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REGULARITIES IN CREEP AND HOT-FATIGUE DATA

By

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Regularities in creep and hot-fatigue data

PART I

- by -

A. Graham M.Sc., D.I.C. and K. F. A. Walles B.A.

SUMMARY

Published experimental results are assembled to support a previously-given theory of uniaxial deformation, and the theory is then used to analyse published data on the creep-rupture and hot-fatigue of engineering materials. The theory enables data for different times and temperatures to be classed together, thereby providing information over a much greater range of times than could practicably be covered by experiments at a single temperature. An underlying numerical pattern common to all the widely different Group VIII materials considered then shows through the experimental scatter. Data for further engineering materials is considered in these terms in Part II.

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TABLES

ILLUSTRATIONS

ILLUSTRATIONS (cont'd)

1.0 Introduction

In the present Report a discussion is given of published experimental results upon the creep and hot-fatigue of heat-resistant steels in terms of a theory which although previously given[1,2,3] is partly developed afresh upon the basis of experimental evidence directly presented. The conclusions are applied in the accompanying Part II[4] to published data for 43 engineering materials.

The study has been guided by a belief that the mechanical properties of engineering materials, despite their microscopic complexity, follow a quantitative pattern in terms of the four primary variables stress strain time and temperature that is simple enough to be understood. Since the pattern is likely to be revealed by direct experiment only in favourable circumstances, the method followed has been to build upon examples believed to be of this kind towards a more complete understanding of the general pattern.

A fact that has come to light during the work, and has progressively affected its direction, is that the range of these variables that may be covered by direct experiment is too small in relation both to the scale of the underlying pattern and scatter in results to define the pattern unambiguously. The working assumption that the underlying pattern is universal, only numerical constants varying from material to material, has had the virtue, to the extent that it has held, of effectively extending experimental ranges by allowing results for different materials and ranges to be classed together. Regular numerical trends are then perceivable through the scatter.

By directly considering the experimentally-found relationships between the primary variables, with a view to describing these quantitatively, the study is complementary to those that attempt to understand behaviour from the metallurgical and atomic points of view. The interdependence of these several complementary studies is considered in some detail in a further Report.

2.0 Simple evidence

Very good straight lines are sometimes observed when relationships between stress σ strain ε and time t, or rate of straining $\dot{\varepsilon}$, at constant temperature T, as measured in creep or tensile tests, are suitably exhibited on log/log paper. A selection of examples taken from the sources indicated in Table I is given in Figures 1(a) to (i). The sources should be consulted for details.

In view of the fact that most results are known to plot rather as continuous curves, a distinctly poor case for a linear relationship is presented by many of the individual sets if considered in isolation. But collectively, especially in view of the several examples in which the fit is undeniably good, they are seen to present a reasonable case for the linear relationship proposed. The simplest assumption to make, on this kind of evidence, in view of the interdependence between the various forms of graph, is that

$$\varepsilon = C\sigma^{\beta}t^{\kappa} \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

where the numerical constants $C\beta\kappa$ may be, in general, neither whole numbers nor rational fractions. This formula, due to Nutting, contains Hooke's law and the Newtonian law of viscosity as limiting special cases, and therefore commends itself to attention[1,2,3].

From a fundamental viewpoint, since the mathematical functions that occur in basic physical theory are almost invariably those that, by their very definition involve only integral or ration-fractional exponents, power laws in which the exponents may be irrational are often regarded as wholly inadmissible. Nevertheless the fit to lines of arbitrary slope in Figure 1 is an experimental fact that requires attention. The problem of whether or not Equation (1) as it stands has any fundamental basis, however, does not need to be answered before (1) is used as a tool to uncover further evidence.

3.0 Further evidence

Figures 2(a) to (h), of which the sources are listed in Table II, show a selection from the multitude of examples to be found in which the points from experiments similar to those for Figure 1 do not fall upon straight lines. The first examples, in Figures 2(a) to (d), have been selected as instances in which the original authors have considered that the points fall upon pairs of straight lines joined by rather abrupt transitions. Without prior evidence of the kind given in the previous figures, the objection that the points fall rather upon continuous curves would be difficult to answer; but, in view of that evidence, the suggestion of pairs of straight lines may be accepted as not unreasonable. Grant and Bucklin[5] and others have provided evidence for drawing more than two straight lines through their data, so that (1) may perhaps be regarded as a special case of a more general formula of the type

$$\varepsilon = C_1\sigma^{\beta_1}t^{\kappa_1} + C_2\sigma^{\beta_2}t^{\kappa_2} + \ldots\ldots \qquad \ldots \qquad (2)$$

in which $C_1\,\beta_1\,\kappa_1\,\beta_2$ ..... are constants. A physical justification of (2) is that Equation (1) representing perhaps the simplest possibility, may be regarded as expressing the effect of a single physical mechanism; if so, then (2) appears to be the simplest more general assumption to represent the joint effect of the many physical mechanisms that must undoubtedly occur in a complex material.

Equation (2), which does not in general represent a set of straight lines one for each term, but a continuous curve, provides a link between the examples of Figures 1, 2(a) to (d), and 2(e) to (h). The curvature is dependent upon the individual values of the constants β and κ and upon the particular pair of variables, the other being held constant, between which a relationship is considered. For experimentally-appropriate values of $\beta_1\,\kappa_1\,\beta_2$ ....., the theoretical curvature is usually very noticeable, in general accordance with experiment, in log strain/log time and log stress/log strain graphs; but in log stress/log time cross-plots of creep data, the curvature according to (2) is often experimentally indistinguishable from a set of linear segments of slope $-\kappa_1/\beta_1$, $-\kappa_2/\beta_2$ .... In these circumstances the magnitudes of the terms in (2) change with stress and time so rapidly in relative importance, that each segment of a graph is determined almost entirely by a single term. These features are discussed in more detail in Part II.

The data in Figure 2(a), (b), (c) for S.590, S.816 and 18 - 8 + Cb steel determines only the ratios κ/β and not κ and β separately, so that (2) cannot be fully applied. The straight lines drawn by the original authors without curved transitions correspond to the approximation that each term contributes separately, each within a particular range, independently of the others. Figure 2(d) for brass requires a formula similar to (2) but with symbols for stress and strain interchanged. This feature needs to be considered in relation to the comments made upon Equation (2) in Section 8.0.

The continuously-curved lines in Figures 2(e), (f), (g), are based upon values of κ_1 and κ_2 taken for Nimonics 80 and 90 to be $\frac{1}{3}$ and 3, as determined by direct experiment, and for G.32 by analogy, and in view of some independent evidence, to be $\frac{1}{4}$ and 4. Figure 2(h), again for Nimoni 90, exhibits the results of a detailed analysis of families of creep curves to establish the values of the constants, and shows, firstly, how the theoretical abruptness of the transition varies with strain, and secondly, how the scatter of the points, which is quite normal for the materials of these figures, may entirely conceal, in a log stress/log time plot, the distinction between abrupt and continuous transitions. The bars on each curve represent the points outside which one of the two terms contributes less than 20 per cent of the total.

It is to be emphasised that these results are not regarded as proving the formula, but as demonstrating its virtues for further consideration.

4.0 Comment

The remainder of the present Report is chiefly concerned with practical creep-rupture data. The bulk of engineering creep data is of this kind partly because the experiments are easy to perform and partly because rupture data is generally more consistent than that relating to the smaller strains of complete creep curves. As they give no information upon the separate values of β and κ (which determine the spread of the curved transitions), they can only be directly analysed by means of (2) to the extent of the individual-term approximation mentioned.

Their suitability for exact analysis is also dependent upon the extent to which the time to rupture under different conditions is affected by differences in specimen extension. For materials such as the Nimonic alloys in which the extension is not large and rupture is preceded by a strongly-accelerating stage of creep, so that large differences of strain are only associated with small differences of time, little error is involved in regarding creep-rupture data as data for constant strain to which (2) may be directly applied; but when these conditions do not hold, unless impracticable corrections were to be made, creep-rupture data would not be expected to obey so simple a formula as (2). It appears nevertheless that creep-rupture data is able to provide significant information.

5.0 Temperature and time

It is necessary to consider how deformation, e.g. creep, is affected by a change of temperature. For guidance in this problem the simplest physical concept is that an increase of temperature, by increasing the rate of atomic vibration, does no more than reduce the time scale of deformation. Although not generally valid in this over-simplified form, the concept is embodied in various practical "time temperature parameters" which, although known to be unreliable for large extrapolations, are nevertheless recognised as giving useful co-ordinations of experimental data for complex materials. These are materials to which, from the microscopic viewpoint, the principle would perhaps least be expected to hold. Once again, however, there is no need to debate whether the general principle is fundamentally correct before an experimentally-supported particular form of it is used as an interim means for gaining further information.

The principle is illustrated in Figure 3 in application to creep-rupture data for S.590 steel (source in Table III). The time to rupture at several different temperatures is plotted on log/log scales (cf. (1)) against the constant stress applied. For a set of points appropriate to a single temperature, a change of time scale is represented by a parallel displacement, say $\Delta \log t$, along the axis of log time, and the stated time-scale principle is that the points for any one temperature may be superimposed upon those for say a lower temperature by a mere displacement to the right towards longer times. In these data, as in most, the evidence for or against the principle is rather meagre in view of the scatter, and also because, owing to the rapid change of strength with temperature and the difficulties of making measurements at very short and long times, the inevitably small ranges of stress covered at the different temperatures overlap over a significant range only when the temperature difference is small. If however the principle is used for what it is worth, it suggests that the points for different temperatures may be relatively displaced to lie upon a single common segmental curve of which that shown in the centre of Figure 6 may give a first impression. If in order to avoid using so dubious a principle, the form of the supposed curve were to be studied at a single temperature, an impracticable number of experiments would be required (owing to scatter) over a wholly impracticable range of times.

Displacement by mere inspection would serve the immediate purpose; but the regularities of displacement are of interest, for the various time-temperature parameters proposed by Larson and Miller, Manson and Hefford, and others may be regarded as formal expressions of the graph of $\Delta \log t$ versus T. The values of $\Delta \log t$ for S.590 in Figure 3 and for materials considered below were however smoothed in accordance with the previously derived parameter[2]:-

$$t(T' - T)^{-A} = \text{constant}, \phi, \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

for stress and strain constant, which is preferred to those mentioned because it attempts to take some account of the experimental fact that behaviour is dependent upon the history of temperature prior to the beginning of an experiment. In (3), A is a constant whose value is not closely defined by any available data, and which has accordingly been standardised, at a value suitable for all materials so far considered, with time in hours and temperature in $^{\circ}$C, at 20. The constant T' was introduced into the theory to represent some average temperature, for the material concerned, which best characterises the previous history of temperature[2].

The graph of $\Delta \log t$ versus T for the S.590 data is shown, together with graphs for other materials considered below, in Figure 4. The curves represent the fit of (3) with best values of T'. Available data for the majority of materials is too sparse and scattered for significant comparisons between the various parameters but a comparison is shown in Figure 5 with points for Nimonic 90. A probable reason why they have been found in particular instances to be inadequate is offered by the results in Section 7.0.

6.0 Numerical regularities

Figure 6 shows the result of applying the procedure just described to creep-rupture data for the materials listed in Table IV where sources of the data are given. The points are plotted to common scales of log stress and log ϕ, and, in order to save space, since only the slopes of the various graphs are of present interest, the absolute positions of the scales are arbitrary. Straight lines have been drawn through the various points for the reasons already indicated, their slopes being determined by the procedure given below. The over-simplified time-temperature principle used is clearly seen to break down where pairs of parallel lines have had to be drawn, but the failure, which is remedied in the next paragraph, will be seen to be of no immediate concern.

The extensive collection of points for the Timken 35-15 steel group rather closely about a single line of slope 0.238. It is then seen that groups of points for several of the other materials fall about lines of slope indistinguishable from that appropriate to the Timken 35-15. Accordingly the lines drawn through all these corresponding groups have been assigned the common slope 0.238.

It is then observed that other groups of points amongst the different materials fall about lines of apparently common slope; and, by grouping the points and determining average values, common slopes of 0.122 and 0.067, shown by the lines drawn, were identified.

Few extensive sets of data other than for creep appear to be available, but Figure 7 exhibits data for rupture by hot-fatigue taken from the sources in Table IV. They relate to tests at zero mean stress, and are seen to be well co-ordinated by the time-temperature variable ϕ, the constants A and T' used, being the same, material for material, as for the creep-rupture data. The time involved in ϕ is the time to rupture, no account being taken of the periodic time which was presumably constant for any one material and not greatly different, on a logarithmic basis, between one material and nother. The positions of the creep lines transferred from Figure 6 are given in Figure 7 for convenient comparisons; but in view of the arbitrary choice of maximum stress to characterise the stress in fatigue, no immediately obvious relation between the stresses and rupture times in creep and hot fatigue is to be expected.

The slope of 0.067 determined from the creep data is now seen to provide a very good fit to groups of points for Rex 337A and Nimonic 90, while the slope of 0.122 is not unsuitable. G.32 is ambiguous in regard to slopes 0.122 and 0.238 but provides fair evidence for the additional slope of 0.032.

It appears from this discussion that, although little evidence of physical law is seen when each set of data is considered on its own, especially when the separate temperatures for each are considered independently, yet, when they are considered collectively, the evidence is strong that the set of slopes 0.238, 0.122, 0.067, 0.032, is common to them all. These numbers do not differ significantly from the rational fractions $\frac{1}{4}$, $\frac{1}{8}$, $^1/16$, $^1/32$, which are members of the series 2^{-n}. It will be noted that the member with n = 0, if arising from $\beta = 1$ ans $\kappa = 1$, corresponds to perfectly viscous behaviour, such as may well be observed near the melting point.

7.0 Presentation of data in terms of improved time-temperature principle

In view of the common slopes found, the lack of a completely satisfactory relationship between time and temperature is seen not to have unduly handicapped progress. The common slopes are now available for use as tools to investigate the relationship in more detail.

The above procedure involving displacement of a set of points for a single temperature, as a whole, along the log time axis is equivalent to the replacement of t in every term of (2) by ϕ defined at (3); but question arises whether the constant T' should be taken, as above, to be the same, for a given material, in every term. A common constant is certainly unlikely if the different segments in Figure 6 and terms in (2) are associated with different physical mechanisms, and indeed for several of the materials of Figure 6, the displacements required by the data are evidently not the same for all segments. The procedure of the previous paragraph may readily be performed for each segment independently, but it is only by the predetermined knowledge of the standard slopes that precision is conferred on the working.

The results obtained by applying this procedure to the materials of Figures 6 and 7, are given in Part II, together with results for other materials. Discussion is here limited to data for the Nimonic alloys which have the present advantage, in addition to that mentioned in Section 4.0, of forming a series of five closely related materials for which a useful quantity of data is available. From the viewpoints of Equation (2), the above discussion involves the independent-term approximation previously mentioned, and the results of an analysis of this kind upon the tabulated data supplied by Mond Nickel for their materials (Table IV) are shown in Figure 8.

The grouping of points into segments (see Part II) is made by deciding for each temperature separately a point of transition above which all points are in principle assigned to one slope and below which to another; but in view of the fact that Equation (2) is being used in an approximate manner, points near a transition are not uniquely assignable to either of the adjacent segments, and in Figure 8, where the segments are shown separately, points near transitions have been bracketed and included in both segments. Values of T' required to superimpose points for different temperatures within each range of stress are conveniently obtained by fitting a master curve[6] of $\Delta \log t$ versus T to the graph of these quantities. The fit is shown by the full lines in Figure 9 in which the points with arrows are explained in Part II. The lines drawn in Figure 8 have standard slopes with the rounded-off values $^1/16$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$. The lines are seen to fit the points rather closely, suggesting that the previous evidence for standard slopes was not merely an illusion permitted by the scatter of the data and imperfections of its treatment.

The values of T' indicated by these Nimonic data appear to be physically significant. In Figure 9, the lines appropriate to the best fitting values of T' and also to these values ±20°C are shown. It is seen from consideration of References 6 and 4 that the preferred values are determined within a range of about this magnitude. It will be recalled that T' was introduced theoretically as some average temperature which best represents the influence of the previous history of temperature before creep testing begins; the values found appear to correspond to temperatures in the manufacturing history of the material.

For segments of slope $\frac{1}{8}$, the experimentally indicated T' for Nimonics 80, 80A, and 90, is 1080°C, and this same temperature is also the specified solution-treatment temperature for these alloys. The exact agreement is co-incidental. For Nimonic 95, the value is 1150°C, and lies between the two solution-treatment temperatures of 1200°C and 1000°C specified for this material. Nimonic 100 has not apparently been tested in the range for which the term in (2) with $\kappa/\beta = \frac{1}{8}$ would make a significant contribution.

In regard to segments of slope $\frac{1}{4}$, the values of T' increase progressively with the increase in strength of the alloys; thus they are 1150, 1200, 1250, 1320, and 1370°C respectively for Nimonics 80, 80A, 90, 95 and 100. They may be compared with the rolling temperatures of these alloys which are known to be in the neighbourhood of 1200°C for Nimonics 80 and 80A and to be higher for the higher members of the series.

For segments of slope $^1/_{16}$, the T' of 910°C for Nimonic 80 and 950°C for Nimonic 80A and 90 may be compared with the temperature of about 950°C above which the main hardening constituents are known to go into solution.

More extensive data for Nimonic 90 presented in Part II indicates the presence of a segment of slope $\frac{1}{2}$ with a T' in the neighbourhood of the softening point. Some evidence of this segment appears in Figure 9.

These results are suggestive only, but they call for further study since the values of T' found, as required by the reasoning that led to T', are close to those at which significant events certainly occurred during the "historical period" prior to testing.

8.0 Discussion

Before the results of the last two paragraphs are provisionally assessed, some comments on the theoretical standing of Equation (2) are appropriate. Apart perhaps from the particular form of functions used, the mathematical form of (2) is like that of a solution of a linear partial differential equation. For the unknown quantity (strain) is represented as the sum of a number of terms, each comprising the product of another quantity (stress) alone with a third quantity (time) alone. Equation (2) may therefore be regarded as in the nature of a solution, for a particular kind of loading, of some more general equation that sets out the relationship between the incremental changes concerned in any kind of loading. This more general equation, were it known, could well express the atomic basis of behaviour. One of the requirements for this point of view is that specific relationships must exist between constants in the different terms of (2). It appears from the above discussion that the ratios of κ to β indeed follow a simple numerical sequence, and to this may be joined the evidence previously found[2], in certain instances, for a simple sequence of the κ, namely 3^n $n = -1, 0, 1$..........

The weight to be put upon these results depends largely upon an assessment of the balance between systematic and random factors, an assessment that has to be made under the incubus of a lack of specific evidence from repeat tests of the magnitude of the random factors. Internal inconsistencies of creep data together with odd results from a few repeat tests clearly show, in view of the presence of four variables and the wide range of each that is relevant, that existing data for almost any single material forms too statistically small a sample upon which to base detailed conclusions of the laws of creep. Laws that appear when the sample is thought to be increased to a significant size by an aggregation of results for different materials are clearly not amenable to checking by reference back merely to the inadequate results for the materials individually. Thus, at present, while statistical methods would answer, if required, the question whether the scatter of points about lines drawn according to the present systematic scheme is greater than or less than that about lines drawn according to some other systematic scheme, they appear unable in the absence of further experiment to answer the absolute question of whether there is or is not a common set of slopes for the various materials. Obviously the scatter about curves freely drawn with free choice of shape through each individual set of points, without regard to other sets, for each material and temperature of testing, will be less than about sets of lines drawn according to any form of pre-assigned relationship. Scatter would be completely absent at the limit, which is very near to technical practice at the moment, where no law at all is pre-assigned and full statistical weight is given to every non-repeated experimental point. But this is the limit when all factors are regarded as systematic, and also that of prodigality of scientific hypothesis in which the number of "freely-adjustable constants", being equal to the number of experimental points, is a maximum.

The present suggestions make smaller demands upon hypothesis.

9.0 Conclusions

It appears from the discussion that

(a) behaviour is governed by a formula like (2), to be regarded as a particular solution of an unknown more general equation that represents incremental behaviour;

(b) in complex Group VIII alloys, the ratios β_1/κ_1, β_2/κ_2, ... have the simple sequence of values 2^n, $n = \ldots\ldots 1, 2, 3, 4, 5, \ldots$;

(c) in each term, time and temperature enter in a combination like that in Equation (3);

(d) in (3), A may for the present be taken as independent of material. Also the T' appear to be specifically related to temperatures in the manufacturing history of the material.

These are offered as a simple set of working hypothesis, in general accordance with the facts, for use until better are found.

Available creep-rupture and hot fatigue data for the above and other materials, 43 in all, which have been analysed in accordance with these principles and generally support them, are presented in Part II. The principles have also been found consistent with the limited available data for about 100 further materials. It is proposed in a later Report to present, for those few materials for which the available data is sufficiently extensive to support analysis, a more detailed analysis of complete families of creep curves.

TABLE III

Sources of data and major constituents of materials of Figures 3 to 7

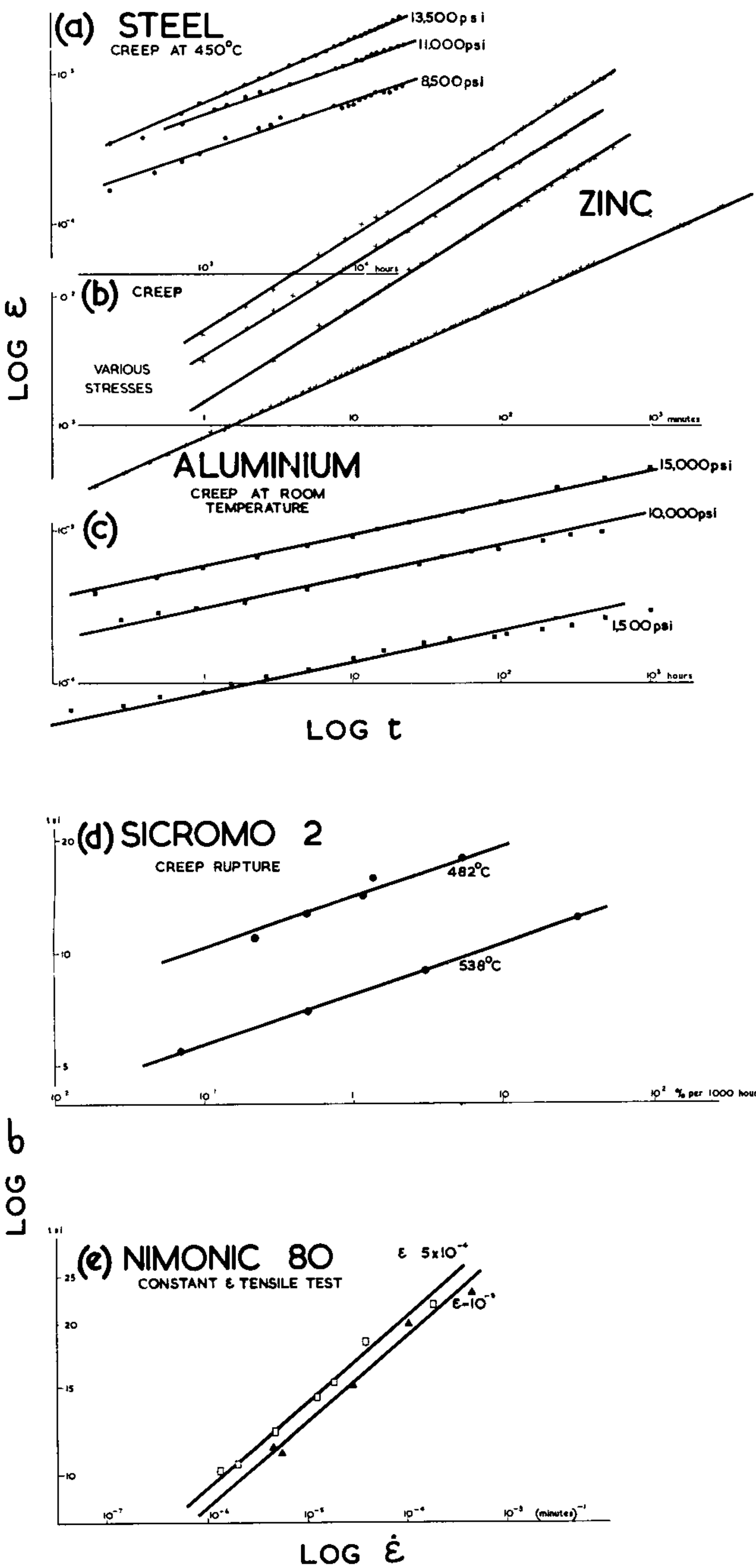
Material	Major constituents	Source of data
S.590	20Ni 20Cr 20Co Bal Fe	Trans. A.S.M. 42, p.720 (1950)
S.816	20Ni 20Cr Bal Co	Trans. A.S.M. 42, p.720 (1950)
Timken 35-15	36Ni 16Cr Bal Fe	Timken Digest, p.184 (1946)
Inconel X	15Cr 7Fe Bal Ni	Inconel X, Int. Nickel Co (1949)
Rex 337A	18Ni 14Cr 7Co Bal Fe	N.P.L. H.T.14/52
Nimonic 90	20Cr 20Co Bal Ni	N.P.L. H.T.35/53
G.32	20Cr 10Ni 13Fe Bal Co	N.P.L. H.T.37/53

TABLE IV

Sources of data for materials of Figures 8 and 9

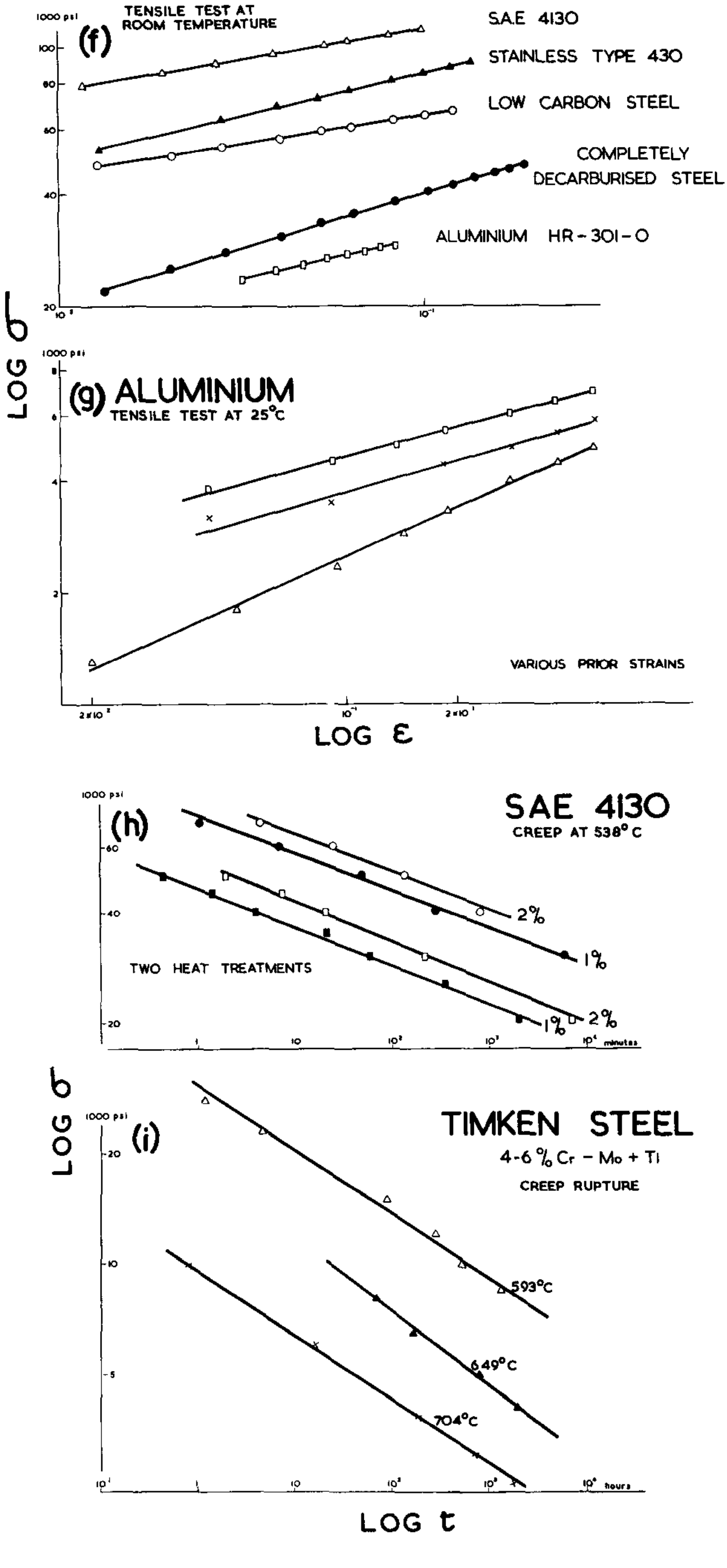
Material	Source of data	
Nimonic 80	The Nimonic Alloys	Henry Wiggin
Nimonic 80A	The Nimonic Alloys	Henry Wiggin
Nimonic 90	The Nimonic Alloys	Henry Wiggin
Nimonic 95	Supplement to the Nimonic Series of Alloys their application to gas turbine design	Henry Wiggin
Nimonic 100	Wiggin Nickel Alloys No. 35 Publication 829	Henry Wiggin

FIG.1(A)(E)



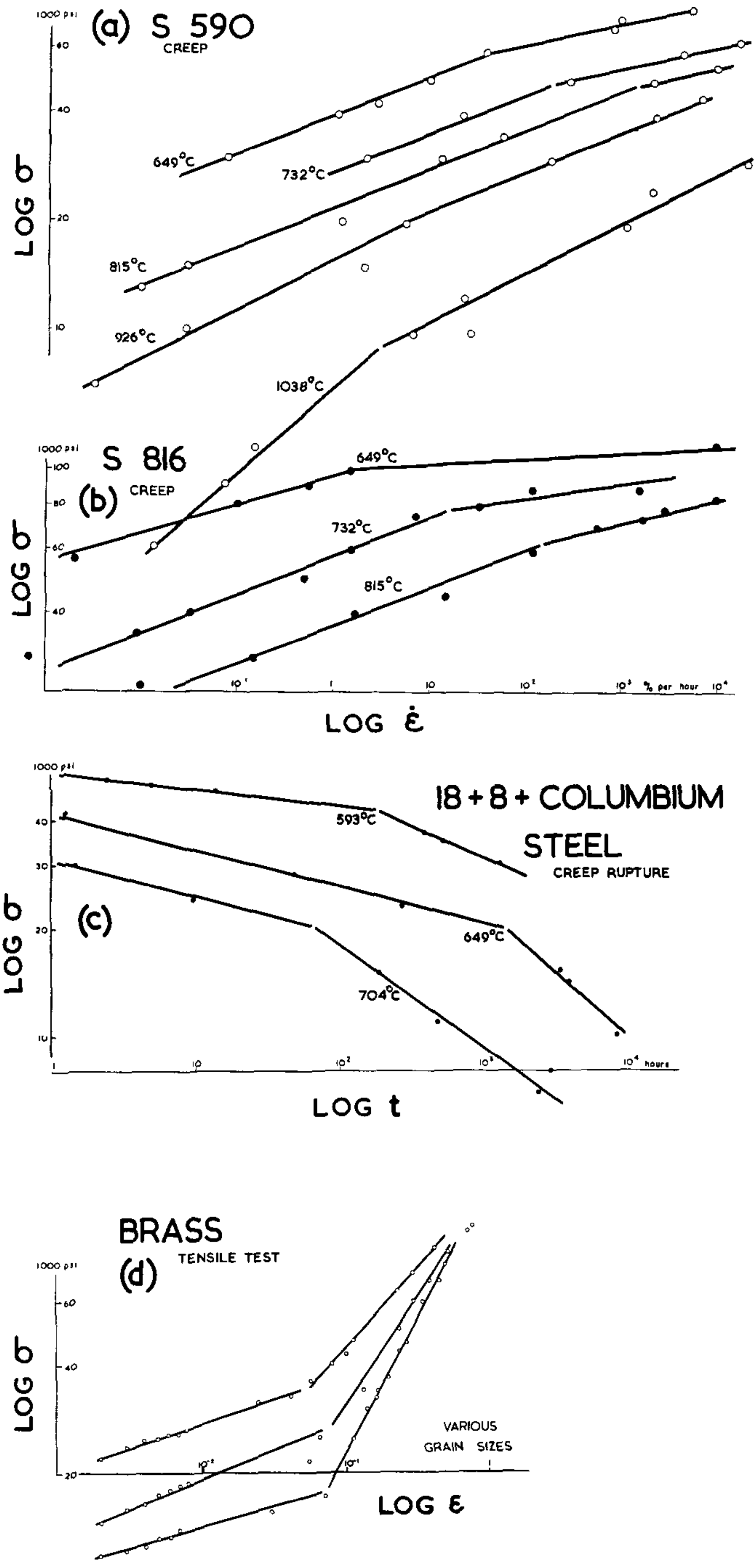
EXAMPLES OF STRAIGHT LINES

ON LOG / LOG PLOTTING



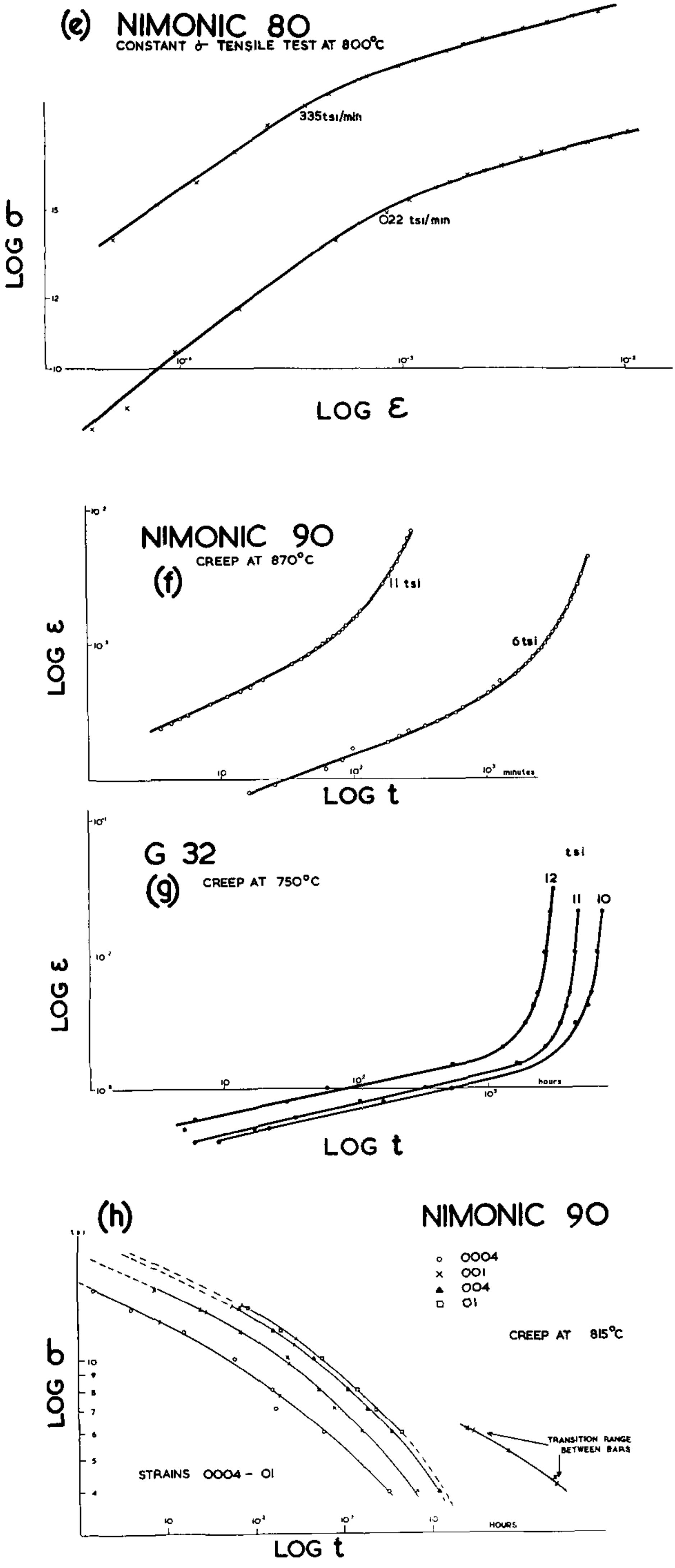
EXAMPLES OF STRAIGHT LINES ON LOG/LOG PLOTTING

FIG. 2 (A)-(D)



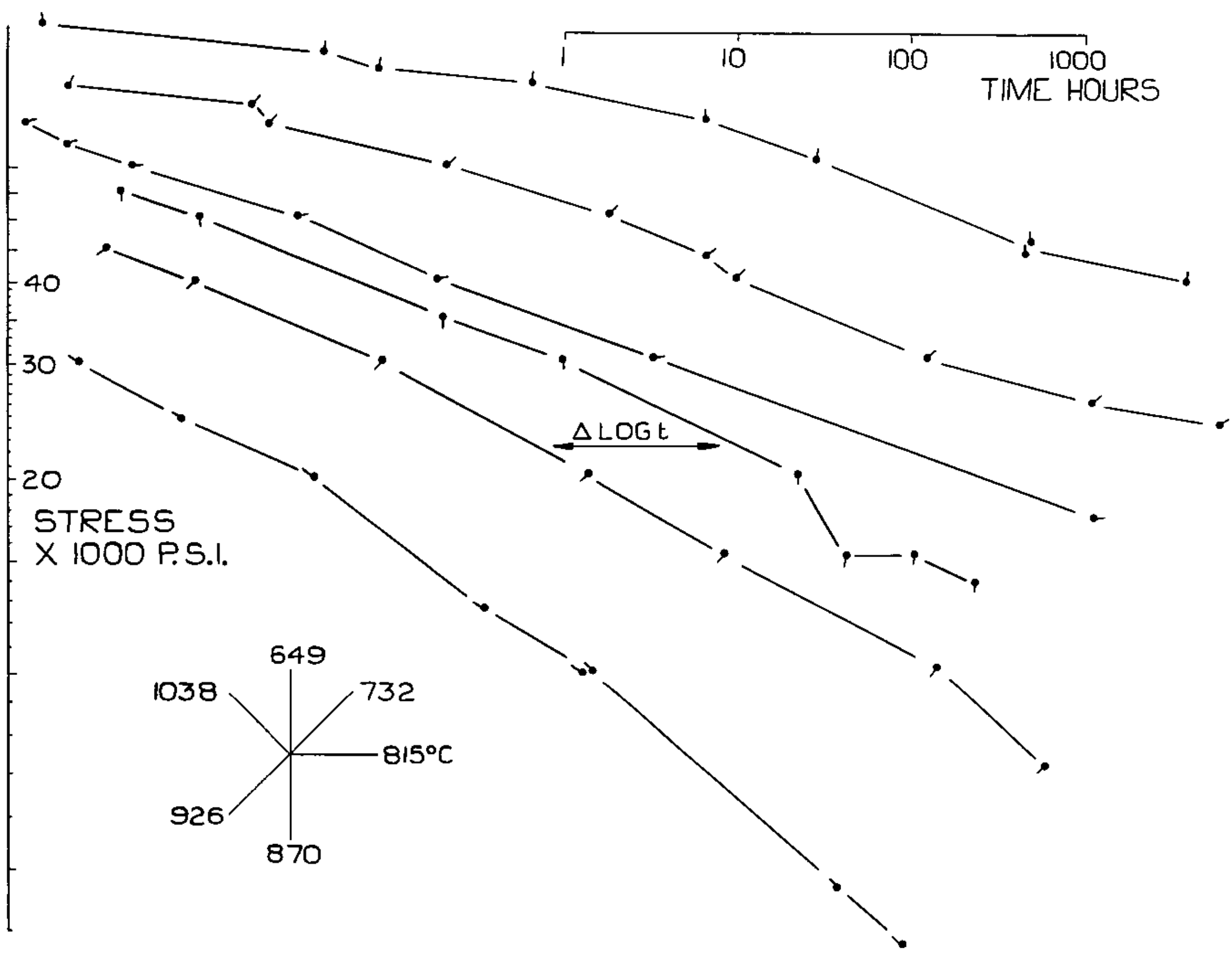
EXAMPLES OF PAIRS OF STRAIGHT LINES ON LOG/LOG PLOTTING

FIG 2(E)-(H)



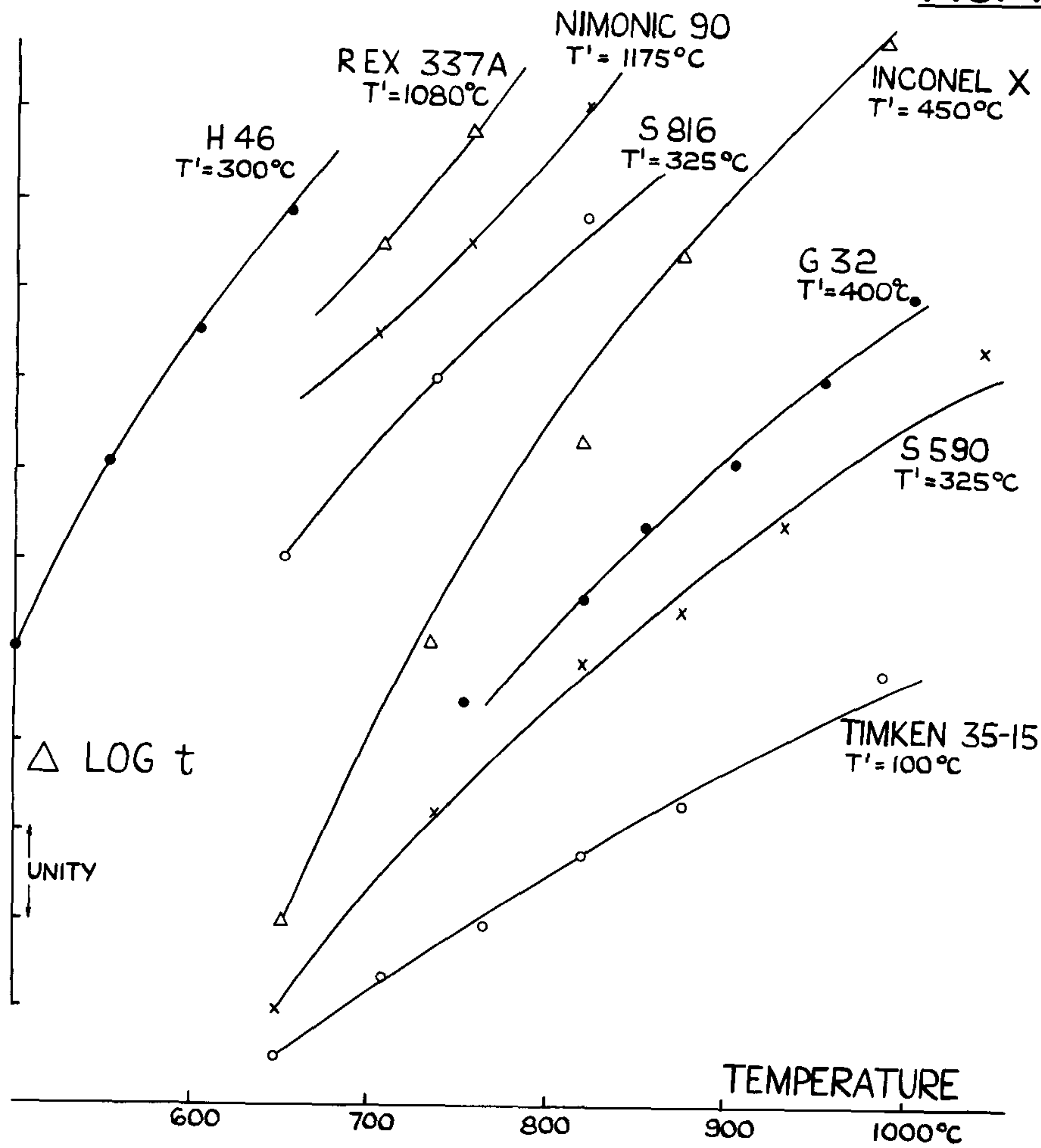
EXAMPLES OF PAIRS OF STRAIGHT LINES ON LOG/LOG PLOTTING

FIG. 3

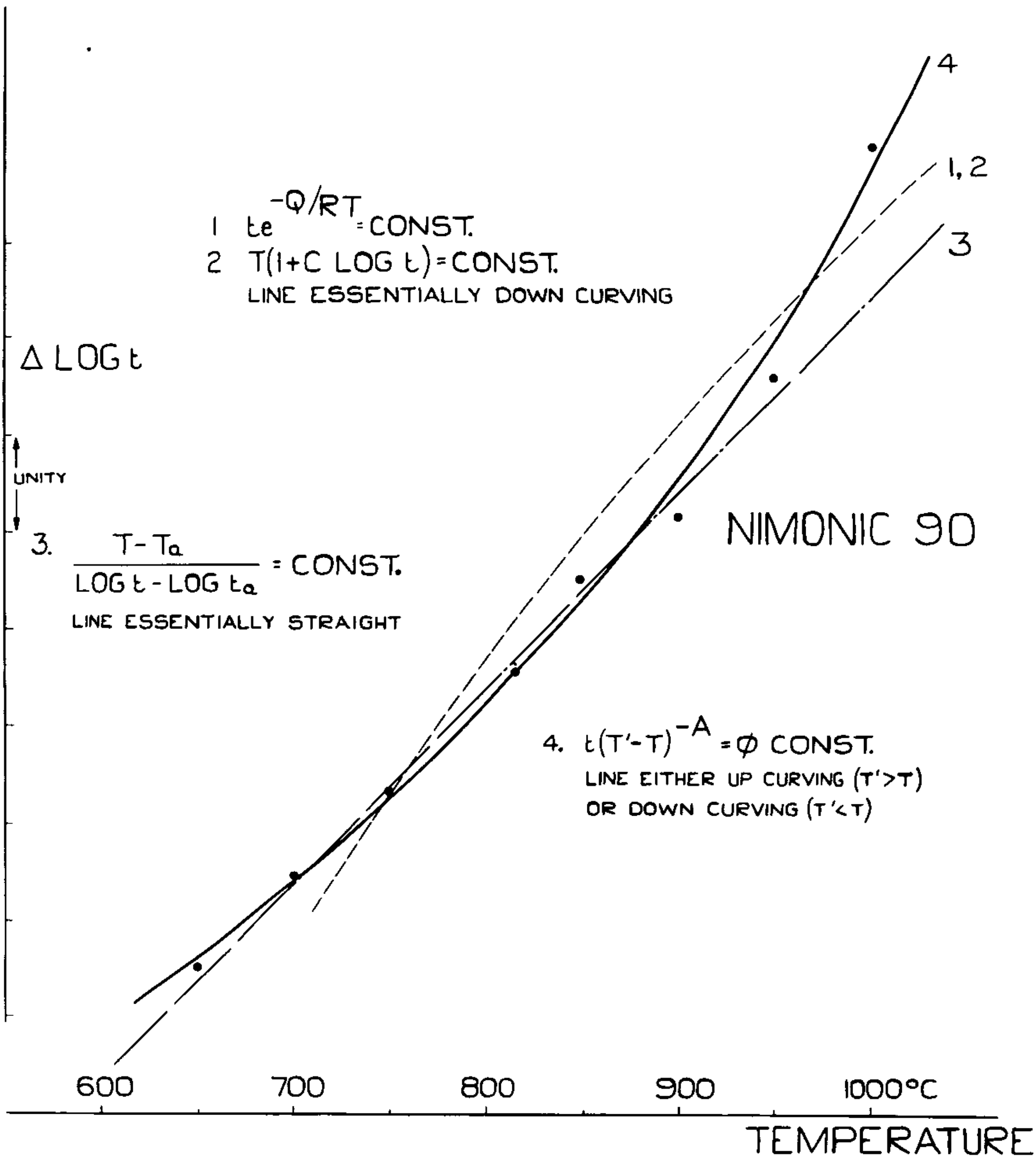


CREEP RUPTURE DATA FOR S 590
AT SEVERAL TEMPERATURES.

FIG. 4

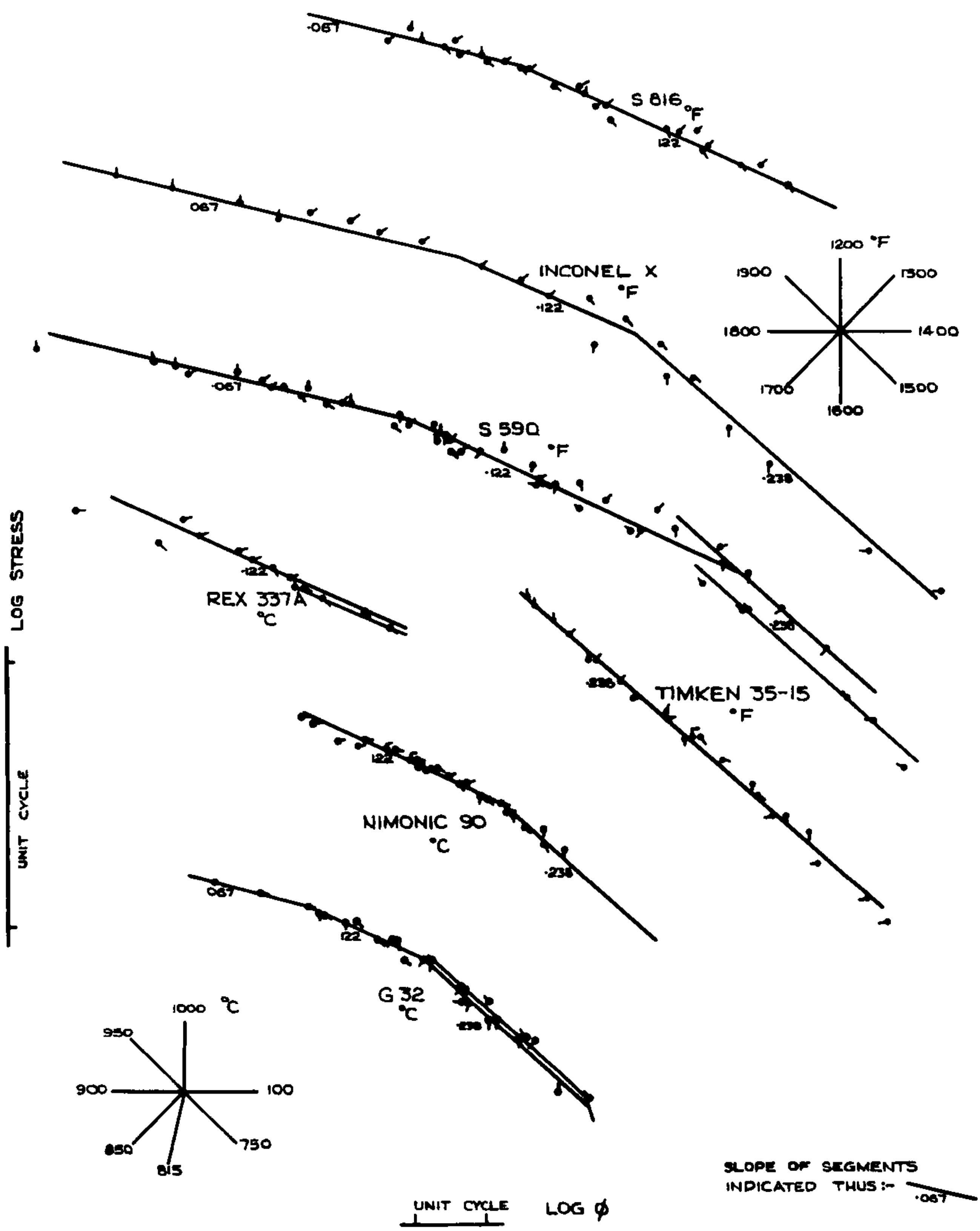


COMPARISON OF PARAMETER Ø
WITH DATA FOR MATERIALS OF FIGS. 3&6



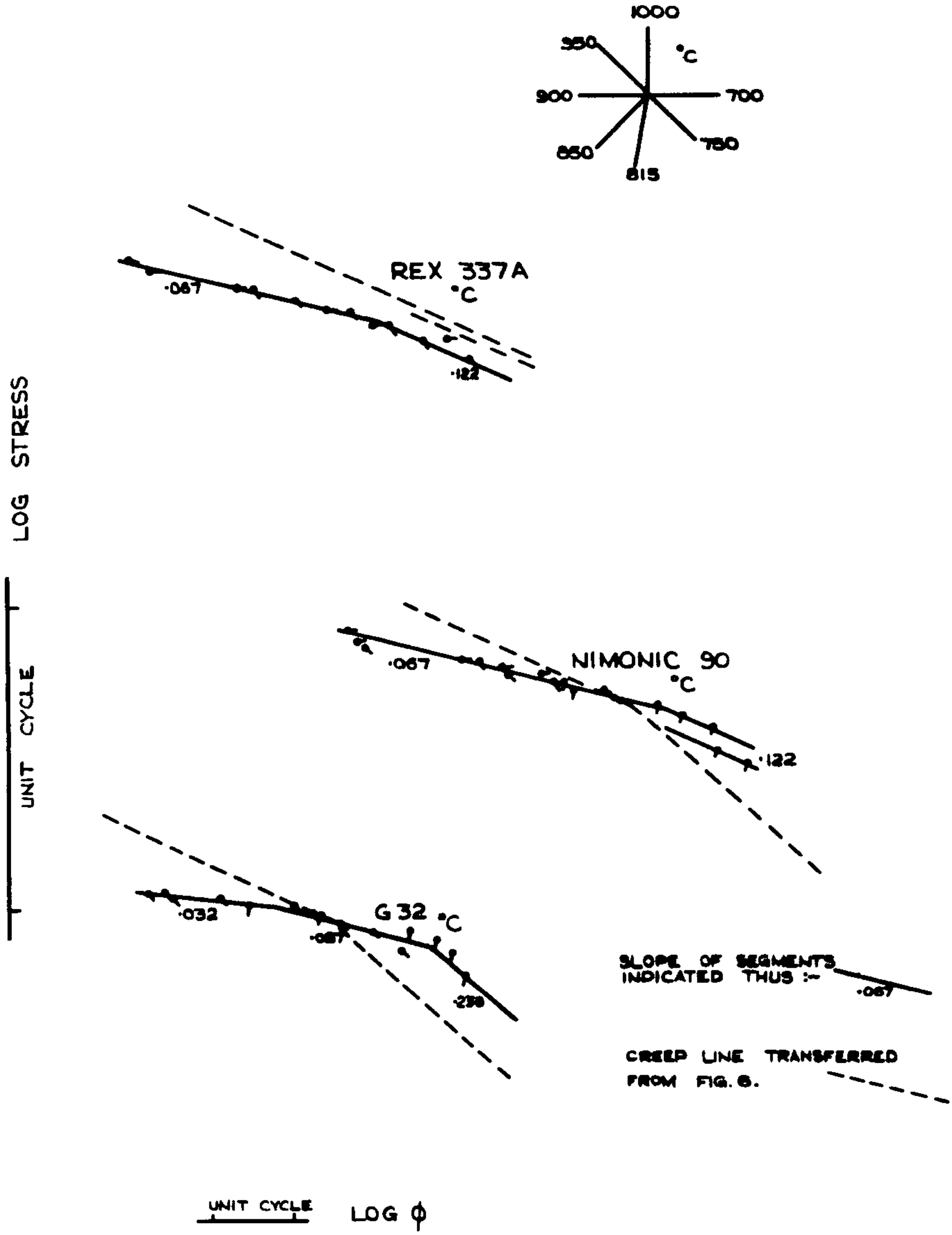
COMPARISON OF TIME-TEMPERATURE PARAMETERS.

FIG. 6.



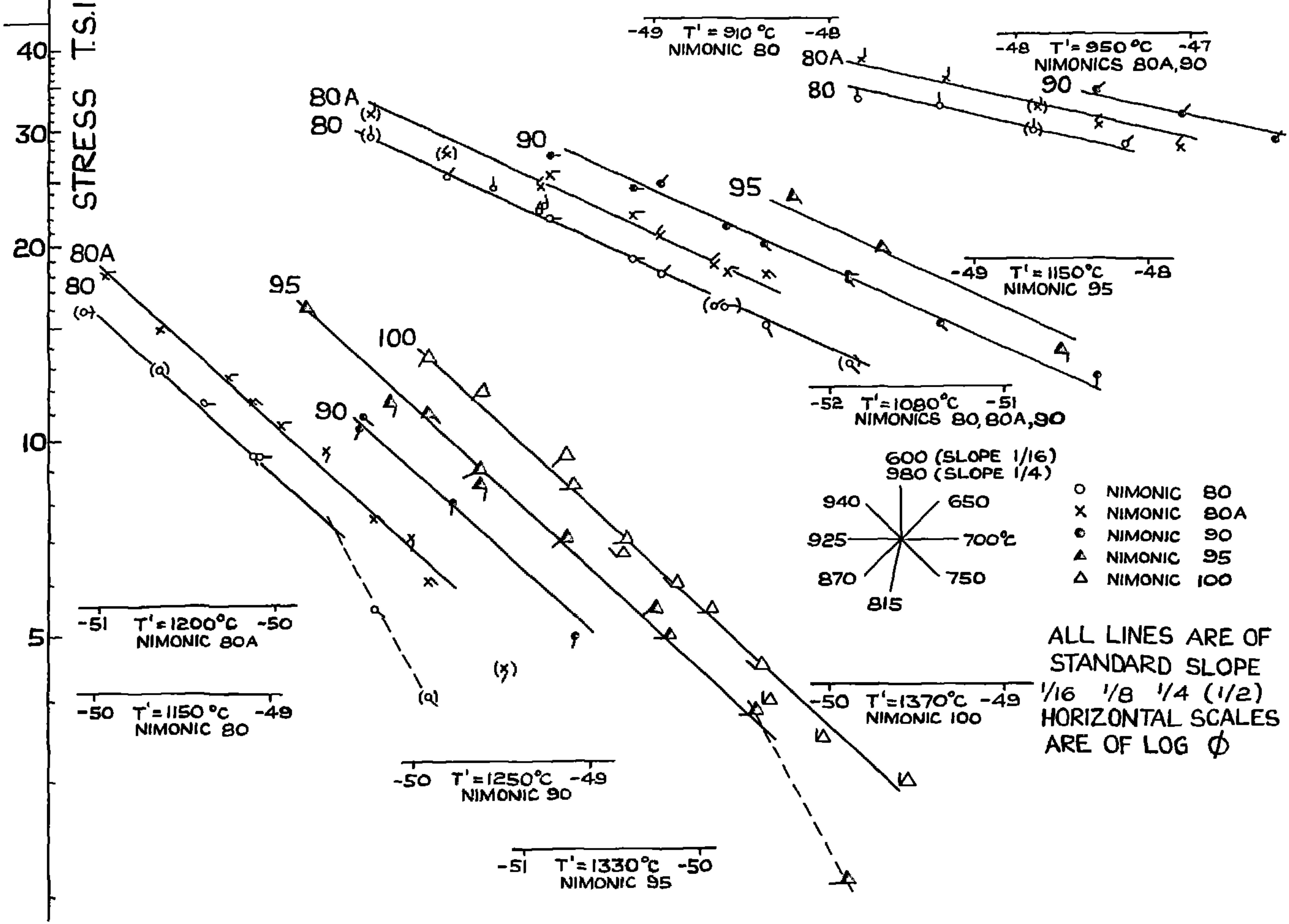
COORDINATION OF CREEP-RUPTURE DATA FOR A NUMBER OF MATERIALS

FIG. 7.

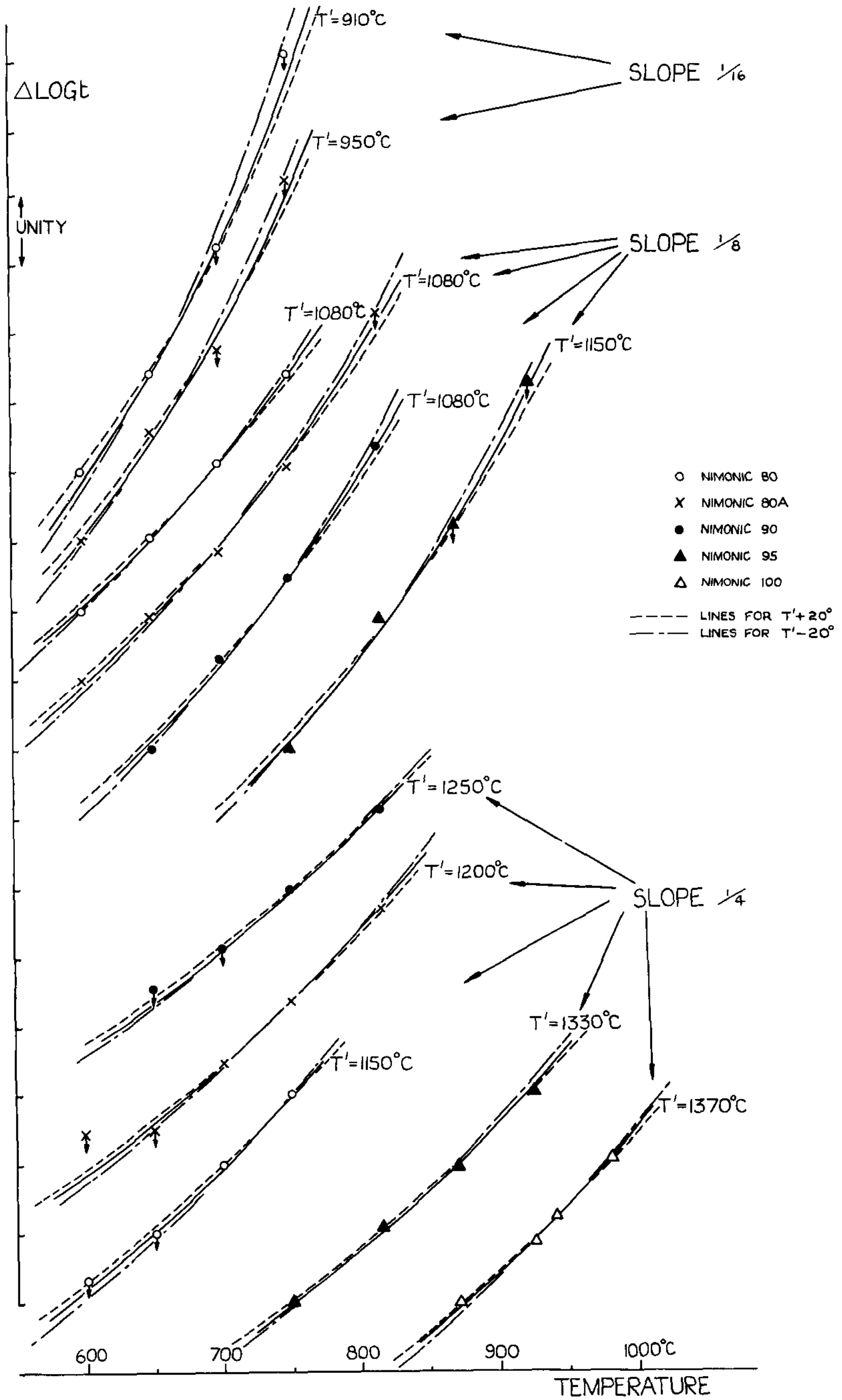


COORDINATION OF HOT-FATIGUE DATA FOR A NUMBER OF MATERIALS

FIG. 8.



CO-ORDINATION OF NIMONIC 80 - 100 CREEP RUPTURE DATA.



ΔLOGt /T PLOT FOR DATA OF FIG. 8
SHOWING COMPARISON WITH Ø

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