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On some Tests Designed to Demonstrate Statistically the Required Mean Life

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By Z. A. Lomnicki

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Summary.

Two Life Tests recently suggested by various Government Agencies are described in sections 1 and 4 respectively and the consequences of their application are discussed. It is shown that to satisfy them with some reasonable probability the manufacturer has to produce equipment of much higher standard than initially stipulated.

Although the tests are meant to be designed so that the probability β of accepting an inferior equipment would be appropriately small, it is shown that the second test does not satisfy this condition leading to the acceptance of inferior equipment with much greater probability (section 7).

1. Lately it has been the practice of some Government Agencies in this country (Ref. 1) and in the U.S.A. (Ref. 2) to demand of the contractor to 'demonstrate statistically' the required Mean Time Between Failures (MTBF) θ_R at some confidence level $1 - \beta$ where β is a small number often referred to as the 'consumer's risk'. The suggested tests are based on the assumption that the failures form a Poissonian Process with a given failure rate $\lambda_R = 1/\theta_R$: a maximum admissible number c of failures in a given testing time T is specified and if the number of failures r in the time interval $(0, T)$ is less or equal to c the equipment is accepted but if c failures occur before the time T is attained, the equipment is rejected.

Since the plan accepts the equipment having the required MTBF only with probability β, it is of interest to the manufacturer to find out what are the consequences of such a test. Clearly, he cannot allow his equipment to be rejected with probability $1 - \beta$, and the only way of avoiding losses is to produce an equipment with high MTBF, say, $\theta^* > \theta_R$ which would enable him to have the equipment accepted with some reasonable probability $1 - \alpha$, where the small number α is referred to as the 'producer's risk'.

2. It is easy to find the value θ^* for a given test procedure and for a given α. Let $L(\theta)$ be the Operating Characteristic Curve of a given test, i.e. the probability of acceptance of the equipment if its MTBF is equal to θ. Since the rule accepts the equipment if the number r of failures in the time interval $(0, T)$ is not greater than c

$$L(\theta) = \sum_{r=0}^{c} (1/r\,!)e^{-T/\theta}(T/\theta)^r. \tag{1}$$

* Replaces A.R.C. 25 953.

The rule is devised in such a way that

$$L(\theta_R) = \beta \tag{2}$$

and, interpolating between the values given in the tables of Poissonian Distribution (cf. e.g. Ref. 3), the value θ^* can be found such that

$$L(\theta^*) = 1 - \alpha. \tag{3}$$

3. Since the proper interpolation between the values given in the tables of Poissonian Distribution can be tedious and lead to some errors it is more convenient to take into account the known fact (cf. Ref. 4, pp. 9-10) that

$$L(\theta) = \sum_{r=0}^{c} (1/r\,!)e^{-T/\theta}(T/\theta)^r = \int_{T/\theta}^{\infty} (e^{-x}x^c/c\,!)dx = \int_{2T/\theta}^{\infty} e^{-u/2}(u/2)^c(1/2c\,!)du. \tag{4}$$

The right-hand side is the probability that a chi-square variate with $2c + 2$ degrees of freedom is greater than $2T/\theta$. This allows us to use the tables of upper percentage points of this chi-square distribution (e.g. table 8 in Ref. 4). If we denote by $\chi_\beta{}^2(2c+2)$ the upper percentage point of this distribution then we have, from (2) and (4), the relation $2T/\theta_R = \chi_\beta{}^2(2c+2)$, i.e.

$$T = (\theta_R/2)\chi_\beta{}^2(2c+2). \tag{5}$$

To satisfy (3) we must have $2T/\theta^* = \chi_{1-\alpha}{}^2(2c+2)$, i.e.

$$\theta^* = \theta_R\chi_\beta{}^2(2c+2)/\chi_{1-\alpha}{}^2(2c+2). \tag{6}$$

Thus, for instance for $\alpha = \beta = 0{\cdot}10$, $c = 4$ and $\theta_R = 1000$ hours we find $\chi_{0\cdot1}{}^2(10) = 15{\cdot}9871$; from (5) $T = 7990$ hours and $\chi_{0\cdot9}{}^2(10) = 4{\cdot}86518$ so that from (6): $\theta^* = 3{\cdot}29.\ \theta_R = 3290$ hours. This means that the manufacturer has to produce the equipment which would have a mean life about $3{\cdot}3$ times longer than required in order to satisfy the acceptance rules of the test with probability 90%.

The above argument was used by Gorski and Epstein (Ref. 2) to evaluate tables giving the values θ^*/θ_R for $c = 1(1)10$; $\beta = 0{\cdot}01, 0{\cdot}05, 0{\cdot}1(0{\cdot}1)0{\cdot}5$; $\alpha = 0{\cdot}01, 0{\cdot}05, 0{\cdot}1(0{\cdot}1)0{\cdot}4$.

4. The results discussed above are applicable to the cases in which a sequential sampling is precluded. However, the tests suggested in this country (cf. e.g. Ref. 1) introduce a kind of sequential procedure. For a given β and a given θ_R a sequence of time moments is given $T_0, T_1, \ldots, T_r, \ldots$ where the values T_r satisfy (5); the equipment is accepted if:

<u>no</u> failure occurs in the time interval $(0, T_0)$,

or no more than <u>one</u> failure occurs in the interval $(0, T_1)$,

or no more than <u>two</u> failures occur in the interval $(0, T_2)$, etc.

....

no more than <u>r</u> failures occur in the interval $(0, T_r)$, etc., etc.

The rule is not well defined: we are told what is the acceptance rule but as regards the rejection the rules are rather vague: 'the test can be discontinued if it becomes evident that the reliability falls very far short'. This can be interpreted only as a rule to reject the equipment if it is not accepted, say, in N steps. Thus the test is well defined by a finite sequence $T_0, T_1, \ldots, T_N$, illustrated by the curve in Fig. 1.

If we represent the number of failures as (integer) ordinates and time as (continuous) abscissae (see Fig. 1) then any test outcome can be represented as a step line jumping one step up at the time of failure occurrence. The times $T_0, T_1, \ldots,$ constitute a boundary and the area to the right of this boundary forms the acceptance region. Thus the test outcome can be regarded as a Poissonian random walk with $\lambda = 1/\theta$ and the probability that the equipment will be accepted at the exactly rth step ($r = 0, 1, \ldots, N$) is equal to the probability that the 'first passage time' τ (which can only coincide with one of the times $T_1, T_2, \ldots$) will occur at T_r.

If $t_1 < t_2 < t_3 \ldots < t_r$ are ordered values of failure times then

$$\begin{aligned}
\mathrm{P}_r &= \text{Probability } (\tau = T_r) = \text{Prob. } (t_1 < T_0, t_2 < T_1, \ldots t_r < T_{r-1}, t_{r+1} > T_r) = \\
&= \int_0^{T_0} \lambda e^{-\lambda t_1}\, dt_1 \int_{t_1}^{T_1} \lambda e^{-\lambda(t_2-t_1)}\, dt_2 \ldots \int_{t_{r-1}}^{T_{r-1}} \lambda e^{-\lambda(t_r-t_{r-1})} e^{-\lambda(T_r-t_r)}\, dt_r = \\
&= e^{-\lambda T_r}\lambda^r \int_0^{T_0} dt_1 \int_{t_1}^{T_1} dt_2 \ldots \int_{t_{r-1}}^{T_{r-1}} dt_r .
\end{aligned}$$

Putting $x_{r-k} = \dfrac{t_k}{T_r}$ $(k = 1, 2 \ldots, r)$ and $d_{r-k} = \dfrac{T_k}{T_r}$ $(k = 0, 1 \ldots, r-1)$ and recalling that the Jacobian of this transformation is equal to $J = T_r^{\,r}$ we obtain:

$$P_r = e^{-\lambda T_r}(\lambda T_r)^r \int_0^{d_r} dx_{r-1} \int_{x_{r-1}}^{d_{r-1}} dx_{r-2} \ldots \int_{x_1}^{d_1} dx_0 . \tag{7}$$

The above integral appears very often in the study of negative exponential distributions [e.g. Ref. 5, formula (15)]. In the next section we shall evaluate it with the aid of a method developed by Daniels (Ref. 6).

5. Let us denote by B_r the integral appearing in (7) multiplied by $r\,!$, i.e.

$$B_r = r\,! \int_0^{d_r} dx_{r-1} \int_{x_{r-1}}^{d_{r-1}} dx_{r-2} \ldots \int_{x_1}^{d_1} dx_0 , \tag{8}$$

and define

$$B_r(x; d_1, d_2, \ldots, d_r) = r\,! \int_x^{d_r} dx_{r-1} \int_{x_{r-1}}^{d_{r-1}} dx_{r-2} \ldots \int_{x_1}^{d_1} dx_0 . \tag{9}$$

When the integrations are carried out in (9) it becomes a polynomial in x of rth degree with coefficients which are functions of the d's. This polynomial is sometimes referred to as the Goncharev Polynomial. Clearly

$$B_r(0; d_1, d_2, \ldots, d_r) = B_r , \tag{10}$$

$$B_r(d_r; d_1, d_2, \ldots, d_r) = 0 , \tag{10a}$$

and

$$dB_r(x; d_1, d_2, \ldots, d_r)/dx = -r\,! \int_x^{d_{r-1}} dx_{r-2} \ldots \int_{x_1}^{d_0} dx_0 = -rB_{r-1}(x; d_1, d_2, \ldots, d_{r-1}). \tag{11}$$

Polynomial (9) can be written explicitly as

$$B_r(x; d_1, d_2, \ldots, d_r) = a_r + a_{r-1}x + a_{r-2}x^2 + \ldots + a_1 x^{r-1} + a_0 x^r . \tag{12}$$

For $x = 0$ this polynomial has the value B_r so that $a_r = B_r$. It vanishes for $x = d_r$ so that d_r is its root. Similarly $d_{r-1}, d_{r-2}, \ldots$ are the roots of consecutive derivatives of this polynomial in view of (11) and (10a). From (9) the rth derivative of $B_r(x; d_1 \ldots d_r)$ is equal to $r\,!(-1)^r$.

Thus:

$$\begin{aligned}
B_r + a_{r-1}d_r + a_{r-2}d_r^2 + \ldots + \quad & a_1 d_r^{r-1} + a_0 d_r^r &= 0 \\
a_{r-1} + 2a_{r-2}d_{r-1} \quad + \ldots + (r-1) & a_1 d_{r-1}^{r-2} + r a_0 d_{r-1}^{r-1} &= 0 \\
\ldots \quad \ldots \quad & \ldots \quad \ldots & \\
(r-1)\,! & a_1 + r\,!\, a_0 d_1 &= 0 \\
& r\,!\, a_0 &= r\,!\,(-1)^r
\end{aligned} \tag{13}$$

This is a system of $(r+1)$ linear equations and, knowing the sequence $(d_1, d_2 \ldots d_r)$ we can find the $(r+1)$ coefficients $(a_r, a_{r-1}, \ldots a_0)$ of (12). This system has a unique solution since its determinant is equal to $2\,!\; 3\,! \ldots r\,! \neq 0$.

Instead of solving (13) and putting the a_i's into (12) we can verify directly that the polynomial

$$B_r(x; d, \ldots d_r) = r\,! \begin{vmatrix} 1 & x & x^2/2 & x^3/3\,! & \ldots & x^r/r\,! \\ 1 & d_r & d_r^2/2 & d_r^3/3\,! & \ldots & d_r^r/r\,! \\ 0 & 1 & d_{r-1} & d_{r-1}^2/2 & \ldots & d_{r-1}^{r-1}/(r-1)\,! \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & 1 & d_1 \end{vmatrix} \tag{14}$$

satisfies these $(r+1)$ conditions. Indeed substituting d_r for x we obtain two identical rows in the determinant (14) and (10a) is satisfied. Similarly, putting d_{r-1} for x into the derivative of (14) we obtain two identical rows and d_{r-1} is shown to be the root of the derivative of (14) etc., etc. The rth derivative of (14) is clearly equal to $r\,!\,(-1)^r$ as it should be.

From (10) putting $x = 0$ in (14) we find:

$$B_r = r\,! \begin{vmatrix} d_r & d_r^2/2\,! & d_r^3/3\,! & \ldots & d_r^r/\mathrm{r}\,! \\ 1 & d_{r-1} & d_{r-1}^2/2\,! & \ldots & d_{r-1}^{r-1}/(r-1)\,! \\ 0 & 1 & d_{r-2} & \ldots & d_{r-2}^{r-2}/(r-2)\,! \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 1 & d_1 \end{vmatrix} \tag{15}$$

6. It follows from (7) that

$$P_r = [e^{-\lambda T_r}(\lambda T_r)^r/r\,!]B_r.$$

provided that we put in (15), T_k/T_r for d_{r-k}.
Writing $A_k = T_k/\theta = T_k\lambda$ we have $d_{r-k} = A_k/A_r$ and

$$P_r = e^{-A_r}/A_r^r \begin{vmatrix} A_0/A_r & A_0^2/2A_r^2 & A_0^3/3\,!A_r^3 & \ldots & A_0^r/r\,!A_r^r \\ 1 & A_1/A_r & A_1^2/2A_r^2 & \ldots & A_1^{r-1}/(r-1)\,!A_r^{r-1} \\ 0 & 1 & A_2/A_r & \ldots & A_2^{r-2}/(r-2)\,!A_r^{r-2} \\ 0 & 0 & 1 & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 1 & A_{r-1}/A_r \end{vmatrix}$$

The general term of this determinant is equal to a_{ij} $(i, j = 1, 2, \ldots, r)$

$$a_{ij} = \begin{cases} A_{i-1}{}^{j-i+1} A_r{}^{i-j-1}/(j-i+1)! & \text{if} \quad j \geqslant i-1 \\ 0 & \text{if} \quad j < i-1 \end{cases}$$

Multiplying every jth column by $j!A_r{}^j$ and dividing every ith row by $(i-1)!A_r{}^{i-1}$ (which is equivalent to multiplying the whole determinant by $A_r{}^r r!$) we obtain a new determinant with terms

$$a_{ij}' = \begin{cases} \binom{j}{i-1} A_{i-1}{}^{j-1+1} & \text{if} \quad j \geqslant i-1 \\ 0 & \text{if} \quad j < i-1 \end{cases}$$

so that

$$P_r = e^{-A_r}/r! \begin{vmatrix} A_0 & A_0{}^2 & A_0{}^3 & \ldots & A_0{}^r \\ 1 & 2A_1 & 3A_1{}^2 & \ldots & rA_1{}^{r-1} \\ 0 & 1 & 3A_2 & \ldots & \binom{r}{2} A_2{}^{r-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & rA_{r-1} \end{vmatrix} \tag{16}$$

for $r = 1, 2, \ldots, N$.

Clearly $P_0 = e^{-A_0}$.

The probability of acceptance in the first N steps is equal to:

$$P(\theta) = P_0 + P_1 + \ldots + P_N \tag{17}$$

and, once the test is defined, this probability is a function of θ since the A's are the functions of θ, $A_k = T_k/\theta$.

7. The test described in section 4 has been suggested under the impression that it would 'demonstrate' the required MTBF with the confidence $1 - \beta$, if the T_r's were chosen according to formula (5). However, this impression is erroneous: since the manufacturer is allowed to continue the test if the equipment fails to be accepted at the kth step ($k < N$), the plan admits a variety of test outcomes which would be excluded in the schemes described in section 1.

This can be seen from the following example. For $\beta = 0{\cdot}10$ formula (5) gives the following values of T_r/θ_R

r	0	1	2	3	4	5	6
T_r/θ_R	2·3	3·9	5·3	6·7	8·0	9·3	10·6

The probabilities P_r of acceptance at the (exactly) rth step under the assumption that the mean life of the equipment is equal to the required θ_R are evaluated from (16) and given below:

r	0	1	2	3	4	5	6
P_r	0·1003	0·0466	0·0316	0·0222	0·0180	0·0146	0·0120

If $N = 6$, $P(\theta_R) = 0{\cdot}2453$.

Thus the equipment having exactly the required MTBF is accepted according to the suggested rules with probability 24·5%. When the MTBF is slightly lower the probability of acceptance β is slightly less than 24·5% and only when the MTBF is substantially lower will the probability of acceptance fall to 10% as intended by the author of the plan.† Although the application of this test is less exacting than that described in section 1, the problem still arises what would be θ^*, the MTBF of the equipment which would enable the manufacturer to have the equipment accepted with some reasonable probability $1 - \alpha$. It would be a tedious task to find the appropriate θ^* for a given α, but it is feasible: the operating characteristic function (17) can be evaluated with the aid of (16) for various values of θ until a value θ^* is found for which $P(\theta^*) = 1 - \alpha$.

In the way of illustration it may be shown that if $\theta = 2\theta_R$ or $3\theta_R$ (if the equipment has a mean life twice or three times longer than required) then

r	0	1	2	3	4	5	6
$P_r(2\theta_R)$	0·3166	0·1644	0·1102	0·0804	0·0615	0·0482	0·0384
$P_r(3\theta_R)$	0·4646	0·2096	0·1189	0·0725	0·0460	0·0298	0·0195

and if $N = 6$, $P(2\theta_R) = 0{\cdot}8197$, $P(3\theta_R) = 0{\cdot}9609$.

† The reason for this discrepancy is that the tests described in section 1 reject all the test outcomes represented (cf. Fig. 1) by the 'paths' which cross the horizontal line of c failures and accept those represented by the 'paths' crossing the vertical line $T = T_c$, whilst the test described in section 4 accepts additional outcomes as e.g. 'no failures in time interval $(0, T_c)$', 'no more than one failure in the interval $(0, T_1)$' etc.

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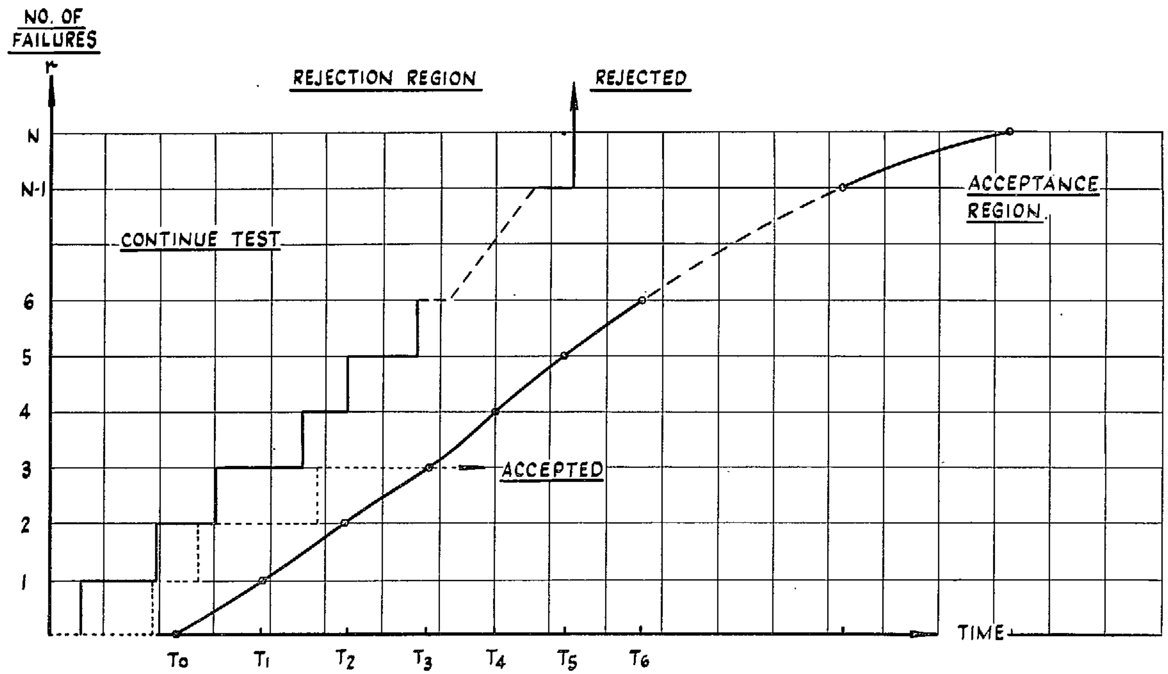

FIG. 1

(94246) Dd. 125874 K 5 12/66 Hw.

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