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Wind-Tunnel Techniques for the Measurement of Oscillatory Derivatives†

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Summary.

This paper discusses the basic principles employed in techniques for the measurement of oscillatory derivatives in wind tunnels, and gives some account of the associated instrumentation. The suitability of the various techniques for different test conditions is also discussed, and brief reference is made to wind-tunnel effects on the measurements.

1. *Introduction.*

The problem of assessing the stability and flutter characteristics of aircraft, and more recently guided missiles, has been met to some extent by the use of theoretical estimates of the aerodynamic derivatives appropriate to the planforms and profiles involved, supported by a limited number of derivative measurements to give confidence in the validity of the theories used. In some cases where large differences exist between measured and theoretical derivatives, as with control surfaces for example, it has been possible to determine numerical factors for application to the theoretical derivatives. Continuing developments in high-speed flight have increased the need for experimental confirmation of theory, which is now required to deal with new shapes and much more complex flow conditions. In the transonic speed range in particular, theoretical estimates of the derivatives are difficult to obtain, even without attempting to estimate effects of profile thickness, shock waves and flow separation, and stability and flutter calculations can only be regarded as satisfactory when based on aerodynamic coefficients which have a measure of experimental support. The same may be said for cases involving large mean incidences or large amplitudes of oscillation.

Although the amount of experimental work on unsteady forces is still very small compared with that done on static forces, realisation of the need has led, during the past two decades, to considerable progress in the development of techniques for the measurement of aerodynamic derivatives. This progress has, of course, been aided considerably by the rapid developments in the science and art of electronics during the same period of time. It is the purpose of this chapter to discuss the basic principles employed in these techniques. The emphasis is on oscillatory measurements in wind tunnels rather than measurements on aircraft in flight, since this subject has been dealt with elsewhere[1].

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2. *The Basic Problem.*

The determination of the aerodynamic derivatives for any system which is in uniform motion relative to the atmosphere, or is subject to the action of a wind stream in a wind tunnel, resolves itself into the measurement of the aerodynamic reaction in each degree of freedom resulting from a displacement in that freedom to give the direct derivatives, and from displacements in each of the remaining degrees of freedom to give the indirect derivatives. Since dynamic derivatives are under consideration, it must be understood that displacement involves not only change in physical position, but also velocity and acceleration, and that corresponding derivatives exist. In the discussion which follows it is assumed that the displacements are sinusoidal oscillations, and it is then convenient to regard the aerodynamic reaction as a vector quantity shifted in phase by an angle ϵ relative to the displacement vector, as indicated in Fig. 1. The problem then becomes that, either of measuring the amplitude and phase of the reaction vector, or of determining the components of this vector in phase and in quadrature with the displacement vector. The latter procedure is often the more convenient.

Both stiffness and acceleration derivatives contribute to the in-phase component, but their effects are physically indistinguishable, and it is now normal practice to define the acceleration derivative in the oscillatory case as being equal to the still-air inertia, which in dynamic stability problems is usually included in the structural inertia. With most techniques for measuring derivatives the results are obtained from the difference between measurements made in a wind stream and in still air, in order to eliminate apparatus reactions, and thus the in-phase component of this difference corresponds to the stiffness derivative as defined above. Normally the acceleration derivative is not measured in a derivative test since a separate experiment is required.

The quadrature component of the reaction vector is proportional to a damping derivative multiplied by the frequency of oscillation; thus the difference obtained from the tests requires correction for the still-air damping on the model, which has been subtracted from the total damping due to the wind. Often this still-air damping is small and may be neglected.

In comparison with static tests the measurement of oscillatory derivatives requires in general the variation of two additional parameters, namely amplitude of oscillation and frequency parameter $\omega = 2\pi fc/V$. A third additional parameter which has been varied only very infrequently is rate of growth or decay of amplitude[2].

In the case of model tests in wind tunnels variation of ω for incompressible-flow conditions can be obtained by variation either of f or V for a given model. Since the latter procedure also varies the Reynolds number, it should be avoided unless there is reason to believe that effects of Reynolds number can be ignored. By using a sufficiently low value of V, full-scale values of ω can be obtained for frequencies well below the natural frequencies of the model and apparatus components, and thus errors due to model distortion and to large inertial reactions may be kept small. When measurements are made in high-speed wind tunnels to determine compressibility effects, V is fixed by the Mach number and cannot be altered to vary ω. Since V is now large, relatively high frequencies must be used to attain values of the frequency parameter approaching full scale. This leads to a number of difficulties which will be mentioned in relation to the various techniques discussed below.

3. *Classification of Techniques.*

The methods employed for the measurement of aerodynamic derivatives fall into two main groups depending on whether the aerodynamic reactions are deduced from the motion of the system resulting from some form of disturbance, or whether they are measured directly by means of force, moment or pressure pickups.* The first group may be further subdivided to distinguish between cases where the free motion is measured after removal of the disturbance and cases where the excited motion is observed during the application of the disturbance. Either or both of the two basic methods may be employed in any apparatus designed for the measurement of derivatives, but on the whole indirect derivatives are determined by the method of measurement of reaction, whilst both methods are suitable for the determination of the direct derivatives at low speeds. At high wind speeds coupled with high frequencies certain forms of the method of measurement of motion are appropriate.

The various techniques are listed below in the groups to which they belong, and they are discussed in this order:

Measurement of motion

Free motion:

(1) Decaying oscillations.

(2) Flutter oscillations.

Excited motion:

(3) Elastic excitation.

(4) Electrical excitation.

(5) Excitation with coupled freedoms.

Measurement of reaction

(1) Free oscillations.

(2) External rigid drive.

(3) Internal rigid drive.

(4) Electrically excited oscillations.

(5) Pressure plotting.

4. *General Remarks on Techniques.*

With the exception of pressure plotting, all the techniques listed above measure overall force or moment, and motion is normally limited to one or, in some cases, two degrees of freedom at a time. An important requirement in apparatus employing these techniques is structural rigidity. Small amounts of structural distortion under the action of the oscillatory aerodynamic loading can give rise to considerable error due to the inclusion in the measurements of coupled aerodynamic and structural reactions from other degrees of freedom[3]. In general the inertial reactions are the chief offenders, and in some cases they may be made negligible by designing to avoid structural coupling from degrees of freedom most likely to be concerned—e.g., by mass balancing. However, the normal procedure is to limit distortion by making the structure sufficiently stiff and by operating well below any resonance frequencies of the system. Where measurements at high speeds and high frequencies are concerned, it may be impossible to satisfy the frequency condition, and it is then sometimes possible to operate between resonance frequencies of the system, without large distortion effects arising. With overall-force methods, however, the model is normally required to behave as a rigid body and its own natural frequencies must lie well above the operating frequencies. Only in

* This classification is somewhat different from that given in Ref. 44, but is convenient for the present discussion.

special cases is it possible to work with a distorting model, and then only if the mode of distortion is unlikely to be influenced to any extent by the aerodynamic loading—e.g., the torsional mode of a two-dimensional model[4]—so that the structural reactions are practically the same both in the wind and in still air.

An arrangement frequently employed in derivative apparatus is the half-span technique, in which the centre-line of a complete finite-aspect-ratio wing becomes the root of a half-wing model and is situated close to a wall of the wind tunnel. Since the model can be attached to the apparatus by a short tongue passing through a hole in the tunnel wall, considerably greater overall rigidity can be attained than with a complete model, which must be supported by some form of linkage system or at high speeds by means of a sting. A further advantage is the larger size of model that can be employed, resulting in a higher Reynolds number and a lower frequency of oscillation for a given frequency parameter. A disadvantage is the presence of the boundary layer at the tunnel wall. The use of a reflection plate in this connection implies a longer tongue and thus some loss of rigidity.

When this technique is used in high-speed wind tunnels it is usually necessary to enclose parts of the oscillating system external to the working section in an airtight box sealed to the wall of the tunnel in order to prevent flow through the hole accommodating the tongue of the model resulting from a pressure difference between the air in the working section and the external atmosphere. A further undesirable effect is flow through the narrow gap between the root of the model and the tunnel wall, and it may be necessary to deal with this either by fixing a boundary-layer fence at the root of the model, or a circular plate flush with the tunnel wall. With the half-span method it is clear from consideration of the image in the tunnel wall that the flow over the half model is identical with that over a complete wing for motions of pitch or vertical translation, but for rolling oscillation of the model about the root chord the correspondence is with symmetrical flapping of the complete wing with a linear mode.

An important feature of derivative equipment, other than that employing 'pressure plotting' or 'internal rigid drive' techniques, is the apparatus damping, which is included in the measurements obtained with aerodynamic loading, and must be measured in still air to enable a correction to be made. The use of journal bearings, and to some extent ball bearings, may contribute heavy friction damping to the apparatus damping if large loads are being supported. Friction forces are objectionable both on account of magnitude and lack of constancy, and they are frequently avoided by using some form of spring bearing in which rotation about the virtual centre of an assembly of flat spring strips is employed. Two examples are illustrated in Fig. 2. The 3-spring assembly has the advantage of not being able to collapse under heavy lateral load, due to the buckling of strips. The rotation of these bearings is limited by stress considerations, especially under side load, when sharp bending of some of the spring strips may occur at their ends.

The essential difference between tests for compressible- and incompressible-flow conditions is the very much higher wind speed in the former which results in general in much heavier aerodynamic forces at Mach numbers ranging up to the lower supersonic values. For this reason a much stiffer model and supporting structure are required than in the low-speed counterpart in order to keep structural distortions within permissible limits. The mass and inertia of the oscillating system are thus increased, and since frequency of oscillation must also be increased to give corresponding values of the frequency parameter, inertial reactions become very large indeed. It thus becomes more difficult to separate out the aerodynamic reactions in overall-force methods, especially where this is attempted by means of a mechanical system producing forces equal and opposite to the inertial forces.

If springs are used to balance inertial reactions, high-speed conditions tend to give very sharply tuned systems in free-motion techniques, and difficulties may arise unless very fine control on frequency is provided, or unless a system is used which automatically oscillates at its natural frequency. Furthermore, the combination of high frequency and large inertia implies the use of springs with a very large elastic stiffness, and experience indicates that this produces heavy apparatus damping forces, probably at the end fittings, which tend to submerge the aerodynamic damping in the higher-frequency tests. Also the elastic stiffness becomes very large compared with the aerodynamic stiffness, and thus the change in natural frequency of the system due to wind loading becomes very small. Where this change is used to determine a stiffness derivative, conditions must be achieved which enable very accurate measurements of frequency to be made.

Finally, when reactions are measured with force pickups, the large inertia and high frequency require these to be very stiff in order that the measuring system should possess a good frequency response. This results in low sensitivity unless a very sensitive type of pickup is used.

5. *Discussion of Individual Techniques.*

In the discussion of the basic principles of measurement which follows, a rigid wing with degrees of freedom in pitch and vertical translation is assumed by way of illustration in most cases, but it will be clear that other configurations may be readily substituted.

The nose-up pitching moment M' about a specific axis and the lift L' are given by the expressions

$$M' = M_\theta \theta + M_{\dot\theta}\dot\theta + M_z z + M_{\dot z}\dot z, \tag{1}$$

$$L' = L_\theta \theta + L_{\dot\theta}\dot\theta + L_z z + L_{\dot z}\dot z, \tag{2}$$

where θ is the pitching displacement and z the downward displacement of the axis (*see* Fig. 3). The coefficients are the aerodynamic derivatives, non-dimensional forms of which are given in the list of symbols.

In designing apparatus to measure these derivatives, the motion is frequently limited to the pitching freedom for simplicity, and arrangements made to measure the direct derivatives M_θ, $M_{\dot\theta}$ and the indirect derivatives L_θ, $L_{\dot\theta}$. If the motion is a sinusoidal oscillation $\theta = \bar\theta e^{jpt}$, the in-phase components of moment and lift are $M_\theta\bar\theta$ and $L_\theta\bar\theta$, whilst the quadrature components are $pM_{\dot\theta}\bar\theta$ and $pL_{\dot\theta}\bar\theta$ respectively.

The derivatives relating to z-motion are deduced by repeating the measurements for a displaced axis position. Displacement of the axis a distance x' downstream introduces a z-displacement $-x'\theta$ at the old axis position, and the pitching moment and lift related to the new axis become

$$M'' = M'_\theta \theta + M'_{\dot\theta}\dot\theta, \tag{3}$$

$$L'' = L'_\theta \theta + L'_{\dot\theta}\dot\theta, \tag{4}$$

where

$$M'_\theta = M_\theta + x'(L_\theta - M_z) - x'^2 L_z, \tag{5}$$

$$M'_{\dot\theta} = M_{\dot\theta} + x'(L_{\dot\theta} - M_{\dot z}) - x'^2 L_{\dot z}, \tag{6}$$

$$L'_\theta = L_\theta - x'L_z, \tag{7}$$

$$L'_{\dot\theta} = L_{\dot\theta} - x'L_{\dot z}. \tag{8}$$

Equations (5) and (7) determine M_z and L_z and (6) and (8) give $M_{\dot z}$ and $L_{\dot z}$. The axes should be spaced well apart for maximum accuracy, but with spring-constrained systems the possibility of divergence with rearward axis positions must be considered.

An alternative procedure is to measure M' and L' for one axis position ($x' = 0$), and M' only for two others ($x' = x'_1$ and x'_2). This leads to two equations of type (5) and two of type (6) for the determination of the z-motion derivatives.

5.1. *Measurement of Motion.*

5.1.1. *Decaying oscillations.*—This free-motion technique has been in use for many years for the measurement of damping in a wide variety of connections, and in its application in the field of aerodynamics it is capable of determining direct damping and stiffness derivatives. In its basic form its merit is simplicity, since excitation is unnecessary, and instrumentation may be elementary.

The arrangement employed is illustrated diagrammatically in Fig. 4 in which a wing is free to rotate about a fixed axis A against an elastic constraint provided by a spring K. After being disturbed the free motion of such a system for small displacements when subjected to the action of a wind stream is governed by the equation

$$I\ddot{\theta} + \kappa\dot{\theta} + \sigma\theta = M_{\theta}\theta + M_{\dot{\theta}}\dot{\theta}\,, \tag{9}$$

where I is the total moment of inertia, κ the apparatus damping coefficient and σ the elastic stiffness for angular displacement about the axis.

The solution of this equation is of the form

$$\theta = \bar{\theta}e^{\lambda t} \tag{10}$$

where

$$\lambda = \mu + jp\,. \tag{11}$$

This is a growing or decaying oscillation (depending on the sign of μ) of frequency $f = p/2\pi$ and with a logarithmic decrement $x = -\mu/2f$ defined as $\log_e$ of the ratio of any maximum displacement to the displacement occurring one half cycle of oscillation later. A negative value of μ corresponds to an amplitude decreasing with time.

Substitution of (10) and (11) in (9) leads to the following expressions for the derivatives

$$M_{\theta} = \sigma - (p^2 + \mu^2)I\,, \tag{12}$$

$$M_{\dot{\theta}} = 2\mu I + \kappa\,. \tag{13}$$

In the simplest form of the technique values of p and μ are derived from an analysis of a record of the motion of the system (including a time scale), whilst σ is obtained either from a static deflection experiment, or from a still-air oscillation test in which p and μ are measured before and after the addition of a known amount of inertia δI. The value of σ is then given by

$$\sigma = \delta I \Bigg/ \left\{ \frac{1}{p_1^2 + \mu_1^2} - \frac{1}{p_0^2 + \mu_0^2} \right\}, \tag{14}$$

where the suffixes 0 and 1 refer to values before and after the addition of δI respectively. Normally μ is small compared with p in still air, and (14) becomes

$$\sigma = \frac{p_0^2 p_1^2 \delta I}{(p_0 - p_1)(p_0 + p_1)}\,. \tag{15}$$

In general I and κ may be written in the form

$$I = I_S + I_A + I_M\,, \tag{16}$$

$$\kappa = \frac{H}{f} + \kappa_A\,, \tag{17}$$

where I_S is the structural inertia and I_A, I_M are still-air inertias relating to apparatus and model respectively. The apparatus still-air damping is represented by κ_A, whilst H/f is the mechanical hysteresis damping (H constant) based on the assumption of constant energy dissipation per cycle independent of frequency. It is normal practice to determine values for I and κ from a wind-off, still-air test made fairly close in time to the wind-on test in order to minimize errors due to drift in the hysteresis damping. This gives from (12) and (13) assuming $M_\theta = M_{\dot\theta} = 0$

$$I_0 = \sigma/(p_0^2+\mu_0^2), \tag{18}$$

$$\kappa_0 = -2\mu_0\sigma/(p_0^2+\mu_0^2), \tag{19}$$

the suffix 0 denoting still-air conditions. These values differ from the required values I and κ, since κ includes κ_M the still-air damping on the model, and I_A, I_M, κ_A in (16) and (17) depend on frequency and air density. The mechanical hysteresis damping also depends on frequency. In going from still-air to wind-on conditions a change in frequency is normally expected, and in high-speed-tunnel tests also a change in density; in fact in this case the air density appropriate to the model will also differ from that for the rest of the apparatus on account of the different temperature in the wind stream. Normally with equipment designed for high-speed tests the changes in I_A and I_M are unimportant, whilst κ_A and κ_M are small compared with the mechanical hysteresis; thus in terms of the still-air measurements I and κ are given by

$$I = I_0, \tag{20}$$

$$\kappa = \frac{f_0}{f}\kappa_0. \tag{21}$$

In order to check the assumptions implied in (20) and (21), and to determine corrections where these are significant, it is necessary to make still-air tests, both with and without the model fitted, for a range of air densities and frequencies. Density may be varied by enclosing the system in an airtight container and varying the pressure, whilst the most satisfactory way of varying frequency is to make known changes in the inertia of the system without altering its geometry. This can be done by replacing masses on the system with others of the same dimensions but made with material of different density. Usually, however, it is sufficiently accurate to ignore the change in geometry due to the addition or removal of masses, since the aerodynamic reactions on these are in general very small. Frequently it is sufficient to determine upper limits to these corrections by omitting the tests with the model removed.

It is important that the structural coefficients containing still-air components should be measured for the same amplitude of oscillation as the wind-on tests. This avoids the possibility of errors arising when the still-air components form appreciable parts of the structural coefficients and are themselves subject to amplitude effects.

Substitution of (18) to (21) in (12) and (13) leads to the expressions

$$M_\theta = \frac{\sigma}{p_0^2+\mu_0^2}\{(p_0+p)(p_0-p)+(\mu_0+\mu)(\mu_0-\mu)\}, \tag{22}$$

$$M_{\dot\theta} = \frac{2\sigma}{p_0^2+\mu_0^2}\left(\mu-\frac{f_0}{f}\mu_0\right) \tag{23}$$

where the derivatives are given directly in terms of the measured values of σ, p, μ, f etc., and it is assumed that the corrections mentioned above are negligible. In general μ and μ_0 are small compared with p and p_0 (for decay to half amplitude per cycle $\mu \sim 0{\cdot}1p$), and thus M_θ is very nearly proportional to the frequency change due to aerodynamic loading, whilst $M_{\dot\theta}$ is proportional to the log. dec. in the wind, assuming μ_0 is small compared with μ.

The common methods employed in wind tunnels for disturbing the system from its position of equilibrium are given below:

(*a*) Application of a mechanical impulse, frequently by striking a spring-loaded plunger normally held clear of the system.

(*b*) Release from a displaced position by rapid withdrawal of a catch.

(*c*) Electrical excitation at resonance with sudden breaking of the exciting circuit.[45]

Methods (*b*) and (*c*) have the advantage of giving positive control over the initial amplitude of oscillation; whilst method (*c*) although involving much more complicated equipment, overcomes the difficulty of obtaining a sufficiently large initial deflection when a very high elastic stiffness is employed in order to attain high frequencies of oscillation.

A simple method of obtaining a record of the decaying motion is to photograph an illuminated point on the system directly with a moving-film camera. Unless the displacements are large, however, the accuracy of the measurements obtained from the film will be low. Greater accuracy is obtainable by means of a focused beam of light reflected on to moving film or sensitive paper from a mirror fixed to the oscillating system in such a way as to indicate angular displacement; this, however, requires a special type of camera. A more flexible technique is to employ some form of displacement pickup which, when fixed to the system, gives an output voltage proportional to displacement. This voltage may be fed into a d.c. cathode-ray oscilloscope and the movement of the spot on the screen recorded with a standard type of moving-film camera, or if the frequency range is suitable a recording galvanometer or pen recorder may be used. Where it is desired to reproduce the decaying transient in terms of voltage, the recording may be made directly on magnetic tape.

The analysis of photographic and pen recordings is normally made by measuring swings ϕ_n defined as displacements between successive stationary values on the record and plotting log ϕ_n against n the number of the swing counting from some datum point. The log. dec. x is proportional to the slope of the resulting curve and is given by

$$x = -\frac{d(\log_e \phi_n)}{dn} = -2\cdot 303\,\frac{d(\log_{10}\phi_n)}{dn}. \qquad (24)$$

If the curve is non-linear the variation of x with amplitude of oscillation may be determined by drawing tangents to the curve at a number of points and measuring their slopes.

It is always advisable to set the datum point a number of cycles after the commencement of the decaying transient in order to allow more heavily damped oscillations of different frequency, which may have been set up by the initial disturbance, to die away. Also in subsonic tests this allows time for the aerodynamic wake to become fully established.

Where lightly damped oscillations are present in unwanted degrees of freedom and they appear in the record of a positively damped signal, an effective way of removing them is to record the transient on magnetic tape and to play back the tape repeatedly in reverse in the form of a complete loop into a tuned filter.[5] Either by adjusting the tape speed or tuning the filter the latter may be made to respond to the required frequency. The output of the filter is then a growing oscillation, which is the required transient in reverse, followed by the response of the filter to the sudden cessation of the growing signal. A record of the output can be made and analysed in the normal way. The essential feature of this technique is the separation in time of the filtered signal and the transient response of the filter itself; these would be superimposed if the playback into the filter were not reversed.

An important consideration in relation to direct recording on magnetic tape is the variation in sensitivity of the tape coating from point to point. This can lead to considerable error in the amplitude of the output signal from the recorder amounting to as much as 50% locally (dropouts). The effect can be avoided by recording a frequency-modulated carrier rather than an analogue representation of the displacements; this, however, requires a high-quality tape recorder with a very constant tape speed (within 0·2%).

The decaying-oscillation method may be used for the measurement of positive or negative damping in both low-speed and high-speed wind tunnels, but the measurement of the stiffness derivative at high speeds and high frequencies may be difficult or impossible on account of the limited number of cycles available. This number may be insufficient for an accurate measurement of the very small frequency change due to wind loading, which results from the large ratio of elastic to aerodynamic stiffness.

A disadvantage of the decaying-oscillation technique in its simplest form is the time and labour involved in measuring and analysing the record of the motion. This has led in recent years to the development of devices designed to measure rate of decay without the need to take a record of the motion, or where a record is required, to facilitate its analysis. Several of these devices are discussed briefly below.

(i) *The Dampometer.*

This is an ingenious electronic instrument[6] in which the damped oscillation is represented by a rotating vector on the screen of a cathode-ray tube. The spot on the screen lies at the outer end of the vector, the length of which is equal to the distance of the spot from its undeflected position at the centre of the screen. The rate of decrease in the length of the radius vector is a measure of the damping.

In order to obtain the required motion of the spot a displacement pickup is used giving a voltage output $v = \bar{v}e^{\mu t}\cos pt$ proportional to the motion of the system. This is applied to the cathode-ray oscilloscope to give the X-deflection, whilst the same signal shifted in phase 90° in the RC network of Fig. 5 gives the Y-deflection proportional to $v = \bar{v}e^{\mu t}\sin pt$. The resulting motion of the spot is the logarithmic spiral illustrated in Fig. 6, the distance of the spot from the centre of the screen being proportional to $\bar{v}e^{\mu t}$, the ordinate of the envelope of the decaying oscillation.

The logarithmic decrement x is determined with the aid of a circular disc which covers the screen of the cathode-ray tube and contains a number of equally spaced radial slots all of the same length and situated at the same distance from the centre. Each time the spot passes a slot on its motion round the spiral a light pulse is picked up by a photocell and recorded by an electronic counter (damping counter). If n is the total count, which corresponds to a decrease in the radius vector from the outer radius of the slots r_1 to the inner radius r_2, and s is the number of slots, the logarithmic decrement is given by

$$x = \frac{s}{2n}\log_e\frac{r_1}{r_2}. \tag{25}$$

The damping counter may be set to open and close an electronic gate at counts n_1 and n_2 where these are under control; this allows a signal of frequency f_c from a valve oscillator to pass to a second counter giving a total count of n_c. The frequency of oscillation of the system is then given by

$$f = \frac{(n_2 - n_1)f_c}{sn_c}. \tag{26}$$

The condition for 90° phase shift in the network of Fig. 5 is that $RC = 1/p$ and x is small compared with π. Provision is made in the apparatus for adjusting the time-constant RC in terms of p. Values of x not small compared with π lead to distortion of the logarithmic spiral and consequent error in the count; this however is not of significance for values of x normally involved in oscillatory-derivative measurements. Errors due to the finite number of slots in the disc are minimised by taking the mean value of repeated measurements.

Effect of amplitude of oscillation on the logarithmic decrement may be examined with the Dampometer by using different degrees of amplification in the oscilloscope, with a disc employing a small radius difference $r_1 - r_2$. This applies the counts to small amplitude ranges at different points in the decaying transient.

The frequency range of the instrument is 0·5 to 500 c/s, but the method could be adapted for frequencies up to the order of 1 Mc/s.

It is obvious that the device can also deal with growing oscillations.

The 'Dampometer' technique is especially convenient where rapid comparison of the mean log. dec. between two given amplitude levels is required for a number of different aerodynamic conditions.

(ii) *Logarithmic circuit.*

Where the frequency is low enough for direct recording on paper by means of a pen recorder or its equivalent, a method which effects a considerable saving in time in the analysis is to feed the decaying signal from the displacement pickup into a logarithmic distorting circuit before recording[7]. This converts the exponential envelope of the oscillation, which is proportional to $e^{\mu t}$, into a linear envelope proportional to μt, and μ is then determined from a measurement of the slope of this envelope.

The basic logarithmic circuit is illustrated in Fig. 7. This is a conventional function generator in which the continuous function is replaced by a series of tangential straight lines the slopes of which are determined by resistances in series with a set of diodes arranged to conduct at different voltage levels. A convenient method of setting up and calibrating this circuit is to feed in a repeated exponential waveform and then to adjust the series resistances to give a linear output as observed on a cathode-ray oscilloscope. The exponential waveform may be generated by arranging for a relay, operated by the a.c. mains, to charge a condenser and discharge it through a resistance. The time-constant RC involved in the discharge gives the calibration constant, since $1/RC = \mu$.

The accuracy of this method clearly depends on the number of diodes employed in the function generator. With six diodes the accuracy is sufficient for many purposes.

Normally the signal fed into the distorting circuit is rectified to make it zero-based in order to utilize the full range of the logarithmic characteristic in determining the slope of the envelope.

(iii) *Integration method.*

This technique was originally designed for analysing magnetic-tape records of decaying oscillations*, but it can be used directly on the signal from the displacement pickup if parts of the analysing equipment are duplicated.

The basis of the method rests on the selection by means of a special counter of a batch of complete cycles from the decaying transient played back from the tape recorder. This batch of cycles is then

* N.P.L., U.K.

rectified and fed into an analogue integrator which presents the integral as a reading on a d.c. voltmeter. The process is repeated on a replay for a different batch, containing the same number of cycles, and from the two meter readings the average logarithmic decrement between the batches is determined. If the first and second meter readings are S_1 and S_2 respectively, and m is the number of cycles between the beginnings of the two batches, the log. dec. is given by

$$x = \left(\log_e \frac{S_1}{S_2}\right) \Big/ 2m. \tag{27}$$

The general arrangement of the apparatus is illustrated diagrammatically in Fig. 8. The signal from the displacement pickup is first amplified to enable the crystal diodes in the rectifier circuit to function efficiently, and the amplified signal is fed to the counter, which is a commercial type employing Dekatrons and coincidence circuits which give an output pulse at each of two preset counts within the range of the instrument. These pulses trigger thyratrons T_A, T_B which energise the windings A, B of two high-speed relays (operating time about 1 ms). The relay contacts control the transfer of the rectified signal to the integrator in such a way that integration commences at the first pulse and ceases at the second; and the phasing of the rectifier is so arranged that the relays operate during the half of the cycle which is cut off, and thus the integral is unaffected by the finite time of switching. The maximum frequency dealt with by this arrangement is about 250 c/s, which is adequate for most aerodynamic problems. Higher frequencies would require a faster counter and some form of electronic gate to control the input to the integrator without altering the d.c. level.

Standard operational d.c. amplifiers are used for amplification and integration of the signal. The rest of the circuit is straightforward, but care must be taken with insulation at certain critical points to avoid drift in the voltmeter readings due to leakage currents. For the same reason an integrating condenser with polystyrene dielectric is used.

The use of an integrating process tends to eliminate effects of noise signals in the recorded transient resulting from excitation of unwanted modes of vibration in the system under investigation; whilst in comparison with methods designed to measure the number of cycles between two amplitude levels, the technique has the advantage of giving weight to every cycle in the range covered if this is split into two adjoining parts containing an equal number of cycles for integration.

5.1.2. *Flutter oscillations.*—The requirement for this free-motion technique is a system with two or more degrees of freedom, for which a number of stable flutter conditions can be obtained equal to the number of degrees of freedom. Although in principle the method could be used for any number of degrees of freedom, as far as is known it has been applied only to two[8, 9], and undoubtedly would become complicated to operate for greater numbers.

By way of illustration the method is discussed in relation to the system shown in Fig. 9, in which the freedoms are pitching displacement about an axis A and vertical translation of the axis. The equations of motion for the system are

$$I\ddot{\theta} + \kappa_\theta \dot{\theta} + \sigma_\theta \theta + P\ddot{z} = M', \tag{28}$$

$$M\ddot{z} + \kappa_z \dot{z} + \sigma_z z + P\ddot{\theta} = Z' = -L', \tag{29}$$

where I, M, P are moment of inertia, mass and product of inertia respectively. The coefficients κ_θ, κ_z relate to structural damping, and σ_θ, σ_z are the elastic stiffnesses, whilst M' and L' are the aerodynamic moment and lift.

When a stable flutter condition is obtained, the motions may be written in the form

$$\left.\begin{aligned} \theta &= \bar{\theta} e^{jpt} \\ z &= \bar{z} e^{j(pt+\epsilon)} \end{aligned}\right\}, \tag{30}$$

where ϵ is the phase angle by which the plunging motion leads the pitching motion. Substitution in (28) and (29), together with the expressions for M' and L' in terms of the non-dimensional derivative coefficients given in the list of symbols, leads on equating real and imaginary parts to zero to the following four equations

$$\omega^2 P \cos\epsilon + \rho S c^2 (m_z \cos\epsilon - \omega m_{\dot{z}} \sin\epsilon) + (\omega^2 I - Y^2 \sigma_\theta + \rho S c^3 m_\theta) r = 0, \tag{31}$$

$$\omega^2 P \sin\epsilon + \rho S c^2 (m_z \sin\epsilon + \omega m_{\dot{z}} \cos\epsilon) - (Y\omega\kappa_\theta - \rho S c^3 \omega m_{\dot{\theta}}) r = 0, \tag{32}$$

$$(\omega^2 M - Y^2 \sigma_z) \cos\epsilon + Y\omega\kappa_z \sin\epsilon + \rho S c (l_z \cos\epsilon - \omega l_{\dot{z}} \sin\epsilon) + (\omega^2 P + \rho S c^2 l_\theta) r = 0, \tag{33}$$

$$(\omega^2 M - Y^2 \sigma_z) \sin\epsilon - Y\omega\kappa_z \cos\epsilon + \rho S c (l_z \sin\epsilon + \omega l_{\dot{z}} \cos\epsilon) + \rho S c^2 \omega l_{\dot{\theta}} r = 0, \tag{34}$$

where $Y = c/V$, $r = \bar{\theta}/\bar{z}$ the amplitude ratio, and ω is the frequency parameter. It is assumed that the indirect structural damping and stiffness are zero. The four equations contain the eight non-dimensional derivative coefficients m_θ, $m_{\dot{\theta}}$, m_z, $m_{\dot{z}}$ and l_z, $l_{\dot{z}}$, l_θ, $l_{\dot{\theta}}$, and in order to determine the values of these in terms of the structural coefficients and the measured flutter data Y, r, ω, ϵ, an additional set of equations is required. This is obtained by repeating the flutter test for a different structural condition such as a change in an elastic stiffness.

Since the two flutter conditions will in general involve different values of wind speed V and frequency parameter ω it must be assumed that the derivative coefficients are independent of both. This assumption may be reasonable at low speeds if the frequency parameter is not very small, provided the changes in V and ω are not large. At high speeds, especially in the transonic range, where compressibility effects become important, the derivatives may be very sensitive to changes in V and to some extent in ω, and the method then becomes unreliable.

Advantages of this technique are that no excitation is required and that complete sets of derivatives are measured. However, it is necessary that stable flutter conditions are achieved, and this may not always be easy in practice. The two sets of equations for determining the derivative coefficients should be well conditioned, and this requires in general a considerable change in amplitude ratio and phase angle between the two tests.

In order to preserve the simplicity of the method, amplitudes, phase angle and frequency have normally been determined from photographic records[8, 9] obtained in the manner described for the method of 'decaying oscillations', but with a double-beam oscilloscope where electromechanical pickups are employed to indicate the two displacements. The direct structural coefficients may also be determined as in the 'decaying oscillation' method, for each degree of freedom in turn, the other being suppressed, whilst P is given by the relation $P = M\bar{x}$, where $\bar{x}$ is the distance of the centre of gravity of the model aft of the axis of oscillation.

5.1.3. *Elastic excitation.*—Commonly known as the method of 'forcing through a spring', this technique can be used to measure the direct derivatives for steady amplitudes of oscillation, and has been applied successfully to low-speed tests[10, 11]. A diagrammatic representation of the system for pitching oscillations is given in Fig. 10, which resembles the arrangement for 'decaying oscillations' (Fig. 4) with the addition of a spring K_1 and mechanical exciter E for applying a sinusoidal moment to the model. This moment is given by

$$M_1 = ls_1 y, \tag{35}$$

where s_1 is the linear stiffness of spring K_1 and y is the downward displacement at the point of attachment of K_1 to the exciter. The equation of motion may be written

$$I\ddot{\theta} + \kappa\dot{\theta} + \sigma\theta = M_\theta\theta + M_{\dot{\theta}}\dot{\theta} + M_1, \tag{36}$$

using the same notation as in equation (9). The angular stiffness σ now relates to the combined effect of K and K_1.

If the exciter motion is sinusoidal, it may be expressed in the form

$$y = \bar{y}e^{jpt}. \tag{37}$$

Applied moment and motion of model may then be written

$$\left.\begin{aligned} M_1 &= \bar{M}_1 e^{jpt} \\ \theta &= \bar{\theta} e^{j(pt-\epsilon)} \end{aligned}\right\}, \tag{38}$$

where ϵ is the phase angle by which the motion of the model lags that of the exciter.

Substitution in (36) leads to

$$\left.\begin{aligned} M_\theta &= \sigma - p^2 I - \frac{\bar{M}_1}{\bar{\theta}}\cos\epsilon \\ M_{\dot{\theta}} &= \kappa - \frac{\bar{M}_1}{p\bar{\theta}}\sin\epsilon \end{aligned}\right\}, \tag{39}$$

where

$$\bar{M}_1 = ls_1\bar{y}. \tag{40}$$

Thus when the structural coefficients of the system are known, the derivatives are obtained in terms of angular frequency p, amplitude ratio $\bar{y}/\bar{\theta}$ and phase angle ϵ between the two motions. In the simplest form of this technique these quantities may be obtained from a photographic record of the motions [10]; but a more convenient method of measurement which has been employed[11] is to use displacement pickups designed to give voltages proportional to the motions. These voltages may be measured by a suitable voltmeter, possibly of the vacuum-tube type, and the phase angle is readily determined with the aid of a d.c. oscilloscope. For this purpose the voltages are fed to the x- and y-plates of the oscilloscope, one directly and the other *via* a calibrated, adjustable phase-shifting network as indicated in Fig. 11. When the phase shifter is adjusted to close up the loop on the oscilloscope screen into a straight line, its setting indicates the phase angle between the two pickup signals directly. Frequency may be determined from a measurement of the r.p.m. of the exciter drive, which can be obtained by means of a stroboscopic disc on the main shaft illuminated by a lamp flashing at a standard frequency.

In principle the structural coefficients could be determined by repeating the test in still air, first with the model attached to give a value for $\sigma - p^2 I$, and then with the model removed or replaced by a framework having negligible still-air damping to obtain a value for κ {Eqns. (39) with $M_\theta = M_{\dot{\theta}} = 0$}. Since these tests should strictly be made with the same amplitude of oscillation of the model as in the wind-on case, difficulties may arise unless a special type of mechanical exciter is used in which the amplitude can be varied continuously during the oscillation. It is normally more convenient to measure the structural coefficients by the methods described for 'decaying oscillations'.

A source of error which may be of importance in some cases arises from the presence of hysteresis in the spring K_1 and its end fixings. This implies that the moment M_1 applied to the model leads the exciter displacement y by a phase angle ϵ_1, and thus ϵ in (39) should be replaced by $\epsilon + \epsilon_1$. A value for ϵ_1 may be obtained from a decaying-oscillation test using spring K_1 with a mass chosen to give the correct frequency. If the test is made *in vacuo* the damping will be due to the hysteresis alone, and ϵ_1 is then given in terms of frequency f_1 and logarithmic decrement x_1 by the relation

$$\sin \epsilon_1 = \frac{2x_1}{\pi}. \tag{41}$$

Since the technique under discussion employs a spring-tuned mechanical system, some general considerations relating to the method may be derived from the basic response formulae for tuned systems. These may be written

$$\left(\frac{\bar{\theta}}{\bar{\theta}_a}\right)^2 = \frac{1 + 4Q^2}{1 + 4Q^2 + 4(1-\gamma^2)^2 Q^4}, \tag{42}$$

$$\tan \epsilon = \frac{2\gamma Q}{1 + 2(1-\gamma^2)Q^2}, \tag{43}$$

where

$$\gamma = \frac{p}{p_a}, \tag{44}$$

$$Q = \frac{p_a I}{\kappa - M_{\dot{\theta}}}, \tag{45}$$

and suffix a denotes values at amplitude resonance. It is assumed that $\kappa - M_{\dot{\theta}}$ and I are constant.

The important parameter is the Q-factor given by (45), since this determines the sharpness of tuning. Apparatus designed for low-speed tests will normally have a low Q-factor and the tuning will be fairly flat. A typical value is $Q = 2 \cdot 5$. On the other hand, designs for high-speed work, especially when reasonably high frequency parameters are required, lead to large Q-factors, and a value $Q = 2000$ is not impossible. Response curves derived from (42) and (43) for these two values of Q are given in Fig. 12. It is clear that excitation near resonance with the sharply tuned system would require a very stable mechanical oscillator, since very small changes in frequency produce large changes in amplitude and phase (20% in $\bar{\theta}$ and 20° in ϵ for a frequency change of $0 \cdot 01$% at $\gamma = 1 \cdot 0003$). A little way from resonance the phase angle has fallen to a small value ($\epsilon = 3°$ at $\gamma = 1 \cdot 005$) and accurate determination of the damping derivative from (39) becomes more difficult.

A special case of this technique is where the system is excited at the amplitude-resonance frequency p_a. The expression for the damping derivative then becomes

$$M_{\dot{\theta}} = \kappa - \frac{\bar{M}_1}{p_a \bar{\theta}_a} \frac{1}{\left(1 + \frac{1}{4Q^2}\right)^{1/2}}, \tag{46}$$

or if Q is not too small, approximately

$$M_{\dot{\theta}} = \kappa - \frac{\bar{M}_1}{p_a \bar{\theta}_a}. \tag{47}$$

This effects a considerable simplification in the technique of measurement, since there is no longer any need to measure a phase angle. However, the stiffness derivative is now given by

$$M_\theta = \sigma - p_a^2 I \left(1 + \frac{1}{2Q^2}\right) \tag{48}$$

the expression on the right being approximately proportional to the change in resonance frequency due to the aerodynamic loading, since $\sigma \approx p_0{}^2 I$. In general this cannot be measured with sufficient accuracy at low speeds on account of the relatively flat tuning associated with systems designed for such tests. An alternative procedure is to excite on phase resonance, but this requires some device to indicate when the phase angle ϵ equals 90°. The derivatives are then given exactly by

$$M_\theta = \sigma - p_p{}^2 I, \tag{49}$$

$$M_{\dot\theta} = \kappa - \frac{\bar{M}_1}{p_p \bar{\theta}_p}, \tag{50}$$

and since the phase angle changes fairly rapidly with frequency for values of Q as low as 2·5, the resonance frequency p_p can be determined with considerable accuracy, and thus a reliable value obtained for the frequency change implicit in (49) corresponding to (48).

If the aerodynamic damping being measured by this method is negative and numerically greater than the apparatus damping, the system becomes unstable. It is then necessary to add damping to the system to make the net damping positive. A convenient method is to use some form of electrical eddy-current damper in which the damping may be varied continuously by adjustment of the magnetic field. The damping is then set to be just positive in the wind, and the aerodynamic damping is derived from the difference between measurements in still air and in the wind with the same eddy-current damping in each case.

5.1.4. *Electrical excitation.*—In its most general form this method[12] is similar to the technique of elastic excitation, but with the mechanical exciter and spring K_1 shown in Fig. 10 replaced by an electrical vibrator D as shown diagrammatically in Fig. 13. The equation of motion is identical with (36), but the exciting moment M_1 is now given by

$$M_1 = lki, \tag{51}$$

where i is the current in the moving coil of the vibrator and k is the force on the coil for unit current. In the normal type of vibrator the coil is suspended by spiders of low stiffness in the radial field of an annular gap in a magnetic circuit. The magnetisation is sometimes permanent and sometimes produced by an energising winding, in which case the constant k will vary to some extent with energising current. It is important that the moving-coil former should be of low- or non-conducting material, otherwise heavy eddy-current damping may swamp the aerodynamic damping being measured.

The current i is supplied by a power amplifier with an input signal from a valve oscillator, and a voltage v_i proportional to i, and thus to M_1, is obtained from a fixed resistance in series with the coil circuit. A voltage v_θ proportional to the displacement θ is given by a displacement pickup fixed to the oscillating system. Measurement of the two voltages and the phase angle by which v_θ lags behind v_i may be made in exactly the same way as described for elastic excitation, and the derivatives are then given by the same equations (39), where

$$\left.\begin{aligned} \bar{\theta} &= k_\theta \bar{v}_\theta \\ \bar{M}_1 &= k_i \bar{v}_i \end{aligned}\right\}, \tag{52}$$

k_θ and k_i being the pickup and vibrator calibration constants respectively, the bar indicating amplitude.

Electrical excitation possesses an advantage over elastic excitation with a fixed eccentric since the excitation, and thus the amplitude of response, can be varied continuously by means of the output control of the valve oscillator. This enables the structural coefficients, which may depend on amplitude, to be determined from an excitation test in still air, as already indicated in Section 5.1.3. A further advantage of electrical excitation is the frequency stability and fineness of frequency control attainable with valve oscillators, which enables satisfactory measurements to be made at higher values of the Q-factor, associated with high-speed conditions, than can be attained with an elastically excited system.

(i) *Electrical excitation at phase resonance.*

Although the general form of the method as described above will give accurate values of the damping and stiffness derivatives, the special case of excitation on phase resonance can be arranged in a particularly convenient form[13] for making the measurements, as indicated in Fig. 14. The resistance R in series with the vibrator coil is shunted by a potentiometer which supplies an adjustable voltage to the Y-amplifier of a d.c. oscilloscope, whilst a velocity pickup on the vibrating system, usually consisting of a coil in a magnetic field, supplies a voltage to the X-amplifier. The oscillator frequency is first adjusted to close up the loop appearing on the oscilloscope screen into a straight line, which brings the exciting current i into quadrature with the displacement and thus sets the system on phase resonance. If the potentiometer is then set to give a null reading on the oscilloscope, the following relation holds

$$biR = k_v\dot{\theta}, \tag{53}$$

where k_v is the velocity-pickup constant and b is the potentiometer reading giving the ratio of output to input voltage. It follows from (51), (38) and (50) that

$$\kappa - M_{\dot{\theta}} = \frac{\overline{M}_1}{p_p\bar{\theta}_p} = \frac{C}{b}, \tag{54}$$

where C is a constant of the apparatus. Thus the damping coefficient is proportional to the reciprocal of the potentiometer reading.

In the more general arrangement of excitation on phase resonance, the damping may be determined either from a measurement of the exciting current[14] or of the electrical power[14, 15] in the vibrator coil. In the latter case, however, certain corrections are required as indicated in the following section. The stiffness derivative results from a measurement of the change in resonance frequency due to the wind loading[15], and is discussed further below.

As with 'elastic excitation' the electrical excitation system becomes unstable when the aerodynamic damping is sufficiently negative. A simple method of applying eddy-current damping in this case is to shunt the oscillating coil of the electrical vibrator by a variable resistance. This effectively converts the vibrator into a combined eddy-current damper and exciter, and by adjusting the resistance a stable system is obtained which may be excited at the required amplitude and phase by the driving amplifier. The exciting current i used in the previous analysis is the current in the oscillating coil and not the total current from the amplifier when a shunt resistance is used. For a given amplitude and frequency the current i is independent of the value of the shunting resistance, provided this is low enough to give stability with no excitation.

(ii) *Electrical self-excitation.*

A special case of electrical excitation on phase resonance, which has certain advantages, arises when the input signal to the vibrator amplifier is derived from a displacement pickup on the oscillating system instead of from a valve oscillator[17, 18, 19]. This leads to the electromechanical feedback system illustrated diagrammatically in Fig. 15. If the phase shifter following the pickup is adjusted to set the exciting current i in quadrature with the displacement, and the amplitude stabilizer is omitted, the exciting moment for a steady oscillation is equivalent to a damping moment which exactly balances the aerodynamic and apparatus damping. This equality is expressed by the relation

$$(\kappa - M_{\dot{\theta}})\dot{\theta} = m_1 \theta, \tag{55}$$

where, referring to Fig. 13

$$m_1 \theta = M_1 = lki, \tag{56}$$

and hence

$$p_p(\kappa - M_{\dot{\theta}}) = m_1. \tag{57}$$

The damping derivative is thus obtained in terms of resonance frequency, amplitude of oscillation and exciting current, since

$$m_1 = \frac{lk\bar{i}}{\bar{\theta}_p} \tag{58}$$

where $\bar{i}$ is the amplitude of the current. When vibration generators with energising windings are used, leading to uncertainty in k, an alternative procedure is to measure the electrical driving power in the vibrating coil with a wattmeter. This measurement, however, requires correction for the power losses due to the ohmic resistance in the coil circuit, to eddy currents in the yoke and to magnetic-hysteresis effects. These losses may be determined from a single power measurement with the oscillating system clamped and a current flowing in the coil of the same magnitude and frequency as when the system was oscillating. If W is the corrected power in watts, the damping is then given by

$$W = 26{\cdot}77 f_p^2 \bar{\theta}_p^2 (\kappa - M_{\dot{\theta}}), \tag{59}$$

where $\bar{\theta}_p$ is in radians.

The apparatus damping coefficient κ may be determined in electrical phase-resonance tests by repeating the measurements in still air for the same amplitude $\bar{\theta}_p$; but identical considerations relating to the effects of the change in frequency, presence of the model and, in the case of high-speed tests, change in air density apply as already discussed for the method of 'decaying oscillations'.

The stiffness derivative M_θ is given by equation (49), or since $\sigma = p_0^2 I$, where p_0 is the still-air phase-resonance frequency,

$$M_\theta = \sigma (f_p + f_0)(f_p - f_0)/f_0^2, \tag{60}$$

where, again, the effect of change in air density on f_0 may need consideration.

A special arrangement of the 'self-excitation' technique is illustrated in Fig. 16. Two spindles A and B are supported by bearings and are free to oscillate about a common axis. The spindles are connected by a torsion bar, and the model is attached to B, whilst the excitation is applied to A in such a way as to excite a free-free oscillation of the system with a node somewhere along the length of the torsion bar. The advantage of this arrangement is that reactions due to twist of the torsion bar are not transmitted to the structure supporting the system, and when the torsion bar is very stiff, as in high-frequency tests, this reduces the apparatus damping. If the torsion bar is clamped directly to a composite supporting structure, very heavy damping may result from distortion of the structure. By arranging for the node to be fairly close to A the acceleration of the vibrator coils may be kept within acceptable limits, but it may be noted that the smaller the movement of these coils, the greater the electrical losses, due to the increased current requirement.

A disadvantage of the arrangement is that the measurement of the stiffness derivative is complicated by shift of the node in going from still-air to wind-on conditions. In this case

$$-M_\theta = c' \frac{1 - a'\Delta}{1 - b'\Delta} \Delta, \tag{61}$$

where $\Delta = p_p^2 - p_0^2$. The coefficients a', b', c' may be taken as constants[33], and they may be determined by measuring Δ for three different values of M_θ simulated by adding known inertias to B.

An essential requirement of the self-excitation method is some form of stabilizer which automatically compensates for any disturbance to the balance of damping forces expressed in equation (55), such as small changes in wind speed or in the loop gain of the system. The result of such changes is a growing or decaying amplitude of oscillation, and to stabilize this some arrangement is required which automatically alters the exciting current to oppose any change in amplitude.

One method, shown in Fig. 17, is to use a resistance bridge with a thermistor in one arm, the input to the bridge being derived from the displacement pickup, and the output feeding the driving amplifier[19]. The bridge is initially set up so that a change in input produces a change of opposite sign in the output as a result of the alteration in resistance of the thermistor due to its change in temperature when the a.c. current alters.

A method which is very rapid in action[17, 18] and avoids the thermal lag associated with thermistors is illustrated in Fig. 18. The signal from the displacement pickup is first amplified and then fed to a Schmitt trigger circuit which produces a constant-amplitude square wave of the same frequency and phase as the input. The fundamental component of this square wave may be regarded as a reference signal with which the pickup signal is compared in a differencing circuit, where a controllable fraction of the input signal is subtracted from the square wave. Harmonic components of the difference signal are removed in a filter circuit, and the remaining fundamental is then used to drive the vibrator amplifier. As with the thermistor circuit, a change in input results in a change of opposite sign in the output, which tends to hold the amplitude of oscillation at a constant value. The tightness of control can be varied by adjusting the fraction of the pickup signal subtracted from the square wave.

A further method for stabilisation is the use of the conventional A.G.C. technique in which the pickup signal is amplified, rectified and smoothed to provide a d.c. potential for biasing back a variable-gain valve in the driving amplifier. Compared with the previous method, however, the control is sluggish. This results from the nature of the filtering employed, which is required to remove all the alternating components and thus involves a much larger time constant than the case where only the harmonics are removed. The A.G.C. method is prone to give rise to relaxation oscillations of the combined electromechanical system unless arrangements are made to balance out automatically signals from the variable-gain valve resulting from changes in its mean anode current.

The main advantage of an electrical self-excitation method arises when dealing with the very sharply tuned systems associated with tests at high speed and high frequency. Since the system automatically oscillates on resonance, sharpness of tuning does not create any difficulty, and the critical adjustment of frequency required when exciting with a valve oscillator is avoided. The adjustment of the phase of the exciting current to bring it into quadrature with the displacement is not critical with the self-excitation method.

In dealing with negative aerodynamic damping, a shunting resistance on the exciter coil may be used as before, but an alternative method is to reverse the connections to the oscillating coil of the vibrator. This reverses the sign of the excitation, and the driving amplifier acts as a damper, its output valves absorbing power. The stabilizer still maintains a constant amplitude. However, care must now be taken that the permissible anode dissipation in the amplifier output valves is not exceeded and that the current waveform is not distorted by the incorrect loading of the valves.

5.1.5. *Excitation with coupled freedoms.*—In principle it should be possible to determine the derivatives for a system with several degrees of freedom by applying excitation to one of them and making measurements of the response near or at each of the resonance frequencies. As in the method of 'flutter oscillations', it must be assumed that the derivatives do not vary with frequency parameter, but in this case the further assumption of independence of V does not arise.

The application of the method to a system with two degrees of freedom is of importance in connection with sting-mounted oscillating models, since although the model may have only one degree of freedom relative to the sting, the latter, due to its flexibility, could introduce an additional freedom which may be difficult to suppress. If pitching and plunging motions are considered, with excitation in the pitching freedom, equations (28) and (29) apply if M' includes an exciting moment M_1, and indirect structural damping and stiffness are zero as before. If the applied moment and the motions are

$$\left.\begin{aligned} M_1 &= \bar{M}_1 e^{jpt} \\ \theta &= \bar{\theta} e^{j(pt-\epsilon_\theta)} \\ z &= \bar{z} e^{j(pt-\epsilon_z)} \end{aligned}\right\}, \tag{62}$$

(28) and (29) lead to the following four equations containing the eight derivatives

$$r(\sigma_\theta - M_\theta - p^2 I) - (M_z + p^2 P)\cos\epsilon_1 + pM_{\dot{z}}\sin\epsilon_1 = rk_1\cos\epsilon_\theta\,, \tag{63}$$

$$rp(\kappa_\theta - M_{\dot{\theta}}) - (M_z + p^2 P)\sin\epsilon_1 - pM_{\dot{z}}\cos\epsilon_1 = rk_1\sin\epsilon_\theta\,, \tag{64}$$

$$r(L_\theta - p^2 P) + (\sigma_z + L_z - p^2 M)\cos\epsilon_1 - p(\kappa_z + L_{\dot{z}})\sin\epsilon_1 = 0\,, \tag{65}$$

$$rpL_{\dot{\theta}} + (\sigma_z + L_z - p^2 M)\sin\epsilon_1 + p(\kappa_z + L_{\dot{z}})\cos\epsilon_1 = 0\,, \tag{66}$$

where $r = \bar{\theta}/\bar{z}$ the amplitude ratio, $k_1 = \bar{M}_1/\bar{\theta}$, ϵ_θ is the phase lag of the pitching motion behind the excitation moment, and $\epsilon_1 = \epsilon_\theta - \epsilon_z$ the phase lag of plunging behind pitching motion. If p, r, k_1, ϵ_1, ϵ_θ are determined near or on the two resonance frequencies, two sets of four equations are obtained, which are sufficient to determine the eight derivatives. The structural coefficients can be measured in the manner indicated for the method of 'flutter oscillations'. It is of course essential with this method that the conditions of the test should give rise to well-conditioned sets of equations.

Excitation with coupled freedoms has found special application for the determination of the indirect derivatives relating to motion in the excited freedom, where the direct derivatives for the coupled freedom have been measured in some other way[9]. This is illustrated by equations (65) and (66). If L_z and $L_{\dot{z}}$ are known, then L_θ and $L_{\dot{\theta}}$ are determined from measurements of amplitude ratio r and phase angle ϵ_1 between the motions, for one frequency p. A knowledge of the excitation is not required, and thus it may take any convenient form, such as excitation by a rigid linkage from a mechanical vibrator. A very simple expression is obtained for the indirect damping if the two motions can be brought into phase ($\epsilon_1 = 0$) by adjustment of the frequency. Then from equation (66)

$$rL_{\dot{\theta}} = -(\kappa_z + L_{\dot{z}})\,. \tag{67}$$

The general considerations relating to low-speed and high-speed tests, elastic and electrical excitation already discussed for single-degree-of-freedom systems may be taken to apply also to the general coupled freedom case. The latter must of course be free from flutter with no excitation. Electrical self-excitation has not yet been developed for derivative systems with more than one degree of freedom.

5.2. *Measurement of Reaction.*

With the exception of pressure plotting, the methods discussed in this section are characterized by the use of force pickups, either supporting the oscillating system or incorporated in the linkage to the exciter. Such pickups may be regarded as stiff springs which deflect when a force is applied, the resulting small displacement being converted into an observable effect, usually by electrical means. When oscillatory forces are measured, these displacements have an undesirable effect, since the resulting small movements of the system introduce inertial reactions which are included with the measured forces. In order to keep such errors small, the resonance frequencies of the system supported by the pickup stiffnesses must be well above the excitation frequency. For example, if a system of mass M is supported by a pickup of stiffness σ and the structural damping is κ, the equation of motion when a force F is applied is of the form

$$M\ddot{z} + \kappa\dot{z} + \sigma z = F. \tag{68}$$

If the force is given by

$$F = \bar{F}e^{jpt}, \tag{69}$$

the steady-state solution gives

$$\sigma\bar{z} = \bar{F}_i = \bar{F}\Big/\left(1 - \frac{p^2}{p_0^2} + j\frac{\kappa}{\sigma}\right), \tag{70}$$

or since κ/σ is usually negligible

$$\bar{F}_i = \bar{F}\Big/\left(1 - \frac{p^2}{p_0^2}\right), \tag{71}$$

where $\bar{F}_i$ is the indicated force. If for this case the excitation frequency p is one-third of the resonance frequency p_0 the response of the measuring system is $12\frac{1}{2}\%$ high compared with a static calibration.

Normally with systems employing force pickups it is desirable to make a dynamic calibration, and this can be carried out by fixing a known mass to the oscillating system, preferably to the model, and measuring its motion directly, for example, with the aid of a travelling microscope and stroboscopic lighting. From this information the applied inertial reaction can be calculated and compared with the value indicated by the measuring system.

Various methods exist for measuring the amplitudes and phase angles relating to pickup outputs, and these are discussed in a later section.

5.2.1. *Free oscillations.*—The method of decaying oscillations described in Section 5.1.1 is used for measuring direct derivatives, but it has been suggested[20] that the indirect derivatives could be determined at the same time by introducing a force pickup at the axis of oscillation A in Fig. 4 to record the lift force. If the motion is given by

$$\theta = \bar{\theta}e^{\mu t}\sin pt, \tag{72}$$

corresponding to equations (10) and (11), the lift may be written

$$L' = \bar{L}'e^{\mu t}\sin(pt+\epsilon), \tag{73}$$

where ϵ is the phase angle by which the lift leads the motion. It follows then from equation (2) that

$$\bar{L}' \cos \epsilon = \bar{\theta}(L_\theta + \mu L_{\dot{\theta}}), \tag{74}$$

$$\bar{L}' \sin \epsilon = \bar{\theta} p L_{\dot{\theta}}, \tag{75}$$

where

$$\frac{\bar{L}'}{\bar{\theta}} = \frac{L_1}{\theta_1} e^{\mu\epsilon/p}, \tag{76}$$

L_1 and θ_1 being peak values separated by the phase angle ϵ. Equations (74) to (76) are sufficient to determine the indirect derivatives L_θ and $L_{\dot{\theta}}$.

In general the reaction recorded by the pickup will include inertial, elastic and apparatus damping forces in addition to the lift. Elastic forces may be made zero if spring K in Fig. 4 is in the form of a torsion spring acting at A, and inertial forces may be removed by mass-balancing the system about A. Apparatus damping and still-air forces may be determined in a similar manner to that described for 'decaying oscillations'.

Where tests for more than one axis position are required in order to determine, in this case, the plunging-motion derivatives a convenient arrangement[19] is shown in Fig. 19 for reducing structural reactions for all axis positions with a single mass-balance condition. The system is mass-balanced about a point B at which two independent springs are attached, one of stiffness σ_z resisting vertical translation, and the other of stiffness σ_θ resisting rotation. The structural reaction F_s taken by the pickup at the axis A is given by the equation

$$I\ddot{\theta} + \kappa_\theta \dot{\theta} + \sigma_\theta \theta = -lF_s, \tag{77}$$

which leads to

$$\{(\mu^2 - p^2)I + \mu\kappa_\theta + \sigma_\theta\}\bar{\theta} = -l\bar{F}_s \cos \epsilon_s, \tag{78}$$

$$(2\mu p I + p\kappa_\theta)\bar{\theta} = -l\bar{F}_s \sin \epsilon_s, \tag{79}$$

where

$$F_s = \bar{F}_s e^{\mu t} \sin(pt + \epsilon_s). \tag{80}$$

If in still air μ is negligible, the in-phase component of F_s becomes zero for the condition

$$p_0^2 I = \sigma_\theta, \tag{81}$$

and there remains only a small quadrature component given by (79). Since p_0 is given by

$$p_0^2(I + Ml^2) = \sigma_\theta + l^2\sigma_z, \tag{82}$$

where M is the mass of the system, condition (81) implies also that

$$p_0^2 M = \sigma_z, \tag{83}$$

hence the still-air, uncoupled frequencies of the system in vertical translation and pitch about B must be made equal, and are then equal to the frequency of oscillation about A. This condition is independent of l, and thus balance between inertial and elastic forces is maintained whatever the position of the axis A.

In the wind the frequency of oscillation differs in general from that in still air, and thus the balance of forces is disturbed. The components of the structural reaction are then given by

$$\{(\mu^2 - \delta(p^2))I + \mu\kappa_\theta\}\bar{\theta} = -l\bar{F}_s \cos \epsilon_s, \tag{84}$$

$$(2\mu p I + p\kappa_\theta)\bar{\theta} = -l\bar{F}_s \sin \epsilon_s, \tag{85}$$

where

$$\delta(p^2) = p^2 - p_0^2. \tag{86}$$

Although the inertial and elastic forces are not completely balanced out, they are considerably reduced.

5.2.2. *External rigid drive.*—The basic arrangement for this method[21, 22, 23, 24], frequently referred to as 'inexorable forcing', is illustrated in Fig. 20. The model pivots about an axis at A and is driven at B by a rigid link connected to a mechanical exciter E. Vertical reaction F_2 at A is measured by a pickup P_2, and the exciting reaction F_1 by a pickup P_1 included in the forcing link.

The equation of motion about A is

$$I\ddot{\theta} + \kappa\dot{\theta} = lF_1 + M', \tag{87}$$

and the balance of vertical forces is given by

$$M\bar{x}\ddot{\theta} = F_1 + F_2 - L', \tag{88}$$

using the notation of Section 5.1.2.

If

$$\left.\begin{aligned} \theta &= \bar{\theta}e^{jpt} \\ F_1 &= \bar{F}_1 e^{j(pt+\epsilon_1)} \\ F_2 &= \bar{F}_2 e^{j(pt+\epsilon_2)} \end{aligned}\right\}, \tag{89}$$

where ϵ_1 and ϵ_2 are the phase angles by which F_1 and F_2 lead the motion, (87) and (88) give the following relations for the derivatives

$$(p^2 I + M_\theta)\bar{\theta} = -l\bar{F}_1 \cos\epsilon_1, \tag{90}$$

$$(\kappa - M_{\dot{\theta}})p\bar{\theta} = l\bar{F}_1 \sin\epsilon_1, \tag{91}$$

$$(-p^2 M\bar{x} + L_\theta)\bar{\theta} = \bar{F}_1\cos\epsilon_1 + \bar{F}_2\cos\epsilon_2, \tag{92}$$

$$L_{\dot{\theta}}p\bar{\theta} = \bar{F}_1\sin\epsilon_1 + \bar{F}_2\sin\epsilon_2. \tag{93}$$

The structural terms in these relations can readily be determined by repeating the measurements in still air at the same density and with the same frequency and amplitude of oscillation as in the wind. Elastic stiffnesses arising from spring bearings are not included in the above equations, as also indirect structural damping, but these would clearly be determined by the still-air test. If still-air damping forces on the model are not negligible they can be determined from an additional still-air test with the model removed, as described for decaying oscillations, and equations (91) and (93).

Inertial reactions are frequently large, especially when the equipment is designed for tests at high speeds and high frequencies, and the aerodynamic forces are then obtained as small differences between large vector quantities having small phase angles between them. This leads to low accuracy, and for this reason it is desirable to balance out the inertial forces automatically. One method of achieving this is to mass-balance about B and use the system of springs described for the previous method (Section 5.2.1), and to oscillate the system in the wind at the still-air phase-resonance frequency for the uncoupled freedoms, indicated by zero in-phase reaction in still air. Equations (90) and (92) for the stiffness derivatives then become

$$M_\theta\bar{\theta} = -l\bar{F}_1\cos\epsilon_1, \tag{94}$$

$$L_\theta\bar{\theta} = \bar{F}_1\cos\epsilon_1 + \bar{F}_2\cos\epsilon_2. \tag{95}$$

In practice difficulty may be experienced in tuning the springs to give relations (94) and (95) simultaneously, and it may then become necessary to measure the direct and indirect derivatives at slightly different frequencies. The order of accuracy to which the frequency must be maintained is usually very high. If for example the inertial reaction is 100 times the aerodynamic stiffness reaction, which is by no means an abnormal case, a departure from resonance of $\frac{1}{2}\%$ in the wind-on condition would lead to an error of 100% in the measurement of the stiffness derivative. Normally the frequency must be maintained well within 0·01%, and in some cases to a much higher order of accuracy.

A difficulty which may arise with a spring-tuned rigidly forced system is overloading of the force pickups by the spring reactions at low frequencies before the mechanical exciter has run up to resonance. This may be overcome by introducing a spring in the forcing link between pickup P_1 and the exciter[20, 22, 23], which also acts as a filter for exciter harmonics. The system is now no longer rigidly forced and must include a displacement pickup to which the measured reactions are referred instead of to the exciter motion. Also a swash-plate type of exciter is required to enable amplitude of oscillation to be adjusted whilst the system is running.

The considerations relating to change of axis position A mentioned in connection with the free-oscillation method (Section 5.2.1) with spring-balanced inertial reactions apply also to externally forced systems with corresponding spring tuning.

An alternative method[26] for balancing out inertial reactions is shown in Fig. 21. The model is oscillated by a linkage system including the horizontal arm CG which pivots about C and is driven by the mechanical exciter at D. The motion is transmitted to the model system *via* the link BG, and the pickup P_1, previously included in the link to B, is now stationary and measures the reaction at C. Inertial reactions at A are first balanced by a mass M_2 fixed between A and B, and satisfying the relation

$$M_2 l_1 l_2 = M(\bar{k}^2 - l\bar{x}), \tag{96}$$

where M, $\bar{k}$ and $\bar{x}$ are mass, radius of gyration and distance of the centre of gravity aft of A for the model system. The equivalent mass M_0 at B is then given by

$$M_0 l = M_2 l_1 + M\bar{x}, \tag{97}$$

and inertial reactions at C may be balanced by a mass M_1 between C and D, where

$$M_1 l_3 l_4 = M_0 l_0 l_5 . \tag{98}$$

This assumes that the mass of the linkage is negligible. If this is not the case its effect can be allowed for by a small adjustment of M_1. Aerodynamic reactions measured at C by pickup P_1 are transformed into reactions at B by the factor $(l_3 + l_4)/l_5$. Reactions at P_1 and P_2 due to spring pivots or springs tensioning BG, which may be long and slender, are normally small, but may be balanced by vertical springs of suitable stiffness attached at points on AB and CD.

The chief advantage of a system in which inertial reactions are balanced by masses instead of by springs is that the condition of balance is maintained for all frequencies of oscillation and it is unnecessary to maintain or measure the frequency to a high order of accuracy. On the other hand a change of axis position introduces a new set of conditions for balance. Also, compared with the spring-balanced scheme, large inertial reactions are present at the mechanical exciter, and special care must be taken to avoid 'hammering' due to backlash.

A third method which has been used for balancing out inertial reactions is to incorporate in the oscillating system an accelerometer which gives an electrical output proportional to acceleration[27, 28]. This can be subtracted from each force-pickup signal to give a difference which is independent of inertial reaction for all frequencies and amplitudes of oscillation. If this method is to be successful, however, it is essential for the accelerometer and force pickups to be very stable and linear devices with negligible phase shift in their outputs.

Two special applications of the method of 'external rigid drive' may be mentioned. The first of these uses the unbalanced arrangement and applies to a model mounted on a short rigid sting[29] as illustrated in Fig. 22. The sting is oscillated by a pair of mechanical exciters on a common driving shaft with their eccentricities in opposite phase, and by adjustment of the relative motions the sting may be made to oscillate about any fixed axis on a line through the mean positions of A and B. Resistance strain-gauge assemblies P_1 and P_2 at two points on the sting measure the bending moments in the sting at these points, and from these measurements the vertical force and moment at the axis of oscillation are calculated.

The second application makes use of the spring-balanced arrangement and relates to vertical translation of a wing with rotation of a flap[30]. The vertical motion of the wing is forced by the exciter as shown in Fig. 23, and the flap motion is linked to the wing motion by an arm at the root of the flap pivoted at A to pickup P_2. Point A corresponds to the axis position in the pitching-wing case, and by making measurements for two positions of A, preferably giving flap motions of opposite phase, all the lift and hinge-moment derivatives relating to wing vertical translation and flap rotation can be derived from the reactions at pickups P_1 and P_2. Basically this arrangement is the same as for a pitching wing, since if the forces in the latter case are referred to some point on AB (Fig. 20) other than the axis A, the motion may be regarded as composed of the two freedoms, pitch and vertical translation coupled together.

A feature of rigid-drive techniques not possessed by other methods is their ability to deal with cases where several parts of a system are to be oscillated together but the forces on some of them are not required. A special case is in two-dimensional tests where the model spans the wind-tunnel working section, and it is required to avoid errors due to the tunnel-wall boundary layers. If the parts of the model entering the boundary layers are separated from the centre portion by very narrow gaps and the three parts are driven with the same amplitude and phase, measurements of the forces on the centre section alone will give a better approach to true two-dimensional results[21].

5.2.3. *Internal rigid drive*.—In this version[31] of the 'inexorable forcing' technique the model and oscillating mechanism together with the driving motor are mounted on a rigid floating frame or 'raft' PQRS in Fig. 24. This frame is prevented from moving out of its own plane by flexible perpendicular struts, whilst motion in its plane is resisted by the force pickups P_1, P_2 which link the frame to a rigid earth. These pickups register reactions F_1, F_2 in the plane of the frame and perpendicular to the direction of the wind stream and axis of the model.

The model is oscillated about A by the exciter E which also drives a rigid arm CD with a motion 180° out of phase with that of the model. By means of a mass M_2 on CD inertial reactions on the frame are completely balanced out, the condition for balance when AB and CD have the same amplitude of motion being

$$\left.\begin{aligned} \bar{k}^2 &= l\bar{x} \\ M_2 l &= M\bar{x} \end{aligned}\right\}, \tag{99}$$

in the notation of the previous section.

An important feature of the internal-drive arrangement is that frictional forces are automatically balanced out, since they always occur in pairs of equal and opposite magnitude acting at the same point, and are confined to the system comprised within the frame. Thus it is unnecessary to use spring bearings or pivots to avoid frictional forces at the pickups, plain bearings being satisfactory if no backlash is present. Normally in a practical design A and C would coincide, and arrangements would also be made to balance out any inertial reactions tending to displace the frame out of its own plane, thus avoiding stresses in the supporting struts.

When a state of balance has been achieved the aerodynamic lift and moment related to axis A are given by

$$L' = F_1 + F_2\,, \tag{100}$$

$$M' = F_1 l_1 - F_2 l_2\,, \tag{101}$$

and the equations for the derivatives become

$$L_\theta \bar{\theta} = \bar{F}_1 \cos \epsilon_1 + \bar{F}_2 \cos \epsilon_2\,, \tag{102}$$

$$L_{\dot{\theta}} p \bar{\theta} = \bar{F}_1 \sin \epsilon_1 + \bar{F}_2 \sin \epsilon_2\,, \tag{103}$$

$$M_\theta \bar{\theta} = \bar{F}_1 l_1 \cos \epsilon_1 - \bar{F}_2 l_2 \cos \epsilon_2\,, \tag{104}$$

$$M_{\dot{\theta}} p \bar{\theta} = \bar{F}_1 l_1 \sin \epsilon_1 - \bar{F}_2 l_2 \sin \epsilon_2\,, \tag{105}$$

using the notation of Section 5.2.2 and assuming that still-air damping forces on moving parts out of the wind are negligible.

Although the 'internal drive' arrangement deals very satisfactorily with structural forces, which are balanced for all frequencies of oscillation, it suffers a disadvantage from the tendency for the 'raft' with all its components to become very massive. This leads to difficulty in maintaining high natural frequencies within the frame system; and also requires force pickups of very high stiffness, and thus low sensitivity, in order to keep up the frequency of the frame as a whole on the pickup stiffnesses to give a satisfactory frequency response. The technique is probably most suited for tests where large forces at low frequencies are to be measured, such as at high subsonic speeds with very low frequency parameters. It is doubtful whether satisfactory operation could be obtained at the higher values of the frequency parameter at such speeds.

The general form of the mechanical system makes it most suitable for the half-model technique, with the frame parallel and close to a wall of the tunnel working section, the components being mounted on one side and the model on the other projecting into the wind stream.

5.2.4. *Electrically excited oscillations.*—The mechanical exciter E in Fig. 20 may be replaced by an electrical vibrator D as illustrated in Fig. 25, and the system can then be driven by a valve oscillator and power amplifier[32] in the manner described in Section 5.1.4. In general the electrical vibrator will not be powerful enough to overcome the inertial reactions in the driving link at the required amplitude of oscillation, and it becomes necessary to balance out these reactions by means of a spring K, either partially or completely, by working near or at the still-air resonance frequency as in excitation methods included under 'Measurement of Motion'. As an alternative to K the spring arrangements discussed in Sections 5.2.1 and 5.2.2 for balancing out inertial reactions at both the pickups P_1 and P_2 may be employed.

With this method the force-pickup outputs are referred in phase to a displacement pickup on the model system, and the wind-on and still-air amplitudes and frequency are made the same. The analysis is then identical with that for 'external rigid drive'.

The advantages of electrical excitation when driving sharply tuned systems, discussed in Section 5.1.4, apply also here.

An alternative is to employ the self-excitation scheme[33, 19] also described in Section 5.1.4. In this case the ideal procedure would be to excite in still air at phase resonance, when the driving force indicated by P_1 is in quadrature with the displacement; and then to adjust the phase and magnitude of the driving current to give the same amplitude and frequency of oscillation in the wind. The driving force is then providing a component which balances the aerodynamic stiffness. The analysis follows the lines indicated above. In practice, however, it may be difficult to make the frequency adjustment, and the system is usually oscillated at the phase-resonance frequency in both cases. The stiffness derivative is then obtained from the frequency difference in the manner described in Section 5.1.4(ii). The direct damping may be obtained either from the output of P_1 or from a measurement of the driving current or power as in Section 5.1.4(ii), whilst the indirect derivatives are given by the output of P_2. Equations (91), (92), (93) are then applicable, but the structural terms in (91) and (92) obtained from the still-air test require correction for the frequency change.

A more practical arrangement of the self-excitation scheme[16] is illustrated in Fig. 26. The oscillating system is mass-balanced about the axis A, and its product of inertia related to A and perpendicular axes in the plane of the wind direction is made zero. The model is fixed to a rigid member supported by two bearings which define the axis of rotation, and force pickups P_2 measure the reactions at the bearings due to lift forces, and thus enable L_θ and $L_{\dot\theta}$ to be determined. The spring K is now in the form of a torsion bar coincident with the axis A. Excitation is applied in the form of a moment by a pair of vibrators working in push-pull and arranged so that their reactions are parallel to the plane of the wind direction. This avoids reactions at the pickups resulting from inequality in the vibrator forces. With this arrangement inertial reactions at the pickups are zero both in the wind and in still air, the remaining reactions in still air being due to apparatus damping forces. The effect of frequency change on these may be investigated in the manner suggested for 'decaying oscillations' by making changes in the inertia of the system without altering the mass-balance and product of inertia. This, incidentally, will also check the frequency effect on any residual in-phase reaction due to imperfect inertia and mass-balance. The effects of change in air density and indirect still-air damping on the model may be investigated in a similar manner to that described in Section 5.1.1. It may be noted that the measured reactions at the two pickups also enable the derivatives relating to rolling moment due to pitching displacement to be determined.

5.2.5 *Pressure plotting*.—All the methods so far discussed measure the overall force on an oscillating model. An alternative method is to measure the distribution of oscillatory pressure over the surface of the model with a suitable arrangement of pressure pickups[34, 14]. Integration of the distributions of in-phase and quadrature components of pressure then enables all the derivatives relating to the particular motion of the model to be determined. In the case of a pitching oscillation, for example, integration of the pressure moment about the axis gives values for $M_\theta\bar{\theta}$ and $pM_{\dot\theta}\bar{\theta}$ whilst integration to give lift determines $L_\theta\bar{\theta}$ and $pL_{\dot\theta}\bar{\theta}$.

Two different arrangements of pressure pickup can be used with this method. In one of these a type of pickup is used incorporating a diaphragm which can be fitted flush with the surface of the model, whilst in the other the pickup is housed within the model and communicates with the surface *via* a short channel. The first method is difficult to apply unless the model is very large compared with the size of the pickup or unless its profile is made up of flat surfaces. The second method is well suited for measurements on model wings, in which case a differential type of pickup can be housed within the profile and connected to corresponding points on the upper and lower surfaces by very short channels as shown in Fig. 27. In this case the pressure difference between the two surfaces, which is normally the quantity required, is measured directly.

Since the pickups themselves may be in oscillatory motion, their response to acceleration is an important factor, and if this is large, still-air measurements must be made and corrections applied as for structural reaction in the 'inexorable forcing' method. Where the type of pickup or the conditions of use lead to negligible response to acceleration, the measurements in the wind give the correct damping forces, whilst the difference between the in-phase components in the wind and in still air at the same frequency give the stiffness forces. It is, of course, desirable that the output from the pickups should not vary with frequency over the range covered in the tests, and that phase shift in their response should be negligible. This implies a high natural frequency for the distorting diaphragm compared with the frequency of the test.

An important feature of the 'pressure plotting' method compared with methods in which overall force is measured is that inertial reactions resulting from distortion of the model under aerodynamic loading are not included in the measurements. Such reactions may lead to serious error in high-speed tests at high frequency, and it is very difficult to estimate corrections.

A field of application in which the method should prove extremely valuable is the measurement of the forces on flexible models with either a free or imposed mode of deformation. In the former case the mode may depend on the aerodynamic loading as well as the structural parameters and will need to be measured, possibly by means of a distribution of acceleration pickups. Derivatives appropriate to the particular mode of motion may then be estimated. Where the still-air mode differs greatly from that in the wind, and the still-air in-phase pressure components are not small, some error may arise in the true stiffness derivative, and it then becomes necessary to control the mode by some means such as multi-point inexorable forcing.

A further advantage of 'pressure plotting' lies in the fact that the distribution of force as well as the total force is obtained. This may be of considerable value, especially for comparison with theory.

A major disadvantage of a method using pressure pickups housed within a model wing is that with the size of pickup at present available a large model and thus a large wind tunnel are required. This rules out many of the smaller high-speed wind tunnels which might be available for derivative measurement. The use of external pickups connected to pressure orifices on the surface of the model is of doubtful value on account of the large attenuation and phase shift in the connecting tubes[35].

Interferometer techniques for the measurement of pressures on oscillating models are limited to two-dimensional tests and would require considerable development to give phase angles with sufficient accuracy.

6. *Instrumentation.*

In discussing the various techniques for measuring oscillatory derivatives, mention has frequently been made of displacement, force and pressure pickups, and measurement of amplitude, phase and frequency, without going into details. Further consideration of these aspects of the techniques is given in this section.

6.1. *Displacement, Force and Pressure Pickups.*

The use of wire resistance strain gauges has been widespread in this connection. Displacement pickups may be formed by cementing the gauges to a thin steel cantilever strip which is deflected by the displacement to be measured. If the displacement is a rotation, this strip may well form part of a spring bearing. Normally an assembly of four gauges would be used, two on either side of the strip, and connected in the form of a bridge to give increased sensitivity. It is important that the natural frequencies of the strip should be high compared with the frequency of oscillation of the model system in order that the mode of bending should not change with frequency. In the case of a pitching oscillation controlled by a torsion bar, the displacement may be indicated by cementing the strain gauges to the bar in such a manner as to give an output proportional to torsion.

If a thick, stiff cantilever is used and a transverse force is applied at the free end, the displacement is very small, and the device forms a force pickup. Similarly a pickup for torque may be in the form of a short, stiff cylindrical bar or tube carrying the strain gauge assembly and introduced into the system at the point where the torque is required to be measured. In some force pickups the members carrying the strain gauges are put into extension or compression by the force as illustrated in Figs. 28 and 29. In the latter the strips are pretensioned by the central screw[22, 23].

In pressure pickups using strain gauges, the latter are normally cemented to a flat diaphragm which deflects under the applied pressure. A special type of strain-gauge pressure pickup[34] suitable for low speeds and relatively low frequencies is illustrated diagrammatically in Fig. 30. The resistance wire in this case is wound round a cylindrical rubber diaphragm, and the pressure difference is applied between the inside and outside of the latter. An advantage of this type of pickup is that it is insensitive to acceleration for oscillations in the direction of the axis of the cylinder.

Commercial pickups available at the present time frequently make use of the unbonded type of strain gauge.

An advantage of resistance strain gauges is that they are small and can usually be situated close together in the form of a bridge, which is relatively insensitive to temperature changes. A disadvantage is that they give a low output which normally requires considerable amplification for measurement.

A principle which is frequently used in pickups is to make them in the form of an electrical capacitance which changes its value under the action of the displacement force or pressure[36, 17]. The capacitance is often included in the tuned circuit of a valve oscillator working at a relatively high frequency (e.g., 2 Mc/s) which is modulated by the changes in capacity. A demodulator then produces output voltages proportional to these changes. The system can be made very sensitive, a typical value being 1 volt output for a change in capacitance of 1 pF.

Displacement pickups are frequently made in the form of a moving-vane air condenser, whilst a typical force pickup[17] is illustrated in Fig. 31. A thick steel disc A is reduced annularly to form a stiff diaphragm with a centre boss to which the force F is applied. This boss, together with an insulated disc B separated from it by a small gap, forms a capacitance which changes in value when the boss moves under the action of the force. A typical value for the width of the gap would be 0·001 in., and the displacement of the boss would be limited to a small fraction of this (1% say) to avoid non-linearity in the response.

In a pressure pickup based on the same principle, the disc A would be replaced by a thin steel diaphragm to which the pressure is applied.

The capacitance-type force and pressure pickups described above are sensitive to temperature gradients which produce structural distortions and can thus alter the width of the narrow gap. This effect can be avoided by designing the pickup with a built-in water-cooling system through which a constant flow of water is maintained[37]. Such a procedure is desirable for tests in high-speed wind tunnels where temperature gradients are normally present.

Other principles which have been used in the design of pickups include variation of inductance, magnetostriction effects, the use of an a.c. energised magnetic circuit in the form of a balanced bridge which is thrown out of balance by the movement of an armature or diaphragm, and a differential transformer with a moving core. Optical methods can be used to convert both large and small displacements into electrical voltages with the aid of photocells and moving masks or varying slit widths which modulate the beam of light. Where electrical analysis is not required, displacement can be recorded directly on moving film or sensitive paper by means of a mirror and beam of light as indicated in Section 5.1.1, and at very low frequencies force can be recorded in a similar way if considerable displacement can be tolerated in the pickup, which could then take the form of a very short, spring-constrained optical lever.

A type of pressure pickup[38] not yet mentioned is formed by cementing a disc of polarized barium titanate to a flat metal diaphragm as shown in Fig. 32. When pressure is applied the barium titanate is strained in such a way that charges of opposite sign appear at the surfaces due to the piezo-electric effect. Electrodes on the surfaces collect the charges and give an output voltage proportional to pressure. This pickup is very sensitive but has several drawbacks. It is sensitive to both acceleration and temperature changes and has a poor low-frequency response. It is possible, however, to reduce temperature effects by special design, and the low-frequency response can be improved by special amplifier circuits.

6.2. *Measurement of amplitude and phase.*

Where pickup signals are recorded photographically or otherwise, amplitude and phase may be measured directly on the record if this includes a reference signal. However, when the signal being investigated contains a considerable amount of noise or 'hash', which is frequently the case in high-speed tests, large errors may arise, especially in the phase angle, which may be small. For this reason it is preferable to make the measurements directly on the pickup signals by means of electrical devices which ignore the noise components.

Measurements of amplitude of signals are frequently made with a valve voltmeter consisting of an amplifier followed by a rectifier and d.c. meter[17]. If this is arranged to measure average value, noise will be eliminated if it is small compared with the signal.

A method for measuring phase directly with a calibrated phase-shifting network[11] has already been discussed in Section 5.1.3, and a commercial instrument is available for direct measurement of phase in which a fraction of each signal equal to a specified voltage is selected and the vector difference measured.

Two methods based on electronic trigger circuits may be mentioned. In the first a bi-stable circuit is used which is triggered at the beginning of each cycle in each of the two signals by means of Schmitt triggers. A voltage level in the bi-stable circuit alternates between two fixed values at each changeover, and the time average of this voltage is thus a function of the phase difference between the signals.

The second method makes use of an electronic, gated counter-chronograph which measures the time interval between two pulses applied to the gate[15, 16]. These pulses are formed at corresponding points on the two signals. The period of the signals is obtained by taking pulses at corresponding points on one signal, and the phase can then be determined from the two time measurements. With the above two methods it is necessary for the signals to be pure sine waves, but in the vector-difference method filtering can be applied to the difference to remove harmonics and noise.

It is often more convenient and useful to measure the vector components of a signal in phase and in quadrature with a reference instead of to measure phase and amplitude directly. When a mechanical oscillator is used to drive a system, the motion of the oscillator forms the reference. In this case a simple method of measuring the components of the pickup output is to rectify the pickup signal by means of a mechanical rectifier formed by a commutator on the driving shaft of the oscillator[22, 23] as illustrated in Fig. 33, and to measure the mean value of the rectified signal with a sensitive, heavily damped d.c. meter. This meter is connected to the commutator segments by slip-rings, and its connections to the pickup are reversed by the commutator when the exciter phase angle is 0° and 180° when using contacts A. Contacts B are set so that this reversal takes place at phase angles 90° and 270°. If the pickup signal is $\bar{v}\sin(pt+\epsilon)$, where ϵ is the phase angle relative to the reference, rectification by contacts A gives a mean voltage

$$v_{\text{in}} = \frac{2}{\pi}\bar{v}\cos\epsilon, \tag{106}$$

which is proportional to the in-phase component of the pickup output, and contacts B give

$$v_{\text{out}} = \frac{2}{\pi}\bar{v}\sin\epsilon, \tag{107}$$

proportional to the quadrature component. With this technique it is important that the pickup signal should be purely sinusoidal, since the mean rectified voltages are affected by odd harmonics.

An alternative method[31] which may be employed with a mechanical exciter when resistance strain gauges are used as pickups is to provide sinusoidal reference currents from generators coupled to the exciter drive, one giving a current in phase with the motion and the other in quadrature. If the in-phase current flows through a strain-gauge bridge and the strain in each arm of the bridge is $\pm\bar{s}\sin(pt+\epsilon)$, the instantaneous output voltage may be written

$$v = C\bar{s}\sin(pt+\epsilon)\bar{i}\sin pt, \tag{108}$$

where C is a constant and the current is $\bar{i}\sin pt$. The mean value of the output voltage is then

$$v_{\text{in}} = \tfrac{1}{2}C\bar{s}\bar{i}\cos\epsilon \tag{109}$$

and is proportional to the in-phase component of the strain. Similarly the quadrature current gives

$$v_{\text{out}} = \tfrac{1}{2}C\bar{s}\bar{i}\sin\epsilon \tag{110}$$

proportional to the quadrature component. The mean voltages may be measured as before. Harmonics in the pickup strain have no effect on these voltages provided the reference currents are sinusoidal. If the pickup strain is purely sinusoidal, the reference currents may be square waves which can be obtained quite simply by means of a suitable commutator on the driving shaft of the exciter[39].

Electrical component-resolvers[35] have been used in some cases*. These are special rotary transformers which produce outputs proportional to the product of the primary input voltage and the sine or cosine of the resolver-shaft position angle. Two of these may be driven by the exciter shaft to provide the in-phase and quadrature references, and the pickup output voltage is applied to the primary windings. The average values of the secondary outputs are then expressed by equations similar to (109) and (110).

An a.c. bridge method[26] for measuring the components of a pickup output when a mechanical exciter is used is shown in Fig. 34. In-phase and quadrature reference voltages obtained from

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* U.S.A.

sinusoidal generators driven by the mechanical exciter can be adjusted by potential dividers and balanced in series against the pickup voltage to give a null reading at the junction of two equal resistances R. If the generator voltages are equal, the settings of the potential dividers are then proportional to the required components. The filter removes 'hash' and harmonics in the pickup signal to enable an accurate balance to be obtained with the indicator, which can be a cathode-ray oscilloscope. This method measures the components of the fundamental in the pickup signal.

When the excitation is not a rigid drive the methods described above for a mechanical exciter are not easy to apply, and a technique employing some form of wattmeter is then convenient[40, 41]. The motion taken as reference will normally supply an in-phase reference voltage $\bar{v}_r \sin pt$ from a displacement pickup, and a quadrature reference voltage $\bar{v}_r \cos pt$ may be obtained from this with a phase-shifting network. If a pickup signal $\bar{v} \sin(pt+\epsilon)$ is supplied to one of the wattmeter circuits and the in-phase reference to the other, a wattmeter reading

$$W_{\text{in}} = C_w \bar{v}_r \bar{v} \cos \epsilon \tag{111}$$

is obtained proportional to the in-phase component of the pickup voltage, C_w being a constant. Similarly the quadrature reference gives a reading

$$W_{\text{out}} = C_w \bar{v}_r \bar{v} \sin \epsilon \tag{112}$$

proportional to the quadrature component. As with the previous method the wattmeter technique measures the fundamental in the pickup signal.

Usually a wattmeter with high input impedances is required in order to avoid the introduction of attenuation and phase shifts in the pickup circuits. One such type[18], illustrated in Fig. 35, uses a pair of thermionic valves with square-law characteristics. If the sum of the pickup and reference signals is applied to the grid of one valve and the difference to the grid of the other, a d.c. potential difference is produced between the anodes proportional to (111) or (112) above and can be measured with a meter. Alternating components are filtered out.

A commercial instrument working on a similar principle makes use of vacuum thermo-junctions, which have approximately square-law characteristics.

For frequencies from 10 to 200 c/s a dynamometer wattmeter can be used[17, 33] with buffer amplifiers having negligible phase shift, but care must be taken that high-frequency noise signals do not produce errors due to transformer effects between the windings.

The setting of the phase-shifting network to give the quadrature reference can conveniently be carried out by feeding both reference signals into the wattmeter and adjusting the phase shifter to give a null reading.

6.3. *Measurement of Frequency.*

Numerous standard methods exist for measuring frequency of oscillation, including recording the signal with a time scale, stroboscopic methods, various frequency bridges, electronic methods in which the signal period is compared with an RC time constant, and beat methods using a valve oscillator and cathode-ray oscilloscope. For convenience and accuracy, however, electronic-counter methods are to be preferred, especially when it is required to measure very small frequency changes.

A satisfactory version of the counter technique[42, 17] requires a timer, which incorporates an electronic counter with a standard-frequency input controlled by a gate, and a separate counter for the signal. The signal counter produces a pulse at some point in either the first or a selected cycle entering, which opens the gate in the timer. The gate is closed again by a second pulse occurring

n cycles later, and thus the time for n signal cycles is presented by the timer as a number of standard-frequency periods. Normally n is a power of 10 and can be selected. If the standard frequency is very stable and high compared with the signal frequency, the latter can be measured to a high degree of accuracy provided it also is stable. For example, measurements to 1 part in 10^6 can be made quite quickly.

7. *Wind-Tunnel Effects.*

The effects of tunnel walls on measurements of oscillatory derivatives in subsonic flow have been investigated theoretically by a number of authors, and some experimental confirmation has been obtained for the two-dimensional case[47, 48, 51, 52], but there appears to be no such confirmation for three-dimensional flow[46, 49, 50]. It has been shown and confirmed that large interference effects on damping can occur at low values of the frequency parameter in two-dimensional tests. In the three-dimensional case the theoretical effects on damping are in general proportionally smaller than those on stiffness, the latter being of the same order as in the static case. This suggests that one of the criteria in selecting size of model should be the estimated static-interference effects. It is a considerable advantage in this respect if the steady forces can be measured on the derivative apparatus, since any unexpected interference effects may then be revealed.

Theoretical investigations of the effects of porous walls have been made[53, 54, 55], and large effects predicted for some conditions. Also some experimental evidence exists that slotted walls may have large effects on measurements of damping, but the conditions for this have not yet been established.

Disturbance to the vortex wake behind the oscillating model may also affect the measured forces[56]. This disturbance may be due to a tunnel fan or to injector slots in a high-speed tunnel cutting off the wake. Theoretical estimates for two-dimensional flow for a forward axis indicate that the effect would be small if the wake is 10 wing chords in length, provided the frequency parameter is not too low.

A further tunnel effect which may occur at high subsonic speeds is acoustic resonance[57]. This arises at frequencies for which disturbances from the model are reflected at the tunnel walls and arrive back at the model in opposite phase. The effect is probably reduced by the use of porous or slotted walls, which would tend to absorb the disturbances.

BASIC NOTATION

b	Potentiometer setting
c	Mean chord of wing
f	Frequency of oscillation
i	Current
k	Vibrator constant
$\bar{k}$	Radius of gyration
l	Linear dimension
p	Angular frequency $= 2\pi f$
p_a	Angular frequency at amplitude resonance
r	Amplitude ratio $= \bar{\theta}/\bar{z}$
s	Strain in resistance strain gauge
s_1	Stiffness of exciter spring
t	Time
v	Voltage
x	Logarithmic decrement
x'	Downstream displacement of axis
$\bar{x}$	Distance of centre of gravity aft of axis
y	Exciter displacement
z	Vertical displacement of axis (positive downwards)
F	Force
I	Moment of inertia
L'	Lift
M'	Pitching moment about axis (positive nose up)
M	Mass
M_1	Exciting moment
P	Product of inertia
Q	Q-factor {equation (49)}
R	Resistance
S	Wing area
V	Wind speed

W		Driving power
Y	$=$	c/V
γ	$=$	p/p_a
ϵ		Phase angle
θ		Pitching displacement (positive nose up)
κ		Apparatus damping coefficient
λ	$=$	$\mu + jp$
μ		Exponential damping factor for decaying oscillations
ρ		Air density
σ		Elastic stiffness
ϕ		Swing between successive stationary values of displacement
ω		Frequency parameter $= 2\pi fc/V$
M_θ, $M_{\dot\theta}$		Direct stiffness and damping derivatives related to pitching displacement
M_z, $M_{\dot z}$		Indirect stiffness and damping derivatives related to vertical translation
L_θ, $L_{\dot\theta}$		Indirect stiffness and damping derivatives related to pitching displacement
L_z, $L_{\dot z}$		Direct stiffness and damping derivatives related to vertical translation

$$\left.\begin{aligned} m_\theta &= M_\theta/\rho V^2 cS \\ m_{\dot\theta} &= M_{\dot\theta}/\rho V c^2 S \\ m_z &= M_z/\rho V^2 S \\ m_{\dot z} &= M_{\dot z}/\rho V cS \\ l_\theta &= L_\theta/\rho V^2 S \\ l_{\dot\theta} &= L_{\dot\theta}/\rho V cS \\ l_z &= cL_z/\rho V^2 S \\ l_{\dot z} &= L_{\dot z}/\rho V S \end{aligned}\right\}\text{non-dimensional forms of derivatives}$$

A bar over a symbol indicates amplitude and a suffix a denotes still-air conditions unless stated otherwise above or in the text.

Suffix ρ denotes values at phase resonance.

Other suffixes are defined in the text.

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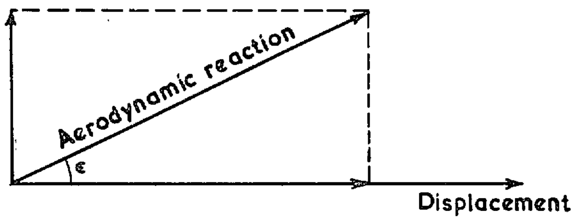

FIG. 1. Aerodynamic vectors.

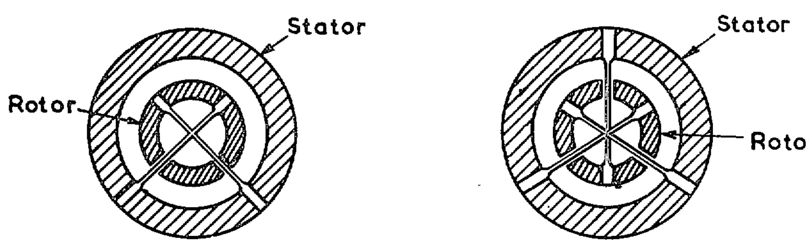

FIG. 2. Spring bearings.

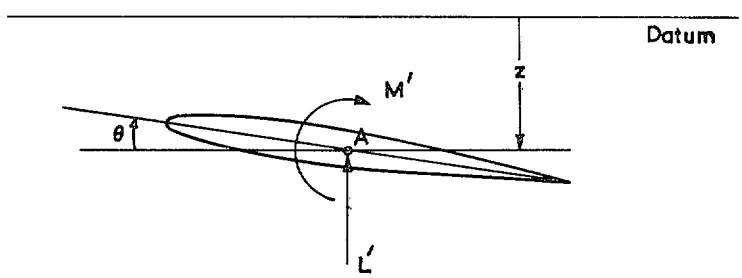

FIG. 3. The system for pitching and vertical translation.

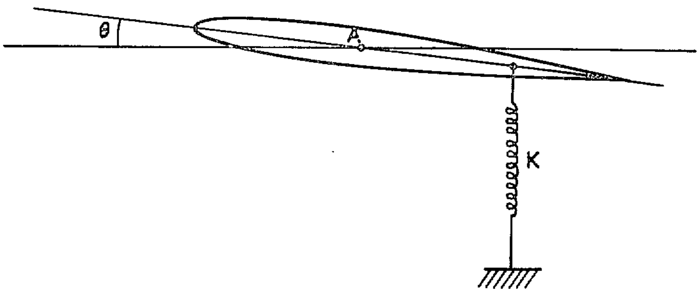

FIG. 4. Decaying oscillations.

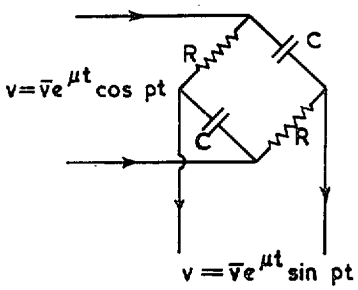

FIG. 5. Phase-shifting network.

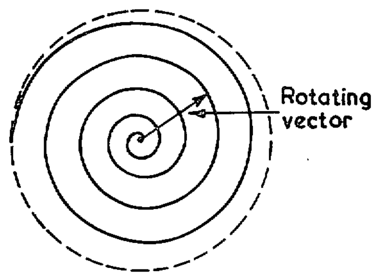

FIG. 6. Logarithmic spiral.

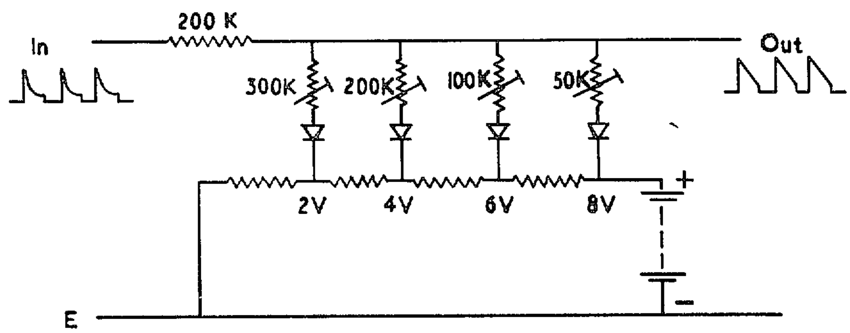

FIG. 7. Logarithmic distorting circuit.

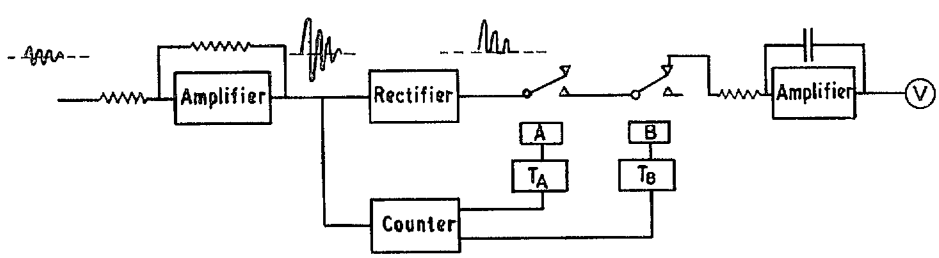

FIG. 8. Integration method for decaying oscillations.

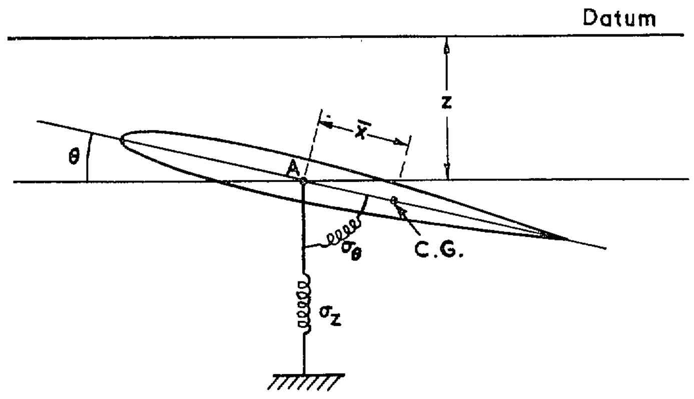

FIG. 9. Flutter oscillations.

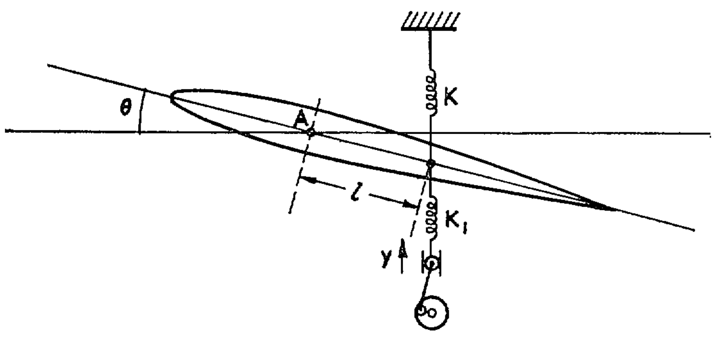

FIG. 10. Elastic excitation.

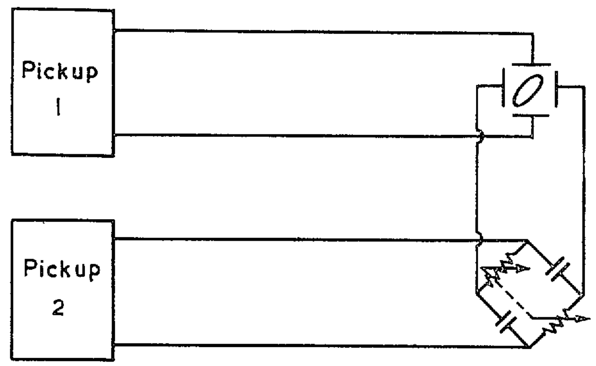

FIG. 11. Measurement of phase.

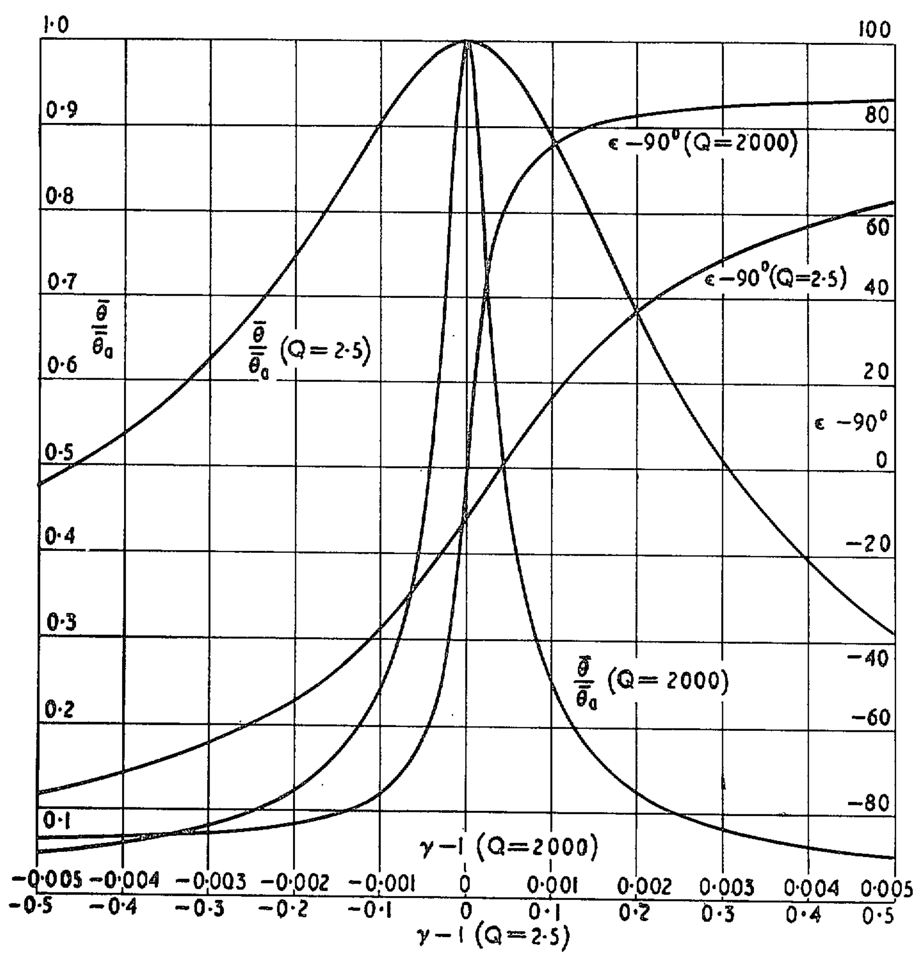

FIG. 12. Response curves for tuned systems.

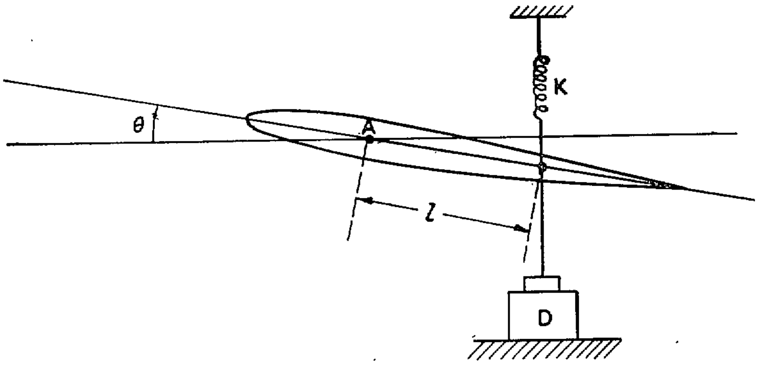

FIG. 13. Electrical excitation.

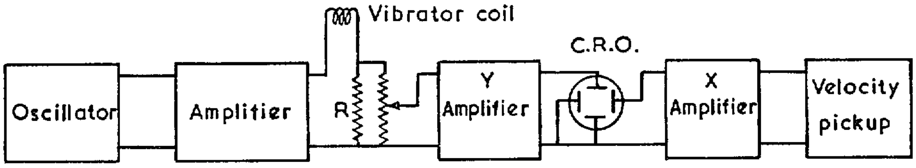

FIG. 14. Electrical excitation at phase resonance.

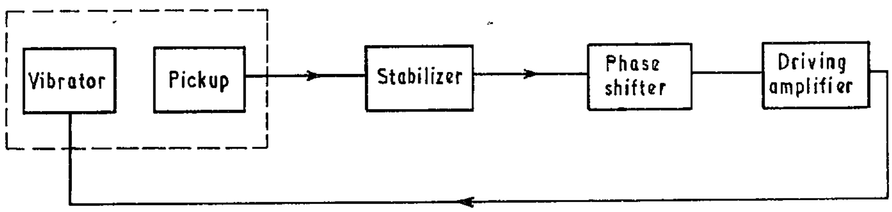

FIG. 15. Electrical self-excitation.

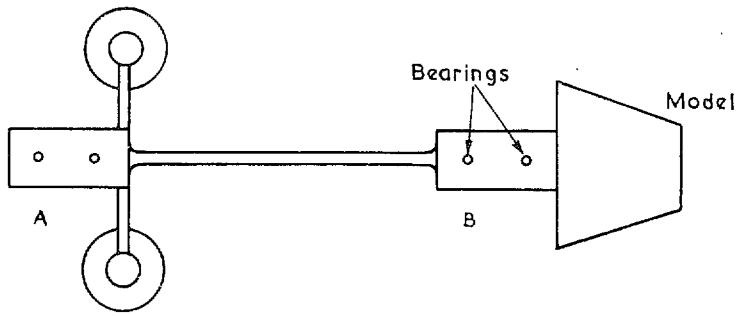

FIG. 16. Free-free system.

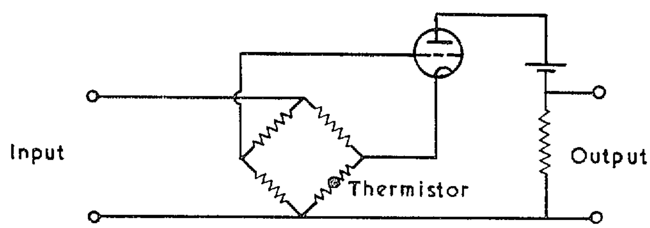

FIG. 17. Thermistor stabilizer.

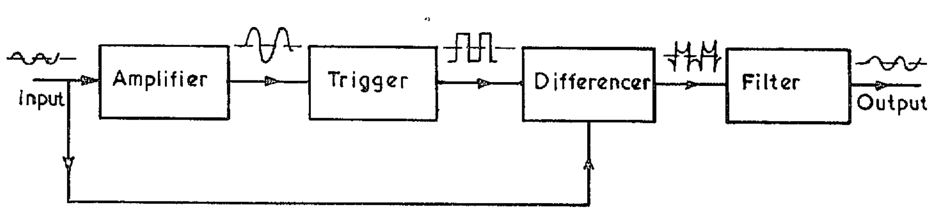

FIG. 18. Quick-action stabilizer.

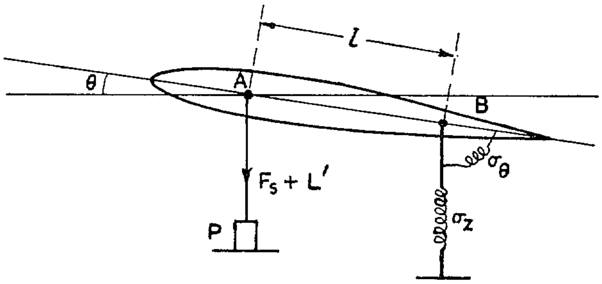

FIG. 19. Balance of inertial reactions with springs.

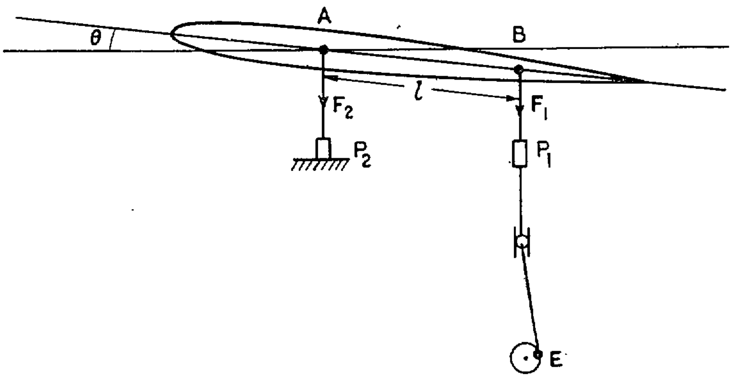

FIG. 20. External rigid drive.

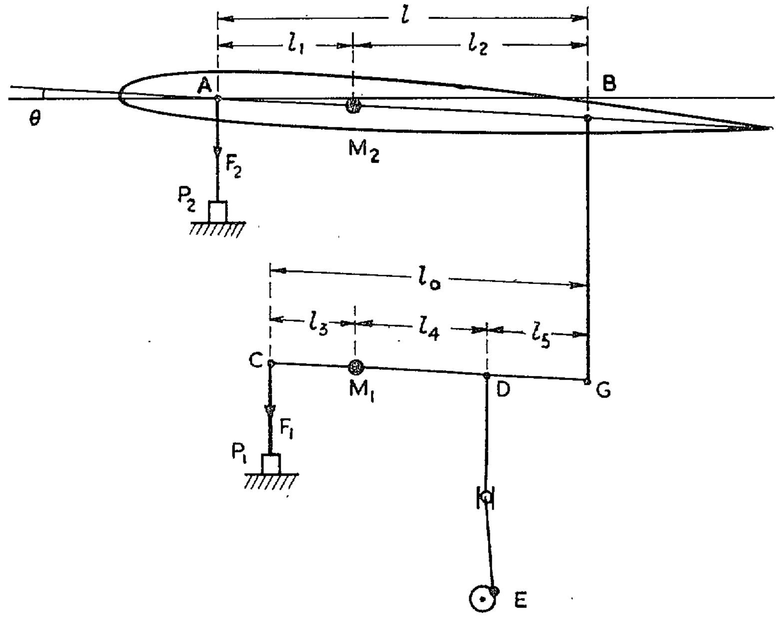

FIG. 21. Linkage for balancing inertial reaction.

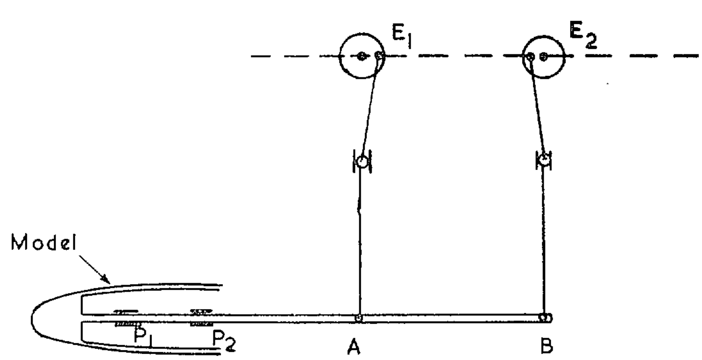

FIG. 22. Sting-mounted system.

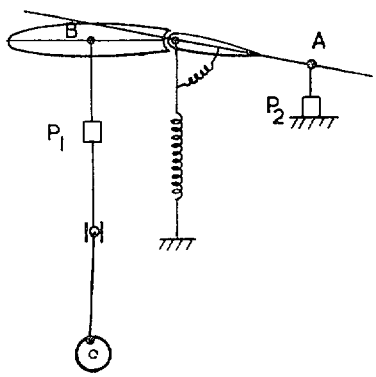

FIG. 23. System with linked freedoms.

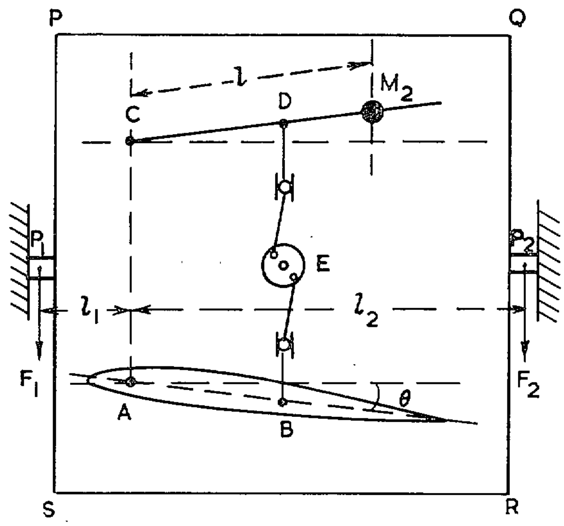

FIG. 24. Internal rigid drive.

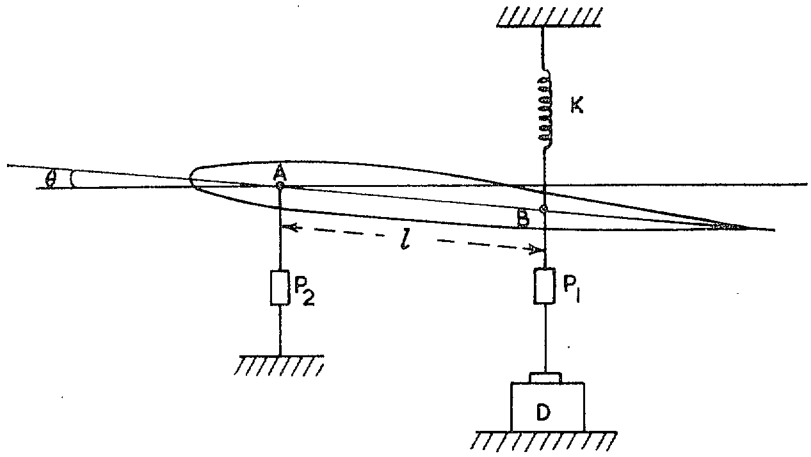

FIG. 25. Electrical excitation for system with force pickups.

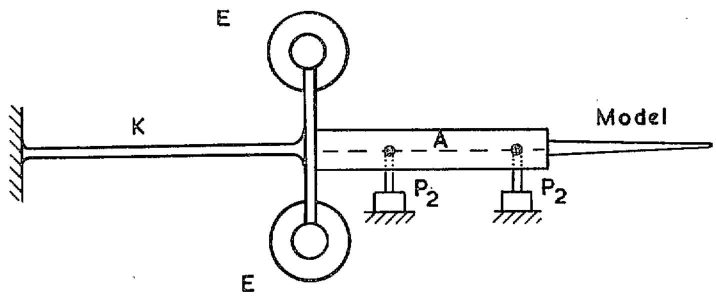

FIG. 26. Practical self-excitation arrangement.

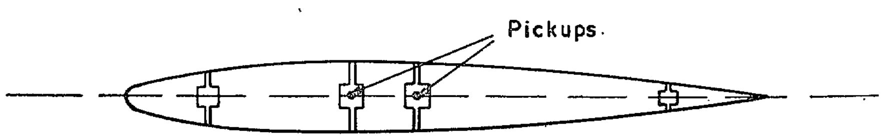

FIG. 27. Pressure-plotting model.

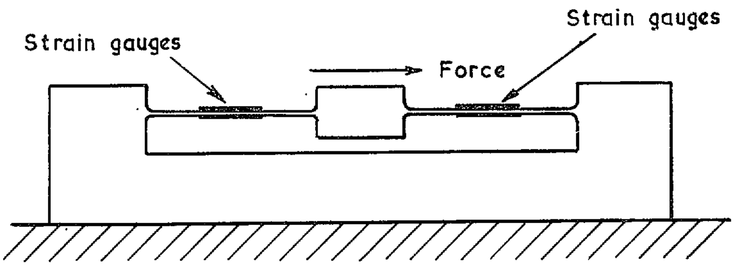

FIG. 28. Strain-gauge force pickup.

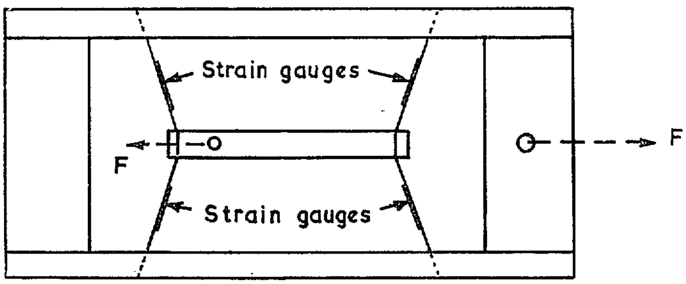

FIG. 29. Strain-gauge force pickup.

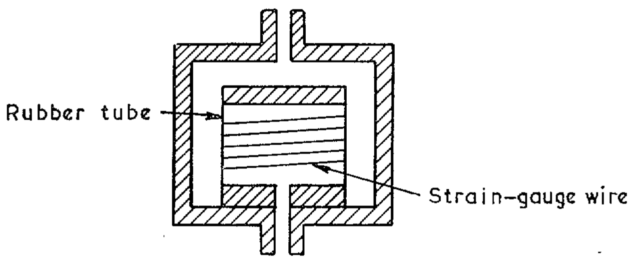

FIG. 30. Strain-gauge pressure pickup.

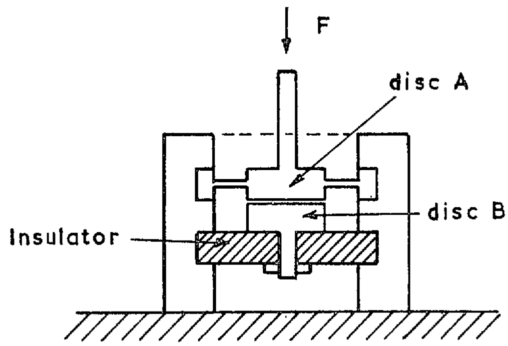

FIG. 31. Capacity force pickup.

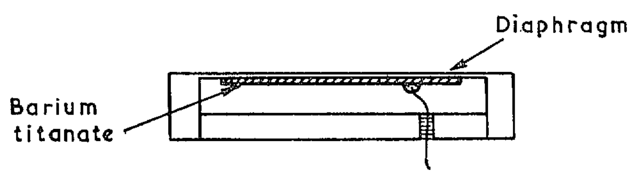

FIG. 32. Flush-type pressure pickup.

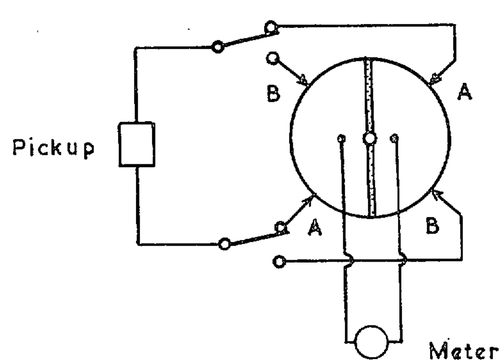

FIG. 33. Commutator rectifier.

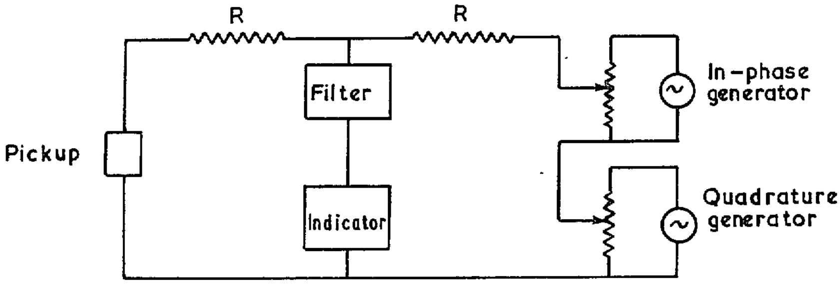

FIG. 34. Bridge measurement of voltage components.

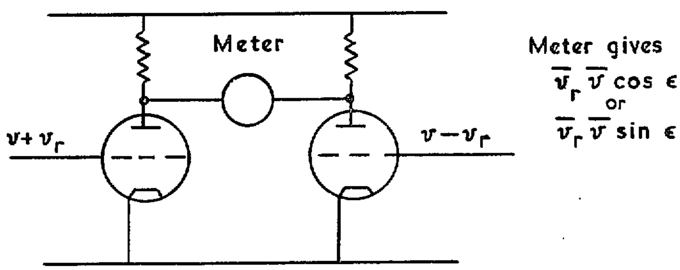

FIG. 35. Electronic wattmeter.

(86837) Wt. 64/1857 K5 5/63 Hw.

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