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On the Flexure of Thin Built-Up Wings

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Summary. When thin built-up wings experience pure flexure, the surfaces deflect towards the neutral axis of the cross-section due to the 'Brazier' effect. The resulting cross-sectional deformation of symmetrical wings with flat faces between adjacent spars is here analysed, together with the corresponding loss in flexural stiffness.

1. *Introduction.* When a thin built-up wing is subjected to flexure, the surfaces tend to deflect towards the neutral axis of the cross-section. For a given applied moment the curvature of the wing is thus greater than the value calculated by the simple engineer's theory of bending. The cross-sectional deformation does not however reduce the stiffness of the wing sufficiently for Brazier[3] buckling to take place, and the initial instability of the wing structure arises from the buckling of the constituent panels.

This Report considers the unbuckled cross-sectional deformation and the consequent loss in stiffness of thin wings, reinforced by spars and ribs, under pure flexure. The Report is restricted to wings of uniform symmetrical cross-section with flat surfaces of constant thickness between adjacent spars.

An analysis is given of the surface deformation and the corresponding loss of stiffness in flexure of wings with no ribs. Curves are presented giving the deformed shape of the surfaces of wings of this type, and the contribution of the surfaces to the stiffness of the cross-section. An approximate analysis is given of the effect of ribs. This analysis, which is also valid for the post-buckling regime provided the deflections are small, is considerably more complicated, and a few specimen results only have been calculated. It is indicated how, with the aid of these specimen results, a reasonable estimate of the effect of the ribs can be made provided the rib spacing is at least twice the spar spacing. The buckling behaviour of the wing surface is also discussed.

2. *Assumptions.* The following assumptions are made:

(i) The cross-sections of the spar webs and of the fillets at the leading and trailing edges do not deform; the chordwise slope of the wing surface adjacent to the spars and fillets is thus unaffected by the deformation of the cross-section of the wing.

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(ii) The chordwise membrane stresses in the wing surface and the anticlastic curvature of the wing, which are both Poisson's ratio effects, can be neglected.

(iii) All deflections lie within the range of validity of 'small deflection' plate theory.

Additional assumptions made when ribs are present:

(iv) The spanwise curvature of the neutral axis of the wing is constant.

(v) Differences in the behaviour of the compression and tension faces do not significantly affect the position of the neutral axis of the wing.

(vi) The ribs are rigid.

3. *Analysis.* The deformation of a flat isotropic spanwise strip on the surface of a thin built-up wing of uniform cross-section is analysed in this Section, together with the consequent effect on the flexural stiffness of the wing. The edges of the strip, which are attached to spars or fillets, are at distances h and ϕh ($\phi \leqslant 1$) from the neutral axis of the wing cross-section, as shown in Fig. 1. The strip, which is of constant thickness t, experiences a longitudinal stress σ_y due to the flexure of the wing. The general equation of the deformation of the strip is then

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{t\sigma_y}{D}\frac{\partial^2 w}{\partial y^2}. \tag{1}$$

Analyses for wings with and without ribs are given in Sections 3.1 and 3.2 respectively.

3.1. *Wing without Ribs under Pure Flexure.* The wing as a whole experiences a constant curvature κ in the y, $\bar{Z}$ plane such that

$$\sigma_y = E\kappa h\bar{Z} \tag{2}$$

where $\bar{Z}$ is measured from the neutral axis of the wing cross-section. The surface of the wing makes an angle η with the neutral axis, hence

$$\frac{\partial^2 w}{\partial y^2} = -\kappa\cos\eta. \tag{3}$$

As the deformation of the wing is independent of y, it is only necessary to consider the region near $y = 0$, where $\partial w/\partial x$ and $\partial^2 w/\partial x^2$ are independent of y, to solve the problem completely. The following expression for w is obtained by integrating Equation (3) twice,

$$w = -\frac{\kappa y^2 \cos\eta}{2} + Cy + h\chi(x)\cos\eta \tag{4}$$

where C is a constant.

The deflection of the strip perpendicular to and relative to the neutral axis is thus

$$h\chi(x)$$

and, if the strip is on the compression side of the neutral axis, the distance of the strip from the neutral axis is given by

$$\bar{Z} = -\left\{1 - (1-\phi)\frac{x}{b} - \chi(x)\right\}. \tag{5}$$

Substituting Equations (2), (4) and (5) in Equation (1), the following linear constant-coefficient differential equation is obtained

$$\frac{d^4\chi}{dX^4} + 4\mu^4\chi = 4\mu^4\{1 - (1-\phi)X\} \tag{6}$$

where

$$\mu^4 = 3(1-\nu^2)\left(\frac{\kappa}{t}\right)^2 b^4,$$

$$b = \text{width of strip}$$

and

$$X = \frac{x}{b}.$$

The general solution to this equation is

$$\chi = 1 - (1-\phi)X + A_1 \cosh\mu X \cos\mu X + A_2 \sinh\mu X \cos\mu X + A_3 \sinh\mu X \sin\mu X + $$
$$+ A_4 \cosh\mu X \sin\mu X .$$

As the strip is attached to spars or fillets at $x = 0$ and $x = b$, the arbitrary constants must be such that

$$\chi = \frac{\partial\chi}{\partial X} = 0 \text{ at } X = 0 \text{ and } X = 1 .$$

Hence

$$A_1 = -1,$$

$$(\sinh^2\mu - \sin^2\mu)A_2 = \tfrac{1}{2}(\sin 2\mu + \sinh 2\mu) - \phi(\cosh\mu \sin\mu + \sinh\mu\cos\mu) + $$
$$+ (1-\phi)\frac{\sin\mu}{\mu}(\sinh\mu + \sin\mu),$$

$$(\sinh^2\mu - \sin^2\mu)A_3 = \sinh^2\mu\cos^2\mu + \cosh^2\mu\sin^2\mu - 2\phi\sinh\mu\sin\mu + $$
$$+ \frac{1}{\mu}(1-\phi)(\tfrac{1}{2}\sin 2\mu - \tfrac{1}{2}\sinh 2\mu + \sinh\mu\cos\mu - \cosh\mu\sin\mu),$$

$$A_4 = \frac{1}{\mu}(1-\phi) - A_2 .$$

If the strip were on the tension side of the neutral axis, Equation (5) would become

$$\bar{Z} = \left\{1 - (1-\phi)\frac{x}{b} + \chi(x)\right\}. \tag{5'}$$

The only effect of this alteration would be to change the sign of χ. The deflected shape is thus the same on the corresponding tension and compression faces.

The value of χ on the spanwise centre-line of the strip, $\chi_{0\cdot 5}$, is plotted in Fig. 2 as a function of $\bar{\kappa}$ for a series of values of ϕ, where

$$\bar{\kappa} = \frac{\kappa b^2}{t}.$$

The deformed cross-section of the strip is plotted in Fig. 3 for a series of values of ϕ and $\bar{\kappa}$.

The strip equilibrates a moment M about the neutral axis of the wing cross-section. The secant stiffness of the strip about this axis is thus given by

$$\frac{M}{\kappa} = \frac{Ebt^3}{12} + Etbh^2\zeta_S,$$

where $Ebt^3/12$ is, to first order, the contribution of the flexural rigidity of the strip about its own mid-surface and where

$$\begin{aligned}\zeta_S &= \int_0^1 \bar{Z}^2 dX = \int_0^1 \{1-(1-\phi)X-\chi\}^2 dX \\ &= \frac{1}{16\mu}[\sinh 2\mu\{(A_1^2+A_2^2)(2+\cos 2\mu)+(A_3^2+A_4^2)(2-\cos 2\mu)\}+ \\ &\quad + \sin 2\mu\{(A_1^2-A_4^2)(2+\cosh 2\mu)-(A_2^2-A_3^2)(2-\cosh 2\mu\}+ \\ &\quad + 2\cosh 2\mu\{A_1A_2(2+\cos 2\mu)+A_3A_4(2-\cos 2\mu)\}+ \\ &\quad + 2\cos 2\mu\{A_2A_3(2-\cosh 2\mu)-A_4A_1(2+\cosh 2\mu)\}+ \\ &\quad + 2(A_1A_3+A_2A_4)(\cosh 2\mu\sin 2\mu-\sinh 2\mu\cos 2\mu)+ \\ &\quad + 2\sinh 2\mu\sin 2\mu(A_1A_2+A_2A_3-A_3A_4+A_4A_1)+ \\ &\quad + 4\mu(A_1^2-A_2^2-A_3^2+A_4^2)-2(3A_1A_2+A_2A_3+A_3A_4-3A_4A_1)].\end{aligned}$$

Similarly the tangent stiffness of the strip about the neutral axis of the wing is given by

$$\frac{dM}{d\kappa} = \frac{Ebt^3}{12} + Etbh^2\zeta_T$$

where

$$\zeta_T = \zeta_S + \bar{\kappa}\frac{d\zeta_S}{d\bar{\kappa}}.$$

The parameters ζ_S and ζ_T are plotted against the wing curvature parameter $\bar{\kappa}$ in Figs. 4 and 5 respectively for a series of values of ϕ.

3.2. *Effect of Ribs on Wing under Pure Flexure.* As the spanwise curvature of the wing varies due to the presence of the ribs, the exact analysis of this deformation would be very complex. An approximate solution can however be obtained by assuming the spanwise curvature of the neutral axis of the wing to be constant. The spanwise curvature of the strip at its boundaries is then constant and is given by

$$\frac{\partial^2 w}{\partial y^2} = -\kappa\cos\eta. \tag{7}$$

The deformed shape of the face as a whole can be expressed as

$$w = -\frac{\kappa y^2\cos\eta}{2} + Cy + h\chi(x, y)\cos\eta. \tag{8}$$

The function $\chi(x, y)$ must satisfy the boundary conditions

$$\left.\begin{aligned}&\chi = \frac{\partial\chi}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = b \\ \text{and} \quad &\chi = \frac{\partial\chi}{\partial y} = 0 \text{ at the adjacent ribs at } y = 0 \text{ and } y = l.\end{aligned}\right\} \tag{9}$$

The distance of the face from the neutral axis of the wing is given by

$$\left.\begin{aligned} \bar{Z} &= -\left\{1-(1-\phi)\frac{x}{b}-\chi(x,y)\right\} \text{ (face in compression)} \\ &\text{or} \\ \bar{Z} &= \left\{1-(1-\phi)\frac{x}{b}+\chi(x,y)\right\} \text{ (face in tension).} \end{aligned}\right\} \qquad (10)$$

Provided the deformation of the surface is small, the spanwise membrane stress is given by

$$\sigma_y = E\kappa h\bar{Z}. \qquad (11)$$

Substituting Equations (8), (10) and (11) in Equation (1), the following equation is obtained

$$\frac{\partial^4\chi}{\partial x^4}+2\frac{\partial^4\chi}{\partial x^2\partial y^2}+\frac{\partial^4\chi}{\partial y^4} = \pm\frac{E\kappa t}{D}\left\{1-(1-\phi)\frac{x}{b}\pm\chi\right\}\left(-\kappa+h\frac{\partial^2\chi}{\partial y^2}\right) \qquad (12)$$

where the upper and lower signs respectively apply to the surface in tension or compression.

It is convenient at this stage to introduce the following non-dimensional parameters and co-ordinates.

$$X = \frac{x}{b}, \qquad Y = \frac{y}{l},$$

$$\alpha = \frac{b}{l}, \qquad \beta = \frac{4\mu^4\alpha^2 l^2}{\kappa}.$$

Equation (12) may now be re-expressed as

$$\frac{\partial^4\chi}{\partial X^4}+2\alpha^2\frac{\partial^4\chi}{\partial X^2\partial Y^2}+\alpha^4\frac{\partial^4\chi}{\partial Y^4} = \pm 4\mu^4\{1-(1-\phi)X\pm\chi\}\left(-1+\frac{\beta}{4\mu^4}\frac{\partial^2\chi}{\partial Y^2}\right). \qquad (13)$$

Neglecting the product term $\beta\chi\frac{\partial^2\chi}{\partial Y^2}$, this equation becomes

$$\frac{\partial^4\chi}{\partial X^4}+2\alpha^2\frac{\partial^4\chi}{\partial X^2\partial Y^2}+\alpha^4\frac{\partial^4\chi}{\partial Y^4} = \pm 4\mu^4\left\{[1-(1-\phi)X]\left(-1+\frac{\beta}{4\mu^4}\frac{\partial^2\chi}{\partial Y^2}\right)\mp\chi\right\}. \qquad (14)$$

This equation may be solved by expanding χ and $[1-(1-\phi)X]$ in terms of the generalised Fourier series

$$\left.\begin{aligned} \chi &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\Phi_m(X)\Psi_n(Y), \\ 1-(1-\phi)X &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn}\Phi_m(X)\Psi_n(Y) \end{aligned}\right\} \qquad (15)$$

where $\Phi_m(X)$ and $\Psi_n(Y)$ are series of orthogonal functions satisfying the necessary boundary conditions, Equation (9). Such a series of functions is the normal modes of vibration of a clamped-clamped beam. These functions have been used in this way by Przemieniecki[4] to solve the problem of thermal stresses in rectangular plates due to temperature variations in their own plane. The following relationships necessary in the present problem are quoted or deduced from Przemieniecki's Paper.

$$\left.\begin{aligned} &\frac{d^4\Phi_r}{dX^4}-p_r^4\Phi_r = 0 \\ &\text{hence,} \\ &\Phi_r(X) = \cosh p_rX-\cos p_rX-q_r(\sinh p_rX-\sin p_rX). \end{aligned}\right\} \qquad (16)$$

p_r is the rth positive root of

$$1 - \cos p \cosh p = 0$$

and

$$q_r = \frac{\cosh p_r - \cos p_r}{\sinh p_r - \sin p_r}.$$

$$\int_0^1 \Phi_r(X)\Phi_s(X)dX = 1, \qquad s = r,$$

$$= 0, \qquad s \neq r.$$

$$(p_r^4 - p_s^4)\int_0^1 \Phi_r \Phi_s'' dX = 4p_r^2 p_s^2 (q_r p_r - q_s p_s)[1 + (-1)^{r+s}], \; r \neq s,$$

$$= p_r q_r (2 - p_r q_r), \; r = s.$$

$$c_{rs} = \int_0^1 \int_0^1 [1 - (1-\phi)X]\,\Phi_r(X)\Psi_s(Y)\,dX\,dY.$$

Substitution of Equations (15) and (16) in Equation (14) gives

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \{a_{mn}(p_m^4 + \alpha^4 p_n^4 + 4\mu^4) \pm 4\mu^4 c_{mn}\}\,\Phi_m \Psi_n + 2\alpha^2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\Phi_m''\Psi_n'' \mp$$

$$\mp \beta[1 - (1-\phi)X]\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\Phi_m \Psi_n'' = 0. \tag{17}$$

Multiplication of Equation (17) by $\Phi_r \Psi_s$ yields, after integration, the following infinite set of linear equations for the coefficients a_{mn}.

$$a_{rs}(p_r^4 + \alpha^4 p_s^4 + 4\mu^4) \pm 4\mu^4 c_{mn} + 2\alpha^2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\int_0^1 \Phi_m''\Phi_r dX \int_0^1 \Psi_n''\Psi_s dY \mp$$

$$\mp \beta \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\int_0^1 [1 - (1-\phi)X]\,\Phi_m \Phi_r dX \int_0^1 \Psi_n''\Psi_s dY = 0.$$

The coefficients a_{mn} converge rapidly to zero as m and n are increased and only the first few modes in each direction need be considered in practice.

It will be observed that, when ribs are present, the deflections of the compression and tension faces of the wing are different even in the unbuckled state. It is thus implicit in this analysis that the differences in the deflections of the two surfaces do not materially affect the position of the neutral axis. Provided this condition is satisfied the analysis is applicable after the onset of buckling of the compression surface.

Specimen results calculated using the above analysis are shown in Fig. 8 and are discussed in Section 5.

Once the deformation of the spanwise centre-line of the strip has been analysed, the consequent loss in stiffness can be found by numerical or graphical integration, as the shape of the deformed cross-section of the strip is virtually independent of the rib spacing. The values of the stiffness parameters ζ_S and ζ_T at any chordwise section can be found by reading off the relevant value of $\chi_{0\cdot5}$ and finding the corresponding value of $\bar{\kappa}$ when no ribs are present by reference to Fig. 2. The values of ζ_S and ζ_T can then be found from Figs. 4 and 5.

4. *Practical Estimation of Loss in Stiffness.* As the loss in stiffness experienced in practice is usually small, it is seldom necessary to solve the equations given in Section 3.2 if the ribs are widely spaced (*i.e.*, l/b is greater than about 2). A reasonable estimate of the loss in stiffness can then be obtained by using the analysis given in Section 3.1 for a wing with no ribs, together with some simple estimate of the effect of the ribs based on the results given in Fig. 8. For example it might be assumed that the deflection of the surface only differs from the value calculated in the absence of ribs within a distance of $0 \cdot 75b$ of the ribs. Between this chordwise section and the rib itself the deflection might then be assumed to vary linearly.

If the above approximate method is used, some estimate of the buckling load of the wing surface must be made independently. A first approximation to the buckling load can be obtained by assuming the strips of sheet constituting the surface to be equivalent to flat strips subjected purely to a longitudinal membrane stress varying linearly across their width. Becker and Gerard[1] have tabulated the buckling-stress coefficients for infinitely long unreinforced strips under pure bending in their own plane, and under pure compression. The ratio of the results for these two loadings when the edges of the strip are built-in differs from the corresponding ratio for simply supported edges by less than a quarter of one per cent. The buckling-stress coefficient k is here defined in such a way that the greatest compressive stress in the sheet is given by

$$k \frac{\pi^2 D}{b^2 t} .$$

The variation of the buckling-stress coefficient with the ratio ϕ of the stresses at the edges of an infinitely long built-in flat strip, which is shown in Fig. 6, has been estimated by factoring and interpolating results derived by Timoshenko[2] for a strip with simply supported edges.

A comparison of the value of the buckling load calculated in this way with that calculated by the full analysis of Section 3.2 is given in Section 5. It is shown that, while this simple method overestimates the buckling load, the simple theory should give reasonable results up to the buckling load calculated by this method.

5. *The Loss of Flexural Stiffness of a Specific Wing Section.* To give an idea of the order of the loss of flexural stiffness likely in practice, the light-alloy two-spar wing cross-section shown in Fig. 7, which has a $2 \cdot 65$ per cent thickness/chord ratio, is now considered. Buckling will occur initially in the skin between the spars. If this skin were in pure compression and $E = 10^7$ lb/in^2 and $\nu = 0 \cdot 3$, the buckling stress in this region would be 56,800 lb/in^2 if there were no ribs present. The value of the parameter $\bar{\kappa}$ equivalent to this stress would be $3 \cdot 78$. The full analysis of Section 3.2 has been computed for this inter-spar box when the rib spacing is four times the spar spacing. The results of this analysis, which are shown in Fig. 8, reveal that buckling actually commences when the wing curvature parameter $\bar{\kappa}$ has a value less than 3. As this example has been particularly chosen to give large values of the cross-sectional deformation, it is apparent from these results that the simple 'unbuckled' theory will give a reasonable estimate of the overall loss in stiffness up to the 'simple buckling load', although this is strictly in the post-buckled regime.

Consider now a bending moment applied to the wing such that the mean stress in the skin reaches 30,000 lb/in^2 at the spars. If the length of the wing is equal to the chord, and if the spanwise curvature varies linearly, this level of stress at the root corresponds to a tip deflection of $9 \cdot 8$ per cent of the chord. For the strips of skin between the spars $\bar{\kappa} = 2$ and, if no ribs are present, the maximum deflection is 10·5 per cent of the distance of the neutral axis of the skin from the neutral axis of the cross-section at the spars. The contribution of these skins to the tangent stiffness of the cross-section is reduced by 30 per cent under this loading. Similarly for the tapered sections $\bar{\kappa} = 1 \cdot 125$, and the contribution to the stiffness of the cross-section is reduced by 8·9 per cent. In this way the overall tangent stiffness of the cross-section is reduced by 19·7 per cent. At half this spanwise curvature the loss in overall tangent stiffness would be 5·4 per cent.

6. *Conclusions.* The unbuckled deflections of the surfaces of a uniform built-up wing in pure flexure, reinforced by ribs and spars, have been analysed, together with the corresponding loss in flexural stiffness. The analysis has been restricted to uniform symmetrical wings in which the undeformed skin surface is flat between adjacent spars. Curves showing the unbuckled deflection of the surfaces of wings of this class with no ribs have been presented. A few specimen results have been calculated from an approximate analysis of the deformation in the presence of ribs. A simple method of estimating the loss of stiffness of any wing of this class in which the rib spacing is at least twice the spar spacing has also been given. It has been shown that wings of practical proportions may experience overall losses in tangent flexural stiffness of the order of 20 per cent.

NOTATION

x, y, z		Cartesian co-ordinates referred to the specimen strip of the wing surface; Ox and Oy lie across and along the strip respectively
$\bar{Z}$		(Distance of the chordwise centre-line of the strip from the neutral axis of the cross-section, measured perpendicular to the latter) $\div h$
X, Y	=	x/b, y/l
b		Breadth of the strip
l		Rib pitch
t		Thickness of the strip
h		Greatest height of the chordwise centre-line of the strip above the neutral axis of the cross-section
ϕ		Least value of $\bar{Z}$ across the undeformed strip
η		Angle between $\bar{Z}$ and z co-ordinates (defined as positive)
κ		Curvature of the wing in the y, $\bar{Z}$ plane
$\bar{\kappa}$	=	$\kappa b^2/t$
w		Total deflection of the strip in the z direction
χ		Non-dimensional function such that $\chi h \cos\eta$ represents the component of w that varies with x
$\chi_{0\cdot5}$		Value of χ along the longitudinal centre-line of the strip
σ_y		Direct stress in the y direction
M		Moment applied to the strip about the neutral axis of the cross-section
α	=	b/l
β	=	$4\mu^4\alpha^2 l^2/\kappa$
μ	=	$\left\{3(1-\nu^2)\left(\frac{\kappa}{t}\right)^2\right\}^{1/4} b$
ζ_S		Secant stiffness parameter, $\dfrac{M}{Etbh^2\kappa} - \dfrac{1}{12}\left(\dfrac{t}{h}\right)^2$
ζ_T		Tangent stiffness parameter, $\zeta_S + \bar{\kappa}\,\dfrac{d\zeta_S}{d\bar{\kappa}}$
Φ_r, Ψ_s		Orthogonal functions of X and Y respectively
m, n, r, s		Suffices used to identify Φ and Ψ functions

a_{mn}		Coefficients of series expansion of $\chi(X, Y)$
c_{mn}		Coefficients of series expansion of $1-(1-\phi)X$
p_r		rth positive root of $1 - \cos p \cosh p = 0$
q_r	=	$\dfrac{\cosh p_r - \cos p_r}{\sinh p_r - \sin p_r}$
k		Buckling coefficient
E		Young's modulus
ν		Poisson's ratio (taken as $0 \cdot 3$ for computational purposes)
D	=	$\dfrac{Et^3}{12(1-\nu^2)}$
A_1, A_2, A_3, A_4		Arbitrary constants used to define $\chi(x)$
C		Arbitrary constant used to define w.

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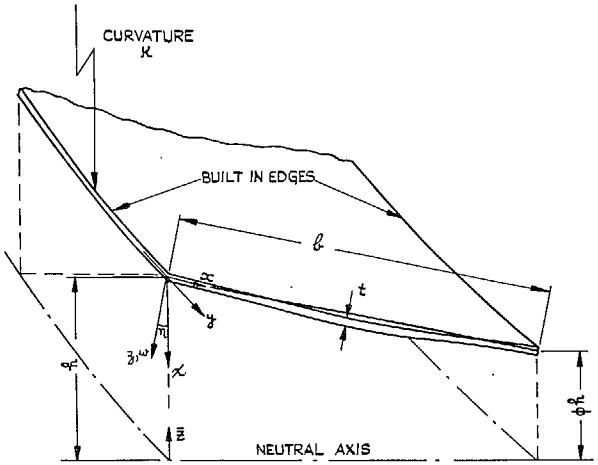

FIG. 1. Typical strip of wing surface with notation.

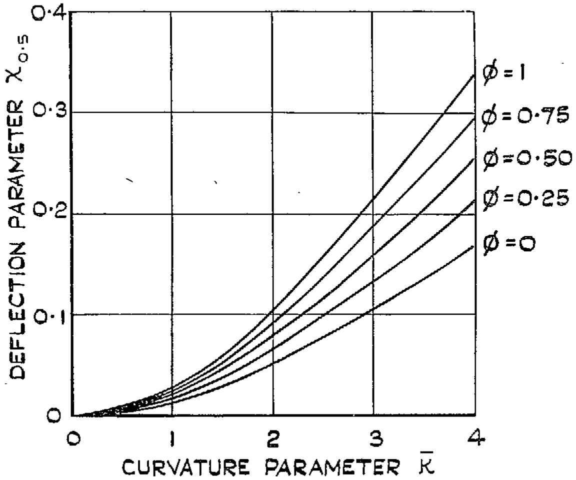

FIG. 2. Deflection of spanwise centre-line of strip without ribs.

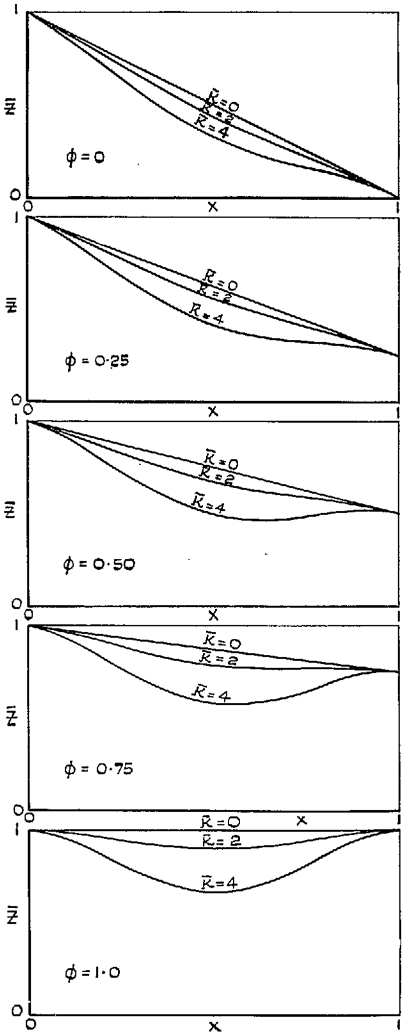

FIG. 3. Deformed cross-section of strip with no ribs.

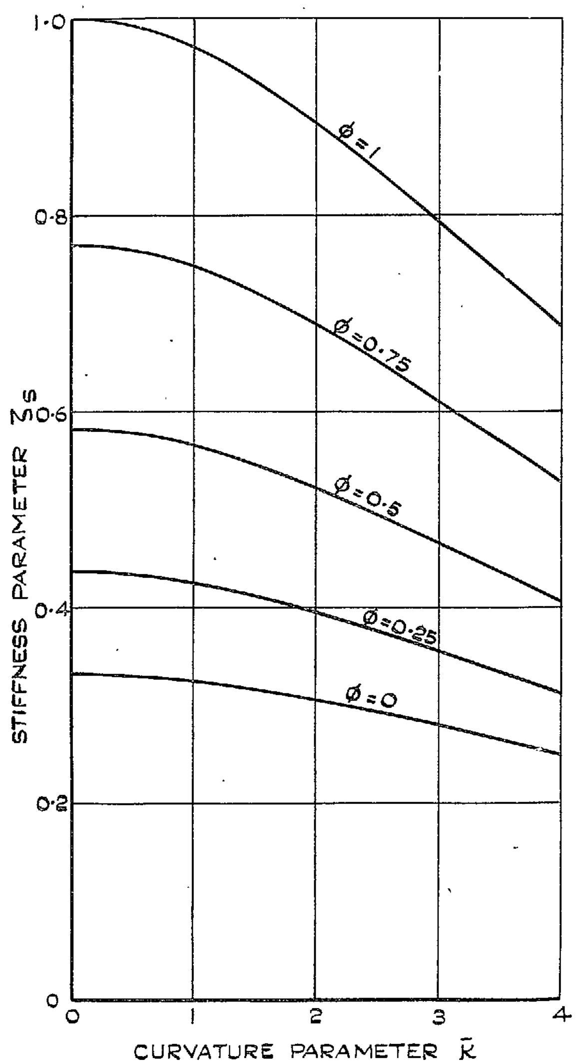

FIG. 4. Secant stiffness parameter of strip with no ribs.

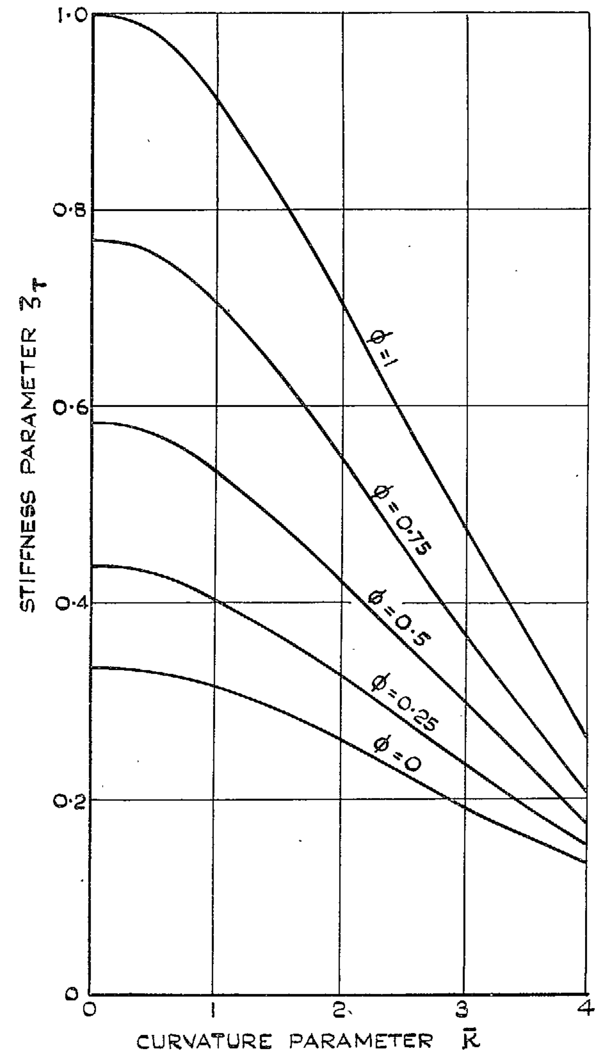

FIG. 5. Tangent stiffness parameter of strip with no ribs.

(84997) Wt. 67/1876 K5 6/62 H.x.

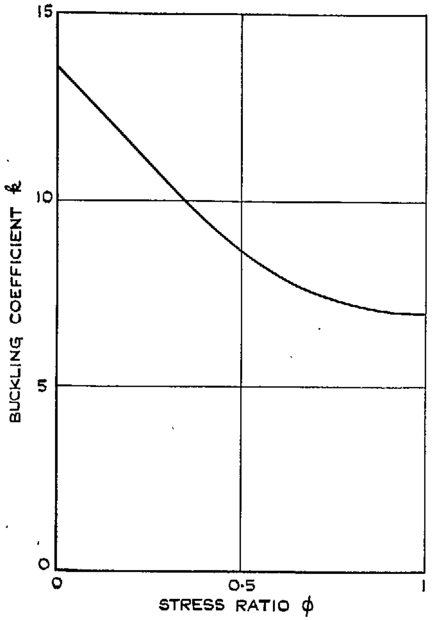

FIG. 6. Variation of buckling coefficient with ϕ foran infinitely long flat strip.

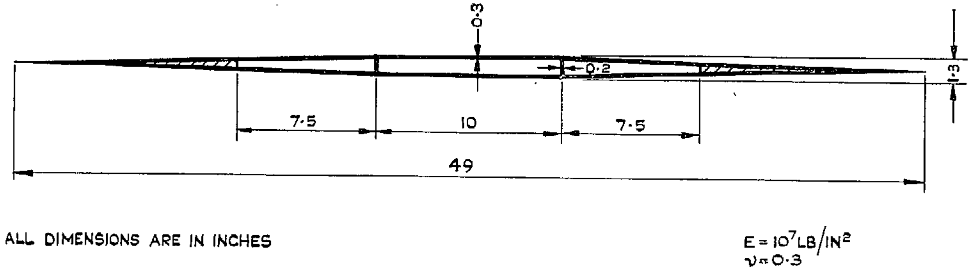

FIG. 7. Doubly-symmetrical light-alloy wing cross-section used as an example.

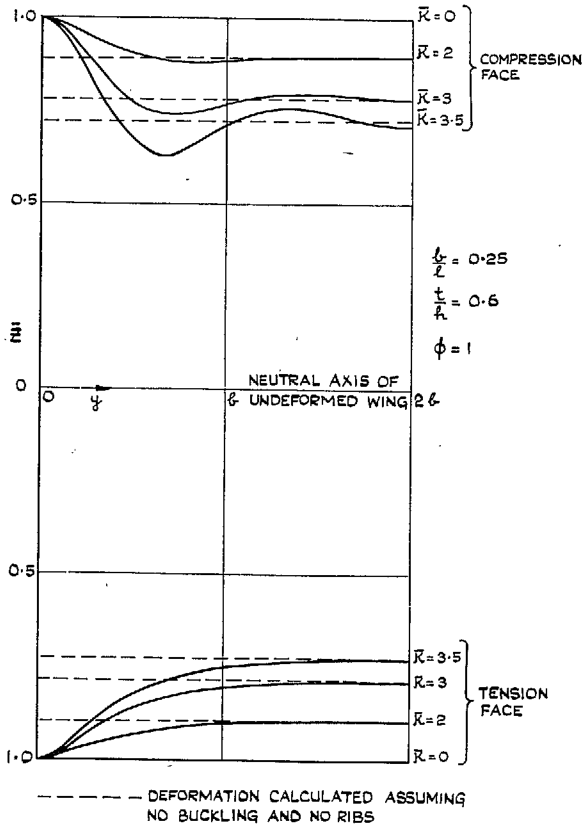

FIG. 8. Deformation of the spanwise centre-line of the surfaces of a specimen rectangular inter-spar box with ribs, under uniform curvature of the neutral axis of the wing.

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