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Flutter Characteristics of a Wing Carrying a Flexibly Mounted Mass

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Summary.

Calculations have been made to investigate the effects on wing flutter of flexibility between a wing and an added mass. Flexibilities in pitch, roll, and yaw have been separately investigated for a mass carried at the tip of a wing. The calculations covered wing sweepback angles from zero to 45°, and a number of mass-loading conditions were considered. The calculations show that flexibility in the mass mounting may result in very low flutter speeds, particularly if the natural frequency of the added mass on its mounting is in the region of the lower wing mode frequencies.

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* Replaces R.A.E. Report No. Structures 261—A.R.C. 22,889.

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1. *Introduction.*

Investigations of the effect of added masses on wing flutter have, in general, assumed that the mass is rigidly attached to the wing. There have, however, been one or two investigations of the effect of a flexibly mounted mass[1, 2] and these have indicated that the flutter characteristics may be considerably influenced by the rigidity of the mounting.

To obtain more information, the calculations described in this paper were made. An idealised model wing was considered, and most of the calculations were concerned with masses carried at the wing tip. Separate flexibilities of the mass mounting in pitch, roll and yaw were investigated over a wide range of mounting stiffnesses, and for various angles of sweepback of the wing. The results show that considerable alteration in flutter characteristics can occur as a result of flexibility in the mounting of the added mass. In particular, the flexibility can lead to very low flutter speeds in cases where the natural frequency of the mass system is of the same order as the natural frequencies of the fundamental bending and torsion modes of the wing. In some cases, however, flutter speeds were increased by the addition of mass flexibility, but in spite of this it would appear that the aim in designing a mass mounting should be to provide as high a natural frequency as possible for the mass system.

The effect of pitching flexibility in the mass system is equally important whatever the sweepback of the wing, but in the case of rolling and yawing flexibilities their effects are more pronounced at the higher sweepback angles.

The possibility of effecting some simplification in predicting the flutter characteristics of the system by considering the flexibly mounted mass in terms of an equivalent rigid mass was investigated. It is shown that this method is not generally applicable to a mass flexibly mounted in pitch, and in view of this, its application to other degrees of flexibility was not investigated.

2. *Details of Calculations*

The wing for which the calculations were made was an unswept, untapered model of four foot span and aspect ratio 8. The inertia properties and stiffness distribution were uniform in a spanwise direction, the inertia axis being at 0·35 chord and the flexural axis at 0·25 chord. The effect of sweepback was investigated by rotation of the wing about the root as in Fig. 1; the tip was assumed to remain parallel to the line of flight, and camber change across the section for bending of, and twist about, the flexural axis was neglected. Further details are given in Table 1.

The attachment of a localised mass at the tip is shown diagrammatically in Fig. 2a. The mass is supported on two arms OO′ and O′M which are at right angles to each other, and whose plane is parallel to the line of flight (irrespective of the wing sweepback). The point O lies on the wing flexural axis. Pitch of the mass due to flexibility of the support is represented by rotation of O′M about O′ in a vertical plane (Fig. 2b), roll of the mass by rotation of OO′ about O in a plane perpendicular to the line of flight (Fig. 2c) and yaw by rotation of O′M about O′ in a plane parallel to the plane of the wing (Fig. 2d).

Four modes of distortion of the wing were used in the calculations. These were:

Mode 1—Fundamental flexure.

Mode 2—Fundamental torsion.

Mode 3—Flexure of the wing when restrained in displacement at the tip.

Mode 4—Torsion of the wing when restrained in twist at the tip.

Detailed consideration of the use of modes of this type is given in Ref. 3 and an example of their use in Ref. 4. Table 2 gives expressions for the mode shapes, and also a list of the uncoupled mode frequencies (with no added mass on the wing). The modes of mass flexibility in pitch, roll and yaw were introduced singly, as there appeared at this stage to be little justification in undertaking the enormous amount of computational work that a comprehensive treatment would have needed.

Two-dimensional incompressible-flow derivatives were used in the aerodynamic terms of the flutter equations, no correction being made for aspect ratio. Aerodynamic effect of the added mass was not allowed for. A frequency parameter $\nu = 0 \cdot 6$ was assumed throughout.

Most of the calculations deal with masses added at the wing tip. A calculation in which a mass is carried at half span is described in Appendix I; this covered a particular case of mass flexibility in which part of the mass was spring mounted in normal translation.

Case 1 was a mass equal to the wing mass with its centre of gravity at the leading edge of the tip section. The pitch, roll and yaw stiffnesses of the mass mounting were each independently varied, with infinite rigidity of the mounting except in the particular flexibility under investigation. The mounting stiffnesses were varied in terms of the mounting frequency, and the frequency range covered at least the first three normal mode frequencies of the system with rigidly mounted mass.

Case 2 differed from the Case 1 mainly in that the mass value was half that of the wing mass. The range of variation of the mounting conditions was the same as for Case 1.

In both these cases sweepback effects were investigated from zero sweepback to 45°. In sweeping an untapered wing by rotation through an angle Λ the chord in the line of flight is increased by sec Λ, and in order to maintain the same relative position of the mass in relation to the wing chord at the attachment point the distances OO′ and O′M of Fig. 2a were increased by sec Λ.

In Case 3 the added-mass centre of gravity was at the wing-tip quarter-chord and the mass value was varied between zero and the wing mass. The wing sweepback was 45° and the effects of pitch and roll flexibility only were investigated.

Full details of the added-mass values for the three cases are given in Table 3.

2. *Results.*

3.1. *Pitching Flexibility.*

The variation of flutter speed and frequency with pitching frequency of the mass may be as seen in Figs. 3 to 5. The flutter speed and frequency of the rigidly mounted mass are shown by dotted lines. The flutter speed and fundamental frequencies of the wing with zero mass are also shown. It may be mentioned here that the terms 'fundamental' and 'overtone' flutter are used to describe types of flutter in terms of the predominant modes. Fundamental flutter is mainly dependent on the fundamental modes of bending and torsion of the wing, and overtone flutter on the higher-order modes. Characteristics of these types of flutter may be found in Ref. 3 and 4 but it is sufficient to note here that low flutter speeds are generally associated with the fundamental type of flutter, and that if this type of flutter is prevented by mass-balancing the wing in its fundamental modes, flutter will occur at a much higher speed, and is a function of the higher-order or overtone modes. It will be seen in Figs. 3 and 4 that there is a drop in flutter speed and frequency for the rigidly mounted mass as the sweepback is increased from 30° to 45°. This is due to a change in the type of flutter for the rigid mass from 'overtone' at the lower sweepback angles to 'fundamental' at 45°.

Figs. 3 and 4 show that where overtone flutter is critical with a rigidly mounted mass, two sharp reductions in flutter speed occur when the mass is flexibly mounted, one when the frequency is very low (3 cycles per second approximately) and one at about 10 cycles per second. It was found that both these low-flutter-speed conditions could be obtained by taking only the fundamental modes of wing bending and torsion in conjunction with the mass pitching freedom. The flutter is therefore essentially a fundamental type which does not occur for this particular mass loading when the mass is rigidly attached to the wing. It would appear that pitching flexibility, when introduced into a system which will flutter in the overtone modes, and is stable in the fundamental modes, has a destabilizing effect on the fundamental modes at certain critical frequencies and may result in very low flutter speeds. It may be assumed that the two critical frequencies are associated with the two fundamental modes, but it is not obvious from the present results how these frequencies may be simply determined without undertaking the calculations.

If the flutter with rigidly attached mass is fundamental in type (as in Figs. 3 and 4, at 45° sweepback) the effect of flexibility may be either favourable or unfavourable. In Fig. 3 there is no effect at all, but in Fig. 4 there is a critical frequency for which a sharp increase in flutter speed is obtained. Fig. 5, which gives the results for a mass of varying magnitude whose centre of gravity is at the wing-tip quarter-chord, shows yet another general pattern. In this case there is only one critical frequency at which a fall in flutter speed occurs. The drop in flutter speed is confined to a narrower band of frequency as the mass is increased, and there is also a marked rise in flutter speed at slightly lower frequencies.

The occurrence of only one minimum in the flutter-speed curves of Fig. 5 and two on those of Figs. 3 and 4 is almost certainly due to the difference in chordwise position of the mass centres of gravity. In Figs. 3 and 4 the centre of gravity is on the wing leading edge so that pure bending of the wing results in pitch of the mass. In Fig. 5 the centre of gravity is on the flexural axis so that no coupling exists between wing bending and mass pitch.

Two conclusions may be drawn from the investigation. Firstly, pitching flexibility may have a destabilizing effect on a type of flutter which is stable when the mass is rigidly fixed to the wing. Secondly, there are no simple rules for determining the critical frequencies of the mass in pitch and the effect will be largely dependent on the inertia coupling between the mass and the wing, and hence on the position and magnitude of the mass.

3.2. *Roll Flexibility.*

The effects of roll flexibility are shown in Figs. 6, 7 and 8. For the unswept wing there is no coupling between the wing torsion modes and the mass roll mode, but as the wing is sweptback, coupling increases between these modes and decreases between wing bending modes and the mass roll mode. Examination of Figs. 6 and 7 for sweepback angles of 0° to 30° shows that there are two distinct characteristics of the flutter curves; firstly there is a slight fall in flutter speed and a rise in flutter frequency as the roll frequency is increased. (This can be seen in Fig. 6 at zero sweepback). Secondly, increasing sweepback introduces another branch of the curve which has a low frequency and results in a pronounced drop in flutter speed at a critical roll frequency (*see* Fig. 6, 15° and 30° sweepback and Fig. 7, 30° sweepback). The first characteristic is probably due to mass roll coupling with the wing bending modes—and probably mainly with the overtone bending mode, since the effect continues up to frequencies in roll beyond the range investigated. The second characteristic is due to mass roll coupling with the wing fundamental torsion mode; this was confirmed in the calculations when it was found possible to obtain curves of this shape with the fundamental modes and mass roll flexibility. The roll flexibility of the mass has a destabilizing effect on the fundamental modes, although with the rigidly mounted mass there was no flutter in the fundamental modes alone. The two cases where flutter occurs in the fundamental modes with a rigid mass (Figs. 6 and 7, at 45° sweepback) show the mass rolling flexibility to have no effect. The results shown in Fig. 8 indicate that the effect of roll flexibility with mass variation is negligible when the mass is large, but can be considerable for the small mass.

The evidence of these results is that rolling flexibility of an added mass is likely to have a more significant effect for a sweptback than for an unswept wing, mainly because coupling with the torsion mode increases with sweepback. The most significant effect will probably occur when the roll flexibility results in flutter of a fundamental type, where (under rigid-mass conditions) flutter of this type did not occur.

3.3. *Yaw Flexibility.*

The inertia coupling between the wing modes and the added-mass flexibility varies with sweepback in the same way for both roll and yaw flexibilities of the mass. One would expect the flutter characteristics produced by these flexibilities to be much the same. Fig. 9 shows the effect of yaw frequency and may be compared with Fig. 6 for roll frequency. Similarity of the results is obvious except that at zero sweepback a U-shaped branch of the yaw curve is obtained which does not appear for the roll case. The yaw frequency of this branch is too high for it to correspond with the branches at 15° and 30° sweepback, and it is likely (by the argument of Section 3.2) to be due to the yaw flexibility coupling with the wing overtone bending mode.

The results shown in Fig. 10 indicate that the yaw flexibility in this case has little or no effect on flutter speed.

The effects of yaw flexibility have not been investigated any further than is shown in Figs. 9 and 10, but on this evidence it seems that the flexibility may be of significance, and would certainly warrant investigation in practical cases.

3.4. *General Comments.*

As a guide to the relation of the critical flexibility frequencies to the wing frequencies, the wing fundamental bending and torsion mode frequencies are shown on all the diagrams. These are the frequencies with no added mass, and although it may be argued that the wing frequencies with a rigidly mounted mass are more relevant for comparison purposes, it is the 'bare' wing frequencies that are usually available to the designer. It will be seen that in general the minimum flutter speeds with pitching flexibility all occur when the pitching frequency is lower than that of the bare-wing fundamental torsion mode. With one exception the same applies to the rolling flexibility case. The exception is seen in Fig. 8 where a low minimum flutter speed occurs at a rolling frequency of the mass slightly higher than that of the wing torsion mode. In the yawing flexibility case shown in Fig. 9 there is again a low minimum flutter speed when the yawing frequency is just above that of the wing torsion frequency. As a rough guide, therefore, it may be said that the flexibility of an added mass should be such that the natural frequency of the mass on its mounting is higher than the fundamental bending and torsion modes of the wing. If this can be achieved there is a fair prospect of avoiding the more serious effects of mass flexibility on flutter.

It is also of interest to examine the flutter-speed levels in relation to the flutter speed of the bare wing. It will be seen in almost every case where significant reductions in flutter speed result from mass flexibility that the minimum flutter-speed values are lower than that of the wing with no added mass. It is apparant that any improvement in flutter characteristics that can be obtained by judicious placing of an added mass may be more than offset if the design of the mounting is inadequate.

4. *Supplementary Investigations.*

4.1. In one of the earliest experimental investigations of the effect of added masses on wing flutter, Lambourne made some tests with flexibly mounted masses[1]. In his analysis he related the mass system at the flutter frequency to an equivalent rigidly mounted mass system using the relationship:

$$m_r = m\left(\frac{\omega_n^2}{\omega_n^2 - \omega^2}\right) \tag{1}$$

where m is the flexibly mounted mass

ω_n is its natural frequency

ω is the flutter frequency

and m_r is the equivalent rigidly mounted mass by which the flexibly mounted mass may be replaced. Lambourne's experiments were made with masses flexibly mounted in pitch about a point on the supporting arm behind the mass, but it was assumed that no rotation of the mass about its centre of gravity occurred; this assumption was justified since the masses used had zero pitching moments of inertia about their centres of gravity (or very nearly so). Since the method of replacing a flexibly mounted mass by an equivalent rigid mass suggests that the flutter characteristics of the flexibly mounted system could be predicted from a knowledge of the rigid system, the possibilities of this were investigated theoretically. The system considered was that shown in Fig. 11 where an added mass m was supported on an arm of length p which was flexibly mounted at O. The stiffness of the support in pitch was K_p and the pitching radius of gyration of the mass about its centre of gravity G was k_p. The natural frequency (ω_n) of the mass system was therefore given by:

$$\omega_n^2 = \frac{K_p}{m(k_p^2 + p^2)} \tag{2}$$

For translation z and pitch α of the wing section at a frequency ω the mass system pitches through an angle θ and the equation of motion is:

$$m\ddot{z}_1 p - m k_p^2 \ddot{\gamma} - K_p \theta = 0 . \tag{3}$$

where $z_1 = (z - p\gamma)$ and $\gamma = (\alpha + \theta)$.

Substituting for z_1, γ and K_p {from equation (2)} gives:

$$\theta = \left\{\alpha - \left(\frac{z}{p}\right)\left(\frac{p^2}{K_p{}^2 + p^2}\right)\right\}\left(\frac{\omega^2}{\omega_n{}^2 - \omega^2}\right). \qquad (4)$$

For an equivalent rigidly mounted mass m_r the lift and moment reactions at O must be equal to those for the flexibly mounted mass, i.e.:

$$m_r \ddot{z}' p - m_r k_p{}^2 \ddot{\alpha} = K_p \theta \qquad \text{(moment)} \qquad (5)$$

and

$$m_r \ddot{z}' = K_p \theta / p \qquad \text{(lift)} \qquad (6)$$

where

$$z' = (z - p\alpha).$$

Clearly, equations (5) and (6) cannot hold simultaneously unless $k_p = 0$. Nevertheless, substituting for θ from equation (4) in (5) and using equation (2) gives:

$$m_r = m\left(\frac{\omega_n{}^2}{\omega_n{}^2 - \omega^2}\right) \qquad \text{(moment)} \qquad (7)$$

Substituting in equation (5) gives:

$$m_r = m\left(\frac{\omega_n{}^2}{\omega_n{}^2 - \omega^2}\right)\left\{\frac{\alpha\left(\frac{k_p{}^2 + p^2}{p^2}\right) - \left(\frac{z}{p}\right)}{\alpha - \left(\frac{z}{p}\right)}\right\} \qquad \text{(lift)} \qquad (8)$$

Equations (7) and (8) indicate that the lift and moment reactions on the wing of an equivalent rigid mass can only be the same as for the flexibly mounted mass if $k_p = 0$ {in which case equations (7) and (8) are identical}.

In the general case, therefore, for which k_p is not zero there would appear to be no simple solution for the mass with pitching flexibility in terms of an equivalent rigid mass. The possibility of replacing masses with roll and yaw flexibility by equivalent rigid masses was not investigated in view of the above result, since pitch flexibility is the more common occurrence, and an equivalent rigid-mass solution for roll and yaw would be of little value without a similar solution for pitch flexibility.

4.2. The possibility of obtaining an approximate solution to a problem of flexible mass mounting by a method similar to that used by Moxon in control-surface flutter calculations[5] was investigated. Moxon considered a wing-aileron system in which the aileron carried a flexibly mounted mass-balance weight. He found that in certain types of flutter involving aileron and mass-balance motions only one of the two normal modes of the aileron-mass-balance system was essential to the oscillation. With a wing mass system and a flexibility of the mass in pitch it might be expected that the effect of the mass flexibility would be obtained mainly from modification of the wing torsion mode. To check if this were so a ternary flutter calculation was made with wing modes of fundamental bending and torsion, and flexibility of the mass in pitch. For two pitching frequencies of the mass the wing torsion and mass pitch modes were normalised to give two uncoupled modes in each case. The binary calculations obtained by taking each of these uncoupled modes with the wing bending mode were then made. In neither case did the binary solutions agree with those obtained from the ternary. It was concluded that all three modes were essential to the flutter condition.

5. *Conclusions.*

5.1. Flexibility in the mounting between an added mass and a wing can lead to considerable alteration of the flutter characteristics. The flutter speed may be quite sensitive to small changes in the natural frequencies of the mass system, particularly if the frequencies are of the same order as those of the wing fundamental bending and torsion modes. Compared with the flutter speed for a rigidly mounted mass, that for the flexibly mounted mass may be either decreased or increased depending on the flexibility of the mounting, but for the cases considered decrease of flutter speed was the more usual.

5.2. Low pitching frequency of the mass system is likely to be the most important practical case, and pitching flexibility, by affecting the wing torsion modes, exerts a strong influence on the flutter characteristics whatever the degree of wing sweepback. In the cases of roll and yaw flexibility, neither motion of the mass couples with wing torsion for an unswept wing, but the coupling will increase with increasing sweepback and it is on the sweptback wing that mass flexibility in roll or yaw has a significant effect.

5.3. In general it was found that the greatest changes in flutter speed due to flexibly mounting the added mass occurred when the wing with rigid mass fluttered in overtone modes. Under these conditions flutter in the fundamental modes of bending and torsion is prevented because of the mass-balancing effect of the added mass. If the mass mounting is flexible the effect may be to destabilize the fundamental modes, giving flutter in these modes at a speed much lower than the overtone flutter speed with the rigidly attached mass.

5.4. From the results of the investigation it must be concluded that rigidity of the added-mass mounting is a desirable feature from the flutter point of view, and that any flexibility merits careful investigation of its effect on the flutter characteristics. As a rule-of-thumb guide, it is desirable that the frequency of an added mass on its mounting shall be greater than that of the bare-wing torsion mode.

REFERENCES

No.	*Author(s)*	*Title, etc.*
1	N. C. Lambourne and D. Weston	An experimental investigation of the effect of localised masses on the flutter of a model wing. Part I. Flutter tests. A.R.C. R. & M. 2533. April, 1944.
2	Kichiji Nishino	Flutter calculations for a wing with an elastically supported concentrated weight. *J. Japan Soc. Aero. Eng.*, Vol. 4, No. 27. April, 1956.
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4	D. R. Gaukroger	A theoretical treatment of the flutter of a wing with a localised mass. *J. R. Ae. Soc.*, Vol. 63, No. 2, p. 95. February, 1959.
5	D. Moxon	Flexible mass-balance arms and control-surface flutter. A.R.C. R. & M. 3167. January, 1958.

APPENDIX

Normal Translation Flexibility Investigation

1. *Introduction.*

A form of added mass flexibility which has not been covered in the main text of the report occurs when part of the mass is flexibly mounted as an anti-vibration measure. The effect of such a mounting on the flutter characteristics of the wing has been investigated for a particular case.

The added mass is attached to the 45° sweptback wing at half span, and the chordwise position of its centre of gravity is at 0·20 chord aft of the wing leading edge (Fig. 12). The added-mass value is 0·625 of the wing mass, and 0·248 of this value is flexibly supported in normal translation. The centre of gravity of the flexibly supported mass coincides with that of the total mass, and the pitching radius of gyration of the total mass is 0·148 of the wing chord.

2. *Calculations.*

Six modes were used in the calculations:

(i) Flexure of the wing in a fundamental mode.

(ii) Torsion of the wing in a fundamental mode.

(iii) Flexure of the wing with an artificial constraint at the localised mass section.

(iv) Torsion of the wing inboard of the added mass.

(v) Torsion of the wing outboard of the added mass.

(vi) Normal translation of the flexibly supported portion of the added mass.

The flutter equations were solved for a range of frequencies of the flexibly supported portion of the added mass from 2 to 50 cycles per second.

3. *Results.*

The variation of flutter speed and frequency with variation of the uncoupled natural frequency of the mass flexibility, is shown in Fig. 13. There are two branches of the flutter curve, the transition between them being denoted by the abrupt change in the flutter frequency. A minimum flutter speed occurs when the mass frequency is slighty lower than its value at the transition. This minimum speed is about 25 per cent below the flutter-speed level when the natural frequency of the mass is very high (50 cycles per second).

The minimum flutter speed occurs when the mass frequency is 10·5 cycles per second. There is no clear case of frequency coincidence to account for the drop in flutter speed.

The flutter mode consists of modes (i), (ii) and (vi) and the overtone modes play little part in the oscillation. Part of the flutter curve was obtained for modes (i), (ii) and (vi) alone and is shown in Fig. 13. The curve agrees quite well with that obtained with six degrees of freedom, particularly in the region of minimum flutter speed. It is interesting that although the type of flutter changes at the transition, the main difference in degree of freedom contribution is that mode (vi) ceases to play an important part in the oscillation when the mass frequency exceeds its value at transition. It is true that modes (iii), (iv) and (v) have some effect on the flutter speed in both branches of the curve; nevertheless both branches are fundamental-type flutter and the distinction between them lies in the contribution of mode (vi).

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The calculations indicate that an added-mass system of the type considered here may lead to a worsening of the flutter characteristics compared to a similar system in which the whole mass is rigidly attached to the wing. In full-scale calculations it would obviously be advisable to take account of any flexibility in the added-mass system; equally it can be seen that small changes in the stiffness of the system can produce large changes in flutter speed, and for this reason the effect of variation of flexibility should be investigated.

TABLE 1

Wing Details (Unswept)

Geometry

Span (root to tip)	$s = 4$ feet
Chord	$c = 1$ foot
Taper ratio (tip chord/root chord)	$= 1$
Aspect ratio ($2s^2$/area)	$= 8$

Inertia

Mass per unit span

$$\overline{m} = 0\cdot0373 \text{ slugs/ft}$$

Mass moment per unit span about reference axis,

$$\overline{m}\overline{x} = 0\cdot00373 \text{ slugs ft/ft}$$

Mass moment of inertia per unit span about reference axis,

$$\overline{m}k^2 = 0\cdot002002 \text{ slugs ft}^2\text{/ft.}$$

Axes

Reference axis	: $0\cdot25c$
Flexural axis	: $0\cdot25c$
Inertia axis	: $0\cdot35c$

TABLE 2

Mode Functions and Frequencies

The values of f and F denote, respectively, the displacement and twist of the flexural axis at a section η where ηs is the distance of the section from the root.

Mode 1 Fundamental flexure $\omega_F = 3\cdot6\ c/s.$

$$f_1 = 1\cdot724822\eta^2 - 0\cdot729936\eta^3$$

Mode 2 Fundamental torsion $\omega_T = 14\cdot5\ c/s$

$$F_2 = \sin\frac{\eta\pi}{2}$$

Mode 3 Overtone flexure $4\cdot34\omega_F = 15\cdot62\ c/s$

$$f_3 = -\ 11\cdot8468\eta^2 + 20\cdot1873\eta^3 - 8\cdot3521\eta^4.$$

Mode 4 Overtone torsion $2\omega_T = 29\cdot0\ c/s$

$$F_4 = \sin\eta\pi$$

TABLE 3

Added-Mass Values

In the following table m/m_W is the added-mass value divided by the wing mass, p/c is the distance of the centre of gravity of the added mass forward of the section 1/4 chord point divided by the wing chord, and r/c is the distance of the centre of gravity of the mass below the plane of the wing divided by the wing mean chord. k_p/c and k_r/c are the pitching and rolling radii of gyration of the added mass about its centre of gravity divided by the wing chord. In all cases the wing chord is in the line of flight.

Case	Sweepback	m/m_W	p/c	r/c	k_p/c	k_r/c
1	0°, 15°, 30°, 45°	1·0	0·25	0·5	0·5	0·1
2	0°, 15°, 30°, 45°	0·5	0·25	0·25	0·5	0·05
3	45°	0·2, 0·4, 0·6, 0·8, 1·0	0	0·2	0·2	0·05

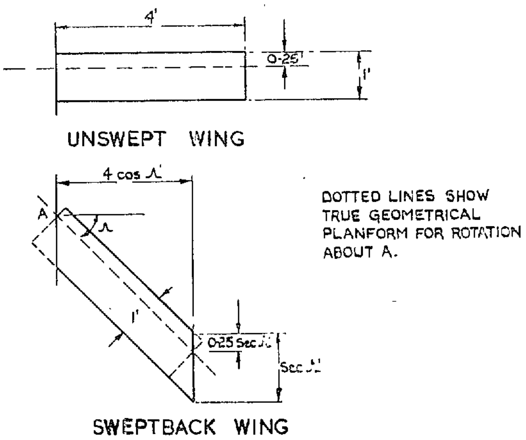

FIG. 1. Details of wing geometry.

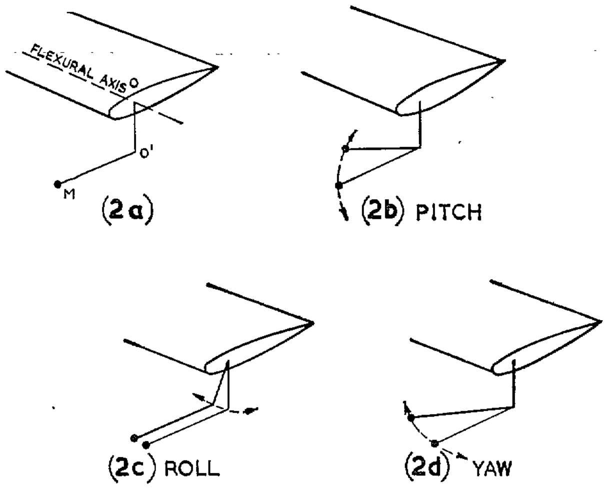

FIG. 2. Mass flexibilities.

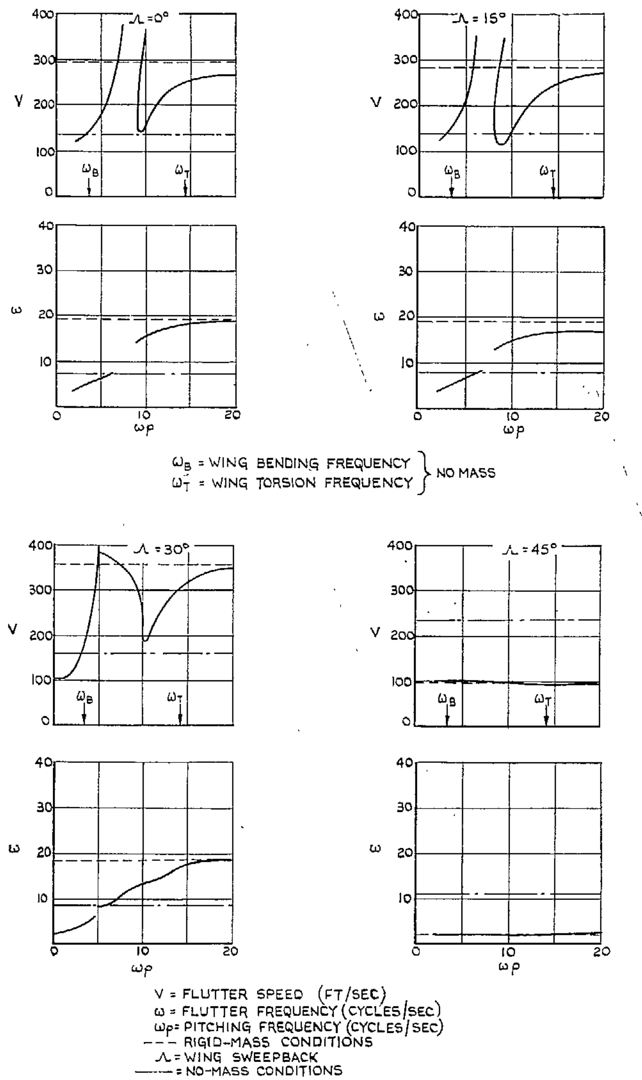

FIG. 3. Variation of pitch frequency—Case 1.

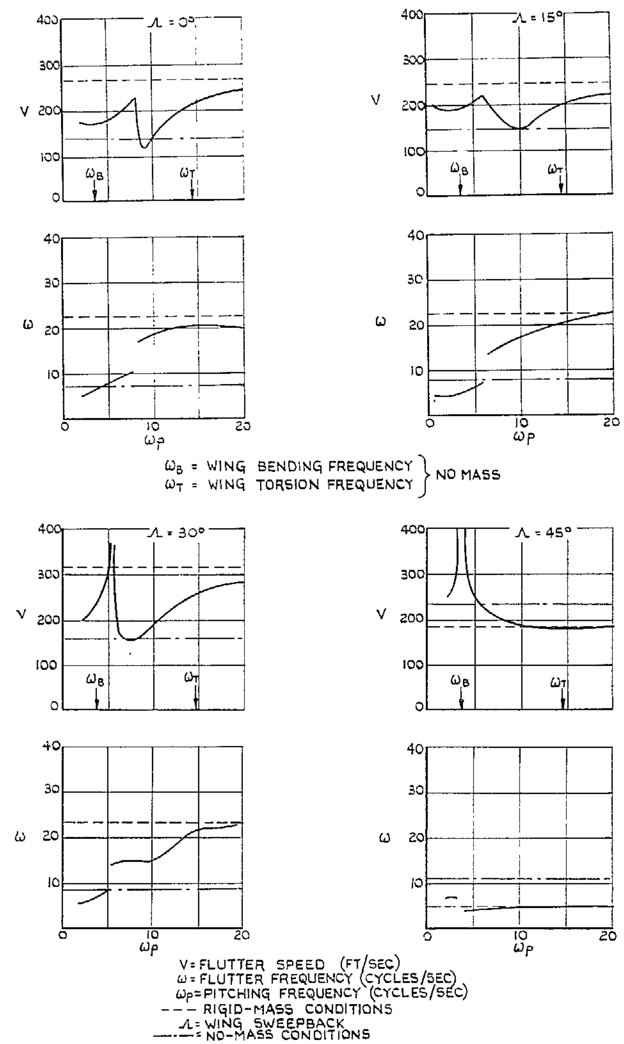

FIG. 4. Variation of pitch frequency—Case 2.

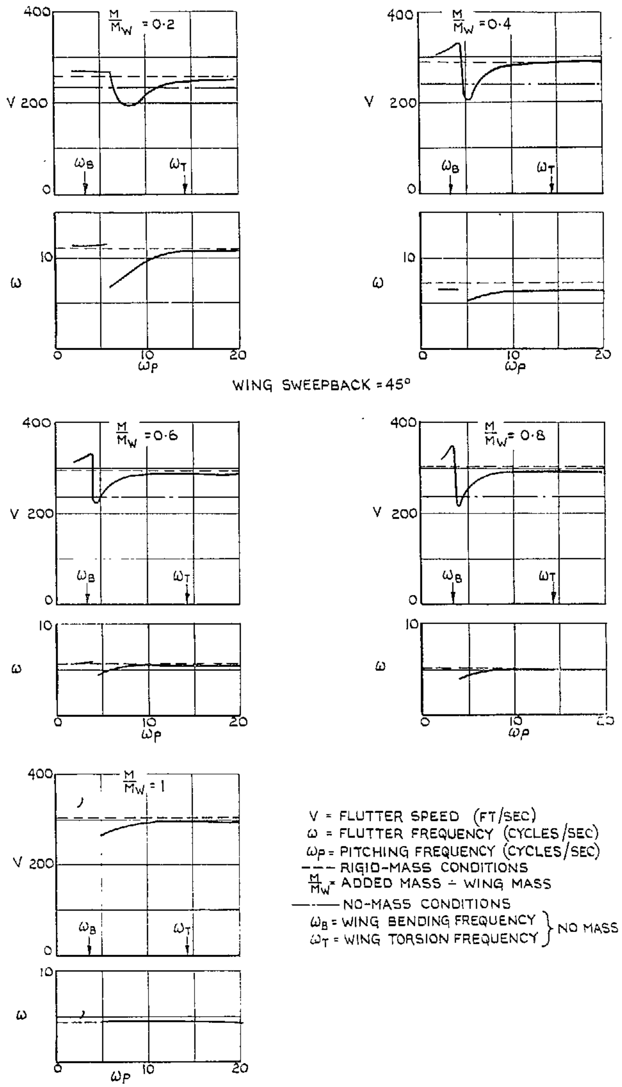

Fig. 5. Variation of pitch frequency—Case 3.

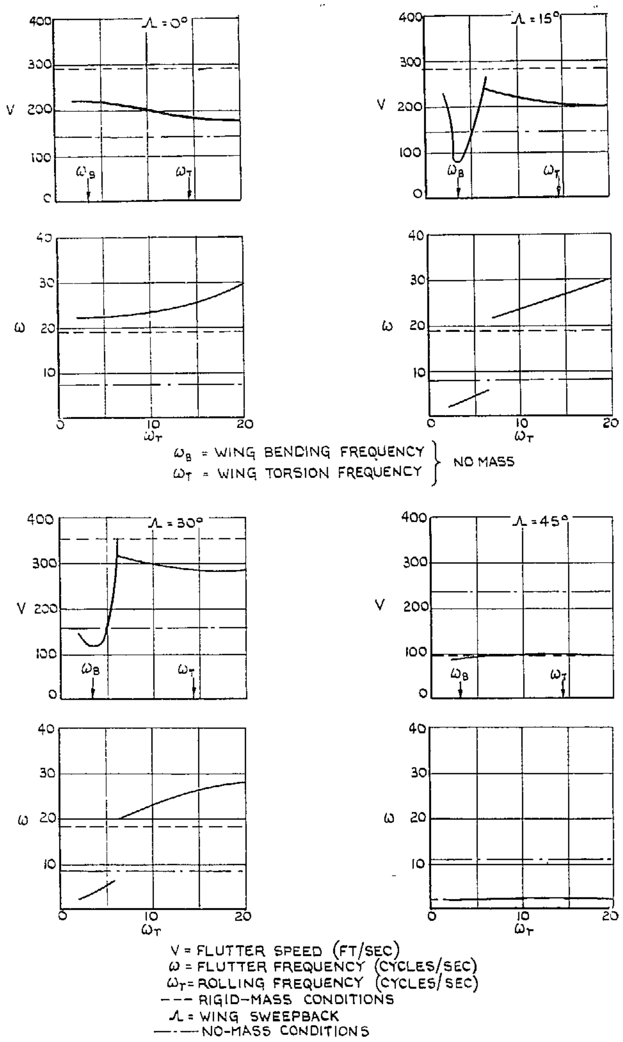

Fig. 6. Variation of roll frequency—Case 1.

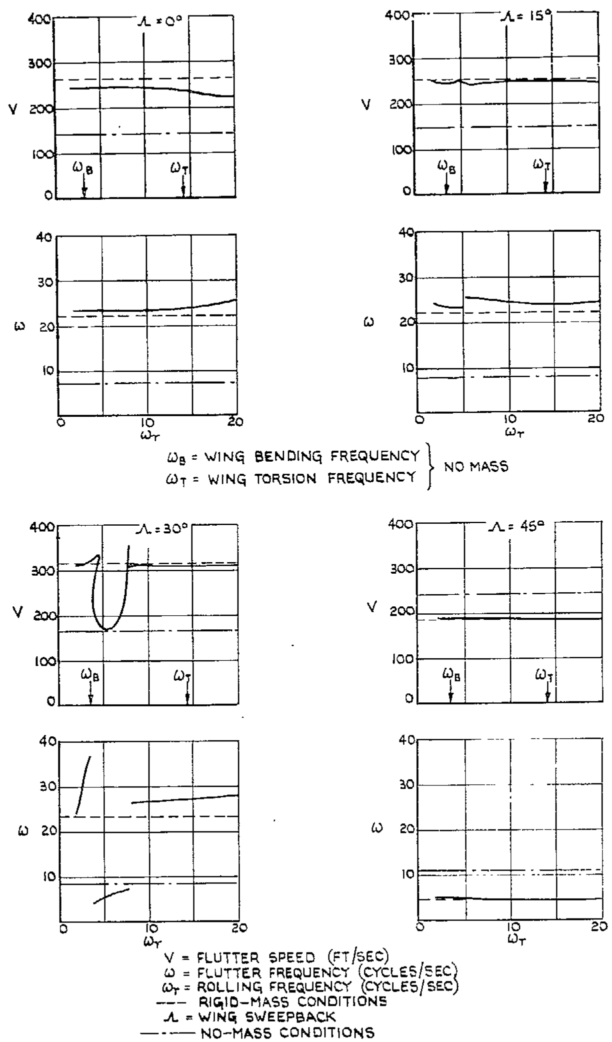

FIG. 7. Variation of roll frequency—Case 2.

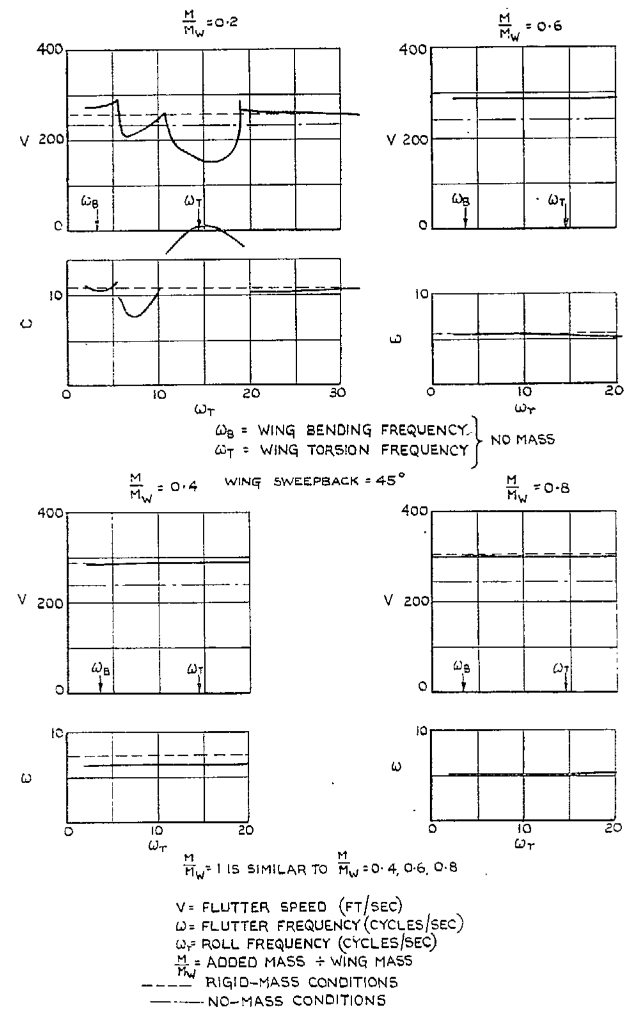

FIG. 8. Variation of roll frequency—Case 3.

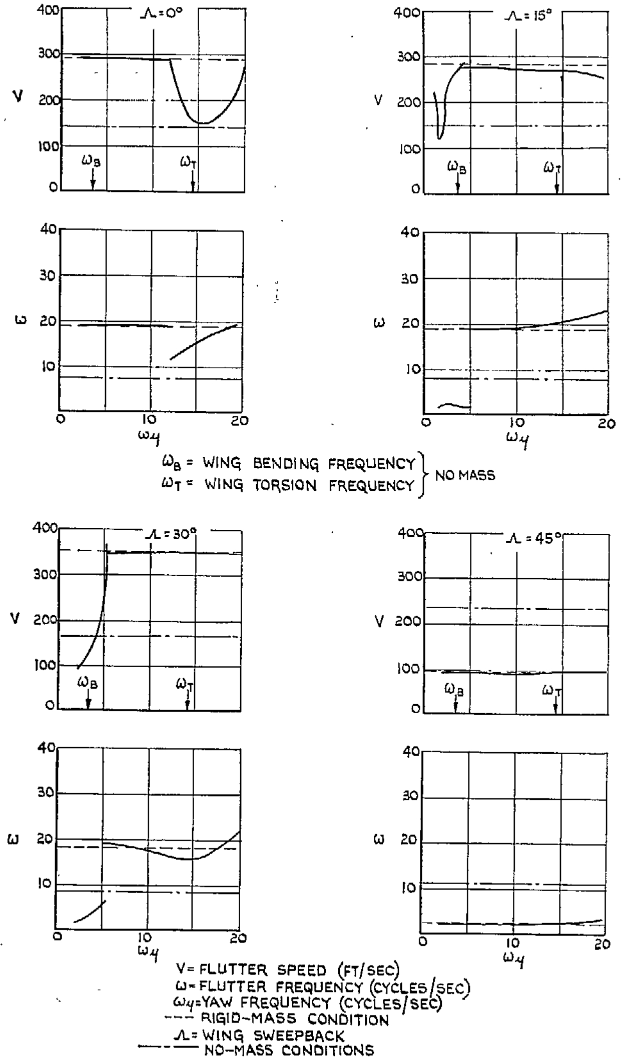

FIG. 9. Variation of yaw frequency—Case 1.

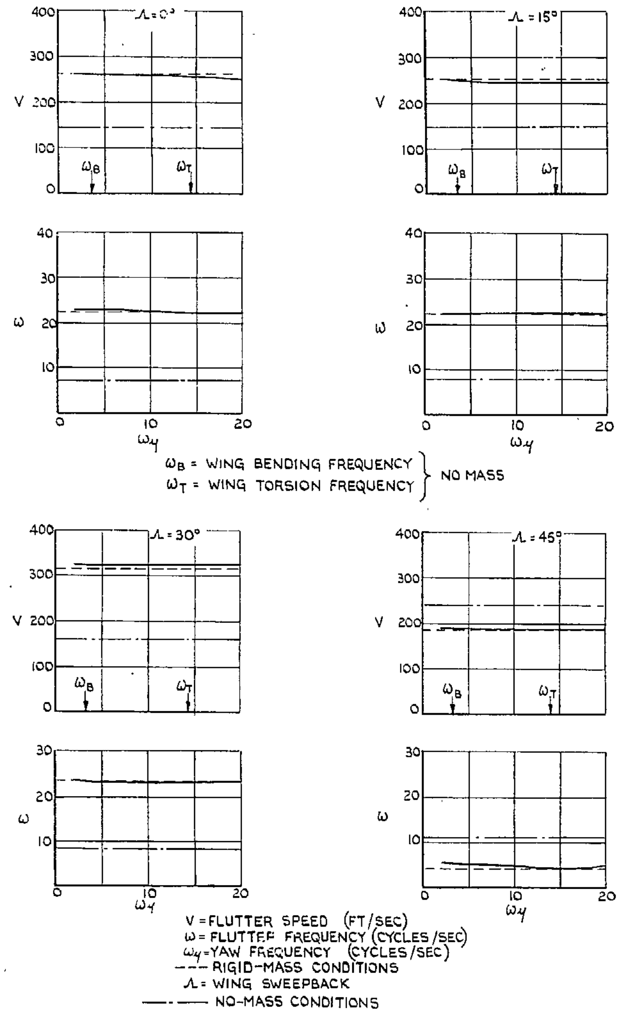

FIG. 10. Variation of yaw frequency—Case 2.

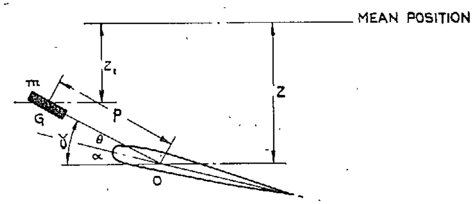

FIG. 11. Mass with pitch flexibility.

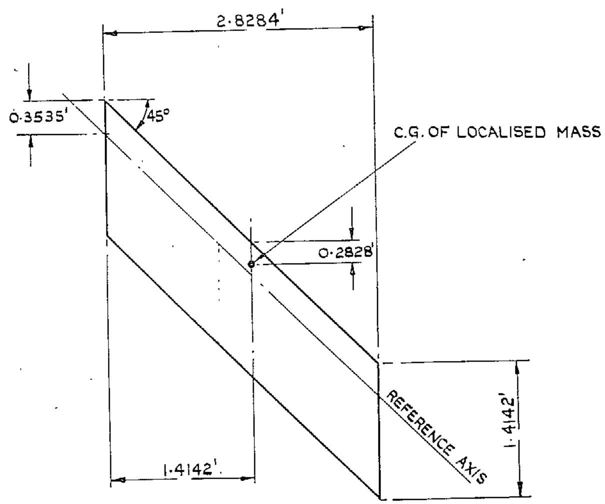

FIG. 12. Wing planform and mass arrangement for normal translation flexibility.

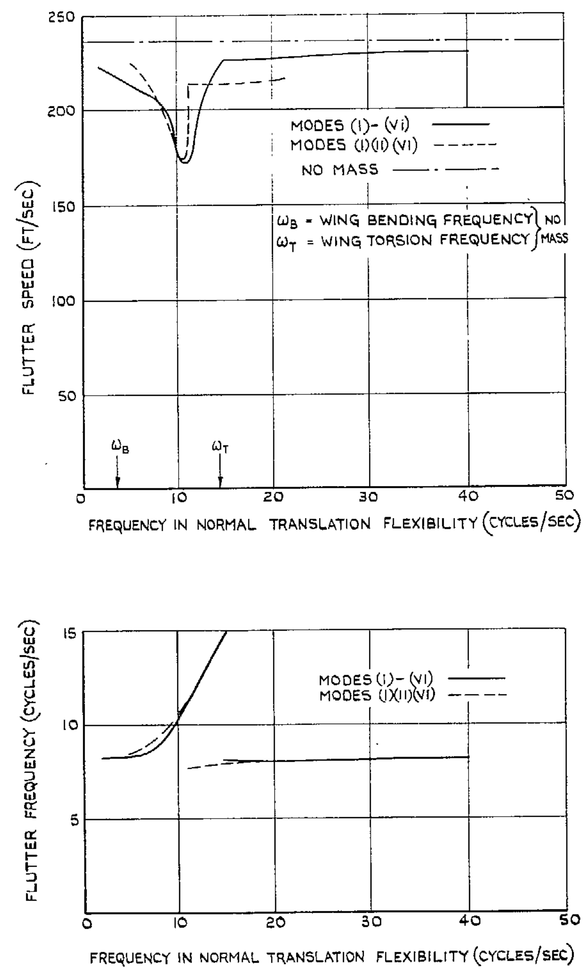

FIG. 13. Variation of normal translation frequency.

(87349) Wt. 64/1857 K.5 7/63 Hw.

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