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The Bursting of Leading-Edge Vortices—Some Observations and Discussion of the Phenomenon

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Summary. The term bursting refers to the structural change from a strong regular spiral motion to a weaker turbulent motion which can occur at some position along a vortex. Various observations of this phenomenon have been recorded during low-speed tunnel investigations of vortex flows generated from sharp, swept back leading edges. These reveal a sudden deceleration of fluid along the vortex axis and expansion of the vortex around a stagnant core downstream of which the flow is turbulent; it is possible at very low stream velocity to see the axial flow, after deceleration, deflect and perform a regular whirling motion before turbulent breakdown. These features together with the effects of bursting on oil-flow patterns and pressures at the wing surface are illustrated and described. Brief reference is made to observations at transonic speeds.

The burst is found to be sensitive to several factors, in particular, an increase of pressure gradient along the vortex seems conducive to the occurrence of a burst. It is suggested that a condition essential for the burst to occur is a low total pressure within the vortex core coupled with an adverse pressure gradient along the axis. When bursting occurs in the flow above a plain wing it is likely that the occurrence is related to the adverse pressure gradient associated with the existence of a trailing edge.

The experiments described in Appendix I show that a similar type of phenomenon can occur in the flow in a vortex tube.

1. *Introduction.* A previous investigation[1] has shown the existence of two régimes of flow in a vortex generated by the sharp leading edge of a sweptback wing at incidence; at the time these were described respectively as an orderly spiral motion and a turbulent spiralling.

The experiments of Peckham and Atkinson[2] using water-vapour condensation trails along the cores of leading-edge vortices in a wind tunnel and those of Elle[3] who observed the cavitation cores in a water tunnel, showed that the changeover occurred as a sudden breakdown or diffusion of the core at a definite position along the axis of the vortex. For the purposes of this Report it is convenient to refer to the sudden breakdown which takes place near the axis of the vortex by the term 'bursting' and to denote the upstream and downstream régimes respectively by the terms laminar vortex' and 'burst vortex'.

Both Peckham[4] and Elle[3] showed that the position along the vortex at which bursting occurred depended primarily on a combination of the angle of sweepback of the leading edge and the incidence

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of the wing; for large angles of sweepback and low incidences the burst occurred in the vortex downstream of the wing, but with an increase of incidence or a decrease of effective sweepback the burst moved upstream to a position above the surface of the wing. In practice vortex bursting may fix an upper limit to the range of incidence and a lower limit to the range of sweepback over which flow with leading-edge vortices can be used; Owen[5] has associated surface pressure fluctuations with a change in the nature of the vortex above the wing.

The observations and experiments to be described in this Report were made at various times within the last two years during a general programme of investigation of leading-edge vortex flow. Although brief reference is made to bursting at transonic speeds, the main observations and measurements have been made with low speeds of flow using several models and various techniques over a wide range of conditions.

During the course of the present research, a number of photographs illustrating vortex bursting have been published by Werlé[6] and theories of vortex breakdown have been put forward by both Jones[7] and Squire[8]. In Ref. 1, Lambourne and Pusey tentatively suggested that the breakdown to turbulence might be due to the separated boundary layer from the upper surface of the wing interfering with the rolling-up of the vortex layer from the leading edge, but this factor is no longer considered to be important to vortex bursting. An examination of the present experimental evidence of bursting, considered together with recent knowledge[9,10] of the structure of a laminar vortex, leads to another explanation of the phenomenon.

2. *List of Models.*

Model	Tunnel
Swept plate (Λ variable) (Fig. 1a)	9 ft × 7 ft Wind Tunnel
Small flat-plate delta, $\Lambda = 65$ deg $c_0 = 8 \cdot 5$ in. (Fig. 1b) .	13 in. × 10 in. Water Tunnel
Large flat-plate delta, $\Lambda = 65$ deg $c_0 = 47 \cdot 5$ in. (Fig. 1b)	9 ft × 7 ft Wind Tunnel
Flat-plate delta, $\Lambda = 50$ deg $c_0 = 3 \cdot 1$ in. . . .	25 in. × 20 in. Transonic Tunnel
Cambered-plate delta, $\Lambda = 80$ deg $c_0 = 8$ in. . .	13 in. × 10 in. Water Tunnel
Water Vortex Tube (Fig. 19)	

3. *Description of Bursting Phenomena.* Before proceeding to detailed observations of bursting phenomena it is useful to refer to the basic characteristics of a laminar vortex. The general nature of a vortex generated by a swept leading edge has received considerable attention both theoretically and experimentally (*see*, for instance, Refs. 1 to 4 and 11 to 15). Each of the pair of vortices for a delta wing is considered to be generated by a conically-rolled vortex sheet from the leading edge. In practice, as shown in Fig. 2, vortex layers spring from the leading edge S_1 and from a line of secondary separation S_2 on the upper surface. The rolling-up of these vortex layers forms the rotational core of the vortex within which, except for the outer regions, the identity of discrete layers is lost.

Flow observations indicate that upstream of the burst, each streamline that enters the vicinity of the vortex after bending around the leading edge has the form of a nearly cylindrical spiral as shown in the diagram.

Observations of vortex bursting made over a wide range of speed suggest that the vortex undergoes an expansion (*see* Fig. 3), which involves a diffusion of the fluid initially flowing along the axis of the laminar vortex and, farther downstream, the formation of a large turbulent core (*see* Fig. 4).

It is of interest to point out that in addition to the pair of main vortices, one or more weak subsidiary vortices are usually located in a region between the leading edge and the separation line S_2 of Fig. 2. In low-speed experiments these vortices have been observed to have a laminar nature close to the apex but to undergo a breakdown to turbulence which seems similar to the bursting of the main vortices. When this has been observed, the breakdown of the subsidiary vortices has taken place well ahead of the bursting of the main vortices. The descriptions and discussions to follow will refer only to the bursting of the main vortices.

Observations of the details of the bursting process for very low speeds of flow (about 2 in. per sec) were made in water using a flat-plate delta having 65 deg sweep and bevelled to 16 deg on the underside. The observations to be described refer to incidences between 21 and 25 deg for which the burst occurred approximately midway between the apex and the trailing edge. The most revealing observations were of the behaviour of dye introduced as a filament ahead of the apex of the model so that it was fed into one or both of the vortices and travelled close to the axis of the laminar vortex, as shown in Fig. 4. With considerable care, it was sometimes possible to obtain a fine filament lying along the axis of the vortex, but usually the dye filament became sheared into a tape which formed a cylindrical helix winding closely around the axis of the vortex. In either case it is convenient to refer to the dye trace as the axial filament of the laminar vortex. There appear to be two possible ways in which the axial filament can behave at the burst, as illustrated in Fig. 4. The lower vortex in the photograph shows an almost axisymmetric arrangement in which the dye has diffused over an ogival region. The upper vortex shows an asymmetrical spiral arrangement in which the dye remains as a discrete filament up to the point of turbulent breakdown. During observations at low speeds, the axisymmetric arrangement has been observed only occasionally and for short durations. The behaviour of the dye filament in the spiral arrangement is shown in more detail in Fig. 5 and can be considered in three successive stages, which proceeding in the downstream direction, are as follows:

(1) A sudden deceleration of the fluid moving along the axial filament.

(2) An abrupt kink where the axial filament is deflected into a spiral configuration which, persisting for a few turns, performs a whirling motion about the central axis.

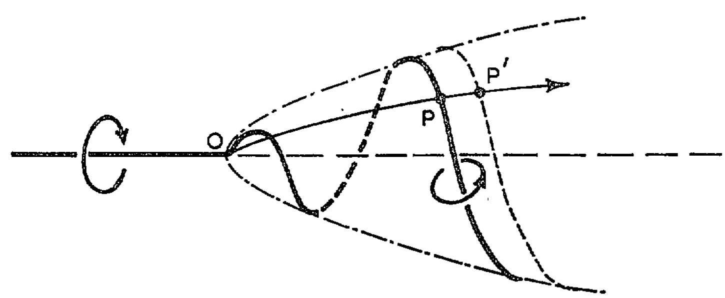
(3) A breakdown to large-scale turbulence.

The motion of dye along the vortex is shown in Fig. 6 by a series of ciné photographs separated by equal time intervals. Measurements of the movement of a dye front derived from film records are shown in Fig. 7 where distance along the vortex axis is plotted against time for two stream velocities. It can be seen that, consistent with other measurements along a laminar vortex[9], the front attains a velocity u_0 greater than that of the free stream V_∞. The high velocity is maintained until, just ahead of the kink there occurs a fairly sudden deceleration leading to a reduced longitudinal velocity in the spiralling region of only about one third of the original value.

At the kink in the axial filament there was often evidence of a cup-like shape in the dye, which probably represented an undeveloped stage in the formation of the axisymmetric arrangement already mentioned. When additional dye was emitted upstream of the burst and close to the central axis of the whirling spiral, a small amount was occasionally carried forward along an undulating path to the kink (Fig. 8a). Thereafter, it was entrained in the vortex motion around the distorted axial filament (Fig. 8b). It would seem that there is a narrow sinuous tongue of reversed flow within the coils of the spiral.

Fig. 9 shows other evidence of this reversed flow. Here, dye emitted from a tube ahead of the model has been introduced into the vortex in the form of a relatively wide cylindrical shell around the axis of the laminar vortex. It can be seen that downstream of the burst the dye from the cylindrical shell has been drawn between the coils of the spiral axial filament and carried upstream.

The spiralling region is best described by considering in turn (*a*) the behaviour of the dye filament considered as a whole, and (*b*) the motion of individual material particles which make up the filament. The whirling filament is essentially a conical spiral of fixed geometry rotating bodily about the original axis of the vortex and having the same sense of rotation as the laminar vortex. At any instant the form of the spiral is of opposite hand (*see* Fig. 5b) to a helical streamline of the laminar vortex. Due to the regular whirling, the flow in the neighbourhood of the spiral has a periodic component. The motion of a particle along the spiral was deduced from analysis of ciné film. The axial movement against time for a dye front as it passes through the spiralling region (Fig. 6) has already been mentioned. From an analysis of the axial movements of the crests of the spiral convolutions, as they appear on the ciné film, and a comparison with the measurements of the movements of dye fronts along the vortex, it can be deduced that, although the spiral rotates as a geometric form, a fluid particle moving along it has hardly any rotation about the central axis until it reaches the beginning of the turbulent breakdown.



This is illustrated in the sketch where the spiral is shown at a certain instant of time with a particular component particle at P; one convolution is shown at a later time when the same particle has moved to a position P′. The line OPP′, traced on an imaginary surface of revolution swept by the rotating spiral filament, shows the path of the particle originally deflected from the central axis at O.

There appears to be some rotation of fluid around the spiral filament itself and this, as indicated in the sketch, has a sense consistent with the reduced or reversed velocity existing within the coils. It would seem that the rotation about the spiral filament provides the mechanism by which external fluid is drawn within the coils of the spiral.

It has already been pointed out that the regular spiralling of the axial filament entails a periodic flow for a limited region downstream of the bursting point. With the flat-plate delta ($c_0 = 8 \cdot 5$ in.)

and for a bursting position approximately midway between the apex and the trailing edge, measurements of f, the frequency of spiralling obtained by stroboscopic observation were as follows:

V_∞	f	f/V_∞
2·4 in./sec	1·3 c/sec	0·56 c/in.
4·0	2·0	0·50
4·6	2·9	0·63

The measurement for 2·4 in./sec agrees closely with the value obtained from analysis of ciné records for the same condition.

Spiralling occurred at flow speeds up to 100 in./sec, the highest speed at which observations were attempted in the water tunnel, but at these higher speeds stroboscopic observation failed to reveal any definite frequencies; it would appear from these tests that, either the spiralling becomes irregular immediately downstream of the kink in the axial filament, or the region of periodic flow has contracted so much that it would not easily be detected by the stroboscopic method used.

A similar spiralling was revealed in slow-motion ciné films of smoke with the swept plate in the wind tunnel for speeds between 5 and 10 ft/sec. In this case the frequency associated with the spiralling was approximately 9 c/sec.

It is evident that high-frequency spiralling which may take place at high speeds of flow would appear both to an observer and in time-exposure photographs as a funnel-shaped diffusion of the axial filament.

With the third stage shown in Fig. 5b, the turbulent breakdown of the spiralling region, there is re-establishment of rotational flow in the vicinity of the central axis and, farther downstream, the flow could be described as a turbulent vortex, which appears weaker than the original laminar vortex.

4. *The Influence of Incidence and Sweepback.* The variable-sweep flat-plate wing shown in Fig. 1a was used to determine the influence of incidence and sweepback, the measurements of the burst position being made by observations of the water-vapour condensation trail obtained at a wind speed of 205 ft/sec. The results are shown in Fig 10 as a graph of l_b, the distance of the burst from the leading apex, against incidence for several values of the leading-edge sweepback. When the burst is in the vicinity of the trailing edge its position is highly sensitive to incidence; with even lower incidences the burst moved outside the range of possible observations. As shown in Fig. 11, it is found that when the distance l_b is plotted against $\gamma \equiv \cos^{-1}(\cos\alpha \sin\Lambda)$, the angle between the leading edge and free-stream direction, the results for the separate angles of sweep show a tendency to collapse to a single curve. It may be noted that γ is closely related to the angle between the vortex axis and the free stream and this angle provides an alternative parameter leading to a collapse of the curves.

When the angle between the leading edge and the flow becomes large (*i.e.*, at high incidence, low sweep or a combination of both), the burst moves to a position very close to the apex of the wing. Almost the whole of the vortex flow above the wing then has the turbulent character of a burst vortex. The disorderly nature of the flow within vortices arising from leading-edge separation for moderately swept wings with rounded leading edges[16] can probably be explained by a burst having occurred very close to the point at which the vortex originates.

The variation of burst position with incidence is also shown in Fig. 12 for a delta planform of 65 deg leading-edge sweepback. The only comparison which can be made with the measurements of Elle[3] refers to the incidence at which the burst occurs above the trailing edge. It will be seen from the Figure that such a comparison shows a large discrepancy which has not yet been explained.

5. *The Influence of Reynolds Number.* To examine the effect of Reynolds number on burst position, wind-tunnel and water-tunnel tests were carried out using the flat delta plates of 65 deg sweepback shown in Fig. 1. Burst positions measured over a wide range of conditions are plotted against incidence in Fig. 12. The wind-tunnel results for a Reynolds number of $4 \cdot 6 \times 10^6$, based on root chord, were obtained from observations of the sudden expansion of water condensation trails, whilst a smoke filament was used at the lower wind-tunnel Reynolds number of $0 \cdot 26 \times 10^6$; observations in water were of dye filaments in all cases.

Whereas the comparisons which can be drawn are of an approximate nature owing to the different techniques which were required, the unknown tunnel-interference effects and possible compressibility effects[9], the range of Reynolds number $0 \cdot 01 \times 10^6 < Re < 4 \cdot 6 \times 10^6$ is sufficiently great to justify the conclusion that the influence of Reynolds number on burst position is small, at least for wings with sharp leading edges. At the same time, it is of interest that a comparison of the curves for $Re = 0 \cdot 01 \times 10^6$ and $Re = 0 \cdot 08 \times 10^6$ shows changes of burst position, which, not being directly attributable to differences in technique or tunnel interference, can be regarded as a genuine effect; it is significant that between these two Reynolds numbers, measurements[9] have revealed a substantial increase in u_0/V_∞, the ratio of the axial to stream velocities, for a laminar vortex. A similar correlation is suggested by Fig. 7 where, for a change of stream velocity at low Reynolds number, a definite movement of the point of sudden deceleration accompanies a large change in the ratio u_0/V_∞.

6. *Effect of Bursting on Surface Flow Patterns.* The effect of a burst on the upper-surface oil-flow patterns is illustrated by the photographs and diagrams of Fig. 13 which relate to the swept plate with 60 deg sweepback. For the condition $\alpha = 10$ deg for which the burst occurs downstream of the trailing edge, the secondary separation line is straight except for an inboard curvature towards the tip. For the condition $\alpha = 20$ deg the burst which now occurs at a position ahead of station C has the effect of causing the secondary separation line to bend towards the leading edge at a position between stations C and D. The downstream distance between the burst and the bend in the secondary separation would seem to be consistent with the outward spread of turbulence from the burst. As shown in the line diagram of Fig. 13, the burst appears to have little effect on the surface-flow pattern inboard of line SS′ which is drawn as a continuation of the secondary separation line associated with the laminar portion of the vortex. Between SS′ and the actual separation line SS there is only slight curvature of the surface flow. The outward movement of the secondary separation line is consistent both with a reduction in the adverse pressure gradient experienced by the flow under the vortex, as suggested in the following paragraph, and with the delay in separation due to increased turbulence.

7. *Effect of Bursting on Surface Pressure.* Fig. 14 shows for the swept plate with 60 deg sweepback the upper-surface pressure distribution measured along lines normal to the leading edge at stations B and D for the three incidences 10, 15 and 20 deg. At 10 deg incidence the burst position is downstream of the wing but for the higher incidences it is above the wing, the positions projected in plan onto the upper surface being shown in the diagram. As would be expected, station B, which is ahead of the burst for each incidence, experiences an increase of peak suction with incidence. The peak suction at station D also increases between incidences 10 and 15 deg for which the burst position remains downstream of the measuring station. By 20 deg incidence the burst has moved well forward of station D and this has led to a pressure distribution which is much flatter than would be expected in the absence of the burst.

Pressure contours for the upper surface show a ridge of high suction which lies approximately along a ray through the apex. In Fig. 15 the pressure along this ridge is shown for two angles of sweepback and several values of incidence. With 70 deg sweepback the burst occurs downstream of the wing for all the incidences shown. It will be seen that under a laminar vortex there is, with increasing distance from the apex, a rising surface pressure and a decreasing pressure gradient. At a fixed position, the pressure gradient increases with incidence. With 60 deg sweepback the burst is still downstream of the wing for 10 deg incidence and by comparison of the curves for the two angles of sweepback at 10 deg incidence, the pressure gradient along the surface is seen to increase with decreasing sweepback. For the higher incidences at 60 deg sweepback the burst moves progressively upstream as indicated. It will be noted that close to the observed position of the burst the gradient is much steeper than that to be expected in the absence of the burst as suggested by the dotted line for 15 deg incidence; this increase in pressure gradient is consistent with the expansion of the flow around the burst. Downstream of the burst the pressure tends to a value which is independent of incidence.

The relation between the position of the burst and the surface pressure is also illustrated by Fig. 16 where $|-C_p|_{peak}/\alpha$ is plotted against the ratio distance of burst from apex to distance of station from apex. For each station the quantity $|-C_p|_{peak}/\alpha$ is approximately constant while the burst remains well downstream but its value falls when, at higher incidence, the burst approaches the measuring station. Unlike the surface-flow patterns the surface pressure begins to be affected even when the burst position is downstream of the measuring station.

8. *Influence of Various Factors on Burst Position.* 8.1. *Perturbations to Flow Upstream of Burst.* Before observations of the details of the bursting process were made, some consideration was given to the possibility that the phenomenon was connected with one or more of the following:

(*a*) the amplification of disturbances travelling downstream along the laminar vortex;

(*b*) the turbulent breakdown of the shear layer from the leading edge;

(*c*) some critical change in the shear layer leaving the upper surface at the secondary separation.

To test these possibilities, several wind-tunnel experiments using the swept-plate model of Fig. 1a were carried out for the same basic conditions $\Lambda = 65$ deg, $\alpha = 22 \cdot 5$ deg, $V_\infty = 180$ ft/sec, at which water-vapour condensation showed the burst to occur just in front of station D.

The tests showed that the modifications set out in the following table had no significant effect on the burst position.

Modification	Purpose
Wire 0·03 in. dia. on lower surface of wing between attachment line and leading edge	Modification of shear layer leaving leading edge by causing thickening of boundary layer
Thin plate 1 in. wide projecting $\frac{5}{16}$ in. from leading edge, in plane of wing surface	Distortion of shear layer from leading edge
Upper-surface spoiler 12 in. long $\frac{3}{8}$ in. high aligned with, and directly below, axis of vortex	To modify secondary separation
Spoiler 18 in. long and 1 in. high on tunnel wall ahead of leading apex	To cause disturbance to flow entering vortex core
Plasticine fairing at leading apex	Modification of shear layer feeding into the axis of vortex

8.2. *Body Introduced along the Axis of Vortex.* Experiments in which bodies of revolution were introduced along the axis of a vortex were carried out in the water tunnel with the delta plate and in the wind tunnel with the swept plate.

A thin body could be inserted in the laminar vortex without any obvious effect on the position of the burst but the insertion of a large body would precipitate a burst. The distance of the burst ahead of a large body inserted along the vortex axis decreased with progressive forward movement of the body ahead of the natural burst position (*i.e.*, the position in the absence of the body). With a similar progressive introduction of a body of only moderate size (*e.g.*, $\frac{1}{2}$ in. dia. rod for the swept-plate model) the burst would finally occur at some position along the body.

It appears that the effect of a body on the burst position depends on the size of the body and its axial position relative to the natural burst. This suggests that useful probe measurements may be possible even close to a natural burst provided the probe is small.

8.3. *Acceleration and Deceleration of Stream.* Experiments with the delta plate in the water tunnel showed that the position of the burst was sensitive to acceleration or deceleration of the stream. The tests were made with the incidence adjusted so that under steady conditions the burst occurred above the wing. The initial flow speeds ranged between 2 and 16 in. per sec and changes of speed were obtained by suddenly altering the setting of the tunnel speed control. As soon as the speed began to increase, the bursting position moved upstream along the vortex and remained at a more forward position whilst the speed was increasing. As the speed approached its new steady value the burst returned to its original position. The opposite effect occurred when the stream was suddenly decelerated, the burst first moving downstream and then gaining its original position as the speed reached a steady value.

8.4. *Removal of Burst by Suction.* Following a suggestion originating from Werlé[6] it was found that a burst occurring above the swept plate in the wind tunnel could be eliminated by suction applied just downstream of the original bursting position. A vacuum-cleaner nozzle facing upstream was found to be effective at very low speeds, the burst being shown by smoke. Once the burst had been 'caught' by the nozzle, the laminar régime could be extended for several feet by gradually moving the nozzle rearwards along the axis of the vortex. Eventually the laminar régime would break down at some point ahead of the nozzle from whence a burst would move forward to the original position.

8.5. *Deflection of Trailing-Edge Flap and Effect of Camber.* The semi-circular tip of the swept plate was removed and replaced by a rectangular piece of sheet metal which could be bent to represent a deflected trailing-edge flap. When the incidence was adjusted so that a burst occurred approximately midway between apex and tip, upwards deflection of the tip flap resulted in a forward movement of the bursting position; a downward deflection led to a rearward position for the burst.

Observations of vortex bursting were provided incidentally during a water-tunnel investigation using a model having an extremely high chordwise camber. The model was a delta plate with 80 deg leading-edge sweepback which had been bent into a circular arc such that the local incidence at the apex differed from the local incidence at the trailing edge by 47 deg.

When the plate was supported so that the local incidence was zero at the apex and 47 deg at the trailing edge, no burst occurred in the neighbourhood of the wing (Fig. 17a). When the incidence was 47 deg at the apex and zero at the trailing edge, bursting occurred at a position about 1/3 root chord from the apex (Fig. 17b).

9. *Compressibility Effects.* No direct attempt to investigate the effects of compressibility on bursting has been made below critical Mach numbers. There was no obvious effect on burst position up to wind speeds of 180 ft/sec at which, due to the low pressures near the axis, compressibility must begin to play a part[9] as evidenced by the appearance of a condensation trail.

A few observations were made at transonic speeds with a sharp-edged delta plate having 50 deg leading-edge sweep. In the side-elevation schlieren photographs of Fig. 18, the leading-edge vortices are manifest because of the change of density gradient across their axes. Over the plate there is a local region of supersonic flow terminated by a shock wave which intersects the leading-edge vortices. Surface oil patterns suggest that due to curvature of the shock, the actual position of intersection is slightly behind the apparent position seen in the photographs. In Fig. 18a for $M = 0 \cdot 90$ an additional expansion of the vortex is seen downstream of the shock wave, and slightly farther downstream, the vortex axis is no longer clearly defined and there is a spread of turbulence. In addition, the surface oil patterns indicate a similar outboard displacement of the line of secondary separation to that found at low speeds. These observations suggest that an occurrence comparable to the low-speed vortex bursting takes place at the shock wave. In view of the sensitivity of bursting to pressure gradient, found at low speeds, it is reasonable to associate a shock-induced burst with the steep pressure rise at the shock wave. With increase of Mach number to $M = 0 \cdot 95$ (Fig. 18b) the position of vortex breakdown has moved rearwards with the shock.

10. *Discussion.* Before advancing a possible explanation of the burst phenomenon it is convenient to discuss the evidence of the experiments already described, also the general nature of a laminar vortex.

10.1. *Factors which Influence the Position of Burst.* For high values of sweepback and low incidences, a burst may occur downstream of the wing and it appears probable that the actual position is then independent of the wing geometry. For higher values of incidence a burst occurs ahead of the trailing edge and its position with respect to the wing then depends on a combination of incidence and sweepback. For the swept plate, the measurements suggest that the distance of the burst from the apex is related to the value of $\sin \Lambda \cos \alpha$.

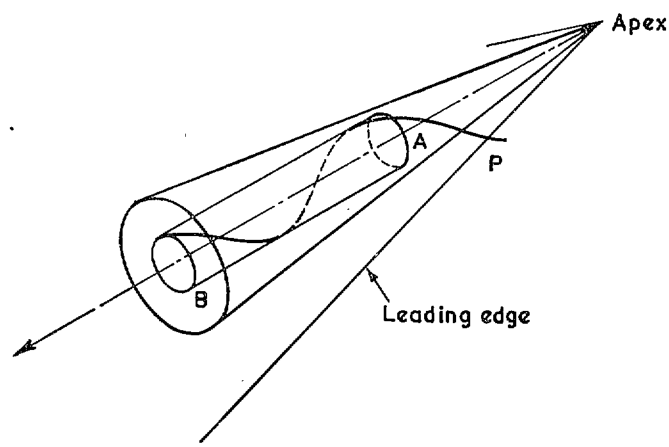
The following discussion concerning the sensitivity of the burst position to additional factors refers only to the condition in which the burst is occurring ahead of the trailing edge.

For a fixed incidence and sweepback, the burst position when related to the planform of the wing is approximately independent of size of wing and, apart from compressibility effects, independent of stream velocity. In other words, the non-dimensional burst position l_b/L is not significantly dependent on a Reynolds number.

The fact that the burst position is not sensitive to small disturbances to the laminar vortex suggests that bursting is not dependent on the amplification of small upstream disturbances.

The experiments have shown conclusively that the burst position is sensitive to the pressure distribution along the vortex. Thus the length of the laminar part of the vortex is extended by a reduction of the longitudinal pressure gradient dp/dl occurring locally either ahead of a downward deflected trailing-edge flap or ahead of the point where suction is applied at the axis of the vortex. An upward deflected flap or a bluff body in the vortex causes an increase in the pressure gradient and both tend to move the burst position upstream. The transient effects of increasing or decreasing the stream speed can also be related to changes in the longitudinal pressure gradient by the following argument.

We shall consider the vortices formed by a delta wing of infinite extent. Each vortex would consist of a rotational core growing in the form of a cone from the apex.



We wish to find the effect of a change of stream speed on the pressure difference between two rings A and B at the extremities of a near-cylindrical stream surface within the rotational core, A and B being fixed relative to the apex of the wing. We regard the rotational core as composed of concentric layers of fluid which has been set in rotation whilst passing across the leading edge and then convected along the length of the vortex. Thus rotation of the rings of fluid at both B and A was determined by the flow in the vicinity of some upstream point P on the leading edge. The radial distribution of pressure across the vortex and hence, to a large extent, the pressure field throughout the vortex core can be regarded as a consequence of the rotational components within the core. We make the assumption that, during a change of stream speed, the form of the vortex is unaltered and that the rate of convection of fluid along the vortex changes instantaneously with change of stream speed.

The final rotational velocities within the core will be proportional to the new stream speed but, due to the manner in which the vortex is formed, the rotational velocity at A will be affected before that at B. Thus A will experience pressure conditions appropriate to the new stream velocity before B. It can be expected that A would reach its new steady condition in an approximate time l_A/u, and B in an approximate time l_B/u, u being the new velocity component along the vortex.

If the change is an increase of speed, then A will experience its full fall of pressure before B, so that after the change is made but before the new steady condition has been attained, the pressure difference $p_B - p_A$ will increase. Thus it may be inferred that for this short period the non-dimensional pressure gradient in the axial direction $\partial C_p/\partial l$ is greater than the steady-motion value. The changes in burst position due to a changing stream speed are then consistent with the previously observed sensitivity of the burst to longitudinal pressure gradient.

The effects of chordwise camber can also be explained by the sensitivity of the burst position to the longitudinal pressure gradient. Where the local incidence of the leading edge decreases with increasing distance from the apex, *i.e.*, with negative chordwise camber, the rotational components fed to the outer annuli would be progressively less with distance downstream than those for an uncambered wing. Thus negative camber would tend to increase the longitudinal pressure gradient.

10.2. *Structure of a Laminar Vortex.* Although a leading-edge vortex is formed by the rolling-up of a vortex layer from the leading edge, it is possible for many purposes to consider the vortex as a core of rotational flow which, being fed by vorticity from the leading edge, increases in size and strength with distance downstream. Over the forward part of the wing the axis of the vortex is approximately straight and at an angle to the free stream but as the trailing edge is approached it curves towards the free-stream direction.

The available measurements[9, 10] which refer to a laminar vortex above the swept plate having 65 deg sweepback and 15 deg incidence show that the rotational velocity component within the core is substantially constant and about $1 \cdot 2V_{\infty}$ except very close to the axis of the vortex where the rotational component is falling to zero. The velocity component parallel to the axis of the vortex increases with approach to the axis, values as high as $3V_{\infty}$ having been measured at the axis.

Following Hall's theoretical vortex structure[13] it is possible to consider the rotational core as consisting of the two parts, an inviscid but rotational annulus surrounding an inner viscous core. In the particular case referred to above, it would appear reasonable to associate the rotational annulus with the region of approximately constant swirl velocity, and the viscous core with the region in which the swirl velocity falls to zero at the axis.

The inner viscous core appeared small in relation to the complete rotational core although precise measurements of its size and rate of growth are not yet available. It would be expected, however, that its growth would be a function of a Reynolds number based on distance from the apex and by analogy with the time rate of growth of a vortex filament[17], it seems likely that the radius is proportional to $(\nu l/V)^{1/2}$.

The rotational annulus would also grow because of the additional vorticity generated at the leading edge. For a delta wing of infinite extent, on the basis of conical flow, the outer radius of the annulus and thus the total strength of the vortex would be proportional to the distance from the apex. For a delta wing of finite extent, we might expect an approximately linear growth of the rotational core over the forward part of the wing followed by a falling rate of increase of vortex strength as the trailing edge is approached. Vorticity from the leading edge is then convected over a greater distance before entering the rotational core and some additional diffusion may be expected in this region. Little or no further increase in strength can be expected in the wake behind the wing.

10.3. *Pressure Distributions for a Laminar Vortex.* Pressure measurements[9] for the wing condition 65 deg sweepback, 15 deg incidence have shown that, for the laminar vortex ahead of the trailing edge, the static pressure at the axis decreases with increasing distance along the vortex, the fall being particularly steep in the neighbourhood of the apex. The total pressure at the axis, after showing a very steep fall close to the apex has a nearly constant value along the length of the vortex. The value of the total pressure at the axis is very low indeed being below the free-stream static pressure by as much as 5 times the stream dynamic pressure (*i.e.*, $\frac{1}{2}\rho V_{\infty}^{2}$). This low value of total pressure at the axis can be associated with a deficiency of rotational velocity brought about by viscous diffusion within the narrow inner core. For fluid at the axis, this diffusion must take place very close to the apex and once this has occurred, the value of total pressure at the axis would be expected to remain fairly constant with distance from the apex as shown by the measurements.

In the absence of more extensive information it is necessary to speculate on the more general form of the pressure distributions along a vortex. Bearing in mind that the vortex is embedded in the type of flow general to the upper surface of lifting bodies, we must expect that above the wing the longitudinal velocity component u outside the vortex core will be greater than the free-stream velocity V_∞ (for $\Lambda = 65$ deg, $\alpha = 15$ deg measurement gives $u = 1 \cdot 1 V_\infty$). Towards the trailing edge we must expect the usual deceleration of this component and an associated pressure recovery together with a general curvature of the flow towards the free-stream direction as indicated by the bending of the vortex axis. Along the axis of the vortex, the pressure distribution would be determined by the balance of three factors, namely:

(i) increasing strength of the vortex tending to provide a falling axial pressure,

(ii) diffusion of vorticity within the viscous core tending to give a rising axial pressure,

(iii) deceleration of the longitudinal component in the irrotational flow tending to cause a rising axial pressure. This factor is associated with the presence of the trailing edge.

For an infinite delta wing, only factors (i) and (ii) are operative and provided the rate of increase of vortex strength is greater than the rate of diffusion in the viscous core, the axial pressure would be expected to fall. As already mentioned, it is reasonable to assume that the radius of the viscous core is proportional to $l^{0 \cdot 5}$; in this case, the above statement is consistent with observations[10] over the forward part of the wing which show a falling axial pressure associated with a growing rotational core having a radius proportional to $l^{0 \cdot 8}$.

Ignoring for the moment the effects of factor (iii), the balance between (i) and (ii) would be expected to change even before the trailing edge is reached because both the rate of generation of vorticity and the rate at which this is added to the vortex are falling. Eventually in the wake we would expect the diffusion of vorticity to be the dominating factor and lead to a rising axial pressure.

The consequences of (iii), a reduction of the longitudinal velocity component and thus a pressure recovery in the irrotational flow, would appear to be as follows. On the ground of continuity a longitudinal retardation leads to an expansion of the stream tubes and thus to an enlargement of the rotational core quite apart from that caused by diffusion; on the basis of constancy of circulation there would then be a general fall of the rotational velocities within the core which augments the pressure rise near the axis. This is illustrated in a simple example in Appendix II which suggests that the longitudinal pressure changes experienced within the core of a vortex are greater in magnitude than those occurring in the outer regions of the vortex. This sensitivity of the vortex core to the outer conditions may be similar to that noted by Hall[18].

In summary, it seems reasonable to expect that along a laminar vortex there would be a falling axial pressure near the apex followed by a rising pressure associated with the position of the trailing edge. In the wake, diffusion of vorticity would lead to a further rise in the axial pressure. In contrast, a low total pressure would be maintained throughout the length of the vortex.

10.4. *Mechanism of Bursting.* Based on the foregoing arguments, it is now possible to advance a plausible explanation of the phenomenon of bursting. An important feature is believed to be the very low total pressure of the fluid close to the axis, which means that the axial flow is easily brought to rest. The anticipated axial distributions of static and total pressure which have been outlined, would lead to the axial fluid being brought to rest either by the pressure rise associated with the existence of the trailing edge or by a subsequent pressure rise in the wake. But a gradual axial pressure rise would imply a gradual retardation of the axial flow which is not in keeping with the observed sudden deceleration immediately ahead of a burst. However, in the light of the analysis in Appendix II, it appears that a gradual pressure rise in the outer regions of the flow surrounding a vortex can produce a much steeper rise along a streamline near the axis. In consequence, the external changes needed to bring the axial flow to rest may be small, the sensitivity depending on the ratio v/u, the rotational to axial velocity components. Furthermore it seems possible that the mutual interactions between axial deceleration, vortex expansion and pressure increase could lead to a process which becomes unstable for certain critical conditions. Indeed the existence of a critical value of the ratio v/u has previously been suggested by Squire[8] from other considerations.

Even in the absence of a critical expansion of the vortex, it is reasonable to suggest that the stagnation of the axial flow, however gradual, would lead to a breakdown of the orderly vortex flow at some downstream position. Once such a breakdown of flow has occurred, it would be possible for fluid from the outer regions to gain access to the vortex axis and little resistance would be offered to the upstream passage of fluid from this relatively high-pressure source.

In brief, it seems plausible to regard the occurrence of a burst above the wing as being due to the usual pressure recovery associated with a trailing edge and to regard the position of the burst as being determined by a balance of various interdependent factors dictated by the geometry of the wing.

In general, if the pressure rise associated with the existence of a trailing edge is insufficient to stagnate the axial flow, a burst would not occur in the vicinity of the wing but may occur during a subsequent axial pressure rise due to vorticity diffusion in the wake. In a wind tunnel, the burst position when downstream of the wing may also be dependent on the pressure rise in the diffuser.

It remains to consider the observed spiralling of the axial filament in relation to the suggested mechanism of bursting. It might be suggested *a priori* that the vortex would expand in an axisymmetric manner about the stagnation point in the axial flow, and it will be recalled that for brief periods (*see* Fig. 4) the behaviour of an axial filament did show such an arrangement. However, it seems probable that an axisymmetric arrangement is unstable so that there is a strong tendency for this to collapse into the spiral configuration. It is therefore suggested that the spiral configuration is a secondary feature in the process of bursting and represents the preferred method by which the remainder of the vortex flow negotiates the stagnated axial flow.

In a broad sense there would seem to be some similarity between vortex bursting and flow separation from a surface in the presence of a positive pressure gradient, since both involve the bringing to rest of low-energy fluid. Also, there is a rather loose analogy between a burst and a normal shock wave in compressible flow, both being associated with an instability in decelerating flow and both providing a sudden increase of pressure.

11. *Conclusions.* 1. The observations have shown that bursting involves a sudden deceleration of the axial flow accompanied by expansion of the vortex around a stagnant core. A short distance farther downstream a breakdown to turbulent flow occurs.

2. At least for low values of Reynolds number, there is usually, between the position of axial deceleration and the turbulent breakdown, a region of periodic flow in which the axial filament performs a regular whirling motion.

3. The presence of a burst above the wing causes a loss of suction locally at the surface and a modification to the position of separation of the surface flow beneath the vortex.

4. When the burst is upstream of the trailing edge, (*a*) its position depends on a combination of incidence and leading-edge sweepback and (*b*) in relation to the geometry of the wing, its position is largely independent of Reynolds number.

5. The burst position is sensitive to the pressure gradient along the vortex, a reduction in the gradient being conducive to a longer laminar vortex.

6. An essential feature for bursting to occur is believed to be a low total pressure at the axis of the laminar vortex.

7. A prerequisite for the flow at the axis of a laminar vortex to stagnate is a positive gradient of static pressure along the vortex.

8. The required positive pressure gradient could be attributable to (*a*) viscous actions within the core of the vortex or to (*b*) deceleration of the flow external to the core. A small change in the external flow may suffice because an external pressure gradient becomes magnified towards the axis of the vortex.

9. It is possible that under certain conditions depending on the ratio of the rotational to axial velocity components, spontaneous expansion of a vortex occurs and provides the pressure rise necessary for stagnation of the axial filament.

10. Once the conditions necessary for the occurrence of a burst have been met, its final position is probably determined by the extent to which fluid from the turbulent region formed downstream of the burst can penetrate upstream along the axis of the vortex.

11. It is suggested that a burst situated above a wing may be attributed to the pressure recovery associated with the existence of the trailing edge. A relation between burst position and pressure distribution would be consistent with the observed effects of incidence and sweepback.

12. Further understanding of vortex bursting is likely to come with increasing knowledge of the structure of a laminar vortex. However, the most useful information would be obtained directly from measurements of pressures within the bursting region.

12. *Acknowledgements.* Messrs. J. F. M. Maybrey and D. F. Bedder made some of the low-speed observations, and observations at transonic speed were made by Mr. G. F. Lee. Acknowledgement is also due to the N.P.L. Central Photographic Section for help in producing photographs and ciné film.

LIST OF SYMBOLS

c_0		Root chord of delta wing
C_p	$\equiv$	$\dfrac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$, pressure coefficient
f		Frequency of spiralling
l		Distance along vortex measured from leading apex
l_b		Distance of burst from leading apex
L		Reference length on model
M		Mach number
p		Pressure
p_∞		Pressure in free stream
R_e	$=$	$\dfrac{V_\infty c_0}{\nu}$, Reynolds number
u		Axial velocity component
u_0		Axial velocity along axis
v		Rotational velocity component
V_∞		Velocity of free stream
x		Chordwise distance from apex of delta wing
α		Angle of incidence
γ	$\equiv$	$\cos^{-1}(\cos\alpha \sin\Lambda)$, angle between leading edge and free-stream direction
Λ		Sweepback of leading edge
ν		Kinematic viscosity
ρ		Density

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APPENDIX I

Experiments with a Vortex Tube

It was considered that an investigation of bursting might be worthwhile with a vortex less complicated than that produced by a swept leading edge. For this purpose the water vortex tube shown in Fig. 19 was constructed on the lines of a design used by Titchener and Taylor-Russell[19] and a series of preliminary observations were made by Gartshore[20]. The speed of water flow in the tube was variable up to 7 in. per sec.

It was found that a breakdown of the vortex flow occurred (*see* Fig. 20) and this bore some resemblance to the bursting of a leading-edge vortex. In particular, there was similarity in the sudden deceleration of the fluid moving along the vortex axis, also in the development of a central core of stagnant flow. Unlike the phenomenon with the leading-edge vortex no spiralling of the axial filament was observed. Increasing the ratio of the rotational to the longitudinal velocity components moved the burst position upstream; this movement is consistent with the behaviour of the burst of a leading-edge vortex when the strength of the vortex is increased by raising the incidence. However, in certain respects, the behaviour of the tube vortex differed considerably from that of the leading-edge vortex. There was some evidence of a Reynolds number effect in that increasing the flow speed or decreasing the viscosity moved the burst upstream, but further experiments are needed before a firm conclusion can be drawn.

The effects of suddenly changing the stream speed were directly opposite to those for a leading-edge vortex. When the flow along the tube was suddenly decelerated the burst immediately moved rapidly upstream (*i.e.*, in the reverse direction to that for a leading-edge vortex). The flow pattern was then obscured by a large-scale unsteadiness from which a new burst emerged downstream of the original position. With a sudden acceleration of the flow, the original burst appeared to be washed downstream and, following a large-scale disturbance similar to that for the decelerated flow, a new burst was established upstream of the original position.

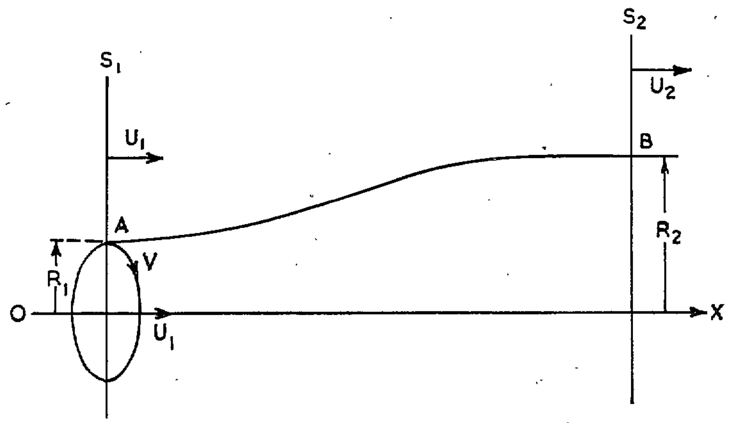
There are, of course, considerable structural differences between a tube vortex and a leading-edge vortex. In generating the tube vortex, angular momentum about the longitudinal axis of the tube is imparted to the fluid during its passage between the swirl vanes. It might be expected that since each ring of fluid around the axis of the tube has received the same angular momentum, the vortex would have the structure of a potential vortex with infinite vorticity concentrated along the axis. In reality the vorticity would be contained within a central core which, as pointed out by Gartshore[20] is fed by the boundary layer on the central body at the inlet to the tube (*see* Fig. 19). Unlike the leading-edge vortex with its feeding vortex sheet from the wing leading edge, the tube vortex would have no increase of strength along its length and, because of diffusion in the core, we might expect an adverse axial pressure gradient offset to some extent by a falling pressure in the irrotational flow due to the growth of boundary layer on the wall of the tube. A further feature peculiar to the tube vortex is the constricting effect of the tube wall which must play a large part in determining both the stability of the vortex flow and the actual form of the burst when it occurs.

In considering the effects of a sudden change of flow rate we note that for any cross section of the laminar vortex, the pressure at the axis is determined by the pressure at the wall together with the distribution of the rotational velocity component along a radius. Further, the rotational components were determined at some earlier time by the passage of fluid between the swirl vanes. Thus, with a change of flow rate, although the flow at the swirl vanes reaches its new condition almost instantaneously, the new steady state at any position along the tube is delayed, the flow in the tube having the new axial velocity combined with an angular momentum corresponding to the original condition. During this period of delay, if the ends of the tube are ignored, the change in the axial velocity may be regarded as a change in the velocity of the observer relative to the fluid. Thus with an increase in this relative velocity, the burst would be expected to move downstream relative to the observer. Later, a discontinuity in the rotational conditions would be convected along the tube after which a burst would appear at a new equilibrium position. This sequence of events is consistent with the observed effects on the burst.

From these experiments it is concluded that although there are fundamental structural differences between a tube vortex and a vortex generated by a sharp leading edge, the study of bursting in a vortex tube provided with means for varying the axial pressure gradient could yield basic information relevant to the understanding of both types of flow.

APPENDIX II

The Effects on a Vortex of Longitudinal Retardation



Let sections S_1 and S_2 represent two stations along an axisymmetric streaming vortex whose axis is OX. Let AB be the edge of the core outside of which the flow is irrotational. The flow is assumed to be incompressible and inviscid between S_1 and S_2. The latter assumption implies that AB represents an axisymmetric stream tube.

The conditions at S_1 are prescribed as follows:

(*a*) the translational velocity component is uniform across the vortex and equal to U_1 and,

(*b*) the core has 'solid body' rotation. That is, the rotational component at radius r_1 is

$$v_1 = V\frac{r_1}{R_1} \text{ (for } r_1 \leqslant R_1) \tag{1a}$$

and

$$v_1 = V\frac{R_1}{r_1} \text{ (for } r_1 \geqslant R_1), \tag{1b}$$

where V is the rotational component at the edge of the core at S_1 and R is the radius of the core.

We postulate that a general retardation of the translational motion occurs downstream of S_1, but that radial equilibrium has been restored ahead of S_2. Retardation implies an expansion of the flow so that any particular stream tube has radii r_1 and r_2 at S_1 and S_2 respectively where $r_2 > r_1$.

Conservation of angular momentum gives the equation, $v_1 r_1 = v_2 r_2$, which allows the rotations at S_2 to be related to those at S_1, thus

$$v_2 = V\frac{r_1}{R_1}\frac{r_1}{r_2} \text{ (for } r_2 \leqslant R_2) \tag{2a}$$

$$v_2 = V\frac{R_1}{r_1}\frac{r_1}{r_2} = V\frac{R_1}{r_2} \text{ (for } r_2 \geqslant R_2). \tag{2b}$$

It can be shown that over the region of irrotational flow at S_2, a uniform translational component U_2 provides the required compatibility between Bernoulli's equation and the condition of radial equilibrium. The application of Bernoulli's equation gives the pressure rise between S_1 and S_2 along a streamline in the irrotational flow,

$$\begin{aligned} p_2 - p_1 &= \tfrac{1}{2}\rho(U_1^2 - U_2^2) + \tfrac{1}{2}\rho(v_1^2 - v_2^2) \\ &= \tfrac{1}{2}\rho(U_1^2 - U_2^2) + \tfrac{1}{2}\rho V^2 \left(\frac{R_1}{r_1}\right)^2 \left[1 - \left(\frac{r_1}{r_2}\right)^2\right]. \end{aligned} \tag{3}$$

For an outer streamline, when $(r_1/R_1) \to \infty$, we have

$$(p_2 - p_1)_\infty = \tfrac{1}{2}\rho(U_1^2 - U_2^2). \tag{4}$$

From Equations (3) and (4) we see that in the irrotational region the pressure rise along an inner stream tube that undergoes an expansion [*i.e.*, $(r_1/r_2) < 1$] is greater than that along an outer stream tube. In particular, the edge of the core suffers a pressure rise which is given by the following equation

$$P_2 - P_1 = \tfrac{1}{2}\rho(U_1^2 - U_2^2) + \tfrac{1}{2}\rho V^2 \left[1 - \left(\frac{R_1}{R_2}\right)^2\right] \tag{5}$$

where P is the pressure at the edge of the core.

We now consider the changes that occur within the core for the purpose of showing that the translational component cannot remain uniform across the core.

A uniform retardation, if it occurred, would lead to a uniform expansion of the flow within the core [*i.e.*, $(r_2/r_1)^2 = U_1/U_2 =$ const.], and therefore from Equation (2a), the distribution of rotational velocity across the core at S_2 would be similar to that at S_1. The pressures p_A at the axis obtained from the condition of radial equilibrium are:

$$\left.\begin{aligned} p_{A1} &= P_1 - \tfrac{1}{2}\rho V^2 \\ p_{A2} &= P_2 - \tfrac{1}{2}\rho V^2 \left(\frac{R_1}{R_2}\right)^2 . \end{aligned}\right\} \qquad (6)$$

From which, by using Equation (5), we have

$$p_{A2} - p_{A1} = \tfrac{1}{2}\rho(U_1^2 - U_2^2) + \rho V^2 \left[1 - \left(\frac{R_1}{R_2}\right)^2\right]. \qquad (7)$$

But Equation (7) is incompatible with the application of Bernoulli's equation to the axial streamline which gives

$$p_{A2} - p_{A1} = \tfrac{1}{2}(U_1^2 - U_2^2). \qquad (8)$$

We may then infer that the retardation cannot be uniform within the core, and that the velocity u_2 at the axis at S_2 must be less than U_2 in accordance with the larger pressure rise given by Equation (7).

The actual retardation which must occur within the core would lead to an expansion of the core which is larger than that given by $(R_2/R_1)^2 = U_1/U_2$. Therefore the actual pressure rise along the axis will be

$$\Delta p_A > \tfrac{1}{2}\rho(U_1^2 - U_2^2) + \rho V^2 \left[1 - \frac{U_2}{U_1}\right]. \qquad (9)$$

Stagnation of the axial flow would occur when

$$\Delta p_A = \tfrac{1}{2}\rho U_1^2 . \qquad (10)$$

Therefore, a sufficient condition for the axial flow to have stagnated between S_1 and S_2 is

$$-\tfrac{1}{2}U_2^2 + V^2\left(1 - \frac{U_2}{U_1}\right) > 0$$

or when $U_2 = U_1 - \Delta U$

$$\left(\frac{V}{U_1}\right)^2 \frac{\Delta U}{U_1} - \tfrac{1}{2}\left(1 - \frac{\Delta U}{U_1}\right)^2 > 0 . \qquad (11)$$

When $(\Delta U/U) < 1$, Relation (11) is satisfied when

$$\left(\frac{V}{U_1}\right)^2 \frac{\Delta U}{U_1} > \tfrac{1}{2} . \qquad (12)$$

Probably as a criterion, Relation (12) is unduly severe, but it does serve to indicate that for large values of (V/U_1), a small change of the translational component in the irrotational flow can be sufficient to stagnate the axial flow.

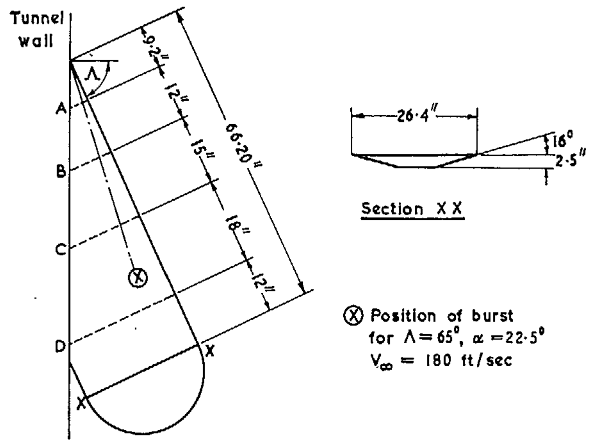
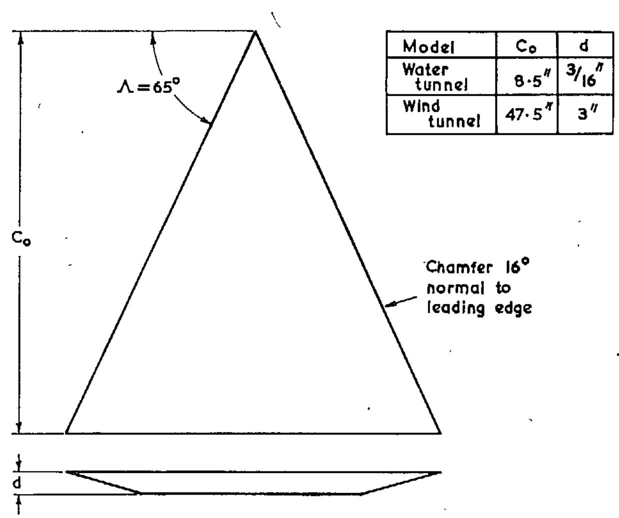


FIG. 1a. Sharp-edged swept plate, 9 ft × 7 ft Wind Tunnel.



Model	Co	d
Water tunnel	8·5″	3/16″
Wind tunnel	47·5″	3″

FIG. 1b. Sharp-edged delta plates.

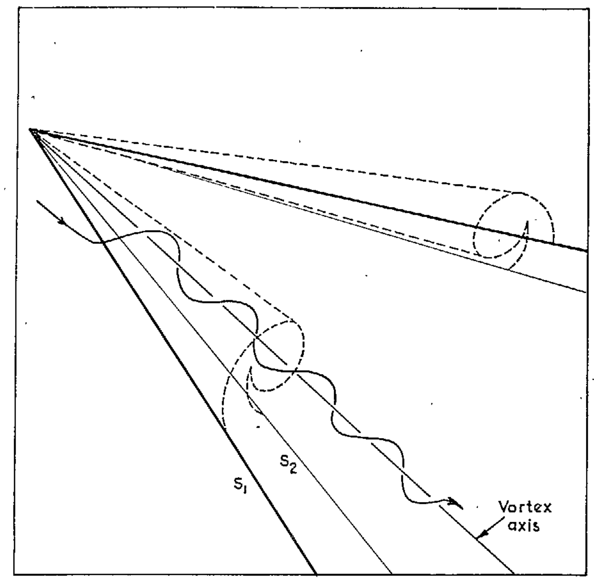


FIG. 2. Formation of laminar vortices for sharp-edged delta plate.

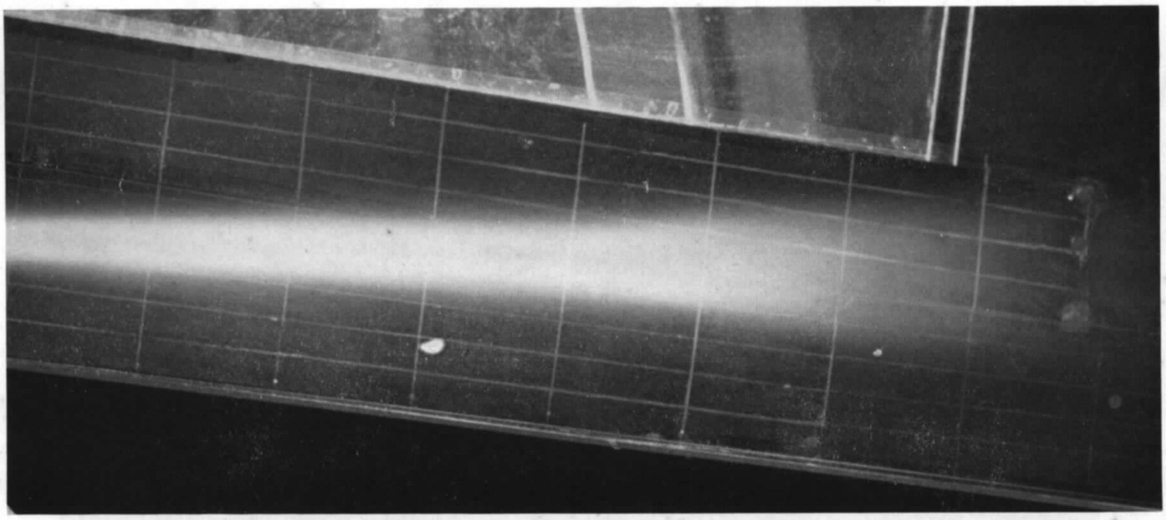


FIG. 3. Expansion and diffusion at burst shown by condensation along vortex. Wind tunnel, 180 ft/sec.

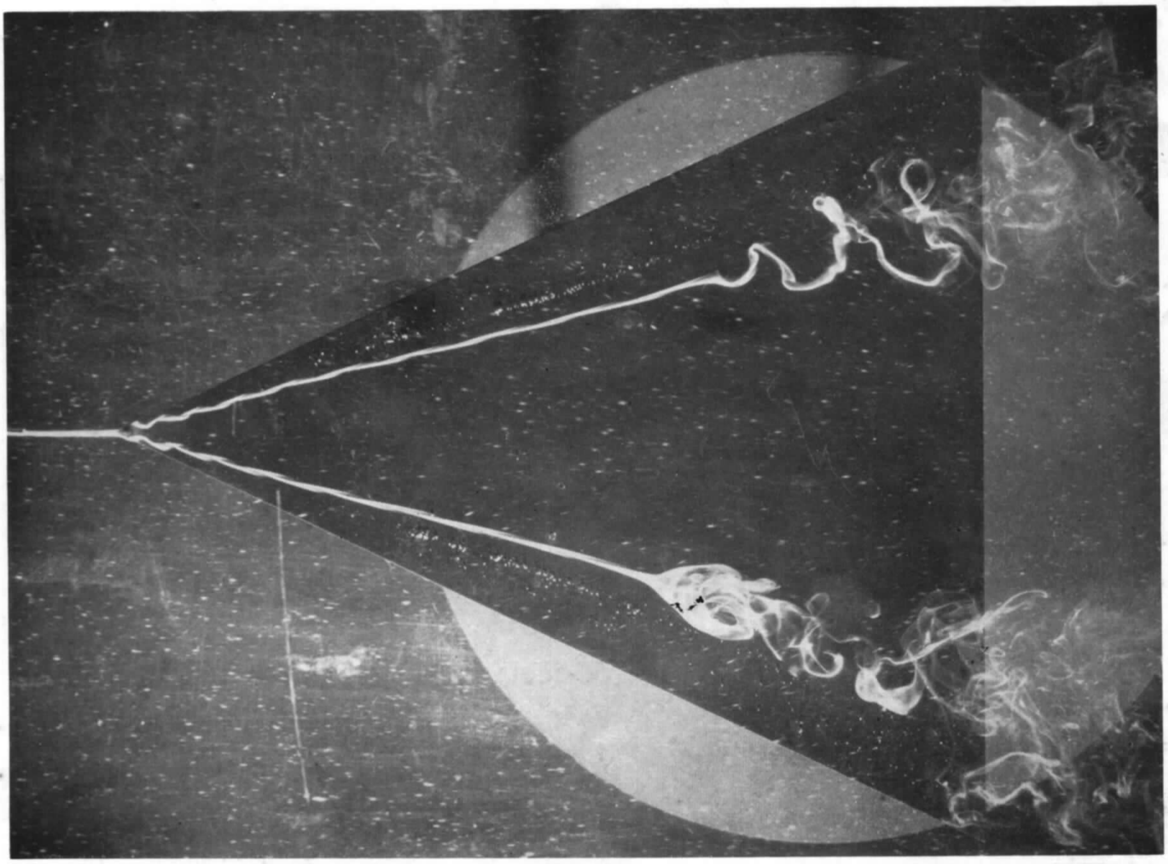


FIG. 4. Axial filaments of dye. Water tunnel, 2 in./sec.

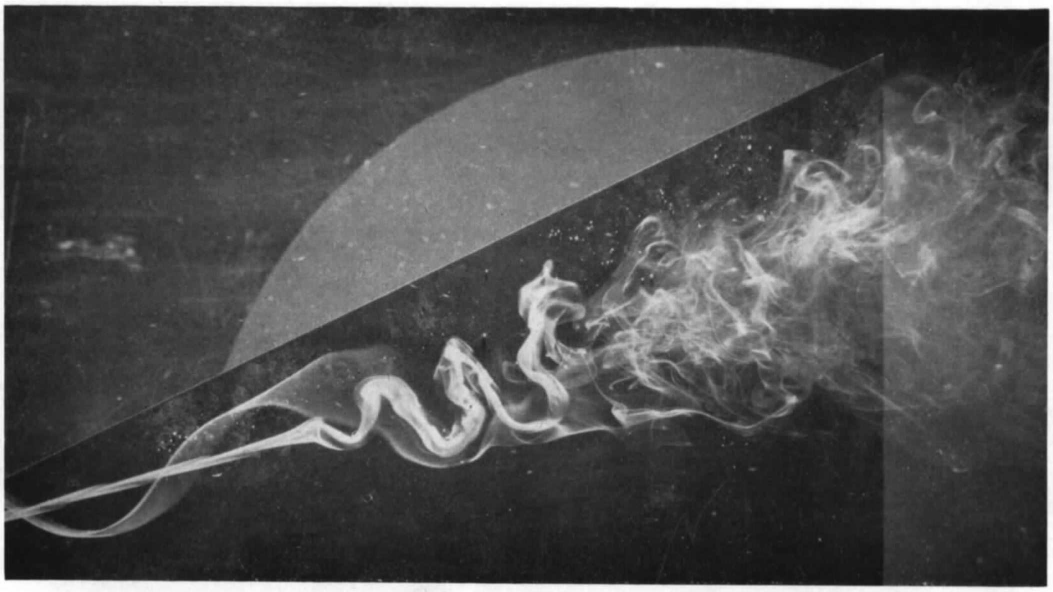


FIG. 5a. Details of bursting process. Water tunnel, 2 in./sec.

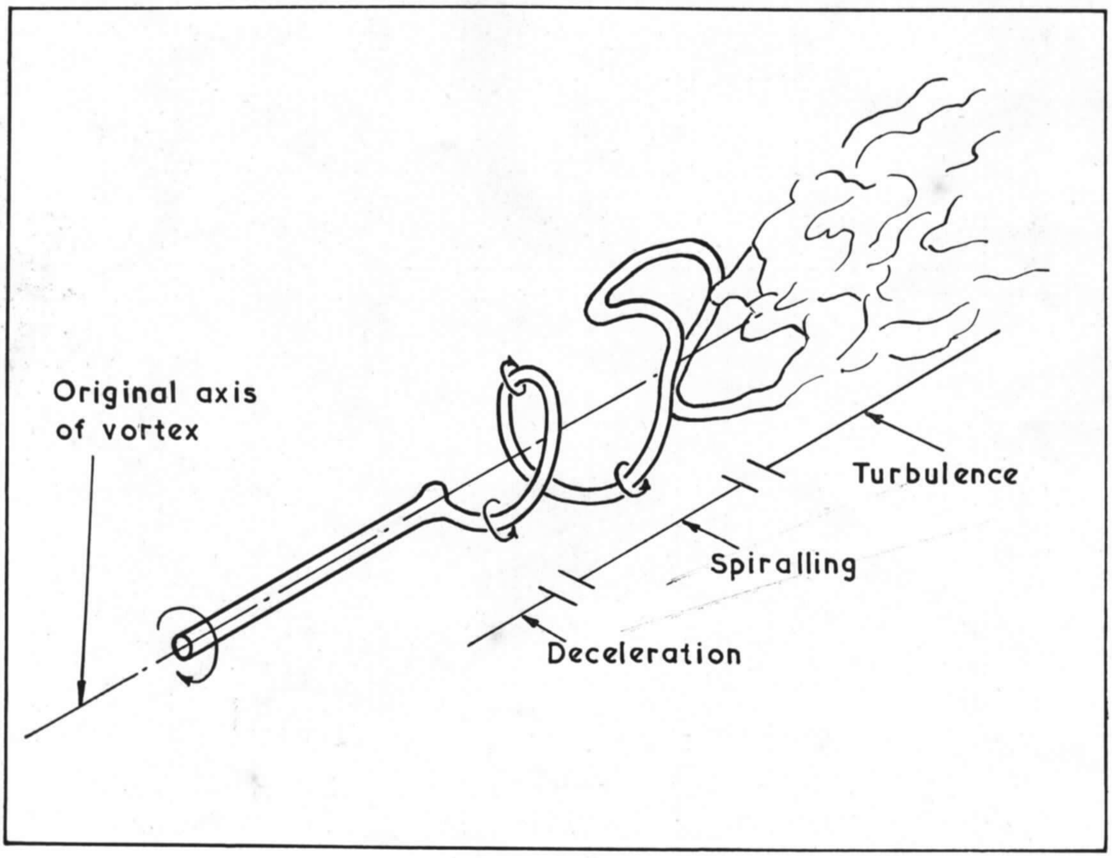


FIG. 5b. Stages in behaviour of axial filament.

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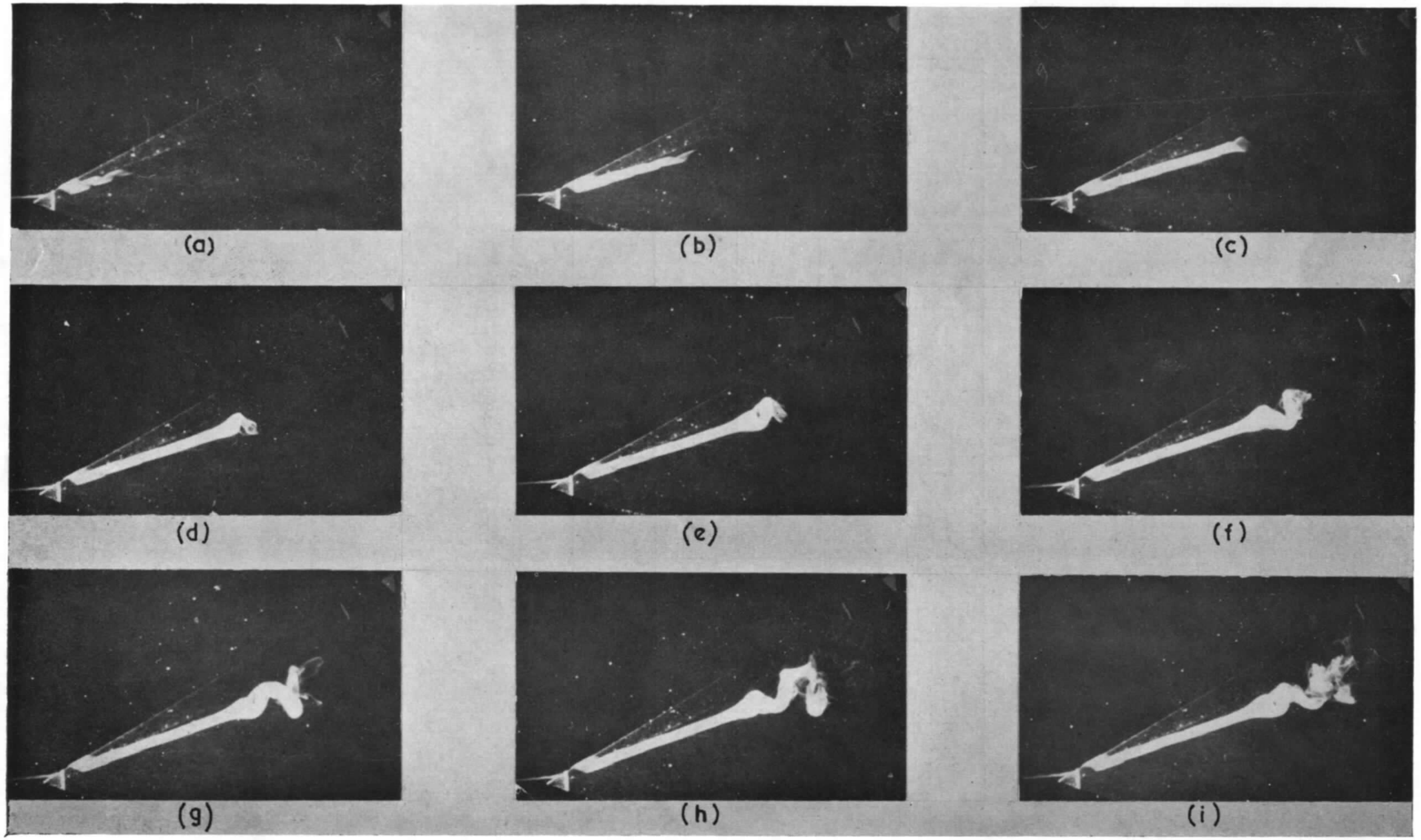

FIG. 6. Progressive movement of dye along vortex. (Equal intervals of time, water tunnel, 2 in./sec.)

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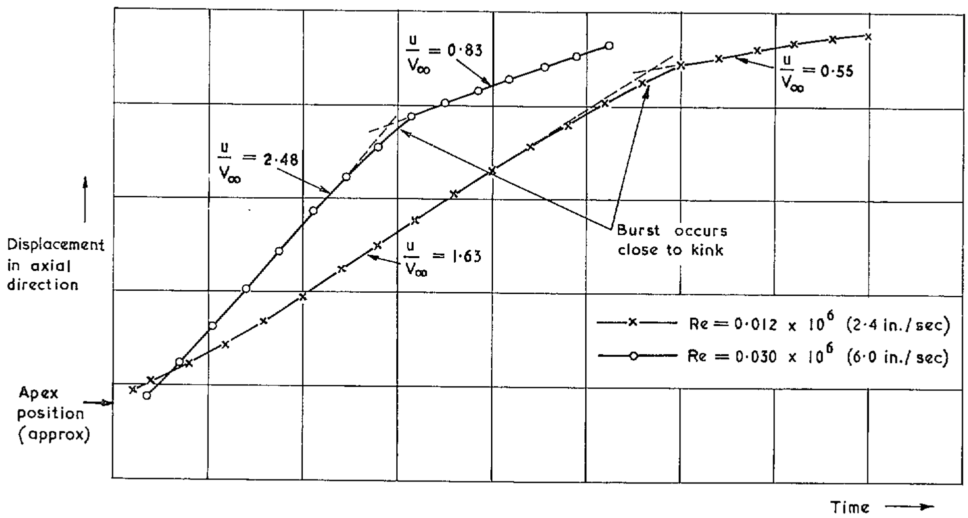
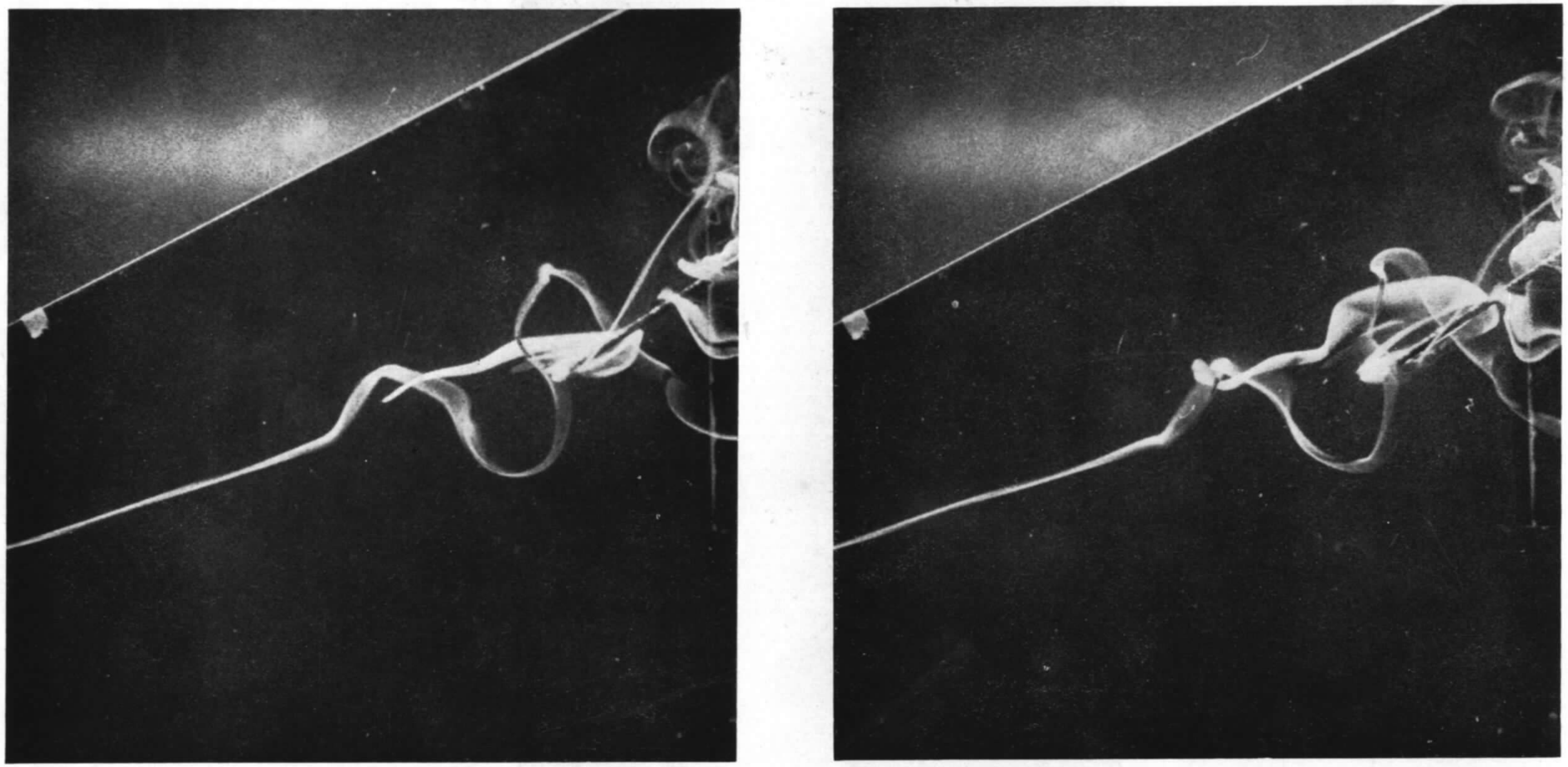


FIG. 7. Time histories of dye fronts introduced along vortex axis. (Delta plate, Λ = 65 deg, water tunnel.)



(a) Dye tongue approaching kink.

(b) Tongue entrained in vortex.

FIG. 8. Progress of reverse flow within spiral. (Sequence from ciné film.)

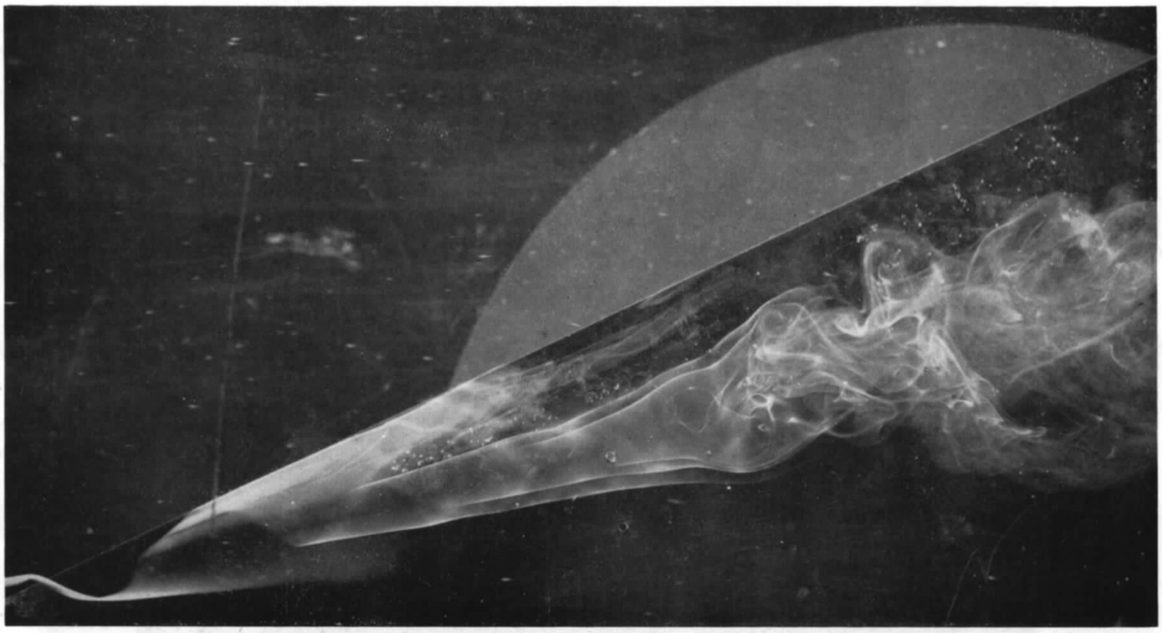


FIG. 9. Entrainment of dye into reversed flow within spiralling region.

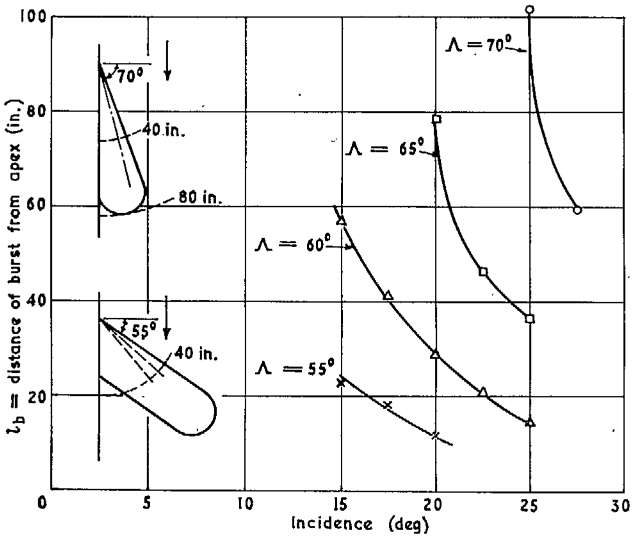


FIG. 10. Variation of burst position with incidence and sweepback. Swept plate, 180 ft/sec.

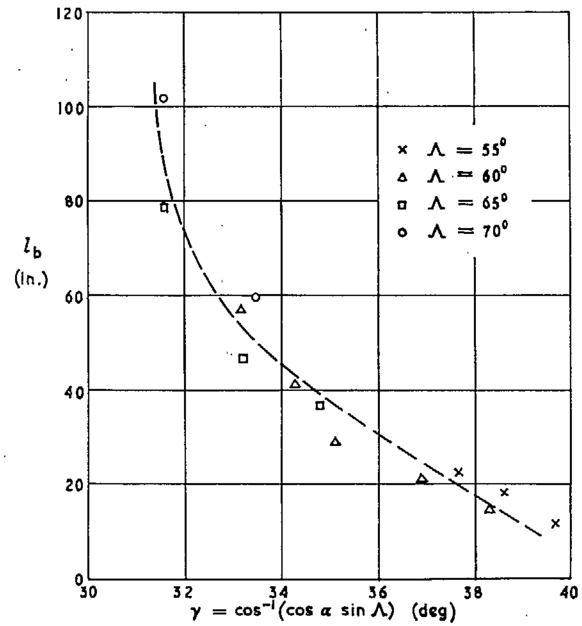


FIG. 11. Burst position plotted against γ, the angle between the leading edge and free stream. Swept plate, 180 ft/sec.

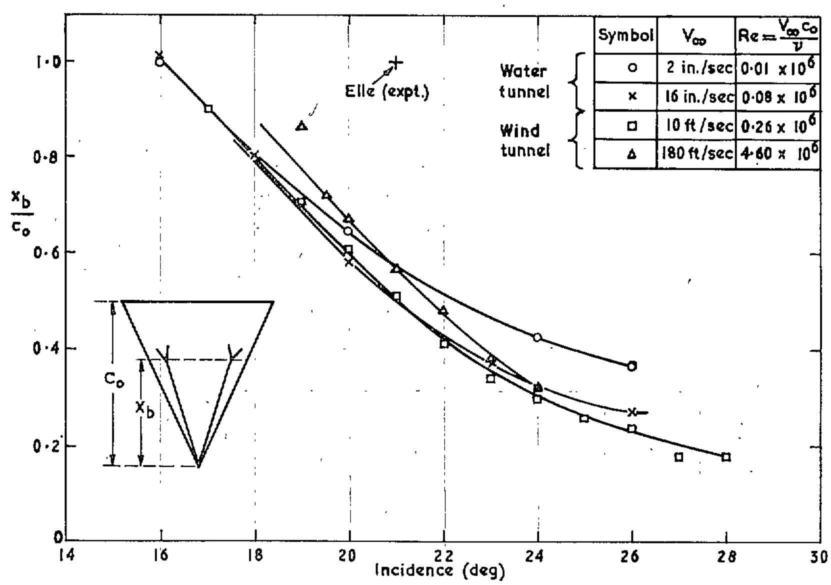
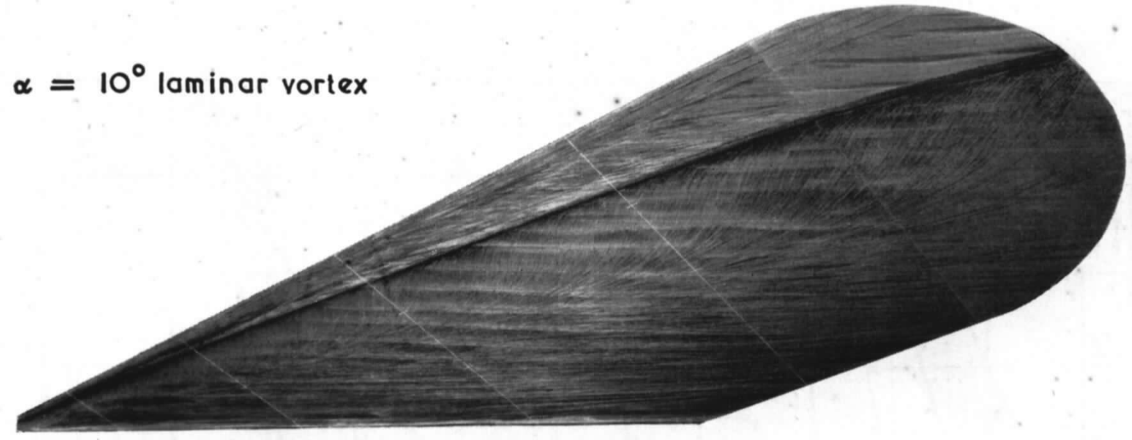
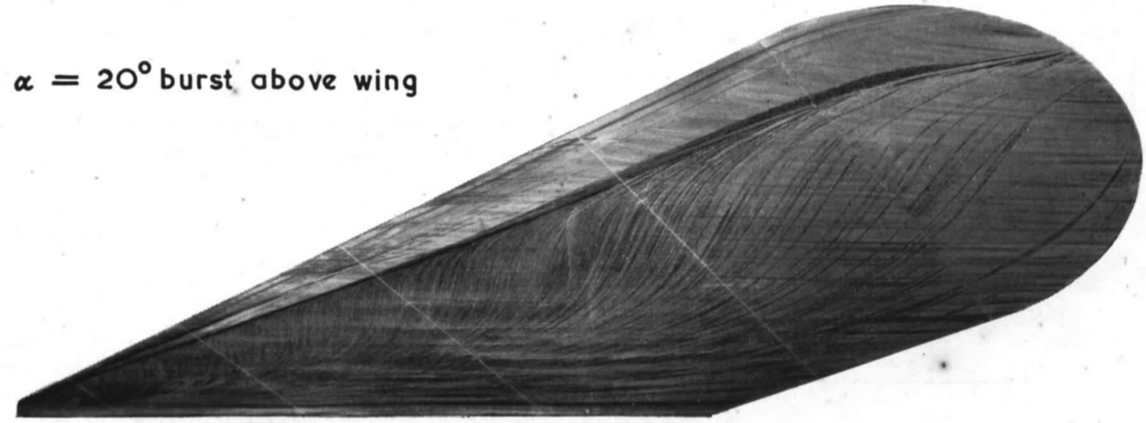


FIG. 12. Burst position vs. incidence for various Reynolds numbers. Delta plate, $\Lambda = 65$ deg.





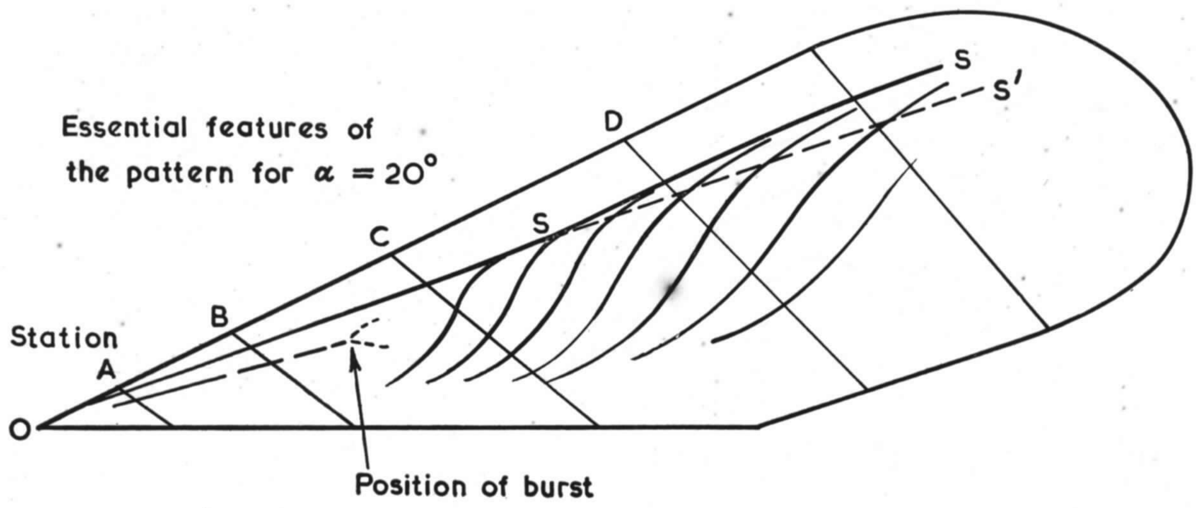


FIG. 13. Effect of burst on surface oil patterns. $\Lambda = 60$ deg, $V_{\infty} = 120$ ft/sec.

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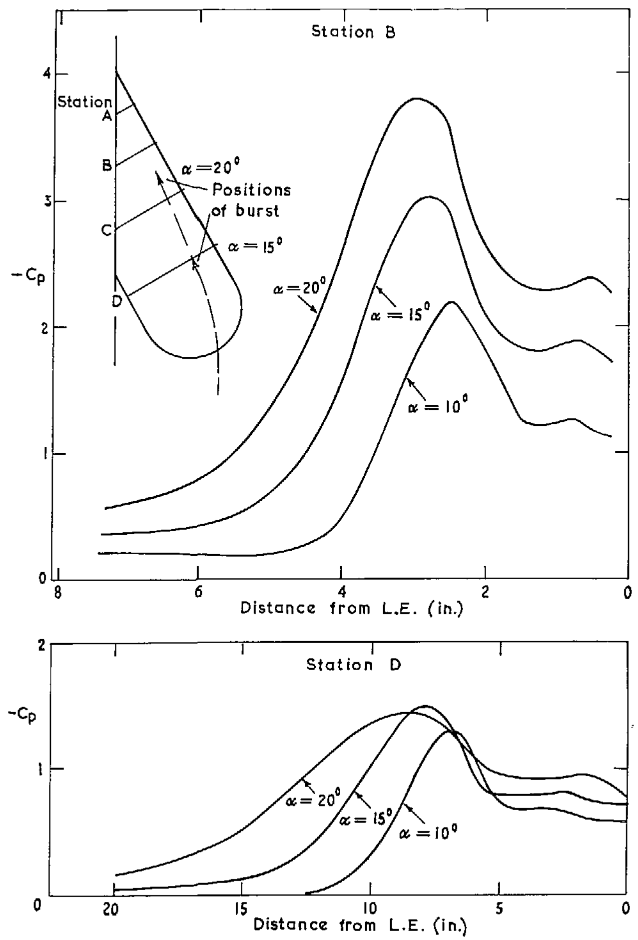


FIG. 14. Surface pressure distributions for three incidences, including the effect of a burst. $\Lambda = 60$ deg, $V_\infty = 120$ ft/sec.

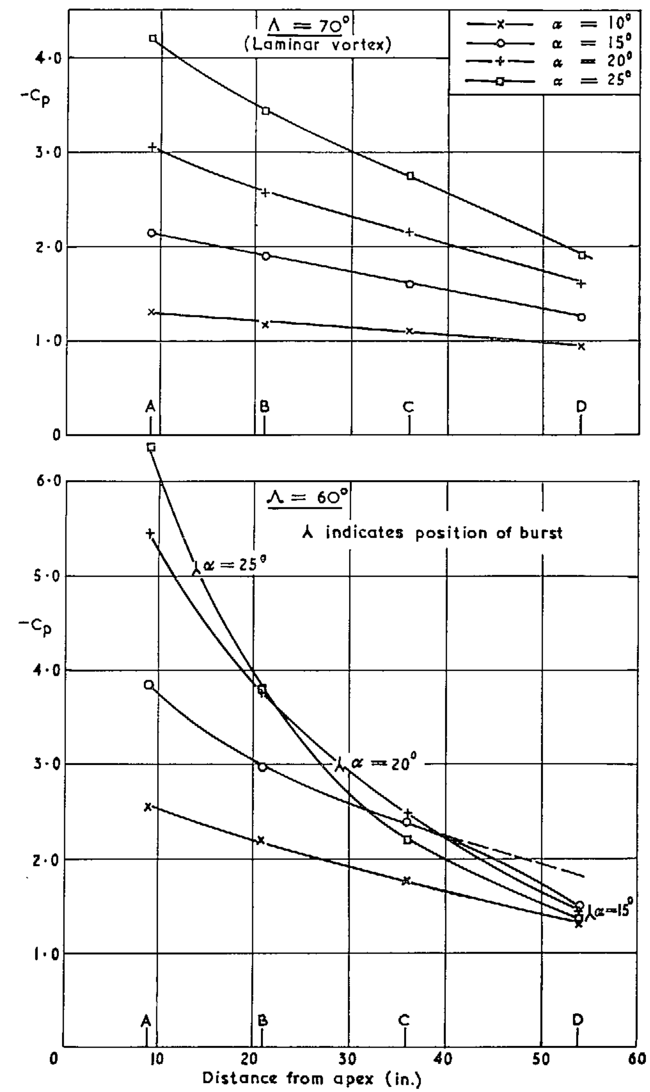


FIG. 15. Surface pressure distribution along suction ridge, including effect of burst. $V_\infty = 120$ ft/sec.

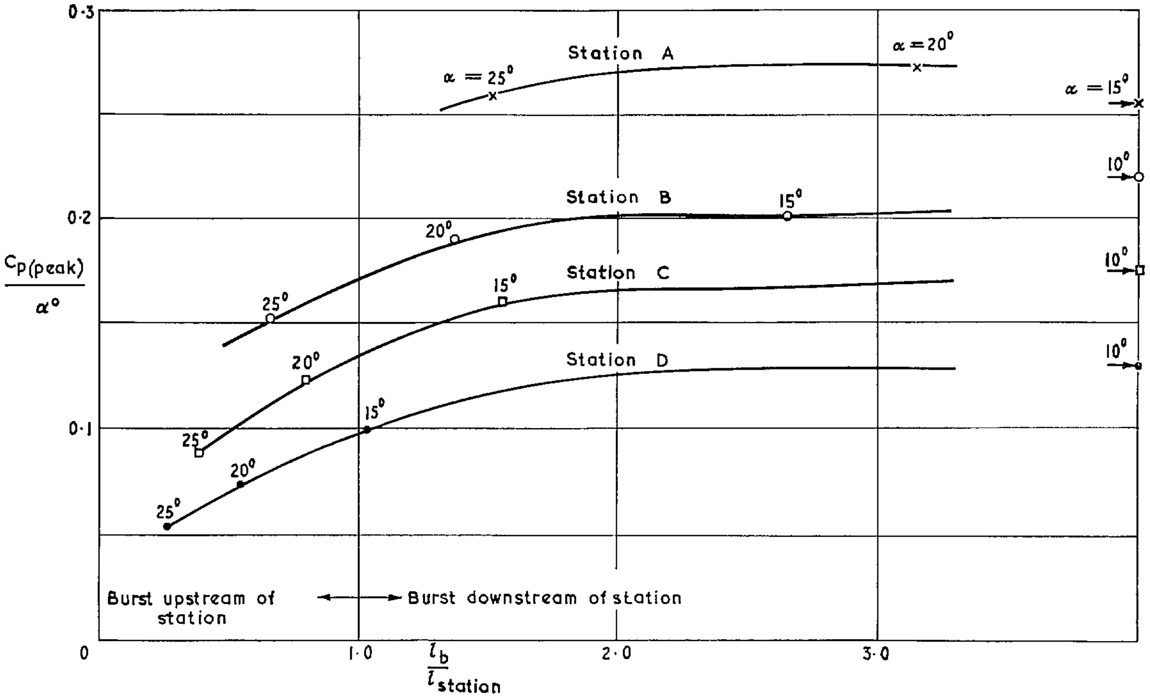
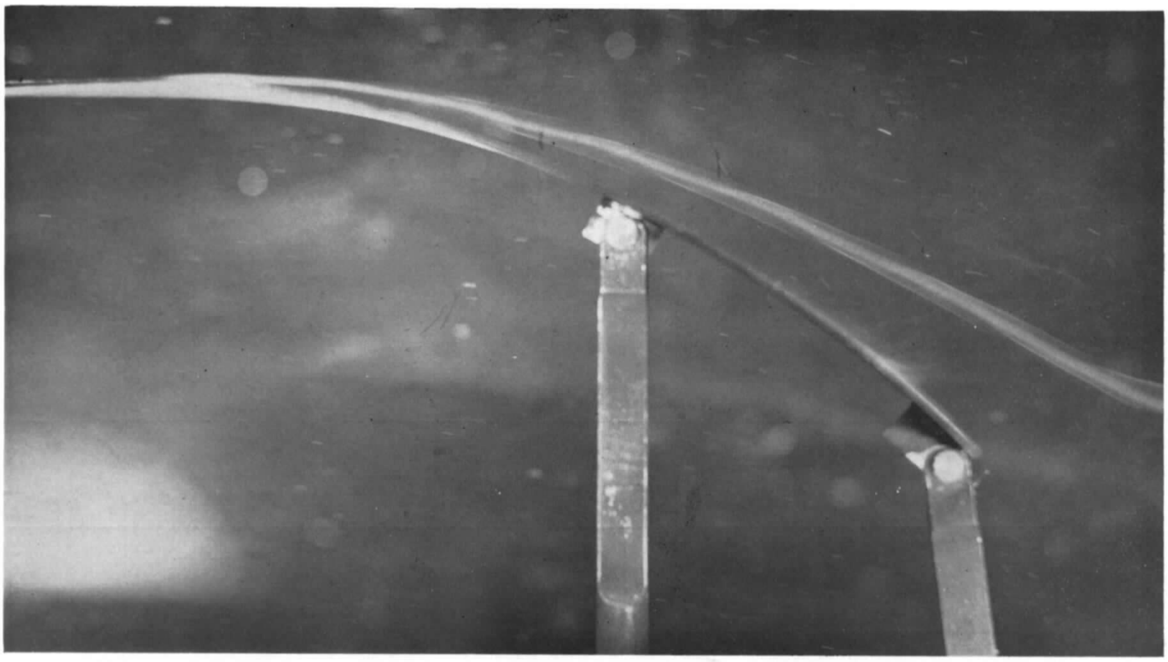
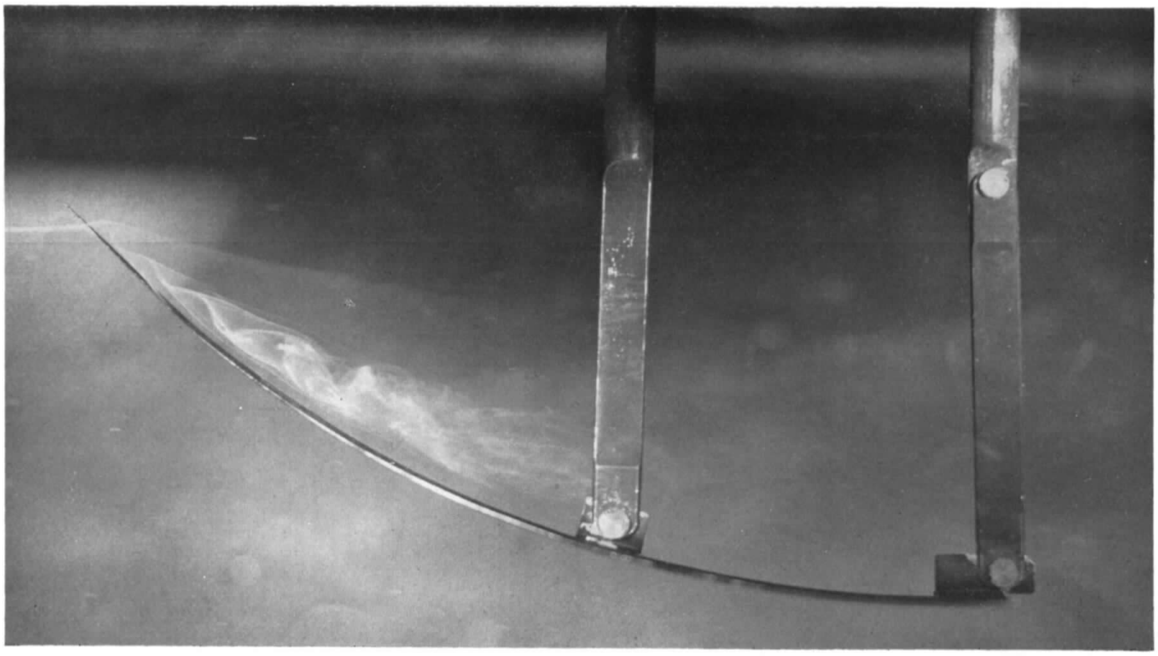


FIG. 16. Change of peak suction as burst moves forward with increasing incidence. Λ = 60 deg, V_∞ = 120 ft/sec.

(84946)

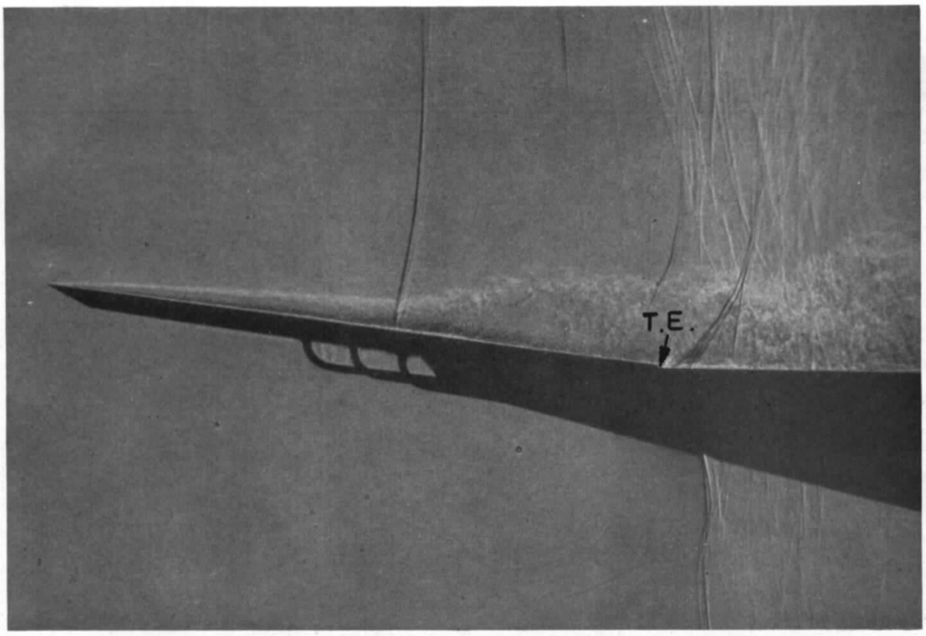


(a) Local incidence increasing with distance from apex.

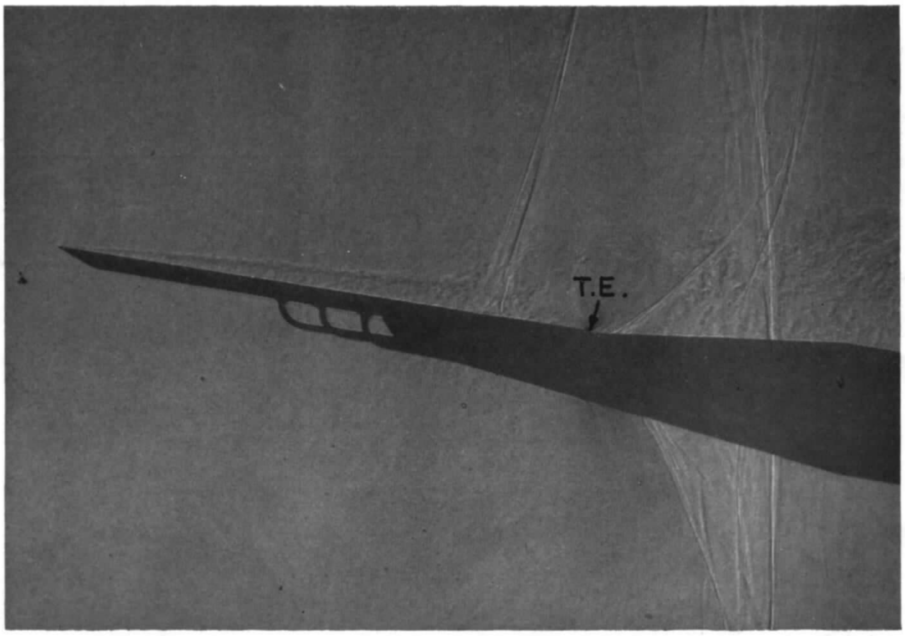


(b) Local incidence decreasing with distance from apex.

FIG. 17. Vortices for cambered delta plate, $\Lambda = 80$ deg. Water tunnel, $V_\infty = 2$ in./sec.



(a) $M = 0 \cdot 90$.



(b) $M = 0 \cdot 95$.

FIG. 18. Interaction of shock wave with leading-edge vortices. Schlieren observation, sharp-edged plate delta. $\Lambda = 50$ deg, $\alpha = 10$ deg.

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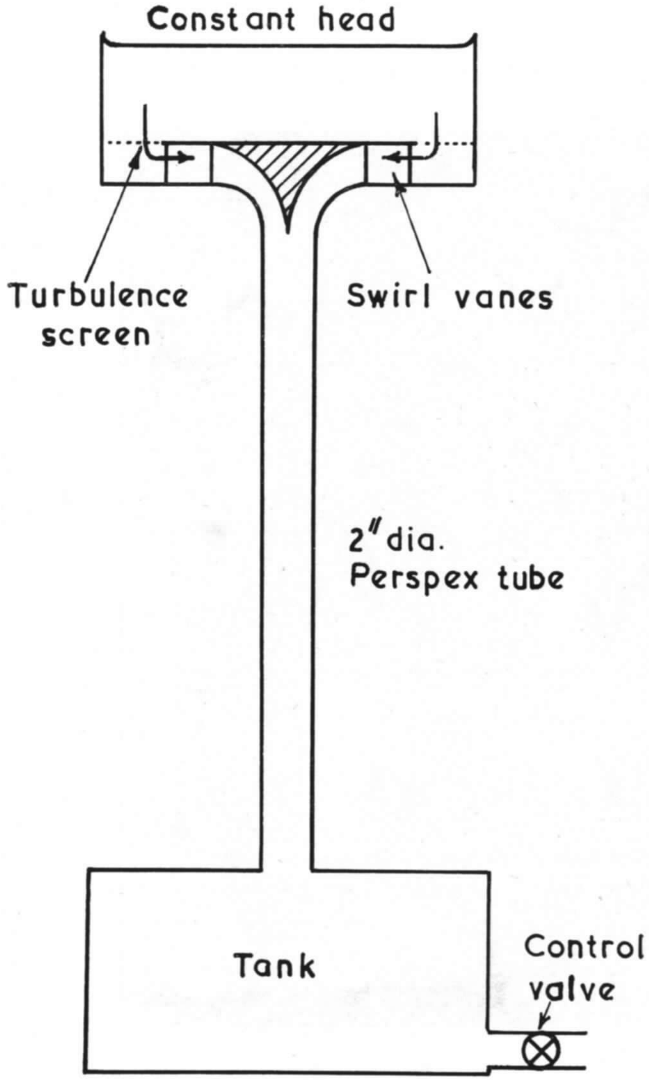

FIG. 19. Schematic diagram of water vortex tube.

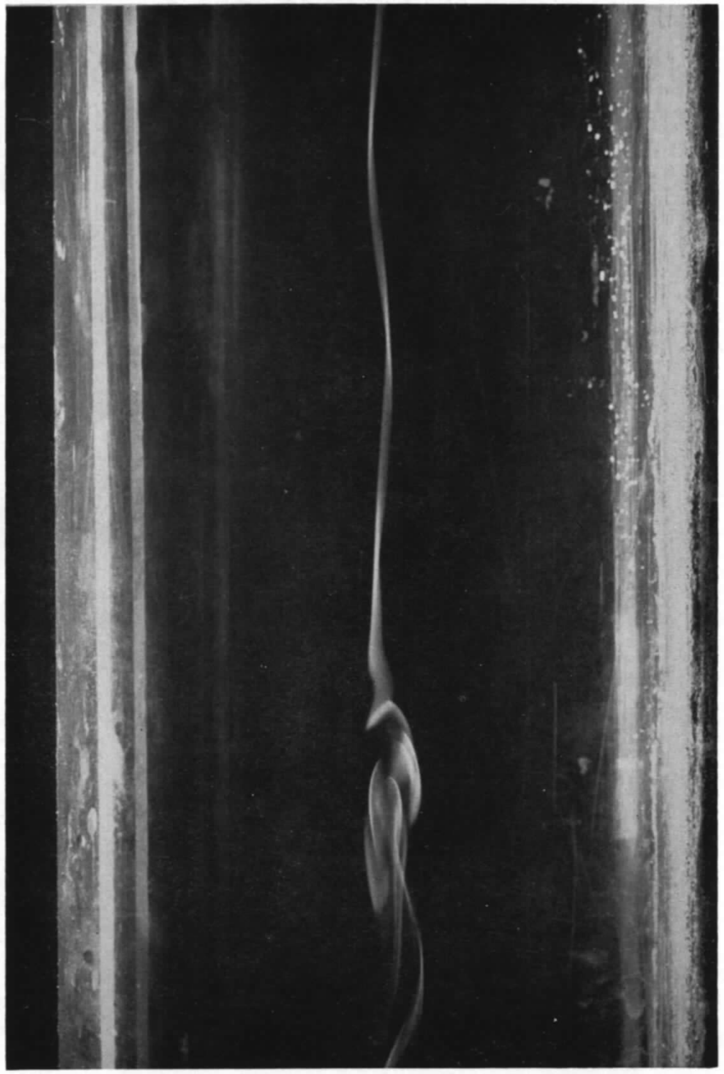

FIG. 20. Burst shown by dye filament in vortex tube.

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