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The Relationship between the Discrete Gust and Power Spectra Presentations of Atmospheric Turbulence, with a suggested Model of Low-Altitude Turbulence

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Summary. The relation between the discrete gust and the spectral presentations of atmospheric turbulence is discussed and the limitations of the discrete gust approach are pointed out.

The available measurements of low-altitude turbulence are analysed. The relationships between the three components of turbulence at low altitude and the effect of wind are considered.

The following model of atmospheric turbulence in temperate climate at 250 ft above the terrain is suggested:

Vertical component

R.M.S. in ft sec^{-1}, σ_w	3·2	6	8·2	12
Proportion of time in σ_w, P	50 per cent	13 per cent	6 per cent	0·6 per cent

Lateral component $\sigma_v = \sqrt{(1\cdot4)}\sigma_w$; with respect to mean wind

Longitudinal component $\sigma_u = \sigma_w$; with respect to mean wind

Assumed spectral shape: $G(\Omega) = \sigma^2 \dfrac{L}{\pi} \dfrac{1 + 3\Omega^2 L^2}{(1 + \Omega^2 L^2)^2}$;

Assumed turbulence scale $L = 1000$ ft

1. *Introduction.* From many practical considerations a knowledge of the structure of low-altitude turbulence is highly desirable. There are at least two problems in the aircraft design field where a more detailed knowledge of low-altitude turbulence is essential. First, in the problem of a high speed, low altitude, long distance flight where not only the airframe and aircrew fatigue but also the controllability of the aircraft in turbulent air have to be considered. Second, the problem of stability and control during the approach, landing and take-off. In the past, when the dynamics of a new design were not too different from past experience, simple, semi-empirical control design requirements were sufficient. At present, however, there are supersonic shapes (*e.g.*, narrow delta) on the one hand and V.T.O.L./S.T.O.L. on the other, where the behaviour in turbulent air may be very different from that occurring in the past requiring, among other things, a more fundamental approach to stability and control at low speeds.

* Previously issued as R.A.E. Tech. Note Aero. 2682—A.R.C. 21,874.

There are programmes of experimental work in this country and in U.S.A. with the object of measuring low-altitude turbulence and correlating it with the meteorological conditions. The results of these experiments will not be available for some time, and it is felt that an effort should be made to establish now some model of low-altitude turbulence based on existing information, which, although incomplete, is quite substantial. The *Hunter* aircraft results obtained from low-altitude flights[1] were analysed by Ray of English Electric[2] with very promising results. Besides the *Hunter* experiment, some information about low-altitude turbulence can be found among the turbulence measurements on *Constellation*[3] and more recently on *Viscount* aircraft[4], and also in Bullen's work on the variation of turbulence with altitude[5].

The description of atmospheric turbulence using the single gust concept is insufficient for the dynamic study of aircraft responses and almost certainly a much more useful description can be given using the concept of the power spectrum. The evaluations of atmospheric turbulence spectra at low altitude obtained from tower and aircraft observations[6, 7, 8] indicate that the assumption of a constant shape of spectra may be valid in application to aircraft dynamics. Attempts have been made to fit analytical expressions with empirical constants to the measured spectra and to correlate them with meteorological parameters[8]. It appears moreover that the analytical expression suggested by N.A.C.A. for high-altitude turbulence is also a good approximation for low levels and only the scale of turbulence is variable with altitude, being roughly proportional to it for low altitudes, say below 1,000 ft.

2. *The Physical Picture of the Power Spectral Density Function of Atmospheric Turbulence.* In order to realise the possible limitation of the power spectral description of atmospheric turbulence, it might be worthwhile to discuss a physical picture of it.

The origin of air turbulence is some form of instability. In the atmosphere this could be either a thermal instability or mechanical instability, like wind shear or flow over obstacles. In wind tunnels the turbulence can be created, for example, by a grid, where mechanical instability produces a multitude of vortices, not unlike Von Karman vortex streets. Turbulence created in this way is roughly of the scale of the grid producing it and is being continuously broken down through the action of inertial and viscous forces into smaller and smaller vortices. Thus the turbulence energy is being transported from the long wavelength towards progressively shorter wavelengths. With decreasing wavelength, the viscous forces become increasingly dominant and dissipation into heat takes place. The power spectrum of air turbulence in Fig. 1 is a plot of energy density against wave number k (in this paper a wave number k is defined as a reciprocal of wave length $k = 1/\lambda$). (*See* Ref. 10.) The energy is added to the air, by some unspecified mechanism, at low values of wave number, Fig. 1, transported towards higher wave numbers and eventually dissipated at very high wave numbers[9]. The part of the power spectrum which is defined mainly by the energy transport is often called the inertial subrange, and the part of viscous dissipation, the viscous subrange. Kolmogoroff's similarity hypothesis states that in the inertial subrange the power of the one-dimensional power spectrum of air turbulence should decrease as $k^{-5/3}$. The experimental evidence seems to be in agreement with this law for a wide range of wavelengths (*see* Ref. 19).

From the experimental evidence it appears that for clear air atmospheric turbulence the inertial subrange begins at a wavelength of the order of a few thousand feet[19]. The inertial subrange merges into the viscous subrange at a wavelength of the order of a centimeter. From the aircraft point of view we are interested only in the wavelength range corresponding to the energy input and inertial subrange.

From this simple physical picture of atmospheric turbulence some general observations about its power spectrum can be made. First of all, it can be expected that in the inertial subrange the turbulence power will always vary as approximately k^{-2} as this is a property of the fluid in general and thus should be independent of the origin of turbulence. This deduction is fully confirmed by the measurements either in the free atmosphere or in wind tunnels. The magnitude of the power spectrum in the inertial subrange will depend of course on the rate of energy input. The shape and magnitude of the power spectrum in the energy input range is more difficult to define, as it will depend on the mechanism by which the turbulence originated. The experimental evidence suggests that the energy input range tends to be flat, independent of the wavelength[19]. It can be expected that moving towards very long waves it is more and more difficult to define the power spectrum function, as this depends on the local mechanism of turbulence creation. It appears that the analytical expression for the one-dimensional vertical component of air turbulence as given by[19]:

$$G(\Omega) = \sigma_w^2 \frac{L}{\pi} \frac{1 + 3\Omega^2 L^2}{(1 + \Omega^2 L^2)^2}; \tag{1}$$

where $\Omega = 2\pi/\lambda = 2\pi k$ space frequency, radians per foot

L scale of turbulence in feet

σ_w root-mean-square of turbulence in ft $\sec^{-1}$

in general fits reasonably well to the experimentally obtained spectra, and for atmospheric turbulence at altitudes above say 1,000 ft, the scale is of the order $L = 1{,}000$ ft. In Ref. 19 the reasons for and advantages of the spectral shape as given by Expression (1) are discussed in some detail. It must be remembered that the Expression (1) is valid only within some range of frequencies Ω (or wavelengths λ). At very high frequencies the spectral density of air turbulence must, due to viscous dissipation, be much smaller than that given by the Expression (1), and at very low frequencies there is no experimental evidence to verify the Expression (1). Fortunately in the range of wavelengths of interest to aircraft applications, say from 10,000 ft down to 10 ft, the above Expression appears to give a satisfactory description of the atmospheric turbulence spectrum.

3. *The Power Spectrum of Atmospheric Turbulence at Low Altitudes, and a Model Assumed for the Present Analysis.* The power spectrum of atmospheric turbulence near the ground surface is a function of many parameters of which the most important appear to be: height above the ground, ground roughness, wind or rather wind shear and air stability (lapse rate)*. From the available experimental results[7,8] and their discussion[6] only very broad conclusions can be drawn at present.

Regarding the vertical component of atmospheric turbulence (w_g), it is best to quote Panofsky[6]:

"To summarise, the 'scale' of vertical velocity is essentially controlled by distance from the ground, being, in fact, proportional to it; the magnitude of the energy, on the other hand, depends on wind speed (or wind shear), stability, roughness, and, especially in stable air, on height."

The power spectra of lateral (v_g) and longitudinal (u_g) components (with respect to wind direction) are much more complex functions of the meteorological parameters largely perhaps because u_g, v_g are not so directly affected by the constraint of the underlying boundary and according to Panofsky their shapes may depend on the origin of the turbulence, *i.e.*, mechanical or thermal.

* These parameters are of course inter-related so that they are not all independent variables.

Taking however a broader, engineering, view, it can be concluded from the examination of the experimental results of Refs. 7 and 8 that the spectra of the vertical component can still be defined by Expression (1), where only the scale L and σ_w are variable.

The present analysis is made for heights below 1,000 ft above the ground; and for these heights it would appear advisable to take as turbulence scale the height above the ground. However, it was felt that no appreciable error would result in practical applications to aircraft dynamics if the scale is assumed constant and equal to $L = 1{,}000$ ft. This assumption has an important advantage, that the same power spectrum function is used for all heights, thus considerably simplifying any numerical studies.

It is fully realised that, when more measurements of low-altitude turbulence become available, the assumption regarding the power spectral shape may need revision.

4. *Description of the Atmospheric Turbulence using the Power Spectrum Concept.* Strictly speaking, the power spectral approach can only be applied to so called stationary processes. A stationary process is a process the statistical properties of which are independent of the time origin. In practice, the statistical properties of atmospheric turbulence are not constant with time but, as long as those changes take place sufficiently slowly, the spectral approach is justified. In the present state of development of spectral techniques it is not very clear how slow these changes in statistical properties have to be, in order to give satisfactory approximation. Experimental evidence seems to indicate that patches of turbulence of the order of a few tens of miles in length can be regarded as statistically stationary.

For a full description of atmospheric turbulence, besides the knowledge of the power spectrum and the associated value of R.M.S. velocity knowledge of the probability of exceeding given values of gust strength (turbulent velocity) is required. From the point of view of engineering expediency, the Gaussian distribution is the obvious choice. Fortunately the measured gust distributions can often be approximated by Gaussian distributions providing that measurements are limited to small enough patches of turbulence.

The large scale statistical measurements of air turbulence covering many thousands of miles are exclusively made with the help of counting accelerometers installed in operational aircraft. Measurements of this type, being a mixture of many patches of turbulence, cannot be described by one Gaussian distribution. Such a large sample can be divided, under assumption of constant spectrum shape, into its Gaussian components with corresponding values of (R.M.S.) turbulence levels and corresponding proportions of time (or distance) attributed to each component.

Many objections, on mathematical and physical grounds, can be raised against such a model of the atmosphere turbulence, but it cannot be denied that this approach seems to work satisfactorily in practice. Furthermore, as we are forced to use the spectral approach in studies of the aircraft dynamic responses, the single gust approach being entirely unsatisfactory, we must have some dynamic picture of the atmospheric turbulence.

A small amount of experimental work is now in progress in the Royal Aircraft Establishment in an effort to clarify some of the doubts in spectral techniques.

The object of the present study was to provide a model of the low-altitude atmospheric turbulence in terms of the power spectral density functions. It was assumed on the basis of Refs. 7 and 8 (*see* Section 3) that the power spectrum is that given by Expression (1). For simplicity of analysis and of future application of results, it was assumed that the turbulence scale $L = 1{,}000$ ft, somewhat contrary to the existing experimental evidence. It was thought that to facilitate a comparison with other results, the model of the turbulence should be presented also in terms of 'single gusts', under appropriate assumptions.

The relationship between the R.M.S. value of aircraft response (usually normal acceleration in g's) and the R.M.S. of the turbulence in feet per second is:

$$\sigma_n = \frac{\rho V a}{2W/S} \times K \times \sigma_w; \tag{2}$$

where σ_n	R.M.S. of normal acceleration, 'g'	
σ_w	R.M.S. of air turbulence, ft sec^{-1}	
ρ	air density,	slug ft^{-3}
V	aircraft speed,	ft sec^{-1}
a	lift slope,	rad^{-1}
W/S	wing loading	lb ft^{-2}
K	gust response factor, spectral.	

The spectral gust response factor, K, is analogous to the single gust alleviation factor F[11] but takes into account more faithfully the dynamic properties of the measuring system (in this case aircraft plus accelerometer) and of the air turbulence. The value of K is a function not only of the aircraft dynamics, but also of the shape of turbulence power spectrum. In the present case, where the shape of spectrum and turbulence scale are assumed constant, the spectral gust response factor K is a function of the measuring system dynamics only. The parameter K is analogous to Fung's parameter $\sqrt{\frac{I(k,s)}{\pi}}$ (*see* Ref. 12 and its nomenclature), except that K includes aircraft pitching and response of the measuring instrument.

The counts of the counting accelerometer tell us how many times per unit of time (or per unit of distance flown) a given value n of acceleration was exceeded. Assuming that a given record is a sum of m Gaussian distributions of R.M.S. acceleration $\sigma_{n\,i}$, each distribution lasting proportion P_i of time (or distance) flown $\left(\sum_1^m P_i = 1\right)$, the number N (say per 1 mile $= 5280$ ft) of accelerations (either positive or negative, but not both positive and negative) can be expressed as:—

$$N(n) = N_0 \sum_{i=1}^{i=m} P_i \exp\left\{-\tfrac{1}{2}\left(\frac{n}{\sigma_{n\,i}}\right)^2\right\} \tag{3}$$

where N_0 can be computed from the power spectrum Φ_n of normal acceleration,

$$N_0 = \frac{5280}{2\pi}\left[\frac{\int_0^\infty \Omega^2\Phi_n(\Omega)d\Omega}{\int_0^\infty \Phi_n(\Omega)d\Omega}\right]^{1/2} \text{; per mile} \tag{4}$$

$\Omega = 2\pi/\lambda = \omega/V$ is the space frequency, rad ft^{-1} and ω is a frequency in rad sec^{-1}. It should be noted that the integral in the denominator of (4) is the mean square of normal acceleration.

(82200)

The N_0 parameter, which can be called a characteristic frequency of a given aircraft in a given turbulence, is a function of the aircraft plus instrument dynamic characteristics and of the shape of the turbulence power spectrum. In the present study therefore the characteristic frequency N_0 varies only with aircraft and instrument characteristics.

One may be tempted to compute the distribution of 'single gusts' from the known spectrum of atmospheric turbulence, as given, *e.g.*, by Expression (1). There are two reasons why such an attempt is entirely futile. First, the integral $\int_0^\infty \Omega^2 G(\Omega)\, d\Omega$ does not converge. The analytical expression for $G(\Omega)$ is not correct for wavelengths less than say 1 cm, where power decreases more rapidly due to viscous dissipation. Second, even if we cut off the spectrum $G(\Omega)$ at say 1 cm assuming $G(\Omega) = 0$ for $\Omega > 6 \cdot 28$ rad cm^{-1}, the calculated value of N_0 will be a few orders higher than that measured by a counting accelerometer. This can be easily explained physically. The actual air turbulence contains many components of very small amplitude but very high frequency. An idealised instrument measuring the turbulence (say some super-sensitive hot wire anemometer with flat frequency response) would actually register a very large number of crossings of given gust strength, thus indicating a very large number of 'single gusts' of that strength. The value of N_0 measured with such an instrument will probably agree reasonably well with estimates based on the truncated expression for $G(\Omega)$. The high frequency components of atmospheric turbulence are strongly attentuated by the aircraft and the measuring accelerometer and the value of N_0 as measured by counting accelerometer and corresponding to the 'single' gust concept, is a characteristic of the aircraft rather than of turbulence. The question arises, how has it been possible to correlate the gust data measured by counting accelerometers on different aircraft, when the single gust concept does not take into account the frequency characteristics of the aircraft defined by N_0? Even if the conversion from the accelerations to the single gust strengths was correct, one would expect to measure many more gusts on a small, high frequency aircraft than on a big, low frequency one. In the course of the present investigation the values of N_0 (expressed in terms of the characteristic wavelength λ_0) were evaluated for different aircraft, and it was found that they do not differ so much as might have been expected. For example, it was estimated that the characteristic wavelength in turbulent air of a *Constellation* aircraft is of the order of $\lambda_0 = 530$ ft (this gives $N_0 \sim 10$ per mile) and of a *Hunter* aircraft approximately $\lambda_0 = 400$ ft*. The values of λ_0 as estimated in Ref. 13 are from 500 to 700 ft being probably for transport aircraft. This agrees pretty well with present estimates, remembering that these values depend to a large extent on the aircraft plus instrument dynamic characteristics at high frequencies. The value of λ_0 for the *Hunter* aircraft was approximately confirmed experimentally. Thus, it can be expected that the value of N_0 does not vary appreciably from aircraft to aircraft. This happy state of affairs is partly due to the sharp cut-off at high frequencies of the counting accelerometers used.

It should be strongly underlined that the above findings cannot be generalised. The measurements of strain, or even measurements of accelerations made on a very flexible aircraft may give values of N_0 many times larger than those quoted in this paper and which it is believed are representative of the past and probably some of the current generation of aircraft. The discrete gust approach is

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* At all-up-weight 14,500 lb, as in Ref. 1, and for an assumed value of static stability. For the same type of aircraft, but at $W = 17{,}000$ lb and with C.G. position as flown during a recent R.A.E. flight test, this wavelength appears to be nearer 450 ft.

not suitable for estimating the response of more lightly damped modes than those on which it is based. The gust distribution based, for example, on fin loads would be entirely different from the distribution based on normal accelerations, even if both vertical and lateral components of air turbulence are identical.

5. *The Relationship between 'Single' Gust and Spectral Representation of Atmospheric Turbulence.* From the measured frequency of normal acceleration $N(n)$, accelerations per mile, and the computed value of the characteristic frequency N_0 it is possible to estimate the values of P_i and $\sigma_{n\,i}$ using Equation (3) (*see* Refs. 2 or 14).

The R.M.S. values of normal acceleration can be converted to R.M.S. values of atmospheric turbulence $\sigma_{w\,i}$ by Equation (2). Thus we have a description of atmospheric turbulence by a group of spectral density functions each of intensity $\sigma_{w\,i}$ and lasting a proportion P_i of the total time (or distance) of flight.

Thus the distribution of discrete gusts measured by an aircraft with known values of N_0 (Equation (4)) and σ_w/σ_n (Equation (2)), can be expressed in terms of the spectral distribution

$$N(w_g) = N_0 \sum_{i=1}^{i=m} P_i \exp\left\{-\tfrac{1}{2}\left(\frac{w_g}{\sigma_{w\,i}}\right)^2\right\}; \qquad (5)$$

It is believed that this approach gives a reasonably true picture of the turbulence. The aircraft dynamic response can be removed entirely from the measurements by the appropriate application of Equations (2) and (4).

In the single gust analysis, the values of normal acceleration are converted directly into single gusts by a formula similar to the Formula (2).

$$n = \frac{\rho V a}{2W/S} \times F \times w_g\,. \qquad (6)$$

It can be shown that the spectral and single gust methods when applied to a new aircraft design, give identical results if and only if the values of N_0 (characteristic frequency of aircraft response in turbulent air) and the ratios (spectral response factor to gust alleviation factor) are the same for the new design as for gust data collecting aircraft.

If the single gust distribution estimates were based on measurements of the response of some lightly damped mode of the aircraft (*e.g.*, wing strain), the number of gusts per mile will be considerably higher than is inferred from counting accelerometer measurements. It cannot be stressed too strongly, that the model of atmospheric turbulence in terms of single gust distribution is only 'an equivalent' model and can be used, strictly speaking, only for an aircraft which has the same values of N_0 and of K/F as the measuring aircraft.

During the present investigation the values of N_0 and K/F were estimated for a range of contemporary aircraft and it was found that the value of N_0 ranges from about 7·5 to 13 positive crossings per mile and the ratio of response factors F/K varies within less than $\pm$ 20 per cent. This order of variation can easily be hidden within the experimental scatter. It can then be concluded that the single gust approach is entirely satisfactory, as long as it is used within the limits of its validity. It cannot be used for the estimates of responses to turbulent air of modes which have widely different frequencies and/or dampings from those of the short period oscillations of conventional aircraft.

6. *Spectral Analysis of Available Data Pertaining to Low-Altitude Turbulence.* For a number of years the N.A.C.A. has been engaged in the re-evaluation of accelerometer data[13, 14, 15, 16] in an attempt to establish a model of atmospheric turbulence based on the spectral concept.

The N.A.C.A. approach can be described briefly as follows. The atmosphere turbulence is divided into two types, which quoting Ref. 13 "one consisting of a severe turbulence condition, represented by turbulence encountered in thunderstorms, and termed 'storm' turbulence and the other consisting of a considerably less severe condition, perhaps representative of conditions in moderately rough clear air, and termed 'non-storm' turbulence". During its operational life the aircraft covers P_1 proportion of distance flown in non-storm turbulence and P_2 proportion of distance flown in storm turbulence*. The intensity of each type of turbulence is defined by a continuous distribution of R.M.S. turbulence σ_w. The suggested analytical expression for probability density distribution of root-mean-square gust velocity is:

$$\hat{f}(\sigma_w) = \sqrt{\left(\frac{2}{\pi}\right)} \frac{1}{b} \exp\left\{-\tfrac{1}{2}\left(\frac{\sigma_w}{b}\right)^2\right\}. \tag{7}$$

Thus the parameter b can be identified as the R.M.S. of turbulence R.M.S.'s, its value being a function of altitude and type of turbulence. The above expression has a great advantage of giving a very simple expression for the frequency distribution of any output quantity, y. Using the nomenclature of the present note, the number of accelerations of intensity n per mile (positive or negative) is given by:

$$N(n) = N_0 \left[P_1 \exp\left(-\frac{n}{b_1 \bar{A}_1}\right) + P_2 \exp\left(-\frac{n}{b_2 \bar{A}_2}\right)\right]; \tag{8}$$

where suffix 1 and 2 refers to non-storm and storm turbulence and $\bar{A} = \sigma_n/\sigma_w$, as given by Equation (2)†.

The shape of the turbulence spectrum assumed by N.A.C.A. is that given by Equation (1) and the turbulence scale assumed was $L = 1{,}000$ ft (thus $\bar{A}_1 = \bar{A}_2$).

In order to obtain some indication of the possible properties of low-altitude turbulence it was decided to utilise the data of Ref. 13, which is based on very extensive experimental information. Unfortunately this approach required extrapolation of the data to low altitudes. Such extrapolation is of doubtful value, as the turbulence parameters vary rapidly with altitude near the ground. The nominal height was chosen to be 500 ft at which height the contribution of storm turbulence can be neglected, remembering that the data refer probably to temperate climate only and the very approximate nature of the extrapolation. From the extrapolation the following values were obtained.

$P = 0 \cdot 4$ proportion of distance flown in turbulence
$b = 5 \cdot 1$ R.M.S. of σ_w of turbulence, ft sec^{-1}
$H = 500$ ft assumed height.

The mean value of R.M.S. turbulence can be estimated in accordance with the definition of Equation (10) from the expression

$$(\sigma_w)_{\text{mean}}^2 = P \int_0^\infty (\sigma_w)^2 \hat{f}(\sigma_w)\, d\sigma_w; \tag{9}$$

and was found to be $(\sigma_w)_{\text{mean}} = 3 \cdot 22$ f.p.s.

The distribution of turbulence R.M.S.'s so obtained is plotted in Fig. 2 for further reference.

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* In this paragraph, the nomenclature of Ref. 13 is used.

† Generally, $\bar{A}_1$ may not be equal to $\bar{A}_2$, if for example, the scale of turbulence in thunderstorms is difference from the scale in clear air turbulence.

The counting accelerometer data of Refs. 3 and 1 were analysed using the approach described previously, and values of P_i and $\sigma_{w\,i}$ were obtained. It should be mentioned that the counting accelerometer data usually contain more positive than negative acceleration increments and it has often been concluded therefrom that the aircraft encounters more positive than negative gusts. Although this might be true, it has never been demonstrated conclusively. The author believes, however, that this effect is mainly due to manoeuvring accelerations, which by their nature are almost entirely positive. This belief is partially verified by experimental evidence obtained during the low-altitude flight experiment being conducted by R.A.E. Thus although the possibility of an asymmetric distribution of positive and negative gusts is not rejected, the data of counting accelerometers are analysed measuring the incremental accelerations from the mean value. This is not necessarily an entirely satisfactory approach, as it does not account for possible negative manoeuvring loads, but at present no better approach can be offered. The counting accelerometer data of Ref. 3 were obtained during final descent for a nominal height of 500 ft. The data of Ref. 1 contain extensive measurements obtained from low flying *Hunter* aircraft, at an average height of 250 ft. The estimates of Ref. 2 are also included, but the values of σ_w are modified to include the effects of pitching motion by appropriate adjustment of the value of parameter K. In addition, a small sample obtained during low-altitude flights by Aero F is included. Table 1 summarises the results of this analysis, which are also plotted in Fig. 2.

It should be noted that the combination of the values of P_i and $\sigma_{w\,i}$ describing given turbulence are somewhat arbitrary. It is possible to fit many combinations of P_i and $\sigma_{w\,i}$ to a set of experimental data, so that a direct comparison of results as shown in Table 1 and Fig. 2 is rather difficult.

To give a better picture of correlation between different data a mean value of R.M.S. turbulence is included in the Table. The mean value of turbulence is defined by:

$$(\sigma_w)_{\text{mean}} = [\sum P_i \sigma_{w\,i}^2]^{1/2}. \tag{10}$$

It can be seen that mean values of R.M.S. turbulence do not differ too widely; it appears however that *Constellation* data[3] indicate a somewhat lower intensity of low-altitude turbulence. It is significant that the *Constellation* and Ref. 13 data refer to a nominal height of 500 ft, when the rest of the data refer to a nominal height of approximately 250 ft.

The data presented in the Table are shown in Fig. 2 as the cumulative probability distribution of air turbulence R.M.S.'s and compared with extrapolated data of Ref. 13. It should be remembered again, that the present analysis takes into account the aircraft plus instrument frequency response as accurately as possible. Thus the present analysis includes the effect of aircraft pitching, while the analysis of Ref. 13 takes into account aircraft plunging motion only using Fung's analysis[14]. It can be expected therefore that the present estimates will give slightly lower numerical values of turbulence R.M.S.'s than those of Ref. 13.

The evaluated distributions of turbulence R.M.S.'s, in Fig. 2, show some degree of consistency and are not too dissimilar from the extrapolated distribution of Ref. 13. It should be pointed out that data labelled 'Aero Department' are based on a small sample. This sample includes flight in very severe turbulence, thus the larger values of σ_w are not representative of average conditions.

7. *Single Gust Presentation.* It was felt that, due to the somewhat arbitrary choice of P_i and $\sigma_{w\,i}$ distributions, no reliable comparisons between the different estimates of low-altitude turbulence

in Fig. 2 could be made. A better comparison can be made by first converting them into the familiar form of gust counts per mile.

It was assumed that all the turbulence models shown in the Table and Fig. 2 are sampled by the same aircraft with the same counting accelerometer and the response of the aircraft and counting accelerometer to continuous turbulence was computed. The calculations were made for a typical transport aircraft, not dissimilar to the *Constellation* aircraft of Ref. 3. The calculated value of the spectral response factor was $K = 0 \cdot 62$ (including pitching) and of positive crossing per mile $N_0 = 10$. Thus the calculated distribution of normal acceleration which would have been experienced by the assumed aircraft plus accelerometer system was converted by the usual method to single gusts. The gust alleviation factor was taken to be $F = 0 \cdot 77$.

It should be pointed out that, for the conversion from the gust spectra to the single gusts, any other conventional aircraft could be assumed with only minor differences in the computed distribution. As mentioned before, if the aircraft have the same values of gust response factor ratios F/K and the same values of zero crossings, N_0, there is no difference between the spectral and single gust approaches.

The calculated single gust distributions, as sampled by the above aircraft, are plotted in Fig. 3 as miles to exceed a given value of a single gust. To check the numerical results of the conversion, the computed single gust distribution based on the spectral distribution from Ref. 3, was plotted and compared with direct gust counts from Ref. 3 in Fig. 4. The computed distribution agrees closely with the mean of the measured up and down gusts confirming the correctness of the analytical procedure.

It can be seen from Fig. 3 that for a range of 'single gusts' between 5 f.p.s. and about 20 f.p.s. there is a remarkable agreement between the estimates based on Refs. 1, 2 and Aero Department data, all these data being for a nominal height of 250 ft. The *Constellation* data, Ref. 3, for a nominal height of 500 ft show more miles per gust of low strength and less at gust strengths above about 15 f.p.s. This is in agreement with our present ideas, as it is known that turbulence intensity increases with decreasing height in the region of smaller gusts but the larger gusts are less frequent at low altitudes which may also imply that turbulence scale is smaller at low heights. The extrapolation from Ref. 13 shows some agreement with respect to overall intensity of turbulence, but the slope of the distribution (Fig. 3) indicates that this extrapolation retained the characteristics of turbulence at higher altitudes.

It should be remembered that for gust strengths less than say 5 f.p.s. the estimates are in error due to the fact that the spectral representation does not include a turbulence component of a very low intensity, say $\sigma_w \sim 1$ to 2 f.p.s. It can be expected on physical grounds that the daily average of low-altitude turbulence must contain this low intensity component, as the conditions of a perfect calm, $\sigma_w = 0$, are extremely rare at low altitude in the daytime.

For discrete gust velocities above say 15 to 20 f.p.s. the present estimates cannot be reliable due to the smallness of the samples considered.

8. *Suggested Model of the Low-Altitude Turbulence.* A low-altitude air turbulence model is proposed in terms of power spectral density functions. It is assumed that the turbulence spectrum is given by the Expression (1) with turbulence scale $L = 1{,}000$ ft. The proposed values of turbulence R.M.S. σ_w and their corresponding proportion of duration P are given in Table 2.

The proposed values provide a good fit to the single gusts distribution of Fig. 3. An attempt was made also to represent the turbulence for gusts less than 2·5 f.p.s., hence $P = 50$ per cent at $\sigma_w = 3 \cdot 2$; the representation for gusts of more than 20 f.p.s. was by an extrapolation of the data in Fig. 3. It should be remembered that it is not very clear at the present up to what values of discrete gust velocities the proposed model is valid, but it is felt that the estimates should be reliable up to gusts of 30 f.p.s.

Table 2 applies only to turbulence at a height of about 250 ft above land during the hours of daylight in a temperate region. It may be mentioned that the mean daylight surface wind speed in East Anglia area is 9·8 knots (3-year average); the yearly wind distribution is shown in Table 3, at the end of this paper.

The calculated single gust distribution (positive or negative, but not both) from the proposed spectral distribution of Table 2, assuming the measuring aircraft characteristics as before, is shown in Fig. 5 and compared with the mean of the distributions of Fig. 3 at 250 ft and with *Constellation* results at 500 ft.

The relative (with respect to 10 f.p.s. gust) frequency of gusts of the proposed distribution is shown in Fig. 6 and compared with the distribution suggested by Bullen in Ref. 5. It can be seen, Fig. 6, that the agreement is very good up to gusts of 20 f.p.s. magnitude. For larger gusts, the proposed distribution shows relatively less gusts than Bullen's curve. This is expected, as the present distribution is for 250 ft above the ground, while Bullen's curve represents the relative distribution at much greater heights.

Fig. 7 shows a comparison of the proposed distribution of gusts (at 250 ft) with the actual direct measurements of 'single gusts' as reported in Refs. 1, 3 and 4. It should be remembered that only Ref. 1 refers to 250 ft altitude. The proposed distribution, being a calculated one, is of course symmetrical with respect to zero gust velocity, and is somewhat more severe (as indicated by the spread of the lines) than the measured distributions.

The mean value of the proposed turbulence model is $(\sigma_w)_{\text{mean}} = 3 \cdot 83$ f.p.s. which is a little higher than the values shown in Table 1. This is mainly due to the fact that the proposed turbulence model contains a large proportion (50 per cent) of low-intensity turbulence ($\sigma_w = 3 \cdot 2$). Reducing the proportion of $\sigma_w = 3 \cdot 2$ from 50 per cent to 30 per cent gives the value of $(\sigma_w)_{\text{mean}} = 3 \cdot 56$ f.p.s., which is in better agreement with Table 2, while the gust distribution for gusts larger than 10 f.p.s. is negligibly affected. From the measured gust counts it is difficult to obtain any information about gusts below 10 f.p.s., and the fitted spectral distributions (Table 1) are in error for small gusts. This can be seen in Fig. 3, where the gust distribution curves, based on fitted spectral distributions, curve up for gusts less than 5 f.p.s. It is believed that due to the lack of resolution of counting accelerometers at very low levels of turbulence, the very small gusts are not accounted for. The proposed spectral distribution (Table 2) attempts to extrapolate linearly the single gusts distribution down to 2·5 f.p.s. (*see* Fig. 5) and hence the large percentage (50 per cent) of low-level turbulence ($\sigma_w = 3 \cdot 2$ f.p.s.) and slightly higher value of $(\sigma_w)_{\text{mean}}$.

9. *The Effect of Wind on the Turbulence.* Results of a number of investigations show that the intensity of turbulence near the ground increases with increasing mean wind speed, U. The tower measurements at Brookhaven[7] indicate that at heights of 300 ft above the terrain the turbulence R.M.S. is proportional to $U^{0 \cdot 8}$, similar results were obtained from aircraft measurements. The Brookhaven terrain appears to be moderately rough, and probably not too dissimilar from the

terrain of the South of England, where the data of Ref. 1 were collected and it was thought worthwhile to correlate these two sets of measurements. Two empirical curves were fitted to Brookhaven data[7], relating turbulence R.M.S. (σ_w) in ft sec^{-1} to the mean wind velocity U ft sec^{-1}.

$$\sigma_w = 0\cdot 235 U^{0\cdot 8}$$

describes the results of tower measurements at 300 ft and

$$\sigma_w = 0\cdot 3 U^{0\cdot 8}$$

represents the aircraft measurements at 400 ft. The first formula is a fit for wind speeds between 12 to 45 f.p.s. (7·1 to 26·6 knots) and the second between 20 to 30 f.p.s. (11·3 to 17·7 knots).

Using the results of Ref. 1 the values of $(\sigma_w)_{\text{mean}}$ were computed for a range of wind speeds. It should be remembered that the computed values of $(\sigma_w)_{\text{mean}}$ are on the low side, due to incomplete description of the low intensity component of the turbulence. Values of turbulence $(\sigma_w)_{\text{mean}}$ so obtained are plotted against the mean wind speed in Fig. 8 and compared with results of Ref. 7.

Some additional data about the effect of wind on turbulence were found in Refs. 17 and 20. In Ref. 17 measurements of wind fluctuation on Blackpool Tower at high wind speeds are mentioned; by fitting a normal distribution to the given data the following relationship between turbulence and wind speed was found

$$\sigma_w = 0\cdot 127U \text{ at 503 ft}$$

$$\sigma_w = 0\cdot 132U \text{ at 210 ft.}$$

The mean Blackpool Tower results as given by the above expressions are plotted in Fig. 8. Ref. 20 is not directly concerned with the turbulence-wind speed relationship, but the basic data are applicable here and have also been plotted in Fig. 8. The measurements at Cardington[18] indicate higher levels of turbulence in relation to the wind speed, but the results are not suitable for the present analysis. The agreement between the four independent sets of data is good, but it should be remembered that the values of $(\sigma_w)_{\text{mean}}$ are not precisely defined. The values of σ_w given in Ref. 7 are thought to have been obtained by integration of the power spectrum, but it is not known for what frequency range. The values of σ_w attributed to the data of Ref. 17 were obtained by fitting a Gaussian distribution to four points, while the mean values of σ_w estimated from the gust counts of Ref. 1 are based on rather small samples and definitely do not contain the contribution of low intensity turbulence. The data of Ref. 20 are somewhat exceptional in that they were obtained at a height of only 2 metres and above very smooth ground. Due to inaccuracies in the vertical velocity measurements no σ_w were obtained and the values used in Fig. 8 are believed to be σ_u. It may be seen from Fig. 9, however, that down to 150 ft $\sigma_u \simeq \sigma_w$ and although the data of Ref. 20 refer to a height of 6 ft it was considered reasonable to include them.

Furthermore it is not clear whether the winds quoted refer to the same height as the turbulence measurements, although it is so assumed in the present work.

In spite of all these reservations it can however be concluded that the turbulence intensity, as measured by its R.M.S. value, increases roughly in proportion to mean wind speed.

10. *Other Components of Turbulence at Low Altitude.* Our present knowledge is too limited to arrive at any definite conclusions about the lateral and longitudinal (with respect to mean wind)

components of atmospheric turbulence at low altitudes. Inspection of the measured spectra of different components presented in Ref. 7 leads us to believe that in practical applications, at least for the time being, the Expression (1) for the turbulence spectrum is a reasonable approximation, provided that the turbulence scale is correctly chosen. Due to lack of reliable information, the scale of turbulence will have to be assumed, for the time being, to be $L = 1{,}000$ ft, even if only for the sake of uniformity with the description of high-altitude turbulence.

In order to obtain some information about the relative magnitude of the three components of air turbulence, the measured values of turbulence R.M.S. as given in Refs. 7 and 21 are plotted in Fig. 9. The measured turbulence R.M.S. are based on measured turbulence spectra which cover the wavelength range from 100 ft to 10,000 ft, which is approximately the range of wavelengths of interest to aircraft engineers.

In Fig. 9 the values of variance of longitudinal component, σ_u, and of lateral component, σ_v, are plotted against the value of variance of vertical components, σ_w. These data cover quite a wide range of turbulence intensities (σ_w from $1 \cdot 7$ f.p.s. to $5 \cdot 5$ f.p.s.) and range of altitudes 150 ft to 2,100 ft.

From the inspection of Fig. 9 it can be concluded that within the wavelength band considered, the longitudinal component is roughly equal to the vertical component ($\sigma_u = \sigma_w$), but the lateral component is definitely larger than the vertical ($\sigma_v > \sigma_w$). It is suggested that for the altitudes considered the following relationship between the intensities of different components of air turbulence can be assumed.

$$\left.\begin{array}{ll} \text{longitudinal} & \sigma_u^2 = \sigma_w^2 \\ \text{lateral} & \sigma_v^2 = 1 \cdot 4\, \sigma_w^2 \end{array}\right\} \text{vertical (with respect to mean wind)}$$

11. *Conclusions.* This Note discusses the two methods of atmospheric turbulence analysis and their presentation: namely discrete gusts and spectral techniques.

It is concluded that the discrete gust approach based on normal acceleration counts is analogous to the spectral approach for quite a wide range of aircraft configurations. It is pointed out that the discrete gust counts do not represent the real air turbulence, but provide a model which can be quite satisfactory for the evaluation of normal accelerations on a future design. However, the discrete gust approach is not suitable for estimating the response of lightly damped modes.

The available measurements of low-altitude turbulence are analysed. The results are presented in terms of turbulence R.M.S. distribution. In the process of R.M.S.'s evaluation, the dynamic characteristics of the measuring aircraft plus accelerometer were, as far as is practically possible, removed. The evaluated spectral distributions were converted to the more familiar form of 'miles per gust' which would be experienced by a typical sampling aircraft. It is concluded that the different estimates are sufficiently consistent to suggest a typical model of low-altitude turbulence, corresponding to average day conditions in temperate areas.

Some evidence is presented that the turbulence intensity at low altitudes varies in proportion to mean wind speed.

It appears that for altitudes less than 2,000 ft and within wavelengths approximately 100 ft to 6,000 ft the longitudinal (with respect to mean wind) and vertical components of air turbulence are about equal, but the lateral component is about $\sqrt{1 \cdot 4}$ times more intense than the vertical component.

12. *Suggested Model of Low-Altitude Turbulence, Summary.*

Height 250 ft above the terrain (land, not sea).

Average day (not night) in temperate climate over the whole year.

Mean wind speed 10 knots.

Vertical component

R.M.S. in ft sec^{-1}, σ_w	3·2	6	8·2	12
Proportion of time in σ_w, P per cent	50	13	6	0·6

Longitudinal component σ_u (parallel to mean wind) the same as vertical component, σ_w.

Lateral component $\sigma_v = \sqrt{(1\cdot 4)}\sigma_w$;

The shape of turbulence spectrum assumed in analysis

$$G(\Omega) = \sigma^2 \frac{L}{\pi} \frac{1 + 3\Omega^2 L^2}{(1 + \Omega^2 L^2)^2}.$$

Turbulence scale assumed in analysis

$$L = 1{,}000 \text{ ft.}$$

13. *Acknowledgement.* The author is indebted to Professor Sheppard for his helpful criticism of this paper.

LIST OF SYMBOLS

a		Lift slope, rad^{-1}
$\bar{A}$	=	σ_n/σ_w. Ratio of R.M.S. normal acceleration to R.M.S. gust velocity[13]
b		R.M.S. of turbulence R.M.S.'s (Ref. 13), ft $\sec^{-1}$
$\hat{f}(\sigma_w)$		Probability density distribution of root-mean-square gust velocity[14]
F		Gust alleviation factor (single gust) SP 970
$G(\Omega)$		Power spectral density function of atmospheric turbulence (vertical component)
H		Height above the ground, ft
k	=	$1/\lambda$. Wave number, cycles per foot
K		Gust response factor, spectral
L		Turbulence scale, ft
n		Normal acceleration in 'g'
N_0		Characteristic frequency of aircraft response to turbulent air (also number of zero crossings with positive slope), cycles per mile
$N(n)$		Number of accelerations n (positive or negative) per mile (number of crossings of n level with positive slope, per mile)
$N(w_g)$		Number of gusts w_g (positive or negative per mile) (number of crossings per mile of w_g gust level with positive slope)
P_i		Proportion of time (or distance) flown under i-th condition
P_1		Proportion of distance flown in non-storm turbulence[13]
P_2		Proportion of distance flown in storm turbulence[13]
S		Wing area, ft^2
U		Wind speed, ft $\sec^{-1}$
V		Aircraft speed, ft $\sec^{-1}$
w_g		Vertical gust velocity, ft $\sec^{-1}$
W		Aircraft weight, lb
$\Phi_n(\Omega)$		Power spectrum of normal acceleration
λ		Wavelength of turbulence, ft
λ_0	=	$\frac{5280}{N_0}$. Characteristic wavelength of aircraft response to turbulent air, ft
ρ		Air density, slug ft^{-3}
$\sigma_w, \sigma_v, \sigma_u$		R.M.S. of air turbulence, vertical, lateral and longitudinal component respectively, ft $\sec^{-1}$
σ_n		R.M.S. of normal acceleration in g
Ω	=	$\frac{2\pi}{\lambda} = \frac{\omega}{V}$. Space frequency, rad ft^{-1}
ω		Frequency, rad $\sec^{-1}$

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TABLE 1

Estimated Distributions of R.M.S. Turbulence

		Source of Information				
		Ref. 3 *Constellation* sample 2952 miles height ~ 500 ft	Ref. 1 *Hunter* sample 4488 miles height ~ 250 ft	Ref. 2 (as Ref. 1) modified to include pitching	Unpublished results of Aero F sample ~ 600 miles height ~ 200 ft	Ref. 13 height ~ 500 ft (extrapolation)
Proportion of distance spent at $\sigma_{w\,1}$	P_1	0·30	0·245	0·475	0·134	Continuous distribution $0{\cdot}4\sqrt{\left(\frac{2}{\pi}\right)}\frac{1}{5{\cdot}1}\exp\left\{-\frac{1}{2}\left(\frac{\sigma_w}{5{\cdot}1}\right)^2\right\}$
Turbulence R.M.S.	$\sigma_{w\,1}$	3·4	4·78	3·25	3·32	
	P_2	0·110	0·0945	0·198	0·152	
	$\sigma_{w\,2}$	6·3	7·4	6·42	4·65	
	P_3	0·011	0·0071	0·009	0·126	
	$\sigma_{w\,3}$	11·2	11·3	11·4	5·83	
	P_4	$5{\cdot}2 \times 10^{-4}$			0·046	
	$\sigma_{w\,4}$	20·8			7·75	
	P_5				0·002	
	$\sigma_{w\,5}$				19·1	
Mean value of $\sigma_w = [\Sigma P_i\, \sigma_{w\,i}^{\,2}]^{1/2}$		3·07	3·41	3·78	3·54	3·22

TABLE 2

Suggested Distribution of R.M.S.'s of Low-Altitude Turbulence (250 *ft*)

σ_w f.p.s.	3·2	6	8·2	12	$(\sigma_w)_{\text{mean}} = 3{\cdot}83$
P per cent	50	13	6	0·6	

TABLE 3

Wind Speeds Distribution

Wind summary for Wattisham calculated over a period of 3 years for the hours 0600 to 1800 hours. Ground wind, 10 metres above the ground

Wind Speed Knots		Percentage of a given wind according to months											
Mean	Limit	J	F	M	A	M	J	J	A	S	O	N	D
0	1	0·4	0·5	1·2	0·7	1·2	0·9	0·9	0·9	1·8	3·1	2·3	1·5
2	1 to 3	7·8	7·3	11·0	5·4	6·2	5·7	4·8	7·1	5·7	11·5	9·5	9·3
5	4 to 6	18·7	19·2	26·6	20·1	23·8	20·5	17·9	21·5	23·4	21·3	18·3	25·9
9	7 to 10	30·2	31·2	36·0	31·7	32·5	33·2	42·4	37·7	33·1	28·7	33·3	34·0
13	11 to 15	24·6	21·9	20·1	23·7	25·6	28·3	26·7	25·7	25·5	21·7	17·6	19·3
18	16 to 21	11·5	12·0	4·2	14·3	7·5	8·8	7·2	6·7	8·4	10·7	13·5	7·4
24	22 to 27	4·7	6·2	0·5	3·0	2·9	2·4	0·1	0·2	1·6	2·6	4·1	1·6
30	28 to 33	1·9	1·7	0·4	1·1	0·3	0·2	—	0·2	0·3	0·3	1·2	0·6
37	34 to 40	0·2	—	—	—	—	—	—	—	0·2	0·1	0·2	0·4
Monthly Average Wind Knots		10·84	10·92	8·4	10·67	9·69	10·06	9·58	9·24	9·63	9·37	9·91	9·09

Yearly average wind 9·783 knots

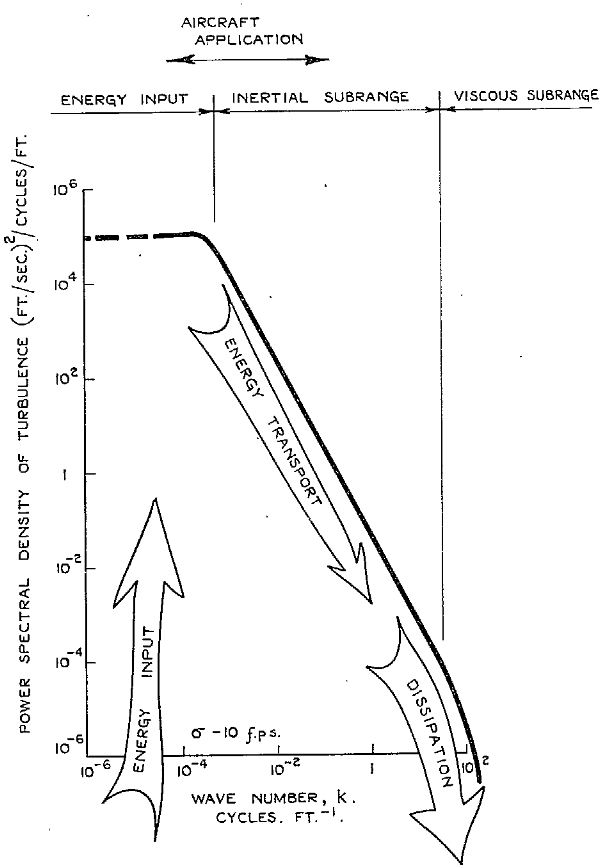

FIG. 1. A diagram of energy flow through the spectrum of atmospheric turbulence.

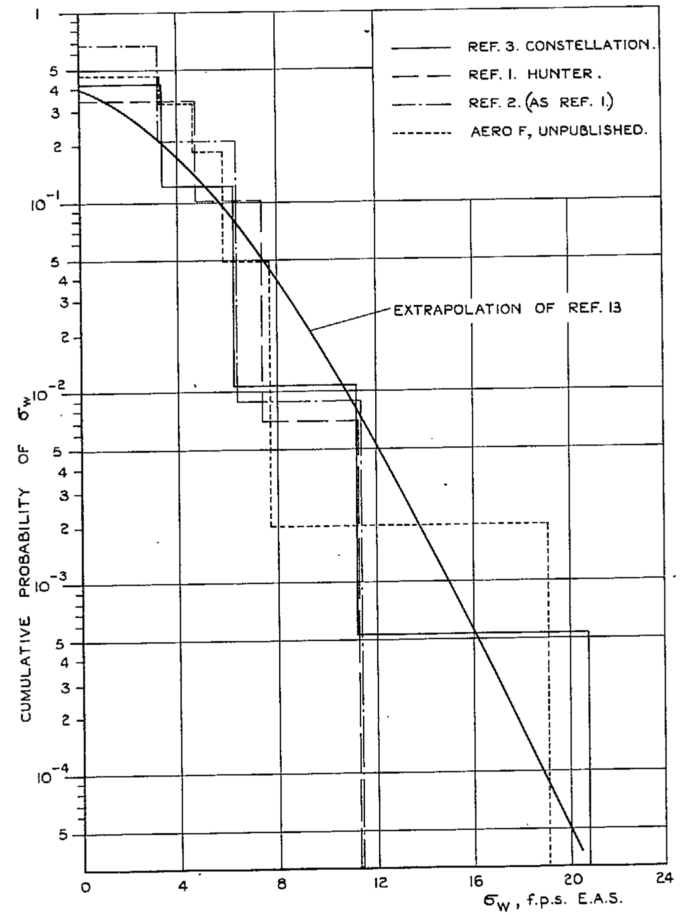

FIG. 2. Cumulative probability distribution of σ_w of atmospheric turbulence at low altitude. Comparison of different estimates.

21

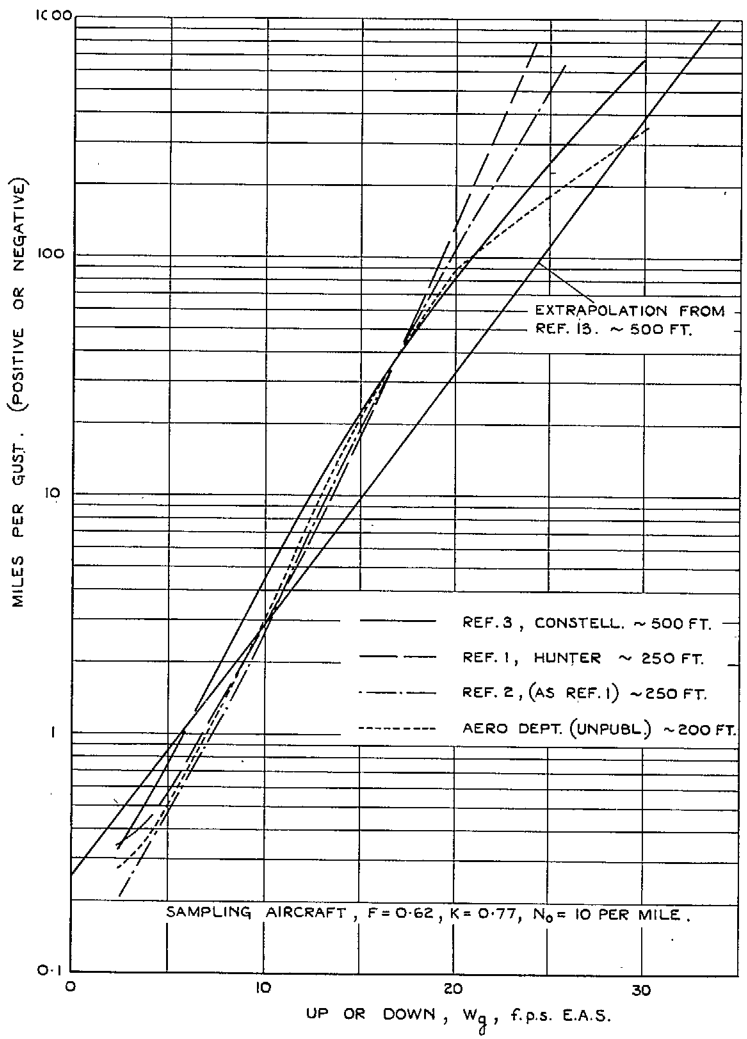

FIG. 3. Discrete gusts distributions calculated from spectral distributions of FIG. 2.

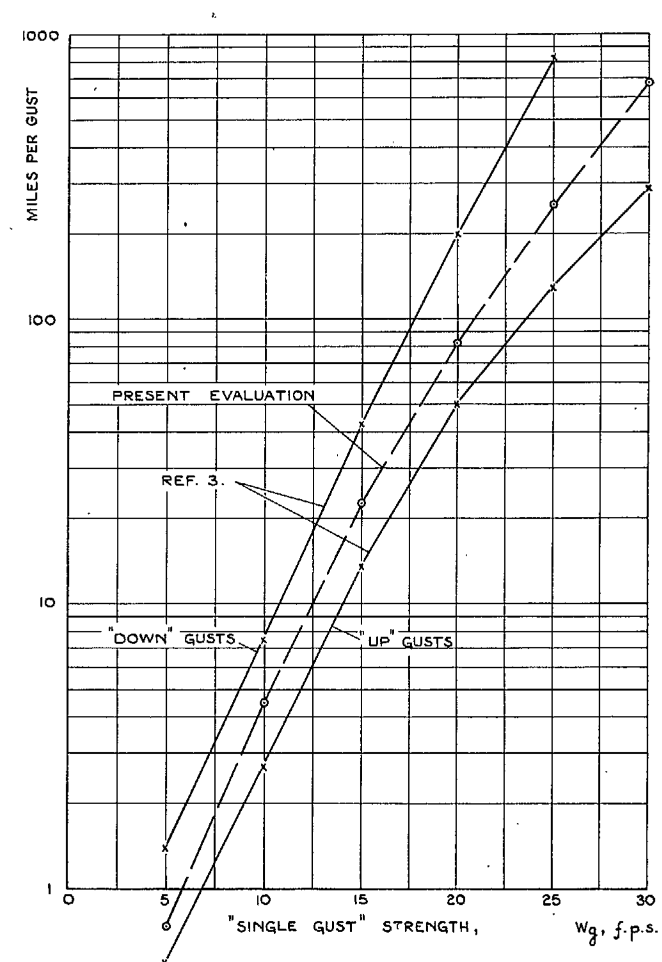

FIG. 4. Comparison between computed and measured distribution of discrete gusts.

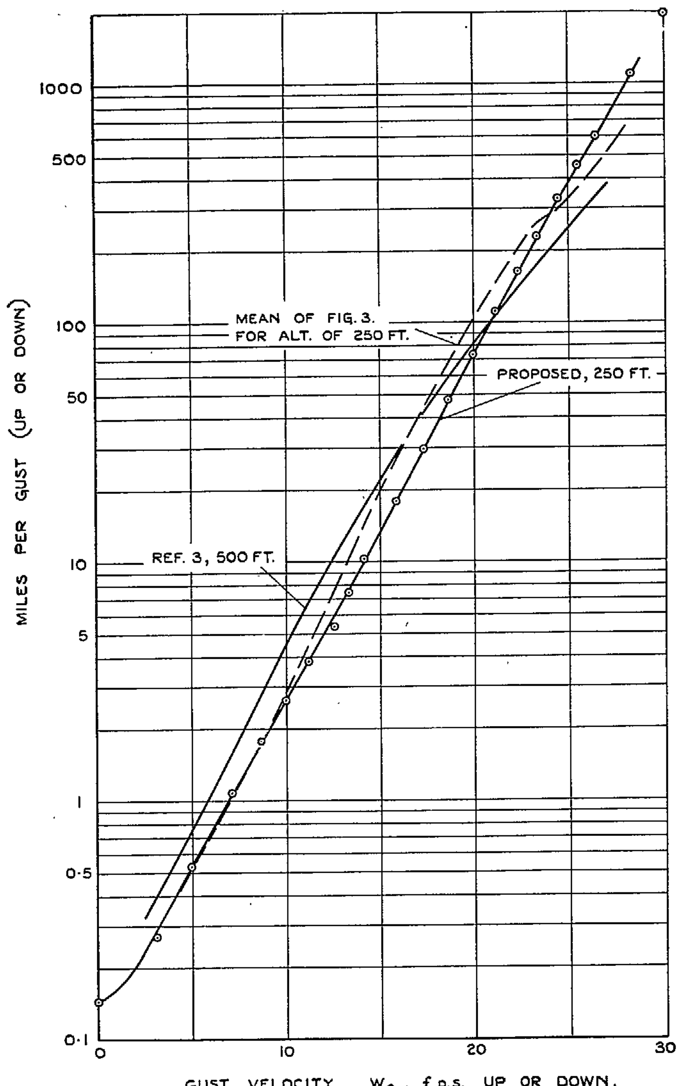

FIG. 5. Discrete gusts distribution computed from proposed spectral distribution, table 2, and comparison with other estimates.

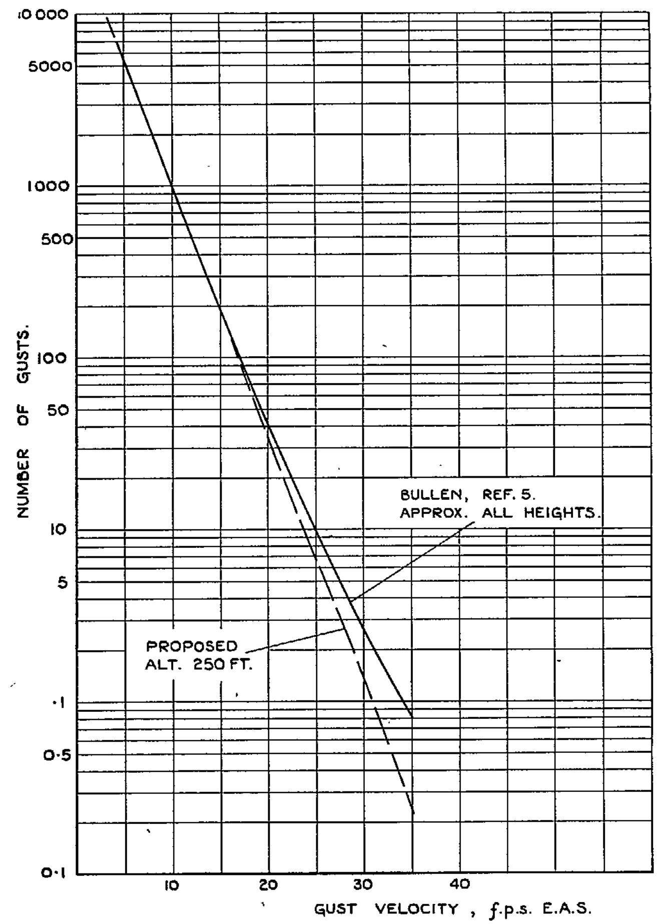

FIG. 6. Number of gusts exceeding different magnitudes per thousand exceeding 10 f.p.s. E.A.S.

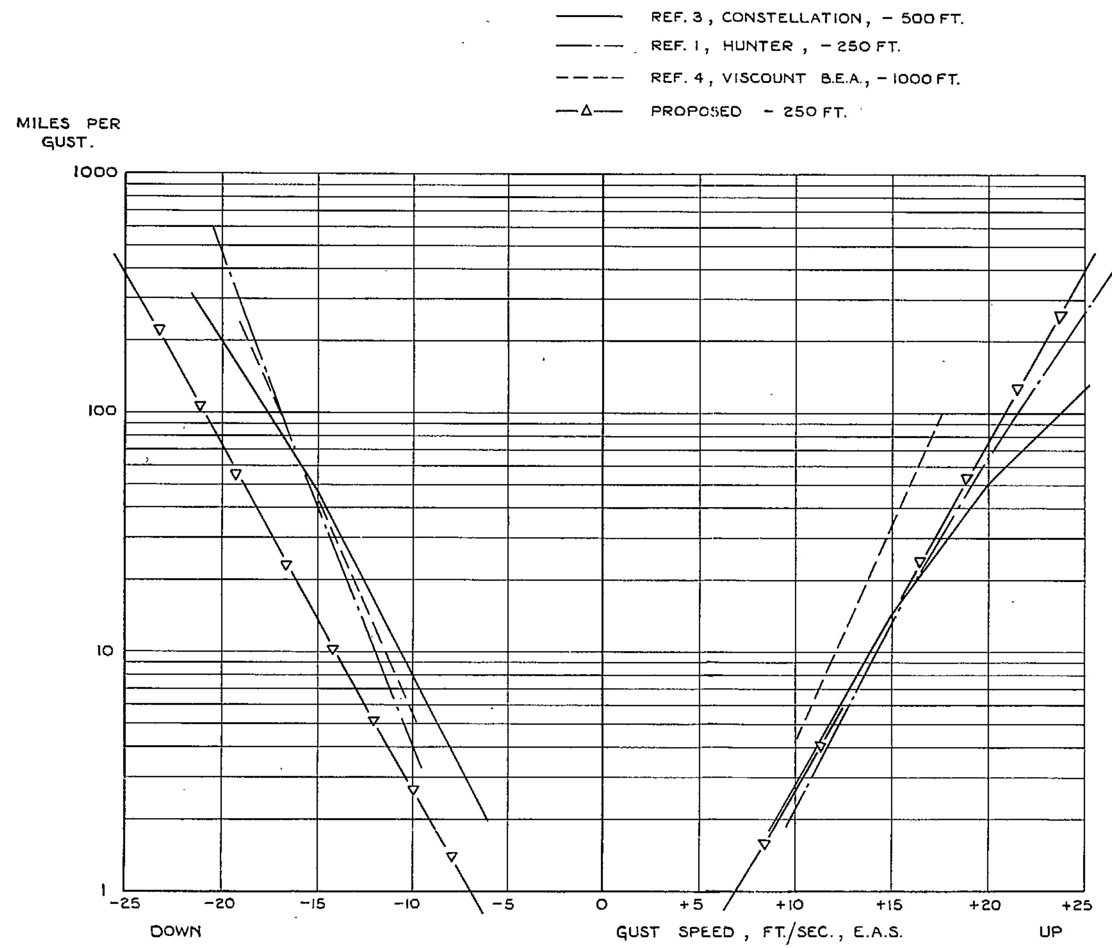

FIG. 7. A comparison in terms of discrete gusts between the proposed model of low-altitude turbulence and available measurements.

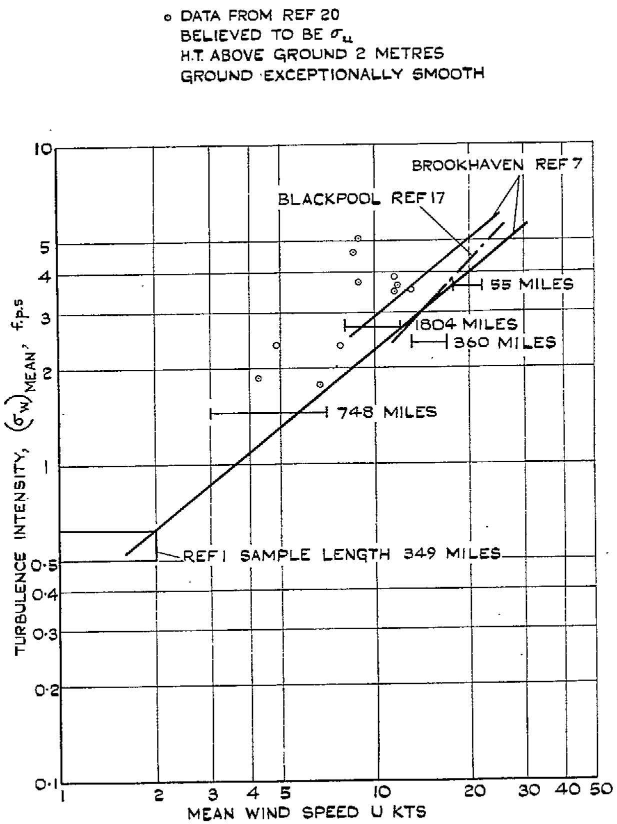

FIG. 8. The effect of wind on atmospheric turbulence at low altitudes.

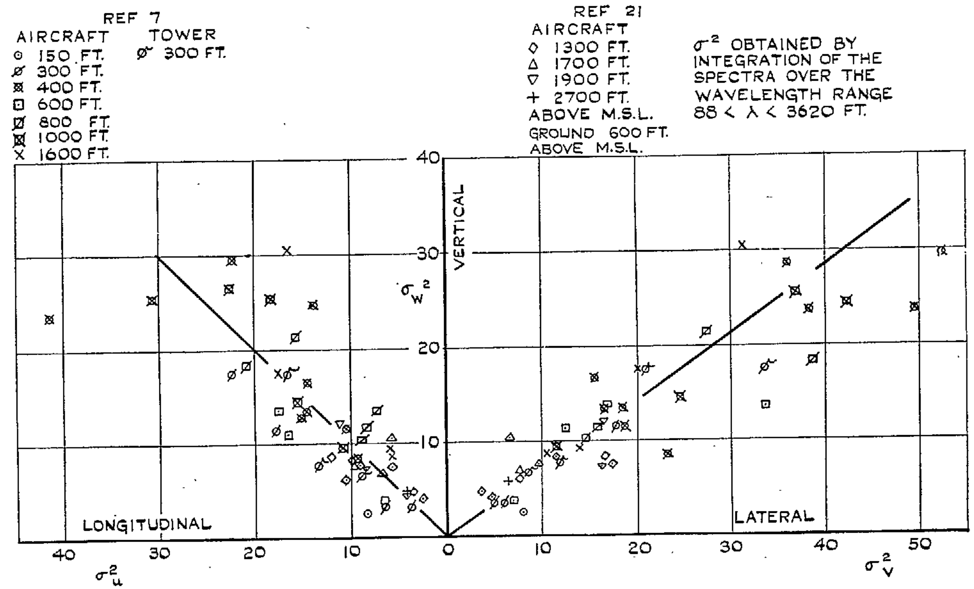

FIG. 9. Relationship between the intensities of the three components of atmospheric turbulence at low altitudes.

(82200) Wt. 66/991 K5 7/61 Hw.

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