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Feed-Back Accelerometer Circuits with a Velocity Output

By R. H. Evans, B.Sc., M.I.E.E., and G. G. Haigh, B.Sc.

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Summary.

Circuits are suggested for modifying the feed-back loop of an accelerometer so that the output voltage is proportional to velocity, that is, an integrating process is incorporated within the loop. The main advantage is that the indicated velocity should be less critical with respect to amplifier drift than in most existing methods of integration. Circuits design is treated in some detail for typical examples of force feed-back accelerometers, including both undamped and viscous damped types, and it is shown that acceptable accuracy and stability should be possible with either type. A comparison is made with other methods of integration, and it is concluded that the method may find application, for example, as part of an analogue inertial navigation system, but that other methods are likely to be preferred in systems incorporating digital computers.

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1. *Introduction.*

Accelerometers of the precision demanded by inertial navigation applications are usually of the force feed-back type, in which the acceleration force on a pendulous mass is balanced by the force produced by a current flowing in a coil situated in the magnetic field of a permanent magnet; the current flowing is then an accurate measure of the acceleration force and hence of the acceleration itself provided both the mass and the magnetic field remain constant. Usually the current is converted to an accurately proportional voltage by means of a precision resistor.

In an inertial navigation system it is velocity rather than acceleration which is required and this may be obtained by integration of the voltage representing acceleration. The integrator may be of various forms, such as electronic analogue (Miller), electronic digital, or electro-mechanical (employing a motor and tachogenerator), the last type being readily adapted to deliver analogue and digital outputs simultaneously. Because of the difficulties associated with accurate integrators it is natural to examine the possibility of modifying the feed-back path of the accelerometer so that it produces an analogue velocity directly; some proposals to this end have been made[1]. This report further analyses some possible circuits and establishes design parameters for such circuits for use with two existing types of accelerometer. Only analogue circuits are considered so that the output must be followed by an analogue to digital convertor if a digital velocity is required.

The report begins with an analysis of the dynamic response of unmodified force feed-back accelerometer loops with special emphasis on the difference in performance between the undamped (air filled) accelerometer and the viscous damped (liquid filled) type. A review is then given of some of the existing methods of integrating the acceleration signal to obtain velocity, including the Miller integrator, a digital integrator and the electro-mechanical method employing a motor and tachogenerator. The general question of modifications to the feed-back loop of the accelerometer is then considered, followed by more detailed design of circuits for the two principal types of accelerometer. Some further details such as the mixing in of external acceleration correction signals are then considered and finally a brief summary of the advantages of the method is given.

2. *Response of Force Feed-Back Accelerometers.*

Before studying the effect of additional elements in the loop it may be useful to analyse the dynamic response of unmodified force feed-back accelerometer loops. Fig. 1 shows the basic arrangement for a single axis accelerometer. If the case is accelerated along the sensitive axis, the pendulous mass tends to move to the opposite direction relative to the case and any displacement from the null position produces a roughly proportional alternating e.m.f. in the attached pick-off coil. After amplification and phase-sensitive rectification this signal is applied to the restoring coil which reacts with the permanent magnet field to restore the mass nearly to the null position, the actual final displacement being inversely proportional to the amplifier gain. If the amplifier gain is sufficiently high the electro-magnetic restoring force on the mass is balancing the acceleration force, and provided both the permanent magnet and the mass are stable the current in the restoring coil will be accurately proportional to acceleration.

The dynamic response may be calculated as follows. Suppose that an acceleration of a cm/sec^2 is applied and at a given instant the mass (M gm) has a displacement relative to the case of x cm from the null position. If the stiffness of the pendulum arm is S dyne/cm and the viscous friction constant is D dynes per cm/sec then the equation of motion without a feed-back circuit would be

$$M\ddot{x}+D\dot{x}+Sx = Ma$$

or

$$\ddot{x}+\frac{D}{M}\dot{x}+\frac{S}{M}x = a \tag{1}$$

Strictly, a minus sign should be inserted before one side or the other of this equation because the pendulum deflection is in the opposite direction to the applied acceleration. A steady acceleration would produce a steady deflection of Ma/S and this would be large since S is small. If the loop is now closed, the pick-off voltage which is proportional to deflection is amplified and a proportional current applied to the restoring coil, so that an additional force term proportional to deflection is added to the left-hand side of the equation of motion; hence the feed-back produces electro-magnetically an additional equivalent stiffness, which in practice is very much larger than S. The amount of feed-back may be expressed as the acceleration equivalent of the restoring force per unit displacement of the pendulum centre of gravity and since it has the dimensions of frequency squared it will be denoted by Ω^2 cm/sec^2 per cm. It may be noted that Ω^2 is the product of the pick-off sensitivity (volts/cm), amplifier gain (mA/volt) and the scale factor of the restoring coil (acceleration per unit current). Then the equation of motion of the pendulum on closed loop becomes

$$\ddot{x}+\frac{D}{M}\dot{x}+\left(\frac{S}{M}+\Omega^2\right)x = a. \tag{2}$$

The acceleration indicated by the instrument is $\Omega^2 x$ (actually measured by the current in the restoring coil). Since Ω^2 is very much larger than S/M, in the steady state the deflection is now a/Ω^2 and the indicated acceleration is equal to the applied acceleration independently of the value of Ω^2. Dynamically the ratio of indicated acceleration to applied acceleration is given by

$$\frac{\Omega^2 x}{a} = \frac{\Omega^2}{p^2 + \frac{D}{M} p + \Omega^2} = \frac{1}{\frac{p^2}{\Omega^2} + \frac{D}{M}\frac{p}{\Omega^2} + 1} \tag{3}$$

where p is the differential operator.

At this point it is convenient to sub-divide the accelerometers under discussion into two classes, namely the air-filled undamped type in which the damping constant D is negligible, and the liquid-filled viscous damped type in which D is large. A widely used example of the former class is the Kearfott/Ferranti Single Axis accelerometer Type 429037–3A which has been fully evaluated[2,3] and an example of the latter class is the Kearfott accelerometer Type 2401 which has also been evaluated[4,5]. For the undamped type, the transfer function, that is, the ratio of indicated acceleration to applied acceleration, reduces to

$$\frac{1}{\frac{p^2}{\Omega^2} + 1}.$$

This represents an undamped oscillatory response to a step change of input, the angular frequency of oscillation being Ω. It may be noted that this natural frequency depends only on the feed-back elements; in particular it is proportional to the square root of amplifier gain and is independent of the mechanical resonant frequency of the pendulum on open loop.

An important parameter to be controlled is the angular deflection of the pendulum, since this determines the sensitivity to cross-acceleration. If this deflection is δ rad per g and the pendulum length from the hinge to the centre of gravity is r cm then it is of interest that

$$\Omega = \sqrt{\frac{g}{\delta r}}. \tag{3a}$$

Thus for a given pendulum length and given sensitivity to cross-acceleration the natural frequency of the closed loop is determined.

The closed-loop transfer function derived above is that corresponding to an open-loop transfer function of Ω^2/p^2, that is the accelerometer itself behaves as a double integrator. Numerical data for the accelerometer are given in Appendix I. Since an undamped oscillation is obviously unsatisfactory it is necessary to add damping electrically, usually by including a phase-advance circuit within the amplifier. The magnification at resonance is then about 2·5, corresponding to about 0·21 critical damping. The resonant frequency is typically about 350 c/s.

For the viscous damped type, the middle term in the denominator of equation (3) predominates over the others in determining the closed-loop response and the closed-loop transfer function approximates to

$$\frac{1}{\frac{Dp}{M\Omega^2} + 1}.$$

This represents a single time-constant response which is intrinsically stable and no electrical damping or stabilising circuit is required. The corresponding open-loop transfer function is $M\Omega^2/Dp$, so that the accelerometer itself behaves as an integrator. In practice the closed-loop response is of a higher order than the first, mainly because of the smoothing circuit after the demodulator in the amplifier. The effect is to produce an approximately second order system in which typically the damping is about 0·7 critical so that there is practically no peak in the frequency response; the bandwidth (3 *db* down) is about 300 *c/s*. Some additional numerical details are given in Appendix I.

An advantage of liquid filling is that the associated amplifier is smaller because it contains no frequency response shaping network; this is particularly significant with the modern use of solid state integrated circuits for the amplifier because the shaping network does not readily lend itself to this technique and it is likely therefore to be disproportionately large. Further, the performance of the accelerometer in the presence of vibration is better because there is no resonant peak in the frequency response; moreover, this improved loop damping can be achieved even with a higher loop gain and hence a smaller pendulum deflection compared with that permissible for the air-filled instrument.

On the other hand, the processes of vacuum pumping, liquid filling and hermetic sealing add considerably to the complications of manufacture. Also, some accelerometers of this type have to be operated at a high controlled temperature to maintain satisfactory values of density and viscosity of the liquid, and performance is not satisfactory at ordinary room temperature.

The liquid filled type would be favoured where it was essential to take advantage of its particular properties. For the integrating-accelerometer which is the subject of this report its characteristics are especially suitable (*see* Section 5.1).

3. *Present Methods of Integration to Obtain Velocity.*

3.1. *Electronic Analogue Integrator.*

This is the well-known Miller integrator shown in Fig. 2b, consisting of a high gain amplifier shunted by a low-loss capacitor C_i, the input signal being applied through the resistor R. The transfer function is derived in Appendix II and is given by

$$\frac{V_0}{V_i} = -\frac{K}{1+KTp}$$

where $T = C_i R$.

This approximates to

$$\frac{V_0}{V_i} = -\frac{1}{Tp}$$

for short times or high frequencies if the amplifier gain K is large. It should be noticed however that if an input is applied so that an output builds up and the input is then reduced to zero, a perfect integrator would retain indefinitely the output at the value reached, but in a Miller integrator the output 'leaks away' exponentially with a time-constant equal to KT. This could be a disadvantage in some circumstances.

For accuracy it is necessary for the components C_i and R to be of high stability and the amplifier must be of high gain and have very low drift. The last requirements is usually achieved by converting the input to an a.c. signal by means of a transistor chopper or a variable capacitance diode and performing most of the required amplification on this a.c. signal, finally reverting to d.c. by means of a phase-sensitive rectifier. The main advantages of this type of integrator are its rapid response, freedom from vibration effects and its long life, all due to the absence of moving parts.

3.2. *Electro-Mechanical Integrator.*

In this type of integrator the shaft of a potentiometer is driven at a speed which is accurately proportional to the input voltage so that the angle turned through by the shaft, and hence the output voltage of the potentiometer, is proportional to the integral of the input voltage. A typical circuit is shown in Fig. 3.

In this circuit the two-phase 400 c/s motor M is driven through an amplifier and 400 c/s modulator G2, and the motor drives the tachogenerator T whose output is fed back through resistor R_2 to the input of the amplifier. If the loop gain is very high, the input point of the amplifier becomes a virtual earth and the tachogenerator must run up to a speed at which the current in R_2 is equal and opposite to that in R_1. Hence, in the steady state, we have

$$V_2 = -\frac{R_2}{R_1} . V_1.$$

Since V_2 is accurately proportional to the speed of the tachogenerator it follows that, provided R_1 and R_2 are sufficiently stable, the speed will be accurately proportional to the input voltage V_1. The motor M drives a precision potentiometer P through an accurate gear train; for overall accuracy it is of course essential for the potentiometer to be excited from a stable voltage source.

Because of the mechanical time-constant of the motor the response of the integrator in its simplest form would be slow; the tachogenerator would be unable to keep in balance with a rapidly changing input voltage, the amplifier G2 would saturate and information would be lost. To avoid this loss of information the first stage of amplification may take the form of a Miller integrator, whose output voltage in effect represents stored demand which is eventually reduced almost to zero when a steady state is reached. With the addition of the Miller amplifier the system now usually has a very lightly damped oscillatory response, and damping may be added electrically, for example by means of a phase-advance circuit after the Miller amplifier (or by modification of the feed-back of the Miller amplifier itself). The forward path of the loop then consists of a Miller integrator, phase-advance circuit, modulator, power amplifier, motor and tachogenerator. Saturation of the power amplifier as the result of a rapidly changing input to the integrator is not important because no information is lost provided the Miller amplifier does not saturate.

One advantage of this type of integrator is that it will store an output indefinitely, i.e. in principle it is a perfect integrator although of course there is some drift in practice, mainly due to drift in the Miller amplifier. By mounting additional potentiometers on the output shaft several independent outputs may be obtained, and an output may be readily multiplied by another variable by supplying the potentiometer with an excitation voltage proportional to that variable; this feature is useful in inertial navigation, where Coriolis terms consisting of the product of a linear velocity and an angular velocity are often required. Finally, by mounting a digitizer on an appropriate shaft the velocity in digital form may be obtained. Disadvantages of the electro-mechanical integrator are that it may be rather large, it requires precision mechanical components as well as precision electrical supplies, and because of its moving parts it is subject to wear.

In order to show the type of performance required, the following figures are given for a recent design of integrator used in an inertial navigation system. The input acceleration range is $\pm 10g$ (represented by ± 10 volts) and the output velocity range is ± 4500 ft/sec represented by $\pm 22{\cdot}5$ volts. The linearity of the tachometer is 0·01 per cent and that of the output potentiometer is 0·1 per cent; the stability of the excitation supply for the potentiometer is 0·05 per cent. The Miller amplifier introduces a drift which at a given temperature over a 3 hour period does not exceed $60 \times 10^{-6}g$ referred to the input; the temperature coefficient of drift is about $25 \times 10^{-6}g$ per C. The integrator frequency response expressed as the transfer function.

$$\frac{\text{INDICATED ACCELERATION}}{\text{APPLIED ACCELERATION}} \text{ or } \frac{\text{INDICATED VELOCITY}}{\text{APPLIED VELOCITY}}$$

has a small (1·2 db up) resonant peak at 4 c/s and the bandwidth (3 db down) is 12 c/s. The velocity lag of the system is such that a constant acceleration of $4g$ produces in the steady state a velocity error of 0·16 ft/sec.

3.3. *Digital Integrator.*

In this system the acceleration signal is converted to a train of pulses whose repetition frequency is proportional to the magnitude of the input, so that a count of the pulses from rest is a measure of the velocity of the vehicle. One of the most promising forms of such as integrator is that described by May[6]. The system consists essentially of an *RC* Miller integrator whose output is reset every time it exceeds a predetermined level; the reset takes the form of a pulse of precise amplitude and duration and of appropriate polarity applied to the input of the integrator. The velocity is then represented by a count of these reset pulses, taking sign into account, in a backward/forward counter or a digital computer.

Fig. 4 presents a block diagram of the system, in which the acceleration signal is the voltage V_i. When a positive input voltage is applied to the integrator its output rises until it exceeds the pre-determined level of the negative Schmitt trigger. The Schmitt trigger opens the gate and allows the start pulse generator to operate the timing bi-stable which in turn closes the appropriate transistor precision switch and a negative reset voltage applied to the integrator input returns its output towards zero. The duration of the reset voltage is determined by the time interval between the start pulse and the following stop pulse from the pulse generator. The stop pulse opens the precision switch *via* the timing bi-stable. When a negative input voltage is applied to the integrator the sequence is the same but operation is now by means of the duplicate set of Schmitt trigger, gate, timing bi-stable, precision switch and voltage reference. Each time a reset pulse is applied to the integrator a voltage pulse appears at the output of one of the bi-stables, positive velocity increments appearing at the output of the upper bi-stable and negative increments at the lower.

In a particular design of this integrator the acceleration range was $\pm 17g$ and each output pulse represented a velocity increment of 0·25 ft/sec to an accuracy of 0·006 per cent (the r.m.s. value of all contributing errors) the largest single error being 0·005 per cent in the time interval between the start and stop pulses. Long term drift is equivalent to an input of about $10^{-4}g$ but short term drift (2–3 hours) is better than $10^{-5}g$.

4. *Intergration Within the Accelerometer Loop; General Considerations.*

A general block diagram for a force feed-back accelerometer and its associated circuits is given in Fig. 5. It is convenient to employ the same units for all signals throughout. In this instance volts have been chosen and accelerations are therefore measured in terms of the equivalent voltage across the restoring coil. More exactly, since it is the current through the restoring coil that is accurately proportional to acceleration, accelerations are measured in terms of the voltage across a stable resistor whose resistance is equal to the nominal resistance of the coil. In practice circuits would be employed which ensured that the coil current rather than voltage bore the desired relationship to the output voltage representing velocity (*see* Section 7). In Fig. 5, e_i represents the applied acceleration, e_0 the acceleration equivalent of the restoring force and v_0 is the voltage output of the system. The function $h_a(p)$ is the transfer function of the accelerometer itself (measured in pick-off volts per restoring coil volt) and $h_e(p)$, $J(p)$ are the transfer functions of the electronic circuits in the forward and feed-back paths respectively. The overall closed-loop transfer function v_0/e_i of the system is given by

$$G = \frac{h_a h_e}{1 + h_a h_e J}$$

so that

$$\frac{1}{G} = \frac{1}{h_a h_e} + J. \tag{4}$$

Evidently if we require G to have a particular form we must choose h_e and J so that they have a relationship to each other given by equation (4). In the case of the conventional accelerometer circuit, v_0 is to be proportional to acceleration so that G is to be a constant; if $h_a\ h_e$ is large then the required condition is that J should be a constant. In this case h_e represents the usual a.c. preamplifier, demodulator, phase-advance circuit if required, and d.c. output amplifier. If, however, we wish the output v_0 to represent velocity then the system transfer function is of the form

$$G = \frac{1}{pT_2}$$

and from equation (4)

$$J = pT_2 - \frac{1}{h_a h_e}.$$

If the forward gain is very high then

$$J = pT_2$$

and the feed-back path must consist of a differentiator. We may employ a practical form of differentiator shown in Fig. 2d having transfer function

$$J = \frac{pT_2}{1 + pT_2/K_2}$$

where $T_2 = C_d R_2$ and K_2 is the open-loop gain of the amplifier, and then endeavour to choose the forward-path electronics h_e so that equation (4) is still met. In this case

$$\frac{1}{h_a h_e} = pT_2 - \frac{pT_2}{1+pT_2/K_2} = \frac{pT_2}{K_2} . \frac{pT_2}{1+pT_2/K_2}$$

so that

$$h_e = \frac{1}{h_a} . \frac{K_2}{pT_2} . \frac{1+pT_2/K_2}{pT_2}. \tag{5}$$

It was shown in Section 2 and Appendix I that the accelerometer transfer function h_a for an undamped accelerometer is equivalent to a double integration and may be written in the form $\frac{1}{p^2 T_1^2}$; for a viscous damped accelerometer it is equivalent to a single integration which may be written in the form $\frac{1}{pT_1}$.

Equation (5) gives the condition for exact integration, but many circuits will produce an approximate integration, the degree of approximation depending on the gain of the forward path at the frequencies of interest and on the extent to which the feed-back function J approaches a true differentiator, this in turn depending on the gain K_2 of its amplifier. In the following Sections circuits are investigated which will approach as nearly as possible to the requirements of equation (5) for the two main types of accelerometer, using the numerical values given in Appendix I and the transfer functions derived in Appendix II. Some estimates are made of the performance of these circuits, including dynamic response, ripple and noise, effect of capacitor leakage, and magnitude of the deflection of the accelerometer pendulum. The viscous damped accelerometer is treated first because this gives the simplest results.

Differentiators such as that proposed for the feed-back path are not in common use because they have a high gain at high frequencies and are therefore easily saturated with ripple and noise from preceding stages. The input to the differentiator in the accelerometer circuits must for this reason be very smooth and this requirement will affect the choice of circuits in the forward path.

5. *Integration Within the Loop, Employing a Viscous Damped Accelerometer.*

5.1. *Choice of Feed-forward Circuit.*

A typical specified range of velocity output is ± 4000 ft/sec (± 2350 knots) and this may be conveniently represented by an output voltage range of ± 25 volts. If this scale is adopted then an acceleration of $1g$ will cause the output to rise at the rate of 0·2 volts/sec and since the scale factor of the restoring coil is 0·8 volts/g a differentiator time-constant (T_2) equal to 4 seconds is required. We may choose the open-loop gain K_2 of its amplifier to be 2000 so that the transfer function of the feed-back differentiator is

$$J = \frac{pT_2}{1+pT_2/K_2} = \frac{4p}{1+0{\cdot}002p}.$$

The required transfer function (h_e) of the electronic circuit in the forward path obtained by substitution of $h_a = \frac{1}{pT_1}$ in equation (5) is

$$h_e = \frac{K_2\,T_1}{T_2} \cdot \frac{1+pT_2/K_2}{pT_2}$$

$$= \frac{T_1}{T_2} + \frac{K_2\,T_1}{T_2^2 p}. \qquad (6)$$

This represents an amplifier and a perfect integrator operating in parallel and the modified integrator of Fig. 2c has approximately the required response, particularly at high frequencies, if the gain of its amplifier is high. When the numerical values $T_1 = 2$ seconds, $T_2 = 4$ seconds, $K_2 = 2000$ are substituted in equation (6) the transfer function h_e becomes

$$h_e = 0{\cdot}5 + \frac{250}{p}.$$

In practice the feed-forward circuit would have as its first stage an amplifier similar to that used in conventional accelerometer circuits, whose function would be (a) to demodulate and smooth the pick-off voltage (b) to raise the output to a high enough voltage level at a low output impedance to act as a suitable input to the following virtual-earth amplifier (the modified integrator circuit). If the voltage gain of this first-stage or buffer amplifier is taken to be 1000 then the transfer function of the modified integrator circuit becomes

$$0{\cdot}0005 + \frac{0{\cdot}25}{p} \quad \text{or} \quad \frac{1+0{\cdot}002p}{4p}.$$

It may be noted that this is the inverse of the differentiator transfer function. Comparison with the transfer function of the modified integrator circuit of Fig. 2c namely

$$\frac{R_2}{R_1} + \frac{1}{C_i\,R_1 p}$$

indicates that $\frac{R_2}{R_1} = 0{\cdot}0005$ and $C_i\,R_1 = 4$ seconds.

Suitable values would be $R_1 = 2$ megohm, $R_2 = 1$ kilohm, $C_i = 2\ \mu F$. We may choose the internal gain of the amplifier to be 1000.

The feed-back differentiator requires a time-constant $T_2 = 4$ seconds and we may choose $R =$ 4 megohm, $C_d = 1\ \mu F$. The complete circuit is shown in Fig. 6a. Figs. 6b and c show the corresponding transfer function block diagram in general and numerical terms respectively.

5.2. *Dynamic Response of System.*

The loop transfer function is given by

$$\frac{e_0}{e_e} = \frac{1}{pT_1} \cdot \frac{T_1 K_2}{T_2} \cdot \frac{1+pT_2/K_2}{pT_2} \cdot \frac{pT_2}{1+pT_2/K_2}$$

$$= \frac{K_2}{pT_2}.$$

Hence the closed loop transfer function is given by

$$\frac{e_0}{e_i} = \frac{1}{1+pT_2/K_2}. \tag{7}$$

This is a single time-constant response which is inherently stable. The transfer function from applied acceleration to output voltage v_0 is

$$\frac{v_0}{e_i} = \frac{1}{1+pT_2/K_2} \cdot \frac{1+pT_2/K_2}{pT_2} = \frac{1}{pT_2}.$$

This is of course the integrating function that the circuit was designed to achieve. Since the transfer function of the modified integrator is the inverse of that of the differentiator the complete loop is equivalent to a conventional accelerometer circuit having an amplifier voltage gain in this case of 1000. The amplifier voltage gain for the conventional circuit is 2000, so that the integrating circuit will have no stability problem but the pendulum deflection per g will be twice the conventional value.

In the above derivation, the limiting form of the transfer function of the modified integrator appropriate to very large values of its amplifier gain K_1 was employed, but it is shown in Appendix III that for $K_1 = 1000$ the employment of the accurate transfer function would make negligible difference.

5.3. *The Effect of Capacitor Leakage Resistance.*

The above calculation assumes a perfect capacitor for the input circuit of the differentiator. In practice the effect of the leakage resistance may be appreciable, although it will be kept to a minimum by the use of special high time-constant capacitors incorporating plastic dielectrics which have been developed for such applications. The effect of the leakage is that the differentiator is now represented by the circuit of Fig. 2e, that is, it functions as a differentiator plus constant multiplier. The result is that at low frequencies the output v_0 of the integrating accelerometer tends to represent acceleration rather than velocity, and a stored velocity at zero acceleration leaks away towards zero with a time constant equal to the capacitor time-constant T_c. A detailed analysis is given in Appendix IV, where it is also shown that compensation for capacitor leakage may be achieved by feeding a suitable fraction of v_0, but with sign reversal, across the capacitor through a resistor into the virtual earth of the differentiating amplifier. The modified circuit is shown in Fig. 7b. Compensation cannot be perfect in practice because of the variable nature of the capacitor leakage, but the effective value of the capacitor time-constant T_c can be substantially increased.

It may be noticed that the Miller integrator discussed in Section 3.1 suffers from similar limitations due to leakage in its capacitor, and indeed all capacitors employed for integrating or differentiating must incorporate very low leakage dielectrics.

5.4. *Ripple and Noise.*

The buffer amplifier following the pick-off will incorporate a degree of smoothing comparable to that of the amplifier in a conventional accelerometer circuit. The following integrator stage has a high attenuation at high frequency and the output voltage v_0 representing velocity is therefore very smooth.

The feed-back differentiator has a transfer function which is the inverse of that of the integrator with the result that the ripple voltage applied to the restoring coil will be the same as that at the output of the buffer amplifier i.e. will be about the same as in a conventional accelerometer circuit.

5.5. *Pendulum Deflection.*

With a buffer amplifier gain of 1000, the pick-off voltage for a steady acceleration of $1g$ will be 0·8 mV, corresponding to a pendulum deflection of 8 arc seconds. Since the accelerometer is employed in conventional circuits up to $20g$ corresponding to 80 arc seconds deflection, the integrating accelerometer should certainly be satisfactory up to $10g$. Of course it is possible to employ a buffer amplifier of higher gain if it is required to measure a higher level of acceleration. It may be noted that at $10g$ the restoring coil voltage will be 8 volts, the output v_0 will be rising at the rate of 2 volts/second, the output of the buffer amplifier will be 8 volts and the pick-off signal will be 8 mV.

When an acceleration is applied and then reduced to zero so that an accumulated velocity is stored on the output there will still be a very small pendulum deflection because the integrator in the forward path under steady state conditions has a gain of 1000 and not infinity as for a perfect integrator. For an indicated velocity of 4000 ft/sec ($v_0 = 25$ volts) the output of the buffer amplifier will be 25 mV and the pick-off voltage will be 25 μV, corresponding to a pendulum deflection of $\frac{1}{4}$ arc second. Thus the integrator in the forward path has the advantage of keeping the pendulum deflection very small under constant velocity conditions as well as providing a high forward gain at low frequencies to give an output which is accurately the integral of applied acceleration. The fact that there is some pendulum deflection at constant velocity means that there must be some restoring coil current and therefore that the output voltage v_0 must be falling. Thus the stored velocity leaks away slowly, as in a Miller amplifier. Since the hinge stiffness of the pendulum is 1700 arc min/g, a deflection of $\frac{1}{4}$ arc-second corresponds to an acceleration of $2{\cdot}45 \times 10^{-6}g$ or a restoring coil voltage of 2 μV. Hence the output v_0 is falling at the rate of 0·5 $\mu V/sec$ and a stored velocity therefore leaks away exponentially with a time-constant of 50×10^6 seconds or 14,000 hours, which is of course far too large to be important.

The total pendulum deflection allowing for that due to acceleration and the much smaller deflection proportional to velocity may be expressed as

$$8a + \frac{v}{16} \text{ arc seconds}$$

where a is the acceleration in numbers of g and v is the velocity in 1000 ft/sec units. This expression applies for steady accelerations or velocities; if a step of acceleration is applied the pendulum deflection builds up exponentially to its steady value of 8 arc seconds/g with a time-constant of T_2/K_2, which in the present case is 2 mS.

6. *Integration Within the Loop, Employing an Undamped Accelerometer.*

6.1. *Choice of Feed-Forward Circuit.*

We may again choose a velocity range of ± 4000 ft/sec represented by ± 25 volts but since the restoring coil scale factor in this type is 1·6 volts/g, a feed-back differentiator having a time-constant T_2 equal to 8 seconds will be required. The amplifier gain K_2 of the differentiator may again be taken equal to 2000. The required transfer function (h_e) of the electronic circuit in the forward path obtained by the substitution of $h_a = 1/p^2 T_1^2$ in equation (5) is

$$h_e = p^2 T_1^2 \frac{K_2}{pT_2} \frac{1+pT_2/K_2}{pT_2}$$

or

$$h_e = \frac{K_2 T_1^2}{T_2^2} + \frac{T_1^2 p}{T_2}. \qquad (8)$$

This represents an amplifier and a perfect differentiator operating in parallel, and the modified differentiator circuit of Fig. 2e has a transfer function approximating to this form, especially at low frequencies, if the gain of its amplifier is high. When the numerical values $T_1 = 0{\cdot}0036$ seconds, $T_2 = 8$ seconds, $K_2 = 2000$ are substituted in equation (8) the transfer function h_e becomes

$$h_e = 0{\cdot}0004 + 0{\cdot}0000016p.$$

Such a severe attenuation of the pick-off signal is clearly quite impracticable since in practice an output voltage v_0 of several volts is required for only a few seconds of pendulum deflection. Thus the conditions of equation (5) for accurate integration cannot be fulfilled for this accelerometer, but any circuit in the forward path which has a high gain at low frequencies will achieve an approximation to an overall integrating effect. Probably the most satisfactory solution is to employ the same circuit as for the viscous damped accelerometer, that is, a modified integrator whose transfer function is the inverse of that of the differentiator, preceded by a buffer amplifier (incorporating a demodulator and smoothing) having a gain approximately equal to that of the amplifier in a conventional circuit. The buffer amplifier would also incorporate a phase-advance circuit in order to achieve adequate system stability. Under these conditions the stability margin and the pendulum deflection would be the same as in a conventional circuit.

6.2. *Dynamic Response of System.*

The transfer function of the buffer amplifier may be written

$$\frac{B(1+p\tau)}{1+pq\tau}$$

where τ is the time-constant of the phase-advance circuit and q is the ratio of low-frequency to high-frequency gain (usually about 0·1). The loop transfer function is then given by

$$\frac{e_0}{e_e} = \frac{1}{p^2 T_1^2} \cdot \frac{B(1+p\tau)}{1+pq\tau}$$

and the corresponding closed-loop transfer function is

$$\frac{e_0}{e_i} = \frac{1+p\tau}{1+p\tau+p^2 T_1^2/B+p^3 T_1^2 q\tau/B}.$$

The output voltage v_0 is given by

$$\frac{v_0}{e_i} = \frac{1}{pT_2} \cdot \frac{(1+pT_2/K_2)(1+p\tau)}{1+p\tau+p^2 T_1^2/B+p^3 T_1^2 q\tau/B} \tag{9}$$

and hence the ratio of indicated velocity to true velocity is given by

$$\frac{pT_2 v_0}{e_i} = (1+pT_2/K_2) \left\{ \frac{1+p\tau}{1+p\tau+p^2 T_1^2/B+p^3 T_1^2 q\tau/B} \right\}.$$

The second expression on the right-hand side is the closed-loop response of the conventional accelerometer system employing the same buffer amplifier, and this is known from tests to approximate to a second order system having a natural frequency of 350 *c/sec* and a damping coefficient of 0·21. Its transfer function may be written in the form

$$\frac{1}{1+0{\cdot}19\times10^{-3}p+0{\cdot}21\times10^{-6}p^2}.$$

Hence the ratio of indicated velocity to true velocity is

$$\frac{1+4\times10^{-3}p}{1+0{\cdot}19\times10^{-3}p+0{\cdot}21\times10^{-6}p^2}.$$

The significance of this expression may be seen by considering in turn the frequency response and the ramp response (i.e. a ramp of velocity, corresponding to a step of acceleration). If we consider sinusoidal applied accelerations of various frequencies and compare the indicated velocities with the true velocities (by substituting $p = j\omega$ in the above expression) we obtain the results plotted in Figs. 8 and 9. The indicated velocity is always high except at very high frequencies; the error is 0·03 per cent at 1 *c/s*; 0·5 per cent at 4 *c/s* and 5 per cent at 12 *c/s*. As mentioned in Section 3.2 a typical electro-mechanical integrator had a peak response of 1·2 *db* (15 per cent) at 4 *c/s* and was 3 *db* (41 per cent) down at 12 *c/s*, so that it appears that the integrating accelerometer circuit is well within the required accuracy so far as frequency response is concerned.

If a $1g$ step of acceleration is applied, the true velocity will be a ramp of slope 32 ft/sec per second. The velocity as indicated by the voltage v_0 will show an oscillatory transient of a few milliseconds duration and will then settle down to a ramp of velocity which is a constant amount above the true velocity as shown in Fig. 10. This constant velocity error is calculated by multiplying the acceleration by a time-constant derived from the system transfer function. This time constant is equal to the difference between the coefficients of p in the numerator and denominator respectively and in the present case is equal to 3·8 *mS*. Thus the velocity error for an acceleration of $1g$ is 0·12 ft/sec; and this is probably acceptable. When the acceleration is removed, the velocity error reduces to zero by means of a transient of a few milliseconds duration, as illustrated in Fig. 10.

The error in indicated velocity (in ft/sec) when a step acceleration of $1g$ is applied is given by the following expression:—

$$0{\cdot}12-e^{-0{\cdot}45t}(0{\cdot}12\cos 2{\cdot}13t+0{\cdot}04\sin 2{\cdot}13t)$$

where t is measured in milliseconds. The velocity error when the acceleration is removed reduces to zero according to the expression

$$e^{-0{\cdot}45t}(0{\cdot}12\cos 2{\cdot}13t+0{\cdot}04\sin 2{\cdot}13t).$$

7. *Some Further Circuit Details.*

Because it is the current through the accelerometer restoring coil rather than the voltage across it that is accurately proportional to acceleration, it is necessary to ensure that it is the current that is the differential of the output voltage v_0 representing velocity. This will be achieved if the differentiator consists of an operational amplifier in which the feed-back is taken from a stable resistor in series with the restoring coil, as shown in Fig. 11a. If this resistor is equal in value to the restoring coil resistance, the transfer function of the differentiator from input voltage to restoring coil voltage is the same as with a direct feed-back except that the effective open-circuit gain of the amplifier is halved. In the calculations of Sections 5 and 6 an effective open-loop gain (K_2) of 2000 was assumed so that in this case it would be necessary to design an amplifier with an actual open-loop gain of 4000.

In an inertial navigation system it is often necessary to integrate not only the acceleration detected by the accelerometer but also various correcting accelerations which are fed in electrically. For example, in the vertical channel a constant $1g$ correction for gravity is required, and in all channels it may be necessary to add Coriolis corrections arising from the fact that the accelerometers are measuring inertial accelerations along axes which are rotating in space. Such a correcting signal may be added in the present system by applying it through a resistor to the virtual earth of the amplifier in the feed-back path, as shown in Fig. 11b. It is shown in Appendix V that the output voltage v_0 does indeed consist of two terms which are the integrals respectively of the applied accelerations e_i and the correction signal E. Of course, several correcting signals may be added in the same way, each through its own input resistor.

8. *Advantages and Disadvantages of Integration Within the Accelerometer Loop.*

The main advantage of integration within the loop by means of a differentiator in the feed-back path is that the indicated velocity is less sensitive to drift in the amplifiers. The amplifiers in the forward path are not critical either in gain or drift provided the gain remains high, and the feed-back amplifier being in a differentiating instead of an integrating circuit is not critical in its drift requirements either, although of course it is essential for the components which define the time-constant to be very stable and for the capacitor to have very low leakage. The main disadvantage is that the system does not easily provide a digital output and the future trend for use in inertial navigation systems incorporating digital computers is therefore more likely to be in favour of the digital integrator of the general type described in Section 3.3. Another disadvantage is that the system is difficult to test because the application of steady acceleration (e.g. known fractions of the gravitational field) would cause the velocity output rapidly to reach saturation. It would probably be necessary to incorporate a switch to change the circuit from the integrating mode to the conventional mode; this would be useful both for testing the accelerometer and as part of the alignment sequence of an inertial platform.

On the question of choice of accelerometer (damped or undamped), the damped type has a dynamic response which enables the system to provide a close approximation to integration over a wide frequency band, the circuit is inherently stable and errors due to vibration are small. The undamped accelerometer gives accurate integration over a narrower frequency band but wide enough for practical purposes, it requires stabilising circuits and is more sensitive to vibration although here again its performance is adequate for many applications.

9. *Conclusions.*

A preliminary analysis indicates that it should be practicable to modify the feed back path of an accelerometer circuit so that the output voltage is proportional to velocity, that is, an integrating process is incorporated in the loop. The feed-back path would consist of an electronic differentiating circuit incorporating a high-gain operational amplifier; in order to achieve adequate accuracy and stability combined with small deflection of the accelerometer pendulum, the forward path should include an electronic integrator, but the requirements for this integrator are not as severe as for an integrator operating outside the loop as in the conventional arrangement. Either a damped or an undamped type of accelerometer may be employed but the former will give greater accuracy and a better damped system.

The main advantage of incorporating integration within the accelerometer loop is that the indicated velocity should be less sensitive to amplifier drift than in most existing circuits. It is likely to find application, however, only in analogue systems; where the inertial navigation system incorporates a digital computer then a process which combines integration and digitisation (Section 3.3) is more likely to be employed.

Provision may be made for mixing in various correcting accelerations (e.g. gravity and Coriolis corrections) when the accelerometer is part of an inertial navigation system. It is suggested that switching may be included so that the circuit may operate either in the integrating mode or the conventional mode, the latter being useful for testing the accelerometer and as part of the alignment sequence of a stable platform.

LIST OF SYMBOLS

a	Applied acceleration (cm/sec^2)
C_d	Capacitance of input capacitor in differentiator (farads)
C_i	Capacitance of feed-back capacitor in integrator (farads)
D	Viscous frictional coefficient in accelerometer (dynes per cm/sec)
E	Additional electrical signal applied to integrating accelerometer, representing corrections, etc. (volts)
e_i	Applied acceleration, scaled to restoring coil volts
e_0	Acceleration equivalent of restoring force, scaled to restoring coil volts
e_e	$e_i - e_0$
G	Overall transfer function of integrating accelerometer. $G = v_0/e_i$
h_a	Transfer function of accelerometer, measured as ratio of pick-off voltage to restoring coil voltage
h_e	Transfer function of electronic circuit in forward path of integrating accelerometer
J	Transfer function of electronic circuit in feed-back path of integrating accelerometer
K_2	Gain of the amplifier in the differentiator
K_n	Gain of last stage of the amplifier in the integrator (used for leakage compensation)
M	Mass of accelerometer pendulum (*gm*)
n	Ratio of R/R_c
p	Differential operator
R	Feed-back resistance in differentiator (ohms)
R_3	Input resistance for signal E (ohms)
R_c	Leakage resistance of capacitor C_d (ohms)
R_n	Input resistance for leakage compensation signal (ohms)
r	Length of pendulum from hinge to centre of gravity (cm)
S	Stiffness of pendulum arm (dynes/cm)
T_1	Time-constant associated with accelerometer (seconds)
T_2	Time-constant of differentiator (seconds) $T_2 = C_d R$
T_c	Time-constant of differentiator capacitor (seconds) $T_c = C_d R_c$
v_0	Output voltage of integrating accelerometer, representing velocity (volts)
x	Displacement of pendulum with respect to accelerometer case (cm)
δ	Angular deflection of the pendulum per g (rad/g)
Ω	The definition of Ω^2 is that it is the strength of the feed-back in the accelerometer loop, measured as the acceleration equivalent of the restoring force per unit displacement of the pendulum. Ω^2 is measured in cm/sec^2 per cm. Ω is shown to be very nearly equal to the natural angular frequency of the closed-loop, measured in radians/second.

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APPENDIX I

Numerical Value of some Important Parameters of Typical Accelerometers.

(a) *Undamped accelerometer Kearfott/Ferranti – Type* 429037–3*A*.

The hinge stiffness (i.e. the slope of the applied acceleration against pendulum deflection curve on open loop) is about 5*g*/rad.; conversely the compliance is 0·2 rad./*g* or 690 arc min/*g*. The natural frequency of the pendulum on open loop is about 4·5 *c/s*. With an excitation voltage of 6 volts 20 *Kc/s* the pick-off sensitivity is 1 mV/arc sec but the excitation is often increased to 18 volts giving a sensitivity of 3 mV/arc sec. The scale factor of the restoring coil is 5 mA/*g*, the resistance of the coil being 650 ohms; a modified low-impedance model has a scale factor of 7·5 mA/*g* and a coil resistance of 215 ohms.

With 6 volts excitation the 5 mA/*g* model is usually employed with an amplifier having a gain (expressed as the ratio of restoring coil current to pick-off voltage) of 2·5 mA/mV. Hence at an acceleration of 1*g* the pick-off voltage is 2 mV and the corresponding pendulum deflection is 2 arc seconds. The range of measurement is usually $\pm 10g$. A read-out resistor of 200 ohms is usually employed in the restoring circuit so that the output scale factor is 1 volt/*g*. If suitable shaping networks are included in the amplifier the resonant frequency of the closed loop system is about 350 *c/s* with a magnification at resonance of 2·5 (corresponding to about 0·21 critical damping). With 18 volts excitation and employing the 7·5 mA/*g* restoring coil the amplifier gain would be adjusted to 1·25 mA/mV, and expressed as a ratio of restoring coil voltage to pick-off voltage this is a voltage gain of 270. The range of measurement is usually $\pm 15g$.

A numerical value for the open-loop transfer function will now be calculated. As indicated in Section 2, the relation between open-loop pendulum deflection and applied acceleration is governed by the equation:

$$\left[p^2+\frac{D}{M}p+\frac{S}{M}\right] x = a.$$

Now in an undamped accelerometer the damping *D* may be neglected, and because the accelerometer is to be employed in a force feed-back loop the hinge stiffness term *S/M* may also be neglected since it makes only a very small contribution to the system response compared with the equivalent stiffness provided electro-mechanically by the feed-back. Thus the open loop transfer function reduces to:

$$\frac{x}{a} = \frac{1}{p^2}.$$

If we measure acceleration in numbers of *g* instead of cm/sec^2 and measure deflection in radians instead of cm then the transfer function becomes

$$\frac{981}{rp^2} \text{ or } \frac{1}{\tau^2 p^2} \text{ rad./}g$$

where *r* is the pendulum length from the hinge to the centre of gravity in cm and

$$\tau^2 = \frac{r}{981}.$$

For this particular accelerometer, τ is calculated to be 0·07 seconds. If we multiply by the pick-off sensitivity we obtain the transfer function in terms of volts/*g*. In closed loop calculations it is convenient to express all quantities in the same units, such as volts, and we will therefore express acceleration in terms of the equivalent voltage across the restoring coil. For a scale factor of 7·5 *mA/g* in a 215 ohm coil this is

1·6 volts/g. The accelerometer transfer function may now be expressed in volts per volt, remembering that the pick-off sensitivity is 3 mV/arc sec or 620 volts/radian. It becomes $\frac{1}{(0{\cdot}07)^2 p^2}\,\frac{620}{1{\cdot}6} = \frac{1}{T_1^2\,p^2}$ pick-off volts per restoring coil volt where T_1 is 0·0036 seconds. This form of the transfer function is employed in the closed loop calculations of Section 5.

(b) *Damped accelerometer Kearfott Type* 2401.

The hinge stiffness is 2g/radian, corresponding to a compliance of 0·5 radian/g or 1700 arc min/g. With an excitation of 6 volts 4 *Kc/s*, the pick-off sensitivity is 0·1 mV/arc sec. The scale factor of the restoring coil is 5 *mA*/g, the coil resistance being 160 ohms (0·8 volts/g). The read-out resistor is usually 100 ohms so that the overall output scale factor is 0·5 volts/g. The accelerometer is usually employed with an amplifier having a gain of 12·5 *mA/mV*. Hence at an acceleration of 1g the pick-off voltage is 0·4 mV and the pendulum deflection is 4 arc seconds. Expressed as a ratio of restoring coil voltage to pick-off voltage the amplifier voltage gain is 2000. The range of the accelerometer is $\pm 20g$ corresponding to a pick-off voltage range of 8 mV and a pendulum deflection range of 80 arc seconds.

In closed-loop operation the behaviour of the accelerometer itself is dominated by the viscous damping forces so that we may take the open-loop transfer function of the accelerometer to be:

$$\frac{x}{a} = \frac{M}{Dp}.$$

As before we shall measure acceleration in numbers of g and deflection in radians and the transfer function then becomes:

$$\frac{1}{\tau p}\ \text{rads.}/g$$

where for this particular accelerometer τ is calculated to be 53 seconds. If acceleration is expressed in terms of the equivalent voltage across the restoring coil (0·8 volts/g) the accelerometer transfer function may be written:

$$\frac{1}{T_1 p}$$

pick-off volts per restoring coil volt, where T_1 is 2 seconds.

APPENDIX II

Response of some Operational Amplifier Circuits.

A circuit having a given transfer function may often be obtained by means of a high-gain amplifier as shown in Fig. 2a, where the external input and feed-back impedances Z_1 and Z_2 are suitable combinations of resistors and capacitors. If the amplifier gain K is very large the voltage at the summing point (V_e) is very small so that this point becomes a virtual earth. Under these conditions the input impedance of the amplifier has only a small effect on the overall response and for simplicity it will be assumed to be infinite. In the general case illustrated in Fig. 2a the response is governed by the equations

$$\frac{V_i - V_e}{Z_1} + \frac{V_0 - V_e}{Z_2} = 0$$

and

$$V_0 = -KV_e.$$

Hence

$$\frac{V_i}{Z_1} + \frac{V_0}{Z_2} = -\frac{V_0}{K}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

or

$$-\frac{V_i}{V_0} = \frac{Z_1}{Z_2}\left(1 + \frac{1}{K}\right) + \frac{1}{K}.$$

Now $1/K$ may be neglected compared with unity so that the transfer function becomes

$$\frac{V_0}{V_i} = \frac{-1}{Z_1/Z_2 + 1/K}.$$

As K approaches infinity the transfer function tends to

$$\frac{V_0}{V_i} = -\frac{Z_2}{Z_1}.$$

If Z_1 is a resistor R and Z_2 is a capacitor C_i as in Fig. 2b, the transfer function is

$$\frac{K}{1 + K C_i R p} \rightarrow \frac{1}{C_i R p} \text{ as } K \rightarrow \infty.$$

This is the Miller integrator circuit. The modified integrator circuit of Fig. 2c has the transfer function

$$\frac{1}{\dfrac{R_1}{R_2 + 1/pC_i} + \dfrac{1}{K}} \rightarrow \frac{R_2}{R_1} + \frac{1}{C_i R_1 p}.$$

Fig. 2d is the circuit of a practical differentiator having transfer function

$$\frac{1}{\dfrac{1}{C_d R_2 p}+\dfrac{1}{K}} = \frac{C_d R_2 p}{1+C_d R_2 p/K} \rightarrow C_d R_2 p.$$

The modified differentiator of Fig. 2e has transfer function

$$\frac{1}{\dfrac{R_1}{R_2(1+C_d R_1 p)}+\dfrac{1}{K}} \rightarrow \frac{R_2}{R_1}+C_d R_2 p.$$

APPENDIX III

Derivation of Transfer Function of Integrating Accelerometer System Incorporating a Viscous Damped Accelerometer

The circuit is shown in Fig. 6. The circuit of the modified integrator in the forward path is shown in Fig. 2c and its transfer function is

$$\frac{1}{\dfrac{R_1}{R_2+1/pC_i}+\dfrac{1}{K_1}} \rightarrow \frac{R_2}{R_1}+\frac{1}{C_i R_1 p} \text{ as } K_1 \rightarrow \infty.$$

The components R_1, R_2 and C_i are chosen so that

$$\frac{R_2}{R_1} = \frac{1}{K_2} \text{ and } C_i R_1 = T_2$$

where T_2 and K_2 are respectively the time-constant and amplifier gain of the differentiator in the feedback path. The buffer amplifier in the forward path is designed to have a gain equal to $T_1 K_2/T_2$ so that the accurate transfer function of the buffer amplifier and modified integrator is

$$\frac{T_1 K_2}{T_2} \frac{1}{\dfrac{1}{\dfrac{1}{K_2}+\dfrac{1}{pT_2}}+\dfrac{1}{K_1}} =$$

$$= \frac{1}{\dfrac{T_2}{T_1 K_2} \dfrac{pT_2}{1+pT_2/K_2}+\dfrac{T_2}{K_1 T_1 K_2}}.$$

Hence the complete loop transfer function is

$$\frac{e_0}{e_e} = \frac{1}{pT_1} \frac{1}{\dfrac{T_2}{T_1K_2}\dfrac{pT_2}{1+pT_2/K_2}+\dfrac{T_2}{K_1T_1K_2}} \frac{pT_2}{1+pT_2/K_2}$$

$$= \frac{1}{\dfrac{pT_2}{K_2}+\dfrac{1+pT_2/K_2}{K_1K_2}}.$$

Thus the closed-loop transfer function is

$$\frac{e_0}{e_i} = \frac{1}{1+\dfrac{pT_2}{K_2}+\dfrac{1+pT_2/K_2}{K_1K_2}}$$

$$= \frac{1}{\left(1+\dfrac{1}{K_1K_2}\right)\left(1+\dfrac{pT_2}{K_2}\right)}.$$

The output voltage v_0 is given by

$$\frac{v_0}{e_i} = \frac{1}{pT_2(1+1/K_1K_2)}.$$

Since $K_1 = 1000$ and $K_2 = 2000$, the effect of the finite value of K_1 for the gain of the amplifier of the modified integrator in the forward path is merely to increase the effective overall integration time-constant T_2 by 0·00005 per cent, which is negligible. It is therefore appropriate in all calculations to employ the limiting form of the transfer function of the modified integrator appropriate to very large values of K_1, namely

$$\frac{R_2}{R_1}+\frac{1}{C_i R_1 p}.$$

APPENDIX IV

The Effect of the Leakage Resistance of the Capacitor in the Differentiator.

In practice the input capacitor of the differentiator in the feed-back loop has a finite leakage resistance, and although this resistance may be very high for plastic dielectrics it may still have a significant effect which has to be compensated by circuit changes.

The circuit of the differentiator including the capacitor leakage resistance R_c is similar to that of Fig. 2e. As shown in Appendix II, its transfer function is

$$\frac{1}{\dfrac{R_c}{R(1+C_d R_c p)}+\dfrac{1}{K}}$$

where R is the feed-back resistance across the amplifier. For large values of the amplifier gain K the transfer function becomes

$$\frac{R/R_c+C_d Rp}{1+C_d Rp/K}.$$

Thus a constant fraction R/R_c of the output v_0 of the integrating accelerometer is fed back to the accelerometer restoring coil in addition to the differentiating term $C_d Rp$ and the result is that the output has a component proportional to acceleration as well as that proportional to velocity. A block diagram of the system is shown in Fig. 7a, where n has been written for the ratio R/R_c. The loop transfer function is now given by

$$\frac{e_0}{e_e}=\frac{1}{pT_1}\cdot\frac{T_1K_2}{T_2}\cdot\frac{1+pT_2/K_2}{pT_2}\cdot\frac{n+pT_2}{1+pT_2/K_2}$$

$$=\frac{K_2}{pT_2}\cdot\frac{n+pT_2}{pT_2}.$$

The closed-loop transfer function is given by

$$\frac{e_0}{e_i}=\frac{1}{1+\dfrac{pT_2}{K_2}\cdot\dfrac{pT_2}{n+pT_2}}$$

and the transfer function from applied acceleration to output voltage v_0 is

$$\frac{v_0}{e_i}=\frac{\dfrac{1+pT_2/K_2}{n+pT_2}}{1+\dfrac{pT_2}{K_2}\cdot\dfrac{pT_2}{n+pT_2}}=\frac{1+pT_2/K_2}{n+pT_2(1+pT_2/K_2)}$$

$$=\frac{1}{\dfrac{n}{1+pT_2/K_2}+pT_2}.$$

The effect of a non-zero value of n is appreciable only at low frequencies, so that we may write

$$\frac{v_0}{e_i} = \frac{1}{n+pT_2} = \frac{1}{pT_2} \cdot \frac{pT_c}{1+pT_c}$$

where T_c is the capacitor time-constant CR_c. Thus the ratio of indicated velocity to true velocity is given by

$$\frac{v_0}{e_i/pT_2} = \frac{pT_c}{1+pT_c}.$$

This means that a stored velocity leaks away with a time-constant T_c. For a capacitor with a good dielectric, T_c may be about 10^5 seconds (10^5 megohm insulation resistance for a 1 μF capacitor). Thus indicated velocity is tending to zero with a time-constant of about 27 hours (an initial rate of fall of 0·1 per cent in 1·7 minutes). Capacitors with time constants as high as 10^6 seconds are now becoming available.

If the inaccuracy due to the capacitor leakage is too large it may be compensated by a circuit modification consisting of feeding a suitable fraction of v_0, but with sign reversal, across the capacitor through a resistor into the virtual earth of the differentiating amplifier. A suitable signal could be obtained from the input of the final stage of the amplifier of the integrator in the forward path, provided this stage was gain stabilised, as shown in Fig. 7b. If the gain of the final stage is K_n and its input is taken through resistor R_n to the virtual earth of the differentiating amplifier, then a compensating current of $-v_0/K_nR_n$ is entering the virtual earth and this will balance the leakage current v_0/R_c if K_n and R_n are chosen so that

$$K_n R_n = R_c.$$

Compensation cannot be exact in practice because the leakage resistance R_c is not constant and moreover R_n itself being of very high value will not be very stable. Enough compensation may be achieved in practice however to increase substantially the effective value of the leakage time-constant T_c.

APPENDIX V

Mixing of Additional Acceleration Signals

In order to obtain the correct velocity it is often necessary to add electrically various correcting accelerations to the accelerometer signal. For example, in the vertical channel of a navigational system it is necessary to subtract the 1g acceleration due to gravity, and in all channels it may be necessary to add Coriolis correcting accelerations arising from the fact that the accelerometers are operating in rotating axes. This may be achieved as in Fig. 11b where the additional acceleration E has been added to the virtual earth of the amplifier in the feed-back path through a resistor R_3. If the voltage across the restoring coil is e_0, the effective open-loop gain of the amplifier is K_2 and the voltage at the virtual earth is e_g, then the performance of the feed-back path is governed by the equations

$$\frac{E-e_g}{R_3} + pC_d(v_0-e_g) + \frac{e_0-e_g}{R} = 0$$

and

$$e_0 = -K_2\, e_g.$$

Hence

$$\frac{E}{R_3}+pC_d v_0+\frac{e_0}{R} = -\frac{e_0}{K_2}\left\{\frac{1}{R_3}+pC_d+\frac{1}{R}\right\}$$

or

$$\frac{R}{R_3}\mathrm{E}+pC_dR\,v_0 = -e_0\left\{\frac{R}{K_2R_3}+\frac{pC_dR}{K_2}+\frac{1}{K_2}+1\right\} .$$

We may neglect $\frac{R}{K_2R_3}$ and $\frac{1}{K_2}$ compared with unity, so that if we write $C_dR = T_2$ and $\frac{R}{R_3} = m$, the expression becomes

$$e_0 = -\frac{pT_2\,v_0+mE}{1+pT_2/K_2}.$$

A block diagram showing the transfer functions of the various parts of the complete loop is given in Fig. 11c, a viscous damped accelerometer being assumed. The relation between the output voltage v_0 and the two input signals e_i and E may be derived from the equations:

$$v_0 = \frac{1}{pT_1}\,\frac{T_1K_2}{T_2}\,\frac{1+pT_2/K_2}{pT_2}(e_i-e_0)$$

and

$$e_0 = \frac{pT_2\,v_0+mE}{1+pT_2/K_2}.$$

Hence

$$\frac{pT_2}{K_2}\,\frac{pT_2}{1+pT_2/K_2}v_0 = e_i-\frac{pT_2\,v_0}{1+pT_2/K_2}-\frac{mE}{1+pT_2/K_2}.$$

Hence

$$pT_2\,v_0 = e_i-\frac{mE}{1+pT_2/K_2}.$$

Since the additional signals such as E are constants or slowly varying quantities the time-constant T_2/K_2 may be neglected (it is only 4 mS) and the output voltage v_0 representing velocity is given by

$$v_0 \simeq \frac{e_i}{pT_2}-\frac{mE}{pT_2}.$$

Thus both the applied acceleration e_i and the added signal E are integrated, as required, since m is a constant equal to R/R_3.

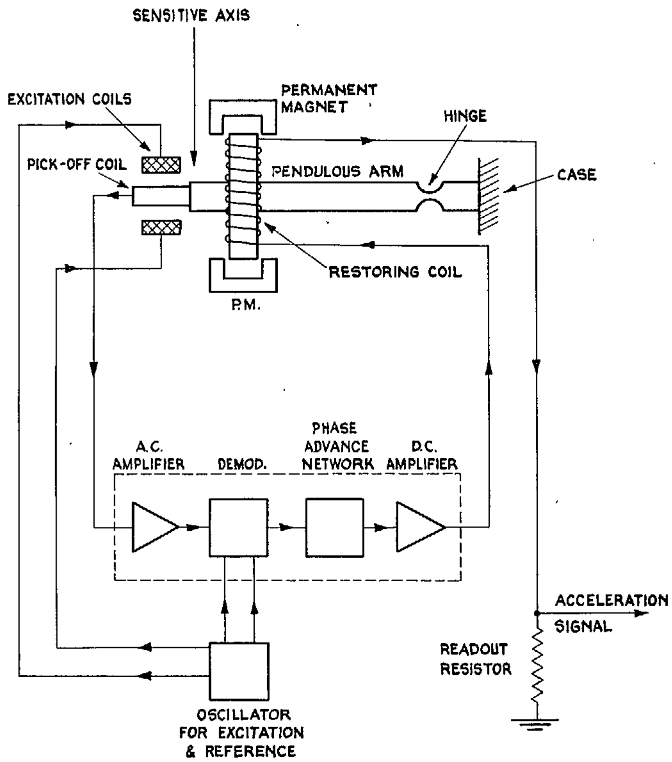


FIG. 1. Schematic of force feed-back accelerometer and associated circuit.

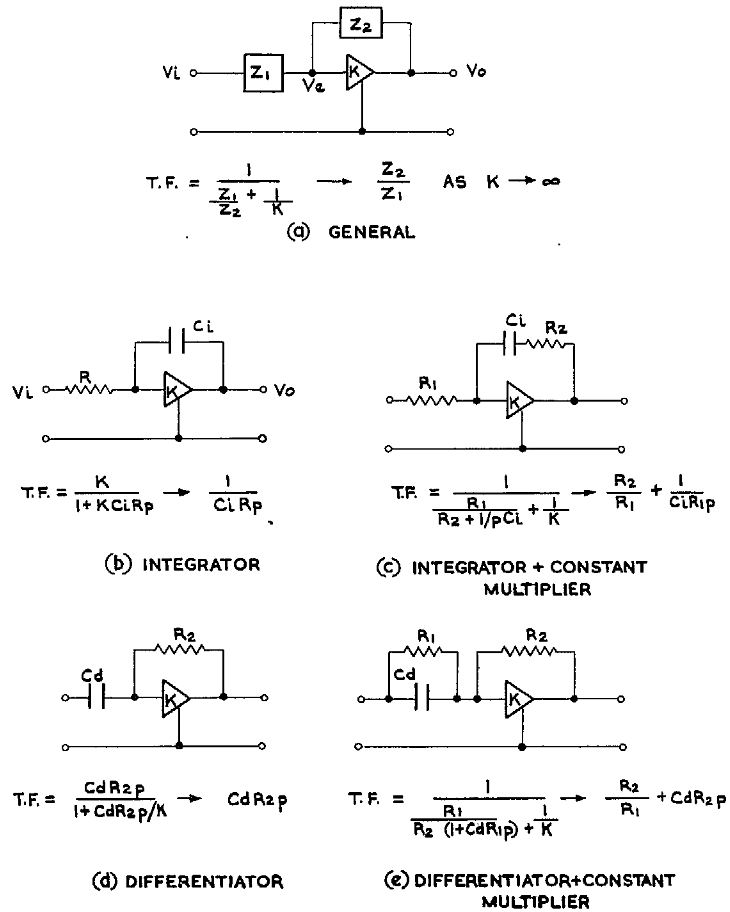
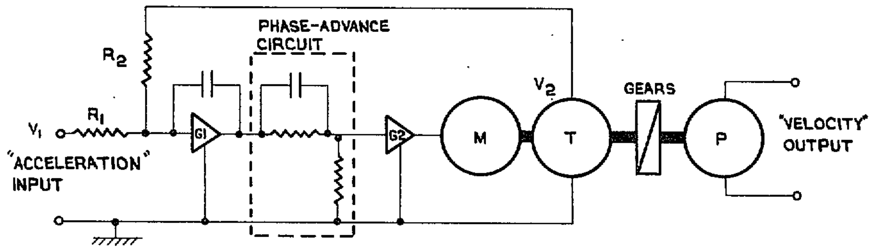


FIG. 2. Operational amplifier circuits.



G1 MILLER AMPLIFIER
G2 MODULATOR AND POWER AMPLIFIER
M TWO-PHASE SERVO MOTOR
T PRECISION TACHO-GENERATOR
P OUTPUT POTENTIOMETER

FIG. 3. Circuit of electro-mechanical integrator.

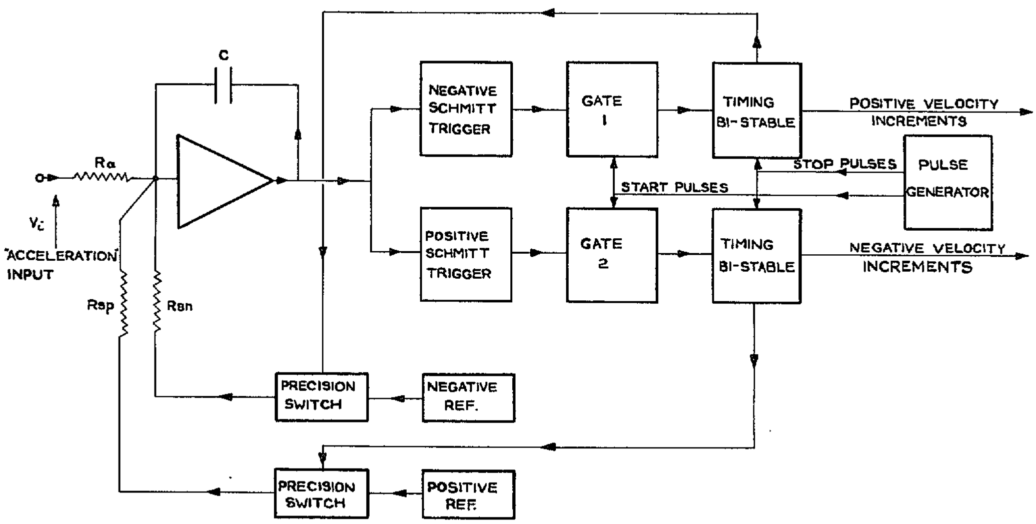


FIG. 4. Block Diagram of Digital integrator.

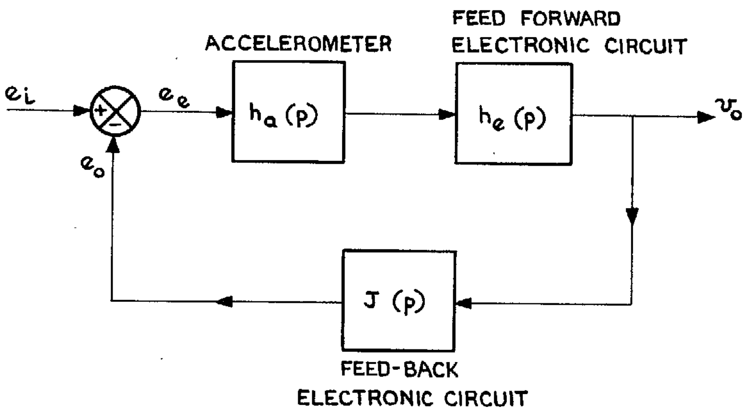


FIG. 5. General block diagram for force feed-back accelerometer.

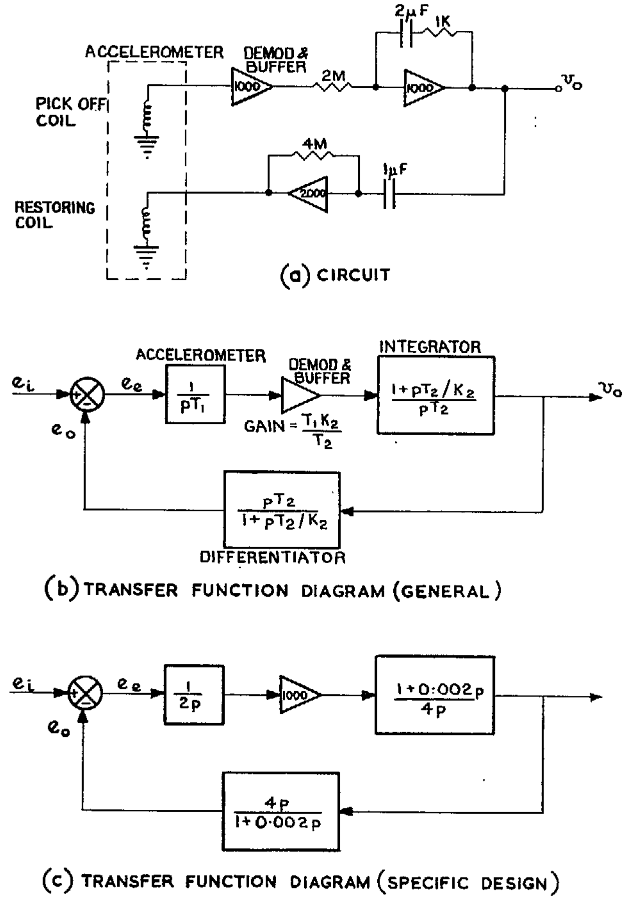


FIG. 6. Proposed circuit for integrating accelerometer, incorporating viscous damped accelerometer.

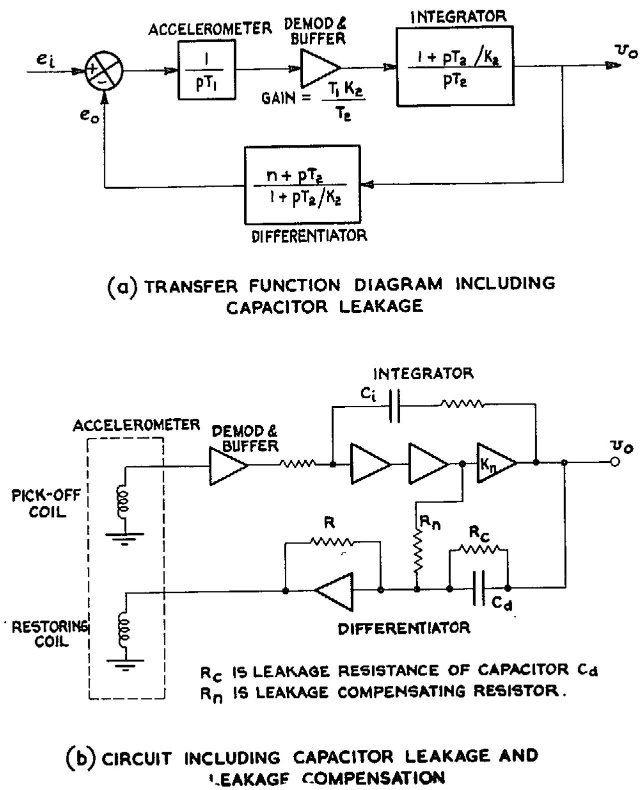


FIG. 7. Effect of capacitor leakage, and compensating circuit.

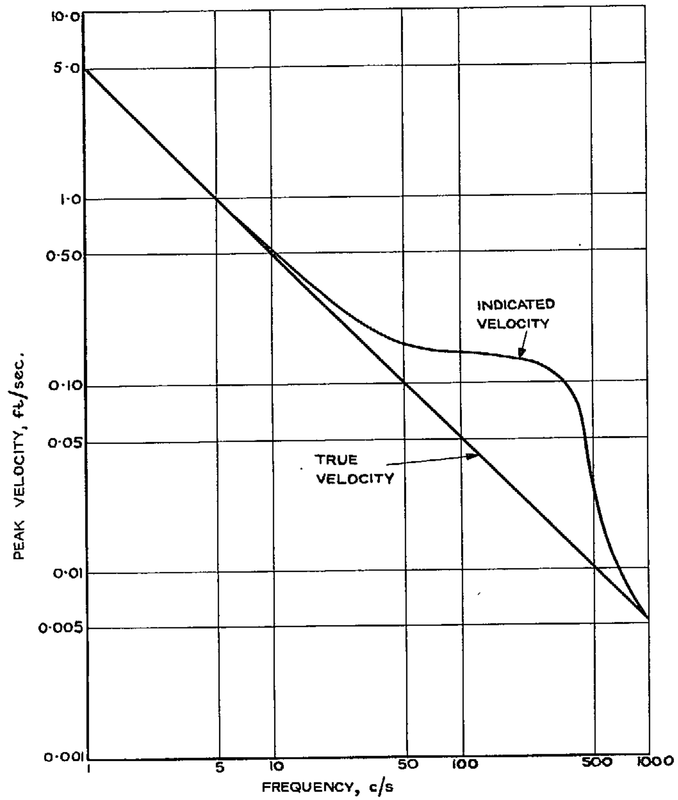


FIG. 8. Indicated velocity for 1g peak sinusoidal Acceleration (undamped accelerometer).

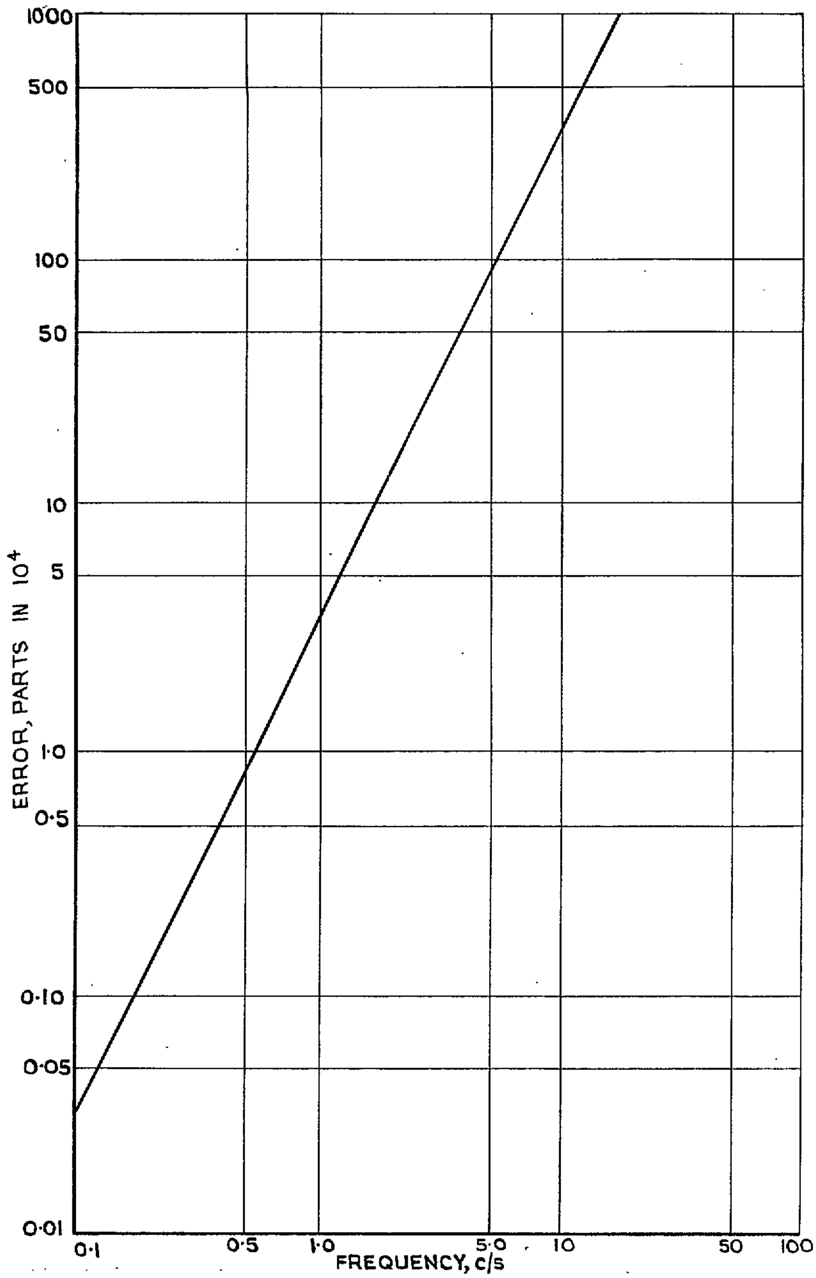


FIG. 9. Error in indicated velocity for sinusoidal acceleration (undamped accelerometer).

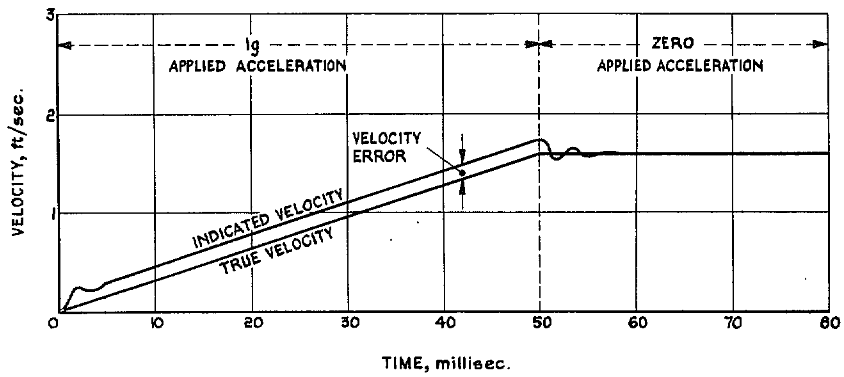


FIG. 10. Response to step applied acceleration. (Undamped accelerometer).

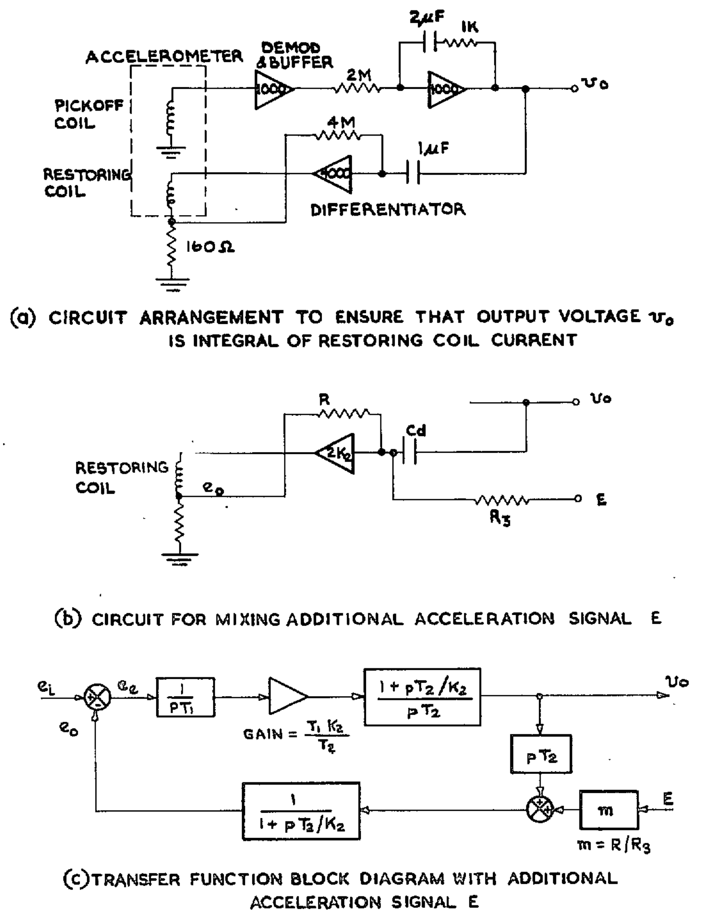


FIG. 11. Some additional circuit details.

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