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Estimates of the Lift Reduction Due to Boundary Layer on Two-Dimensional Aerofoils

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Summary.

Experimental values of the reduction in lift curve slope due to boundary layer on two-dimensional aerofoils in incompressible flow are compared with estimates derived from the Royal Aeronautical Society Data Sheet and estimates calculated by Spence's theoretical method. The Data Sheet is shown to under-estimate the lift slope in some cases where transition is not fixed at the leading edge and it is suggested that this is due to the effect upon lift of the movement of transition position with incidence. It is proposed that the Data Sheet should be modified to indicate the limiting values between which the slope of the lift curve should lie. No more accurate method of assessing the effect of transition movement seems possible but it is thought that the uncertainty will be small in full scale practical cases.

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1. *Introduction.*

An aerofoil in viscous flow develops a boundary layer, which in general is thicker on the upper surface than the lower. The circulation about the aerofoil is reduced below the value which would be developed in inviscid flow; the reduction in circulation can be calculated if the boundary layer profile at the trailing edge of the aerofoil is known. Spence (Ref. 1) has developed a method of calculating this and hence predicting the loss of lift due to boundary layer from the aerofoil geometry alone. The basic theory applies to any aerofoil, but the method is at present only applicable to symmetrical aerofoils. It is assumed that transition from a laminar to a turbulent boundary layer occurs instantaneously at some point, with unchanged momentum thickness; the available calculation methods break down if the adverse pressure gradients on the aerofoil are large, which restricts the range of incidences for which estimates of the lift reduction can be made.

Although this calculation method is quite straightforward a much simpler method is needed for many practical purposes and this is provided by a Royal Aeronautical Society Data Sheet (Ref. 2). This is primarily based on an analysis of experimental results made by Garner (Ref. 3) and gives curves of the reduction in lift curve slope factor as a function of aerofoil trailing-edge angle for various Reynolds numbers and various assumed transition positions. The validity of some of these curves has been challenged; in particular Spence and Beasley (Ref. 4) have pointed out some discrepancy between results calculated by Spence's method and those derived from the Data Sheet for the same conditions. The comparison given in Ref. 4 is repeated here as Fig. 1 and it is seen that the discrepancy between the two sets of curves for some combinations of Reynolds number and transition position exceeds the quoted accuracy ($\pm 5\%$) of the Data Sheet. The present note explores the reason for this discrepancy: further calculations by Spence's method are presented and compared with experimental data, including those used in the construction of the Data Sheet, and proposals are made for a revision of the Data Sheet.

Although the loss of lift due to boundary layer is essentially a simple concept, some care is needed in defining terms and properly specifying transition conditions in both theoretical and experimental work. Some definitions are given in Section 2 and evidence of the effect of several variables on transition position and a discussion of the effect of transition position upon lift slope are given in Section 3. Three sets of experimental data are examined in Section 4 and compared with the predictions of the Data Sheet and with results calculated by Spence's method. The comparisons with experiments are summarized and interpreted collectively in the last part of Section 4. In Section 5 the difficulties inherent in any method of estimating lift reduction due to boundary layer are discussed and proposals are made for changes in the Data Sheet.

2. *Definitions.*

The slope of the lift coefficient curve with incidence is denoted by a_1. The value of a_1 calculated on the basis of the Kutta-Joukowski hypothesis is denoted by $(a_1)_T$. In Ref. 2 this quantity (there called $(a_1)_{0T}$) is shown as a function of thickness-chord ratio and trailing edge angle and all values of $(a_1)_T$ used in this Report have been read from the appropriate curve of Ref. 2.

The trailing edge angle, τ, is defined for a symmetrical section by:

$$\tau = 2 \tan^{-1}\{(y_{0\cdot 9c} - y_{0\cdot 99c})/0\cdot 09c\} \tag{1}$$

where c is the chord length of the aerofoil and $y_{0\cdot 9c}$ and $y_{0\cdot 99c}$ are the semi-thickness ordinates at positions 0·9 and 0·99 chord lengths from the leading edge respectively. For a cambered section τ is given by the trailing edge angle of the symmetrical section having the same thickness distribution.

The chordwise positions of transition measured from the leading edge are denoted by X_{T_u} and X_{T_l} for the upper and lower surfaces respectively; X_T without a lower suffix is the mean value of X_T and X_{T_l}.

Spence's method actually gives the lift reduction factor at a given incidence, that is $C_L/(C_L)_T$, where $(C_L)_T$ is the theoretical lift coefficient, based on the Kutta-Joukowski hypothesis. For a symmetrical aerofoil:

$$(C_L)_T = (a_1)_T \, \alpha, \tag{2}$$

where α is the incidence. $C_L/(C_L)_T$ can be expected to be a good approximation to $a_1/(a_1)_T$ only when the lift varies nearly linearly with incidence and is zero at zero incidence. Spence's method cannot, therefore, be used to estimate the lift curve slope for sections on which the transition at zero incidence occurs asymmetrically, thus producing lift, unless the loss of lift is calculated at more than one incidence.

3. *The Behaviour and Significance of Transition Position.*

The position of transition is of vital importance in determining the thickness of the boundary layer at the trailing edge and hence the lift reduction factor. As a guide to the behaviour of transition position some experimental results will be briefly considered here. Fig. 2 illustrates the movement of transition position with varying incidence for certain aerofoils and the effect on transition position of some other variables. Fig. 2a shows the forward shift of transition due to increased Reynolds number observed by Brebner and Bagley (Ref. 5) on an RAE 101 section of 10% thickness-chord ratio. Fig. 2b, which is due to Bryant, Halliday and Batson (Ref. 6), provides an example of the adverse effect of increased tunnel turbulence upon transition position, the NPL 7′ No. 2 tunnel being less turbulent than the 7′ No. 3 tunnel. The experimental evidence (Refs. 6, 7 and 8) to be discussed in Section 4.1 was mostly obtained on aerofoil models having trailing edge flaps set at zero deflection. The presence of a gap at the hinge was found to affect the position of transition; this is illustrated in Fig. 2c which is taken from Ref. 7. It is remarkable that the transition position is changed even when the hinge gaps on both surfaces are well within the region of turbulent flow.

On the evidence of Fig. 2 transition occurs further forward on the upper surface than on the lower surface except at zero incidence or when fixed by irregularities on the aerofoil surfaces. The difference in boundary layer thickness at the trailing edge produced by the asymmetry of the transition positions reduces the lift slope and this effect should be taken into account in any method of estimating lift slope when transition is not fixed. The Data Sheet does not make clear how transition position is defined but a study of the sources (Ref. 3) of the Data Sheet curves reveals that the quoted transition position refers to X_T, the mean of the positions of transition on the upper and lower surfaces. This definition does not distinguish between, say, a case with transition at mid-chord on both surfaces and, taking an extreme example, one with transition at the leading edge on the upper surface and at the trailing edge on the lower surface. Spence's method would give a very much lower value of $a_1/(a_1)_T$ for the latter case and although the accuracy of the method may not be established this result is certainly qualitatively correct. Fig. 3 has been prepared from the results of calculations made using Spence's method on an RAE 101 section of 10% thickness-chord ratio at an incidence of 2 degrees and a Reynolds number of 1 million; these results are tabulated in Ref. 4. Curves of constant $C_L/(C_L)_T$ are plotted against X_{T_u} and X_{T_l} and two sets of orthogonal lines have been drawn to represent constant values of X_T and χ, where χ is the difference between X_{Tl} and X_{T_u}. χ may be regarded here as a measure of the change of transition position with incidence while X_T is the position of transition at zero incidence, assuming that X_{T_u} decreases with incidence at the same rate as X_{T_l} increases. It is clear from Fig. 3 that $C_L/(C_L)_T$ changes slowly with χ constant but rapidly with X_T constant. On this evidence, the movement of transition with change of incidence controls the value of $a_1/(a_1)_T$ and hence is a more important parameter in estimating this ratio than the actual positions of transition at any one incidence.

4. *Comparisons between Experimental Lift Curve Slopes and Estimates from the R.Ae.S. Data Sheet and Spence's Method*

4.1. *RAE* 102 *and NPL sections.*

The Data Sheet curves are based largely on the results of wind tunnel tests at the National Physical Laboratory which were conducted to investigate, among other things, the slope of the lift curve of two-dimensional aerofoils. The results are reported in Refs. 6, 7 and 8 which refer respectively to tests on a family of original NPL sections of 15% thickness-chord ratio, an RAE 102 section of 10% thickness-chord ratio, and a similar section modified to have a cusped trailing edge, referred to as RAE 102C. A feature of most of the sections tested was the presence of flaps; these were found to affect the position of transition and the slope of the lift curve although set at zero deflection for the tests discussed here. In Fig. 2c, taken from Ref. 7, the variation of transition position with the location of the hinge is shown. Included in this figure is a curve for an aerofoil without flap; no details of the measured lift curve slope are available for this aerofoil although it is stated in Ref. 7 that the lift slope was similar to that of the aerofoil with the larger flap.

The five aerofoils of Ref. 6 differ from each other in respect of their trailing edge angles. Tests were made with transition either free or fixed by means of wires at a series of chordwise stations; values of lift slope are plotted against position of transition in Ref. 6. Fig. 4 shows $a_1/(a_1)_T$ plotted against the position of the transition wires for the five trailing edge angles; $a_1/(a_1)_T$ for the aerofoils without wires is also shown. Most of the points are based on several values, obtained for different flap sizes, which lay within $\pm 1\frac{1}{2}$ per cent of the values shown.

Uncertainty about the behaviour of transition in individual cases makes a detailed explanation of the variation of $a_1/(a_1)_T$ with the position of the transition wires impossible but consideration of the probable behaviour of lift slope on a hypothetical aerofoil may be instructive. Suppose we have an aerofoil on which transition occurs naturally at 50 per cent chord at zero incidence and at 40 per cent chord on the upper surface and at 60 per cent chord on the lower surface at an incidence of 2 degrees. If a_1 is estimated from the experimental values of $C_L/(C_L)_T$, measured at an incidence of 2 degrees, for a range of positions of symmetrically disposed transition wires then a curve similar to that of Fig. 5 should result. In the range AB transition is fixed symmetrically by the wires and a_1 should vary nearly linearly with transition movement; if transition did not occur except when forced by wires then a_1 should continue to vary linearly beyond B, as indicated by the broken line. Between B and C transition occurs naturally at 40 per cent chord on the upper surface, thus producing asymmetry and greater loss of lift which reaches a maximum at C. Further rearwards movement of the wires beyond C will not affect a_1 as transition occurs forward of the wires on both surfaces in the range CD. The relative values of a_1 at D and A were chosen arbitrarily.

The behaviour of $a_1/(a_1)_T$ in Fig. 4 may be seen to conform closely with the curve of Fig. 5. There is no significant difference between the values for the wing without wires and with wires aft of mid-chord which suggests that all these values are for similar transition conditions so that the changeover from fixed to free transition would occur somewhere between $0{\cdot}53c$ and $0{\cdot}65c$, supposing that the wires were being moved towards the trailing edge. The lift slopes have been estimated over a small incidence range in order to restrict transition movement but even a slight movement of transition is sufficient to establish asymmetric boundary layer conditions and so increase the loss of lift. This introduces a significant uncertainty in the quoted experimental results.

Experimental values of a_1 for the RAE 102 and RAE 102C sections are plotted against transition position in Ref. 7 and 8 respectively. Values of a_1 obtained with transition not forced by wires are included and are treated as cases with transition occurring well aft, the lift slope being estimated over a very small range of incidence. But, as stated above, the small movement of transition that occurs, shown here in Fig.2c, cannot be ignored and the values obtained are for free transition and cannot be grouped with points obtained with fixed transition. In one or two cases the values of a_1 with transition free are rather higher than might be expected but as the incidence range has been restricted to $\pm 1\frac{1}{2}$ degrees these values must be very liable to experimental error. The points obtained with transition wires conform reasonably well with the model sketched in Fig. 5, when considered in conjunction with curves of free transition position.

There is very little increase in the values of a_1 between the points for wires at $0{\cdot}1c$ and those for wires further aft which suggests that transition was held at the wires on both surfaces only in the former case. An increase in the values of a_1 occurred in the results of Ref. 6 as the wires were moved aft of $0{\cdot}1c$ and the contrast between the two sets of results may be associated with the different turbulence levels in the tunnels in which the experiments were conducted; the experiments of Refs. 7 and 8 were conducted in a tunnel of higher turbulence level with the result that transition may have been held back to mid-chord over a smaller range of incidence.

Since the Data Sheet curves are based largely on the above experimental results it appears that the curves for transition nominally at mid-chord include the effect of transition movement. Spence's method, assuming symmetrical transition, can therefore be expected to give rather higher values of $a_1/(a_1)_T$ than those given by the Data Sheet and this may explain, at least partly, the discrepancy shown in Fig. 1a. Values of $a_1/(a_1)_T$ have been calculated by Spence's method for an RAE 101 section of 10 per cent thickness-chord ratio for plausible asymmetric transition conditions and are given as isolated points in Fig. 1a, they are seen to give better agreement with the Data Sheet curves than the curves calculated assuming symmetrical transition.

4.2. *RAE* 101 *Section.*

In Ref. 5 Brebner and Bagley give the results of wind tunnel tests on an RAE 101 section of 10 per cent thickness-chord ratio at Reynolds numbers of $1{\cdot}7 \times 10^6$ and $3{\cdot}4 \times 10^6$. Transition was either fixed by means of wires or allowed to occur naturally; in the latter case its position was observed by means of liquid film evaporation. The curves obtained for the movement of transition with incidence are reproduced here in Fig. 2a. Lift coefficients were calculated from normal and tangential force coefficients found by graphical integration of the measured pressure distributions. Incidence was measured by the movement of a light beam reflected from a small mirror on the wing and is stated to be accurate to about 0·01 degrees. Values of incidence and lift coefficient from Ref. 5 are reproduced here in Table 1, together with theoretical lift coefficients, reduction of lift factors, and transition positions. The experimental lift coefficients for a Reynolds number of $1{\cdot}7 \times 10^6$ are mean values between positive and negative incidences. The theoretical lift coefficients were calculated from equation (2) and the free transition positions read from Fig. 2a. The transition positions are therefore for the wing without wires except where transition is directly fixed by the wire, that is when the wire is upstream of the position of natural transition.

Values of the reduction-in-lift factor, $C_L/(C_L)_T$, from Table 1 are plotted against $(C_L)_T$ in Fig. 6. It can be seen that within the incidence range $-4° \leqslant \alpha \leqslant 4°$ $C_L/(C_L)_T$ remains roughly constant except in the case where transition is fixed on one surface only. The asymmetry of the transition positions at zero incidence in this case produces a positive lift and hence the lift reduction factor tends to infinity. Brebner and Bagley found the experimental lift coefficient at zero incidence to be 0·011. The variation in the value of $C_L/(C_L)_T$ does not imply that the lift curve is not straight; the scatter of the experimental values of C_L plotted against α is in fact no greater than that for the other cases of Fig. 6.

For the cases where $C_L/(C_L)_T$ is nearly constant horizontal lines have been drawn to represent the probable approximate values of $a_1/(a_1)_T$ in the range $-4° \leqslant \alpha \leqslant 4°$; these lines are biased in favour of the points for higher incidences which should be more accurate since any error in measuring either incidence or lift coefficient is independent of both quantities. The values obtained in this way are:

$$R = 1{\cdot}7 \times 10^6 \quad \begin{array}{ll} \text{Transition free} & : a_1/(a_1)_T = 0{\cdot}89 \\ X_{T_u} = X_{T_l} = 0{\cdot}15c & : a_1/(a_1)_T = 0{\cdot}92 \end{array}$$

$$R = 3{\cdot}4 \times 10^6 \quad \begin{array}{ll} \text{Transition free} & : a_1/(a_1)_T = 0{\cdot}89 \\ X_{T_u} = X_{T_l} = 0{\cdot}15c & : a_1/(a_1)_T = 0{\cdot}89. \end{array}$$

An increase in Reynolds number would be expected to produce an increase in the slope of the lift curve but in these experiments it appears that no significant increase occurred when transition was free, while with fixed transition the lift slope decreased slightly with an increase in Reynolds number. No explanation of this result can be offered, other than the supposition that it is due to unsuspected experimental error; it may be noted that the method of obtaining C_L from the observed pressures implies that the experimental error at $R = 1{\cdot}7 \times 10^6$ was greater than at $R = 3{\cdot}4 \times 10^6$.

Values of lift reduction factor have been calculated by Spence's method for the incidences and transition positions quoted in Table 1 and are shown by the square symbols in Fig. 6. The agreement with the lines representing the experimental lift reduction factor is seen to be good at small incidences. Large adverse pressure gradients at higher incidences reduce the validity of the theoretical results.

Values of $a_1/(a_1)_T$ have been estimated from the Data Sheet for those cases where $a_1/(a_1)_T$ should approximate to $C_L/(C_L)_T$ and are represented by broken lines in Fig. 6. In all cases the estimated values are lower than the experimental points, the discrepancy being greater for the two cases where transition is fixed symmetrically, which is consistent with the analysis of Section 4.1, although free transition would have only a small effect for $X_T = 0{\cdot}15c$.

4.3. NACA sections.

In Ref. 9 Riegels has collected together experimental data for a large number of NACA sections from several sources. Using experimental lift slopes from Ref. 9 and theoretical lift slopes estimated by the method of Section 2, values of the ratio $a_1/(a_1)_T$ have been computed and are plotted against tan $(\tau/2)$ in Fig. 7 for the five Reynolds numbers, ranging from 10^6 to 9×10^6. Also shown in Fig. 7 are a pair of solid lines representing the lift reduction factor estimated according to the Data Sheet and a pair of broken lines representing this factor estimated by Spence's method. The former were computed by the formula on which the curves of the Data Sheet are based, given by Garner in Ref. 3. The lower curve is for $X_T = 0$, thus transition is at the leading edge on both surfaces; the upper curve is for $X_T = 0{\cdot}5c$, thus the mean position of upper and lower surface transition is at mid-chord. The broken lines were drawn from interpolation between the appropriate curves of Fig. 1, which were based on calculations made for RAE 101 sections of varying thickness-chord ratios at an incidence of 2 degrees. The lower curve is for transition at the leading edge and the upper is for transition at mid-chord on both surfaces. There is, then an important difference between the upper curves of each pair in that one is for transition fixed symmetrically while the other is for transition free and, on the analysis of Section 4.1, based on experiments in which transition did in fact move. The difference between the two curves is not therefore a cause for concern as the lift reduction with transition fixed must be expected to be less than that with transition free.

Much of the data used in Fig. 7 was derived from cambered sections. The Data Sheet does not specifically exclude cambered sections but Spence's method in its present form is applicable only to symmetrical sections. In Ref. 7 it is shown that the lift slope of an RAE 102 section of 10% thickness-chord ratio was increased by the addition of camber. But a wider study of the effect of camber, based on families of cambered sections from Ref. 9, shows that camber has no consistent effect upon lift slope, in some cases increasing and in others reducing it. Therefore to simplify the comparison with the estimated lift reduction factors Fig. 8 has been prepared using only data from symmetrical sections, all of which are also used in Fig. 7.

There is some inconsistency in the estimation of lift curve slope between the several sources of the data given in Ref. 9. A revised tunnel wall constraint correction was used in some cases and the slope of the lift curve was estimated sometimes at the design incidence and sometimes over the range between zero incidence and the design incidence. In Ref. 10 experimental lift curve slopes are given for 15 NACA aerofoils, some of them cambered; the slopes were estimated in a consistent way from data which were all corrected for tunnel wall constraint by the most recent method. Lift reduction factors have been calculated from the experimental lift slopes and are plotted against tan $(\tau/2)$ in Fig. 9. Comparison with Fig. 7 shows that the general distribution of values of $a_1/(a_1)_T$ is hardly altered by the removal of all points except those based on data from Ref. 10. There seems no reason, therefore, to discredit the general impression given by Fig. 7, and hence Fig. 8 also, on account of inconsistency between the sources of data.

The scatter of points apparent in Fig. 8 at all Reynolds numbers could be due to one, or a combination, of the following factors:

(i) Experimental error.

(ii) Variation of transition position in cases without nose roughening.

(iii) Dependence of the lift reduction factor on parameters additional to the trailing edge angle.

Experimental results are shown in Fig. 8 for a Reynolds number of 6×10^6 for aerofoils with and without leading edge roughening. There is seen to be a marked reduction in the amount of scatter in the case with leading edge roughening, suggesting that much of the scatter in the case with free transition is due to variations in the behaviour of transition between the different aerofoils. The effect of leading edge roughening upon $a_1/(a_1)_T$ in individual cases is obscured by the experimental error in this factor, which may well be greater than the variation due to transition movement. In Fig. 10 the change in lift reduction factor, $\delta[a_1/(a_1)_T]$, associated with leading edge roughening is plotted against tan $(\tau/2)$ for each section of Fig. 8. Also shown is a curve of the difference between the curves for $X_T = 0$ and $X_T = 0{\cdot}5c$ derived from the Data Sheet. It may be seen that although there is a wide scatter amongst the experimental points the Data Sheet curves give a reasonable estimate of the average change of slope; values of X_T for the experimental cases are not available but Fig. 2 suggests that $0{\cdot}5c$ is of the right order for this Reynolds number and turbulence level. The actual values of $a_1/(a_1)_T$ given by the Data Sheet at this Reynolds number and at 3×10^6 and 9×10^6 seem, however, to be too low. The value of X_T is unlikely to be much greater than $0{\cdot}5c$ so that the curves for $X_T = 0{\cdot}5c$ might be expected to be a rough upper bound of the experimental points; the curves for $X_T = 0$ need not necessarily be the lower bound as an aerofoil with transition near the leading edge on the upper surface and further aft on the lower surface may have a smaller lift slope than the same aerofoil with transition at the leading edge on both surfaces. Examination of Figs. 8e, 8f and 8h shows the opposite state of affairs in that no points fall significantly below the curve for $X_T = 0$ while a large number lies above the curve for $X_T = 0{\cdot}5c$. On the other hand the curves estimated by Spence's method bound nearly all the points but this is not entirely satisfactory as the upper curve, being for transition fixed symmetrically, might be expected to be some way above all the experimental points.

At Reynolds number of 10^6 and 2×10^6 lift curve slopes were available for only two symmetrical sections, both of 9 per cent thickness-chord ratio. A comparison between the figures including and the figures excluding cambered sections at higher Reynolds numbers shows no marked overall change in the distribution of experimental points; thus it seems reasonable, in the absence of sufficient data for symmetrical sections, to compare the calculated curves with experimental points that include values for cambered, sections. Fig. 7 rather than Fig. 8, will therefore be considered, bearing in mind that the scatter of points may be increased by the different effects of camber present. At Reynolds numbers of 10^6 and 2×10^6 there is a marked contrast with higher Reynolds numbers in the relative success of the two methods of estimating $a_1/(a_1)_T$. When leading edge roughening is present, Figs. 7b and 7d, the Data Sheet method provides a good estimate of $a_1/(a_1)_T$ while Spence's method overestimates it. For free transition Spence's method again overestimates while the Data Sheet curves appear not to make sufficient allowance for variation of transition position. In Ref. 10 a sharp increase in the rate of change of lift curve slope with Reynolds number was noted as the Reynolds number fell below 3×10^6; there is nothing in Spence's method to reproduce this effect so that when it occurs experimentally this method cannot give good agreement at Reynolds numbers both above and below 3×10^6.

4.4. *Interpretation of comparisons with experimental results.*

The analysis of the NPL tests in Section 4.1 suggested that the Data Sheet includes the effect of transition movement in the curves for $X_T = 0{\cdot}5c$. Spence's method agrees well with the Data Sheet (which itself naturally agrees with the NPL experiments) for $X_T = 0{\cdot}5c$, assuming free transition, but gives rather higher values of $a_1/(a_1)_T$ for transition at the leading edge. The comparison with the experimental results of Brebner and Bagley, which do not include cases with transition fixed at the leading edge, showed that Spence's method agreed well at both Reynolds numbers considered and with fixed or free transition, whereas the Data Sheet gave reasonable agreement only with free transition. The comparison with the NACA experimental results showed that the Data Sheet gave good agreement at Reynolds numbers of 10^6 and 2×10^6 when transition was fixed at the leading edge but predicted too small a range of values for free transition. Spence's method gave poor agreement, particularly at the lower Reynolds number.

At higher Reynolds numbers the Data Sheet gave reasonable predictions when transition was fixed at the leading edge but again predicted too small a range of values for free transition. Spence's method agreed well with experiment, particularly in respect of the range of values with free transition.

Together these results suggest that the Data Sheet gives reasonable estimates when transition is fixed at the leading edge but can predict too small a value of $a_1/(a_1)_T$ when transition is free. It was shown in Section 3 that the movement of transition with incidence is an important parameter in determining lift slope and it seems likely that this is the source of the variation in values of $a_1/(a_1)_T$ obtained experimentally with free transition, the Data Sheet giving the effect of transition movement only as measured in the NPL experiments. Spence's method generally gives good agreement when transition conditions are known and predicts the range of values well when the position of transition is uncertain but overestimates the lift slope at Reynolds numbers of 2×10^6 and below; no explanation of this last result can be offered.

5. *Difficulties in Estimating Reduction in Slope of Lift Curve and Suggested Modifications to Data Sheet.*

On the evidence of Section 4 reasonable estimates of reduction-in-lift factors can be obtained from the Data Sheet at all Reynolds numbers provided that transition is at or near the leading edge. For other transition conditions the Data Sheet seems in many cases to underestimate the reduction-in-lift factor and there is some evidence to suggest that this is due to the use of the mean transition position, X_T, rather than the difference between the transition position, χ, as a measure of the effect of transition. To estimate the additional reduction in lift curve slope due to movement of transition position requires a knowledge of both $dC_L/d\chi$ and $d\chi/d\alpha$. Empirical formulae for $dC_L/d\chi$ have been suggested by Bryant and Garner (Ref. 3 and 11) but do not adequately account for incidence or Reynolds number, both of which seem likely to affect the change in displacement thickness of the boundary layer at the trailing edge caused by a movement of transition and hence the value of $dC_L/d\chi$. However if more evidence of the variation of $dC_L/d\chi$ with lift slope, trailing edge angle, transition position and Reynolds number were available it might be possible to prepare curves which would enable more accurate values of $a_1/(a_1)_T$ to be estimated provided that $d\chi/d\alpha$ was known. The movement of transition with incidence must depend on several factors, including detailed pressure distribution, Reynolds number, turbulence level and surface roughness. It is possible that curves could be drawn to represent the effect on $d\chi/d\alpha$ of the first two of these factors, the detailed pressure distribution perhaps being replaceable by some geometrical feature such as trailing edge angle, but there seems no likelihood of taking into account turbulence level and surface roughness which together must be of fundamental importance. The most that it seems possible to do is to estimate the maximum variation, due to movement of transition position, of $a_1/(a_1)_T$ for a given section. Thus the highest possible values of $a_1/(a_1)_T$ would correspond to no change of transition position with incidence, that is for symmetrical transition on an uncambered section. The lowest possible value of $a_1/(a_1)_T$ is not so easily defined since the case where transition moves to the leading and trailing edges with the slightest change of incidence, besides not occurring in practice, would also produce a pronounced non-linearity in the lift curve near zero incidence. But the experimental evidence of Section 4 suggests that the value of $a_1/(a_1)_T$ for free transition occurring at mid-chord at zero incidence is greater than or equal to the value for transition fixed at the leading edge; this latter case might therefore be used to provide the lower limit for $a_1/(a_1)_T$. In the absence of reliable experimental data for transition fixed at mid-chord the curves calculated by Spence's method for transition at $0{\cdot}5c$ could be used as an upper bound.

We would then have two sets of curves, as in the present Data Sheet and shown in Fig. 1, but the curves for transition at $0{\cdot}5c$ would be calculated by Spence's method for transition fixed symmetrically. There is some evidence in Section 4 that the curve for a Reynolds number of 10^6 and transition at $0{\cdot}5c$ should be lowered, perhaps by about 0·03. The choice of these higher and lower bounds for estimates of $a_1/(a_1)_T$ results in a wide range of values but on the evidence of Section 4 this is no more than realistic. Transition on the upper surface is unlikely to occur far from the leading edge at full scale flight Reynolds numbers and may sometimes be fixed by a surface discontinuity associated with a nose flap or other high lift device; in these cases any variation of χ would arise from movement of the lower surface transition position alone and the estimate of $a_1/(a_1)_T$ could be biased towards the lower curve with some confidence.

Where the behaviour of transition position is not clearly defined it seems to be difficult to estimate the lift reduction factor, either from charts or by Spence's method, with sufficient accuracy to allow useful comparisons between different aerofoil sections to be made.

6. *Conclusions.*

Experimental values of the reduction in lift curve slope due to boundary layer effects are compared with estimates derived from the Royal Aeronautical Society Data Sheet and estimates calculated by the method of Spence. The Data Sheet is shown to provide reasonable estimates when transition is fixed at the leading edge but to be sometimes inaccurate for other transition conditions. Spence's method is also shown to work well for transition at the leading edge, except for Reynolds numbers below 3×10^6, and to give reasonably accurate estimates when transition occurs well aft, provided that the actual positions of transition at a given incidence are known. The interpretation of the experimental evidence is summed up in Section 4.4, and in Section 5 it is suggested that the failure of the Data Sheet to give accurate estimates for transition not at the leading edge is partly due to the use of the mean position of transition as a parameter rather than the difference between the positions of transition. There seems little hope of preparing general curves to predict either the movement of transition with incidence or the effect of transition movement upon lift. Changes to the Data Sheet are proposed that would give an indication of the limiting values between which the reduction of lift slope factor should lie for an aerofoil with transition not at the leading edge. The uncertainty would be small for aerofoils with transition near the leading edge on one or both surfaces; most full scale cases are in this category. Where the behaviour of transition position is not clearly defined it seems to be difficult to estimate the lift reduction factor, either from charts or by Spence's method, with sufficient accuracy to allow useful comparisons between different aerofoil sections to be made.

LIST OF SYMBOLS

a_1	Slope of lift curve
$(a_1)_T$	Slope of lift curve from Kutta-Joukowski hypothesis
c	Aerofoil chord
C_L	Lift coefficient
$(C_L)_T$	Lift coefficient from Kutta-Joukowski hypothesis
R	Reynolds number
t	Maximum thickness of aerofoil
x	A distance measured from the leading edge in a chordwise direction
X_{T_l}	Position of transition on the lower surface, measured from the leading edge
X_{T_u}	Position of transition on the upper surface, measured from the leading edge
X_T	$\frac{1}{2}(X_{T_u}+X_{T_l})$
α	Incidence
τ	Trailing edge angle, defined in Section 2
χ	$X_{T_l}-X_{T_u}$

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TABLE 1

$C_L/(C_L)_T$ and Transitions Positions from Reference 5

(a) $R = 3{\cdot}4 \times 10^6$

		Transition free				X_{T_u} free, $X_{T_l} = 0{\cdot}15c$				$X_{T_u} = X_{T_l} = 0{\cdot}15c$			
$\alpha°$	$(C_L)_T$	C_L	$C_L/(C_L)_T$	X_{T_u}/c	X_{T_l}/c	C_L	$C_L/(C_L)_T$	X_{T_u}/c	X_{T_l}/c	C_L	$C_L/(C_L)_T$	X_{T_u}/c	X_{T_l}/c
−2·20	−0·260	−0·232	0·892	0·58	0·35	−0·225	0·865	0·58	0·15	−0·232	0·892	0·15	0·15
−1·16	−0·137	−0·124	0·904	0·55	0·43	−0·116	0·846	0·55	0·15	−0·123	0·897	0·15	0·15
−0·12	−0·014	−0·009	—	0·50	0·50	0	—	0·50	0·15	−0·010	—	0·15	0·15
0·92	0·109	0·099	0·911	0·43	0·57	0·109	1·003	0·43	0·15	0·902	0·098	0·15	0·15
1·96	0·232	0·210	0·906	0·35	0·60	0·222	0·958	0·35	0·15	0·206	0·889	0·15	0·15
3·00	0·355	0·315	0·888	0·11	—	—	—	0·11	0·15	0·317	0·894	0·11	0·15
4·04	0·478	0·426	0·892	0·05	—	0·437	0·915	0·05	0·15	0·428	0·896	0·05	0·15

(*b*) $R = 1{\cdot}7 \times 10^6$

		Transition free				$X_{T_u} = X_T = 0{\cdot}15c$			
$\alpha°$	$(C_L)_T$	C_L	$C_L/(C_L)_T$	X_{T_u}/c	X_{T_l}/c	C_L	$C_L/(C_L)_T$	X_{T_u}/c	X_{T_l}/c
1·02	0·121	0·111	0·920	0·54	0·67	0·111	0·920	0·15	0·15
2·05	0·242	0·218	0·900	0·44	0·72	0·222	0·916	0·15	0·15
3·07	0·363	0·324	0·893	0·32	0·76	0·333	0·918	0·15	0·15
4·09	0·483	0·430	0·890	0·11	0·85	0·435	0·900	0·11	0·15

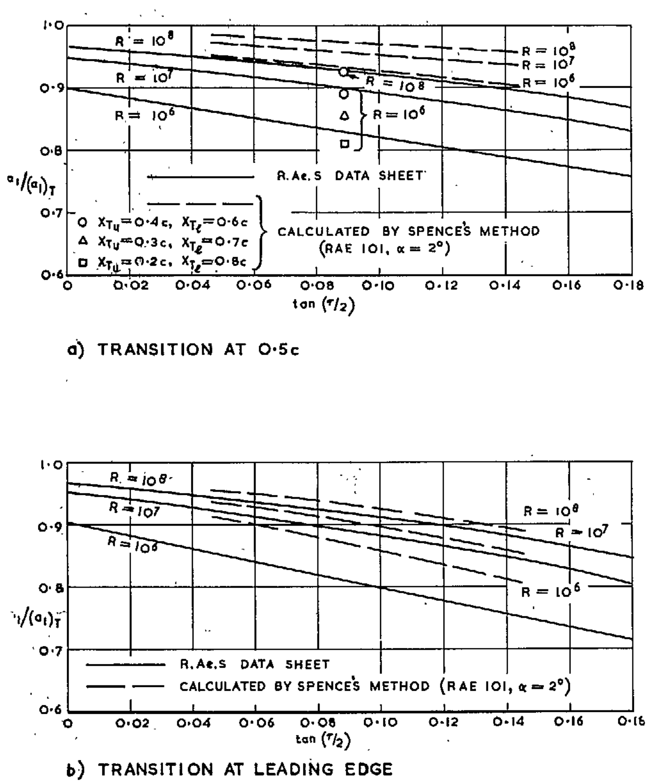

FIG. 1. Comparison between *R.AeS* Data Sheet wings 01.01.05 and curves calculated by Spence's method.

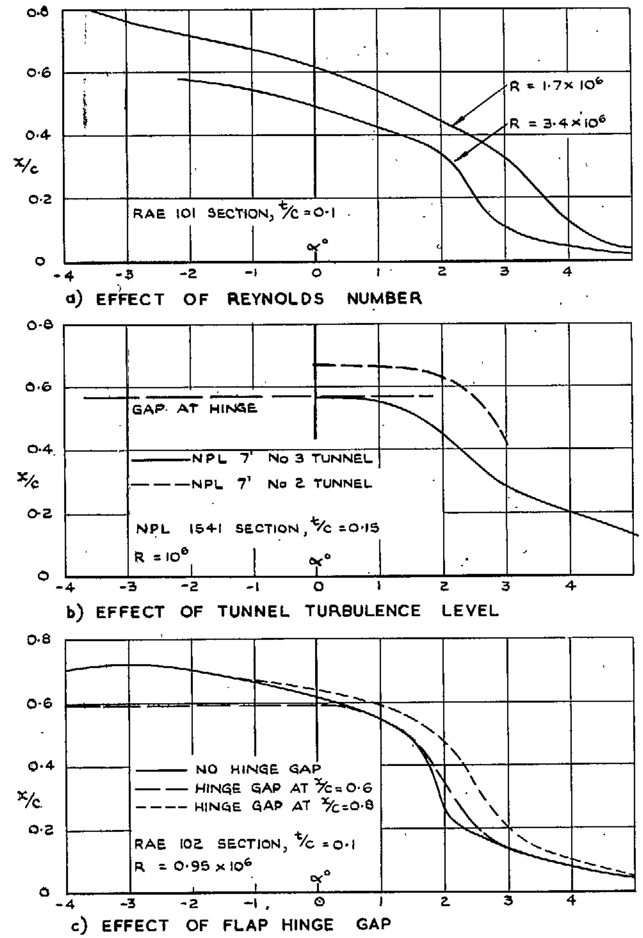

FIG. 2. Movement of transition position.

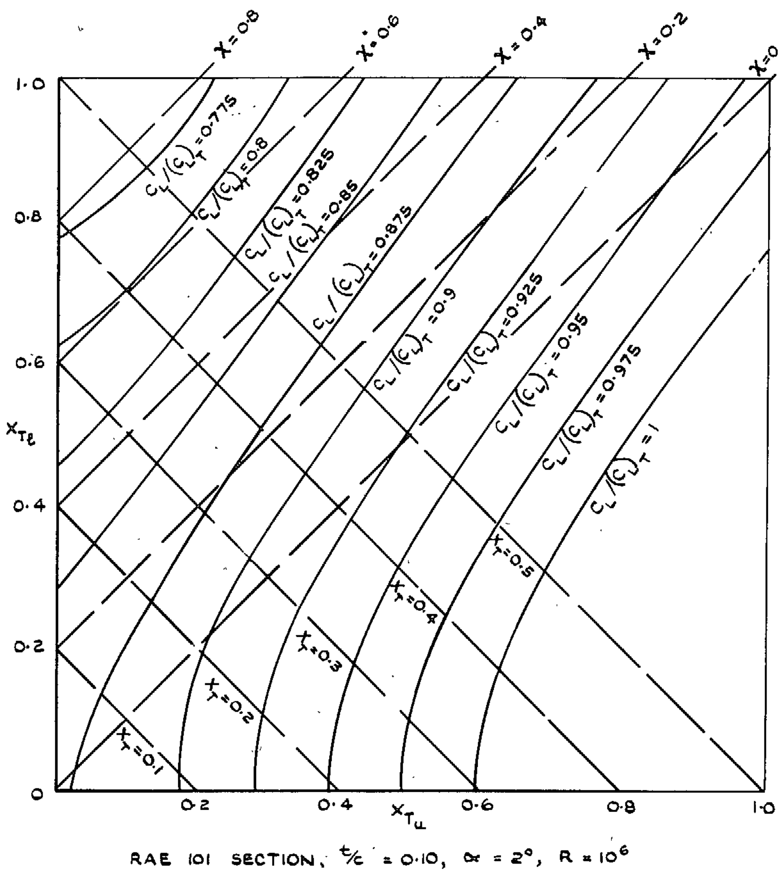

FIG. 3. Variation of $C_L/(C_L)_T$ with transition position, Calculated by Spence's method.

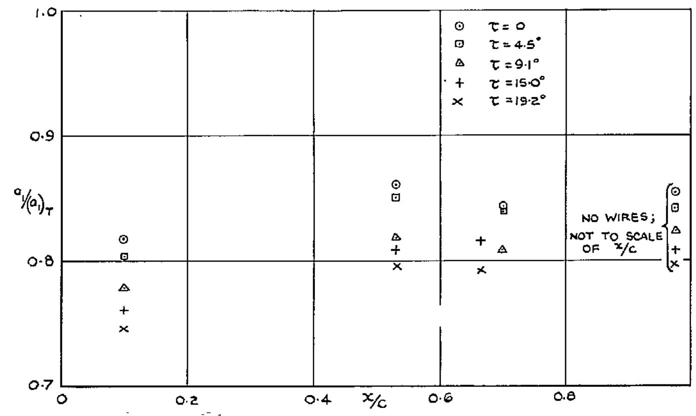

FIG. 4. $a_1/(a_1)_T$ from experiments of reference 6.

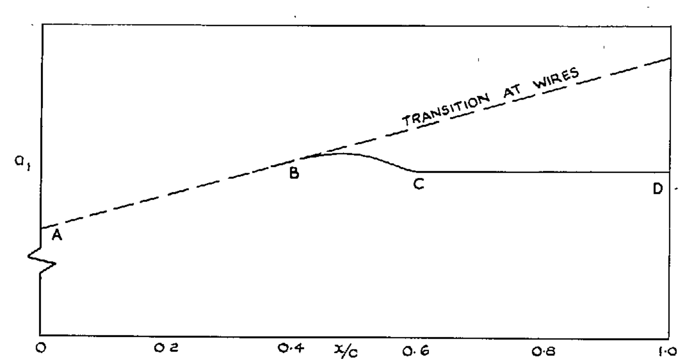

FIG. 5. Variation of a_1 with movement of transition wires.

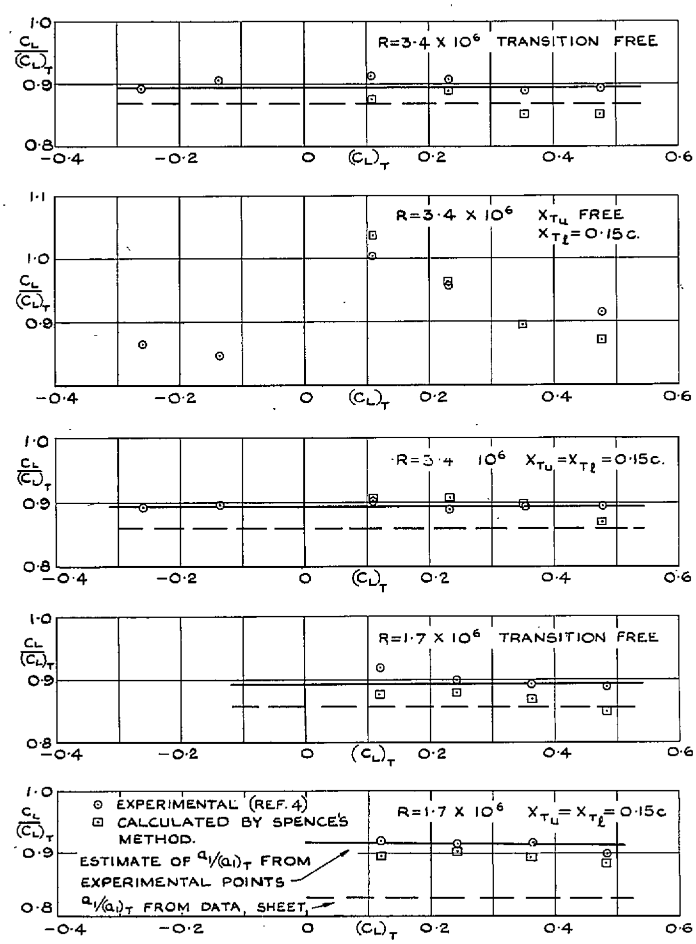

FIG. 6. $C_L/(C_L)_T$ from experiments of reference 5

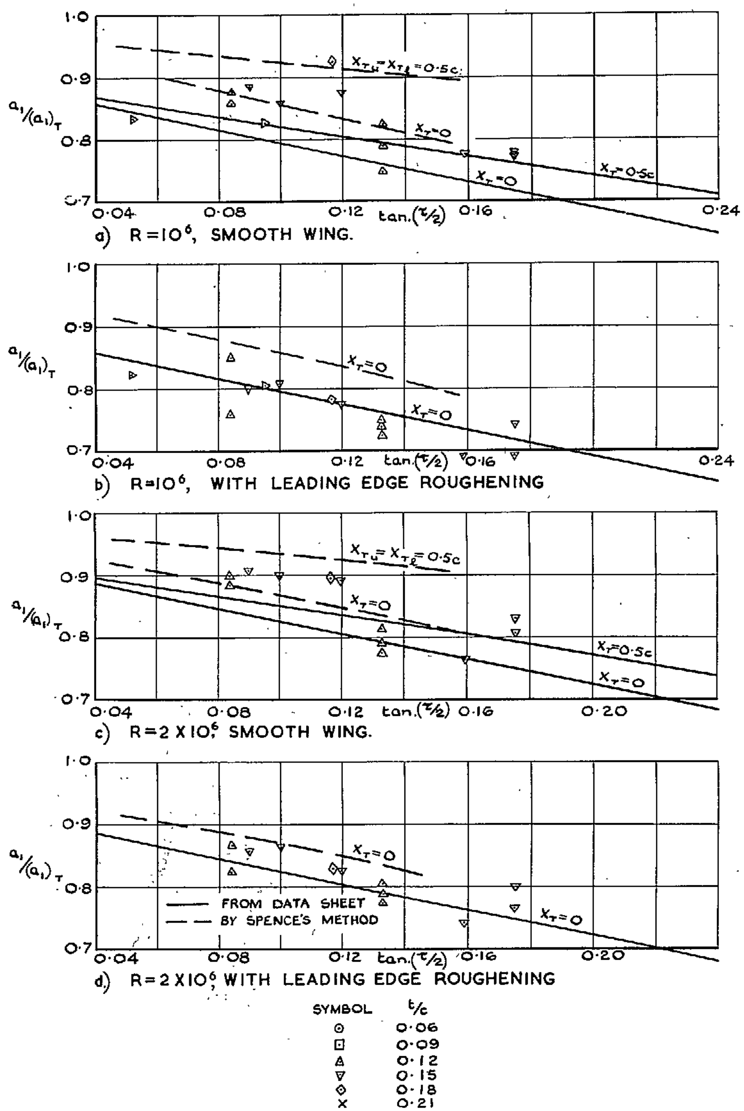

FIG. 7. $\alpha_1/(\alpha_1)_T$ for NACA sections from Ref. 9.

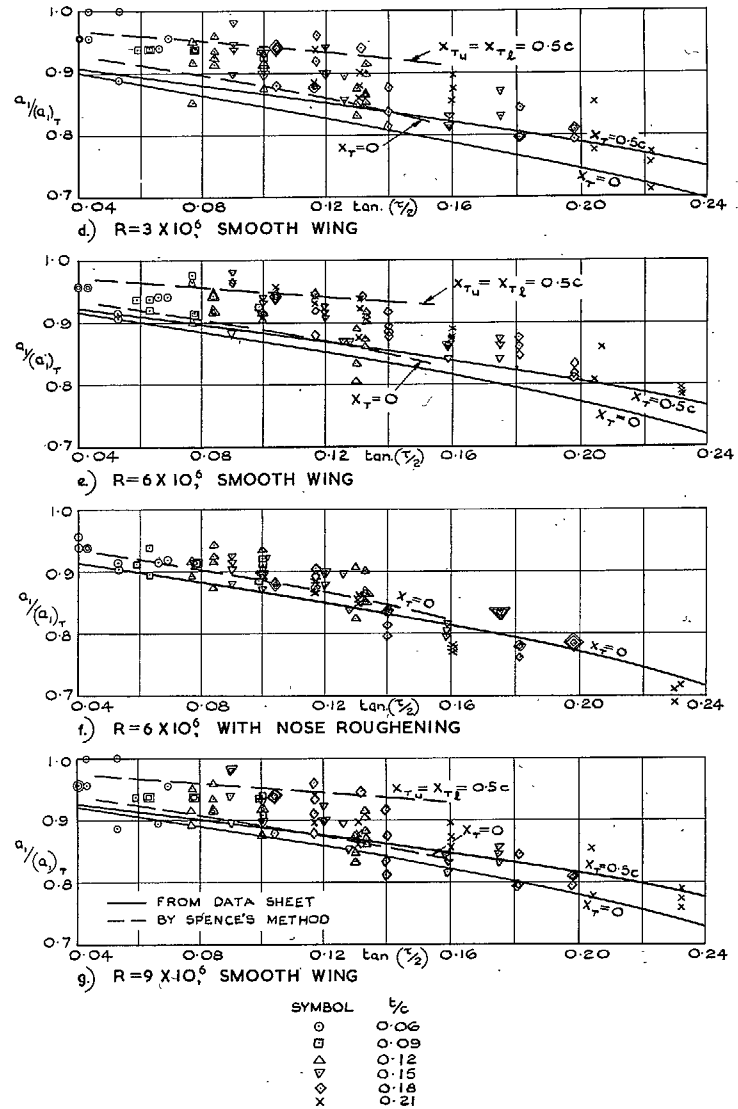

FIG. 7. (contd.). $\alpha_1/(\alpha_1)_T$ for NACA sections from Ref. 9.

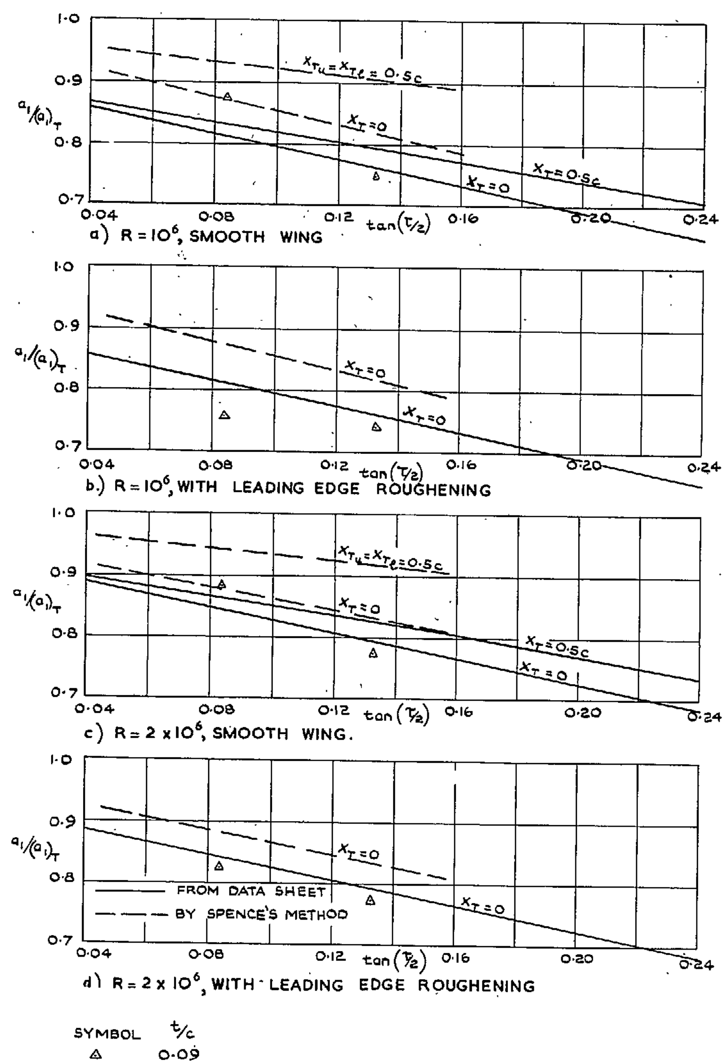

FIG. 8. $a_1/(a_1)_T$ for symmetrical NACA sections from Ref. 9.

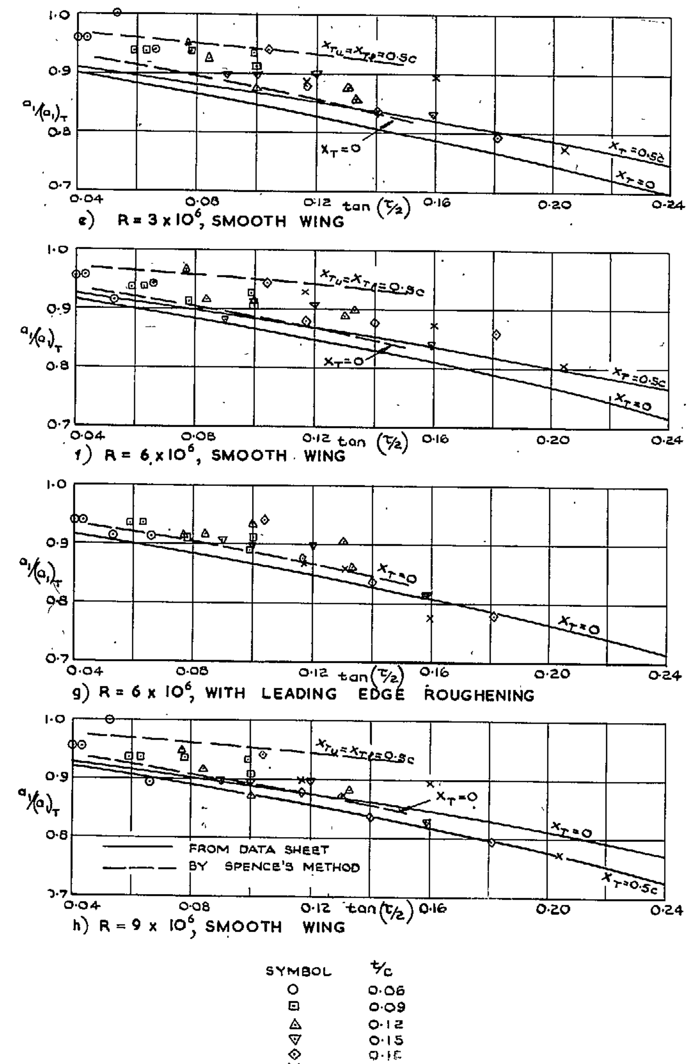

FIG. 8 (contd.). $a_1/(a_1)_T$ for symmetrical NACA sections from Ref. 9.

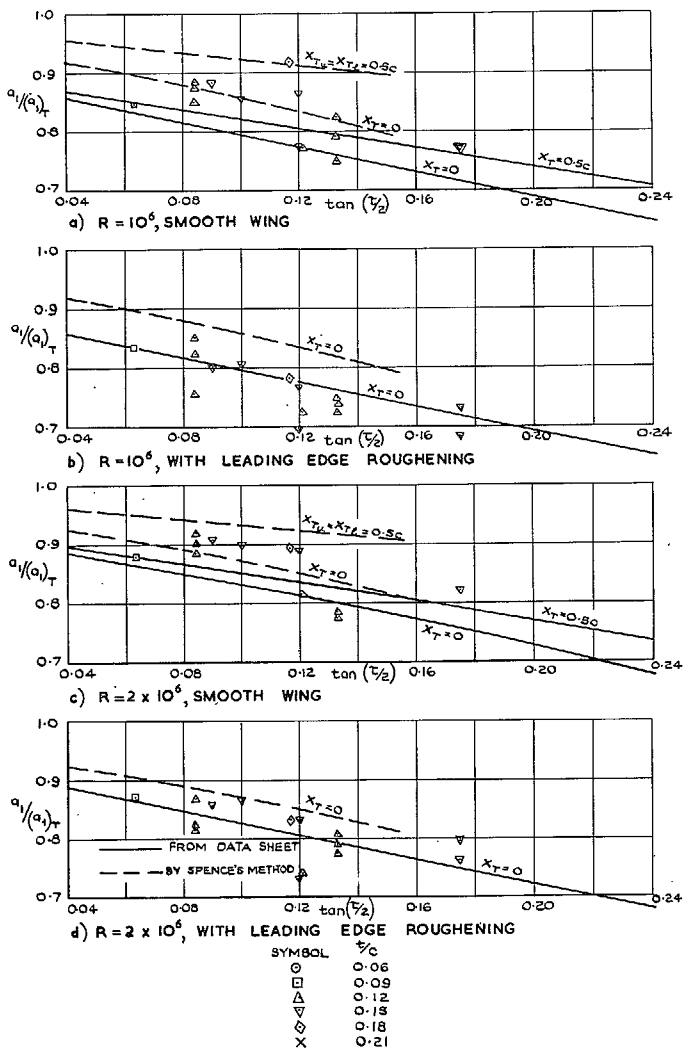

FIG. 9. $a_1/(a_1)_T$ from NACA sections from Ref. 10

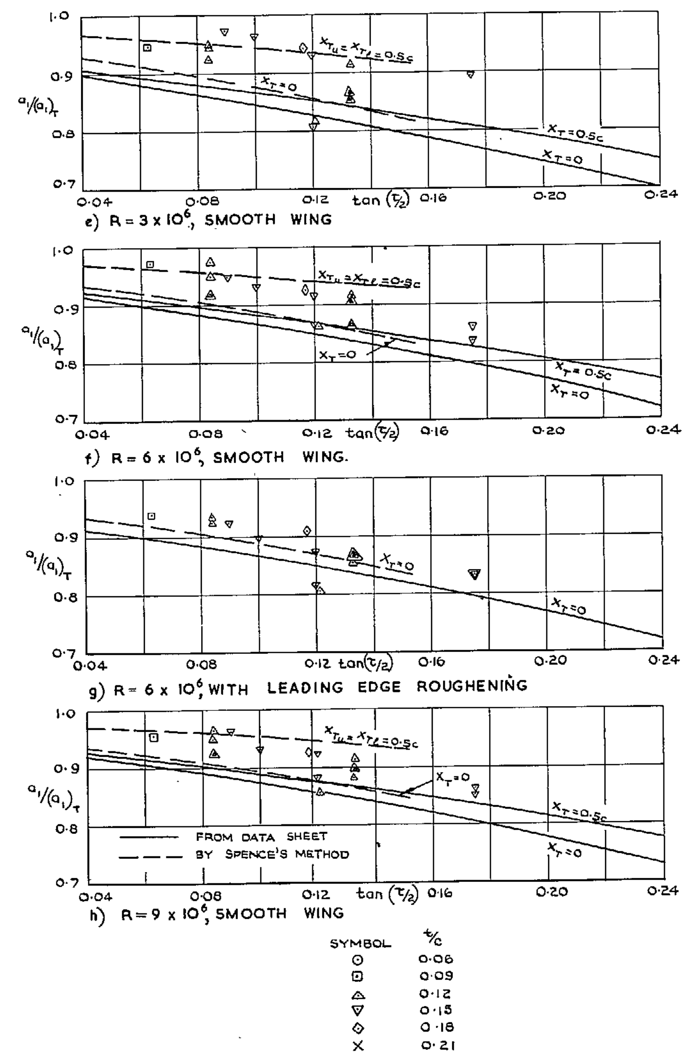

FIG. 9 (contd.). $a_1/(a_1)_T$ for NACA sections from Ref. 10.

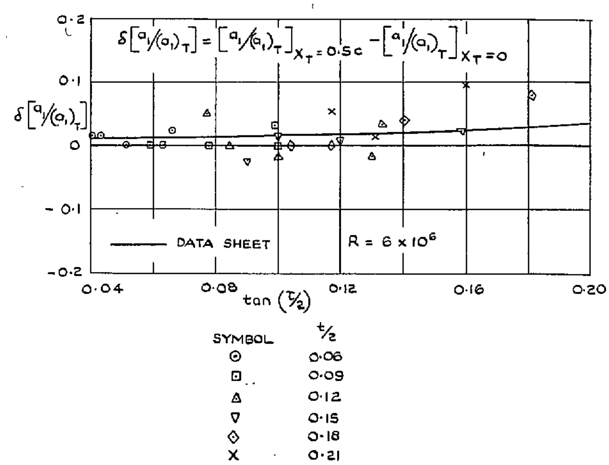

FIG. 10. Reduction of $a_1/(a_1)_T$ with leading edge Roughening.

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