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Summary.

The transfer of heat between a fluid and a cavity in a solid surface over which it flows is discussed from the viewpoint of the closed 'inner boundary layer'. A (constant pressure) integral condition is derived from the energy equation which specifies the net flux of heat through the cavity at all Mach numbers. For a simplified viscosity-temperature function, velocity and temperature profiles are obtained as functions of Mach number and Prandtl number, while heat-transfer coefficients are developed in terms of these parameters.

1. *Introduction.*

Following Squire[1] we describe the closed motion which occurs in cavities and steps in solid surfaces, etc., as 'cavity flow' and consider it to consist of a boundary layer surrounding an inviscid core. Analyses and experimental results for this type of flow have been given for a square cavity by Mills[2,3]. The heat transfer occurring in this type of motion may be of interest and has some practical importance as in the low-speed motion of a fluid over a transversely finned surface (e.g. nuclear reactor rod), and in high Mach number flow over grooves and slots in aerodynamic surfaces.

Interaction between the inner cavity flow and the outer flow takes place in a mixing region which is commonly turbulent. For the purposes of the present calculation the mechanics of this interaction process will be neglected, and the outer flow 'separated' from the inner flow by a dividing streamline along which the velocity and temperature are taken as constant. In reality, of course, re-attachment of the outer flow accompanied by a strong pressure rise will take place in a zone near the downstream top corner. It is assumed that this zone is very small in length compared with the breadth of the cavity, so that these assumptions are reasonably realistic.

The present analysis will embody the hypothesis that the vorticity and temperature of the core tend to constant values once the flow becomes steady. The validity of this has been demonstrated theoretically by Batchelor[4,5]; and verified to some extent experimentally by Mills[2,3] in the case of vorticity for the flow in rectangular cavities.

As an example of this type of problem we shall consider the steady, two-dimensional flow in a square cavity. The methods may be applied to other shapes provided the inner boundary layer closes and remains substantially attached to the walls of the cavity.

* Replaces A.R.C. 26 049.

2. *Governing Equations and Conditions of Closure.*

We consider the boundary-layer motion to be of constant pressure so that the equations of momentum and energy become in von Mises form,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial \psi}\left(\mu\rho u \frac{\partial u}{\partial \psi}\right) \tag{1a}$$

$$\frac{\partial T}{\partial x} = \frac{1}{\sigma}\frac{\partial}{\partial \psi}\left(\mu\rho u \frac{\partial T}{\partial \psi}\right) + \frac{\mu\rho}{C_p} u \left(\frac{\partial u}{\partial \psi}\right)^2 \tag{1b}$$

where

$$\rho u = \partial\psi/\partial y \tag{1c}$$

and the shear stress τ and local heat-transfer rate q take the forms

$$\tau = \mu\rho u \frac{\partial u}{\partial \psi} \qquad q = -\frac{\mu\rho C_p}{\sigma} u \frac{\partial T}{\partial \psi} \tag{1d}$$

$\sigma = \mu C_p/k$ is the Prandtl number.

In the following discussion we take the product $\mu\rho$ = constant, that is $\mu \propto T$ from the perfect gas law. From (1d) this implies that the shear stress and heat-transfer rate are the same functions (of x, ψ) at high Mach number as at low speeds (*see* also Lighthill[7]), and this circumstance is implicit throughout our argument.

The non-dimensional quantities denoted by * are now introduced (*see* Fig. 1),

$$\left.\begin{aligned} &u = U_0 u^*, \quad x = \frac{2b}{\pi}x^*, \quad y = by^*, \quad \rho = \rho_0\rho^*, \quad \mu = \mu_0\mu^*, \quad k = k_0 k^*, \\ &\mu^*\rho^* = C, \quad T = T_0 T^*, \quad \psi = [\mu_0\rho_0 U_0(2b/\pi)C]^{1/2}\psi^* \end{aligned}\right\} \tag{2}$$

whereupon equations (1a, b) become

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial \psi}\left(u \frac{\partial u}{\partial \psi}\right) \tag{3a}$$

$$\frac{\partial T}{\partial x} = \frac{1}{\sigma}\frac{\partial}{\partial \psi}\left(u \frac{\partial T}{\partial \psi}\right) + Eu\left(\frac{\partial u}{\partial \psi}\right)^2 \tag{3b}$$

where

$$E = M_0^2(\gamma - 1) \tag{3c}$$

and we have finally dropped the *.

Boundary Conditions: on u and T these may be written (*see* Fig. 1)

periodicity $\quad u(x, \psi) = u(x+2\pi, \psi) \quad T(x, \psi) = T(x+2\pi, \psi) \quad$ all x, ψ (4a)

walls $\quad u(x, 0) = 1 \quad T(x, 0) = 1 \quad \frac{3\pi}{4} < x < \frac{5\pi}{4}$ (4b)

$\quad u(x, 0) = 0 \quad T(x, 0) = \beta \quad -\frac{3\pi}{4} < x < \frac{3\pi}{4}$ (4c)

infinity $\quad u(x, \infty) = U \quad T(x, \infty) = T_\infty \quad$ all x (4d)

together with the additional condition that the velocity and temperature gradients vanish at the outer edge of the boundary layer,

$$\left(u\frac{\partial u}{\partial \psi}\right)_{\psi\to\infty} = \left(u\frac{\partial T}{\partial \psi}\right)_{\psi\to\infty} = 0. \tag{4e}$$

Closure Conditions.

Before proceeding to give solutions of the above equations we derive important general conditions resulting from the fact that the boundary layer must close. On integrating (3a, b) around any closed streamline we find

$$\int_0^{2\pi} \frac{\partial^2 u^2}{\partial \psi^2}\, dx = 0 \tag{5}$$

$$\frac{1}{\sigma}\int_0^{2\pi} \frac{\partial}{\partial \psi}\left(u\,\frac{\partial T}{\partial \psi}\right) dx + E\int_0^{2\pi} u\left(\frac{\partial u}{\partial \psi}\right)^2 dx = 0 \tag{6}$$

in view of the periodicity conditions (4a). Two integrations of equation (5) with respect to ψ and use of the boundary conditions (4) will show for any streamline

$$\int_0^{2\pi} u^2 dx = \text{constant.} \tag{7}$$

This result provides the value of the velocity U at the edge of the boundary layer ($\frac{1}{2}$ in the present case) and in turn the constant vorticity of the core with the aid of Stokes' theorem[3]. Forms of equations (5), (7) have been first reported by Batchelor[4], Wood[8].

Condition (6) throws light on the heat flux through the cavity; for on utilising the second equations of (1d), (4e) we find after one integration with respect to ψ

$$\int_0^{2\pi} q(x, \psi) dx = \sigma E \int_\infty^{\psi} \int_0^{2\pi} u\left(\frac{\partial u}{\partial \psi}\right)^2 dx\, d\psi. \tag{8}$$

This equation, which is in effect the energy conservation law for this problem, has two immediate consequences:

(I) If there is no frictional heating the net flux of heat across any closed streamline is zero; in particular for $\psi = 0$ this means heat given up at the walls is convected with the boundary layer and delivered to the free stream along the dividing streamline, and *vice versa**.

(II) With frictional heating there is a net flux of heat measured by the right-hand side of (8).

3. *High Speeds.*

A solution of the linearised form of (3a) suggested by Wood[8], namely,

$$\frac{\partial u^2}{\partial x} = U\frac{\partial^2 u^2}{\partial \psi^2} \qquad U = \tfrac{1}{2} \tag{9}$$

has been obtained by Mills[3] with boundary conditions (4):

$$u^2(x, \psi) = \frac{1}{4} - \frac{2}{\pi}\sum_{m=1}^{\infty}\left(\frac{\sin \frac{3}{4} m\pi}{m}\right) e^{-\sqrt{m}\,\psi} \cos\left(mx - \sqrt{m}\,\psi\right). \tag{10}$$

This form is invariant at all speeds under the hypothesis $\mu\rho$ = constant; that the velocity profiles change with Mach number follows since ψ involves the density ρ and the meaning of u in terms of the physical co-ordinate y is different.

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* (I) is of course valid regardless of the viscosity-temperature function; furthermore the product $\mu\rho$ would appear under the integral signs on r.h.s. in equation (8) if no restriction were placed on this function {cf. equation (1b)}.

It is well known that the quadratic function $T = a' + b'u + c'u^2$ with $2c' + E = 0$ satisfies equations (3) when the Prandtl number is unity. We shall take equation (10) to be a sufficiently accurate solution of (3a), (4) for the present heat-transfer calculations. (A method of solution of the non-linear problem is to be found in Mills[3].)

Adiabatic Wall.

Suppose the walls of the cavity to be perfectly heat insulating so that $u = 0$, $dT/du = 0$; $u = 1$, $T = 1$ at $\psi = 0$. Solving for a', b', c', we find the temperature distribution to be given by

$$T = (1+\tfrac{1}{2}E) - \tfrac{1}{2}Eu^2 \tag{11}$$

and the adiabatic temperature rise by $T_{ad} = 1 + (\gamma - 1)M_0^2/2$ in exact parallel with the well known solution for a flat plate. The uniform temperature of the core becomes $T_\infty = 1 + 3(\gamma-1)M_0^2/8$, since $u \to \frac{1}{2}$ as $\psi \to \infty$.

Heat Transfer.

Next suppose that heat can be transferred to and from the fluid along the walls and also along the dividing streamline. With boundary conditions as in (4) we find the solution

$$T = \beta + (1-\beta+\tfrac{1}{2}E)u - \tfrac{1}{2}Eu^2. \tag{12}$$

The uniform temperature of the core now becomes $T_\infty = \frac{1}{2}(1+\beta+\frac{1}{4}E)$. The quantities of heat transferred along the dividing streamline and walls are respectively

$$Q_0 = -\int_{3\pi/4}^{5\pi/4}\left(u\frac{\partial T}{\partial\psi}\right)_0 dx = -(1-\beta-\tfrac{1}{2}E)\int_{3\pi/4}^{5\pi/4}\tau_0 dx \tag{13a}$$

$$Q_w = -\int_{5\pi/4}^{11\pi/4}\left(u\frac{\partial T}{\partial\psi}\right)_0 dx = +(1-\beta+\tfrac{1}{2}E)\int_{3\pi/4}^{5\pi/4}\tau_0 dx \tag{13b}$$

where the skin-friction integrals are given by

$$\int_{3\pi/4}^{5\pi/4}\tau_0 dx = -\int_{5\pi/4}^{11\pi/4}\tau_0 dx = -\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\sin^2\frac{3}{4}m\pi}{m^{3/2}} = -\frac{2}{\pi}\frac{\sqrt{2}+1}{2\sqrt{2}}\left(1-\frac{1}{2^{3/2}}\right)\zeta(3/2)$$
$$= -0{\cdot}91757 \tag{14}$$

on utilising (1d) and (10). $\zeta(3/2) = 2{\cdot}612375$ is the Riemann zeta function for argument 3/2 (*see* Ref. 6). (The first equality of (14) follows since for dynamic equilibrium there is zero net torque on the fluid in the cavity.)

Direction of Heat Flux.

The direction of the heat flow through the cavity may be determined by adhering to the convention that the flux is positive if into the cavity, *i.e.*, in the direction of ψ increasing, and noting that the shear stress τ_0 is negative everywhere along the dividing streamline. With the aid of (13) and (14) we may then distinguish the following régimes. The upper arrows represent the heat transferred by temperature difference, the middle the frictional heat transfer and the lower their resultant.

(i) $0 \leqslant \beta < 1 - \frac{1}{2}E \quad (E < 2)$.

$$\begin{array}{ccccc} & \dashrightarrow & & \dashrightarrow & \\ \text{Free stream} & \leftarrow & \text{boundary layer} & \rightarrow & \text{walls.} \\ & \rightarrow & & \dashrightarrow & \end{array}$$

There is thus a net transfer of heat from free stream to walls.

(ii) $\beta = 1 - \frac{1}{2}E \quad (E < 2)$.

$$\begin{array}{ccccc} & \rightarrow & & \rightarrow & \\ \text{Free stream} & \leftarrow & \text{boundary layer} & \rightarrow & \text{walls.} \\ & 0 & & -\rightarrow & \end{array}$$

There is no flux across the dividing streamline (adiabatic condition there) and a quantity equal to the heat generated by friction passes out through the walls.

(iii) $1 - \frac{1}{2}E < \beta < 1 + \frac{1}{2}E$

$$\begin{array}{ccccc} & \rightarrow & & \rightarrow & \\ \text{Free stream} & \leftarrow- & \text{boundary layer} & -\rightarrow & \text{walls.} \\ & \leftarrow & & --\rightarrow & \end{array}$$

The direction of the net heat flux depends on whether β is greater or less than unity, the latter case being illustrated by the arrows. When β is unity, only frictional heat is involved, half of which passes to the walls and half to the free stream.

(iv) $\beta = 1 + \frac{1}{2}E$

$$\begin{array}{ccccc} & \leftarrow & & \leftarrow & \\ \text{Free stream} & \leftarrow & \text{boundary layer} & \rightarrow & \text{walls.} \\ & \leftarrow- & & 0 & \end{array}$$

An amount equal to the heat generated by friction passes out to the free stream and there is no flow through the walls (adiabatic wall condition); the wall temperature will rise to the adiabatic value T_{ad} quoted earlier.

(v) $\beta > 1 + \frac{1}{2}E$.

$$\begin{array}{ccccc} & \leftarrow- & & \leftarrow- & \\ \text{Free stream} & \leftarrow & \text{boundary layer} & \rightarrow & \text{walls.} \\ & \leftarrow-- & & \leftarrow & \end{array}$$

This is the reverse of situation (i).

The above results are not unexpected on *a priori* physical grounds, for the temperature of the core can never increase beyond all bounds for finite Mach number. We observe that the net flux of heat $Q_0 + Q_w$ is independent of the temperature ratio β, yet this quantity determines its direction, while the proportions of the whole flowing to the walls and free stream depend on both Mach number and β. For fixed $E < 2$ ($M_0 < \sqrt{5}$, air) the situtation represented by (i) may of course reverse itself of its own accord through stages (ii) to (iv) if the heat delivered to the walls is not removed.

Velocity and Temperature Profiles (Adiabatic Wall).

For the adiabatic wall condition (iv) the velocity and temperature profiles are shown for various Mach numbers, Figs. 2 and 3. The physical co-ordinate y has been retrieved from the stream-function representation with the aid of (10) and (11) and the formulae

$$Hy = \int_0^{\psi} \frac{d\psi}{\rho u} = \int_0^{\psi} \left(\frac{T}{u}\right) d\psi \tag{15}$$

where $H^2 = (\pi/2C)[U_0\rho_0 b/\mu_0]$, that is $\pi/2C$ times the Reynolds number R_0 of the flow based on the speed U_0 and fluid properties μ_0, ρ_0 on the dividing streamline, and the cavity breadth b. The profiles may similarly be obtained from (12) in the heat-transfer case.

Together with the usual oscillatory behaviour of this type of boundary-layer flow, we note the flat-plate characteristics of thickening of the velocity boundary layer and tendency to a linear profile with increasing Mach number at the wall station 0 (circled). Somewhat similar, but less pronounced behaviour is noted at the dividing streamline station 2 (circled). With regard to the temperature profiles, Fig. 3, we again observe features akin to those of the ordinary flat-plate case at station 0 (circled), while in contrast at station 2 (circled) the temperature increases with y until at the core a uniform temperature is reached intermediate between the wall temperature and that on the dividing streamline.

4. *Low Speeds.*

It is noted in passing that the high-speed case gives correctly zero net flux of heat through the cavity when $M_0 \to 0$ {*cf.* (I) and equations (13)}. The situations likeliest to be of physical interest will now involve arbitrary Prandtl number rather than the latter being (in the neighbourhood of) unity.

Mathematically the problem amounts to finding a solution of the (homogeneous) equation (3b) without the frictional term and satisfying boundary conditions (4). Even if (10) is accepted as the velocity distribution this linear boundary value problem remains one of some complexity. One could replace u in (3b) by its r.m.s. value $\frac{1}{2}$ and determine a solution for the temperature similar in form to (10) and involving a square-root dependence on Prandtl number. This suffers from the defect, however, that the solution does not satisfy the integral condition (I). We might try to follow Lighthill's[7] procedure of replacing the velocity profile by its tangent near $\psi = 0$, but this leads to difficulties due to the change of sign (under the square root) of the boundary shear stress, and consequent change in character of the approximated differential equation. Notwithstanding, if we use Blasius' value or some weighted equivalent for this stress then we should have a solution for the temperature involving modified Bessel functions of order 1/3 as eigenfunctions and a cube-root dependence on Prandtl number.

In the light of the above remarks the present author was forced to consider an Oseen linearisation of the energy equation, namely (in dimensional quantities),

$$\frac{\partial T}{\partial x} = \frac{\mu}{\sigma \rho U} \frac{\partial^2 T}{\partial y^2} \qquad U = \tfrac{1}{2} U_0 \tag{16}$$

which correctly yields zero net flux of heat through the cavity. The solution of (16) satisfying $T = T_0$ along the dividing streamline and $T = T_w$ along the walls may be written

$$\theta = \frac{T - T_w}{T_0 - T_w} = \frac{1}{4} - \frac{2}{\pi} \sum_{m=1}^{\infty} \left(\frac{\sin \frac{3}{4} m\pi}{m} \right) e^{-\sqrt{m}\, Y(\sigma)} \cos\left(mx^* - \sqrt{m}\, Y\right) \tag{17}$$

where

$$Y = \tfrac{1}{2}\sqrt{\sigma} \left[\frac{\pi U_0 b \rho}{2\mu} \right]^{1/2} \frac{y}{b} \quad \text{and } x^* \text{ is defined as earlier.}$$

Temperature profiles derived from this equation are shown in Fig. 4 for a range of Prandtl numbers. We observe that the thermal boundary layers become thinner with increase in Prandtl number as in E. Pohlhausen's solution for a flat plate. That the thermal and velocity boundary layers do not reduce to the same curve when $\sigma = 1$ is a consequence of the different forms (Mises, Oseen) of the linearised differential equations solved for the velocity and temperature.

On evaluation of the local heat-transfer rate—$k(\partial T/\partial y)_0$ from (17), it is found with the aid of (14) that the Nusselt numbers for the dividing streamline and walls take the forms

$$(\bar{N}u)_{0,w} = \pm \sqrt{\left(\frac{2}{\pi}\right)}\, 0\cdot91757 \sqrt{R}\sqrt{\sigma}. \tag{18}$$

Heat Transfer—All Speeds.

In view of the principal corollary of the $\mu\rho$ = constant hypothesis, *viz.*, constancy of τ and q we now propose with the aid of (13), (14) and (18) the following heat-transfer coefficients valid for all speeds,

$$(\bar{N}u)_{0,w} = \pm\, 0\cdot91757 \sqrt{\left(\frac{2C}{\pi}\right)}\, \frac{T_0}{\Delta T} \sqrt{R_0}\sqrt{\sigma}\left[1 - \beta \mp \frac{\gamma - 1}{2}\sqrt{\sigma}\, M_0^2\right] \tag{19*}$$

where ΔT is some suitable non-zero temperature difference. (The low speed $\Delta T = T_0 - T_w$ would cease to give a meaningful result at high speeds for $\beta = 1$.) Conversely (18) represents the correct limiting case of (19) for zero Mach number and constant fluid properties ($C = 1$). Equation (19) with $\sigma = 1$ follows directly from (13), (14).

5. *Concluding Discussion.*

The solutions developed for high speeds while retaining the inherent oscillatory character of this type of flow show features akin to compressible flow over a flat plate with constant pressure: thickening of the velocity boundary layer and an almost linear profile as the Mach number becomes large, particularly at the wall station 0 (circled) in the adiabatic case. The ultimate temperature of the core has been shown to be a simple function of both Mach number and temperature ratio β, and certain conditions have been derived which determine in direction and proportions to the walls and free stream the net flux of heat through the cavity. Heat-transfer coefficients have been developed for all speeds.

The particularly simple form of these results arises from the assumed constancy of the density $\times$ viscosity function at all speeds. A more accurate representation of the viscosity-temperature function will involve an analysis of greater complexity but this will still contain the same crucial information regarding the heat flux through the cavity.

The interaction process between the main stream and the cavity flow region and the resulting high rate of mixing must certainly influence the various exchanges of heat energy. However, it is felt that embedded in the more complex phenomenon including the mixing process will still be found the basic mechanism of heat exchange *via* the inner boundary layer as illustrated above.

Another neglected factor will be the complex of shock waves and shock-wave boundary-layer interactions that must attend separated flows of this nature. There may also be a tendency to enhance the local separation and re-attachment effects in the cavity corners which are already present in the low-speed case [2,3]. Then there is the question whether the boundary layer is laminar or turbulent and the intricate inter-relationships between this circumstance and the shock-wave boundary-layer

* The frictional part of (19) exhibits the correct dimensional dependence on σ, E as demanded by the integral condition (8). Numerically, the condition will not be exactly obeyed since a linearised solution, equation (10), has been used for the velocity distribution. Inverting the argument, (8) might prove to be a useful condition for rendering unique any approximate attempt at a comprehensive solution $u\,(x, \psi, \sigma, E)$, $T(x, \psi, \sigma, E)$.

interactions. In a gross manner a turbulent boundary-layer flow of the present type could be analysed by employing an eddy viscosity and eddy heat conductivity with some attendant hypothesis of very simple form (*see* low-speed case[2, 3]).

At low speeds the linearised Mises forms of the boundary-layer equations though yielding a reasonably accurate description of the velocity boundary layer[3] fail to describe correctly the heat flux through the cavity, and resort to an Oseen linearisation is necessary to preserve this cardinal feature of the thermal phenomenon. It does not appear possible to predict the constant temperature of the core from the exact thermal equation without first determining the details of a solution. This contrasts with our ability to determine the constant vorticity of the core directly from the exact momentum equation (*via* the value of the velocity at the outer edge of the boundary layer). The value $\frac{1}{4}$ for θ_∞ (or $T_\infty/T_0 = \frac{1}{4} + \frac{3}{4}\beta$) as yielded by the Oseen linearisation then may reasonably be in question; it could readily be tested by experiment, however.

Finally, it may reasonably be questioned whether the assumptions of (a) constant velocity, temperature along a (straight) dividing streamline (b) constant pressure within the cavity would seriously impair the correspondence of the present analysis to reality. In an actual cavity significant departures from (a) may occur at low Reynolds numbers and from (b) at high speeds due to abrupt flow direction changes at corners. However, it should still prove possible to take these effects into account in an analysis of the present type. To this end we note that the present methods, though applied for definiteness to a square cavity, are in fact independent of the boundary shape inasmuch as the boundary layer encircles a single vortex and remains attached to the cavity walls; and furthermore we can encompass readily non-uniform velocity $U_0(x)$, temperature $T_0(x)$ and pressure $p(x)$ (at least at low speeds) by expanding these in Fourier series (for an example of this *see* Ref. 3). For high speeds some progress should be possible in this direction on first utilising an Illingworth transformation. Thus in principle we can continuously deform the boundary streamline $\psi = 0$ so as to approach more realistically that in an actual cavity and at the same time incorporate these arc-distance (x) dependences into the solution. In fairness, however, a full solution of this nature would be a difficult undertaking even using linearised equations, for in surmounting the usual difficulties we have additionally to find a solution of 'shear-wave' character, as distinct from a 'similarity' solution.

Nevertheless, it is hoped that the present much simplified analysis may give a useful overall description of this somewhat complicated (thermal) phenomenon, and that the heat-transfer coefficients may be of some guidance in practical situations.

LIST OF SYMBOLS

b		Breadth of cavity
x, y		Orthogonal curvilinear co-ordinates
ψ		Stream function
u		Velocity component in direction of x
T		Temperature
q		Local heat-transfer rate
τ		Shear stress
U_0, T_0		Constant velocity and temperature along dividing streamline
U, T_∞		Constant velocity at edge of boundary layer and constant temperature of core
ρ, μ, k		Density, coefficient of viscosity and coefficient of thermal conductivity of fluid
C_p		Constant-pressure specific heat of fluid
γ		Ratio of latter to constant-volume specific heat
C		Constant in density $\times$ viscosity function
σ	=	$\dfrac{\mu C_p}{k}$, Prandtl number
M_0		Dividing streamline Mach number
E	=	$(\gamma - 1)M_0^2$, Eckert number
β	=	T_w/T_0, wall to dividing streamline temperature ratio
a', b', c'		Constants in quadratic temperature function
Q_0, Q_w		Quantities of heat transferred along dividing streamline and walls of cavity
R_0	=	$\dfrac{U_0 b \rho_0}{\mu_0}$, dividing streamline Reynolds number
$\bar{Nu}$	=	$Q/k_0 \Delta T$, average Nusselt number; ΔT temperature difference
θ	=	$\dfrac{T - T_w}{T_0 - T_w}$, low-speed temperature ratio.

N.B. (i) Quantities not dimensionless in above list are made so according to the scheme of equation (2) and are used in the text without a distinguishing mark for convenience. Reversion to dimensional quantities is mentioned explicitly.

(ii) Suffices $_0$ and $_w$ refer to conditions along the dividing streamline and walls respectively.

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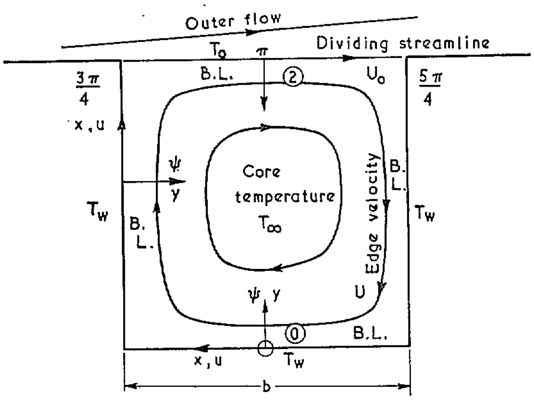

Fig. 1.

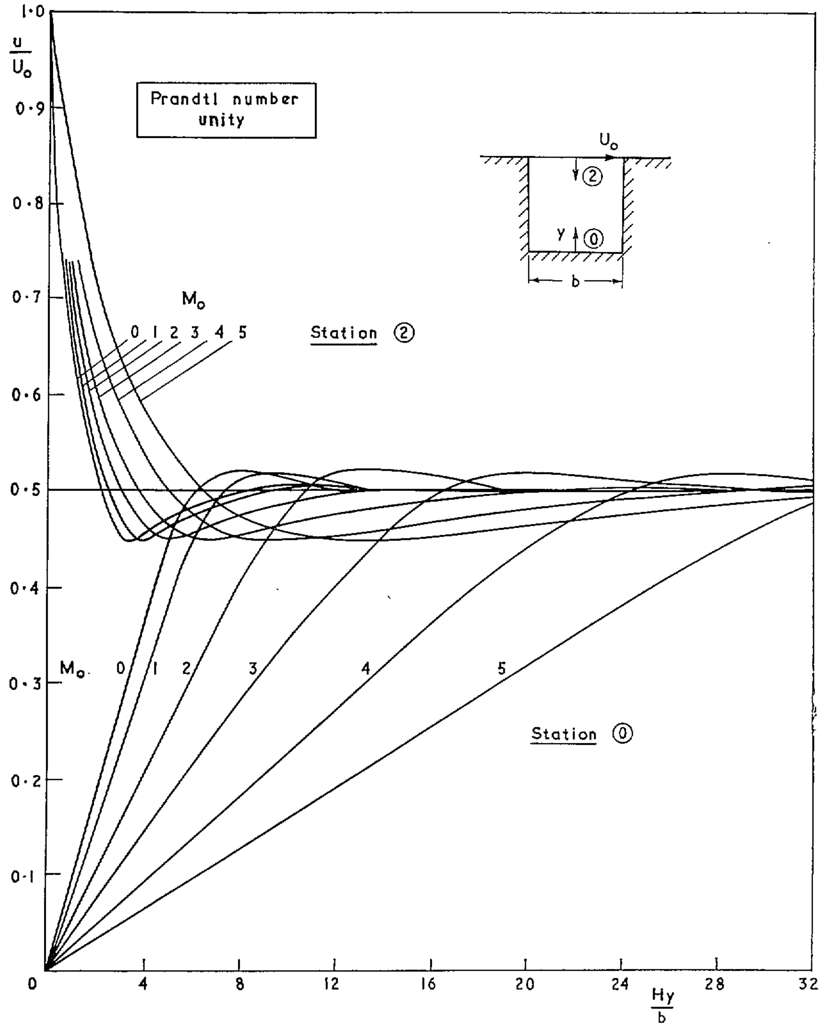

Fig. 2. Velocity profiles as functions of Mach number—adiabatic wall.

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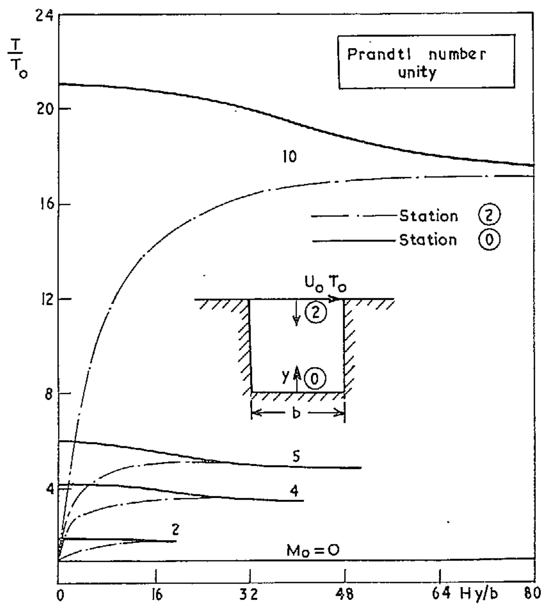

FIG. 3. Temperature profiles as functions of Mach number—adiabatic wall.

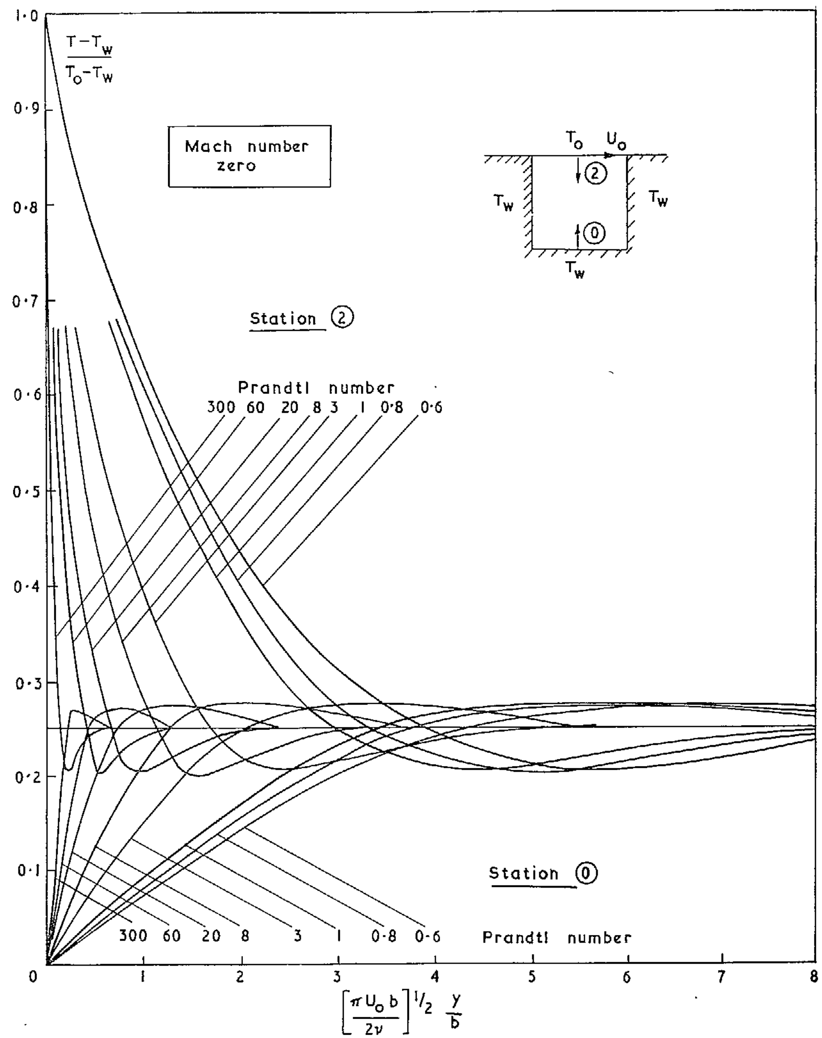

FIG. 4. Velocity profiles as functions of Prandtl number.

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