R. & M. No. 2544
(6259)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Induced Drag of Flapped Elliptic Wings with Cut-out and with Flaps that Extend the Local Chord

By

A. D. YOUNG, B.A.

LONDON: HIS MAJESTY'S STATIONERY OFFICE
1951
PRICE 2*s* 6*d* NET

The Induced Drag of Flapped Elliptic Wings with Cut-out and with Flaps that Extend the Local Chord

By

A. D. Young, B.A.

Communicated by the Principal Director of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 2544*

February, 1942

Summary.—Calculations have been made of the induced drag of flapped elliptic wings covering a range of aspect ratios from about 4 to about 12, a range of flap spans from 0·2 to 1·0 of the wing span, and a range of flap cut-outs from 0 to 0·6 of the wing span. The results are presented in charts in a convenient form for application. Calculations have also been made of the induced drag of elliptic wings of an aspect ratio of about 6 with flaps that extend the local chord by 40 per cent when in operation; flap spans of 0·26, 0·5 and 0·77 of the wing span were examined.

It is concluded that for a given net flap span and lift increment minimum induced drag will be obtained with a cut-out of about 0·1 wing span. The effect of local chord extension due to a flap was found to be negligible.

These results apply strictly to elliptic wings but they probably apply with fair accuracy to wings of taper ratio of the order of 2 : 1.

1. *Introduction.*—Although there is a considerable and growing body of literature dealing with the theoretical lift distributions, induced drags, etc., of flapped wings, the effects on induced drag of flap cut-out or of flaps that increase the local chord when in operation do not appear to have been considered in any detail. Both these points are of some practical importance; flap cut-outs are frequently inevitable on aircraft because of the presence of the fuselage, and modern high-lift flaps generally involve some extension of the local chord in their operation (*e.g.* Fowler flaps). Calculations have therefore been made to investigate their effect in a comprehensive range of cases for untwisted elliptic wings of various aspect ratios. Elliptic wings have been chosen because the calculations are then considerably less laborious than for wings of non-elliptic plan form, and the results can be taken to apply fairly closely to wings of taper ratio of the order of 2 : 1.

2. *Theory and Scope of Calculations.*—The theory upon which the calculations depend is described in detail in the Appendix. It is based on the assumption that the wing can be replaced by a lifting line and follows closely the procedure usual to such investigations. Briefly, the spanwise distribution of the circulation is expressed as a Fourier series from which a series for the spanwise distribution of downwash is deduced. These series are then substituted in the fundamental Joukowski relation between the lift developed at any point and the circulation there. The resulting basic equation is then solved to satisfy the given chord and geometric incidence distributions along the flapped and unflapped parts of the wings. It is assumed that the ratio local flap chord/local wing chord is constant along the flap span, and that the effect of the flaps is equivalent to a constant change of geometric incidence along the flap span resulting in a local lift increment due to the flap independent of the local geometric wing incidence.

* R.A.E. Report Aero. 1732—received 4th November, 1942.

In the case of the elliptic wing with non-extending flaps, with or without cut-outs, the basic equation is easily solved to give all the coefficients of the assumed Fourier series, from which the induced drag can be calculated. For the calculations described in this note it was assumed, following Hollingdale[1] (1936), that sufficient accuracy was attained by neglecting all but the first eight terms of the series. By adopting the familiar form for the induced drag, viz. :—

$$C_{Di} = \frac{C_L^2}{\pi A}(1 + \delta) , \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (1)$$

it was found that the quantity δ, allowing for the effect of the flaps, could be expressed in the simple and useful form

$$\delta = K\left(\frac{\Delta C_L}{C_L}\right)^2 , \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (2)$$

where K is a constant depending only on the aspect ratio A of the wing and the span and cut-out of the flap, ΔC_L is the increment in the lift coefficient due to the flap, and C_L is the total lift coefficient of the wing and flap. Calculations of the factor K were made for values of A/a_0 of $\frac{2}{3}$, $1 \cdot 0$ and $2 \cdot 0$ (a_0 is the lift-incidence slope of the wing for infinite aspect ratio and is approximately equal to 6), and covered a range of flap spans up to the full wing span and a range of cut-outs up to $0 \cdot 6$ wing span.

The basic equation for the Fourier coefficients in the case of the elliptic wing with extending flaps could not be solved directly. It was solved, however, by a method of successive approximation which, fortunately, was found to be rapidly convergent. The calculations were made for an aspect ratio given by $A/a_0 = 1 \cdot 0$ and a flap which extended the local wing chord by 40 per cent ; three flap spans $0 \cdot 26$, $0 \cdot 50$ and $0 \cdot 77$ of the wing span were considered.

3. *Results and Discussion.*—(*a*) *Effect of flap cut-out.*—The calculated values of the factor K for the various flap spans and cut-outs examined are shown in Figs. 1, 2 and 3, corresponding to the three aspect ratios $A/a_0 = \frac{2}{3}$, $1 \cdot 0$ and $2 \cdot 0$ ($A = 4$, 6 and 12 approximately). The results are also given in Table 1.

TABLE 1

Values of K for Flapped Elliptic Wings

$\frac{A}{a_0}$	Overall flap span	No cut-out	0·1 cut-out	0·2 cut-out	0·4 cut-out	0·6 cut-out
2·00	0·2	5·080				
	0·4	1·740	2·592	4·11		
	0·6	0·667	1·040	1·889	5·022	
	0·8	0·198	0·472	0·988	2·537	6·382
	1·0	0	0·219	0·588	1·684	4·129
1·00	0·2	3·171				
	0·4	1·237	1·524	2·215		
	0·6	0·480	0·629	1·068	2·677	
	0·8	0·136	0·274	0·603	1·520	3·758
	1·0	0	0·120	0·361	1·125	2·861
$\frac{2}{3}$	0·2	2·346				
	0·4	0·944	1·117	1·563		
	0·6	0·375	0·472	0·782	1·983	
	0·8	0·103	0·198	0·421	1·168	2·933
	1·0	0	0·088	0·274	0·916	2·303

It will be seen that for a given overall flap span (*i.e.* distance between the outboard ends of the flap) the factor K increases with flap cut-out; hence, provided the strength (ΔC_L) of the flap remains constant, the induced drag will increase with cut-out. If, however, we consider the variation of K keeping the net flap span constant as shown by the dotted curves of Figs. 1, 2 and 3, it will be seen that K is a minimum when there is a flap cut-out of about 0·1 wing span. This is, fortunately, of the order of the flap cut-out that generally occurs in practice.

An examination of these results has shown that to a very close order of approximation, for any given size of cut-out up to 0·2 wing span, the factor K could be expressed as

$$K = K_f \, . \, K_A \, , \qquad \qquad (3)$$

where K_f is a function of the overall flap span, and K_A is a function of the aspect ratio. The functions K_f and K_A for cut-outs of 0, 0·1 and 0·2 wing span are shown in Fig. 4. It will be seen that the factor K_A increases steadily with aspect ratio in every case.

(*b*) *Effect of local extension of chord due to flap operation.*—In this case, as in the case of flapped non-elliptic wings, the general expression for δ cannot be reduced to the simple form of equation (2); nevertheless, it is shown in the Appendix that, for any given flap and wing arrangement, δ is a function of $\Delta C_L/C_L$ only.

The calculated values of δ for the three flap spans investigated are shown in Fig. 5 plotted as functions of C_{L0}/C_L (*i.e.* $1 - \Delta C_L/C_L$), where C_{L0} is the lift coefficient of the wing alone. For comparison, the dotted curves have been drawn showing the corresponding variation of δ for flaps that do not extend the local wing chord. It will be seen that for the 0·259 span flaps the two sets of results differ only by about 5 per cent, for the 0·5 span flaps the agreement is even closer, and for the 0·766 span flaps the two curves are indistinguishable from each other. Since high lift flaps of such large chord extension as that considered (40 per cent) are unlikely to be used with spans less than about 0·5 wing span, it can be concluded that for all practical purposes the effect of such chord extension on induced drag can be neglected. It is not anticipated that flaps of much larger extension are ever likely to be used, nor does it seem likely that if the calculations had been made for other aspect ratios or had included cut-outs the effect of local chord extension would have been found to be any more marked.

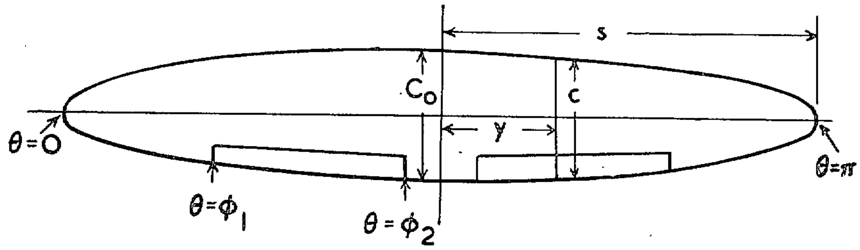
4. *Conclusions.*—For flapped wings of elliptic plan form (or taper ratio of the order of 2 : 1), it is found that :—

(*a*) With flaps of a given net flap span the induced drag is a minimum with a cut-out about 0·1 wing span in size.

(*b*) For all practical purposes the effect on the induced drag of local chord extension due to the operation of the flaps can be neglected.

APPENDIX

(a) *Effect of Flaps and Flap Cut-out on Induced Drag*



Let $2s$ and C_0 be the span and maximum chord, respectively of the wing,
y the spanwise distance of a section from the minor axis,
c the local chord at this section,
α the incidence from the no-lift angle of the unflapped wing,
β the effective change in incidence due to the flap along the flapped part of the wing.

Then we can write

$$\left.\begin{aligned} y &= -\, s \cos \theta \\ c &= C_0 \sin \theta \end{aligned}\right\}, \quad \ldots \qquad (1)$$

where θ varies from 0 to π along the span of the wing.

We may note that the area (S) and aspect ratio (A) of the ellipse are given by

$$\left.\begin{aligned} S &= \pi s C_0/2 \\ \text{and} \qquad A &= 8s/\pi C_0 \end{aligned}\right\}. \quad \ldots \qquad (2)$$

Consider the port half of the wing, then if we assume that the flap extends from $\theta = \phi_1$ to $\theta = \phi_2$, the incidence distribution can be represented by

$$\bar{\alpha} = \alpha + f(\theta) \cdot \beta \,, \quad \ldots \qquad (3)$$

where $f(\theta) = 0$ for $0 \leqslant \theta < \phi_1$ and $\phi_2 < \theta \leqslant \pi/2$,

$= 1{\cdot}0$ for $\phi_1 < \theta < \phi_2$.

The circulation K at a point θ is then expressed in the usual way as a Fourier series

$$K = 4sV \, \Sigma \, A_n \sin n\theta_1 \,, \quad \ldots \qquad (4)$$

where n takes only odd integral values. The induced velocity w at θ can then be shown to be given by (*see* Glauert[2])

$$w \sin \theta = V \, \Sigma \, n \, A_n \sin n\theta \,. \quad \ldots \qquad (5)$$

The aerodynamic incidence at θ is given by

$$\alpha_a = \bar{\alpha} - \frac{w}{V} \,, \quad \ldots \qquad (6)$$

and the local lift coefficient is

$$c_L = a_0 \alpha_a = a_0 \, (\bar{\alpha} - w/V) \,, \quad \ldots \qquad (7)$$

where a_0 is the slope of the lift *vs* incidence curve in two-dimensional flow.

Joukowski's relation connecting the lift (L) per unit span and circulation at any point is

$$\varrho V K = L = c_L \times \tfrac{1}{2}\rho V^2 c \, .$$

Hence,

$$2K = cVc_L = a_0 cV\left(\bar{a} - \frac{w}{V}\right). \qquad (8)$$

Substituting in (8) the expressions for K and w given by (4) and (5), we obtain

or

$$\Sigma A_n \sin n\theta \left[8s + \frac{na_0 c}{\sin\theta}\right] = a_0 c\bar{a}$$

$$\Sigma A_n \sin n\theta \, [n\mu_0 + 1] = \mu_0 \bar{a} \sin\theta \, , \qquad (9)$$

where

$$\mu_0 = a_0 \frac{C_0}{8s} \qquad (10)$$

$$= \frac{a_0}{\pi A} \, .$$

From (9) we have

$$\int_0^{\pi/2} \Sigma A_n \sin n\theta \, (n\mu_0 + 1) \sin n\theta \, . \, d\theta = \int_0^{\pi/2} \mu_0 \bar{a} \sin\theta \, . \, \sin n\theta \, . \, d\theta$$

or

$$\left.\begin{aligned} A_n (n\mu_0 + 1) \, . \, \frac{\pi}{4} &= \mu_0 \alpha \frac{\pi}{4} + \beta \int_0^{\pi/2} \mu_0 f(\theta) \sin^2\theta \, . \, d\theta \, , \\ &\qquad \text{for } n = 1, \\ &= \beta \int_0^{\pi/2} \mu_0 f(\theta) \sin\theta \, . \sin n\theta \, . \, d\theta \, , \\ &\qquad \text{for } n \geqslant 3. \end{aligned}\right\} \qquad (11)$$

The form of these equations allows us to separate the contributions of the wing and flap by writing

$$A_n = a_n \alpha + b_n \beta \, , \qquad (12)$$

and we obtain

$$a_1 = \frac{\mu_0}{\mu_0 + 1} \, , \quad a_3 = a_5 = - - - - = 0 \qquad (13)$$

and

$$b_n = b_n(\phi_1) - b_n(\phi_2) \, ; \qquad (14)$$

where

$$\left.\begin{aligned} b_n(\phi) &= \frac{\mu_0}{\mu_0 + 1} \cdot \frac{2}{\pi}\left[\frac{\pi}{2} - \phi + \frac{\sin 2\phi}{2}\right], \\ &\qquad \text{for } n = 1, \\ &= \frac{\mu_0}{n\mu_0 + 1} \frac{2}{\pi}\left[\frac{\sin(n+1)\phi}{n+1} - \frac{\sin(n-1)\phi}{n-1}\right] \\ &\qquad \text{for } n \geqslant 3. \end{aligned}\right\} \qquad (15)$$

It will be seen that

$$b_n(\pi/2) = 0,$$

i.e.

$$b_n = b_n(\phi_1) \, ,$$

when there is no cut-out. From equations (13), (14) and (15) we can calculate, for any required value of μ_0 (or A/a_0) and flap span and position, the corresponding values of the Fourier coefficients and hence we can determine the lift distribution, induced drag, etc. Following Hollingdale[2] it was considered sufficiently accurate for these calculations to take account of only the first eight terms of the series (*i.e.* the terms up to and including A_{15}).

At any point θ, the local c_L is given by

$$c_L = \frac{2K}{cV} = \frac{8s}{c} \Sigma A_n \sin n\theta \ .$$

Hence, the overall C_L of the wing and flap is

$$C_L = \frac{16s^2}{S} \int_0^{\pi/2} \Sigma A_n \sin n\theta \ . \sin \theta \ . \ d\theta \ ,$$

$$= \pi A \ . \ A_1 = \pi A \ (a_1\alpha + b_1\beta) \ . \qquad (16)$$

It follows that the lift coefficient increment due to the flap is

$$\Delta C_L = \pi A \ b_1\beta \ . \qquad (17)$$

At any point θ, the local induced drag coefficient is

$$c_{Di} = c_L \ . \ w \ ,$$

$$= \frac{8s}{c} \Sigma A_n \sin n\theta \ \Sigma \frac{n \ A_n \sin n\theta}{\sin \theta} \ .$$

Therefore, the overall induced drag coefficient is

$$C_{Di} = \frac{16s^2}{S} \int_0^{\pi/2} \left[\Sigma A_n \sin n\theta \ . \ \Sigma n \ A_n \sin n\theta \right] . \ d\theta$$

$$= \pi A \ \Sigma \ n \ A_n^2$$

$$= \pi A \left[(a_1\alpha + b_1\beta)^2 + \sum_3^{\infty} n \ b_n^2 \beta^2 \right] . \qquad (18)$$

Hence

$$C_{Di} = \frac{C_L^2}{\pi A} (1 + \delta) \ , \qquad (19)$$

where

$$\delta = \left(\frac{\pi A}{C_L}\right)^2 \sum_3^{\infty} n \ b_n^2 \beta^2 = \left(\frac{\Delta C_L}{C_L}\right)^2 \sum_3^{\infty} \frac{n b_n^2}{b_1^2} \ .$$

or

$$\delta = K \left(\frac{\Delta C_L}{C_L}\right)^2 ,$$

where

$$K = \sum_3^{\infty} \frac{n b_n^2}{b_1^2} \ . \qquad (20)$$

It will be seen that K is a function only of the flap span and position and is independent of the wing incidence or flap setting.

(*b*) *Effect of Local Chord Extension Due to Flap*.—The notation is the same as that adopted above. It is assumed that there is no flap cut-out and that the ration of the flap chord to the local wing chord is constant along the span of the flap.

Let C_0' be the effective chord length at the mid-span position when the flap is extended.

As before, we write

$$K = 4sV \ \Sigma \ A_n \sin n\theta \ ,$$

and we obtain

$$w \sin \theta = V \ \Sigma \ n \ A_n \sin n\theta \ .$$

The port flap extends from $\theta = \phi_1$ to $\theta = \pi/2$, and hence

$$\left.\begin{aligned} c &= C_0 \sin\theta,\ \bar{a} = \alpha, &&\text{for } 0 \leqslant \theta < \phi_1, \\ \text{and} \quad c &= C_0' \sin\theta,\ \bar{a} = \alpha + \beta, &&\text{for } \phi_1 < \theta \leqslant \pi/2. \end{aligned}\right\} \quad \ldots \quad \ldots \quad (21)$$

By substituting as before the series for K and w in the Joukowski relation we obtain eventually

$$\Sigma A_n \sin n\theta \left(n + \frac{1}{\mu}\right) = \bar{a} \sin\theta\,, \quad \ldots \quad (22)$$

where

$$\left.\begin{aligned} \mu &= \mu_0 &&\text{for } 0 \leqslant \theta < \phi_1 \\ &= \mu_0' = a_0 C_0'/8s &&\text{for } \phi < \theta \leqslant \pi/2. \end{aligned}\right\} \quad \ldots \quad (23)$$

Equation (22) cannot be solved outright for the coefficients A_n in the simple manner that equation (9) above was solved, because of the discontinuity at ϕ_1 in the value of μ that now occurs. A process of successive approximation was therefore adopted as follows. Equation (22) was written in the form

$$\left.\begin{aligned} \Sigma A_n \sin n\theta \left[n + \frac{1}{\mu'}\right] &= \alpha \sin\theta - \Sigma A_n \sin n\theta \underset{\text{for } 0 \leqslant \theta < \phi_1,}{\left(\frac{1}{\mu_0} - \frac{1}{\mu'}\right)}, \\ &= (\alpha + \beta) \sin\theta - \Sigma A_n \sin n\theta \underset{\text{for } \phi_1 < \theta \leqslant \pi/2,}{\left(\frac{1}{\mu_0'} - \frac{1}{\mu'}\right)}, \end{aligned}\right\} \quad \ldots \quad (24)$$

where μ' is some convenient value chosen intermediate between μ_0 and μ_0'.

We then have that

$$\int_0^{\pi/2} \Sigma A_n \sin n\theta \left(n + \frac{1}{\mu'}\right) . \sin n\theta \, d\theta = \int_0^{\phi_1} \left[\alpha \sin\theta - \Sigma A_n \sin n\theta \left(\frac{1}{\mu_0} - \frac{1}{\mu'}\right)\right] \sin n\theta \,.\, d\theta$$
$$+ \int_{\phi_1}^{\pi/2} \left[(\alpha + \beta) \sin\theta - \Sigma A_n \sin n\theta \left(\frac{1}{\mu_0'} - \frac{1}{\mu'}\right)\right] \sin n\theta \,.\, d\theta\,. \quad (25)$$

Hence,

$$A_n \left(n + \frac{1}{\mu'}\right) . \frac{\pi}{4} = D + \beta \int_{\phi}^{\pi/2} \sin\theta \,.\, \sin n\theta \, d\theta$$
$$- \left[\left(\frac{1}{\mu_0} - \frac{1}{\mu'}\right) \int_0^{\phi} (\Sigma A_n \sin n\theta) \sin n\theta \, d\theta\right.$$
$$\left. + \left(\frac{1}{\mu_0'} - \frac{1}{\mu'}\right) \int_{\phi}^{\pi/2} (\Sigma A_n \sin n\theta) \sin n\theta \, d\theta\right] \quad \ldots \quad (26)$$

where $D = \alpha \frac{\pi}{4}$ for $n = 1$, and $D = 0$ for $n \neq 1$.

We can again write

$$A_n = a_n \alpha + b_n \beta\,.$$

The first approximation is obtained by neglecting the terms in the square brackets in equation (26) and then we have

$$\left.\begin{aligned} a_1' &= \frac{\mu'}{\mu' + 1}\,, \\ a_3' &= a_5' = 0\,, \\ b_1' &= \frac{2}{\pi}\frac{\mu'}{\mu' + 1}\left[\frac{\pi}{2} - \phi_1 + \sin 2\phi_1\right], \\ b_n' &= \frac{2}{\pi}\frac{\mu'}{n\mu' + 1}\left[\frac{\sin (n+1)\,\phi_1}{n+1} - \frac{\sin (n-1)\,\phi_1}{n-1}\right]. \end{aligned}\right\} \qquad (27)$$

The second approximation is obtained by substituting these values in the terms in the square brackets in equation (26) *i.e.*

$$\left.\begin{aligned} a_1'' &= a_1' - \frac{4}{\pi}\cdot\frac{\mu'}{\mu' + 1}\left[\left(\frac{1}{\mu_0} - \frac{1}{\mu'}\right)\int_0^{\phi_1} a_1' \sin^2\theta\,.\,d\theta \right. \\ &\quad \left. + \left(\frac{1}{\mu_0'} - \frac{1}{\mu'}\right)\int_{\phi_1}^{\pi/2} a_1' \sin^2\theta \cdot d\theta\right], \\ a_n'' &= \frac{4}{\pi}\frac{\mu'}{n\mu' + 1}\left[\left(\frac{1}{\mu_0} - \frac{1}{\mu'}\right)\int_0^{\phi} a_1 \sin\theta\,.\,\sin n\theta\,d\theta \right. \\ &\quad + \left(\frac{1}{\mu_0'} - \frac{1}{\mu'}\right)\int_{\phi_1}^{\pi/2} a_1' \sin\theta\,.\,\sin n\theta\,d\theta\,, \\ b_n'' &= b_n' - \frac{4}{\pi}\frac{\mu'}{(n\mu' + 1)}\left[\left(\frac{1}{\mu_0} - \frac{1}{\mu'}\right)\int_0^{\phi_1} (\Sigma\, b_n' \sin n\theta) \sin n\theta\,.\,d\theta \right. \\ &\quad \left. + \left(\frac{1}{\mu_0'} - \frac{1}{\mu'}\right)\int_{\phi_1}^{\pi/2} (\Sigma\, b_n' \sin n\theta) \sin n\theta\,d\theta\right]. \end{aligned}\right\} \qquad (28)$$

A third approximation is similarly obtained by substituting the values a_n'', b_n'' in the terms in the square brackets in equation (26) and so on.

In some cases the integrals involved could be simplified and evaluated analytically, but in most cases they had to be evaluated graphically. Fortunately, in every case examined the process was found to be very rapidly convergent, three stages of successive approximation were quite sufficient for the accuracy required. As a check, in one case the coefficients were calculated twice, using two values of μ' differing as widely as possible from each other. After three stages the two resulting sets of coefficients did not differ significantly. As a further rigorous check, in every case the final coefficients were substituted back in the basic equation (22) to see how closely they satisfied it. Fig. 6, for example, compares the calculated spanwise distribution of $\Sigma b_n \sin n\theta\ (n + 1/\mu)$ for the $0{\cdot}77$ span extending flap with the distribution $g(\theta)$, where $g(\theta) = 0$, when $0 \leqslant \theta < 40$ deg. (ϕ_1), and $g(\theta) = \sin\theta$, when 40 deg. $< \theta \leqslant \pi/2$. The agreement is satisfactory and is typical of the agreement generally required in such problems.

As before, we have

$$\text{Overall } C_L = \pi A\ A_1 = \pi A\ (a_1\alpha + b_1\beta)\,. \qquad (29)$$

With the flap retracted

$$a_1 = \frac{\mu_0}{\mu_0 + 1} = a_{10},\ \text{say},$$

hence

$$C_{L0} = \pi A\ a_{10}\,.\,\alpha\,,$$

and

$$\Delta C_L = \pi A\ (a_1 - a_{10})\,\alpha + \pi A b_1\beta\,. \qquad (30)$$

Also, the total induced drag is given by

$$C_{Di} = \pi A\ \Sigma n\ A_n^2$$

$$= \frac{C_L^2}{\pi A} (1 + \delta) ,$$

where

$$\delta = \frac{\pi^2 A^2}{C_L^2} \sum_3 n\ A_n^2$$

$$= \frac{\pi^2 A^2}{C_L^2} \left[\alpha^2 \sum_3 n\ a_n^2 + \beta^2 \sum_3 n\ b_n^2 + 2\alpha\beta \sum_3 n\ a_n\ .\ b_n\right] . \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

But

$$\left.\begin{aligned} \alpha &= \frac{C_{L0}}{\pi A\ a_{10}} , \\ \beta &= \frac{1}{\pi A\ b_1} \left[\Delta C_L - C_{L0} \left(\frac{a_1 - a_{10}}{a_{10}}\right)\right] . \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

and

Hence

$$\delta = \left(\frac{C_{L0}}{C_L}\right)^2 \sum_3 n \left(\frac{a_n}{a_{10}}\right)^2 + \left(1 - \frac{C_{L0}}{C_L} \frac{a_1}{a_{10}}\right)^2 \sum_3 n \left(\frac{b_n}{b_1}\right)^2$$

$$+ 2 \left(\frac{C_{L0}}{C_L}\right) \left(1 - \frac{C_{L0}}{C_L} \frac{a_1}{a_{10}}\right) \sum_3 n \frac{a_n\ b_n}{a_{10}\ b_1} . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

This expression enables us to calculate δ given the coefficients a_n and b_n and the quantity C_{L0}/C_L.

REFERENCES

No.	Author.	Title, etc.
1	Hollingdale	Aerodynamic Characteristics of Tapered Wings with Flaps and Slots. R. & M. No 1774. (1936).
2	Glauert	Aerofoil and Airscrew Theory. (2nd Edition (1947), p. 139).

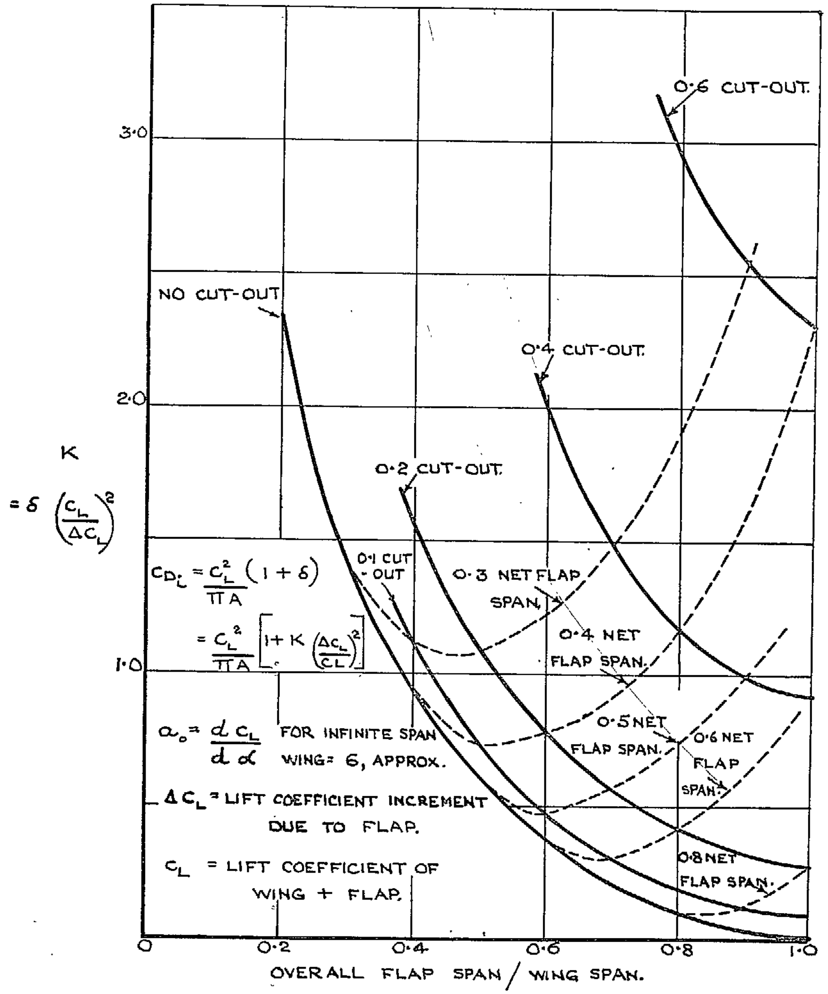


FIG. 1. Factor K. $A/a_0 = \frac{2}{3}$

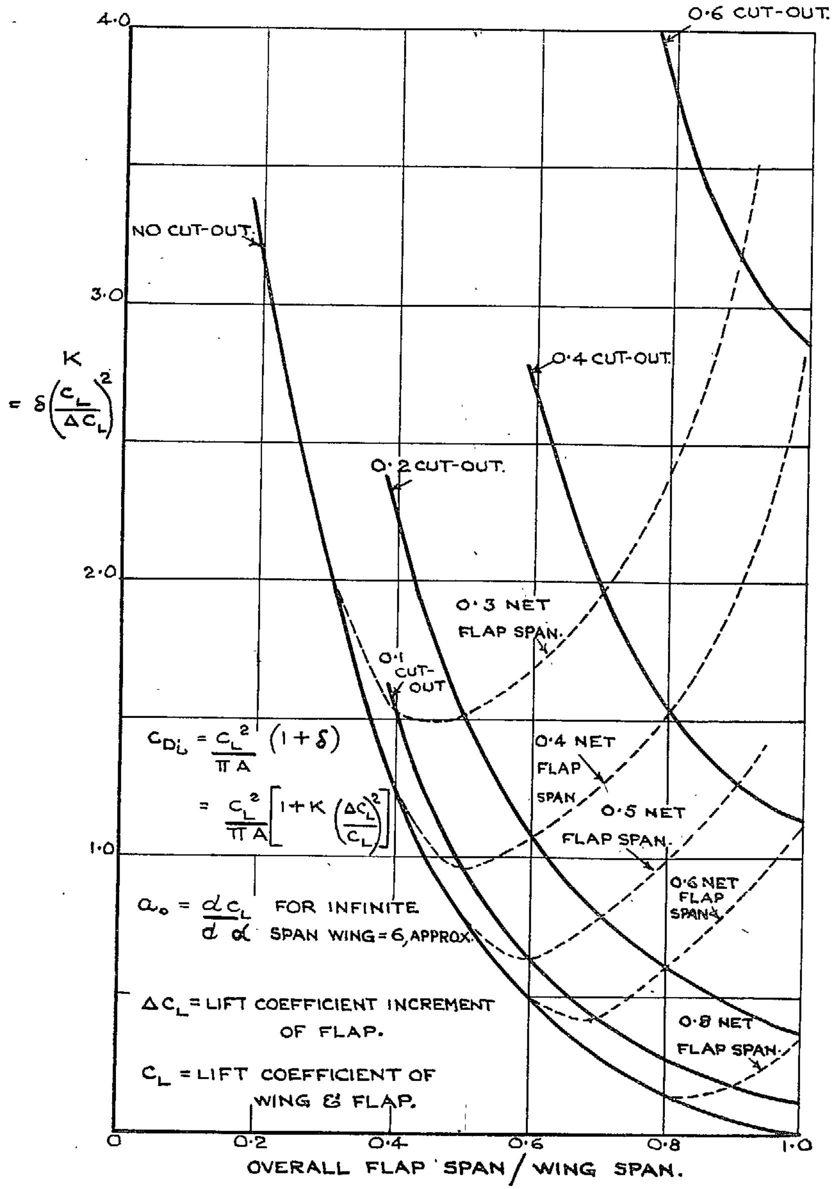


FIG. 2. Factor K. $A/a_0 = 1$.

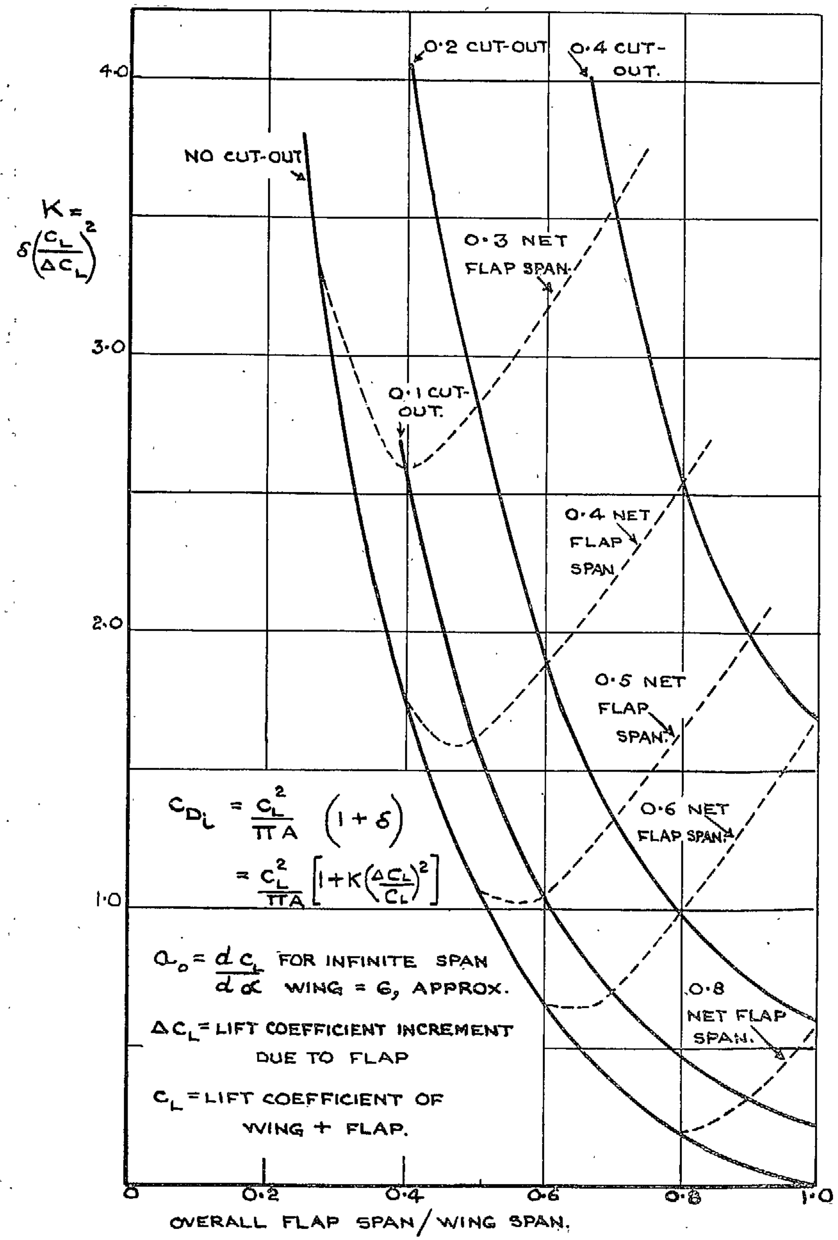


FIG. 3. Factor K. $A/a_0 = 2\cdot 0$

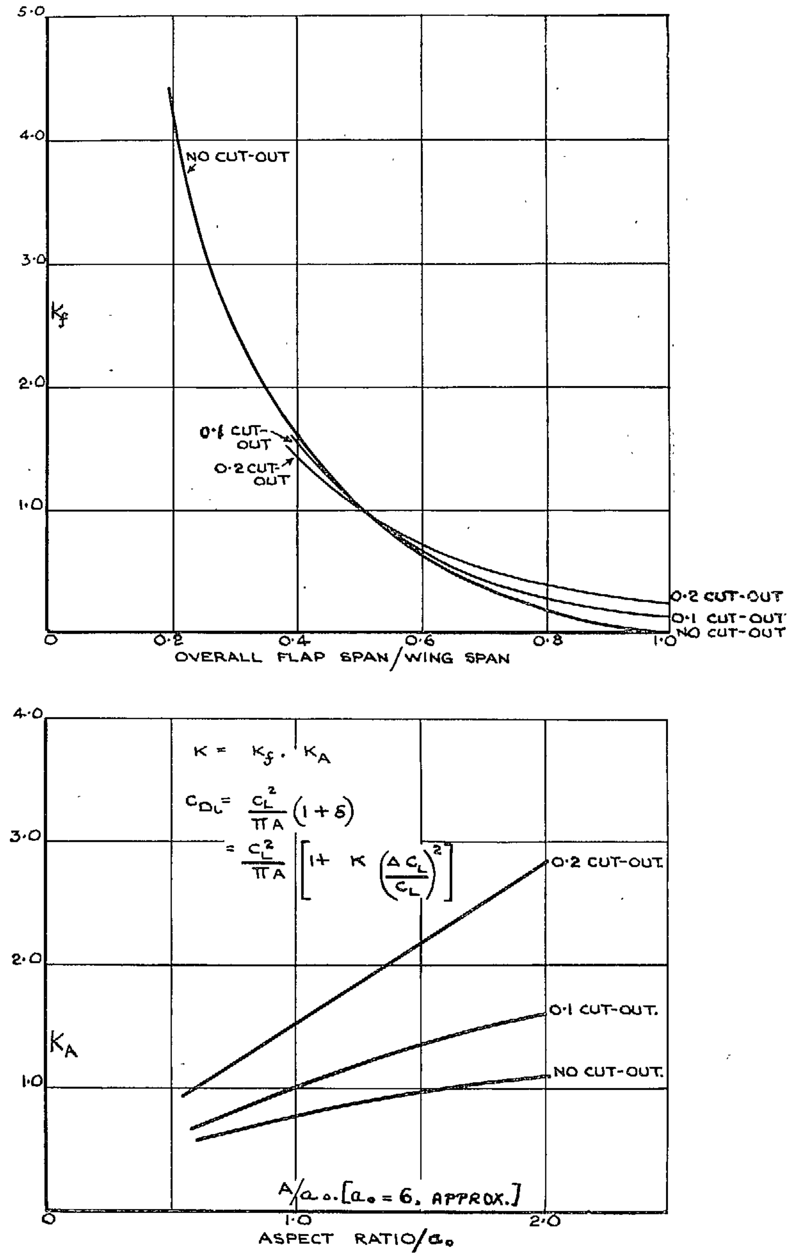


FIG. 4. Effect of Aspect Ratio and Flap Span on Induced Drag Factor K.

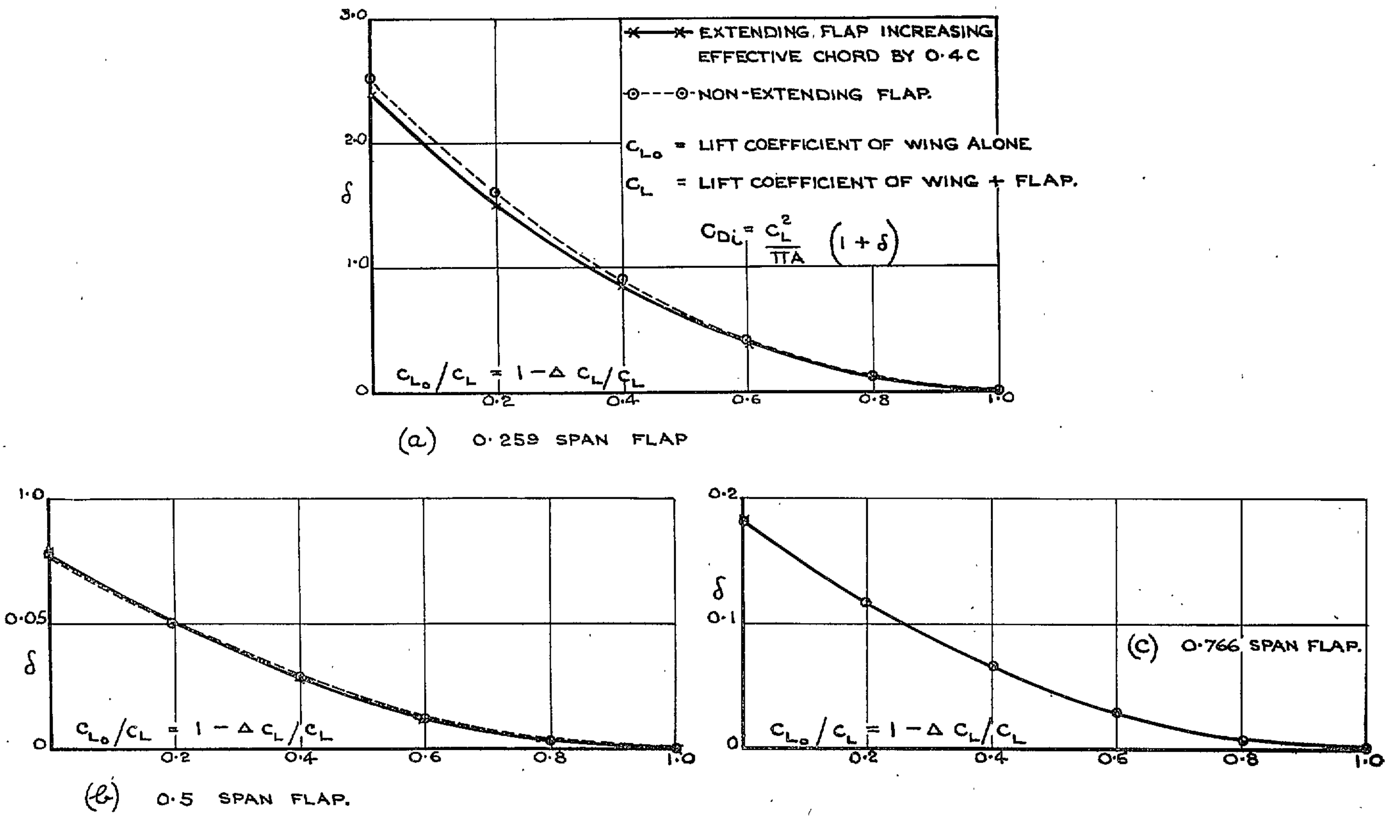


FIG. 5. Comparison of Calculated Effects on Induced Drag of a Flap that Increases the Local Effective Chord by 0·4c and of a Normal Flap. (Elliptic Wing, $A/a_0 = 1$.)

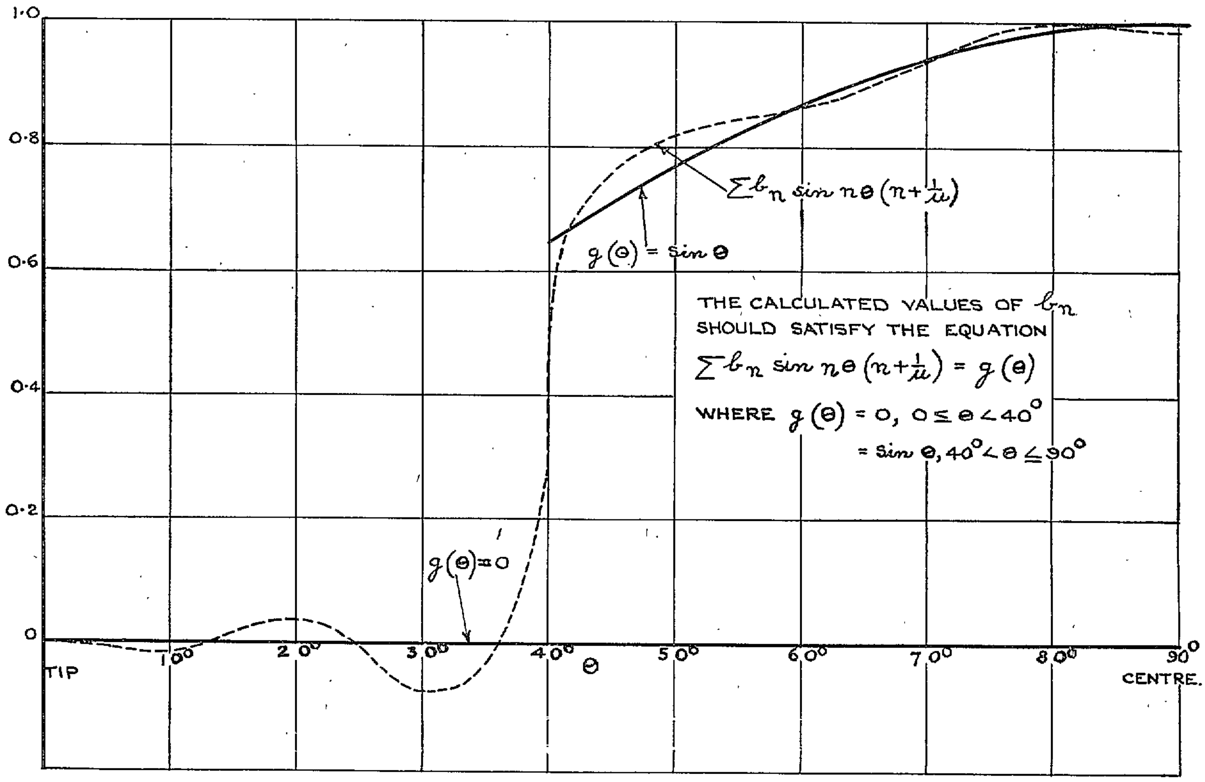


FIG. 6. Accuracy of Calculated Flap Coefficients for 0·77 Span Flap Extending Local Wing Chord by 40 per cent. $A/a_0 = 1·0$.

(93251) Wt. 13/806 K.5 2/51 Hw. PRINTED IN GREAT BRITAIN

R. & M. No. 2544
(6259)
A.R.C. Technical Report

Publications of the Aeronautical Research Committee

TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COMMITTEE—

1934–35 Vol. I. Aerodynamics. 40*s.* (40*s.* 8*d.*)
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40*s.* (40*s.* 8*d.*)

1935–36 Vol. I. Aerodynamics. 30*s.* (30*s.* 7*d.*)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30*s.* (30*s.* 7*d.*)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 9*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50*s.* (50*s.* 10*d.*)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 9*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60*s.* (61*s.*)

1938 Vol. I. Aerodynamics General, Performance, Airscrews, 50*s.* (51*s.*)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30*s.* (30*s.* 9*d.*)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (50*s.* 11*d.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (64*s.* 2*d.*)

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COMMITTEE—

1933–34	1*s.* 6*d.* (1*s.* 8*d.*)
1934–35	1*s.* 6*d.* (1*s.* 8*d.*)
April 1, 1935 to December 31, 1936.	4*s.* (4*s.* 4*d.*)
1937	2*s.* (2*s.* 2*d.*)
1938	1*s.* 6*d.* (1*s.* 8*d.*)
1939-48	*In the press*

INDEXES TO THE TECHNICAL REPORTS OF THE ADVISORY COMMITTEE ON AERONAUTICS—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1*s.* 3*d.* (1*s.* 5*d.*)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1*s.* (1*s.* 2*d.*)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1*s.* (1*s.* 1*d.*)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1*s.* 3*d.* (1*s.* 4*d.*)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1*s.* 3*d.* (1*s.* 4*d.*)

Prices in brackets include postage.

Obtainable from

His Majesty's Stationery Office

York House, Kingsway, LONDON, W.C.2 429 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or through any bookseller.

S.O. Code No. 23-2544