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**Inflated Mobile Lifting Structures:
Analysis of Membrane Construction and Design
Optimisation for Oval Planforms**

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Summary.

A stress analysis has been made of inflated mobile lifting structures which are considered as membranes supporting loads attached at a seam or seams on the underside. The initial treatment discusses a structure of infinite length in which the end effects can be ignored, the ends being treated later as a body of revolution.

The most efficient structure is shown to be that which encloses the load with minimum base width and height of support seam, and for any such given configuration and load, the bulk of tent material is proportional to the linear dimensions.

For a given height and base, the horizontal position of the load support seam is not critical.

Application to a construction with strengthening cords or inflated ribs is discussed, and some examples of practical design shapes are given.

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* Replaces R.A.E. Tech. Note No. Mech. Eng. 361—A.R.C. 24 670.

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1. *Introduction.*

An inflated mobile lifting structure is essentially a membrane attached to the periphery of a base containing an orifice. On inflating the membrane or tent with a steady supply of air it is possible to lift it, and the reaction between the weight of the system and the ground or water surface is taken by the excess pressure over the base orifice and is retained against the flow of air escaping under the base. In this state the effective friction between the structure and the surface is negligible, and the tent can be moved horizontally with little force.

With such an inflated membrane it has been found that substantial loads can be hung from the upper surface with low internal pressure and moderate flow of air, while retaining transverse mobility of low resistance. Fig. 1 illustrates a structure of this type.

The purpose of this report is to outline the design of an efficient membrane for lifting a load in the way described.

2. *The Inflated Lifting Membrane in Comparison with Other Devices.*

So far, inflated lifting structures of tent type have been made by adapting the rescue dinghy. This essentially provides a lifting membrane and an inflatable hoop base made of proofed fabric. Design is simple because the cross-section consists always of circular arcs. However, the device must exhibit some deformation about the load attachment points. This is one of the factors to be studied.

In contrast to the lifting membrane it is possible to use a stiff platform to carry the payload, and this platform which is airtight has a lip or curtain projecting below its periphery. The curtain, which may be stiff or flexible, is designed to allow air to be retained under pressure below the platform. The air escapes under the curtain at a rate appropriate to the equilibrium pressure. A stiff platform designed to carry a concentrated load can be made comparable in weight with a tent, but the inflated membrane can be packed away into a small space, which can be a very useful feature.

The hovercraft is a special case of a pneumatically lifted platform in which the curtain is replaced by a peripheral jet of air directed downwards and inwards to maintain the whole craft clear of the ground. The essential practical distinction between the true hovercraft and the simple inflated lifting structure is that whereas the hovercraft obtains a ground clearance of the order of feet, requiring power of the order of that for an aircraft, the simple tent has virtually no clearance and requires power similar to that of a waterborne barge.

Although the capabilities of the inflated structure in clearing rough ground and water are not as good as those of the hovercraft, it is nevertheless considered that the design problems can be solved and advantage taken of the cheapness of this vessel.

3. *Range of Investigation.*

The section of a long thin lifting system, in which the influence of the ends can be neglected, has been analysed. The analysis has been aimed at the provision of design data and to determine whether any optimum conditions in the design exist. At the same time an attempt has been made to draw conclusions with regard to physical and mechanical factors in the design which cannot be revealed clearly in the more complicated analysis of a symmetric system formed by a generator of revolution.

In the first place the treatment considers a unit length of a long load supported by a fabric curtain suitably attached at a seam or seams along the underside of the upper surface. In this case the tensions due to the ends of the device do not affect the study of a section. However, if the load is supported by cords spaced along the length at points where the tent is braced by cords over the top, the effect of the lengthwise tensions must be discussed.

Extension of the treatment considers this case, and also, for the membrane without bracing (or generator) cords, the design of the tent ends as a complete body of revolution.

4. *Stress Analysis of Inflated Tents.*

4.1. *General.*

An extensive range of shapes for inflated tents is amenable to stress analysis. Those which are formed from surfaces of revolution using generator cords as in parachutes have been analysed by Dr. W. G. S. Lester¹ but, for the purposes of illustrating the fundamental physical and mechanical principles the simplest case will first be considered: that of a section of an infinitely long hemi-cylindrical structure remote from the ends.

In a completely flexible construction it is necessary to maintain tensile continuity all round the section of the tent. The base aperture will, therefore, be supposedly covered by porous net so that the tension per unit length can be maintained all round. The imporous length of section will be substantially longer than the porous section. In tents which are surfaces of revolution a central uncovered aperture is permissible in a fully flexible structure because the distributed stresses can terminate in a hoop tension.

4.2. *The Section of a Long Tent.*

Fig. 2 is a cross-section of a hemi-cylindrical portion of an inflated tent which has re-entrant points A, A' from which the sectional load, Mg , is supported. Above and below the points A and A' the tent section will be in tension and curved so that the membrane is in equilibrium with the pressure difference. The tent can be considered as an upper section AOA' and two wall sections ABC and A'B'C', symmetrical about the centreline OO'.

It must be remembered that the tent, however long, must have ends to retain the pressure so that there is a distributed tension normal to the plane of the diagram. The integration of this tension around the envelope must balance the pressure over the area enclosed by it. However, if the section has no curvature normal to the plane this tension has no influence on the shape of the section. The basic equation for the equilibrium of stressed membrane subjected to a pressure difference p is

$$\frac{T_p}{\rho_p} + \frac{T_n}{\rho_n} = p \quad (1)$$

where the suffices p and n refer to planar and normal components of Fig. 2. If ρ_n tends to infinity the term T_n/ρ_n becomes insignificant.

4.3. *Equilibrium of Pressure, Load and Tensions.*

In analysing the section of Fig. 2 the tension is considered as distributed and the equations will relate to a tension over unit length normal to the plane of the figure. The load will be a long bar normal to the section which is supported by membranes attached to seams normal through A and A' and the support of the weight per unit length is represented by the equations which will follow.

In Fig. 2, between A and C and between A and A' the tension is constant but changes at A and A'. Therefore, under the pressure p , of inflation the arcs ABC, AOA' and A'B'C' will be arcs of circles. If a is the radius of the cap and b the radius of the walls, then from equation (1):

$$\frac{T_a}{a} = \frac{T_b}{b} = p. \quad (2)$$

For the tent to just lift, the supported weight, Mg per unit length must equal the product of the pressure and the area $2c$ (aperture times unit length)

$$2pc = Mg. \quad (3)$$

At the points of re-entry, A and A' there are equations of equilibrium of vertical and horizontal force components:

$$T_a \sin \alpha + T_b \sin \beta = F \sin \gamma = Mg/2 \quad (4)$$

$$T_b \cos \beta - T_a \cos \alpha = F \cos \gamma = \frac{Mg \cot \gamma}{2} \quad (5)$$

where T_a and T_b relate respectively to the tension in the cap, AOA' and the wall ABC, A'B'C', F is the tension in the suspension cords and the angles α , β and γ are as illustrated in Fig. 2.

It should be noted that the arrangement illustrated represents a tent partly braced by the two load suspension cords, and does not represent the simplest system for a preliminary analysis. Two separate loads exerting only material forces at A and A' could be considered.

4.4. Configuration of Section.

It is possible to eliminate load, tensions and pressure and obtain equations in terms of c and radii and angles of curvature and thus obtain equations of configuration, by substituting in (4) and (5) the values for load and tension given in (2) and (3):

$$ap \sin \alpha + bp \sin \beta = cp \quad (6)$$

$$bp \cos \beta - ap \cos \alpha = cp \cot \gamma \quad (7)$$

The pressure p , is a factor which will cancel from equations (6) and (7).

If the braced support is removed, leaving vertically acting loads at A and A' equation (7) is replaced by

$$b \cos \beta - a \cos \alpha = 0. \quad (8)$$

To reduce the above equations further it is necessary to consider the boundary conditions and the basis of optimisation which will result in practical designs.

5. Tent Design.

5.1. Design Parameters.

It would appear that the most important design condition for a tent is the ratio of the total length of fabric over a cross-section, $2S$, to the distance across the aperture, $2c$. A second design condition is whether the load is supported from a central point O, or from two (or possibly more) points A and A' symmetrically placed about O. The third condition is the height clearance H , which can be expressed as the height of A above the base as a fraction of c .

From Fig. 2, the length S is equal to the sum of the arcs AO and ABC

$$S = (\pi + \beta)b + \alpha a \quad (9)$$

and

$$H = b(1 + \cos \beta). \quad (10)$$

It is convenient to refer all linear dimensions to the half-base aperture, c . Let

$$\frac{S}{c} = \lambda, \quad \frac{b}{c} = \mu, \quad \frac{a}{c} = \nu \quad \text{and} \quad \frac{H}{c} = \zeta.$$

Then from (9) and (10)

$$\lambda = \mu(\pi + \beta) + \nu\alpha \quad (11)$$

and

$$\zeta = \mu(1 + \cos \beta). \quad (12)$$

From equation (6)

$$\nu \sin \alpha + \mu \sin \beta = 1. \quad (13)$$

Although a low value of λ will in general lead to a tent of small inflated bulk, it is also desirable that the bulk and weight of the tent should be small when packed. It is reasonable to assume that the total fabric bulk, B , will be proportional to the product of the tensions and the arc lengths, that is to

$$T_b \times (\pi + \beta)b + T_a \times \alpha a.$$

Eliminating the tensions, as before, we have

$$B \propto \frac{Mg}{2c} \{b(\pi + \beta)b + a\alpha a\}$$

or in non-dimensional coefficients

$$B \propto \frac{Mg}{2} c \{\mu^2(\pi + \beta) + \nu^2\alpha\}.$$

The configuration is completely determined by α , β , μ and ν and the scale of the tent by c . Hence for a given configuration,

$$B \propto Mg \times c.$$

It follows that the base width should be made as small as the physical dimensions of the load will allow.

Let η be a factor proportional to the bulk (or tent weight) per unit load, per unit base width, so that

$$\eta = \frac{\mu^2(\pi + \beta) + \nu^2\alpha}{2}. \quad (14)$$

5.2. Optimisation of Parameters.

5.2.1. *General.*—It is now possible to consider the optimisation of the coefficients λ , ζ and η in terms of α , β , μ and ν , and the number of support points. The general case leads to a great range of possible values of the parameters, consideration of which is outside the scope of this report.

5.2.2. *Simplified cases with equal tensions.*—In a practical tent design it would be reasonable to use the same fabric for the wall and cap, thus the designed tensions in both parts should be the same, and the radii a and b will be equal.

By making a equal to b , however, $v = \mu$ and from (14)

$$\eta = \frac{\mu^2(\pi + \alpha + \beta)}{2}$$

and from (13)

$$\mu = \frac{1}{(\sin \alpha + \sin \beta)}.$$

Hence, when $a = b$,

$$\eta = \frac{\pi + \alpha + \beta}{2(\sin \alpha + \sin \beta)^2}. \quad (15)$$

Also, when $a = b$, from (11)

$$\lambda = \mu(\pi + \alpha + \beta).$$

Hence, substituting for μ ,

$$\eta = \frac{\lambda}{2(\sin \alpha + \sin \beta)} \quad (16)$$

$$\lambda = \frac{\pi + \alpha + \beta}{(\sin \alpha + \sin \beta)}. \quad (17)$$

Inspection of equation (15) indicates that η will be low for values of α and β approaching $\pi/2$, and equation (16) suggests that low values of η will also lead to low values of λ .

Eliminating μ from equation (12) gives, when $a = b$,

$$\zeta = \frac{1 + \cos \beta}{\sin \alpha + \sin \beta}. \quad (18)$$

It has already been shown (in Section 5.1) that the half base c should be as small as practicable. Therefore, on the basis of the minimum value of c to enclose the design load, there will be a minimum value of ζ which will give sufficient height clearance. From equation (18) it is clear that low values of λ and η which require α and β of about $\pi/2$ will lead to low values of ζ . In other words, a relatively high load results in a bulky and heavy tent.

Considering a tent with two-point suspension, if the load object is rigid with points of attachment on its edges at A and A', then the possibility of γ being other than $\pi/2$ will not affect the following considerations (which are based only on the vertical components at A and A'). If it is not rigidly connected at A and A' then the lines of action of the forces F must meet at the load at an acceptable height above the base, that is

$$\zeta \geq v \sin \alpha \tan \gamma.$$

This is a special case for consideration by the designer, but as it will not apply in the majority of tents, detailed discussion is omitted from this report.

Two other special cases, of simplified parameters, which are considered below are:

(1) Two-point suspension with $\alpha = \beta$, or separate loads.

(2) Single-point suspension. This can be considered as an extension of the two-point case when α tends to zero and the cap eventually disappears so that A and A' coincide.

To enable the parameters to be quickly chosen in design work, a graphical solution of equations (15), (17) and (18) has been used. In Figs. 3, 4 and 5 the variation of η , λ and ζ with α and β is given.

Figs. 3 and 4 show that the desirable qualities of low fabric bulk (η) and low arc length (λ) are obtainable over quite a wide range of values of both α and β . Thus, while the lowest values of

both these coefficients occur at around $\alpha = \beta = 75^\circ$, there is very little increase of η when α and β lie within the range of, say 60° to 90° or of λ when α and β are between about 45° and 90° .

Fig. 5 shows that high values of ζ which may sometimes be required lead to undesirably high η and λ , because these large values of ζ occur at relatively small angles α and β . In order to show the minimum for η which can be obtained for any desired height coefficient ζ , Fig. 6a has been drawn (by reference to the values of η obtained from Fig. 3). Again, there is not a great change in η over certain variations in α and β , but values of α and β which correspond with the minimum values of η are plotted in Figs. 6b and 6c.

5.2.3. *Tents for particular requirements.*—Configuration of the tent may now be decided by reference to Figs. 3 and 4, except as regards scale, because with a fixed (arbitrary) radius to represent a and b , the angles α and β determine the two arcs. The value of $\mu = b/c$ can then be found graphically. Examples of tents to fulfil several sets of requirements are given in Fig. 7.

(A) *Lightest possible tent on a given base with a load of given weight.*—Fig. 6 shows that η is a minimum for $\zeta = 0.66$ or less and with $\alpha = \beta = 80^\circ$. This tent is shown in Fig. 7a, and it is useful to remember that, as shown in Fig. 3a, both α and β may differ appreciably from the optimum value without much increase in η . Thus, if the suspension point for the load is required farther from the centreline than is shown, α may be increased and β reduced to some extent to achieve this configuration.

(B) *Lightest possible tent with single-point suspension.*—Single-point suspension results for any value of β when $\alpha = 0$ (Fig. 7b). Here again, although the lightest tent is obtained with β of approximately 80° , η is virtually unchanged if β lies within the range 75° to 90° .

(C) *Tent of greatest height to tent-weight ratio.*—For a maximum value of ζ/η we must choose the point where the line OA on Fig. 6a is tangential to the curve. The resulting configuration is that of Fig. 7c. For this requirement also there is some latitude in the value of ζ , that is, the height to tent-weight ratio is near its maximum over a range of values of ζ of from about 1 to 1.3. It is clear, however, that larger values of ζ carry an increasing weight penalty.

(D) *Tents for very wide loads.*—For loads having a value of ζ of less than 0.6 it may be worthwhile to consider tents with more than one point of suspension on each side of the centreline. Fig. 7d shows one such tent with two points of suspension each side and using the value of 80° for β and for all other arc semi-angles.

Let there be N suspension points on each side of the centreline, and let all radii of arc and all the semi-angles of arc be respectively b and β .

Then

$$\eta = \frac{T\lambda}{Mg} \quad \text{and} \quad \frac{T}{b} = \frac{Mg}{2c}$$

$$\lambda = \frac{S}{c} = \frac{b(\pi + 2N\beta)}{c}$$

and

$$c = b2N \sin \beta.$$

Hence

$$\eta = \frac{\pi + 2N\beta}{8N^2 \sin^2 \beta} \tag{19}$$

and this leads to minimum values of η all at the value of 80° for β , as follows:

N	η
1	0.76
2	0.28
3	0.16
4	0.12

$$\zeta = \frac{b(1 + \cos \beta)}{2c}$$

and with equal radii and angles,

$$\zeta = \frac{1 + \cos \beta}{2N \sin \beta} = \frac{0.60}{N} \quad \text{when } \beta = 80^\circ.$$

With $\beta = 80^\circ$, for minimum values of η , the maximum available values of ζ are

N	ζ
1	0.6
2	0.3
3	0.2
4	0.15

When one of the N suspension points is on the centreline, this will be effectively a half point on each side. For example, in the single-point suspension of Fig. 7b, $N = 0.5$. This leads, with $\alpha = 85^\circ$, to the result already obtained:

$$\eta = 2.34, \quad \zeta = 1.1.$$

5.3. Application to Generator-Cord Construction.

Pressure acting on the tent ends will result in forces normal to the plane of the section of Fig. 2. With membrane construction it may be assumed that the resulting normal loads due to the segmental areas ABC, AOA' and A'B'C'' are taken by the membrane, and the normal loads due to the trapezium area CAA'C' are taken by cords at the four corners. The normal tensions in the membrane will then never exceed the planar tensions and will generally be about half as much.

If the tent is constructed in the manner of a parachute, using 'generator' cords to react practically all the planar tensions, and the membrane which fills the areas between these cords is allowed to billow out with a curvature of radius ρ_m , then the relation between membrane radius, tension and pressure is

$$\frac{T_m}{\rho_m} = p.$$

As in parachute design, ρ_m may be made small relatively to a and b , so that weaker and lighter fabric may be used. Although there may not always be a net gain in lightness of the tent as a whole, as indicated in Appendix I, the use of this construction may be justified on the grounds of availability of materials and manufacturing techniques. Avoidance of heavy fabric will also help in packing of the tent for storage. The resulting 'accordion' shape could tend to oscillate longitudinally unless

restrained by rigid attachments at the points AA'. At CC' there will be cords normal to the section to contain the forces exerted by the end portions. There must also be a cord at O to join the upper ends of the generator cords from the end portions.

On the assumption that ρ_n is small, the same principles of optimisation of the design will apply as for the membrane construction.

5.4. Tent Ends.

Tent ends will generally lend themselves to a semi-circular planform. With membrane construction they will be bodies of revolution of the section of Fig. 2, with sections formed from circular arcs.

For the cap, the planar and normal radii are equal, leading to equal planar and normal tensions by equation (1):

$$T_a = \frac{ap}{2}. \quad (20)$$

For the wall, maximum planar tension T_b occurs as a vertical tension at the line of maximum diameter

$$\begin{aligned} T_b &= p\pi\{(b+c)^2 - c^2\}/2\pi(b+c) \\ &= \frac{bp}{2} \left(\frac{b+2c}{b+c} \right) \\ &= \frac{bp}{2} \left(\frac{\mu+2}{\mu+1} \right). \end{aligned} \quad (21)$$

Hence, although the cap requires fabric of only half the strength needed in the centre portion of the tent, the wall may require somewhat stronger fabric, particularly if μ is small. However, even if μ is zero the required strength does not exceed the value for the centre portion (bp). With $\mu = 1$, strength required is 75%, and with $\mu = 0.5$, 85% of that for the centre part.

Considering the normal tension T_{bn} for the wall, from (1)

$$\begin{aligned} p &= \frac{T_b}{b} + \frac{T_{bn}}{b_n} \\ &= \frac{T_b}{b} + \frac{T_{bn}}{b+c}. \end{aligned}$$

Hence, substituting for T_b as in (21) and for b and c in terms of μ ,

$$T_{bn} = bp/2.$$

Equilibrium of the end sections requires that, if $M'g$ is the load carried by the two ends,

$$p\pi c^2 = M'g.$$

The total vertical pressure forces acting on the cap and the part of the wall between the point of re-entry and the position where $\beta = 0$ may be equated to $p\pi c^2$, i.e.

$$p\pi(a \sin \alpha)^2 + p\pi\{c^2 - (c - b \sin \beta)^2\} = p\pi c^2 \quad (22)$$

and on substituting for a , b and c with μ and ν :

$$\nu^2 \sin^2 \alpha + 2\mu \sin \beta - \mu^2 \sin^2 \beta = 1. \quad (23)$$

Let η' be the bulk factor for the ends. This will be the sum of the products of fabric tensions and areas per unit load and base width for the cap and wall:

$$\text{Area of cap} = 2\pi a^2(1 - \cos \alpha) \quad (24)$$

$$\text{Area of wall} = 2\pi b[c(\pi + \beta) + b(\cos \beta + 1)]. \quad (25)$$

{The derivation of (24) and (25) is given at Appendix II.}

From (24) and (25),

$$\begin{aligned} \eta' &= \frac{T_a \times \text{cap area} + T_b \times \text{wall area}}{Mg'c} \\ &= \frac{a^3(1 - \cos \alpha) + b^2 \left(\frac{\mu + 2}{\mu + 1} \right) [c(\pi + \beta) + b \cos \beta + 1]}{c^3}. \end{aligned} \quad (26)$$

Considering the case where $a = b$ and $\nu = \mu$ this becomes

$$\eta' = \mu^3(1 - \cos \alpha) + \mu^2 \left(\frac{\mu + 2}{\mu + 1} \right) [\pi + \beta + \mu(\cos \beta + 1)]. \quad (27)$$

Equation (23) has been solved for a range of values of α and β , and the results have been applied to equation (27) to determine η' , with $a = b$. Fig. 8 shows that η' is a minimum when both α and β are 90° , but that there is only a small increase in its value provided that α and β lie within the range 60° to 90° .

This result is fortunate in being in accord with the result for the centre section (Fig. 3). Because the end sections have a larger value of η' than the value of η for the centre, it will be preferable to choose the best possible value of η' at the expense of a slightly worse value of η —unless the tent is very long.

However, it may be considered hardly worthwhile to use special light fabrics for the ends. If one weight of fabric is to be used throughout, making a equal to b and using this same radius for ends—to match the centre, then for the purpose of fabric bulk calculation we can put

$$T_b = T_a = ap.$$

Hence with one fabric weight throughout, and $b = a$,

$$\begin{aligned} \eta' &= \frac{ap\{2\pi a^2(1 - \cos \alpha) + 2\pi a[c(\pi + \beta) + a(\cos \beta + 1)]\}}{p\pi c^3} \\ &= 2\mu^3(1 - \cos \alpha) + 2\mu^3(\pi + \beta) + 2\mu^3(\cos \beta + 1) \\ &= 2\mu^3(2 - \cos \alpha + \cos \beta) + 2\mu^3(\pi + \beta). \end{aligned} \quad (28)$$

Fig. 9 has been calculated from (28) for a range of values of α and β , and shows the same general variation as before. η' is low when both α and β lie between about 60° and 90° . The penalty of using the unnecessarily strong fabric in the end parts is apparent in that η' is roughly 30% greater than when fabrics of the minimum required strength are used (Fig. 8). For low values of β the penalty becomes greater.

Loads in the centre and ends of a tent should be evenly distributed according to the base aperture areas in these parts.

If the centre is of length nc , total fabric bulk will then be proportional to

$$\eta 2ac^2 + \eta' \pi c^2$$

or with a given base, to

$$2n\eta + \pi\eta'.$$

In a typical tent where $n = 4$, use of light fabric in the ends leads to a saving of only around 12% in total bulk.

5.5. Conditions During Inflation.

During the initial inflation, with the load resting on the ground, and with reduced pressure, the points AA' of Fig. 2 will be held down by the load so that the lower canopy or wall is flatter and wider. This results in a larger area in contact with the ground, and the load begins to lift at a pressure below the equilibrium value. Both b and c may be different from the equilibrium values, and μ will vary during inflation.

With a given value of S_b for the wall, at equilibrium

$$S_b = b(\pi + \beta).$$

At reduced pressure, the effective base is wider by some amount δc —the amount of the wall which lies on the ground, and if b_1 and β_1 denote values of b and β in this condition,

$$S_b = b_1(\pi + \beta_1) + \delta c.$$

At equilibrium

$$\lambda = \frac{b(\pi + \beta)}{c} \quad (29)$$

and at the reduced pressure

$$\lambda_1 = \frac{b_1(\pi + \beta_1) + \delta c}{c + \delta c}. \quad (30)$$

δc can never be negative, and hence because the numerators in (29) and (30) are equal, λ_1 must never exceed λ . That is to say, the required value of λ_1 at any stage of inflation must not exceed the value of λ which has been pre-supposed for the equilibrium state.

From Fig. 4 it is simple to choose parameters which satisfy this requirement. For example, with $\alpha = 90^\circ$ and $\beta = 45^\circ$ as a proposed equilibrium configuration, it is seen that the required value of λ_1 is 0.32. The required value falls as β rises up to approximately 70° but rises again to 0.32 when β reaches 90° . Therefore, if enough fabric is provided to satisfy the configurations for $\beta = 45^\circ$ and 90° there will be more than sufficient to satisfy all intermediate stages of inflation, and it will be possible to lift the load over this range.

Over the lifting range where the effective base is increased, the required pressure is reduced, so that, as soon as the load leaves the ground, the tent will tend to expand rapidly at first and lift the load with little additional inflow of air.

5.6. Stability.

Because the friction forces between the structure and the ground at equilibrium are very small, it is to be expected that side forces will merely cause the device to move bodily sideways rather than to overturn. However, if the base collides with a projection on the ground (Fig. 10) a horizontal deceleration force P will act at the centre of gravity of the load. For the device to resist overturning, the resultant, Q , of P and Mg must pass through the centroid, G , of the base area in contact with

the ground. The limiting condition is reached when the edge of the orifice at D' is about to rise and cause deflation, and the opposite edge of the base at C is on the ground. The approximate shape of the ground contact area is then as shown in the lower, plan view of Fig. 10.

It is worth noting that such a tent may also be supported on water, without the need for a buoyancy base tube such as provided for dinghy inflation. Collapse of the tent will only occur if tilted to the point where some part of the base orifice begins to rise above the level of the water under the base. A model of the section of Fig. 7c is quite stable.

The position of G can be estimated, but its accuracy depends on the correctness of the estimate of the shape of the area in ground contact. Furthermore, the side force P will alter the values of the forces F , in the load support cords, and will thus adversely change the shape of the canopy and the position of the load relative to the base. In practice it will almost certainly be necessary to maintain the shape by guys from the load to the base as indicated in Fig. 1.

Owing to the great flexibility of devices of the type now considered, it would appear impracticable to attempt the calculation of deformation under load. There exists the alternative of practical tests with scale models, giving due consideration to the effects of scale changes or rigidity.

The elasticity of the loaded tent, together with its inertia will result in certain natural frequencies of oscillation. These oscillations will normally be held in check by the use of guys from the load to the base, leaving relatively harmless, small fluttering motions in the parts of the fabric more remote from the guys and load cords. A further problem is the tendency of the base to flutter with a vertical motion which could lead to a serious waste of air. Possible remedies of this fault include a weighted base and an air-spoiling curtain or skirt attached to the base or regulation of the air supply, but little experience has yet been obtained of such devices.

6. *Conclusions.*

Use of the graphical results in this report should enable a design to be optimised for any application of membrane construction.

In general there are no significant differences as regards bulk of fabric required between the membrane and 'generator cord' constructions.

For applications which favour 'generator cord' construction the same methods of optimisation apply to the central part of the structure. Detailed design of the body-of-revolution ends in this construction follows the principles given in Ref. 1.

The complexity of stability calculations which would result from the great flexibility in all directions of these structures renders it necessary to assess the stability with scale models.

LIST OF SYMBOLS

a	Radius of curvature of cap
b	Radius of curvature of wall
c	Width of semi-base aperture
F	Tension in load support cords per unit sectional length
H	Height of upper ends of load support cords above base
Mg	Load per unit sectional length
Mg'	Load on the two end portions
p	Air inflation pressure at equilibrium
S	Length of fabric in semi-cross-section
T	Fabric tension per unit sectional length
α	Angle of cap to horizontal at re-entry
β	Angle of wall to horizontal at re-entry
γ	Angle of load support cords to horizontal
ζ	Height coefficient (H/c)
η	Fabric bulk coefficient for central portion
η'	Fabric bulk coefficient for end portions
λ	Fabric length coefficient (S/c)
μ	Wall radius coefficient (b/c)
ν	Cap radius coefficient (a/c)
ρ	Radius of curvature, general

Suffices

a	Pertaining to the cap
b	Pertaining to the wall
m	Pertaining to the fabric gore
p	Planar
n	Normal
1	Pertaining to partial inflation

REFERENCE

<i>No.</i>	<i>Author</i>	<i>Title, etc.</i>
1	W. G. S. Lester . . .	Inflated mobile lifting structures: analysis of generator-cord construction for circular planforms. A.R.C. R. & M. 3377. July, 1962.

APPENDIX I

Comparative Weights of Loaded-Cord and Membrane Structures

Consider a simple inflated hemispherical structure of radius a , supporting a load Mg contained in a hem spaced evenly round the base. It will be assumed that all materials have the same strength to weight ratio.

For a loaded-cord construction we may assume that the area of fabric is practically that of the hemisphere: although the flattened 'Taylor' shape reduces the area, the billowing fabric gores increase it.

$$\text{Total cord strength} = Mg$$

$$\text{Cord length} = 1.3a \text{ (for Taylor shape with } n = 8)$$

$$\text{Hence total cord weight} \propto aMg$$

$$\text{If } \rho_m \text{ (gore radius)} = \frac{a}{2}$$

$$\text{Gore fabric strength} = \frac{1}{2} Mg/2\pi a$$

$$\text{Fabric area} \simeq \frac{4}{2} \pi a^2$$

$$\text{Hence fabric weight} \propto \frac{a}{2} Mg$$

and

$$\text{Total weight} \propto 1.8 aMg$$

on the assumption that no strength and no weight is required in the gore fabric in the planar direction. In fact the fabric weight will be rather greater, leading to a probable figure of $2aMg$ for the structure weight factor.

For a membrane construction with flat panels having the warp running from base to top,

$$\text{Warp strength} = Mg/2\pi a$$

and weft must be the same, giving a fabric strength $Mg/\pi a$.

$$\text{Fabric area} = \frac{4}{2} \pi a^2.$$

Hence fabric (and total) weight $\propto 2aMg$ – substantially the same as for the loaded-cord construction.

APPENDIX II

Area of Surface of the Cap and Wall of the Tent Ends

Cap.

This is a zone of a sphere of radius a and height,

$$h = a(1 - \cos \alpha).$$

$$\text{Surface area of a gore} = 2\pi ah$$

$$= 2\pi a^2(1 - \cos \alpha).$$

Wall.

This is a part of a torus (Fig. 11), which if complete (the case where $\beta_0 = \pi$) would be of area

$$4\pi^2 bc.$$

Consider a narrow annular strip of width $b\delta\beta$ and circumference $2\pi(c - b \sin \beta)$.

Its area is

$$2\pi(c - b \sin \beta)b\delta\beta.$$

The whole area is, for the range $\beta = \beta_0$ to $\beta = -\pi$,

$$2\pi b \int_{-\pi}^{\beta_0} (c - b \sin \beta) d\beta = 2\pi b [c(\pi + \beta) + b(\cos \beta + 1)].$$

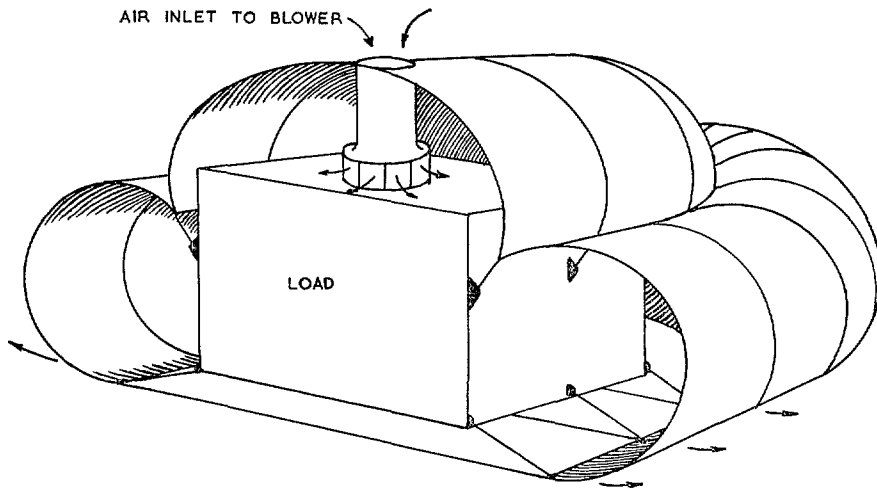


FIG. 1. Part sectional view of structure.

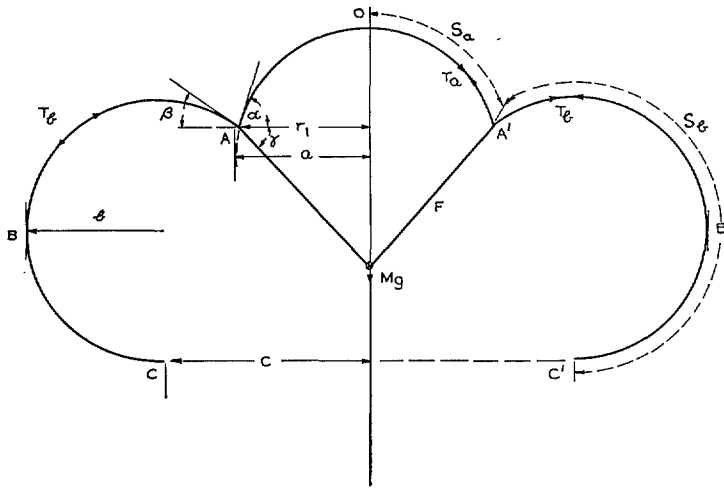


FIG. 2. Cross-section of inflated structure.

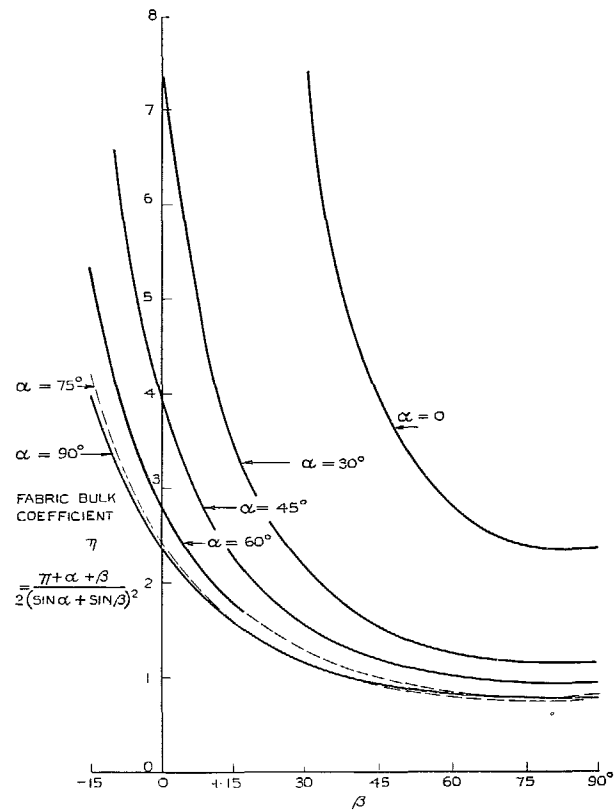


FIG. 3. Variation of fabric bulk coefficient, η , with α and β when $a = b$.

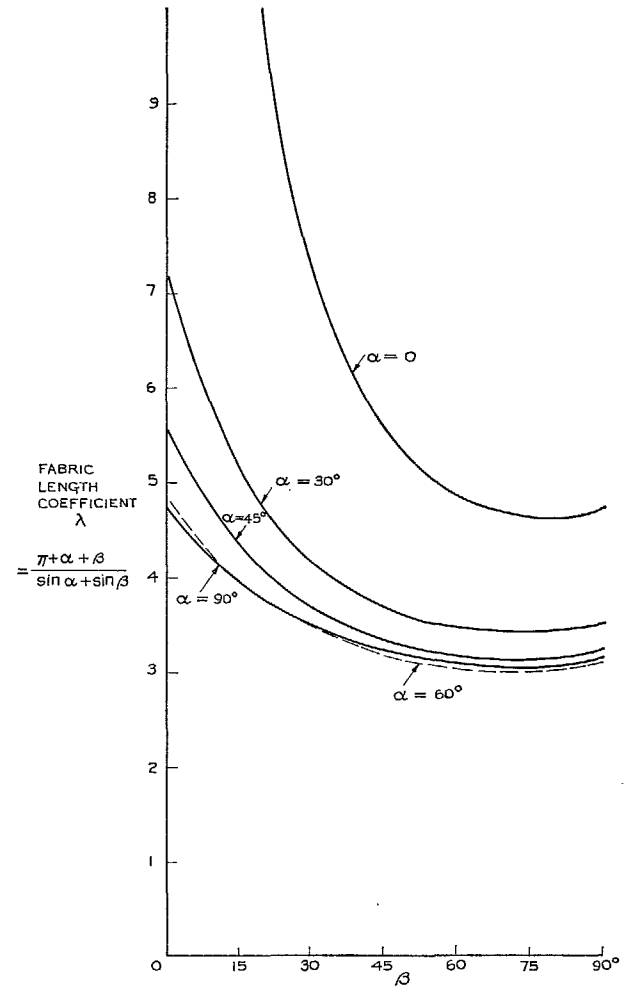


FIG. 4. Variation of fabric length coefficient, λ , with α and β when $a = b$.

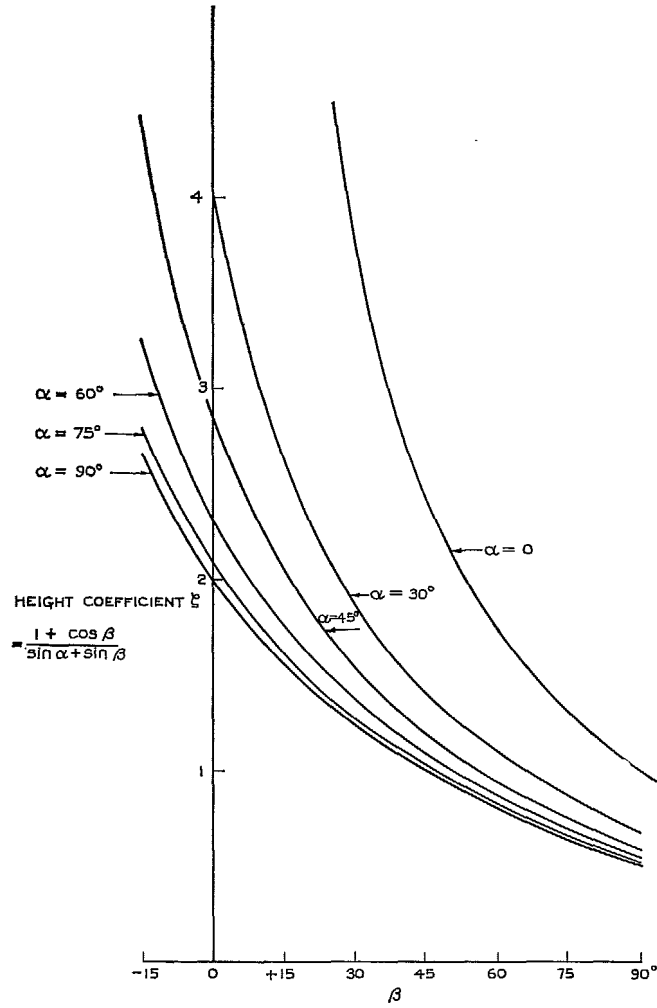


FIG. 5. Variation of height coefficient, ζ , with α and β when $a = b$.

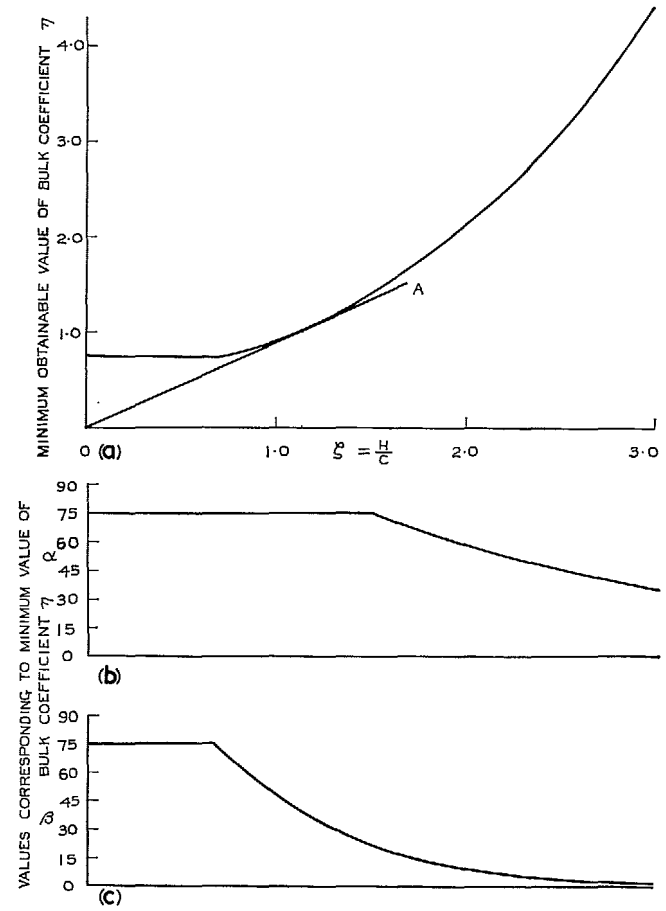


FIG. 6. Variation of minimum obtainable bulk coefficient, η , with height coefficient, ζ (and corresponding variation of α and β when $a = b$).

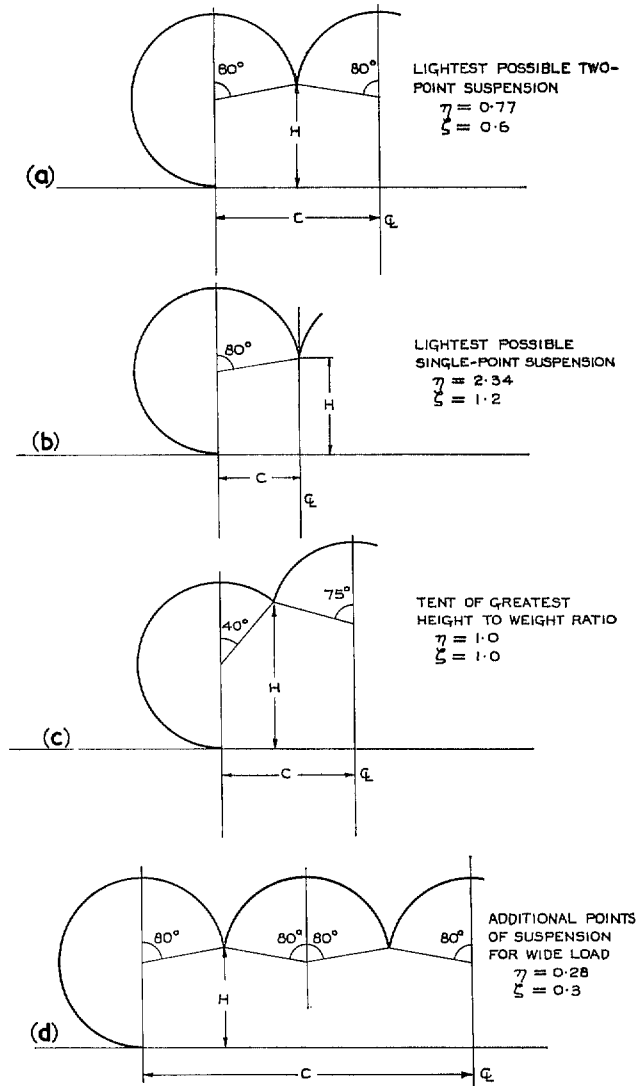


FIG. 7. Examples of tents for various requirements.

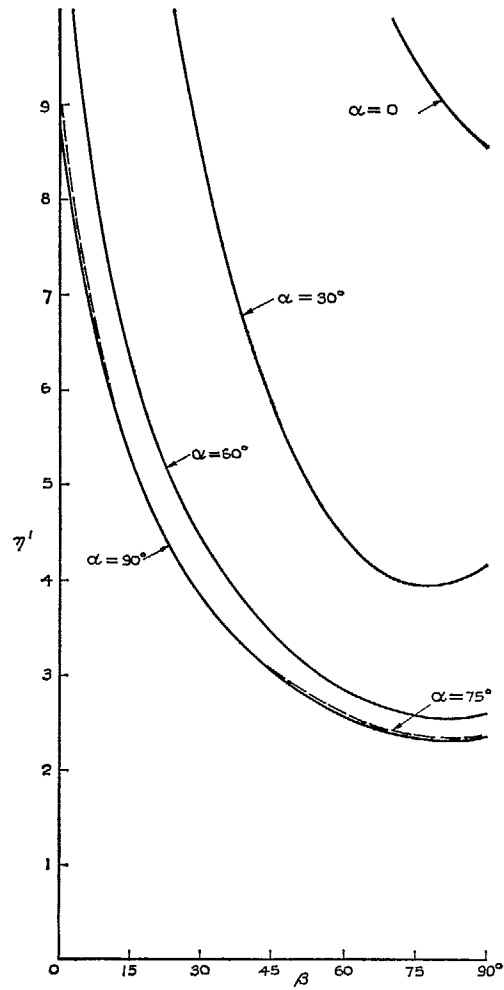


FIG. 8. Variation of end bulk coefficient, η' , with α and β when $a = b$ (using minimum strength fabrics for cap and wall).

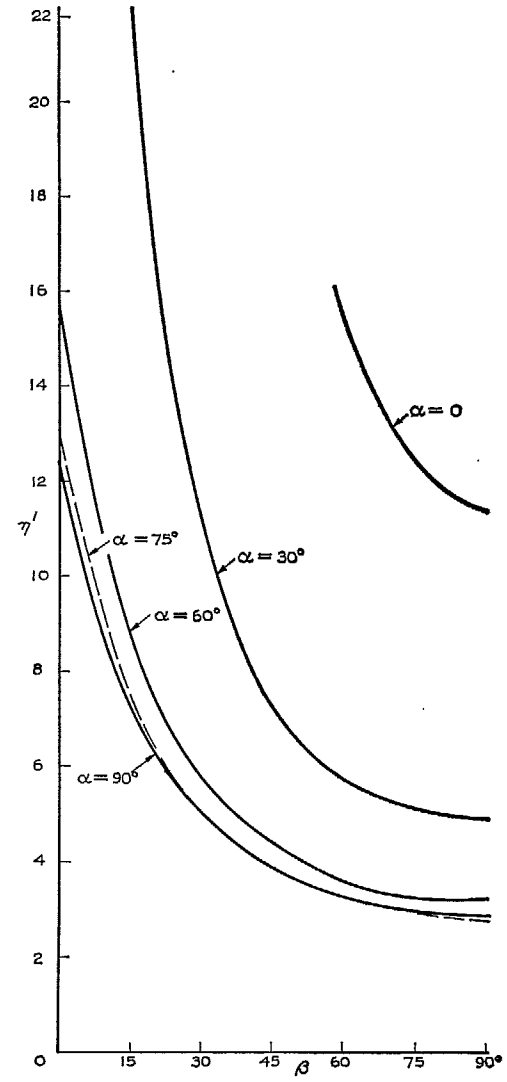


FIG. 9. Variation of end bulk coefficient, η' , with α and β when $a = b$ (with same fabric strength as for centre portion).

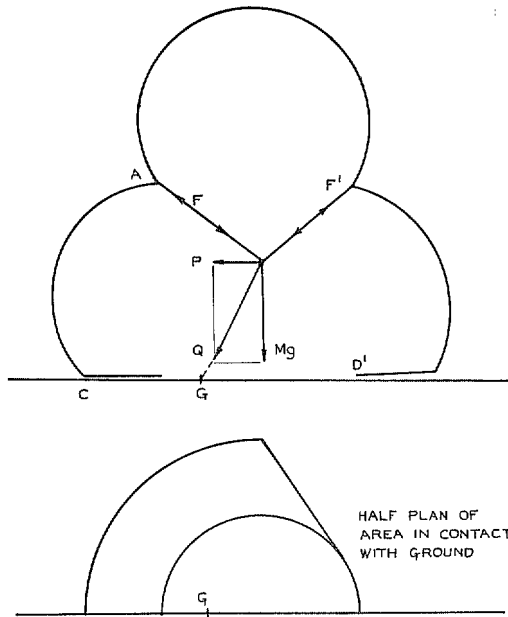


FIG. 10. Structure under side load.

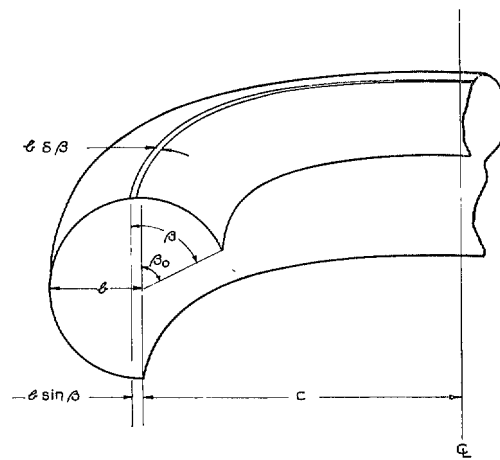


FIG. 11. Toroidal surface of tent end wall.

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