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Summary. The problem of calculating the rotary lateral stability derivatives for a straight wing with a thin jet emerging at the trailing edge is considered. Based on Maskell and Spence's analysis of the effect of finite aspect ratio on lift and drag a solution is described which shows that the presence of the jet has considerable effect on the stability derivatives.

1. *Introduction.* In a paper by Maskell and Spence[1] the relationship is derived between the lift of a finite-span aerofoil with a jet emerging from the trailing edge and the lift of the corresponding two-dimensional aerofoil. Many features of his arguments suggest that there is a possibility of applying the basic concepts of classical aerofoil theory in many problems of the so-called jet-flapped aerofoil when the jet-momentum coefficient is small. In what follows we pursue this further by considering the problem of calculating the rotary stability derivatives of such a wing, and show that the same line of reasoning is consistent with the result given in Ref. 1 for the lift of the wing in steady symmetrical flow.

The derivatives are calculated on the basis of the usual quasi-steady flow conditions, and the trailing vortices are assumed to be straight and not curved. It is considered that this approach should provide the better approximation to the derivatives involved in the oscillatory motion of an aircraft.

The results show that the presence of the jet has a marked effect on all the derivatives considered. The rolling-moment derivative due to rolling (damping in roll), l_p, increases almost linearly with C_J, the jet-thrust coefficient. The rolling- and yawing-moment derivatives due to yawing, l_r and n_{ri} both increase in magnitude with C_J, and with increasing wing incidence and jet deflection. The effect of the jet on the yawing-moment derivative due to rolling, n_p, is even more marked, as there is a change in sign for C_J's larger than a certain value depending on the aspect ratio. All the results given reduce to those for the plain wing as the coefficient C_J tends to zero.

It is possible to use the same method for calculating other derivatives, *e.g.*, the rolling-moment derivative due to sideslip, l_v, for a straight wing with dihedral.

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The basis of the calculations described is such that whilst it is expected that the trends in the derivatives with increase of the jet-momentum coefficient, C_J, are reliably predicted the numerical values may become less reliable for the larger C_J values.

Throughout the present calculations the jet momentum (J) has been assumed constant along the span of the wing. No consideration has been given to the possible adjustment of the derivatives by deliberately designing for varying J, as this should await experimental confirmation of the present calculated values and assessment of what is desirable.

2. *Theory.* It is clear that there is a contribution to the lift of the jet-flapped wing of finite span due to the downward deflection of the jet at an infinite distance downstream of the wing. No such contribution is present in the two-dimensional flow since the jet direction at infinity will coincide with the stream direction. Accordingly in seeking to establish the equation which relates the local lift to the induced incidence due to the trailing vorticity, and corresponds to the simple relationship,

$$C_L = a_0(\alpha - \alpha_i)\,, \tag{1}$$

of the plain wing we must remove this direct jet effect. Denote by L_n, the lift so obtained, and let $C_{L\,n}$ be the corresponding lift coefficient, so that

$$C_L = C_{L\,n} + C_J\alpha_\infty\,. \tag{2}$$

From Maskell and Spence's analysis we have,

$$C_{L\,n} = C_{L\,\infty} - \alpha_i \frac{\partial C_{L\,\infty}}{\partial \alpha} - C_{J\,i}\frac{\partial C_{L\,\infty}}{\partial C_J}\,.$$

Introducing the notation $\mu_1 = \partial C_{L\,\infty}/\partial \tau$, $\mu_2 = \partial C_{L\,\infty}/\partial \alpha$ (μ_1 and μ_2 being function of C_J only) and substituting for $C_{J\,i}$, we have that

$$C_{L\,n} = \mu_1\tau + \mu_2\alpha - \mu_2\alpha_i - \frac{\alpha_i}{\tau+\alpha}\,C_J\{\dot{\mu}_1\tau + \dot{\mu}_2\alpha\}$$

$$\approx \mu_1\tau + \mu_2\alpha - \alpha_i(\mu_2 + \dot{\mu}_1 C_J)\,, \tag{3}$$

where the dot denotes differentiation with respect to C_J, and α/τ is assumed sufficiently small to allow terms $(\alpha/\tau)\,(\dot{\mu}_1 - \dot{\mu}_2)$ and $O\,(\alpha^2/\tau^2)$ to be neglected. Equation (3) is the equation we seek, and it reduces to Equation (1) as $C_J \to 0$. Furthermore it is readily shown that $\alpha_i = C_{L\,n}/\pi A$, for elliptic spanwise distribution of lift.

Let $\Gamma(\theta)$ denote the circulation corresponding to the nett lift coefficient $C_{L\,n}$, and expand it in the well-known Fourier sine series

$$\Gamma(\theta) = 4sV\sum A_n \sin n\theta\,,$$

where $y = -s\cos\theta$.

We consider a wing having semi-span s in motion with angular velocity p in roll, and r in yaw. This motion implies that the relative velocity of a section distance y from the centre chord line is $V - ry$, and that there is a change of effective incidence of $py/(V-ry) = py/V$ (*see* Fig. 1) and of effective jet-momentum coefficient. If J, the jet momentum, is constant across the span, then the local jet-momentum coefficient is

$$C_J(y) = C_J \frac{V^2}{(V-ry)^2}$$

$$= C_J \left(1 + \frac{2ry}{V}\right)$$

and so

$$\mu_1(y) = \mu_1 + \frac{2ry}{V} C_J\dot{\mu}_1 ,$$

$$\mu_2(y) = \mu_2 + \frac{2ry}{V} C_J\dot{\mu}_2 .$$

Then, by using Equation (3), we have that the circulation function is

$$\Gamma(\theta) = \frac{c}{2}\left[(V-ry)\left\{\left(\mu_1 + \frac{2ry}{V} C_J\dot{\mu}_1\right)\tau + \left(\mu_2 + \frac{2ry}{V} C_J\dot{\mu}_2\right)\left(\alpha + \frac{py}{V}\right)\right\} - \right.$$

$$\left. - V\alpha_i\left\{\mu_2 + \frac{2ry}{V} C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + \frac{2ry}{V} C_J \frac{\partial}{\partial C_J}(C_J\dot{\mu}_1)\right\}\right]. \qquad (4)$$

With $A_n = a_n\alpha + b_n + c_n\tau$, say, we may write Equation (4) in the form

$$4sV\Sigma(a_n\alpha + b_n + c_n\tau)\sin n\theta$$

$$= \frac{cV}{2}\left[\left(1 + \frac{rs}{V}\cos\theta\right)\left\{\left(\mu_1 - \frac{2rs}{V}\cos\theta C_J\dot{\mu}_1\right)\tau + \left(\mu_2 - \frac{2rs}{V}\cos\theta C_J\dot{\mu}_2\right)\alpha - \mu_2\frac{ps}{V}\cos\theta\right\} - \right.$$

$$\left. - \frac{\Sigma n(a_n\alpha + b_n + c_n\tau)}{\sin\theta}\sin n\theta\left\{\mu_2 + C_J\dot{\mu}_1 - \frac{2rs}{V}\cos\theta(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1)\right\}\right]. \qquad (5)$$

This may be separated into two parts (*cf.* Gdaliahu[2]),

$$\Sigma(a_n\alpha + c_n\tau)\sin n\theta = \frac{c}{8s}\left\{\mu_1\tau + \mu_2\alpha - \Sigma n(a_n\alpha + c_n\tau)\frac{\sin n\theta}{\sin\theta}(\mu_2 + C_J\dot{\mu}_1)\right\} \qquad (6)$$

and

$$\Sigma b_n \sin n\theta = \frac{c}{8s}\left\{\frac{rs}{V}\cos\theta(\mu_1\tau + \mu_2\alpha) - \frac{2rs}{V}\cos\theta(C_J\dot{\mu}_1\tau + C_J\dot{\mu}_2\alpha) - \frac{ps}{V}\cos\theta\mu_2 + \right.$$

$$+ \Sigma n(a_n\alpha + c_n\tau)\frac{\sin n\theta}{\sin\theta}\frac{2rs}{V}\cos\theta(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1) -$$

$$\left. - \Sigma nb_n\frac{\sin n\theta}{\sin\theta}(\mu_2 + C_J\dot{\mu}_1)\right\}. \qquad (7)$$

These equations imply a general type of spanwise distribution of circulation, and correspond to the two Equations (15) and (16) of Gdaliahu's paper[2]. To obtain reasonably simple closed forms for the derivatives we restrict further analysis to wings having elliptic loading in symmetric flight, so that Equation (6) gives that

$$c = \bar{c} \sin \theta ,$$

and

$$\begin{aligned} \bar{C}_{L\,n} &= \pi A(a_1\alpha + c_1\tau) \\ &= \frac{\pi A(\mu_1\tau + \mu_2\alpha)}{\pi A + \mu_2 + C_J\dot{\mu}_1} \end{aligned} \tag{8}$$

which is consistent with Maskell and Spence's first order result for

$$\bar{C}_L = \bar{C}_{L\,n} + C_J\alpha_\infty = \frac{(\pi A + 2C_J)C_{L\,\infty}}{\pi A + \mu_2 + C_J\dot{\mu}_1}$$

Thus we have only the coefficient b_2 in Equation (7), given by

$$\begin{aligned} b_2(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1) = \frac{rs}{V}\bigg\{ & \frac{\mu_1\tau + \mu_2\alpha}{2} - C_J(\dot{\mu}_1\tau + \dot{\mu}_2\alpha) + \\ & + (a_1\alpha + c_1\tau)(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1)\bigg\} - \frac{ps\mu_2}{V\,2}, \end{aligned} \tag{9}$$

which may be written

$$b_2 = b_{2r}\frac{rs}{V} + b_{2p}\frac{ps}{V},$$

where

$$\begin{aligned} b_{2r}(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1) = & \frac{(\mu_1\tau + \mu_2\alpha)}{2} - C_J(\dot{\mu}_1\tau + \dot{\mu}_2\alpha) + \\ & + (a_1\alpha + c_1\tau)(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1) \end{aligned} \tag{10}$$

and

$$b_{2p}(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1) = -\frac{\mu_2}{2} \tag{11}$$

If $C_{L\,\infty}$ is the lift on the wing of infinite span, *i.e.*, $C_{L\,\infty} = \mu_1\tau + \mu_2\alpha$, we have that

$$b_{2r}(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1) = C_{L\,\infty}\left\{\frac{1}{2} + \frac{(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1)}{(\pi A + \mu_2 + C_J\dot{\mu}_1)}\right\} - C_J\frac{\partial C_{L\,\infty}}{\partial C_J} \tag{10a}$$

and

$$b_{2p}(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1) = -\frac{1}{2}\frac{\partial C_{L\,\infty}}{\partial \alpha}. \tag{11a}$$

3. *Calculation of Rotary Lateral Stability Derivatives.* 3.1. *Rolling-Moment Derivatives.* In order to evaluate the rolling moment, we have to consider the moment of the total lift on a section of width δy of the span (*see* Fig. 1). On using the definition of the circulation Γ as that associated with the nett lift (Equation 2), we have that

$$\text{Lift on section} = \{\rho(V - ry)\Gamma(y) + J\alpha_\infty\}\,dy,$$

where J is the momentum flux in the jet per unit span and so the total rolling moment on the wing is given by

$$-\int_{-s}^{+s}\{\rho(V - ry)\Gamma(y) + J\alpha_\infty\}\,y\,dy.$$

Now $\Gamma(y) = 4sV\Sigma A_n \sin n\theta$ when $y = -s\cos\theta$, and

$$\alpha_\infty = \frac{2\Sigma nA_n \sin n\theta}{\sin\theta}$$

in the usual way, so that the rolling moment, in coefficient form, is

$$C_l = \frac{\text{moment}}{\frac{1}{2}\rho V^2 S2s}$$

$$= \int_0^\pi \left\{ \left(1 + \frac{rs}{V}\cos\theta\right) \frac{4}{S}\Sigma A_n \sin n\theta + \frac{J}{\frac{1}{2}\rho V^2 S2s} 2\Sigma nA_n \left[\frac{\sin n\theta}{\sin\theta}\right] \right\} s^2 \sin\theta\cos\theta\, d\theta .$$

If we take $J =$ constant, with $C_J = \dfrac{J2s}{\frac{1}{2}\rho V^2 S}$, then

$$C_l = \frac{2s^2}{S}\int_0^\pi \left(1 + \frac{rs}{V}\cos\theta\right)\Sigma A_n \sin n\theta \sin 2\theta\, d\theta +$$

$$+ \frac{C_J}{4s^2} 2s^2 \int_0^\pi \Sigma nA_n \sin n\theta\cos\theta\, d\theta$$

$$= \frac{A}{2}\int_0^\pi \left\{\Sigma A_n \sin n\theta\sin 2\theta + \frac{rs}{V}\Sigma A_n \sin n\theta \sin 2\theta\cos\theta\, d\theta\right\} +$$

$$+ \frac{C_J}{2}\int_0^\pi \Sigma nA_n \sin n\theta\cos\theta\, d\theta$$

$$= \frac{\pi A}{4}\left\{A_2 + \frac{rs}{2V}(A_1 + A_3)\right\} + \frac{C_J}{2}\Sigma \frac{n^2}{(n^2-1)} A_n(1+\cos n\pi)$$

The A_n's have been used in the form

$$A_n = a_n\alpha + b_n + c_n\tau$$

$$= a_n\alpha + b_{nr}\frac{rs}{V} + b_{np}\frac{ps}{V} + c_n\tau ,$$

so that for the case of elliptic loading under consideration we have

$$C_l = \frac{\pi A}{4}\left\{b_{2r}\frac{rs}{V} + b_{2p}\frac{ps}{V} + \frac{rs}{2V}(a_1\alpha + c_1\tau)\right\} +$$

$$+ \frac{C_J}{2}\frac{8}{3}\left(b_{2r}\frac{rs}{V} + b_{2p}\frac{ps}{V}\right). \qquad (12)$$

Thus, l_p, the rolling-moment derivative due to rolling, is

$$l_p = \frac{\partial C_l}{\partial\left(\frac{ps}{V}\right)}$$

$$= \frac{\pi A}{4} b_{2p} + \frac{4}{3} C_J b_{2p}$$

and l_r, the rolling-moment derivative due to yawing is

$$l_r = \frac{\partial C_l}{\partial\left(\frac{rs}{V}\right)} = \frac{\pi A}{4} b_{2r} + \frac{\pi A}{8}(a_1\alpha + c_1\tau) + \frac{4}{3} C_J b_{2r}$$

On substituting for b_{2p} and b_{2r} from Equations (10a) and (11a), and using the relationship between C_L and $C_{L\,\infty}$ given by Maskell and Spence[1] (which is consistent with that obtained in Equation (8) between $C_{L\,n}$ and $C_{L\,\infty}$) the derivatives become

$$l_p = -\left(\frac{\pi A}{4} + \frac{4}{3}\,C_J\right)\frac{\mu_2}{2(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1)}$$

or

$$\frac{lp}{A} = -\frac{\pi}{8}\,\frac{\mu_2}{(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1)}\left\{1 + \frac{16C_J}{3\pi A}\right\} \tag{13}$$

and

$$l_r = \left(\frac{\pi A}{4} + \frac{4}{3}\,C_J\right)\left[\frac{C_{L\,\infty}\left\{\frac{1}{2} + \frac{(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1)}{\pi A + \mu_2 + C_J\dot{\mu}_1}\right\} - C_J\,\frac{\partial C_{L\,\infty}}{\partial C_J}}{(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1)}\right] + \frac{\bar{C}_{L\,n}}{8}$$

or

$$\begin{aligned}\frac{l_r}{\bar{C}_L} = \frac{1}{8(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1)}\left(\frac{\pi A}{\pi A + 2C_J}\right)\Bigg\{\Bigg[&\left(1 + \frac{16C_J}{3\pi A}\right)\Big(\pi A + \mu_2 + C_J\dot{\mu}_1 + \\ &+ 2(C_J\dot{\mu}_2 + C_J\dot{\mu}_1 + C_J{}^2\ddot{\mu}_1)\Big) + \pi A + 2\mu_2 + 2C_J\dot{\mu}_1\Bigg] - \\ &- 2(\pi A + \mu_2 + C_J\dot{\mu}_1)\left(1 + \frac{16C_J}{3\pi A}\right)\frac{C_J}{C_{L\,\infty}}\,\frac{\partial C_{L\,\infty}}{\partial C_J}\Bigg\}\,.\end{aligned} \tag{14}$$

In the limit as $C_J \to 0$, when $\mu_2 \to a_0$ and $\mu_1 \to 0$, these expressions reduce to

$$\frac{l_p}{A} = -\frac{\pi}{8}\,\frac{a_0}{(\pi A + 2a_0)}$$

and

$$\frac{l_r}{\bar{C}_L} = \frac{1}{8}\,\frac{(2\pi A + 3a_0)}{\pi A + 2a_0}\,,$$

the well-known formulae for the rolling derivatives of an elliptically loaded plain wing*.

3.2. *Yawing-Moment Derivatives.* The yawing moment will arise from the drag on a section of width δy of the span (*see* Fig. 1) where

$$\text{Drag on section} = \{-\,\rho(py - w)\Gamma(y) + \tfrac{1}{2}J\alpha_\infty{}^2 + D_0\}\,dy\,.$$

D_0 is the profile drag per unit span. The term $\frac{1}{2}J\alpha_\infty{}^2$ is the first-order difference between J and $J\cos\alpha_\infty$, and represents the effect of deflecting the jet at infinity in the three-dimensional flow.

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* It is interesting to note that in the case $C_J \to 0$ with $\alpha \to 0$ also (*i.e.*, $\bar{C}_L \to 0$), the factor $(C_J/C_{L\,\infty})\,(\partial C_{L\,\infty}/\partial C_J)$ in Equation (14) has the limits 0 or $\frac{1}{2}$ according as C_J or α tends to zero first. However, this does not affect the rolling-moment derivative l_r, since l_r tends to zero as $\bar{C}_L$, and the difference arises only in the limit of $l_r/\bar{C}_L$.

The first term is the component of the resultant force corresponding to the circulation $\Gamma(y)$, *cf.* lift in Section (3.1) and Equation (2). So the total yawing moment is

$$\int_{-s}^{+s} \{-\rho(py-w)\Gamma(y) + \tfrac{1}{2}J\alpha_{\infty}^{2} + D_0\} y\, dy\,.$$

Thus C_n, the yawing-moment coefficient, is given by

$$\begin{aligned} C_n = & -\int_0^{\pi} \left\{\frac{ps}{V}\cos\theta + \Sigma n A_n \left[\frac{\sin n\theta}{\sin\theta}\right]\right\} \frac{4s^2}{S} \Sigma A_n \sin n\theta \cos\theta \sin\theta\, d\theta - \\ & -\frac{C_J}{2}\int_0^{\pi} \Sigma n A_n \left[\frac{\sin n\theta}{\sin\theta}\right] \Sigma m A_m \left[\frac{\sin m\theta}{\sin\theta}\right] \sin\theta\cos\theta\, d\theta + \\ & + \int_{-s}^{+s} C_{D0}\left(1-\frac{ry}{V}\right)^2 \frac{c}{2Ss} y\, dy\,, \end{aligned}$$

where C_{D0} is the sectional profile-drag coefficient.
i.e.,

$$\begin{aligned} C_n = & -A\int_0^{\pi}\left\{\frac{ps}{4V}\Sigma A_m [\cos(m-2)\theta\cos\theta - \cos(m+2)\theta\cos\theta] + \right. \\ & \left. + \tfrac{1}{2}\Sigma\,\Sigma m A_m A_n [\cos(m-n)\theta\cos\theta - \cos(m+n)\theta\cos\theta]\right\} d\theta - \\ & -\frac{C_J}{2}\int_0^{\pi}\{A_1^2\sin\theta\cos\theta + 4A_1A_2\sin 2\theta\cos\theta + \ldots\} d\theta - \frac{rs}{V}\int_{-s}^{+s}\frac{C_{D0}}{Ss^2} cy^2\, dy \\ = & -\frac{\pi A}{8}(A_1+A_3)\frac{ps}{V} - \frac{\pi A}{4}\Sigma m A_m (A_{m-1}+A_{m+1}) - \\ & -\frac{C_J}{2}\left\{4A_1A_2\frac{4}{3} + \ldots\right\} - \frac{rs}{V}\int_{-s}^{+s}\frac{C_{D0}}{Ss^2} cy^2\, dy\,. \end{aligned}$$

For the elliptically loaded wing, and for the case when C_{D0} = constant = $\bar{C}_{D0}$ = profile drag of wing, this expression reduces to

$$\begin{aligned} C_n = & -\frac{\pi A}{8}(a_1\alpha + c_1\tau)\frac{ps}{V} - \frac{\pi A}{4} 3(a_1\alpha + c_1\tau)\left(b_{2r}\frac{rs}{V} + b_{2p}\frac{ps}{V}\right) - \\ & -\frac{8}{3}C_J(a_1\alpha + c_1\tau)\left(b_{2r}\frac{rs}{V} + b_{2p}\frac{ps}{V}\right) - \frac{rs}{V}\frac{\bar{C}_{D0}}{4}, \end{aligned} \tag{15}$$

where the substitution $A_n = a_n\alpha + b_{nr}(rs/V) + b_{np}(ps/V) + c_n\tau$ has again been made. From Equation (15), the yawing-moment derivative due to rolling is seen to be

$$\begin{aligned} n_p = & \frac{\partial C_n}{\partial\left(\frac{ps}{V}\right)} \\ = & -(a_1\alpha + c_1\tau)\left\{\frac{\pi A}{8} + \frac{3\pi A}{4}b_{2p} + \frac{8C_J}{3}b_{2p}\right\} \end{aligned}$$

and the yawing-moment derivative due to yawing, n_r, is conveniently expressed as the sum of $n_{r\,i}$, that part of n_r due to the induced drag, and $n_{r\,0}$, that part of n_r due to the profile drag, so that

$$n_{r\,i} = \frac{\partial(C_{n\,i})}{\partial\left(\frac{rs}{V}\right)}$$

$$= -(a_1\alpha + c_1\tau)\left\{\frac{3\pi A}{4}\,b_{2r} + \frac{8C_J}{3}\,b_{2r}\right\}$$

and

$$n_{r\,0} = \frac{\partial C_{n\,0}}{\partial\left(\frac{rs}{V}\right)}$$

$$= -\frac{\overline{C}_{D\,0}}{4}.$$

The value of $(a_1\alpha + c_1\tau)$ is given in Equation (8) and b_{2p} and b_{2r} are given by Equations (10) and (11), so that the yawing-moment derivatives become

$$n_p = -\frac{\overline{C}_{L\,n}}{\pi A}\left\{\frac{\pi A}{8} + \left(\frac{3\pi A}{4} + \frac{8C_J}{3}\right)\frac{-\mu_2}{2(\pi A + 2\mu_2 + 2\dot{\mu}_1 C_J)}\right\}$$

or

$$\frac{n_p}{\overline{C}_L} = -\frac{1}{8}\,\frac{\left\{\pi A - \mu_2 - 2C_J\dot{\mu}_1 - \frac{32}{3\pi A}\,C_J\mu_2\right\}}{\{\pi A + 2\mu_2 + 2\dot{\mu}_1 C_J\}}\left\{\frac{\pi A}{\pi A + 2C_J}\right\} \tag{16}$$

and

$$n_{r\,i} = -\frac{\overline{C}_{L\,n}}{\pi A}\left\{\frac{3\pi A}{4} + \frac{8C_J}{3}\right\}\frac{C_{L\,\infty}}{(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1)} \times$$

$$\times\left\{\frac{1}{2} + \frac{C_J\dot{\mu}_1 + C_J\dot{\mu}_2 + C_J{}^2\ddot{\mu}_1}{\pi A + \mu_2 + C_J\dot{\mu}_1} - \frac{C_J}{C_{L\,\infty}}\,\frac{\partial C_{L\,\infty}}{\partial C_J}\right\}$$

or

$$A\,\frac{n_{r\,i}}{\overline{C}_L{}^2} = -\frac{3}{8\pi}\left(\frac{\pi A}{\pi A + 2C_J}\right)^2\frac{(1 + 32C_J/9\pi A)}{(\pi A + 2\mu_2 + 2C_J\dot{\mu}_1)} \times$$

$$\times\left\{\pi A + \mu_2 + C_J\dot{\mu}_1 + 2(C_J\dot{\mu}_1 + C_J\dot{\mu}_2 + C_J{}^2\ddot{\mu}_1) - \right.$$

$$\left. - \frac{2C_J}{C_{L\,\infty}}\,(\pi A + \mu_2 + C_J\dot{\mu}_1)\,\frac{\partial C_{L\,\infty}}{\partial C_J}\right\} \tag{17}$$

with

$$\frac{n_{r\,0}}{\overline{C}_{D\,0}} = -\frac{1}{4}. \tag{18}$$

These again reduce to the derivatives for an elliptically loaded plain wing when $C_J \to 0$, becoming

$$\frac{n_p}{\overline{C}_L} = -\frac{1}{8}\,\frac{(\pi A - a_0)}{(\pi A + 2a_0)},$$

$$A\,\frac{n_{r\,i}}{\overline{C}_L{}^2} = -\frac{3}{8\pi}\,\frac{(\pi A + a_0)}{(\pi A + 2a_0)}$$

and

$$\frac{n_{r\,0}}{\overline{C}_{D\,0}} = -\frac{1}{4}.$$

4. *Discussion and Conclusions.* Examination of Figs. 2 to 5 and Equations (12), (13), (16) and (17) shows that all the rotary lateral stability derivatives are affected by the presence of the jet. The derivatives are presented in Figs. 2 to 5 in the form of plots of $-l_p/A$, $-l_p$, l_r/C_L, l_r/τ, n_p/C_L, n_p/τ, $An_{r\,i}/C_L^2$ and $n_{r\,i}/\tau^2$ against C_J for various values of α/τ and aspect ratio†. The first graph in each pair is included so that a direct comparison with the values for a wing without a jet flap ($C_J = 0$) is possible, and the second graph shows the derivatives for a given wing and flight condition.

The damping-in-roll ($-l_p$) is seen in Fig. 2 to increase almost linearly with C_J, and is independent of α and τ. The rolling-moment derivative due to yawing, l_r, also increases with C_J, and with increasing α/τ, but Fig. 3 shows that l_r/C_L decreases sharply as C_J increases from zero to $1 \cdot 0$ for $\alpha/\tau \neq 0$. The different limits in l_r/C_L as C_J and α/τ tend to zero independently also appear on Fig. 3 (*see* footnote Section 3.1).

Of the derivatives considered, the yawing moment due to rolling is most affected by C_J, since n_p/C_L changes sign above some value of C_J which decreases with decreasing aspect ratio. It may be noted in Fig. 4 that n_p/C_L is independent of α/τ, but that the magnitude of n_p/τ increases as α/τ increases. The function $-An_{r\,i}/C_L^2$ is similar to l_r/C_L, decreasing sharply as C_J increases from zero to $1 \cdot 0$, with a more gradual change for $C_J > 1 \cdot 0$, and the different limits as C_J and α/τ tend to zero again appear. The derivative $n_{r\,i}$ itself increases in magnitude with increasing C_J and α if τ is constant.

It may be of interest to consider a particular jet-flapped wing, and compare the derivatives for this wing with those for a plain wing at the same incidence, and also for the plain wing developing the same lift. Calculations for a wing of aspect ratio 6, at 6 deg incidence, with a jet deflected at 60 deg to the chord line, and with $C_J = 2 \cdot 0$ yield the results given in Table 1.

No attempt is made here to assess the effect on lateral stability and response of the aircraft of the changes in the magnitude of the derivatives as compared with conventional aircraft, nor indeed can this be done adequately before the contributions to the various lateral derivatives of the fin of a jet-flapped aircraft have been estimated. It may prove quite inadequate to take the contributions as for a fin in combination with a plain wing on the following grounds. The vortex sheet shed by a wing is essentially unstable and commences to roll up after leaving the wing. It has been shown[3, 4] that this rolling-up process is essentially complete at a distance e behind the wing given by,

$$\frac{e}{S} = \frac{K^*}{C_L} \text{ (K^*, a constant depending on planform and spanwise loading)}$$

$$= 0 \cdot 56 \frac{A}{C_L} \text{ for elliptic loading‡.}$$

Owing to the much greater values of C_L associated with the jet-flapped wing the rolling-up process occurs much more rapidly than for a plain wing of the same aspect ratio. Thus $e \ll s$ if $C_L \gg 0 \cdot 56A$, which for $A = 6$ is $C_L \gg 3 \cdot 36$, from which it follows that the vortex sheet is completely rolled-up ahead of the usual tail positions for much of the C_L range of the jet-flapped wing of moderate aspect ratio.

† Values of $C_J > 1 \cdot 0$ are considered, in order to show the trend of the variation of the derivatives with C_J, but it should be noted that Equation (3) is liable to become less accurate as C_J increases.

‡ It is possible that the constant K^* may depend on the jet momentum. Any such effect would merely alter the level of C_L involved in the relationships which follow, and would not affect the general conclusion.

Furthermore if the spanwise distribution of circulation is elliptic the strength of these rolled-up vortices is simply directly proportional to the lift coefficient, and accordingly are many times more powerful than those associated with the plain wing. The presence of these strong vortices in the neighbourhood of a tail demand consideration as to their effect on the fin in asymmetric flight conditions. However, the effect of the vortex strength may be offset by the fact that the jet and so the wake, will be at a greater distance below the fin than the wake of a plain wing (*cf.* calculations of Ref. 5). We have only discussed the more probable condition of the trailing vorticity but the problem is present whether we assume the rolled-up state or not.

The method of the present Paper seems adequate for dealing with the calculation of many of the wing stability derivatives, but further work is necessary on the calculation of the rolling moment produced by asymmetric deflection of the jet.

In view of the importance of the jet effect on the lateral stability derivatives as predicted by the present calculations it seems advisable to ensure that an experimental check should be available as soon as possible.

LIST OF SYMBOLS

a_0		Lift-curve slope of infinite span wing without jet-flaps
A		Aspect ratio
A_n	=	$a_n\alpha + b_n + c_n\tau$, coefficients of Fourier series for the circulation corresponding to the nett-lift coefficient
c		Wing chord
$\bar{C}_{D\,0}$		Profile-drag coefficient for finite wing
C_J		Jet-momentum coefficient
C_L		Local lift coefficient
$\bar{C}_L$		Total lift coefficient
$C_{L\,n}$		Local nett-lift coefficient
$\bar{C}_{L\,n}$		Total nett-lift coefficient
$C_{L\,\infty}$		Lift coefficient of infinite span wing
C_l		Rolling-moment coefficient
C_n		Yawing-moment coefficient
l_p	=	$\dfrac{\partial C_l}{\partial\left(\dfrac{ps}{V}\right)}$, rolling-moment derivative due to rolling
l_r	=	$\dfrac{\partial C_l}{\partial\left(\dfrac{rs}{V}\right)}$, rolling-moment derivative due to yawing
n_p	=	$\dfrac{\partial C_n}{\partial\left(\dfrac{ps}{V}\right)}$, yawing-moment derivative due to rolling
n_r	=	$\dfrac{\partial C_n}{\partial\left(\dfrac{rs}{V}\right)}$, yawing-moment derivative due to yawing
$n_{r\,0}$		The part of n_r due to the profile drag
$n_{r\,i}$		The part of n_r due to the induced drag
p		Angular velocity in roll
r		Angular velocity in yaw
s		Semi-span of wing

α		Incidence of wing
α_e		Effective incidence of wing
α_i	=	$\alpha - \alpha_e$, induced incidence at wing
α_∞		Downwash angle at infinity downstream
μ_1	=	$\dfrac{\partial C_{L\infty}}{\partial \tau}$, a function of C_J only
μ_2	=	$\dfrac{\partial C_{L\infty}}{\partial \alpha}$, a function of C_J only
τ		Angle of ejection of the jet, relative to the wing chord
$\Gamma(y)$		Circulation corresponding to the local nett-lift coefficient

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TABLE 1

Comparison of Derivatives for Three Wings

Aspect ratio $= 6 \cdot 0$	Wing incidence $= 6$ deg
Jet-thrust coefficient $= 2 \cdot 0$	Jet deflection $= 60$ deg

Wing I Wing without jet flap, at incidence of 6 deg.

Wing II Wing without jet flap, but with lift coefficient equal to that of jet-flapped wing.

Wing III Wing with jet flap, at incidence 6 deg.

Wing	C_L	l_p	l_r	n_p	$n_{r\,i}$
I	0·49	−0·471	0·11	−0·025	−0·004
II	4·95	−0·471	1·11	−0·248	−0·390
III	4·95	−0·816	0·64	−0·046	−0·065

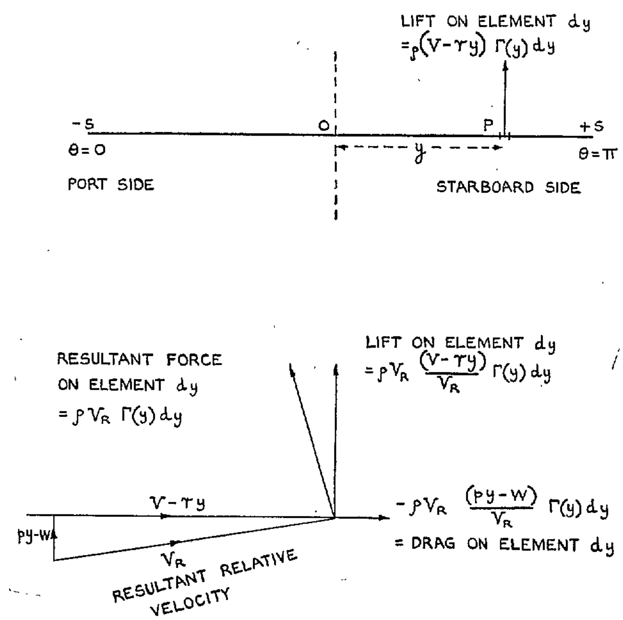

FIG. 1. Notation and resolution of the force on an element of wing due to the circulation $\Gamma(y)$ into components (lift and drag).

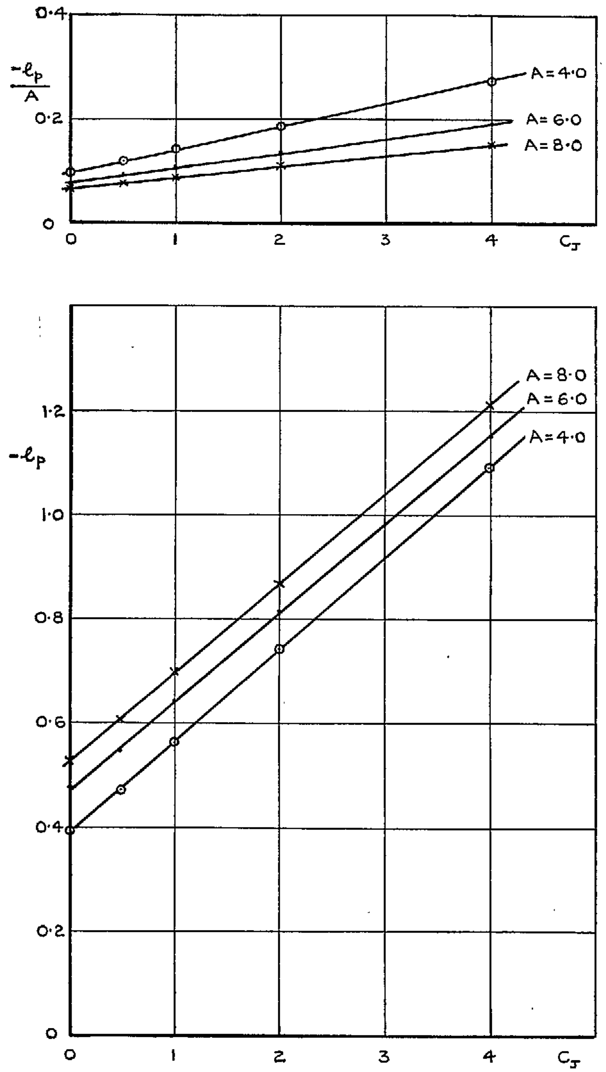

FIG. 2. Rolling-moment derivative due to rolling, l_p.

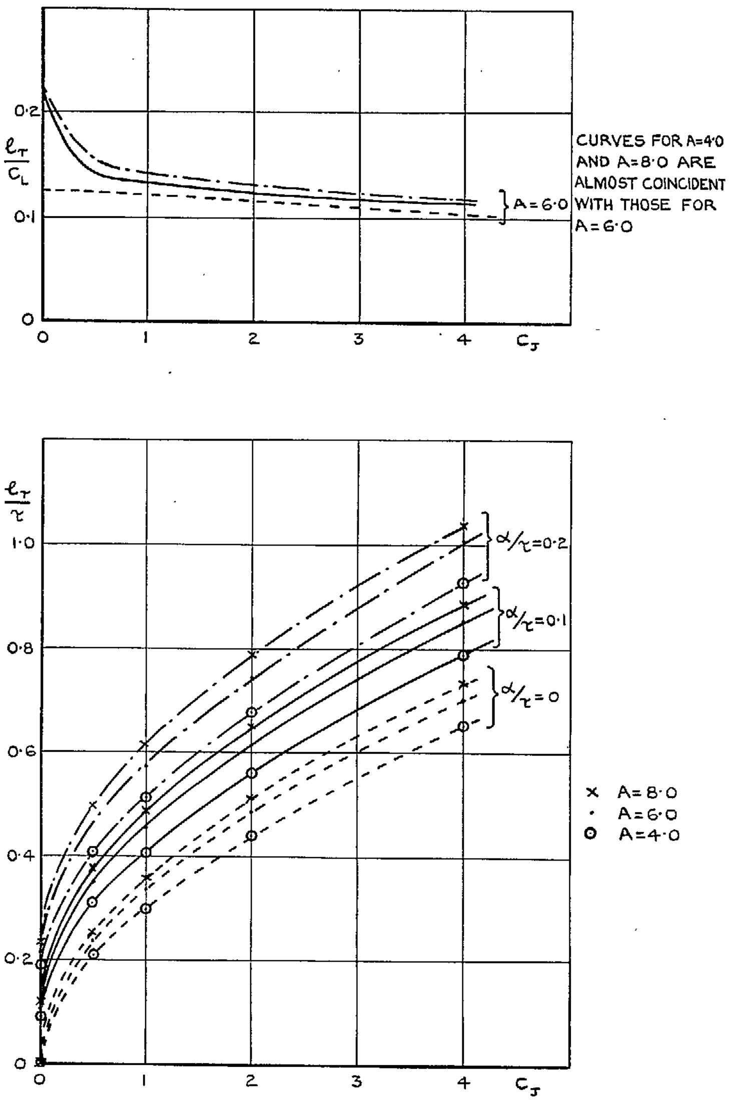

FIG. 3. Rolling-moment derivative due to yawing, l_r.

(84804) Wt. 67/1876 K.5 6/62 Hw.

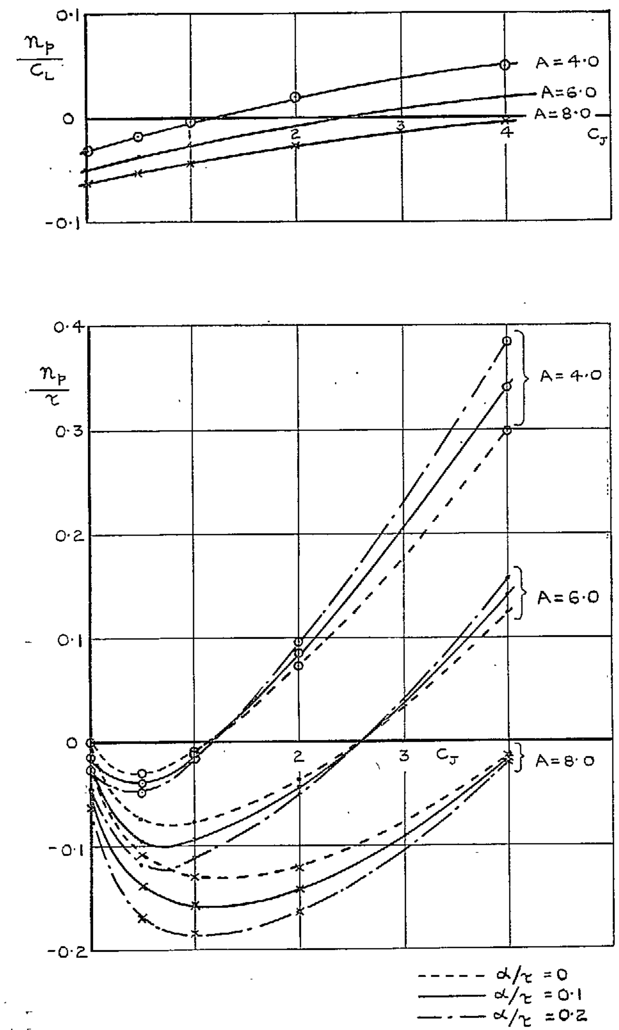

FIG. 4. Yawing-moment derivative due to rolling, n_p.

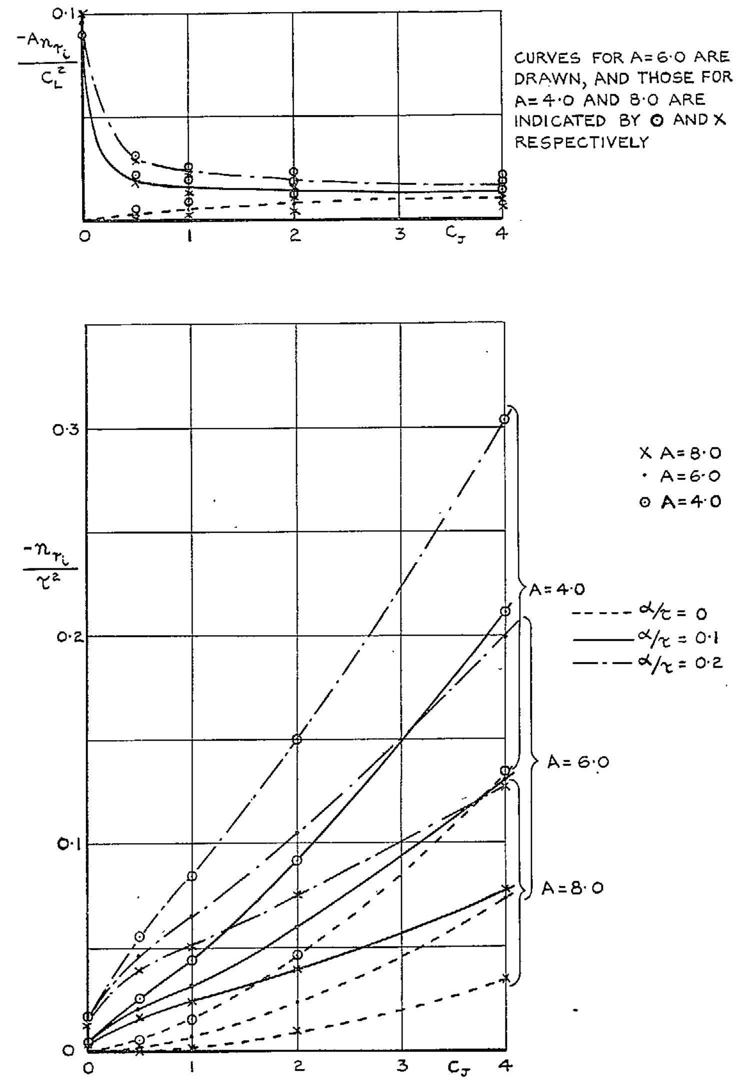

FIG. 5. Yawing-moment derivative due to yawing, n_{ri}.

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