C. C.P. 727. September 1961.
61	J. C. Cooke	Separated supersonic flow. R.A.E. TN Aero 2879. A.R.C. 24 935. March 1963.
62	P. Carrière and M. Sirieix ..	Résultats récents dans l'étude des problemes de melange et de recollement. *Proc. 11th Int. Cong. of App. Mech.*, Munich. August/September, 1964.
63	H. McDonald	A study of the turbulent separated flow region occurring at a compression corner in a supersonic flow. *J. Fluid Mech*, Vol. 22, pp. 481–505. 1965.
64	H. McDonald	The turbulent supersonic base pressure problem: a comparison between a theory and some experimental evidence. *Aeronaut. Quart*, Vol. XVII, pp. 108–126. 1966.
65	B. S. Stratford	The prediction of the separation of the turbulent boundary layer. *J. Fluid Mech.* Vol. 5, pp. 1–16. 1959.

No.	*Author(s)*	*Title, etc.*
66	A. A. Townsend	The development of turbulent boundary layers with negligible wall stress. *J. Fluid Mech.* Vol. 8, pp. 143–155. 1960.
67	E. Reshotko and M. Tucker	Effect of a discontinuity on turbulent boundary layer thickness parameters with application to shock induced separation. NACA TN 3454; May 1955.
68	N. Curle	*The laminar boundary layer equations.* Oxford University Press. 1962.
69	L. Crocco and L. Lees ..	A mixing theory for the interaction between dissipative flows and nearly isentropic streams. *J. Aero. Sci.*, Vol. 19, pp.649–676. October 1952.
70	J. Rom	Supersonic two-dimensional base type flows. Technion (Israel), T.A.E. Rep. 27; March 1963.
71	J. Rom	Analysis of the near-wake pressure in supersonic flow using the momentum integral method. Technion (Israel), T.A.E. Rep. 35; September 1964.
72	J. Rom	Theory for supersonic two-dimensional laminar base-type flows using the Crocco-Lees mixing concepts. *J. Aero/Space Sci*, Vol. 29, p. 963. 1962.
73	J. Rom and M. Victor ..	Base pressure behind two-dimensional and axially-symmetric backward facing steps in a turbulent supersonic flow. *Israel Journal of Technology*, Vol. 2, No. 1, pp. 50–57. 1964.
74	J. J. Jones	The effect of base bleed on the base pressure, lift, drag and pitching moment of a 10 per cent thick blunt-base airfoil at a Mach number of 2·72. NASA RM L54K10. January 1955.
75	J. F. Nash and J. Callinan ..	The use of a ventilated cavity as a means of increasing base pressure at supersonic speeds. 1965. (N.P.L. paper to be issued).
76	H. A. Smith and R. F. Schaefer	Aerodynamic characteristics at Reynolds numbers of $3 \cdot 0 \times 10^6$ and $6 \cdot 0 \times 10^6$ of three airfoil sections formed by cutting off various amounts from the rear portion of the NACA 0012 airfoil section. NACA TN 2074. April 1950.
77	T. H. Moulden	(Unpublished N.P.L. work).
78	B. Thwaites (Ed.)	*Incompressible aerodynamics.* Oxford University Press. 1960.

No.	*Author(s)*	*Title, etc.*
79	J. J. Ginoux	On the existence of cross flows in separated supersonic streams. TCEA TN 6. February 1962.
80	M. Sirieix	Pression de culot et processus de mélange turbulent en écoulement supersonique plan. *La Recherche Aeronautique*, No. 78, p. 13. September/October 1960.
81	L. Fuller and J. Reid	Experiments on two-dimensional base flow at $M = 2{\cdot}4$. A.R.C. R. & M. 3064. February 1956.
82	A. Naysmith	Heat transfer and boundary-layer measurements in a region of supersonic flow separation and reattachment. A.R.C. 20 601. May 1958.
83	E. S. Love	Base pressure at supersonic speeds on two-dimensional airfoils and on bodies of revolution with and without fins having turbulent boundary layers. NACA TN 3819. January 1957.
84	K. L. Goin	Effects of planform, airfoil section, and angle of attack on the pressures along the base of blunt-trailing-edge wings at Mach numbers of 1·41, 1·62 and 1·96. NACA RM L52D21. September 1952.
85	A. F. Charwat and J. K. Yakura	An investigation of two-dimensional supersonic base pressures. *J. Ae. Sci.*, Vol. 25, p. 122–128. 1958.
86	A. F. Charwat, J. N. Roos, F. C. Dewey and J. A. Hitz ..	An investigation of separated flows: Part 1. The pressure field. *J. Aero/Space Sci.*, Vol. 28, p. 457–470. 1961.
87	H. Thomann	Measurements of heat transfer and recovery temperature in regions of separated flow at a Mach number of 1·8. F.F.A. Rep. 82. 1959.
88	J. W. Cleary and G. L. Stevens	The effects at transonic speeds of thickening the trailing edge of a wing with a 4 per cent thick circular arc airfoil. NACA RM A51J11. December 1951.
89	R. H. Sawyer and F. L. Dawn	Measurements through the speed of sound of static pressures on the rear and sides of a wedge by the NACA wing-flow method. NACA RM L8B13. July 1948.
90	D. Beastall and H. Eggink ..	Some experiments on breakaway in supersonic flow (Part II). R.A.E. TN Aero 2061. 1950.
91	M. A. Beheim	Flow in the base region of axisymmetric and two-dimentional configurations. NASA TR R–77. 1961.

No.	Author(s)	Title, etc.
92	D. R. Chapman, W. R. Wimbrow and R. H. Kester	Experimental investigation of base pressure and blunt-trailing-edge wings at supersonic velocities. NACA Rep. 1109. 1952.
93	M. A. Badrinarayanan ..	An experimental investigation of base flows at supersonic speeds. *J. R. Ae. Soc.*, Vol. 65, p. 475–482. 1961.
94	R. J. Stalker	Sweepback effects in turbulent boundary-layer shock-wave interaction. *J. Aero/Space Sci.*, Vol. 27, pp. 348–356. 1960.
95	A. E. Stevens	A wind tunnel investigation at supersonic speeds of the pressure drag of a two-dimensional step. (Unpublished work). 1959.
96	J. J. Ginoux	Effect of gas injection in separated supersonic flows. TCEA TN 7. February 1962.
97	F. N. M. Brown and V. P. Goddard	The effect of sound on the separated laminar boundary layer. University of Notre Dame, Indiana, Dept. of Aeronautical Engineering, Contract NSF G11712. June 1963.
98	B. P. Quinn	Relationship between drag and periodic vortex shedding. *AIAA Journal*, Vol. 3, pp. 542–543. 1965.
99	J. F. Nash	Turbulent boundary-layer behaviour and the auxiliary equation. AGARD Specialists' Meeting. 'Recent developments in boundary layer research', Naples. May 1965. AGARDograph 97.
100	P. Bradshaw and D. Ferriss	The response of a retarded equilibrium turbulent boundary layer to the sudden removal of pressure gradient. A.R.C.26 758. March 1965.
101	A. Roshko and G. L. Thomke	Observations of turbulent reattachment behind an axisymmetric downstream–facing step in supersonic flow. Douglas Rep. SM–43069. April 1965.
102	H. McDonald and J. A. P. Stoddart	On the development of the incompressible turbulent boundary layer. B.A.C. (Preston Div.), Rep. Ae225. March 1965, also A.R.C. 27 647.
103	T. J. Mueller and J. M. Robertson	A study of the mean motion and turbulence downstream of a roughness element. *Modern Developments in Theoretical and Applied Mechanics*, Vol. 1, Plenum Press. 1963.

No.	*Author(s)*	*Title, etc.*
104	A. R. Wazzan	A review of recent developments in turbulent supersonic base flow. *AIAA Journal.* Vol. 3, pp. 1135–1138. June 1965.

Additional references.

Although the following have not been specifically mentioned in the body of the paper they are thought to be of sufficient importance to merit being listed here.

Subsonic periodic flows:

105	J. H. Gerrard	An experimental investigation of the oscillating lift and drag of a circular cylinder shedding turbulent vortices. *J. Fluid Mech.*, Vol. 11, pp. 244–256. 1961.
106	G. E. Gadd	Two-dimensional separated or cavitating flow past a flat plate normal to the stream. A.R.C. C.P. 697. November 1962.
107	B. J. Vickery	Fluctuating lift and drag on a long cylinder of square cross-section in a smooth and in a turbulent stream. *J. Fluid Mech.* Vol. 25, pp. 481–494. 1966.

Turbulent mixing, etc.:

108	W. L. Chow and H. H. Korst	On the flow structure within a constant pressure compressible turbulent jet mixing region. NASA TN–D–1894. April 1963.
109	P. G. Hill, U. W. Schaub and Y. Senoo	Turbulent wakes in pressure gradients. *J. Appl. Mech.*, Vol. 30(E), pp. 518–524. 1963.
110	K. S. Wan	Turbulent wake characteristics with different eddy viscosity coefficients. *AIAA Journal*, Vol. 2, No. 1, pp. 121–122. 1964.
111	R. M. Halleen	A literature review on subsonic free turbulent shear flow. Dept. of Mech. Eng., Stanford Uni. AFOSR – TN.5444. April 1964.
112	E. J. Plate	The drag on a smooth flat plate with a fence immersed in its turbulent boundary layer. *Fluids Eng. Conf.*, Philadelphia. May 1964. ASME Paper 64–FE–17.
113	G. Kleinstein	Mixing in turbulent axially symmetric free jets. *J. Spacecraft and Rockets*, Vol. 1, pp. 403–408. 1964.

No.	*Author(s)*	*Title, etc.*
	Supersonic base flows:	
114	B. S. Stratford and .. .. M. C. Neale	Some tests with trapped vortices in supersonic flow. A.R.C. CP.716. September 1960.
115	K. Karashimna	Base pressure on two-dimensional blunt-trailing-edge wings at supersonic velocities. Univ. of Tokyo, Aero. Res. Inst. Rep. 368. October 1961.
116	T. J. Mueller and Y. T. Chin	Base pressure analysis for annular plug nozzles. United Aircraft Corp. Rep.D.210889–9. February 1965.
117	H. McDonald	An analysis of the turbulent base pressure problem in supersonic axisymmetrical flow. *Aero. Quart.* Vol. 16, pp. 97–121. 1965.
	Thermal effects:	
118	R. A. Seban and A. M. Levy	The effect of a downstream splitter plate on the heat transfer from a circular cylinder normal to an airstream. WADC Tech. Rep. 57–479. ASTIA Doc. AD.155765. August 1957.
119	H. Thomann	Measurements of the recovery temperature in the wake of a cylinder and of a wedge at Mach numbers between 0·5 and 3. F.F.A. Rep. 84. 1959.
120	R. A. Seban, A. Emery and .. A. Levy	Heat transfer to separated and reattached subsonic turbulent flows obtained downstream of a step. *J. Aero/Space Sci*, Vol. 26, No. 12, pp. 809–814. December 1959.
121	A. F. Charwat, F. C. Dewey Jr., J. N. Roos and J. A. Hitz	An investigation of separated flows. Part II. Flow in the cavity and heat transfer. *J. Aero/Space Sci.*, Vol. 28, pp. 513–527. 1961.
122	W. L. Chow and H. H. Korst	Influence on base pressures by heat and mass additions. ARS Journal, Vol. 32, No. 32, No. 7, pp. 1094–1095. July 1962.
123	L. H. Townend	Some effects of stable combustion in wakes formed in a supersonic stream. R.A.E. TN.Aero. 2872. A.R.C.24 891. March 1963.
124	R. A. Seban	Heat transfer to the turbulent separated flow of air downstream of a step in the surface of a plate. *J. Ht. Transf.* Vol. 86(C), No. 2, p. 259–264. 1964.

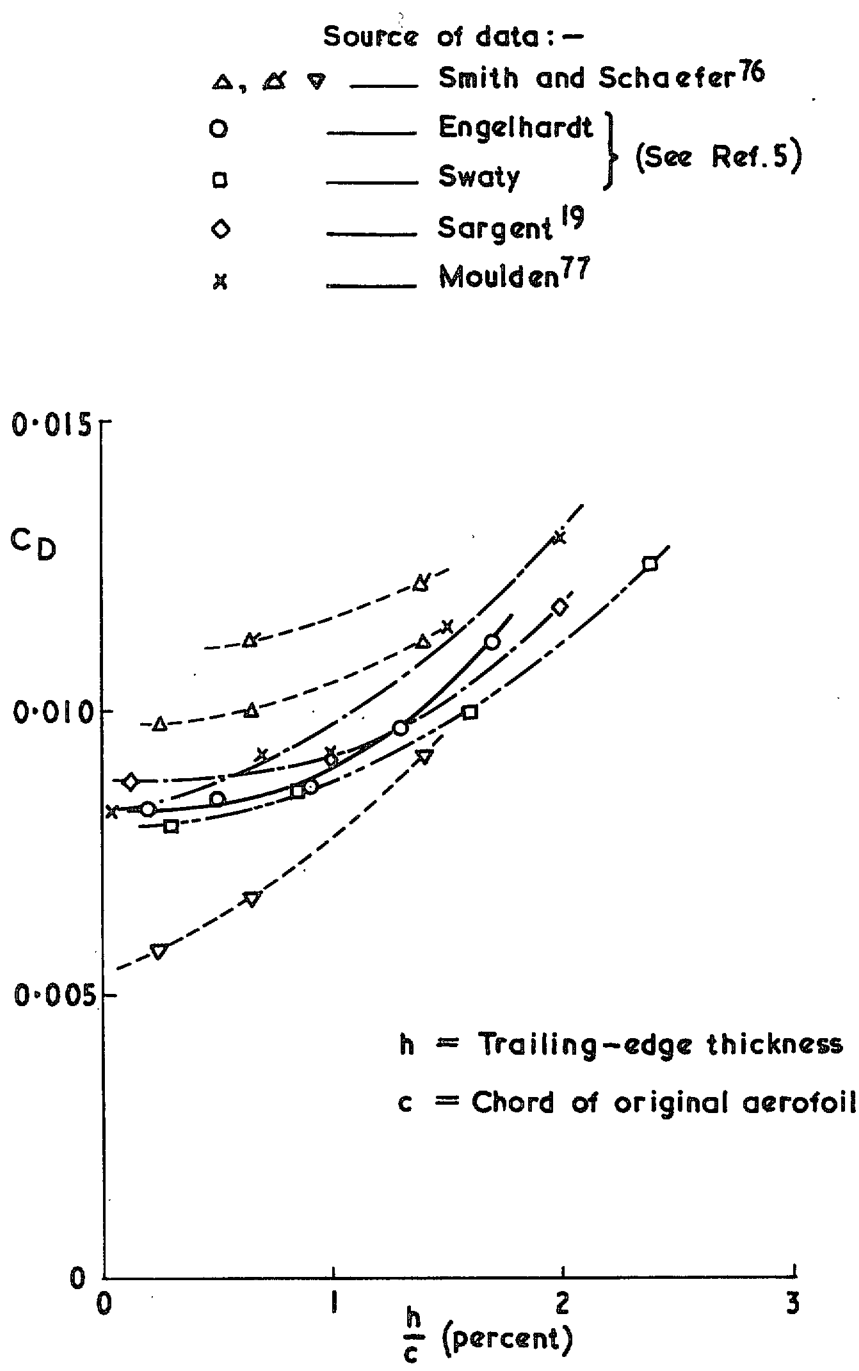

FIG. 1. Effect on drag of thickening or cutting back the trailing edge of a two-dimensional aerofoil – low speeds.

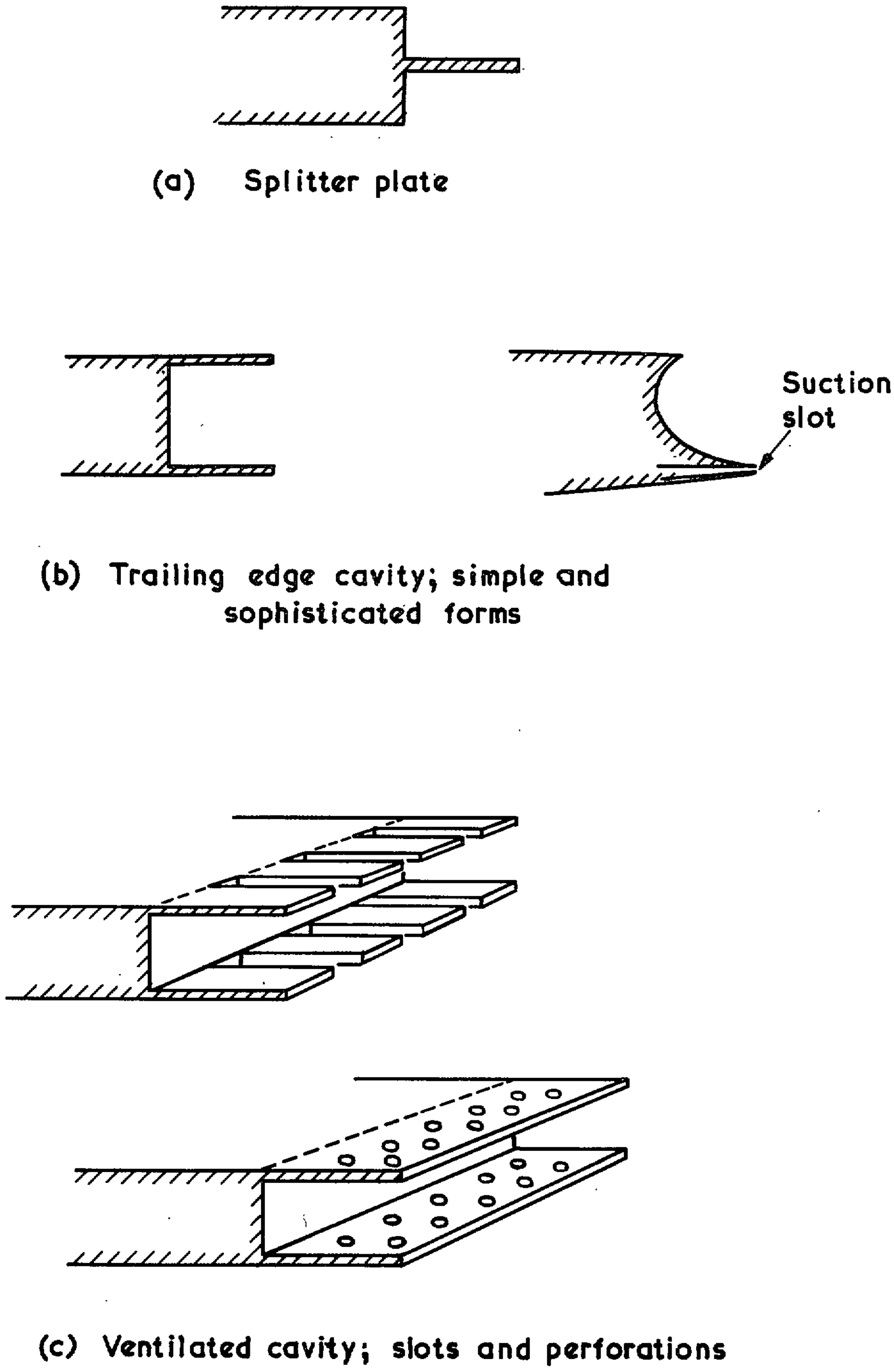

FIG. 2. Various devices for weakening the vortex street and increasing base pressure at subsonic speeds.

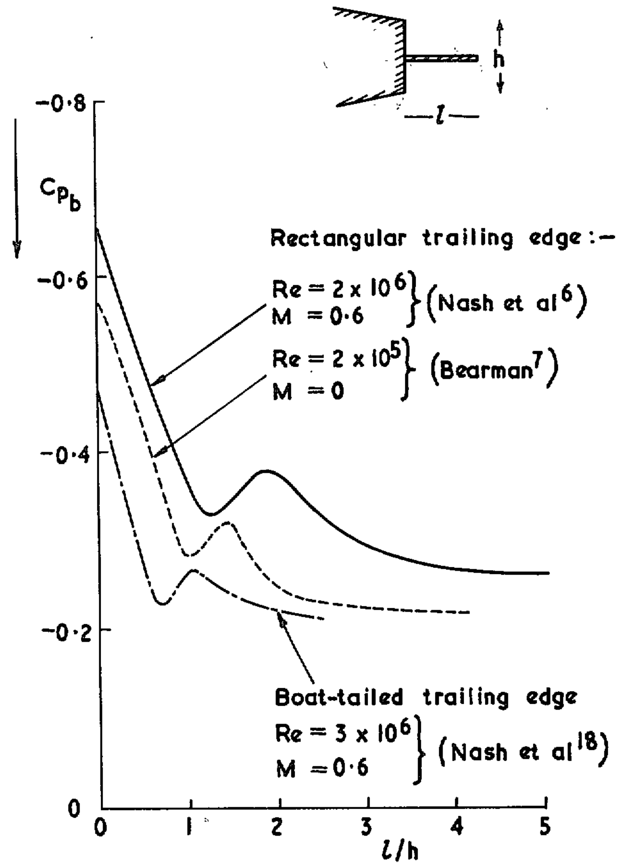

FIG. 3. Effect of a splitter plate on base pressure at subsonic speeds.

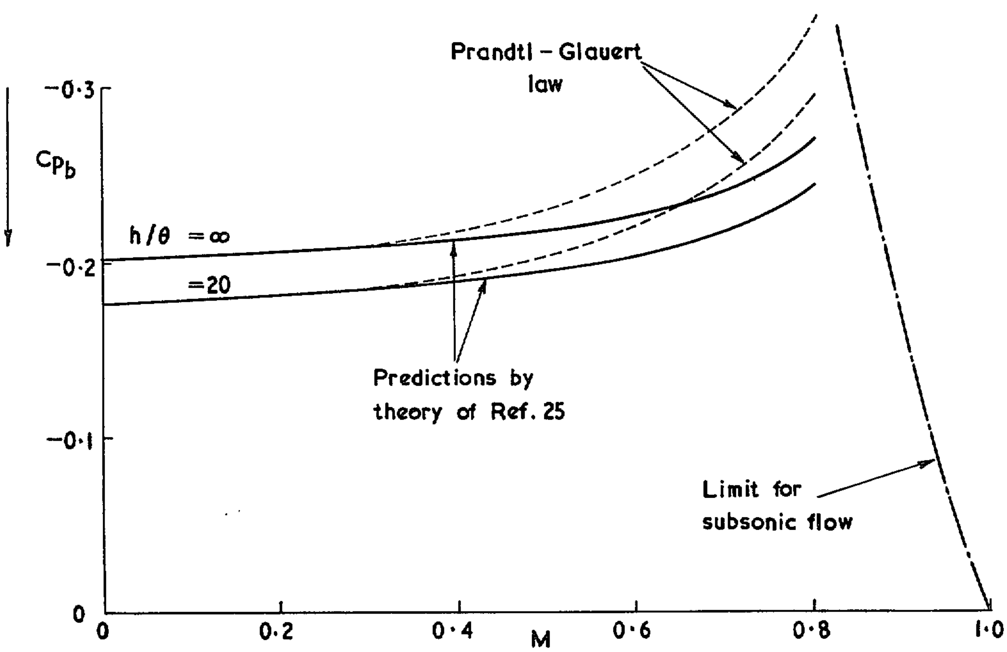

FIG. 4. Variation of base pressure coefficient with Mach number, (subsonic speeds, no vortex street, long section with rectangular trailing edge – i.e. no boat-tailing).

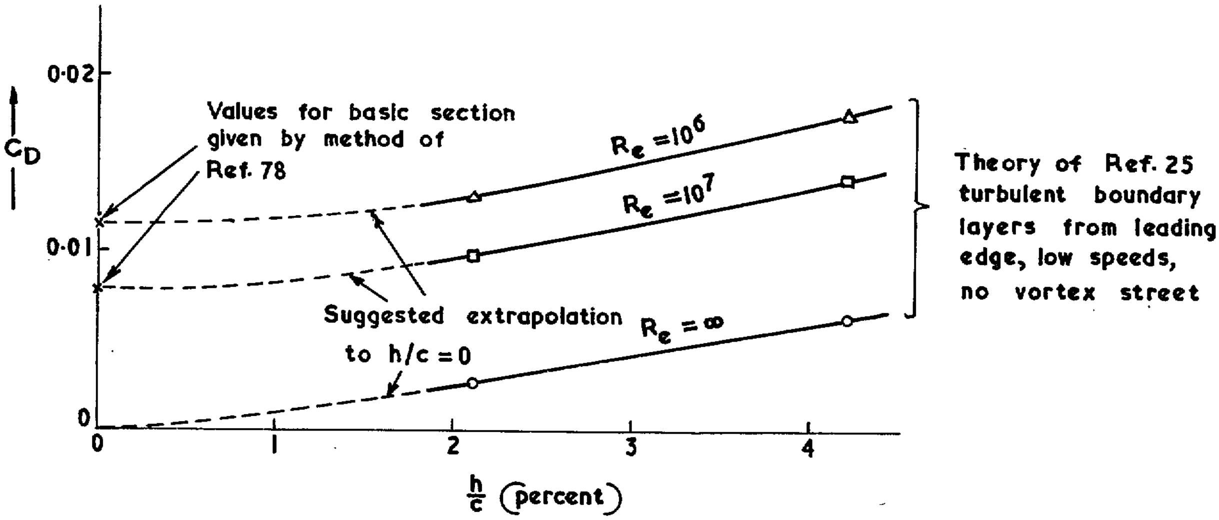

FIG. 5. Increase of C_D with trailing-edge thickness. Calculations for sections of 10 per cent thickness-chord ratio derived by truncation of R.A.E. 103 aerofoil (cf. data in Fig. 1.).

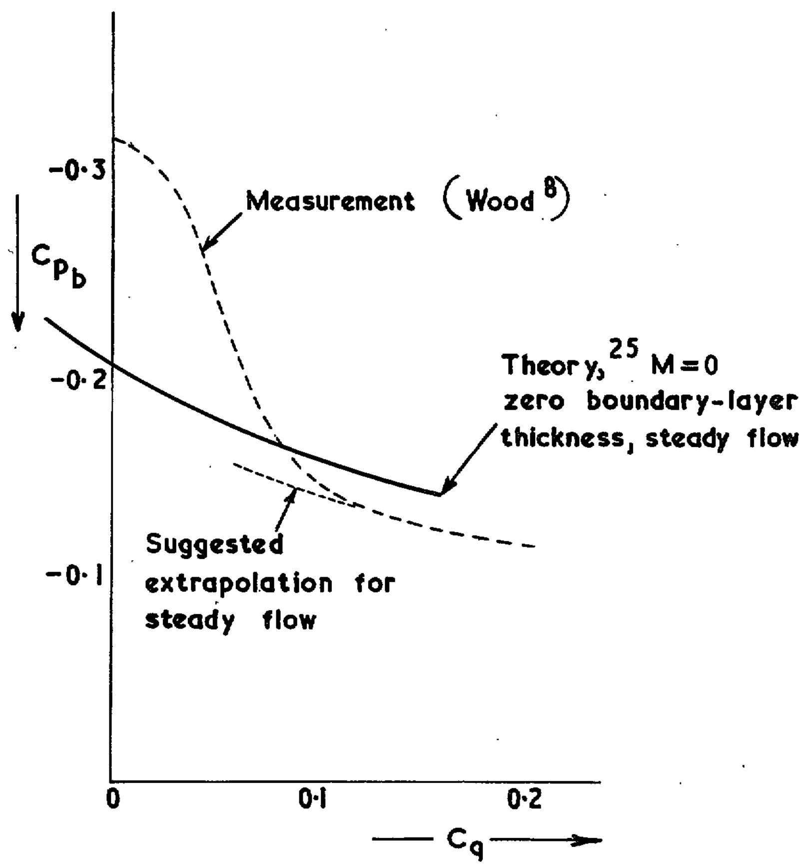

FIG. 6. Effect of base bleed at low speeds, long, parallel – sided section.

$$C_q = \frac{\text{bleed mass flux}}{\rho u_\infty h}$$

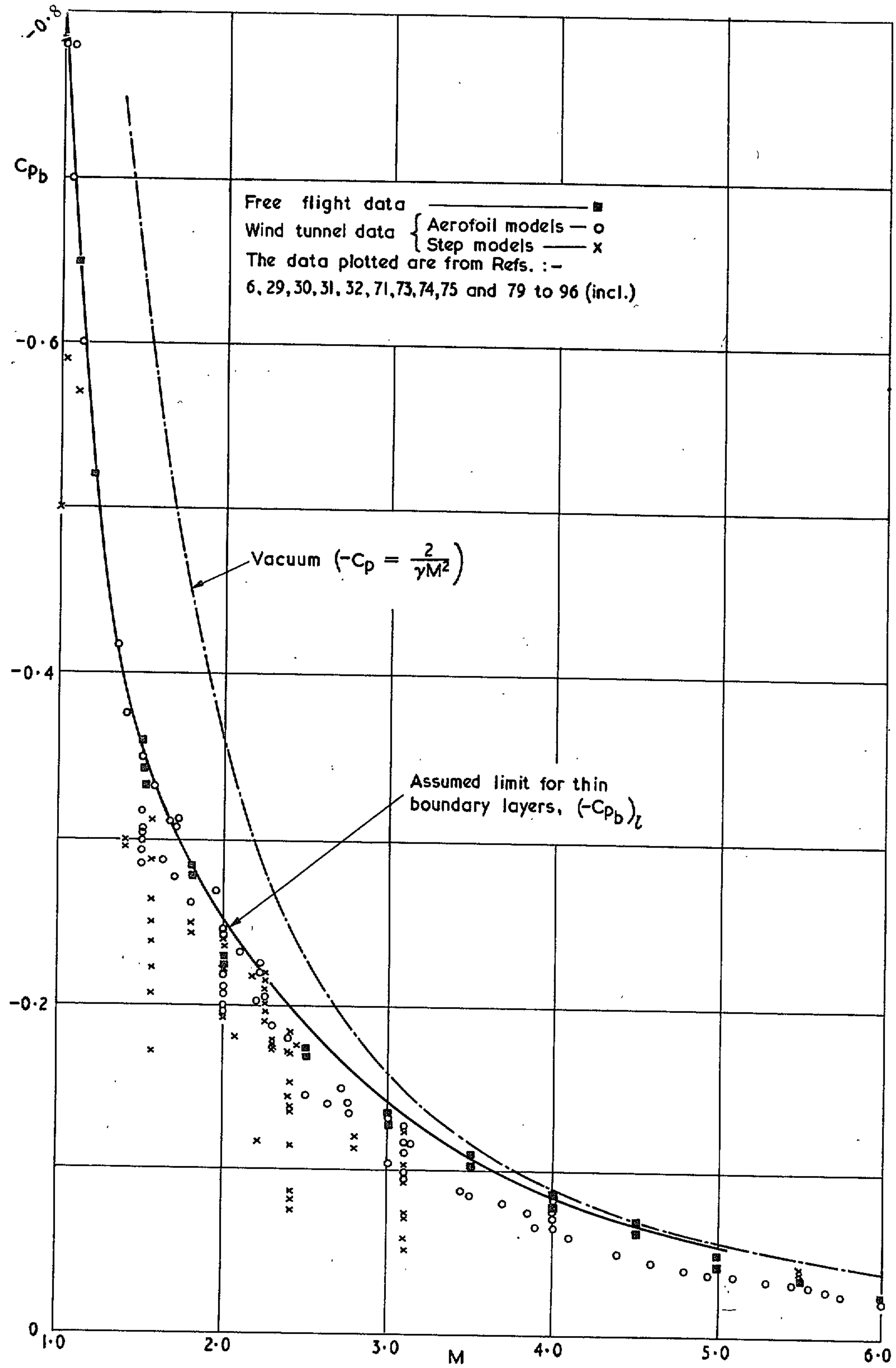

FIG. 7. Base pressure measurements at supersonic speeds.

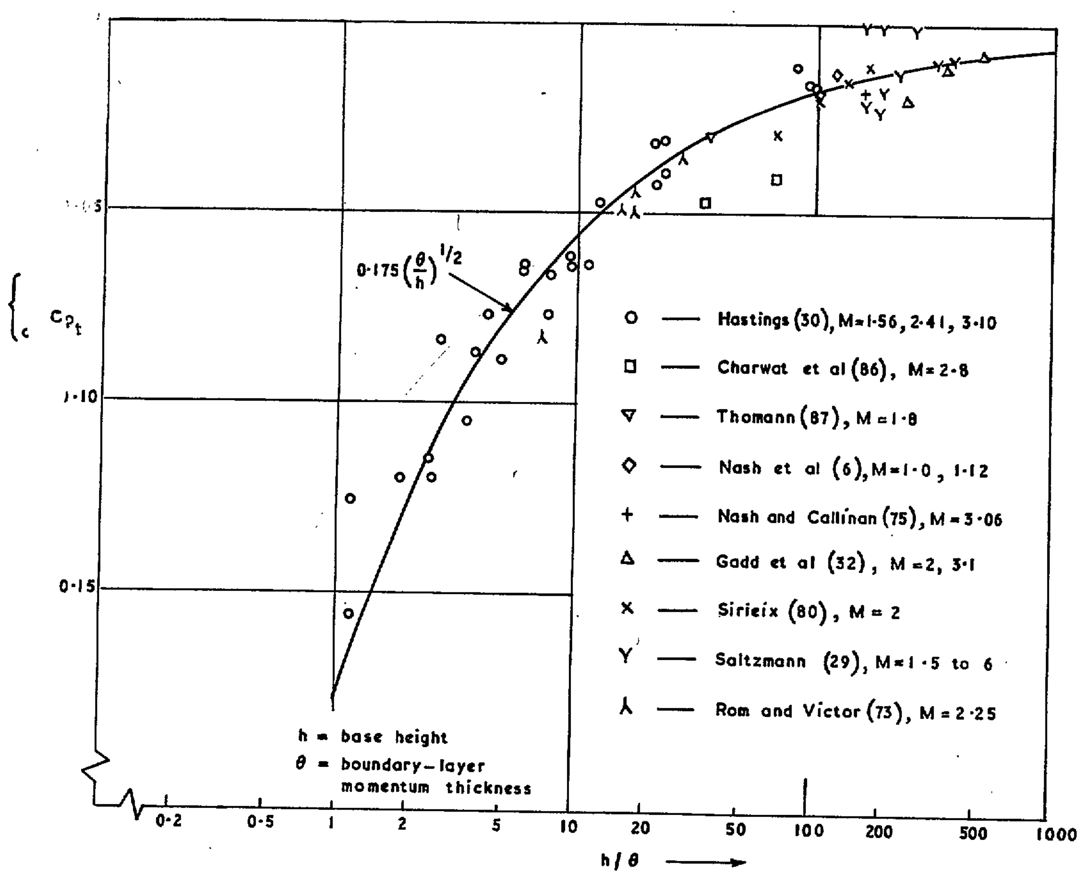

FIG. 8. Correlation of base pressure data for supersonic flow. $(C_{p_b})_l$ is shown as a function of Mach number in Fig. 7 and tabulated in Table 2.

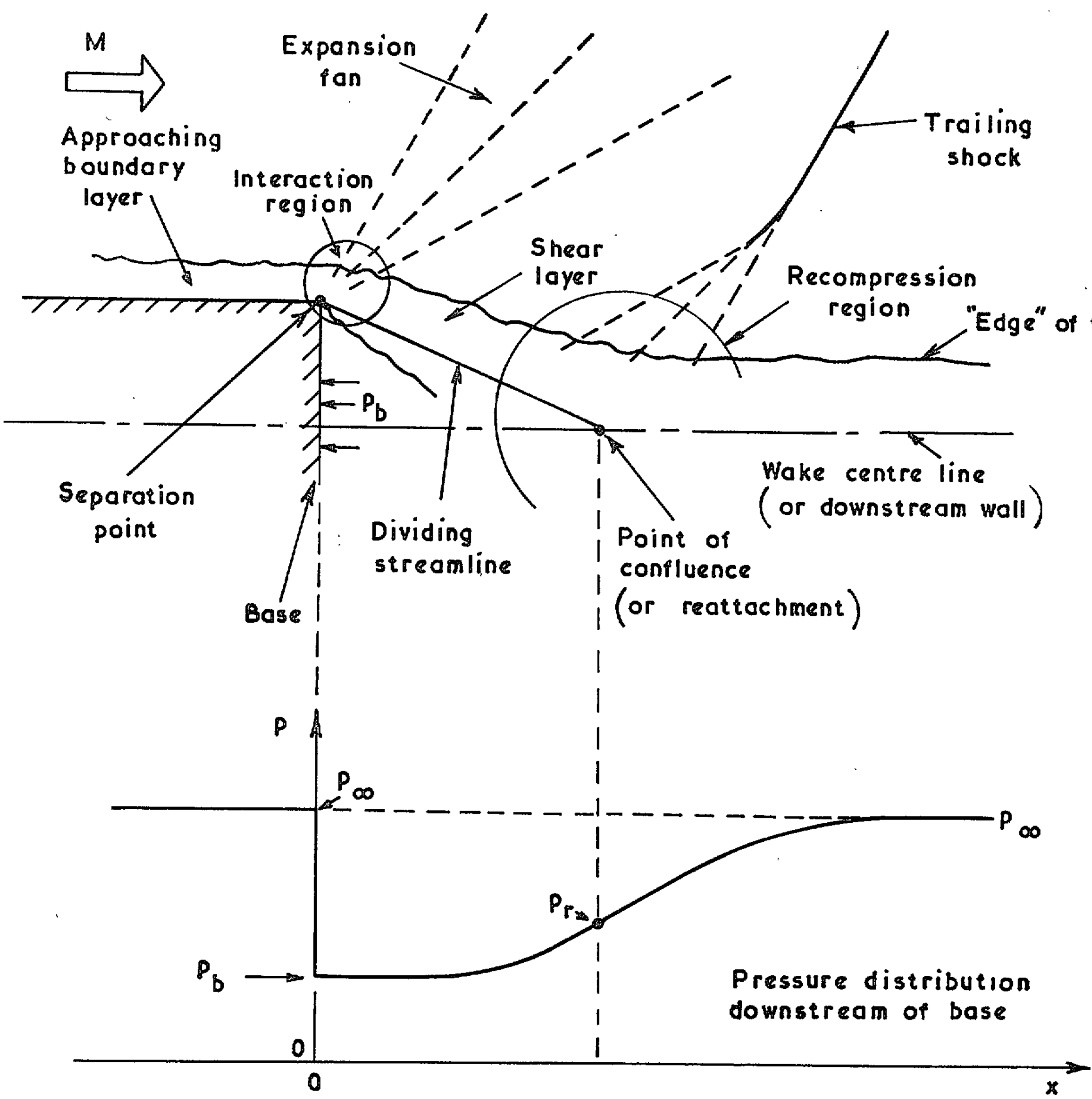

FIG. 9. Model of supersonic base flow.

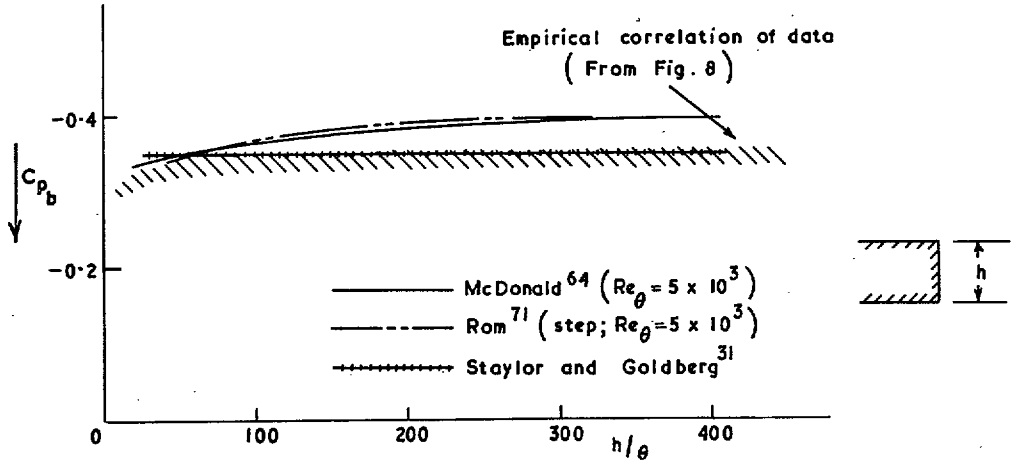

FIG. 10. Comparison of models for predicting base pressure, $M = 1{\cdot}5$.

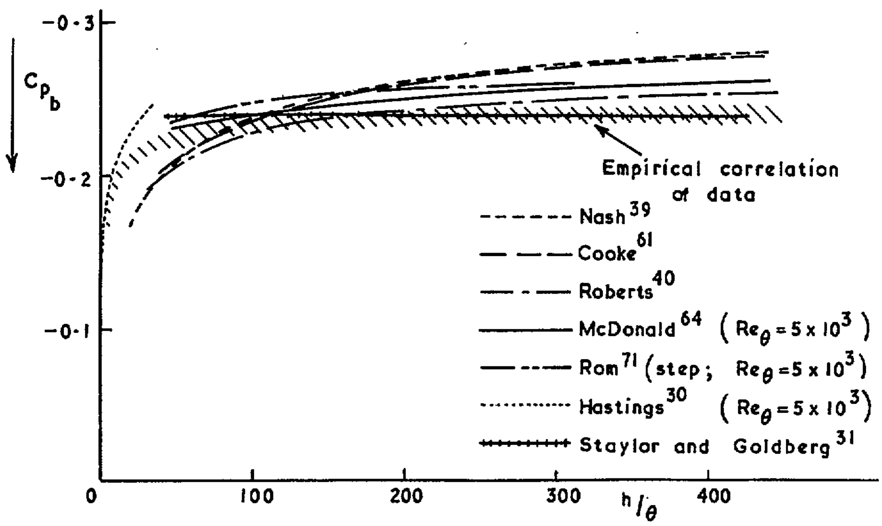

FIG. 11. Comparison of methods for predicting base pressure, $M = 2$.
(Note change of vertical scale as compared with Fig. 10).

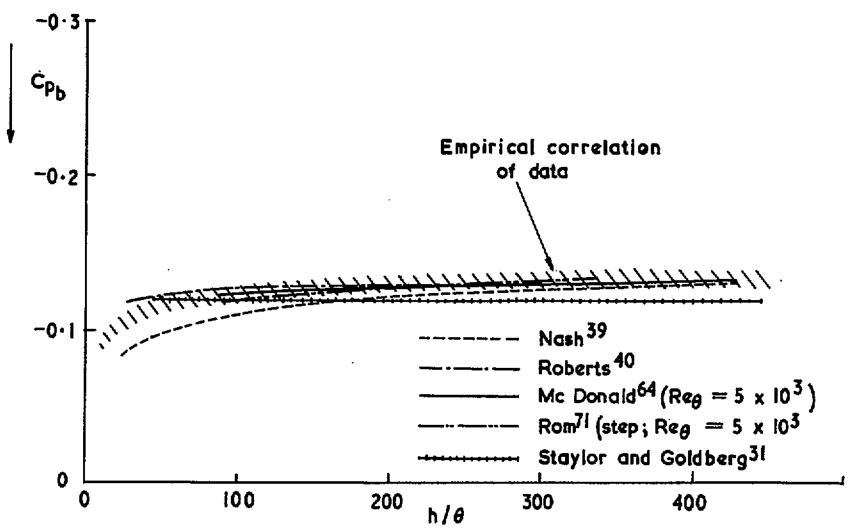

FIG. 12. Comparison of methods for predicting base pressure, $M = 3$.

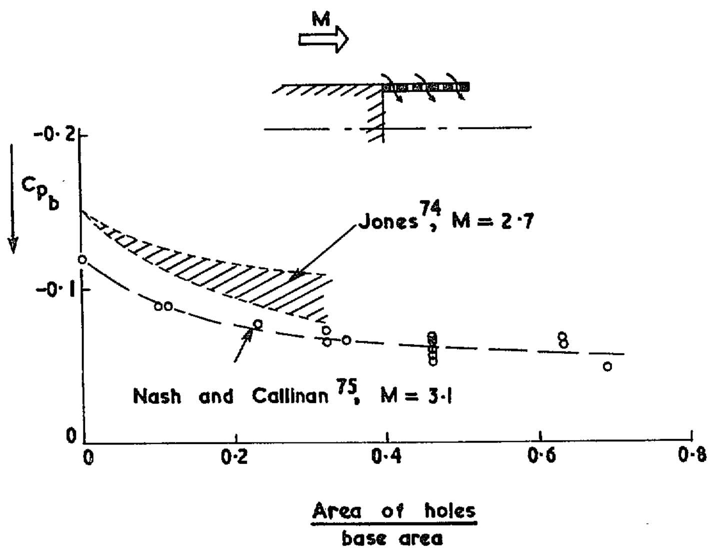

FIG. 13. Effect of perforated cavity on base pressure at supersonic speeds.

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