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An Analysis of Two-Dimensional Turbulent Base Flow, Including the Effect of the Approaching Boundary Layer

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Summary.

The present position of the theory of supersonic turbulent base flow is reviewed and arguments are put forward which suggest that the theory of Korst and others is incorrect in certain respects.

The analysis of two-dimensional turbulent base flow is formulated on the basis of a modified recompression criterion which is more consistent with experimental observation than the one previously used. At supersonic speeds examples are submitted to illustrate the use of the theory to predict the effect of an approaching boundary layer on base pressures and satisfactory agreement with measurements is obtained so long as the thickness of the boundary layer is small.

The theory is also shown to account for certain features of the steady base flow at subsonic and transonic speeds, e.g. the abrupt fall in base pressure which occurs near sonic velocity.

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1. *Introduction.*

1.1. The subject of two-dimensional supersonic base flow has received a great deal of attention in recent years from both the experimenter and the theoretician (*see* Ref. 1, for example), and the basic mechanisms which control such characteristics as the base pressure are to a large extent understood. The early attempts to construct models to represent certain features of the supersonic base flow have now given way to two principal lines of approach, the method of Crocco and Lees and that derived from the flow model proposed by Chapman.

Of these two the former[2] sets out to describe the essential balance which must be maintained between the external stream, which behaves according to the laws of an inviscid isentropic flow, and the viscous flow in the wake which is represented by integral relations that take some account of the mixing process. The Crocco-Lees theory is concerned mainly with the overall features of the base flow and was of particular use in accounting for the variations of base pressure with Mach number and Reynolds number.

The model of supersonic base flow suggested by Chapman[3] was more detailed and correspondingly more limited in scope. It had an advantage in that it rested on the dissection of the flow field into its constituent parts each of which could be understood more readily. The solution of the complete flow could then be built up from an analysis of the behaviour of each part under conditions which simulated to some extent the influence of the neighbouring regions on it.

From the initial flow model Chapman *et al* went on[4] to discuss the laminar-base-flow solution for the special case when the thickness of the boundary layer approaching the base was zero. (*See* Fig. 1.) The authors were realistic in emphasising the importance which this restriction had on the interpretation of the results of the solution and carefully arranged experiments were devised to check the theory.

In the case of turbulent base flow, Korst[5] and Kirk[16]* employed largely the same arguments to establish a method for predicting base pressures and for demonstrating the increase in base pressure which could be achieved by allowing fluid to bleed at low velocity into the wake—base bleed. Korst's theory has been used as the basis of a number of modifications to cover special cases of base flow, for example when a high-velocity jet is issuing from the base[6, 32], and in the presence of non-adiabatic bleed[7]. The theory of closed base flows[5], i.e. in the absence of bleed, predicts the variation of base pressure with Mach number for the restricted case when the thickness of the boundary layer approaching the base is zero. It compares very favourably[1] with a large number of measurements of base pressure on sections for which the ratio of boundary-layer thickness to base height is small. Moreover of the very few systematic experiments to measure the variation of base pressure with boundary-layer thickness, two[8, 9] produced results which suggested that, at a given Mach number, as the boundary-layer thickness tended to zero the base pressure approached the value predicted by Korst, and in a third[10] the extrapolation indicated a lower value of base pressure which could be attributed to cross-flow effects[11] associated with the small span of the model.

It would appear then that both the fundamental principles which form the basis of the theory of Korst and the details of the solution are supported by observation and that the principles are sufficiently understood for the theory to be extended readily to the more general classes of base flow, for example, to explain the variation of base pressure with Reynolds number. The objects of

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* The work on base flows was carried out by Kirk in 1954 but the paper reporting it was not issued by the Royal Aircraft Establishment until December, 1959.

the present paper are first to put forward arguments which suggest that this is by no means the case, and then to formulate a modified method which is more consistent with experimental observations and should lend itself to more general application.

1.2. The first of these arguments is simple. Both the experiments and the theory indicate that as the thickness of the boundary layer tends to zero the base pressure falls to some minimum value —the limiting base pressure[1]—which is a function only of Mach number. The consequence of this observation is that, at a given Mach number, base pressures below the limiting value can be reached only by the use of such devices as base suction (negative bleed), or when cross-flow or three-dimensional effects in the wake cause suction conditions to be approached. Thus the measurement of base pressures, lower than the predicted limiting values, which cannot be traced to the action of one of these agencies must necessarily cause serious doubts to be cast on the validity of the method.

Most of the tests on turbulent base flow at supersonic speeds either have been made at Reynolds numbers high enough to ensure that the transition point was well forward of the trailing edge of the section, or roughness elements have been used to fix transition ahead of the base. Under these conditions a well-developed turbulent boundary layer will have had time to grow up on the surface before it reaches the base, and very small values of the ratio of boundary-layer thickness to base height will be difficult to reach. It is then conceivable that conditions approaching the limiting base flow, and hence measurements of base pressure close to the limiting base pressure, have rarely been achieved. As the Reynolds number is decreased however, the transition point moves rearward towards the trailing edge and the ratio of boundary-layer thickness to base height is reduced resulting in a fall in the base pressure[12]. Further reduction of the Reynolds number allows transition to be delayed to some point in the wake and the base pressure to rise again. At a particular Reynolds number transition to turbulent flow will take place very close to, but upstream of, the trailing edge giving a turbulent base flow in which the thickness of the approaching boundary layer is at a minimum. On a fairly thick section under these conditions the experimental limiting base pressure may be approached.

Tests on the transitional base flow have been reported by Gadd *et al*[12] and by Van Hise[14]. In the former the span of the model was small and cross-flow effects may have been present. In the experiments by Van Hise however care was taken to minimise the influence of three-dimensional effects and the investigator was satisfied that over the portion of the model span where measurements were made the flow was sensibly two-dimensional. Base pressures substantially lower than the theoretical limiting values were measured for a range of Reynolds number. As an example, at a Mach number of 2·22 the ratio of limiting base pressure to ambient static pressure computed from the theory of Korst is 0·30 whereas values as low as 0·23 were recorded on ogive models of thickness-chord ratio 1/8; similar differences are found in the base pressures at Mach numbers of 1·95 and 2·92.

In the transonic speed range it is observed that the base pressure on two-dimensional sections falls to a local minimum[1]; moreover very little variation of base pressure is found with changes of the ratio of trailing-edge thickness to chord. The reasons for this latter effect will be discussed later but a useful result can be deduced from it at this stage. It is apparent that the increase of base pressure on sections in the low-supersonic speed range due to the presence of the boundary layer on the surface is small and it is plausible that the experimental limiting base pressure can again be approached. It was thought initially that the low base pressures encountered on blunt-trailing-edge sections near sonic velocity were associated with the persistence from subsonic speeds of periodic effects in the wake. Tests have now shown that at Mach numbers only slightly in excess of 1·0 the influence of unsteady phenomena is small and similar low pressures have been observed on a backward-facing step[15] over which the flow was essentially non-periodic. At a Mach number of $1 \cdot 1$, for example, the base-pressure coefficient was found to be $-0 \cdot 57$ compared with the value of $-0 \cdot 42$ calculated from the theory of Korst, or alternatively the ratio of base pressure to ambient static pressure was $0 \cdot 52$ compared with $0 \cdot 64$ predicted by the theory.

1.3. The second argument concerns the extension of the theory to include the effects of the approaching boundary layer. The effect of the presence of the boundary layer on base pressure was discussed by Kirk[16], who also formulated the theory of limiting base flow independently of Korst. While it was shown from the form of the solution that the predicted increase of base pressure with boundary-layer thickness would be qualitatively correct no computed results were presented. The present author has since performed some calculations[17] on the basis of Kirk's theory and found that the computed base pressure on a section with a given ratio of boundary-layer thickness to base height was considerably higher than the measured values for turbulent base flow. A similar lack of success was evident in the method of Carrière and Sirieix[18] which was an extension of Korst's theory but which used largely the same approach as Kirk's in dealing with the boundary-layer effect. The theory of limiting turbulent base flow due to Korst has also been extended to include the effect of the approaching boundary layer by Karashima[19]. The results were shown to compare very favourably with the measurements of Chapman *et al*[4] for laminar base flow, or at least under conditions where the transition point was downstream of the trailing edge of the body. The persistence of regions of laminar flow into the wake necessarily gives rise to a higher base pressure than would be supported by a fully turbulent base flow (*see* Section 1.2 above), and for this reason the comparison was not a valid test of the theory.

It is apparent that in turbulent base flow, the variation of base pressure, with the thickness of the boundary layer approaching the trailing edge cannot be accounted for quantitatively by an extension of the method of Korst as it stands, the theory indicating substantially higher values of base pressure for given boundary-layer thickness than are observed experimentally. The really important point which emerges from a study of the complete base-flow solution is however as follows. The form of the variation of base pressure with boundary-layer thickness derived from the theory indicates that the limiting base pressure cannot be estimated successfully by an extrapolation from measurements made at small but finite values of the ratio of boundary-layer thickness to base height. In the neighbourhood of the limiting condition the curve of base pressure against boundary-layer thickness at a given Mach number has both a large slope and a large negative second derivative and it is clear that a linear extrapolation of the curve from some position of finite boundary-layer thickness[9, 10] could result in a serious over-estimation of the limiting base pressure. The apparent agreement between the values of limiting base pressure predicted from Korst's theory on the one hand and from measurements with finite boundary-layer thickness on the other is thus undermined.

1.4. To sum up then we can review the present position of the theory of supersonic turbulent base flow. The method of Korst relates to the limiting condition when the thickness of the boundary layer on the surface of the body tends to zero, and as such should predict the lowest base pressure which can be supported by a turbulent base flow in the absence of suction. In two sets of conditions measurements can be made of base pressures substantially lower than the theoretical minimum values; at low supersonic speeds (Mach numbers less than about $1 \cdot 4$), and at higher supersonic speeds over the Reynolds number range in which the transition point is close to, but upstream of, the trailing edge. At moderate and high supersonic speeds when the transition point is well forward of the trailing edge the measured base pressures on thick sections are found to agree well with the values given by the theory. However no tests have been made on a section on which the thickness of the boundary layer was zero, and it is by no means certain that a simple extrapolation of the existing results can be made to zero boundary-layer thickness. Moreover if a correction is applied to the theoretical value to take account of the appropriate boundary-layer thickness the base pressure is then severely over-estimated.

On this evidence we believe that the methods of Korst, Kirk and others do not predict the true limiting base pressure. It becomes increasingly clear that the true limiting base pressure must be essentially lower than is indicated by either the theory or by the body of experimental results with which so favourable a correlation is obtained. The fact that the theory agrees so well with the measured base pressures on a variety of sections with a small but finite value of the ratio of boundary-layer thickness to base height is explained not in terms of the boundary-layer effect being small, but by recognising that the theory over-estimates the limiting base pressure by just the amount necessary to account for the boundary-layer effect in a large number of typical cases. This is of course consistent with the fact that extensions of the theory to predict the increase in base pressure brought about by various forms of bleed have shown such good agreement with experiment. None of these methods has taken deliberate account of the approaching boundary layer but an increment in base pressure roughly equal to the increase brought about by a typical boundary-layer thickness has, so to speak, been 'built in' to the results; the further effects of bleed are then correctly predicted.

1.5. In an attempt to remedy the serious deficiency thus revealed we shall proceed to make a thorough re-examination of the assumptions which form the basis of the theory of turbulent base flow and to determine the stage at which the observed effects deviate from them.

The scope of the associated analysis will not be restricted to the supersonic speed range but will be applicable also to the class of base flows generated by the subsonic flow over a backward-facing step. The flow in the wake of a blunt-trailing-edge aerofoil at subsonic speeds is generally dominated by the break-up of the wake into periodic vortices and cannot be represented by a steady-flow model of the form to be considered. It has long been recognised however[4] that if the shedding of periodic vortices can be inhibited the subsonic base flow closely resembles that at supersonic speeds and can be treated in an analogous manner. The subsonic flow past a step is a good example of a steady (or at least aperiodic) base flow and it will be seen that several of its features, such as the abrupt fall in base pressure as the Mach number approaches unity[15] can be explained in terms of steady-flow effects.

At supersonic speeds the flow on either side of the plane of symmetry in the wake behind a blunt-trailing-edge section is essentially similar to the flow past a backward-facing step in an otherwise plane boundary, and for continuity with the low-speed case the analysis as a whole will be framed with reference to the flow over a step. It will be shown that there are grounds for suspecting that the value of a parameter appearing in the solution may be influenced by the presence of the wall downstream of the base region, and for this reason the base pressure on an isolated aerofoil section need not be exactly the same as that on a step under similar conditions. At present however there is insufficient experimental evidence to permit any definite conclusion to be drawn on this particular point.

2. *The Flow Model.*

In the present exercise we are concerned with the analysis of the class of base flows generated by the flow past a backward-facing step in an otherwise plane boundary (Fig. 2). The stream approaching the step is regarded as steady, and uniform except close to the wall where a boundary layer is assumed to have developed. The flow separates at the corner, S, and reattaches to the downstream surface at R enclosing a bubble of slowly circulating fluid. In the cavity the fluid velocity is low, and the pressure essentially constant and equal to the base pressure on the step. The external quasi-inviscid flow is divided from the dissipative region by a free-shear layer which has its origin in the boundary layer approaching the separation point. It is assumed that the flow in the shear layer approximates to that generated by the constant-pressure turbulent mixing of a stream with a fluid at rest. In the supersonic case (Fig. 2b) the separating boundary layer interacts with a strong centred expansion springing from the corner and account must be taken of the modification to the velocity profile before the subsequent development of the free-shear layer can be computed. At subsonic speeds the upstream influence of the low pressure in the cavity spreads to an appreciable distance and the negative pressure gradient imposed on the approaching boundary layer is at least an order of magnitude less severe. As a first approximation it may therefore be possible to neglect the effect of the pressure gradient on the boundary-layer characteristics.

The separated shear layer reattaches to the downstream surface in a region of high positive pressure gradient (Fig. 2). This abrupt rise in pressure has the effect of reversing part of the fluid in the shear layer to flow back into the cavity whilst allowing the fluid with higher velocity to escape from the base region and continue its passage downstream. An essential balance must be maintained between the mass of fluid scavenged from the cavity by entrainment in the shear layer and the mass of fluid reversed into the region by the pressure rise at reattachment. This can be expressed in terms of the mean streamline pattern by the requirement that the same streamline which separated from the corner should reattach to the downstream surface. The mass balance in the cavity can be disturbed artificially by allowing a continuous bleed of fluid through the base from some external source. In this case the separation streamline will be distinct from the reattachment streamline (Fig. 3) to allow a mass flow equal to the bleed rate to escape from the region. In principle both base bleed and base suction (negative bleed) can be considered although in the present context the latter would appear to be of little more than academic interest. The important point to be established at this stage is that bleed is a mechanism for providing a measure of control over the selection of the reattachment streamline in contrast with the case in the absence of bleed when the reattachment streamline is determined from the outset by continuity requirements.

It has been shown that the reattachment pressure rise results in the division of the flow in the free-shear layer into two streams (Fig. 3) one of which returns into the cavity whilst the other continues in the main-stream direction to form a new boundary layer on the downstream surface. The two streams are separated by the reattachment streamline on which the fluid is brought to rest when it reaches the wall. The flow in the reattachment zone is characterised by a complex interplay between viscous and pressure forces, and in this respect certain differences in form might be manifest depending on the nature, whether laminar or turbulent, of the mixing process. A few attempts have been made to analyse the mechanism of reattachment (Ref. 20, for example) but as yet no really adequate theory has been formulated and it is usual to employ arguments similar to those used by Stratford[22] in his treatment of the separating boundary layer, namely, that the viscous and pressure-gradient effects can be applied in turn to the fluid on a particular streamline. In the present problem it is assumed that, during its passage along the length of the free-shear layer, the fluid on the reattachment streamline gathers momentum and that this mixing process continues in the same way through the reattachment region, as though the pressure gradient were not present, until the streamline interects the wall. The recompression which takes place in reality, resulting in the retardation of this fluid to rest, is then taken into account by equating the total pressure attained by the fluid on the reattachment streamline to the static pressure on the downstream surface at the reattachment point. It can be argued (Ref. 50) that the total pressure of fluid particles on the reattachment streamline is likely to increase as they approach the wall. However in the absence of a more detailed analysis at best a free parameter could be introduced to allow for this effect, and the necessity for the inclusion of such a parameter will be judged by an examination of the final results. The analysis will therefore proceed on the assumption of a quasi-isentropic compression along the reattachment streamline.

What cannot be substantiated, however, and this is believed to be the key to the failure of earlier methods, is the assumption used in all known previous attempts to analyse base flow on the basis of the present model, that the pressure rise to reattachment can be equated to the difference between the base pressure and the final recovery pressure far downstream, which is itself assumed to be equal to either the free-stream ambient pressure[4, 16, 18] or to a slightly lower pressure to take account of the losses resulting from the passage of the external stream through the trailing shock[5, 6, 7, 18, 19, 21, 23]. This reattachment criterion, if taken literally, would indicate that at supersonic speeds (Fig. 2b), the reattachment point lies at the top of the pressure rise. Now quite apart from any doubts as to the physical possibility of reattachment taking place at a point of zero pressure gradient, experiments on both supersonic base flow[10, 24, 31] and shock-wave boundary-layer interaction[25, 26, 27] are in agreement that the point of zero shear stress, i.e. reattachment point, is reached before the pressure has risen to its maximum value. The discrepancy between the observed behaviour of the flow and the assumptions made in the theory seems to have been recognised[51], but the justification of the criterion of reattachment hitherto employed was that it appeared to lead to accurate prediction of base pressures in supersonic turbulent base flow. The discussion in Sections 1.2 and 1.3 above should serve to demonstrate that there is no longer a case for its retention.

It is apparent that reattachment takes place at a point of strong positive pressure gradient and that a substantial residual pressure recovery is achieved downstream of the position of zero shear stress. This is at once to be expected since the reinstatement of the boundary layer on the downstream surface, involving a rapid decrease of displacement thickness, requires the imposition of a positive pressure gradient[28]. The flow near the reattachment point can be compared with the flow past a concave corner of the appropriate included angle to provoke incipient separation[29]. In this case also part of the total pressure rise takes place downstream of the point of zero shear stress at the corner and is associated with the change in direction of the external stream and the rehabitation of the boundary-layer velocity profile.

At subsonic speeds the concave curvature of the external streamlines and the resulting increase in the stream-tube area near the reattachment point give rise to an overshoot in the pressure distribution along the downstream surface and the pressure rise through the reattachment region is greater than the difference between free-stream static pressure and the base pressure. (*See* Fig. 2a.) It is then possible, and indeed it will be shown later to be often the case, that reattachment will take place at a point where the local static pressure is in excess of the ambient pressure in the free stream.

It is clear from these considerations that the base-flow solution depends on a parameter additional to those previously recognised, namely, the ratio of the pressure rise to reattachment to the difference between free-stream static pressure and the base pressure. In principle the acceptance of this ratio as a further variable does not constitute a major complication but the problem arises of determining its value under given conditions of Mach number, Reynolds number and any other factor on which it may depend. It may be possible to estimate the value of the parameter analytically on the lines used in the treatment of shock-wave boundary-layer interaction and there is scope for a considerable amount of future work in this direction. For the present we shall rest with pointing out the relevance of this parameter and attempt to assess its value from available measurements. (Section 7 below.)

3. *The effect of an Abrupt Expansion on the Boundary Layer.*

It was pointed out in the discussion of the flow model (Section 2 above) that at supersonic speeds the passage of the boundary layer through the centred expansion at the corner could be expected to give rise to a substantial modification of the velocity profile, and that in order to take proper account of the boundary-layer effect it would be necessary to compute its characteristics as they are presented to the development of the mixing layer, i.e. immediately downstream of the expansion fan. This problem which is similar to that of the viscous flow past a convex corner[33, 34] is outside the province of conventional boundary-layer theory since the streamwise gradients are not small compared with those in the transverse direction, and in fact is even more complex than it might appear since the interaction between the pressure field and the entropy gradient in the viscous layer gives rise to the generation of both expansion and compression waves[13]. While in principle the problem could be tackled by making some plausible assumption about the path of the sonic line in the boundary layer and using rotational characteristics in the supersonic region[35] such a Herculean task is hardly justified in the present context.

The fact that the interaction length is short, of the same order as the boundary-layer thickness, and that the pressure gradients are orders of magnitude greater than those normally encountered in boundary-layer phenomena, suggests that the viscous effects may be taken into account simply insofar as they generate the initial velocity profile. The resolution of the subsequent velocity and temperature fields can then be regarded as due to pressure forces alone. We shall further assume that the flow along any streamline through the interaction can be approximated adequately by one-dimensional isentropic-flow relations.

For an iso-energic shear flow, conditions on some streamline can be expressed in terms of those at the edge of the layer in the form

$$u^2 + 2c_p T = u_e^2 + 2c_p T_e$$

or

$$1 - u^{*2} = \frac{2c_p}{u_e^2}(T - T_e), \tag{3.1}$$

where

$$u^* = u/u_e .$$

Denoting conditions at the beginning and end of the interaction (Fig. 4) by suffices $_1$ and $_2$ respectively, two equations may be formed

$$1 - u_1^{*2} = \frac{2c_p}{u_{e_1}^2}(T_1 - T_{e_1}) \tag{3.2a}$$

and

$$1 - u_2^{*2} = \frac{2c_p}{u_{e_2}^2}(T_2 - T_{e_2}). \tag{3.2b}$$

Equations (3.2) are linked by the isentropic expansion along each streamline from pressures P_1 to P_b which are assumed to be constant through the boundary layer and shear layer respectively. Hence we may write

$$\frac{T_2}{T_1} = \frac{T_{e_2}}{T_{e_1}} = \left(\frac{P_b}{P_1}\right)^{\gamma-1/\gamma}, \tag{3.3}$$

and from equations (3.2a), (3.2b) and (3.3)

$$\begin{aligned} \frac{1-u_2^{*2}}{1-u_1^{*2}} &= \frac{u_{e_1}^{\ 2}\,T_{e_2}}{u_{e_2}^{\ 2}\,T_{e_1}} \\ &= \frac{M_{e_1}^{\ 2}}{M_{e_2}^{\ 2}}. \end{aligned} \tag{3.4}$$

Thus it has been possible to show that within the framework of the assumptions made the velocity of fluid on a particular streamline at the end of the interaction depends only on its initial velocity and the ratio of the Mach numbers in the stream outside the viscous layer.

It will be shown in Section 4 below that the development of the free-shear layer from an initial boundary layer is defined if the momentum thickness, ϑ, at separation is specified. This quantity may be computed in terms of the momentum thickness, θ, of the boundary layer approaching the corner using equation (3.4). Defining a stream function

$$\frac{\partial\psi}{\partial x} = \rho v,\ \frac{\partial\psi}{\partial y} = \rho u, \tag{3.5}$$

the momentum thicknesses of the viscous layer immediately upstream and downstream of the interaction are given by

$$\left.\begin{aligned} \rho_{e_1}u_{e_1}\theta &= \int_{\psi_S}^{\infty}(1-u_1^*)\,d\psi \\ \text{and} \qquad & \\ \rho_{e_2}u_{e_2}\vartheta &= \int_{\psi_S}^{\infty}(1-u_2^*)\,d\psi, \end{aligned}\right\} \tag{3.6}$$

where ψ_S is the streamline representing the wall. In the turbulent boundary layer most of the mass flux passes through stream tubes along which the mean velocity is nearly equal to the velocity in the external stream. Writing

$$1 - u_1^* = z,$$

where z is small and

$$M_{e_2}^{\ 2} = \frac{1}{r}M_{e_1}^{\ 2},$$

we have from equation (3.4)

$$\begin{aligned} u_2^* &= \{1 - rz(2-z)\}^{1/2} \\ &= 1 - \frac{rz}{2}(2-z) - \frac{r^2z^2}{2.4}(2-z)^2 - \dots, \\ &\{\text{for } rz(2-z) < 1\}; \end{aligned}$$

and finally

$$1 - u_2^* = r(1-u_1^*) - \frac{r}{2}(1-r)(1-u_1^*)^2 + \dots. \tag{3.7}$$

Substituting this result into equation (3.6) an expression for the downstream momentum thickness can be obtained in the form

$$\rho_{e_2}u_{e_2}\vartheta = r\int_{\psi_S}^{\infty}(1-u_1^*)d\psi - \frac{r}{2}(1-r)\int_{\psi_S}^{\infty}(1-u_1^*)^2 d\psi + \dots$$

$$= r\int_{\psi_S}^{\infty}(1-u_1^*)d\psi - \frac{r}{2}(1-r)\left\{2\int_{\psi_S}^{\infty}(1-u_1^*)d\psi - \int_{\psi_S}^{\infty}(1-u_1^{*2})d\psi\right\} + \dots . \quad (3.8)$$

The integrals in equation (3.8) define characteristics of the boundary layer upstream of the interaction:

$$\int_{\psi_S}^{\infty}(1-u_1^*)d\psi = \rho_{e_1}u_{e_1}\theta$$

$$\int_{\psi_S}^{\infty}(1-u_1^{*2})d\psi = \rho_{e_1}u_{e_1}\delta^{**},$$

(kinetic-energy thickness),

and so on. Hence equation (3.8) becomes

$$\rho_{e_2}u_{e_2}\vartheta = r\rho_{e_1}u_{e_1}\left\{\theta - \frac{1-r}{2}(2\theta - \delta^{**}) + \dots\right\}. \quad (3.9)$$

In general for a turbulent boundary layer the term in $2\theta - \delta^{**}$ and higher terms are small and to a first approximation we can use the very simple result

$$\frac{\rho_{e_2}u_{e_2}\vartheta}{\rho_{e_1}u_{e_1}\theta} = r = \frac{M_{e_1}^2}{M_{e_2}^2}. \quad (3.10)$$

Hence for a given value of the approach Mach number, M_{e1}, one can determine the variation of the momentum-thickness parameter

$$\frac{\rho_{e_2}u_{e_2}\vartheta}{\rho_{e_1}u_{e_1}\theta}$$

with the base pressure ratio, since the Mach number, M_{e2}, of the flow outside the free-shear layer is given at once from isentropic relations in the form

$$\frac{P_b}{P_1} = \left[\frac{1+\frac{\gamma-1}{2}M_{e_1}^2}{1+\frac{\gamma-1}{2}M_{e_2}^2}\right]^{\gamma/(\gamma-1)}. \quad (3.11)$$

As an example, for an approach Mach number of 2·0 the present result obtained from equations (3.10) and (3.11) is compared in Fig. 5 with the method of Kirk[16] which is valid only for small changes in pressure, and that of Ref. 18 which retains integral terms in the final expression which must be evaluated for each set of conditions. While being very much easier to incorporate into the complete base-flow solution the present method appears to be in satisfactory agreement with the other two. Moreover it is worth noting that equation (3.10) does not involve the shape of the initial boundary-layer velocity profile but the theory specifies only that the profile should be 'full'.

4. *The Free-Shear Layer.*

4.1. The boundary layer on the upstream surface separates at the corner to generate a free-shear layer which forms the demarcation between the external stream and the slowly circulating fluid in the cavity. So long as the boundary-layer thickness at separation is not large compared with the height of the step, the velocities of fluid in the cavity are small and a region of almost constant pressure extends from the step to a point where the fluid in the shear layer first impinges on the downstream surface. Past this region the development of the mixing layer continues without any appreciable influence from the wall below the cavity. At supersonic speeds the streamlines in the flow adjacent to the shear layer are straight and the pressure gradients both along and normal to the streamlines in the layer are zero. In this case especially, the mixing layer can be analysed without significant loss of accuracy on the basis of a flow model which considers the constant-pressure mixing between a uniform external stream and a fluid at rest, with initial conditions to represent the profile of the boundary layer after it has passed through the centred expansion at the corner.

At subsonic speeds the streamlines outside the shear layer are not straight and the change in curvature from one streamline to another gives rise to a transverse pressure gradient. Nevertheless if the thickness of the shear layer is small compared with its length and with the radius of curvature of the adjacent streamlines it should be possible to neglect the effect of these factors on the mixing process. It will therefore be assumed that the development of the velocities along streamlines in the layer can be computed from the analysis of the constant-pressure flow model proposed for the supersonic case. At subsonic speeds however there is no abrupt deflection of the flow at the corner and the velocity profile of the approaching boundary layer represents directly the initial conditions imposed on the subsequent development of the mixing layer.

4.2. It was shown in Ref. 36 that except in the region close to the separation point the development of a free-shear layer from a turbulent boundary layer, and in particular the variation of the velocities along typical streamlines, could be represented to some considerable accuracy by the simple method proposed by Kirk[16]. The comparison of the latter method with more detailed calculations was made for the case of incompressible flow but it was pointed out that there was no obvious reason why the same should not be true for flow at higher Mach numbers. It was demonstrated that the real shear layer developing from an initial boundary layer could be replaced by an equivalent asymptotic shear layer growing over a greater distance from zero thickness, and that the distance between the origin of the equivalent system and the separation point could be equated to a simple multiple of the boundary-layer momentum thickness. In this way it is possible to make reference to all the results for the asymptotic mixing layer and to allow for the effects of the approaching boundary layer by nothing more than a simple shift of the origin. (*See* Fig. 7.)

The problem of the asymptotic turbulent shear layer at low speeds has been investigated in some detail by Tollmien[37] and Görtler[38], and Abramovich[39] has discussed certain features of the problem in compressible flow up to the speed of sound. It has since been recognised that the result originally put forward by Görtler as a first approximation, that is, the 'error function' velocity profile,

$$u^* = \frac{1}{2}\left(1 + \operatorname{erf}\frac{\sigma y}{x}\right) \tag{4.1}$$

is a good fit to the measured velocity profiles over quite a wide Mach number range[10, 13], so long as the value of the constant σ is chosen appropriately. There is some evidence that the value of σ

increases with Mach number (Fig. 6), indicating a reduced rate of spread of the shear layer, although the scatter in the available data is large. In the absence of more reliable measurements it would seem that at best a linear interpolation could be made as suggested by Korst and Tripp[32]:

$$\sigma = 12(1+0\cdot 23M_{e_2}). \tag{4.1a}$$

The basis of Kirk's approximation is that equation (4.1) can be employed to describe the mean velocity field in the pre-asymptotic turbulent mixing layer if x, the distance from the separation point, is replaced by $x + x'$ (Fig. 7). Thus the scale of the velocity profile at a certain distance x from separation can be increased to take account of the initial thickness of the layer. The distance x' is assumed to be proportional to the momentum thickness θ of the boundary layer at separation, or in the supersonic case to the momentum thickness ϑ of the layer after it has negotiated the centred expansion at the corner. For consistency ϑ will be used throughout the analysis in this section but it should be remembered that the meaning of ϑ is interpreted as above.

The constant of proportionality, k, between x' and ϑ is determined [36] from the argument that over the distance x' the equivalent shear layer growing from zero thickness should have attained a momentum thickness, Θ, equal to ϑ. The momentum thickness, $\Theta(x)$, of a free-mixing layer is defined by

$$\rho_{e_2}u_{e_2}\Theta = \int_{\psi_B}^{\infty}(1-u^*)d\psi\,; \tag{4.2}$$

where

$$\psi_B(x) = \psi_{\rm ref} - \int_{-\infty}^{y_{\rm ref}}\rho u dy \tag{4.3}$$

(*see* Fig. 8), and $\psi_{\rm ref}$ is any reference streamline. The integrals can be evaluated assuming, say, the error-function velocity profile {equation (4.1)} and the relation between velocity and density in the shear layer for unit Prandtl number and zero heat transfer. In this way the constant k can be found as a function of Mach number, the value at low speeds being around 30.

In the present analysis it is not usually necessary to compute the actual value of k in a particular case but use can be made of a result which may be derived from the following arguments. Within the framework of the assumption of a free-shear layer in a semi-infinite fluid it may be stated that the total momentum of the mean flow downstream of the separation point is conserved. If the equivalent mixing layer at a station corresponding to the real separation point, i.e. a distance x' from its origin, is to generate velocity profiles at a point downstream which are identical to those in the real flow, it must represent a total momentum equal to that in the boundary layer at separation. This equality may be expressed as follows: if ψ_h is a streamline in the external stream, at $x = 0$

$$\left[\int_{\psi_{B0}}^{\psi_h} u^* d\psi\right]_{\substack{\text{Equiv.}\\ \text{Shear}\\ \text{Layer}}} = \left[\int_{\psi_S}^{\psi_h} u^* d\psi\right]_{\substack{\text{Boundary}\\ \text{Layer}}}$$

$$= \Psi,\ \text{say}, \tag{4.4}$$

where $u_e\Psi$ is the total momentum flux in the shear layer between the streamline ψ_h and the dead-air region, and $\psi_B = \psi_{B0}$ for the equivalent layer at $x = 0$.

Now from the definitions of ϑ and Θ

$$\rho_{e_2}u_{e_2}\vartheta = \int_{\psi_S}^{\psi_h}(1-u^*)d\psi = \psi_h - \psi_S - \Psi, \tag{4.5}$$

and at $x = 0$,

$$\rho_{e_2}u_{e_2}\Theta = \int_{\psi_{B0}}^{\psi_h}(1-u^*)d\psi = \psi_h - \psi_{B0} - \Psi. \tag{4.6}$$

Hence if Θ is put equal to the momentum thickness ϑ of the boundary layer at separation it follows firstly that

$$\psi_{B0} = \psi_S \tag{4.7}$$

and secondly that if the 'median streamline'[36] is defined as

$$\psi_M = \psi_h - \Psi \tag{4.8}$$

then

$$\psi_M - \psi_S = \psi_M - \psi_{B0} = \rho_{e_2}u_{e_2}\vartheta. \tag{4.8a}$$

Now the asymptotic turbulent free-shear layer grows linearly with distance from its origin and for the equivalent mixing layer in the present problem we may express the local velocity u^* in terms of the stream function and the position x by a relation of the form

$$\psi_M - \psi = \rho_{e_2}u_{e_2}(x+x')f(u^*), \tag{4.9}$$

where the form of f depends on the shape of the velocity profile and on the Mach number, M_{e_2}, of the external stream. The lower boundary of the flow field in the $x - \psi$ plane, i.e. where $u^* = 0$, is represented by the curve $\psi_B(x)$ which is defined by

$$\psi_M - \psi_B = \rho_{e_2}u_{e_2}(x+x')f(0) \tag{4.10}$$

and at $x = 0$

$$\psi_M - \psi_{B0} = \rho_{e_2}u_{e_2}x'f(0).$$

Hence the distance x' between the origin of the equivalent mixing layer and the real separation point is given by

$$x' = \frac{\psi_M - \psi_{B0}}{\rho_{e_2}u_{e_2}f(0)} = \frac{\vartheta}{f(0)}, \tag{4.11}$$

and eliminating x' between equations (4.9) and (4.11) we obtain

$$\frac{\psi_M - \psi}{\rho_{e_2}u_{e_2}} = xf(u^*) + \vartheta\frac{f(u^*)}{f(0)}, \tag{4.12}$$

or finally, since

$$\psi_M - \psi_S = \rho_{e_2}u_{e_2}\vartheta,$$

$$\frac{\psi_S - \psi}{\rho_{e_2}u_{e_2}} = xf(u^*) - \vartheta\left\{1 - \frac{f(u^*)}{f(0)}\right\}. \tag{4.12a}$$

Hence the mass flux between the separation streamline and any adjacent streamline can be expressed in terms of functions which relate to the velocity profile of the asymptotic turbulent mixing layer, and the momentum thickness ϑ of the initial boundary layer.

4.3. The datum from which the function $f(u^*)$ is measured, is the median streamline. In the asymptotic free-shear layer growing from zero thickness, the velocity on the median streamline is constant, equal to u_c^* say, and ψ_M is then sometimes termed the 'constant-velocity streamline'.

The median streamline is defined by equation (4.8), and if the velocity profile is expressed in its similarity form

$$u^* = u^*(\zeta)$$

where $\zeta = \sigma y/\bar{x}$ and $\bar{x}$ is the distance from the origin of the asymptotic mixing layer, the location, ζ_M, of the median streamline is given by

$$\int_{\zeta_M}^{\zeta_h} \rho^* u^* d\zeta = \int_{-\infty}^{\zeta_h} \rho^* u^{*2} d\zeta \tag{4.13}$$

where ζ_h is some station in the stream outside the shear layer. Hence with the density and velocity related by the equation

$$\rho^* = \left[1 + \frac{\gamma - 1}{2} M_{e_2}{}^2(1 - u^{*2})\right]^{-1} \tag{4.14}$$

the position of the median streamline and the velocity, u_c^*, of the fluid on it are known as functions of the Mach number, M_{e_2}, of the external stream and the shape of the velocity profile.

This latter dependence is fairly critical and special care needs to be taken. The use of the error function to represent the velocity profile is a powerful method of computing the velocities at various stations in the shear layer and except towards the edges of the layer where there is some deviation, good correlation with experiment is obtained. However, the form of equation (4.13) is such that the small errors incurred by this approximation are magnified and the value of u_c^* derived from the error-function velocity profile, at low speeds, is $0 \cdot 62$ compared with $0 \cdot 58$ computed by the more precise methods of Tollmien and Görtler. This would seem to indicate that at higher Mach numbers also the calculation of u_c^* on the basis of the error function, as was done in Ref. 5, would lead to some overestimation of the correct values.

An alternative indication of the trend of u_c^* with Mach number can be obtained by making what appears to be a gross oversimplification of the velocity profile but which nevertheless at low speeds leads to a very precise value of u_c^*. If the velocity profile is approximated by a linear relation of the form

$$u^* = \tfrac{1}{2}(1 + a\zeta)$$

where a is some constant, the value of u_c^* is given by

$$u_c^{*2} = \frac{1}{c^2}\left[1 - \frac{1 - c^2}{e^2}\left(\frac{1 + c}{1 - c}\right)^{1/c}\right] \tag{4.15}$$

where

$$c^2 = \frac{\frac{\gamma - 1}{2} M_{e_2}{}^2}{1 + \frac{\gamma - 1}{2} M_{e_2}{}^2},$$

which at low speeds reduces to

$$u_c^* = \frac{1}{\sqrt{3}} = 0 \cdot 578 .$$

Equation (4.15) is plotted in Fig. 9 and compared with the variation of u_c^{*2} with Mach number computed from the error function (Ref. 5). It is seen that there is a considerable discrepancy between the two curves. Further detailed calculations and indeed precise experiments on free-shear layers, are required before a more reliable assessment of the increase of u_c^* with Mach number can be made.

For the present the calculations throughout the subsonic range will be based on the value of u_c^* correct for incompressible flow, i.e. 0·58, and for supersonic speeds u_c^* will be assumed to follow the law

$$u_c^{*2} = 0\cdot 348 + 0\cdot 018 M_{e_2} \tag{4.16}$$

which lies between the values given by the error function and the linear velocity profile. (*See* Fig. 9.)

4.4. Hence in principle at least, the median streamline ψ_M can be identified and its associated velocity u_c^* determined. The function $f(u^*)$ is then given by

$$f(u^*) = \frac{1}{\sigma}\int_{\zeta}^{\zeta_M} \rho^* u^* \, d\zeta . \tag{4.17}$$

Once the value of u_c^* is known to the necessary precision the value of $f(u^*)$ is not unduly sensitive to the form of the velocity profile and no significant loss of accuracy will be incurred by representing the profile by the error function or even by simplifying it still further over that part where the velocity gradient is nearly linear. In fact this latter is to some extent justified since the variation of f is required principally over the range of u^* which corresponds to small values of ζ. Initially however the error-function velocity profile

$$u^* = \tfrac{1}{2}(1 + \operatorname{erf} \zeta) \tag{4.18}$$

is assumed.

The function $f(u^*)$ is defined by equation (4.9) and since the stream function is given by

$$\psi = \int \rho u \, dy = \frac{\bar{x}}{\sigma}\int \rho u \, d\zeta ,$$

f can be expressed in terms of the velocity profile by

$$f(u^*) = \frac{1}{\sigma}\int_{\zeta}^{\zeta_M} \rho^* u^* \, d\zeta . \tag{4.19}$$

Making a change of variable from ζ to u^* we obtain

$$f = \frac{1}{\sigma}\int_{u^*}^{u_c^*} \rho^* u^* \frac{d\zeta}{du^*} \, du^* ,$$

and extracting the derivative from equation (4.18)

$$f = \frac{\sqrt{\pi}}{\sigma}\int_{u^*}^{u_c^*} \rho^* u^* e^{\zeta^2} \, du^*$$

which for small values of ζ becomes

$$f = \frac{\sqrt{\pi}}{\sigma}\int_{u^*}^{u_c^*} \rho^* u^* \, du^* . \tag{4.20}$$

Now from equation (4.14)

$$u^*du^* = \frac{d\rho^*}{(\gamma-1)M_{e_2}{}^2\rho^{*2}}$$

and hence a further change of variable may be made from u^* to ρ^* and equation (4.20) reduces to

$$f = \frac{\sqrt{\pi}}{(\gamma-1)\sigma M_{e_2}{}^2}\int_{\rho^*}^{\rho_c^*}\frac{d\rho^*}{\rho^*} = \frac{\sqrt{\pi}}{(\gamma-1)\sigma M_{e_2}{}^2}\log\lambda, \tag{4.21}$$

where

$$\lambda = \frac{\rho_c^*}{\rho^*} = \frac{1+\dfrac{\gamma-1}{2}M_{e_2}{}^2(1-u^{*2})}{1+\dfrac{\gamma-1}{2}M_{e_2}{}^2(1-u_c{}^{*2})}. \tag{4.22}$$

Thus for small values of ζ it has been possible to express the function f in this simple way in terms of the logarithm of a parameter λ which can be found easily from the velocity ratio u^*. Equation (4.12a) involves also the limiting value of f as u^* tends to zero and in this case the condition of small ζ is strictly violated. Nevertheless the term in $f(0)$ is small, and looking ahead to the final results which will be derived from the present analysis it can be shown that the error incurred by computing $f(0)$ from equation (4.21) is negligible until the ratio of boundary-layer thickness to step height becomes large and then the validity of the method in general is questionable. Hence it will be assumed that both $f(u^*)$, for the range of u^* in which we are interested, and $f(0)$ can be evaluated in this way.

4.5. To sum up then we are now in a position, using equations (4.12a), (4.21) and (4.22), to determine the variation of the flow parameters along selected streamlines in the shear layer, for given conditions of the Mach number, M_{e_2}, of the stream just outside the layer and the momentum thickness, ϑ, of the boundary layer at separation. Before proceeding to apply the reattachment criterion however it will be necessary to attach some value to the length, l, of the shear layer. In the discussion of the flow model, in Section 2 above, it was suggested that the mixing process in the shear layer could be regarded as continuing until the layer impinged on the downstream surface. At supersonic speeds since the mixing layer proceeds along an almost straight path from the corner and the angle of declination of the layer relative to the wall is determined at once in terms of the Prandtl-Meyer angles of the flow outside the layer, the length l is given approximately by

$$l = \frac{t}{\sin(\nu_{e_2}-\nu_{e_1})}, \tag{4.23}$$

where t is the height of the step and ν is the Prandtl-Meyer angle corresponding to the local Mach number of the external stream.

At subsonic speeds in principle a relation could be derived for the external flow between the length of the cavity and the base pressure using free-streamline theory. At present however no suitable data are available for substitution in the analysis and the length l of the mixing layer will be left as a free parameter.

5. *The Reattachment Region.*

The base-flow solution is closed by the reattachment condition. It has been seen (Section 2 above) that the total pressure on the reattachment streamline ψ_R can be equated to the static pressure P_r at the reattachment point, on the downstream surface, and that this pressure is related to the ambient static pressure and the base pressure by an expression of the form

$$\frac{P_r - P_b}{P_1 - P_b} = N, \tag{5.1}$$

where the value of N remains to be found.

Without at the moment attempting to attach a value to N some useful relations can be established. Along the streamline ψ_R the pressure rise from P_b to P_r is associated with the fall in the velocity from u_R to zero and an increase in the density from ρ_R to some value ρ_r, say. If this compression is assumed to be quasi-isentropic we can at once write

$$\frac{\rho_r}{\rho_R} = \left(\frac{P_r}{P_b}\right)^{1/\gamma}. \tag{5.2}$$

Now for unit Prandtl number, the recovery temperature along the wall is the same as that in the cavity and hence the density of fluid in the cavity, ρ_b say, is related to ρ_r by the ratio of the pressures:

$$\frac{\rho_b}{\rho_r} = \frac{P_b}{P_r},$$

and from the last two equations

$$\frac{\rho_b}{\rho_R} = \left(\frac{P_b}{P_r}\right)^{(\gamma-1)/\gamma}. \tag{5.3}$$

This equation can be expressed in terms of the density parameter λ, {*see* equation (4.22)}. If

$$\lambda_R = \frac{\rho_c}{\rho_R} \text{ and } \lambda_b = \frac{\rho_c}{\rho_b}$$

we have

$$\lambda_R = \lambda_b \left(\frac{P_b}{P_r}\right)^{(\gamma-1)/\gamma}. \tag{5.4}$$

The quantity λ_b is a function only of M_{e_2} since

$$\lambda_b = \frac{1 + \dfrac{\gamma - 1}{2} M_{e_2}{}^2}{1 + \dfrac{\gamma - 1}{2} M_{e_2}{}^2(1 - u_c{}^{*2})}, \tag{5.5}$$

and $u_c{}^*$ is either assumed constant or varies with M_{e_2} according to equation (4.16). The parameter λ_R is a measure of the pressure rise to reattachment and equation (5.4) may be used to determine the value of the function f on the reattachment streamline. From equations (4.21) and (5.4)

$$f(u_R{}^*) = \frac{\sqrt{\pi}}{(\gamma-1)\sigma M_{e_2}{}^2} \log\left\{\lambda_b \left(\frac{P_b}{P_r}\right)^{(\gamma-1)/\gamma}\right\}, \tag{5.6}$$

and also

$$\frac{f(u_R{}^*)}{f(0)} = 1 - \frac{\log\left(\dfrac{P_r}{P_b}\right)^{(\gamma-1)/\gamma}}{\log \lambda_b}. \tag{5.7}$$

It now remains to apply the reattachment condition to the known development of the shear layer at a distance l from separation. The reattachment streamline is located within the shear layer by the continuity requirement in the cavity. Assuming a bleed mass flux q into the cavity from an external source (base bleed) the streamline ψ_R is specified by

$$\psi_S - \psi_R = q \tag{5.8}$$

and from equations (4.12a) and (5.8), with the values of $f(u_R^*)$ and $f(u_R^*)/f(0)$ given by equations (5.6) and (5.7), the base-flow solution for subsonic flow is given formally by

$$q = \rho_{e_2} u_{e_2} \left[l f(u_R^*) - \theta \left\{ 1 - \frac{f(u_R^*)}{f(0)} \right\} \right], \tag{5.9}$$

$$= \rho_{e_2} u_{e_2} \left[\frac{\sqrt{\pi} l}{(\gamma - 1)\sigma M_{e_2}^2} \left\{ \log \lambda_b - \log \left(\frac{P_r}{P_b} \right)^{(\gamma-1)/\gamma} \right\} - \theta \frac{\log \left(\frac{P_r}{P_b} \right)^{(\gamma-1)/\gamma}}{\log \lambda_b} \right]. \tag{5.10}$$

At supersonic speeds the correction must be made for the change in momentum thickness of the boundary layer as it passes through the expansion fan at the corner. Also the length l of the shear layer can be expressed in terms of the step height t and the Prandtl-Meyer angles associated with the Mach numbers M_{e_1} and M_{e_2}. From equations (3.10), (4.12a) and (4.23) we have

$$q = \rho_{e_2} u_{e_2} \frac{t f(u_R^*)}{\sin(\nu_{e_2} - \nu_{e_1})} - \rho_{e_1} u_{e_1} \theta \frac{M_{e_1}^2}{M_{e_2}^2} \left\{ 1 - \frac{f(u_R^*)}{f(0)} \right\}, \tag{5.11}$$

$$= \rho_{e_2} u_{e_2} \frac{\sqrt{\pi} t \left\{ \log \lambda_b - \log \left(\frac{P_r}{P_b} \right)^{(\gamma-1)/\gamma} \right\}}{(\gamma - 1)\sigma M_{e_2}^2 \sin(\nu_{e_2} - \nu_{e_1})} - \rho_{e_1} u_{e_1} \theta \frac{M_{e_1}^2}{M_{e_2}^2} \frac{\log \left(\frac{P_r}{P_b} \right)^{(\gamma-1)/\gamma}}{\log \lambda_b}. \tag{5.12}$$

The procedure for computation is as follows:

(*a*) For a given approach Mach number M_{e_1} values of M_{e_2} are chosen corresponding to a range of values of the base pressure ratio P_b/P_1 {equation (3.11)}. The parameters σ (equation (4.1a), u_c^* {equation (4.16)} and λ_b {equation (5.5)} are functions of M_{e_2} and can be computed, and also the quantities $(\rho_{e_2} u_{e_2}/\rho_{e_1} u_{e_1})$ and $(\nu_{e_2} - \nu_{e_1})$.

(*b*) A value of N {equation (5.1)} is assumed and for each value of P_b considered in (*a*) the reattachment pressure P_r is found and the value of the functions $f(u_R^*)$ and $f(u_R^*)/f(0)$ computed, {equations (5.6), (5.7)}.

(*c*) If the bleed rate q is specified, equations (5.10) or (5.11) give the boundary-layer momentum thickness necessary to produce this level of base pressure.

(*d*) If the boundary-layer momentum thickness is specified, equations (5.10) or (5.11) give the variation of base pressure with bleed mass flow.

6. *Special Cases.*

In this way the theory has been formulated to predict not only the base pressure as determined by the thickness of the boundary layer upstream of the base, but also the increase of base pressure which can be achieved by the continuous bleed of fluid into the cavity. Earlier methods have demonstrated the extent to which the effect of base bleed can be estimated theoretically, and we shall now illustrate the use of the theory to predict the variation of base pressure with the thickness of the approaching boundary layer.

6.1. *No Bleed* ($q = 0$).

If q is placed equal to zero in equations (5.10) and (5.12) expressions are obtained for the boundary-layer momentum thickness necessary for a given base pressure ratio P_b/P_1 to be achieved. For subsonic flow

$$\frac{\theta}{l} = C_1 \frac{A(A-B)}{B} \tag{6.1}$$

where

$$A = \log \lambda_b$$

$$B = \log \left(\frac{P_r}{P_b}\right)^{(\gamma-1)/\gamma}$$

$$C_1 = \frac{\sqrt{\pi}}{(\gamma-1)\sigma M_{e_2}^{\,2}},$$

and for supersonic flow

$$\frac{\theta}{t} = C_2 \frac{A(A-B)}{B} \tag{6.2}$$

where

$$C_2 = \frac{\rho_{e_2} u_{e_2} M_{e_2}^{\,2}}{\rho_{e_1} u_{e_1} M_{e_1}^{\,2}} \frac{C_1}{\sin(\nu_{e_2} - \nu_{e_1})},$$

$$= \left(\frac{M_{e_2} T_{e_2}}{M_{e_1} T_{e_1}}\right)^3 \frac{C_1}{\sin(\nu_{e_2} - \nu_{e_1})}.$$

T_{e_1} and T_{e_2} are the values of the stream temperature corresponding to the Mach numbers M_{e_1} and M_{e_2} respectively, and the ratio of the specific heats γ is taken as $1 \cdot 4$.

At high subsonic speeds, a centred expansion develops at the corner as soon as the Mach number of the stream outside the free-shear layer exceeds unity and account should be taken of the effect of the expansion on the boundary-layer momentum thickness. The geometry of the cavity, and thus the value of l/h, is not however specified in terms of the Prandtl-Meyer angles until the stream approaching the step reaches sonic velocity. There is then an intermediate range of Mach number, from approximately $0 \cdot 85$ to $1 \cdot 0$, when the solution should be written

$$\frac{\theta}{l} = C_3 \frac{A(A-B)}{B}, \tag{6.3}$$

where

$$C_3 = \left(\frac{M_{e_2} T_{e_2}}{M_{e_1} T_{e_1}}\right)^3 C_1, \text{ for } \gamma = 1 \cdot 4.$$

It is to be remembered that in the subsonic cases, equations (6.1) and (6.3), the solution contains two parameters which are as yet unspecified, the length l of the free-shear layer, and the reattachment parameter N which appears in the quantity B. The present theory makes no attempt to assess the value of these terms analytically. The length of the shear layer would appear to depend both on Mach number and on the base pressure ratio. The reattachment parameter N which is also involved in the solution {equation (6.2)} for supersonic base flow will be seen to be principally a function of Mach number.

6.2. *Limiting Base Flow* ($\theta = 0$).

As the thickness of the boundary layer approaching the step decreases to zero, the base pressure tends to its limiting value and the solution becomes simply

$$A = B,$$

i.e.

$$\frac{P_r}{P_b} = (\lambda_b)^{\gamma/(\gamma-1)}, \tag{6.4}$$

or in terms of N {equation (5.1)}

$$\left(\frac{P_b}{P_1}\right)_{\mathrm{lim}} = \frac{1}{1 + \frac{1}{N}\left\{(\lambda_b)^{\gamma/(\gamma-1)} - 1\right\}}. \tag{6.5}$$

λ_b is given by equation (5.5) for the value of M_{e_2} obtained by assuming the stream of Mach number M_{e_1} to expand isentropically from pressure P_1 to P_b {equation (3.11)}. It is noted that the limiting-base-flow solution does not involve the length of the free-shear layer and can therefore be applied directly to the subsonic case without further reference to empirical factors.

7. *Some Calculated Results and Discussion.*

7.1. We have seen that the base-flow solution for both subsonic and supersonic flow depends on a reattachment parameter which must for the present be assessed empirically. Some data on this parameter have been collected from the results of experiments on various types of reattaching flows and are presented in Fig. 10. The scatter is considerable but a definite trend of N with free-stream Mach number is clearly evident. At subsonic speeds reattachment generally takes place at a point where the surface pressure is greater than the ambient static pressure (Fig. 2a) as is shown by the values of N being greater than unity. In incompressible flow the value of N would appear to be in the region of 1·6 and with increase of Mach number through the subsonic range its value is seen to fall slowly followed by an abrupt decrease as sonic velocity is approached[15].

At supersonic speeds there is no significant overshoot in the static pressure along the downstream surface and the pressure at reattachment is less than the static pressure in the free stream (Fig. 2b). The tests of Ref. 15 extended to a Mach number of 1·10 by which time the value of N had fallen to 0·35. This figure also appears to be a rough mean of the available results for supersonic reattaching flows. There is a need for systematic tests to measure the reattachment parameter in the supersonic range and to investigate any dependence it may have on factors other than Mach number such as the thickness of the reattaching boundary layer. It would also be interesting to know whether the operation of base bleed would have any appreciable effect on its value.

It is worth noting at this stage that since the pressure recovery which takes place downstream of the reattachment point in the supersonic case is assumed to be associated with the rehabilitation of the velocity profile, it is by no means certain that the conditions which determine the value of N for the flow past a step will be precisely similar to those which determine the value of the corresponding parameter for the case of the flow in the wake of a blunt-trailing-edge aerofoil. Indeed it could be argued that the absence of the solid boundary in the latter case must necessarily lead to a redistribution of local shear stress in the recompression region and affect the ability of fluid to negotiate the residual pressure rise from the point of stagnation onwards.

7.2. The variation of base pressure with boundary-layer momentum thickness at a Mach number of $2 \cdot 0$ is illustrated in Fig. 11. The theoretical curve is computed from equation (6.2) taking $N = 0 \cdot 35$ and assuming u_c^* to vary according to equation (4.16). The predicted values are compared with the available data on the base pressure on backward-facing-step models from tests by Sirieix[10], Thomann[31] and Badrinarayanan[24], and it is seen that the measure of agreement obtained is very encouraging. Also presented in Fig. 11 are some data on the base pressure on blunt-trailing-edge aerofoil sections.† From the tests by Gadd *et al*[12] a point is shown representing the minimum base pressure reached when, with increasing Reynolds number, the transition point moves upstream past the trailing edge. (*See* Section 1.2 above.) The momentum thickness has been estimated on the assumption of laminar flow on the body surface. A similar data point has been extracted in this way from the tests on transitional base flow by Van Hise[14] by an interpolation between his results for $M = 1 \cdot 95$ and $2 \cdot 22$. These cases demonstrate the extent to which small values of the ratio of boundary-layer momentum thickness to base height can be achieved during the transitional phase, and it is observed that the measured base pressures under these conditions are in good agreement with the values predicted by the present theory.

For the case when the transition point is well forward of the trailing edge, Chapman *et al*[8] present the results of a large number of tests on aerofoil sections in which the ratio of boundary-layer momentum thickness to base height was varied over a wide range. The momentum thickness was not measured during the tests but an assessment of it can be made since the Reynolds number is quoted and the flow can be assumed turbulent downstream of the transition wire fixed to the models. The mean curve derived from the results in this way is shown in Fig. 11, and lies below the values computed from the theory and the data points from Refs. 24 and 31 relating to the base pressure on step models. It is extremely unlikely that an error of sufficient magnitude could have been made in the estimation of the momentum thickness to account for such a discrepancy and the measurements of Chapman *et al* should be reliable.

The test by Thomann[31] was carried out at a Mach number of $1 \cdot 8$ and therefore for a true comparison with the other data in Fig. 11 a correction would need to be made to the base pressure to take account of the difference in Mach number. According to the present theory however the correction would not be large at this relatively high value of θ/t and at most the data point in question could be displaced downwards by about $0 \cdot 015$ in P_b/P_1 and would remain distinct from the values given by Chapman *et al.* Thus if this result and that of Badrinarayanan[24] are to be believed it would appear to be an indication of a general effect that when the thickness of the boundary layer is large

† For the purposes of comparing the results of the aerofoil sections with those of the backward-facing-step models the height t is in the former case identified with the trailing-edge semi-thickness.

the variation of base pressure with momentum thickness is not the same for the blunt-trailing-edge aerofoil and the backward-facing step. As suggested above a mechanism does exist for some divergence between the values of N for the two cases which could lead to a difference in base pressure, but it would be premature to come to any definite conclusion on the evidence currently available and further experimental results are required. If the value of N and hence the base pressure is influenced by the presence of a solid boundary in the recompression region it would be a simple test to insert a thin splitter plate into the wake behind a blunt-trailing-edge section in a supersonic flow and observe any change in base pressure which might be brought about.

If further experiments do not show a significant difference between the results for steps and isolated sections it would seem that the validity of the present theory deteriorates when the ratio of boundary-layer thickness to base height ceases to be small. Indeed this would not be unexpected since several of the assumptions then become questionable. For instance the equivalence between the free-shear layer developing from an initial boundary layer and one developing over a greater distance from zero thickness is reasonably precise only at distances from separation greater than some multiple of the initial thickness. At low speeds[36], for distances from the separation point less than about 80 times the boundary-layer momentum thickness Kirk's approximation predicts a lower velocity on typical streamlines than is found from a more detailed analysis. In the present problem at high speeds, as the thickness of the approaching boundary layer increases the ratio of the length of the shear layer to the momentum thickness falls and the velocity on the reattachment streamline may tend to be underestimated also. This will lead to an overestimation of the base pressure under these conditions which is qualitatively consistent with the trend observed in Fig. 11.

Some further calculations based on the present theory are presented in Fig. 12. At a Mach number of 2·3 it is seen that there is a significant discrepancy between the predicted values and the measurements of Fuller and Reid[9]. However it is difficult to interpret this since the experimental data in question are not in line with measurements at neighbouring Mach numbers. It must be stressed that the scarcity of reliable data on base pressures is acute, and until more systematic tests are carried out under conditions where the ratio of boundary-layer thickness to base height is relatively small a really valid comparison with the theory cannot be attempted. In Fig. 12a two data points are shown derived from the tests on transitional base flow at $M = 2 \cdot 22$ by Van Hise[14], and are seen to be in good agreement with the predicted values.

At a Mach number of 3 (Fig. 12b) the experimental data, which include base pressures measured by Gadd *et al*[12] on models at the Reynolds number for which the transition point was located at the trailing edge, correlate more closely for small values of θ/t and do follow the trend established by the theoretical curve. Comparing this diagram with Fig. 11 it would appear that the stage at which, with increasing boundary-layer thickness, the theory begins to overestimate the base pressure, is reached earlier as the Mach number is increased. The theory can be brought into line with the experimental results by accepting a progressive reduction in the value of N with increase of the thickness of the boundary layer at separation. However it is conceivable that the flow model on which the present method is based becomes less valid at the higher Mach numbers, and in this respect we may note the findings of Charwat and Yakura[13] that strong interactions between the component parts of the model become increasingly apparent at a Mach number of 3. Even viewed in this light the theory is seen to be a substantial advance from earlier methods[16, 18, 19] in which the significance of the parameter N was not considered. That of Carrière and Sirieix[18] is typical of these and is illustrated in Fig. 12b.

Also shown in Figs. 11 and 12 are the points given for $\theta/t = 0$ by the theory of Korst[5] and it is seen that a number of data points lie below the level indicated by this method. The limiting base pressures derived from the present analysis are substantially lower than the predictions of Korst and are not violated, according to the evidence available, by the occurrence of cases where lower base pressures can be measured.

Before leaving this discussion on the comparison between the available data and the predictions of the present theory it must again be stressed that the validity of some of the measurements could be questioned. Indeed we have already pointed out (Sections 1.1 and 1.2 above) that in a few tests the small span of the models employed may have led to significant cross-flow effects and that these can provide a mechanism for some decrease in the base pressure from the values appropriate to strictly two-dimensional flow. However the justification of the present method does not depend on these data, and moreover the fact that in general satisfactory correlation is found between the results of these tests and those obtained under more ideal conditions tends to suggest that the effect of cross flows on base pressure may not always be large.

7.3. The apparent success with which certain base pressures measured [12, 14] in the transitional Reynolds number range have been correlated with the results of the present theory leads us to attempt a further exercise. It was found that the minimum base pressure reached when the transition point was located on the aerofoil surface but close to the trailing edge could be predicted from the theory by estimating the boundary-layer momentum thickness on the assumption of laminar flow on the body surface. With increase of Reynolds number from this condition the transition point moves upstream over the body and the increasing thickness of the boundary layer at the trailing edge brings about a rise in base pressure[2, 12]. It would seem then that if the boundary-layer growth could be estimated during this phase the increase in base pressure with Reynolds number on a particular section could be predicted.

Standard methods are available (e.g. Ref. 47 for low speeds) for computing the momentum thickness at the trailing edge of an aerofoil once the position of the transition point is specified. From the results of Gadd *et al*[12] and Van Hise[14] the transition Reynolds number appropriate to their test conditions is known since it is equal to the chord Reynolds number at which the minimum base pressure was reached, in fact in the former paper the stage at which the transition point moved on to the body is indicated. Assuming the transition Reynolds number to remain constant the extent of turbulent flow can be assessed at any higher chord Reynolds number and hence the momentum thickness of the boundary layer at the trailing edge computed.

The variation of θ/t, where t is the trailing-edge semi-thickness, with Reynolds number for the wedge section in Ref. 12 has been calculated (on the basis of flat-plate theory) and is shown in Fig. 13a. The transition Reynolds number indicated by T was taken as $0 \cdot 8 \times 10^6$. For these values of θ/t the corresponding variation of base pressure with Reynolds number, derived from the present method, is seen to be in reasonable agreement with the measurements. The correlation would incidentally have been marginally better if the transition Reynolds number had been taken at the actual minimum in the measured curve, i.e. approximately 10^6, rather than the value quoted in the paper. This of course raises the issue that transition from laminar to turbulent flow occurs in fact not at a point but over some finite distance and if for the purposes of an approximation an equivalent 'transition point' must be assumed there can be a measure of latitude in its selection according to the particular problem. The discontinuity of slope in the predicted variation of base pressure with

Reynolds number is also of course introduced as a result of the assumption of a definite point of transition.

A similar comparison between the measured and the theoretical variation of base pressure with chord Reynolds number for the test on an ogive model by Van Hise[14] is illustrated in Fig. 13b for a Mach number of 2·22. Here the transition Reynolds number is taken as $2 \cdot 4 \times 10^6$ which corresponds to the point of minimum base pressure. The predicted increase of base pressure with Reynolds number is in good agreement with the measured values except for a slight difference in overall level of approximately 0·02 in P_b/P_1. In view of the number of uncertainties in the present calculations, such as the variation of σ and u_c^* with Mach number or the value of N, a discrepancy of this magnitude is not to be considered significant.

These two examples serve to illustrate the use of the present theory to predict, firstly the minimum base pressure reached in the transitional Reynolds number range when turbulent flow first occurs at the trailing edge of the section, and secondly the variation of base pressure with increasing chord Reynolds number from this condition. It is important to note the consequence of the higher transition Reynolds number in the tests by Van Hise as compared with that in the tests by Gadd *et al.* In this latter the transition Reynolds number was $0 \cdot 8 \times 10^6$ and the onset of turbulent flow at the trailing edge occurred while the laminar boundary layer was still relatively thick. Thus the value of θ/t did not fall below 0·012 and the minimum base pressure ratio was approximately 0·32. In the experiment by Van Hise however laminar flow was preserved up to a Reynolds number of $2 \cdot 4 \times 10^6$ allowing the thickness of the boundary layer to continue decreasing. Hence when the transition Reynolds number was finally reached the value of θ/t was as small as 0·007 (the ratio of trailing-edge thickness to chord was the same in the two cases), and the transition to turbulent flow resulted in the fall of the base pressure, even allowing for the difference in Mach number, to a substantially lower value.

Thus it would appear that the longer laminar flow is preserved, (in terms of Reynolds number), the lower will be the base pressure when the onset of turbulence does take place. This fact should be taken into account when boundary-layer laminarization schemes for blunt-trailing-edge wings are being considered.

7.4. The present theory has been applied to the problem of predicting the variation of base pressure with boundary-layer thickness for both backward-facing steps and blunt-trailing-edge aerofoils at supersonic speeds. The agreement between theory and experiment has been very encouraging especially for the cases when the ratio of boundary-layer thickness to base height was not large. We shall now attempt to use the theory to explain certain features of the subsonic flow past a backward-facing step.

It was seen in Section 7.1 above that the variation of the reattachment parameter N with Mach number through the subsonic and transonic ranges was marked and it will be shown that this will dominate the variation of base pressure. In this respect the calculations will depend to a large extent on empirical data and this dependence will be all the more so since it is recalled that the base-flow solution for subsonic speeds in all but the limiting condition involves an additional free parameter, namely the length of the turbulent mixing layer. It will not therefore be possible to make any predictions of the base pressure in general cases at this stage, but the validity of the analysis will be checked against the results of an experiment on the flow past a step by Nash *et al*[15]. In this series of tests the base pressure on a backward-facing step was recorded and measurements were made of the location of the reattachment point and its position with respect to the pressure rise through the reattachment region. Hence a set of data was obtained relating the base pressure, the length of the free-shear layer and the value of N through the Mach number range $0\cdot4$ to $1\cdot1$.

The variation of N with Mach number derived from this experiment was presented in conjunction with other data in Fig. 10 and has been referred to already. (*See* Section 7.1.) Using these values the variation of limiting base pressure with Mach number can be computed from equation (6.5). At subsonic speeds the base pressures can be expressed more conveniently as coefficients and the results are presented in this form in Fig. 14. The limiting base pressures are indicated by the chain-dotted curve. It is to be noted at this stage that the abrupt fall in the base pressure near sonic velocity is accounted for by the theory and is in fact directly associated with the similar decrease in the value of N.

The length of the free-shear layer, or to be more precise its projection on the downstream surface, over this range of Mach number is shown (inset) in Fig. 14. Through the subsonic range the value of l/t is seen to increase steadily reaching a maximum of 11 but shortly after the flow adjacent to the cavity becomes sonic the shear layer begins to deflect sharply at the corner and the value of l/t decreases again. As soon as the flow in the free stream becomes supersonic the wake geometry is known in terms of the Prandtl-Meyer angles of the external stream and the empirical values of l/t are no longer required.

Hence with these values of N and l/t the present theory is used to estimate the base pressures corresponding to the appropriate boundary-layer thickness. In the experiment of Ref. 15 the ratio of the boundary-layer momentum thickness to the height of the step was found to be in the region of $0\cdot026$ and this value has been assumed in the calculations. The base pressures are computed for the three flow régimes. In the range where the flow is everywhere subsonic the solution is given by equation (6.1) and is valid up to a free-stream Mach number of approximately $0\cdot86$. When the flow is everywhere supersonic equation (6.2) is used. In the intermediate range of mixed flow the wake geometry is not yet specified and the calculations must continue to rely on the measured values of l/t. However the effect of the abrupt expansion at the corner is beginning to be felt and the correction must be applied to take account of the decrease in momentum thickness suffered by the boundary layer as it negotiates the expansion. This is provided for in equation (6.3) which is seen (Fig. 14) to link the other two solutions for wholly subsonic and wholly supersonic flow. In all these calculations the values of σ and u_c^* correct for incompressible flow were assumed.

In this way the variation of base pressure with Mach number through the subsonic and transonic ranges has been computed in this particular case, and the measure of agreement with the experimental points is demonstrated in Fig. 14. It is observed that both the extent and the position, with respect to Mach number, of the transonic fall in base pressure has been estimated with good precision and it would appear that the reasons for its occurrence are understood and can be represented adequately in the present flow model. The abrupt decrease in base pressure is seen to be associated with the change in the pressure distribution through the recompression region at reattachment brought about by the establishment of supersonic flow outside the reattaching boundary layer. With the disappearance of the overshoot (Fig. 2a) in the pressure distribution along the downstream surface reattachment takes place at a point where the pressure is below free-stream ambient pressure, and in order to accommodate the pressure rise to reattachment determined by conditions in the free-shear layer the base pressure is forced to decrease.

A further point worth mentioning concerns the magnitude of the increase in base pressure from the limiting values brought about by the presence of the boundary layer upstream of the base.

The effect of the initial boundary layer on the reattachment process, and hence on the base pressure, arises insofar as the velocity profile at separation causes the development of the free-shear layer to depart from its asymptotic form. As the length of the shear layer increases therefore, the influence of the initial perturbations is diminished and the base pressure will decrease towards its limiting value. In the case under consideration the length of the free-shear layer increases to a maximum at a Mach number of $0 \cdot 95$ after which it falls again as supersonic flow is established and the layer begins to deflect sharply at the corner, and over the short Mach number range in which the values of l/t are high the presence of the boundary layer is seen (Fig. 14) to result in only a small increase in base pressure. This result might explain to some extent the fact (*see* Section 1.2 above, and Ref. 1) that at Mach numbers near unity the base pressure on aerofoil sections does not vary significantly with decrease of the ratio of trailing-edge thickness to chord until values of this ratio as low as $0 \cdot 02$ are reached.

Through the subsonic speed range the base-pressure coefficient appears to be relatively constant and the predicted level is in satisfactory agreement with the measurements. The present analysis could be claimed to go some way towards providing a method for estimating the base pressure on rearward-facing steps. If the values of N derived from the experiment of Ref. 15 are representative the limiting base pressures can be assessed, and if data become available on the length of the free-shear layer under different conditions the variation of base pressure with boundary-layer thickness could be readily predicted. In general, at subsonic speeds the wake behind a blunt-trailing-edge aerofoil is dominated by periodic effects and the present flow model does not adequately represent the true picture. However if techniques are perfected for inhibiting the formation of vortices and steady flow can be achieved, the base pressures on aerofoil sections could be predicted on the basis of the present theory.

7.5. At this stage it is well to examine more closely the implications of the main point made in the present paper, namely the significance of the parameter N. It has been seen that for turbulent base flow at both subsonic and supersonic speeds the acceptance of values of N different from unity not only pays more attention to the observed behaviour of the flow in the recompression region but leads to a greatly improved representation of the effect of the approaching boundary layer on base pressure. Moreover it enables the theory to explain more readily certain anomalies arising from the concept of the limiting base pressure. However aside from the discussion presented in Section 1 above, it could still be argued that the assumption of $N = 1$ in the analysis is more well-founded than is indicated just by the agreement obtained in the supersonic case between the method of Korst and the body of experimental data to which we have already referred.

Indeed although for the present we have dealt with the turbulent case it remains to look for a moment at the relevance of these results to the laminar base flow. At supersonic speeds the variation with Mach number of the limiting base pressure has been computed by Chapman *et al*[4] on the assumption, to use the present notation, that N is equal to unity. If in the case of turbulent base flow it is conceded that conditions of zero boundary-layer thickness at separation are impossible to realise in practice, it is important to remember that Chapman *et al* were able to devise experiments on laminar 'base flow' in which separation was provoked at the leading edge of the models in order to reduce the influence of the initial boundary layer to a minimum. The fact that good agreement was obtained between the base pressures measured under these conditions and the values predicted by the theory is highly significant and some indication of an explanation is certainly called for.

While further examination of this point is necessary it would seem at first sight that the difficulty could be resolved by one of two arguments; either the assumption of $N = 1$ is valid for laminar flow, or again the true limiting condition had not in fact been approached in the experiments in question. The first is more unlikely since there is no obvious reason why the laminar reattachment should be qualitatively different from that in turbulent flow, and indeed the data on the value of N submitted in Fig. 10 include a result obtained in tests on laminar flow[27]. In the context of the latter suggestion a definite conclusion must await some calculations to determine the extent to which the limiting base pressure can be approached under experimental conditions. If the value of $\partial P_b/\partial \theta$ in the neighbourhood of the limiting condition is an order of magnitude greater in the laminar case than the present analysis indicates for turbulent base flow (and this is not implausible), one might well be led to question the assertion in Ref. 4 that the boundary-layer effect was negligible. Moreover the term 'leading-edge separation' itself needs careful interpretation.

At low subsonic speeds the theory of steady base flow makes no distinction between the limiting base pressures in laminar and turbulent flow since the velocity on the median streamline under asymptotic conditions is in both cases approximately $0 \cdot 58$. As an exercise the theory of supersonic laminar base flow was extrapolated to $M = 0$ by Chapman *et al*[4], and the value of the limiting base-pressure coefficient so derived, $-0 \cdot 526$, was found to agree well with Roshko's measurements[48] of the base pressure on the leeward side of bluff cylinders when the wake was stabilised by a splitter-plate. Now in the supersonic case the implication of the present theory was that the true limiting base pressure was lower than that predicted by previous methods. At subsonic speeds however the available evidence suggests that the value of N is greater than unity and therefore the theoretical limiting base pressure ought to be higher than that indicated by the pressure coefficient computed by Chapman *et al.* If the values of N at low speeds suggested by the data in Fig. 10 are representative of conditions relevant to the flow past bluff bodies, it would be necessary to examine more closely the discrepancy between the predictions of the present method and the experimental results of Roshko. It might be argued that the length of the splitter plates used in these tests was insufficient to completely stabilise the wake but similar low base pressures were measured on the rear of a flat plate normal to the stream by Arie and Rouse[49] when the length of the splitter plate (in terms of the height of the model) was considerably greater. More probably the values of N indicated in Fig. 10 are not representative of the subsonic flow past bluff shapes. The extent of the overshoot in the pressure distribution would appear to be dependent on the thickness of the reattachment shear layer, and it could be argued that the value of N will decrease with increase of thickness of the shear layer. In the case of bluff sections the length, and hence the thickness at reattachment, of the free-shear layer are proportionately greater than the corresponding values for the flow past a step. Thus for a bluff shape a reduced value of N could be to some extent anticipated, and the observation of lower base pressures in this case would not necessarily undermine the essential argument put forward in the present paper.

8. *Conclusions.*

(*a*) The present position of the theory of supersonic turbulent base flow has been reviewed and attention is drawn to a number of shortcomings. In particular an examination has been made of experimental data which appear to evade Korst's predictions of the limiting minimum base pressure approached at a given Mach number as the thickness of the boundary layer upstream of the base tends to zero. On the basis of this exercise and of considerations of the magnitude of the boundary-layer effect in other results it is suggested that the theory of Korst is incorrect and that the true limiting base pressures are substantially lower than the values predicted by that theory.

(*b*) The analysis of two-dimensional turbulent base flow has been formulated in general on the basis of a flow model which considers the passage of a stream past a backward-facing step in a otherwise plane boundary. In contrast with the reattachment criterion used in previous attempts at the solution the total pressure on the reattachment streamline is equated not to the final recovery pressure far downstream of the base but to the static pressure at the reattachment point which is left to be determined. Experimental results on reattaching flows are collected which indicate that reattachment does not generally take place at a point where the local static pressure is equal to the final recovery pressure and a parameter N is introduced which is the ratio of the pressure rise to reattachment to the difference between ambient static pressure and the base pressure. At subsonic speeds N is shown to take values greater than unity while at supersonic speeds the value of N is smaller and, on the evidence available, appears to lie in the region of 0·35.

(*c*) At Mach numbers between 2 and 3 the theory is used to predict the variation of base pressure with boundary-layer momentum thickness and satisfactory agreement is obtained with the available data, at least until the ratio of boundary-layer thickness to base height ceases to be small, if N takes the value 0·35. It is pointed out that a mechanism exists by which the base pressure on a step may not be the same as that on an isolated aerofoil section under nominally identical conditions. The experimental evidence can be interpreted as indicating that this is true when the thickness of the boundary layer approaching the base is large, but any definite conclusion on this matter would at this stage be premature.

Attention is drawn to the extent to which the low base pressures observed in the transitional Reynolds number range can be attributed to the very small values of the ratio of momentum thickness to base height which can then be reached. It is shown that both the minimum base pressure and the rise of base pressure with increasing chord Reynolds number from this condition can be estimated by the present theory once the transition Reynolds number is known.

(*d*) At subsonic speeds the validity of the present flow model is checked against the results of an experiment on a backward-facing step by Nash *et al*[15], and the basic assumptions made in the analysis for subsonic flow appear to be supported. At transonic speeds the abrupt fall in base pressure which occurs as sonic velocity is approached can be accounted for on the basis of the present theory and is seen to be associated with a similar decrease in the value of the parameter N referred to above.

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LIST OF SYMBOLS

x, y	Cartesian co-ordinates
u, v	Velocity components
P	Pressure
T	Temperature
M	Mach number
ρ	Density
c_p	Specific heat
γ	Ratio of specific heats
ψ	$= \int \rho u \, dy$, stream function
ν	Prandtl-Meyer angle corresponding to Mach number M
θ	Momentum thickness of boundary layer approaching base
ϑ	Momentum thickness of boundary layer immediately downstream of centred expansion
Θ	Momentum thickness of free-shear layer
Ψ	Integral defined by equation (4.4)
σ	Parameter related to rate of spread of shear layer {equation (4.1)}
x'	Distance between virtual origin of equivalent shear layer and separation point
$\bar{x}$	Distance from origin of equivalent asymptotic free-shear layer
ζ	$= \dfrac{\sigma y}{\bar{x}}$
l	Length of free-shear layer
t	Height of step; equivalent to semi-thickness of blunt trailing edge
q	Base-bleed mass flux
$f(u^*)$	Non-dimensional stream function defined by equation (4.9)
λ	Density ratio defined by equation (4.22)
A, B, C_1, C_2, C_3	Functions defined in Section 6
N	Reattachment (or recompression) parameter {equation (5.1)}

Subscripts

$_e$	Conditions outside viscous layer
$_1$	Conditions far upstream
$_2$	Conditions in free-shear layer
$_b$	Conditions in cavity behind base
$_r$	Conditions at reattachment point
$_0$	Conditions at $x = 0$

Conditions on specific streamlines:

$_S$	Separation streamline
$_R$	Reattachment streamline
$_M$	Median streamline
$_c$	Conditions on median streamline of asymptotic free-shear layer
h	Conditions on reference streamline outside free-shear layer

Ratios of quantities in the viscous layer to those at the edge of the layer are denoted by an asterisk, e.g. $u^* = u/u_e$, $\rho^* = \rho/\rho_e$.

Conditions in the free stream are denoted by subscript $_{e_1}$, e.g. M_{e_1}, u_{e_1}.

The pressure coefficient C_p is defined by $C_p = \dfrac{P - P_1}{\frac{1}{2}\rho_{e_1} u_{e_1}{}^2}$.

The term 'external stream' is used to refer to the quasi-inviscid flow adjacent to the viscous flow in the boundary layer and free-shear layer.

REFERENCES

No.	Author(s)	Title, etc.
1	J. F. Nash	A review of research on two-dimensional base flow. A.R.C. R. & M. 3323. March, 1962.
2	L. Crocco and L. Lees	A mixing theory for the interaction between dissipative flows and nearly isentropic streams. *J. Ae. Sci.*, Vol. 19, p. 649. October, 1952.
3	D. R. Chapman	An analysis of base pressure at supersonic velocities and comparison with experiment. N.A.C.A. Tech. Note 2137. July, 1950.
4	D. R. Chapman, D. M. Kuehn and H. K. Larson	Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transition. N.A.C.A. Report 1356. 1958.
5	H. H. Korst	A theory for base pressures in transonic and supersonic flow. *J. App. Mech.*, Vol. 23, No. 4, p. 593. December, 1956.
6	W. L. Chow	On the base pressure resulting from the interaction of a supersonic external stream with a sonic or subsonic jet. *J. Ae. Sci.*, Vol. 26, p. 176. March, 1959.
7	R. H. Page and H. H. Korst ..	Non-isoenergic turbulent compressible jet mixing with consideration of its influence on the base pressure problem. Proc. 4th midwest Conf. Fluid Mech., Purdue University. September, 1955.
8	D. R. Chapman, W. R. Wimbrow and R. H. Kester	Experimental investigation of base pressure and blunt-trailing-edge wings at supersonic velocities. N.A.C.A. Report 1109. 1952.
9	L. Fuller and J. Reid	Experiments on two-dimensional base flow at $M = 2 \cdot 4$. A.R.C. R. & M. 3064. February, 1956.
10	M. Sirieix	Pression de culot et processus de mélange turbulent en écoulement supersonique plan. *La Recherche Aeronautique*, No. 78, p. 13, O.N.E.R.A. September-October, 1960.
11	J. J. Ginoux	On the existence of cross flows in separated supersonic streams. T.C.E.A. Tech. Note 6, Belgium. February, 1962.
12	G. E. Gadd, D. W. Holder and J. D. Regan	Base pressure in supersonic flow. A.R.C. C.P.271. March, 1955.
13	A. F. Charwat and J. K. Yakura ..	An investigation of two-dimensional supersonic base pressures. *J. Ae. Sci.*, Vol. 25, p. 122. February, 1958.
14	V. van Hise	Investigation of variation in base pressure over the Reynolds number range in which wake transition occurs for two-dimensional bodies at Mach numbers from $1 \cdot 95$ to $2 \cdot 92$. N.A.S.A. Tech. Note D-167. November, 1959.

No.	*Author(s)*	*Title, etc.*
15	J. F. Nash, V. G. Quincey and J. Callinan	Experiments on two-dimensional base flow at subsonic and transonic speeds. Report in preparation.
16	F. N. Kirk	An approximate theory of base pressure in two-dimensional flow at supersonic speeds. R.A.E. Tech. Note Aero.2377. December, 1959.
17	J. F. Nash	Unpublished work.
18	P. Carrière and M. Sirieix ..	Facteurs d'influence du recollement d'un écoulement supersonique. Proc. 10th int. Congr. Appl. Mech., Stresa. September, 1960. O.N.E.R.A. Memo. Tech. 20. 1961.
19	K. Karashima	Base pressure on two-dimensional blunt-trailing-edge wings at supersonic velocity. University of Tokyo, Aero. Research Institute Report 368. October, 1961.
20	S. B. Savage	An approximate analysis for reattaching turbulent shear layers in two-dimensional incompressible flow. McGill University, Montreal. Mech. Eng. Res. Lab. Report Ae.3. September, 1960.
21	M. A. Beheim	Flow in the base region of axisymmetric and two-dimensional configurations. N.A.S.A. Tech. Report R-77. 1961.
22	B. S. Stratford (with an Appendix by G. E. Gadd)	Flow in the laminar boundary layer near separation. A.R.C. R. & M. 3002. November, 1954.
23	M. A. Beheim, J. L. Klann and R. A. Yeager	Jet effects on annular base pressure and temperature in a supersonic stream. N.A.S.A. Tech. Report R–125. 1962.
24	M. A. Badrinarayanan	An experimental investigation of base flows at supersonic speeds. *J. R. Ae. Soc.*, Vol. 65, p. 475. July, 1961.
25	S. M. Bogdonoff, C. E. Kepler and E. Sanlorenzo	A study of shock wave turbulent boundary layer interaction at $M = 3$. Princeton University, Dept. of Aero. Eng. Report 222. July, 1953.
26	I. E. Vas and S. M. Bogdonoff ..	Interaction of a shock wave with a turbulent boundary layer at $M = 3 \cdot 85$. Princeton University, Dept. of Aero. Eng. Report 294. A.F.O.S.R. Tech. Note 55–199. April, 1955.
27	R. J. Hakkinen, I. Greber, L. Trilling and S. S. Abarbanel	The interaction of an oblique shock wave with a laminar boundary layer. M.I.T. Fluid Dynamic Research Group. Tech. Report 57–1. May, 1957.

No.	*Author(s)*	*Title, etc.*
28	M. H. Bloom	On moderately separated viscous flows. *J. Ae. Sci.* (Readers' Forum), Vol. 28, p. 339. April, 1961.
29	H. H. Pearcey	Shock-induced separation and its prevention by design and boundary-layer control. (Part IV of *Boundary layer and flow control.* Editor G. V. Lachmann.) Pergamon Press. 1961.
30	C. Bourque and B. G. Newman ..	Reattachment of a two-dimensional jet to an adjacent flat plate. *Aero. Quart.*, Vol. 11, Part 3, p. 201. August, 1960.
31	H. Thomann	Measurements of heat transfer and recovery temperature in regions of separated flow at a Mach number of 1·8. F.F.A. Report 82, Sweden. 1959.
32	H. H. Korst and W. Tripp ..	The pressure on a blunt trailing edge separating two supersonic two-dimensional air streams of different Mach numbers and stagnation pressures, but identical stagnation temperatures. Paper presented at the midwest Conf. on Solid and Fluid Mech., University of Michigan. April, 1957.
33	M. L. Robinson	An experimental investigation of laminar boundary-layer separation in the presence of a convex corner. Report in preparation.
34	H. K. Zienkiwicz	An investigation of boundary-layer effects on two-dimensional supersonic aerofoils. College of Aeronautics Report 49. December, 1951.
35	K. R. Ananda Murthy and A. G. Hammitt	Investigation of the interaction of a turbulent boundary layer with Prandtl-Meyer expansion fans at $M = 1 \cdot 88$. Princeton University, Dept. of Aero. Eng. Report 434. A.F.O.S.R. Tech. Note 58–839. August, 1958.
36	J. F. Nash	The effect of an initial boundary layer on the development of a turbulent free shear layer. A.R.C. C.P.682. June, 1962.
37	W. Tollmien	Berechnung turbulenter Ausbreitungsvorgänge. *Z.A.M.M.*, Vol. 6, p. 468. 1926. (Translated as N.A.C.A. Tech. Memo. 1085. 1945.)
38	H. Görtler	Berechnung von Aufgaben der freien Turbulenz auf Grund eines neuen Näherungsansatzes. *Z.A.M.M.*, Vol. 22, p. 244. 1942. (Translated as R.T.P. Trans. 2234. 1944.) A.R.C. 7,947.
39	G. N. Abramovich	The theory of a free jet of a compressible gas. Central Aero. Hydrodynamical Institute, Moscow, Report 377. 1939 (Translated as N.A.C.A. Tech. Memo. 1058. 1944.)

No.	*Author(s)*	*Title, etc.*
40	H. Reichardt	Gesetzmässigkeiten der freien Turbulenz. *V.D.I.-Forschungsheft*, No. 414, 1942. (Translated as R.T.P. Trans. 1752, A.R.C. 6,670.)
41	H. W. Liepmann and J. Laufer ..	Investigation of free turbulent mixing. N.A.C.A. Tech. Note 1257. August, 1947.
42	L. J. Crane	The laminar and turbulent mixing of jets of compressible fluid. Part II. The mixing of two semi-infinite streams. *J. Fluid Mech.*, Vol. 3, Part I, p. 81. October, 1957.
43	N. H. Johannesen	The mixing of free axially symmetric jets of Mach number 1·40. A.R.C. R. & M. 3291. January, 1957.
44	P. B. Gooderum, G. P. Wood and M. J. Brevoort	Investigation with an interferometer of the turbulent mixing of a free supersonic jet. N.A.C.A. Report 963. 1950.
45	E. T. Pitkin and I. Glassman ..	Experimental mixing profiles of a Mach 2·6 free jet. *J. Ae. Sci.* (Readers' Forum), Vol. 25, p. 791. December, 1958.
46	J. S. Thale and R. O. Fredette ..	Dynamics of packages deployed from high performance rockets. *Z. Flugwiss.*, Vol. 10, Part 6, p. 229. June, 1962.
47	H. B. Squire and A. D. Young ..	The calculation of profile drag of aerofoils. A.R.C. R. & M. 1838. November, 1937.
48	A. Roshko	On the drag and shedding frequency of two-dimensional bluff bodies. N.A.C.A. Tech. Note 3169. July, 1954.
49	M. Arie and H. Rouse	Experiments on two-dimensional flow over a normal wall. *J. Fluid Mech.*, Vol. 1, Part 2, p. 129. July, 1956.
50	G. E. Gadd	(Private communication.)
51	D. R. Chapman and H. H. Korst	Discussion on 'theory for base pressures in transonic and supersonic flow'. *J. App. Mech.*, Vol. 24, No. 3, p. 484. 1957.
52	A. F. Charwat, J. N. Roos, F. C. Dewey and J. A. Hitz	An investigation of separated flows: Part I. The pressure field. *J. Ae. Space Sci.*, Vol. 28, p. 457. June, 1961.
53	R. C. Hastings	Unpublished work.

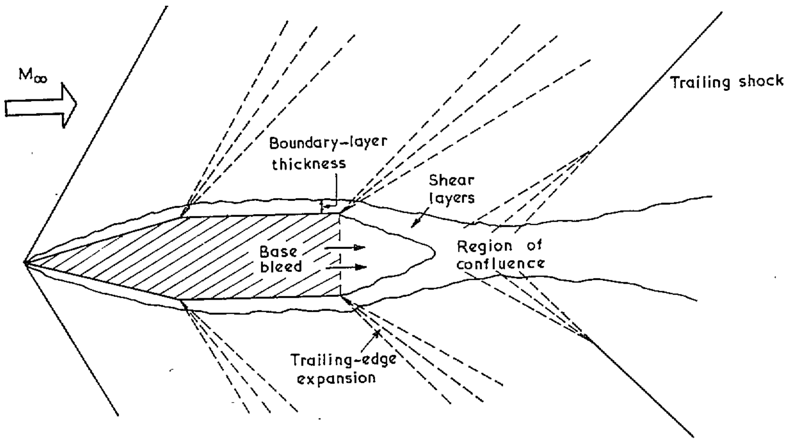

FIG. 1. Supersonic flow past a blunt-trailing-edge section.

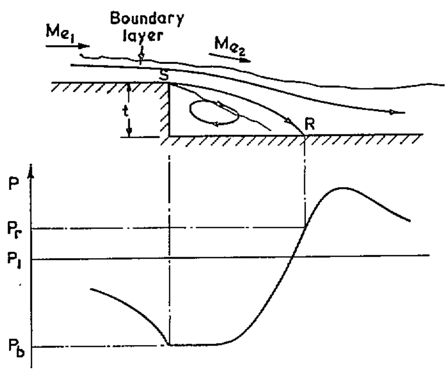

FIG. 2a. Subsonic flow over a step.

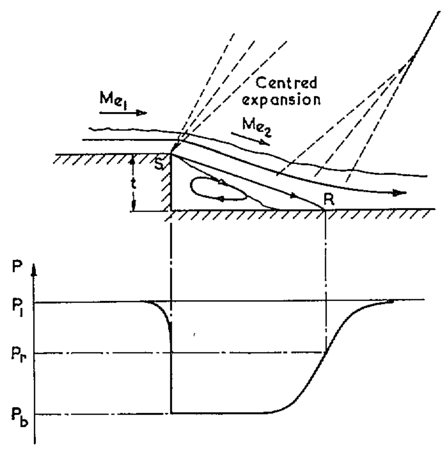

FIG. 2b. Supersonic flow over a step.

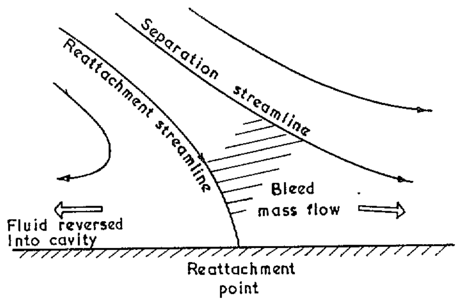

FIG. 3. Flow near reattachment point.

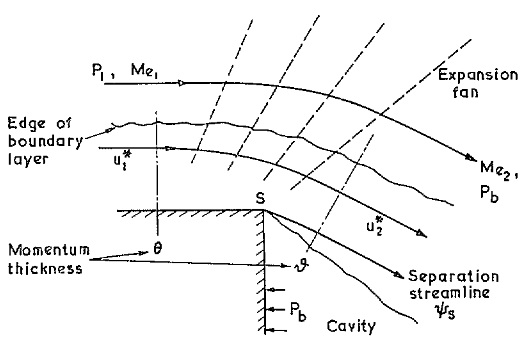

FIG. 4. Model of supersonic flow at corner.

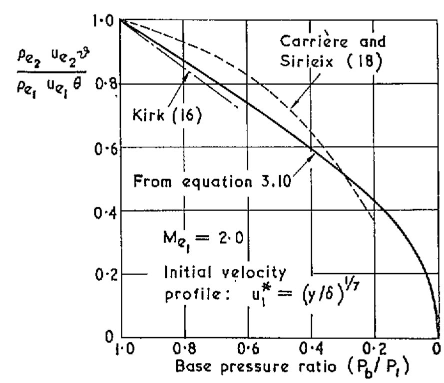

FIG. 5. Effect of abrupt expansion on boundary-layer momentum thickness.

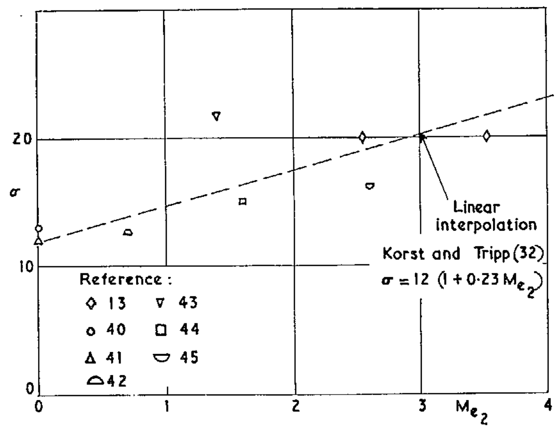

FIG. 6. Variation of σ with Mach number.

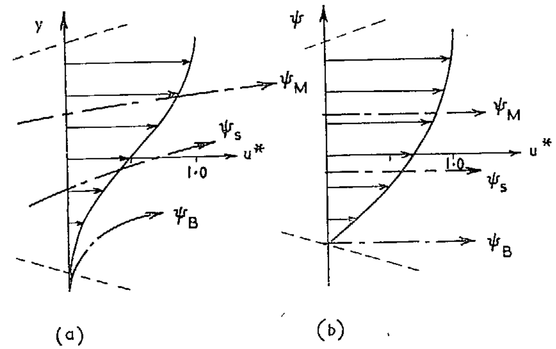

FIG. 8. Shear-layer velocity profiles in terms of (a) y and (b) stream function.

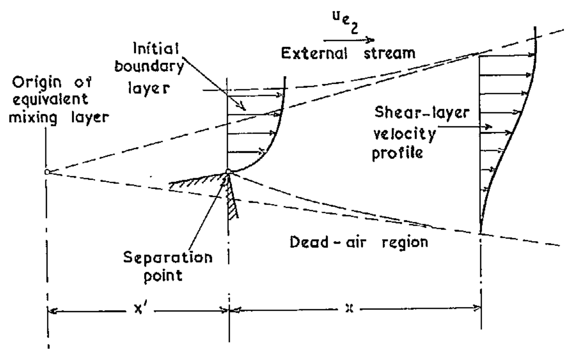

FIG. 7. Model for analysis of free-shear layer.

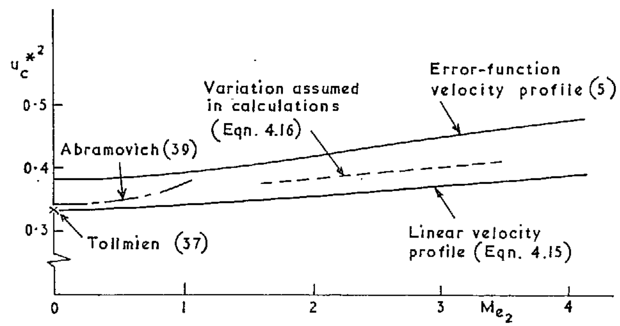

FIG. 9. Variation of u_c* with Mach number.

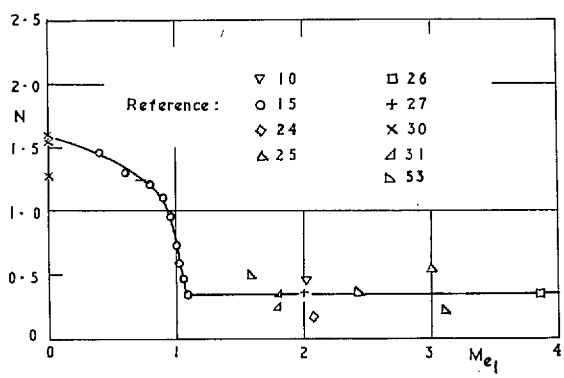

FIG. 10. Variation of N with Mach number.

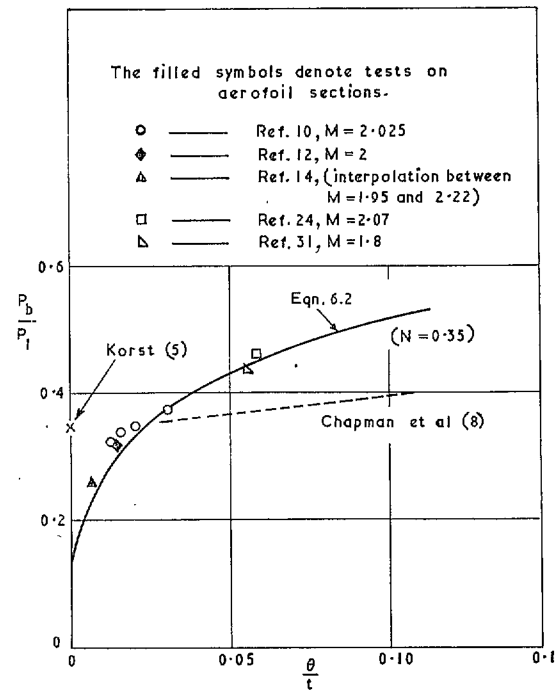

FIG. 11. Variation of base pressure with boundary-layer momentum thickness ($M_{e_1} = 2\cdot0$).

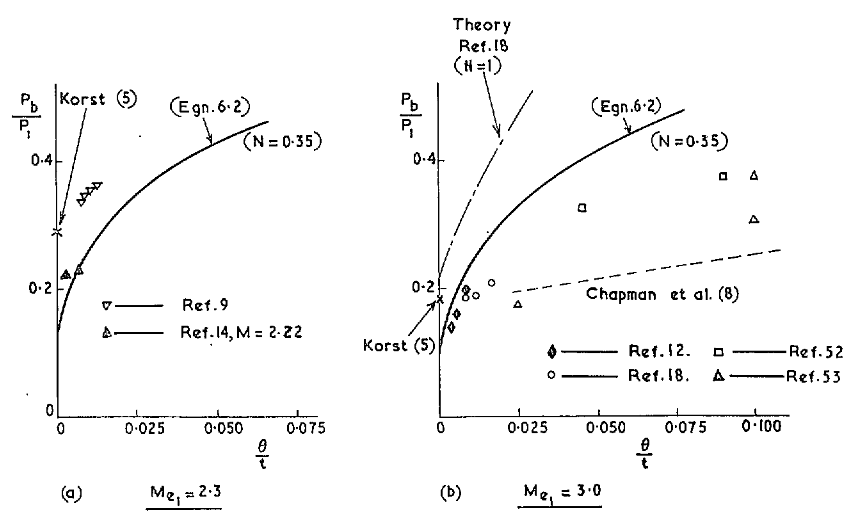

FIG. 12. Variation of base pressure with boundary-layer momentum thickness ($M_{e_1} = 2\cdot3$ and $3\cdot0$).

(87996) Wt. 65/1418 K.5 11/63 Hw.

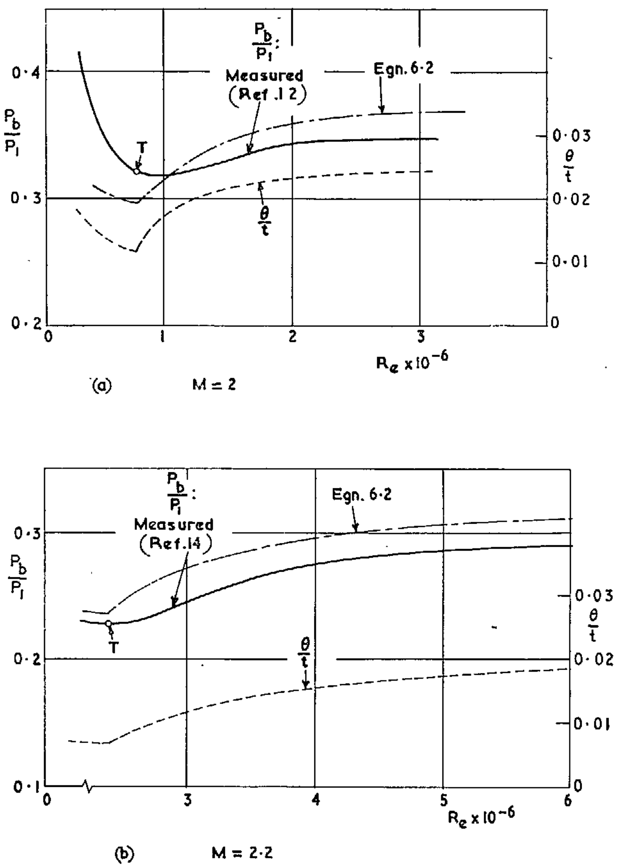

FIG. 13. Variation of base pressure on aerofoils with chord Reynolds number.

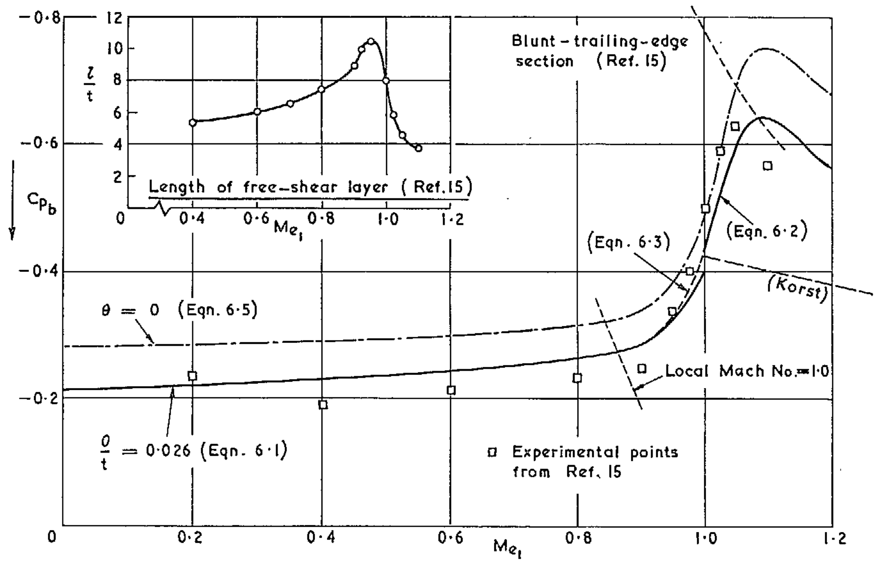

FIG. 14. The base pressure on a step at subsonic and transonic speeds. Comparison between theory and experiment.

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