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An Equivalence Law Relating Three- and Two-Dimensional Pressure Distributions

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An Equivalence Law Relating Three- and Two-Dimensional Pressure Distributions

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Summary.

Two wings may be said to have equivalent pressure distributions if the distribution of M_n, the component of surface Mach number normal to the local isobars, is the same in both cases. A simple approximate method is given of calculating M_n from a knowledge of the pressure distribution, and some satisfactory comparisons with a more accurate theory are obtained. It is then possible to define an equivalent two-dimensional pressure distribution from a knowledge of the three-dimensional distribution, or *vice versa*.

1. *Introduction.*

In choosing the design pressure distribution for a swept wing at high speeds, or in analysing a measured pressure distribution on an existing wing, the concept of the 'equivalent two-dimensional wing' is of great importance; it is through this concept that improvements in two-dimensional section design can be incorporated in a real three-dimensional design, and it has also been used in predicting transonic effects on three-dimensional wings. For a swept wing of infinite extent and constant chord, the relationship with the corresponding two-dimensional section (which in this case is simply the section of the swept wing normal to the leading edge) is well known; the pressure coefficient on the swept wing (at a freestream Mach number M_1), is given by

$$C_p = C_p' \cos^2 \Lambda ,$$

where Λ is the angle of sweep and C_p' the corresponding pressure coefficient in the two-dimensional flow at the Mach number $M_1 \cos \Lambda$. But for a finite wing, particularly when taper is present, it is much less simple to define an equivalent two-dimensional pressure distribution, and it is the purpose of this paper to clarify the position as far as is at present possible.

The salient feature of the equivalence previously mentioned between an infinite yawed wing and its normal section is that in the two flows the components of local Mach number normal to the surface isobars are the same in both cases. It is proposed to extend this definition to a general three-dimensional wing. That is, the flow at a particular spanwise station over a given swept wing, at a freestream Mach number M_1, is said to be equivalent to the flow over a two-dimensional section at some reduced Mach number M_n^* provided that the local Mach number $M'(\xi)$ at a fraction ξ of the chord from the leading edge of the two-dimensional section is equal to $M_n(\xi)$, the component of local Mach number normal to the local isobar at the same fraction of the chord on the swept wing. The importance of this definition is that in an effectively transonic flow over a swept wing the principal non-linear features, and in particular the appearance and strength of the shock waves, may be expected (according to Bickley's criterion) to be governed by the distribution of M_n over the wing surface.

† Replaces N.P.L. Aero. Report No. 1028—A.R.C. 23,952.

Unfortunately, even when the pressure distribution over a three-dimensional wing is specified (measured or assumed) it is not immediately possible to calculate the distribution of M_n. The reason for this is that, though a knowledge of C_p does imply a knowledge of the *magnitude* of the local surface velocity $\mathbf{q}$ (assuming isentropic flow), the *direction* of $\mathbf{q}$ is still unknown, and this is of course required before M_n can be found. The next two sections will be devoted to a discussion of methods for calculating M_n; the first gives a simple approximate method and the second a more accurate one. The second method simplifies appreciably when the isobars follow the lines of constant ξ (as would often be assumed in an ideal design), and in particular it is possible to calculate the way in which the critical pressure coefficient (corresponding to $M_n = 1$) varies with the local angle of isobar sweep.

2. *Approximate Theory.*

An assumption which has sometimes been made in attempting to resolve the difficulty mentioned above is as follows. Suppose that at any point on the wing surface the local isobar makes an angle Λ with the normal to the freestream direction. Then we assume that the relation between M_n and the pressure coefficient C_p is locally the same as that for an infinite yawed wing of sweep Λ, which is of course itself equivalent to a two-dimensional wing with pressure coefficient $C_p' = C_p \sec^2 \Lambda$ at a freestream Mach number $M_1' = M_1 \cos \Lambda$. Using the standard relation between local Mach number and pressure coefficient for isentropic flow, we obtain the following expression for M_n, the component of local Mach number normal to the isobar:

M dist^n

$$
\begin{aligned}
1 + \tfrac{1}{2}(\gamma-1)M_n^2 &= \frac{1 + \frac{1}{2}(\gamma-1)M_1'^2}{(1+\frac{1}{2}\gamma M_1'^2 C_p')^{(\gamma-1)/\gamma}} \\
&= \frac{1 + \frac{1}{2}(\gamma-1)M_1^2 \cos^2 \Lambda}{(1+\frac{1}{2}\gamma M_1^2 C_p)^{(\gamma-1)/\gamma}}. \qquad (1)
\end{aligned}
$$

In this way, the distribution of M_n along the chord can be calculated from a knowledge of the local pressure coefficient C_p and isobar sweep Λ.

It is interesting to note that exactly the same result can be obtained from the alternative assumption that the component of local surface velocity *along* the isobar is $U_1 \sin \Lambda$, where U_1 is the freestream velocity (i.e. that the perturbation velocity is normal to the isobar); this assumption is of course true for an infinite yawed wing, but not necessarily in general. To prove this, we note that the component of velocity normal to the isobar is then given by

$$
q_n^2 = q^2 - U_1^2 \sin^2 \Lambda. \qquad (2)
$$

Dividing by the square of the local velocity of sound a, it follows that

$$
\begin{aligned}
M_n^2 &= M^2 - \frac{U_1^2}{a^2} \sin^2 \Lambda, \text{ where } M = \frac{q}{a} \text{ is the local Mach number} \\
&= M^2 - M_1^2 \sin^2 \Lambda \, \frac{a_1^2}{a^2}.
\end{aligned}
$$

But

$$
\frac{a_1^2}{a^2} = \frac{1 + \frac{1}{2}(\gamma-1)M^2}{1 + \frac{1}{2}(\gamma-1)M_1^2}, \text{ and so}
$$

$$
1 + \tfrac{1}{2}(\gamma-1)M_n^2 = 1 + \tfrac{1}{2}(\gamma-1)M^2 - \tfrac{1}{2}(\gamma-1)M_1^2 \sin^2 \Lambda \, \frac{1 + \frac{1}{2}(\gamma-1)M^2}{1 + \frac{1}{2}(\gamma-1)M_1^2}. \qquad (3)
$$

Making use of the fact that

$$\frac{1+\frac{1}{2}(\gamma-1)M^2}{1+\frac{1}{2}(\gamma-1)M_1^{\,2}} = \frac{1}{(1+\frac{1}{2}\gamma M_1^{\,2}C_p)^{(\gamma-1)/\gamma}},$$

equation (1) follows immediately.

As an example of the use of equation (1), we can at once calculate the value of the critical pressure coefficient, by putting $M_n = 1$; we obtain

$$\tfrac{1}{2}\gamma M_1^{\,2}C_{p\,\mathrm{crit}} = \left(\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M_1^{\,2}\cos^2\Lambda\right)^{\gamma/(\gamma-1)} - 1\,. \tag{4}$$

This agrees with the standard result, quoted for example by Bagley[1]. Though originally derived for an infinite untapered wing, it has been used to define local values of $C_{p\,\mathrm{crit}}$ for a tapered wing where the local sweep varies along the chord.

We are now in a position to define an equivalent two-dimensional pressure distribution. We choose a suitable mean sweep angle Λ^* and corresponding mean Mach number

$$M_n^{\,*} = M_1\cos\Lambda^*,$$

and specify that the distribution of local Mach number, at a freestream Mach number $M_n^{\,*}$, shall be the same as the distribution of M_n, given by equation (1) above. The equivalent two-dimensional pressure coefficient is therefore C_p', given by

$$1+\tfrac{1}{2}(\gamma-1)M_n^{\,2} = \frac{1+\frac{1}{2}(\gamma-1)M_n^{\,*2}}{(1+\frac{1}{2}\gamma M_n^{\,*2}C_p')^{(\gamma-1)/\gamma}}, \tag{5}$$

so that

$$\frac{1+\frac{1}{2}(\gamma-1)M_n^{\,*2}}{1+\frac{1}{2}(\gamma-1)M_1^{\,2}\cos^2\Lambda} = \left(\frac{1+\frac{1}{2}\gamma M_n^{\,*2}C_p'}{1+\frac{1}{2}\gamma M_1^{\,2}C_p}\right)^{(\gamma-1)/\gamma} \tag{6}$$

Equation (6) may be written

3D 2D

$$\left.\begin{aligned} 1+\tfrac{1}{2}\gamma M_1^{\,2}C_p &= (1+\tfrac{1}{2}\gamma M_n^{\,*2}C_p')f(M_1,\Lambda,\Lambda^*) \\ \text{where}\quad f &= \left\{\frac{1+\frac{1}{2}(\gamma-1)M_1^{\,2}\cos^2\Lambda}{1+\frac{1}{2}(\gamma-1)M_1^{\,2}\cos^2\Lambda^*}\right\}^{\gamma/(\gamma-1)}. \end{aligned}\right\} \tag{7}$$

[Note that if the standard ratio of static pressure to total head for isentropic flow is

$$\frac{p}{H} = P(M), \text{ where } P = \{1+\tfrac{1}{2}(\gamma-1)M^2\}^{-\gamma/(\gamma-1)}, \text{ then}$$

$$f = P(M_1\cos\Lambda^*)/P(M_1\cos\Lambda).]$$

It is sometimes convenient to think, as an intermediate stage, of an equivalent infinite yawed wing (at the same freestream Mach number) of sweep Λ^* and pressure coefficient

$$C_p^{\,*} = C_p'\cos^2\Lambda^*.$$

This is related to the pressure coefficient on the actual wing by equation (7), which becomes

$$C_p = \frac{f-1}{\frac{1}{2}\gamma M_1^{\,2}} + fC_p^{\,*}. \tag{8}$$

TRU has programme!

In a design problem, we shall normally specify that the local isobar sweep Λ is the same as that of the line ξ = constant, so that Λ will decrease linearly from Λ_0 at the leading edge to Λ_1 at the trailing edge. Now $f < 1$ when $\Lambda > \Lambda^*$

$$f = 1 \text{ when } \Lambda = \Lambda^*$$

$$\text{and } f > 1 \text{ when } \Lambda < \Lambda^*.$$

Thus a typical yawed-wing pressure distribution will be distorted by the effect of taper (in order to keep the same distribution of M_n) as sketched in Fig. 1. Since C_p and C_p^* are usually fairly small, the dominant term in equation (8) is $(f-1)/\frac{1}{2}\gamma M_1^2$, which ensures in particular that the pressures at the leading edge are correctly related in the two cases.

The local wing loading $l\{= C_p(\text{lower}) - C_p(\text{upper})\}$ for the tapered swept wing is related to the value l^* for the yawed wing by

$$l = fl^*. \tag{9}$$

Somewhat surprisingly, this means that the loading on the tapered wing must be reduced relative to the yawed wing near the leading edge, and increased near the trailing edge; but the effect is normally smaller than that of the first term in equation (8), which affects both upper and lower surfaces equally. In a practical design procedure it is usually the upper surface that is of primary importance, so that it will often be sufficient to adjust the thickness and loading to obtain the correct correspondence with the given two-dimensional section on the upper surface only, without paying too much attention to achieving the same correspondence on the lower surface as well.

So far nothing has been said about the choice of the mean angle of sweep Λ^*. At first sight it appears possible to choose this arbitrarily, so long as it lies between the leading-edge sweep Λ_0 and trailing-edge sweep Λ_1. If we start from the pressure distribution on a given tapered wing, then for any value of Λ^* we shall arrive at a corresponding two-dimensional pressure distribution on a hypothetical wing section, still to be determined. But if Λ^* is not chosen correctly then this two-dimensional section will either close before the trailing edge is reached (if Λ^* is too small) or have a finite trailing-edge thickness (if Λ^* is too large). Since there is no simple criterion whether a given pressure distribution determines a closed section, this question can only be solved by experience; though again in practice it is often necessary to achieve complete equivalence between two and three dimensions only over the forward part of the sections, so that Λ^* may be chosen somewhat larger than suggested by the above considerations.

3. *Improved Theory.*

The calculation of the separate components of velocity in potential flow at the surface of a wing from a knowledge only of the pressure distribution has been considered by Der and Raetz[2]. If ξ_1 and ξ_2 are orthogonal curvilinear co-ordinates on the wing surface and h_1, h_2 are the corresponding metrical coefficients (such that an element of length ds on the surface is given by $ds^2 = h_1^2 d\xi_1^2 + h_2^2 d\xi_2^2$), then the component of vorticity normal to the surface is

$$\Omega_n = \frac{1}{h_1 h_2}\left[\frac{\partial}{\partial \xi_1}(h_2 u_2) - \frac{\partial}{\partial \xi_2}(h_1 u_1)\right],$$

where u_1, u_2 are the components of velocity in the ξ_1 and ξ_2 directions respectively. If the external flow is assumed to be irrotational, then $\Omega_n = 0$, so that

$$\frac{\partial}{\partial \xi_1}(h_2 u_2) = \frac{\partial}{\partial \xi_2}(h_1 u_1), \tag{10}$$

and also

$$u_1^2 + u_2^2 = q^2(\xi_1, \xi_2), \tag{11}$$

where q^2 is a given function of ξ_1 and ξ_2, obtained from the pressure distribution by means of the isentropic equation

$$\left.\begin{aligned} 1 + \tfrac{1}{2}(\gamma-1)M_1^2\left(1 - \frac{q^2}{U_1^2}\right) &= \left(\frac{p}{p_1}\right)^{(\gamma-1)/\gamma} \\ &= (1+\tfrac{1}{2}\gamma M_1^2 C_p)^{(\gamma-1)/\gamma} \end{aligned}\right\} \tag{12}$$

where p_1 is the freestream static pressure. For a given wing shape it is first necessary to specify the co-ordinate system; in a general case this may be a matter of some complexity, but for a wing with straight leading and trailing edges and similar streamwise sections (so that the surface is conical with respect to the tip) a natural co-ordinate system is as shown in Fig. 2 (cf. Ref. 2). If S is the tip then we take ξ_2 to be the distance of the point P from the tip along the generator PS. The co-ordinate ξ_1 orthogonal to ξ_2 may be chosen to be the angle θ between the generator PS and the leading edge *when the surface is developed into a plane.* In this case we have

$$h_1 = \xi_2, \; h_2 = 1,$$

so that equation (10) becomes

$$\frac{\partial u_2}{\partial \xi_1} = u_1 + \xi_2 \frac{\partial u_1}{\partial \xi_2}. \tag{13}$$

It is still necessary to determine ξ_1 and ξ_2 in terms of the Cartesian equation of the surface. If the root section (assumed to be of unit chord) is

$$\begin{cases} z = f(x) & (0 \leqslant x \leqslant 1) \\ y = 0, \end{cases}$$

and the tip S is $(x_0, y_0, 0)$ then the equation of the wing surface is

$$z = \left(1 - \frac{y}{y_0}\right) f(\xi) \tag{14}$$

where $\xi = (xy_0 - x_0 y)/(y_0 - y)$ is the usual chordwise co-ordinate. Then clearly

$$\xi_2 = \sqrt{\{(x-x_0)^2 + (y-y_0)^2 + z^2\}},$$

and it can easily be shown that

$$\theta = y_0 \int_0^{\xi} \frac{\sqrt{[1 + \{f'(\xi)\}^2]}}{(x_0 - \xi)^2 + y_0^2 + f^2(\xi)}\, d\xi. \tag{15}$$

For a thin wing it will almost always be possible to neglect the terms in z^2 and f^2 in (15), giving

$$\xi_2 \simeq \sqrt{\{(x-x_0)^2 + (y-y_0)^2\}}$$

$$\theta \simeq y_0 \int_0^{\xi} \frac{\sqrt{[1 + \{f'(\xi)\}^2]}}{(x_0 - \xi)^2 + y_0^2}\, d\xi,$$

but the additional approximation of neglecting the term in $\{f'(x)\}^2$, which is equivalent to replacing the wing surface by its projection on the plane $z = 0$, may not always be justifiable, particularly near a rounded leading edge. At the leading edge, or more precisely along the 'quasi-stagnation line' there (the line of minimum velocity), the velocity component u_1 is zero and $u_2 = q$, so that initial data for u_1 and u_2 are known along this line. It is therefore possible to integrate equations (11) and (12) numerically, starting from this line (which is either at or very close to the leading edge $\xi_1 = 0$).

For a given velocity distribution of general type the numerical problem is still quite difficult; but if the whole or part of a wing is such that the pressure (and hence q) is constant along the generators (ξ_1 = constant), then a considerable simplification occurs. Such a condition would of course be assumed for a wing designed so that the chordwise pressure distribution is independent of spanwise position. In this case u_1 and u_2 will both be functions of ξ_1 (or θ) only so that equation (13) becomes simply

$$\begin{aligned} \frac{du_2}{d\theta} &= u_1 \\ &= \sqrt{(q^2 - u_2^2)}, \end{aligned} \tag{16}$$

with the initial conditions $u_1 = 0$

$$q = -u_2 = U_1 \sin \Lambda_0 \text{ at } \theta = 0.$$

This equation can be solved numerically by standard methods; alternatively we can assume as a first approximation that $u_2 = -U_1 \sin(\Lambda_0 - \theta)$, and use equation (16), in the form

$$u_2 = -U_1 \sin \Lambda_0 + \int_0^\theta \sqrt{(q^2 - u_2^2)} d\theta \tag{17}$$

to obtain successive approximations to u_2 by iteration. The required velocity component normal to the isobar is then simply

$$u_1 = \sqrt{(q^2 - u_2^2)}.$$

The local velocity of sound is given by Beronelli's equation in the form $a^2/U_1^2 = (1/M_1^2) + \frac{1}{2}(\gamma - 1)(1 - q^2/U_1^2)$, so that the Mach number component $M_n = u_1/a$ can be found at once. The equivalent yawed-wing pressure distribution can then be found from M_n as described in Section 2.

The result of applying this procedure to a typical pressure distribution is shown in Fig. 3. The wing chosen has a leading-edge sweep $\Lambda_0 = 71°$ and trailing-edge sweep $\Lambda_1 = 66°$, and the freestream Mach number is 2·0; the value of the sweep Λ^* for the equivalent yawed wing has been taken to be 70°. The result of the approximate theory of Section 2 {equation (8)} is included for comparison; it appears that in this case the error involved in the assumptions of Section 2 are small. (The difference between θ and $\Lambda_0 - \Lambda$ has been neglected.)

4. *Calculation of the Critical Pressure Coefficient.*

The approximate equation (4) suggests that the critical pressure coefficient is a unique function of the freestream Mach number and the local isobar sweep. This is not precisely true, and in fact the exact value of the critical pressure must depend also on the whole pressure distribution upstream of a critical point. It is interesting to consider a wing of tapered planform such that the pressure along the lines of constant ξ_1 is exactly critical from just downstream of the leading edge; this is obviously important in the design of three-dimensional wings which correspond to 'roof-top' sections in two dimensions. In this case the theory of Section 3 enables an explicit solution to be obtained, as follows.

Putting $M_n = 1$, we get

$$\frac{u_1^2}{U_1^2} = \frac{a^2}{U_1^2} = \frac{1}{M_1^2} + \tfrac{1}{2}(\gamma-1)\left(1 - \frac{q^2}{U_1^2}\right).$$

Since $u_1^2 = q^2 - u_2^2$, it follows that

$$\frac{u_1^2}{U_1^2} = \left(\frac{\gamma-1}{\gamma+1}\right)\left(\mu^2 - \frac{u_2^2}{U_1^2}\right),$$

where

$$\mu^2 = 1 + \frac{2}{(\gamma-1)M_1^2}.$$

Substituting in equation (16), we obtain

$$\frac{d}{d\theta}\left(\frac{u_2}{U_1}\right) = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\sqrt{\left(\mu^2 - \frac{u_2^2}{U_1^2}\right)}. \tag{18}$$

At $\theta = 0$, we can assume that conditions *are* the same as on an infinite wing of sweep Λ_0, so that

$$u_2 = -U_1 \sin\Lambda_0 \text{ when } \theta = 0.$$

Equation (18) can then be integrated to give

$$\frac{|u_2|}{U_1} = \mu \sin\left\{\theta_0 - \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\theta\right\}, \tag{19}$$

where

$$\theta_0 = \sin^{-1}\left(\frac{1}{\mu}\sin\Lambda_0\right).$$

Then

$$\frac{q^2}{U_1^2} = \frac{2}{\gamma+1}\left\{\frac{1}{M_1^2} + \tfrac{1}{2}(\gamma-1) + u_2^2\right\}$$

$$= \frac{2}{\gamma+1}\mu^2\left\{\tfrac{1}{2}(\gamma-1) + \sin^2\left[\theta_0 - \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\theta\right]\right\},$$

and the critical pressure coefficient follows from equation (12). We obtain finally

$$\tfrac{1}{2}\gamma M_1^2 C_{p\,\mathrm{crit}} = \left[\frac{2}{\gamma+1}\{1 + \tfrac{1}{2}(\gamma-1)M_1^2\}\cos^2\left\{\theta_0 - \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\theta\right\}\right]^{\gamma/(\gamma-1)} - 1. \tag{20}$$

Since the actual shape of the wing that will produce this hypothetical pressure distribution is not specified, θ is not known precisely in terms of the wing geometry, and it is reasonable to take $\theta = \Lambda_0 - \Lambda$ (the value for a flat wing). With this assumption, the value of $C_{p\,\mathrm{crit}}$ given by equation (20) can be compared with that given by simple sweep theory {equation (4)}. The two agree of course for $\Lambda = \Lambda_0$, but for $\Lambda < \Lambda_0$ the simple theory slightly overestimates the value of $(-C_{p\,\mathrm{crit}})$. An example is shown in Fig. 4 for $M_1 = 2\cdot 0, \Lambda_0 = 70°$. It is interesting to note that in this case $C_{p\,\mathrm{crit}}$ should be zero for $\Lambda \simeq 61°$, not 60° as might be expected. The reason for this apparent anomaly is that, though when $C_p = 0$ the *magnitude* of the velocity vector $\mathbf{q}$ is equal to U_1, the *direction* of $\mathbf{q}$ is not necessarily the same as that of the freestream.

5. *Conclusions.*

The examples shown in Figs. 3 and 4 suggest that the error in using the simple sweep theory of Section 2 is unlikely to be serious, unless the taper is large; and in that case the whole idea of equivalence with a yawed-wing or two-dimensional pressure distribution is probably not a sensible one. But there is no doubt that for wings of small or moderate taper the concept introduced in this paper is a valid one and should be considered when selecting the design pressure distribution for a tapered swept wing. Unfortunately, however, it gives no help in specifying the actual shape of the wings involved; and though the problem of finding the shape from the pressure distribution is comparatively easy in the two-dimensional case, it is clear that more research needs to be done on the three-dimensional aspects, with particular reference to the thickness problem.

LIST OF SYMBOLS

x, y, z		Cartesian co-ordinates
x_0, y_0		Co-ordinates of effective wing tip (S in Fig. 2)
ξ_1, ξ_2		Curvilinear co-ordinates (*see* Section 3)
ξ		Distance from leading edge divided by local chord
u_1, u_2		Components of velocity in the directions ξ_1, ξ_2
h_1, h_2		Metrical coefficients (*see* Section 3)
U_1		Freestream velocity
q		Total velocity $\{= \sqrt{(u_1^2+u_2^2)}\}$
Λ		Angle of sweep (suffices $_0$, $_1$ denote leading and trailing edges respectively)
$\Lambda_{1/4}$		Sweep of quarter-chord line
Λ^*		Sweep of equivalent yawed wing
M		Mach number
M_1		Freestream Mach number
M_n^*	=	$M_1 \cos \Lambda^*$
M_n		Component of Mach number normal to local isobar
p		Static pressure
p_1		Freestream static pressure
H		Total pressure
C_p		Pressure coefficient on three-dimensional wing
C_p^*		Pressure coefficient on equivalent yawed wing
C_p'	=	$C_p^* \sec^2 \Lambda^*$, pressure coefficient on equivalent two-dimensional section
a		Velocity of sound
f		Function defined by equation (7)
l		Wing loading $\{= C_p(\text{lower}) - C_p(\text{upper})\}$
l^*		Loading of equivalent yawed wing
θ		Angle defined in Section 3 (*see* Fig. 2)
γ		Ratio of specific heats
μ	=	$1 + \dfrac{2}{(\gamma-1)M_1^2}$
θ_0	=	$\sin^{-1}(\mu^{-1} \sin \Lambda_0)$

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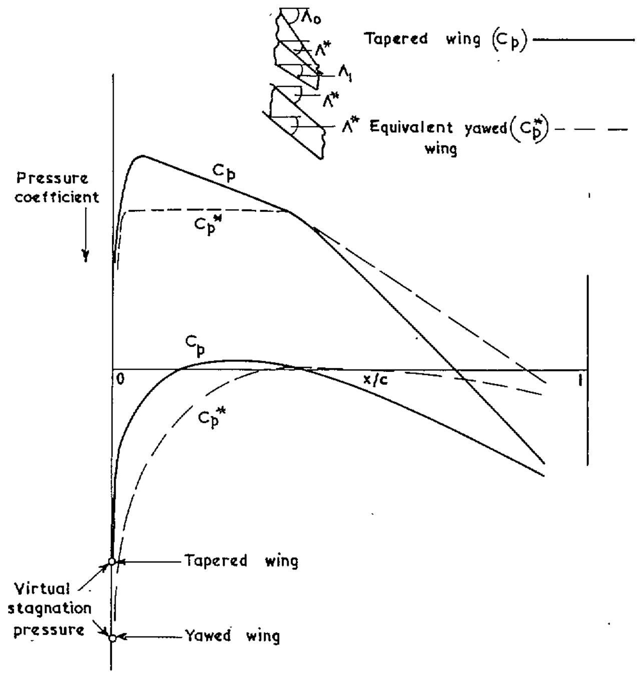

FIG. 1. Pressure distribution for a tapered wing and equivalent yawed wing.

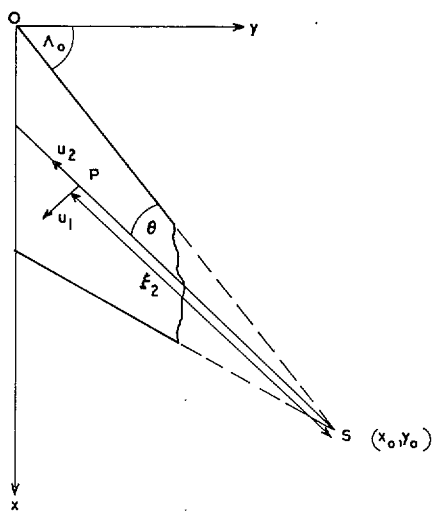

FIG. 2. Sketch showing co-ordinate system.

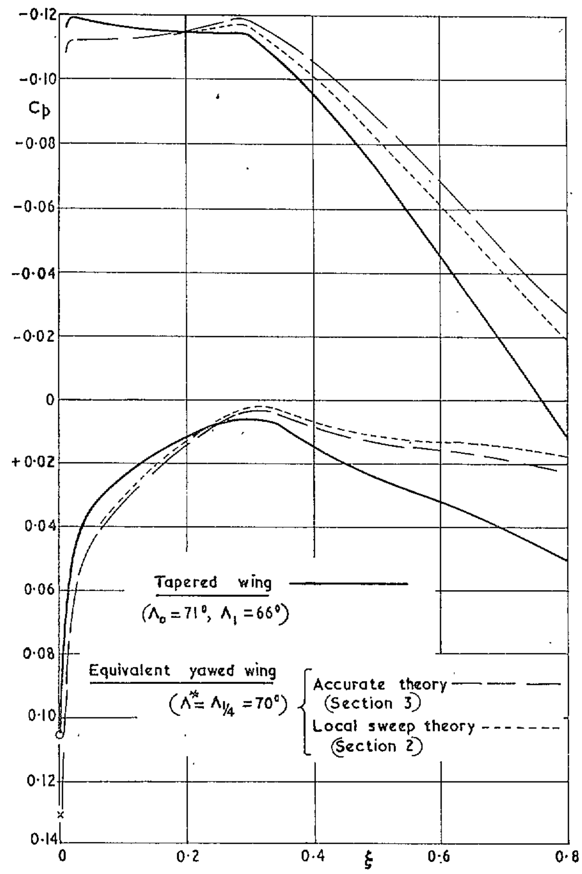

FIG. 3.

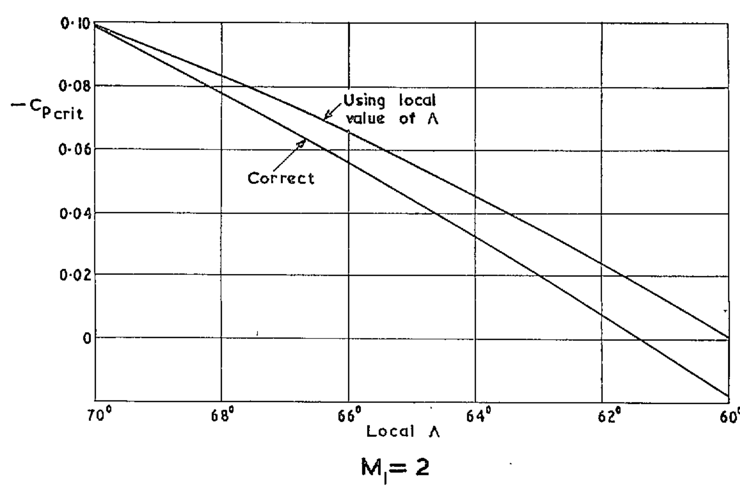

FIG. 4. Effect of taper on critical pressure coefficient.

(88991) Wt. 65/1418 K.5 2/64 Hw.

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