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Aileron Control of Small-Aspect-Ratio Aircraft; in particular, Delta Aircraft

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Summary.—The aerodynamic and inertia characteristics associated with small-aspect-ratio wings are shown to affect aileron control, causing a tendency towards excessive aileron power combined with poor initial response. Simple formulae are given for the determination of the critical parameters and the effects of these on some aileron manoeuvres are analysed. Touchiness of lateral trimming is also expected to add to the handling difficulties of such aircraft.

Finally, design criteria are discussed for the attainment of optimum aileron control.

1. *Introduction.*—The trend in modern fighter wing design is generally towards lower aspect ratios and increasing sweepback, culminating in the delta plan-form. These characteristics result in a marked reduction in damping in roll (l_p) and in an increase in l_v compared with the straight-wing design. Considerable changes have also occurred in the spanwise weight distribution causing a relative increase in the inertia in roll (i_A).

The aileron-response characteristics, being functions of these parameters, have consequently undergone considerable changes when compared with those of more conventional aircraft. Pilots' complaints in this field have, in fact, initiated the present investigation.

In this report criteria will be deduced for the aileron response characteristics and their relation to the geometric, aerodynamic and inertia features of the aircraft.

Response calculations will be presented to illustrate the implications of these criteria on the manoeuvrability in roll and on the effect of aileron control during lateral oscillations.

2. *Rolling-Response Characteristics.*—2.1. *Equations of Motion.*—Considering three lateral degrees of freedom and aileron movement (ξ) the lateral motion of an aircraft is described by the differential equations

$$\left.\begin{aligned} \rho/2\, V^2Sb \left\{l_v\beta + l_r\frac{rb}{2V} + l_p\frac{pb}{2V} + l_\xi\xi\right\} &= A\dot{p} - E\dot{r} \\ \rho/2\, V^2Sb \left\{n_v\beta + n_r\frac{rb}{2V} + n_p\frac{pb}{2V} + n_\xi\xi\right\} &= C\dot{r} - E\dot{p} \\ \rho/2\, V^2S \left\{2y_v\beta + C_L\sin\phi\right\} &= mV(r - \dot{\beta}) \end{aligned}\right\} . \quad \ldots \quad \ldots \quad (1)$$

Aero-elastic distortions are disregarded in the present investigation which will have its main implications in the lower part of the speed range.

* R.A.E. Tech. Note Aero. 2264, received 9th April, 1954.

2.2. *Steady Rate of Roll.*—To obtain a steady state of roll in response to aileron deflection ξ, the variables of the aircraft motion are assumed to have attained steady values, *i.e.*,

$$\dot{p} = \dot{r} = \dot{\beta} = r = 0 . \qquad (2)$$

This is strictly a hypothetical condition, however, since the theory is restricted to small disturbances and in rolling ϕ will necessarily reach large angles before a steady state is obtained.

With the above assumption the side-force equation is eliminated and equations (1) are reduced to

$$\left.\begin{aligned} l_v\beta + l_p \frac{pb}{2V} + l_\xi \xi = 0 \\ n_v\beta + n_p \frac{pb}{2V} + n_\xi \xi = 0 \end{aligned}\right\} \qquad (3)$$

Equations (3) can be solved for p to give the steady rate of roll, p_∞, for unit aileron deflection :

$$\frac{p_\infty}{\xi} = -\frac{2V}{b}\frac{l_\xi^*}{l_p^*} = -\frac{2}{\sqrt{\sigma}}\frac{V_i}{b}\frac{l_\xi^*}{l_p^*} \qquad (4)$$

l_p^* and l_ξ^* are ' effective ' damping in roll and aileron power respectively and are defined as

$$l_p^* = l_p\left(1 - \frac{n_p}{l_p}\frac{l_v}{n_v}\right) . \qquad (5a)$$

It should be noted that for the range of not too large bank angles, *i.e.*, where $d \sin \phi / dt = p$, the effect of gravity and thus the side-force equation may be treated in a manner similar to that used in the above analysis. This gives an alternative expression for the effective damping in roll :

$$l_p^* = l_p\left(1 + \frac{l_v}{l_p}\left\{\frac{C_L}{2}\left(\frac{i_C}{n_v} + i_E\right) - \frac{n_p}{n_v}\right\}\right) . \qquad (5b)$$

Unless there is an exceptionally large negative value of i_E, the effect of the additional C_L terms will be to increase the effective damping.

In the numerical work quoted later in this report equation (5*a*) has been used throughout and it should be noted that this results in rather pessimistic answers for the delta aircraft when only small manoeuvres are concerned.

The effective ailerons power is in either case :

$$l_\xi^* = l_\xi\left(1 - \frac{n_\xi}{l_\xi}\frac{l_v}{n_v}\right) . \qquad (6)$$

Equation (4) expresses the known relation that steady aileron response varies in proportion with aileron power and speed and that it is inversely proportional to the span and to l_p.

The variations of l_p with aspect ratio, taper ratio and sweep, as obtained from Ref. 1, are plotted in Figs. 1 to 3. l_p is seen to be substantially reduced due to sweep and also with small aspect ratio and taper ratio.

Sweepback also reduces the aileron power for a given size of control, as shown in Fig. 4 (Ref. 2). However, for lateral control during cross-wind take-off and landing of an aircraft with large sweep (*i.e.*, with large l_v at high C_L values), ailerons may have to be provided which are more powerful than those of a corresponding straight-wing design. The effective l_ξ^* applicable to dynamic aileron control, however, is normally reduced by adverse aileron drag, n_ξ, according to equation (6) and this reduction increases with l_v.

Equation (5) shows that positive yawing moment due to rate of roll, n_p, reduces the effective damping in roll. The main contributor to positive n_p is a high set fin which is a typical feature of the tailless delta aircraft. Ref. 3 suggests that this effect is generally reduced by sidewash.

With no methods of estimation available, however, sidewash is neglected in the numerical examples given later in this report and the results may therefore be somewhat pessimistic.

2.3. *Initial Acceleration in Roll.*—The initial acceleration in roll, $\dot{p}$, in response to a sudden aileron application is obtained from equation (1) with the initial conditions at $t = 0$:

$$\beta = r = p = \phi = 0 \qquad (7)$$

$$\left.\begin{aligned} \rho/2\, V^2 Sb\, l_\xi \xi &= A\dot{p} - E\dot{r} \\ \rho/2\, V^2 Sb\, n_\xi \xi &= C\dot{r} - E\dot{p} \end{aligned}\right\} \qquad (8)$$

If these equations are solved for $\dot{p}$, the initial acceleration per unit aileron angle is obtained as :

$$\frac{\dot{p}_0}{\xi} = V_i^2 \frac{\rho_0 g}{(W/S)(b/2)} \frac{l_\xi^\circ}{i_A^\circ}, \qquad (9)$$

with

$$l_\xi^\circ = l_\xi \left(1 + \frac{n_\xi}{l_\xi} \frac{i_E}{i_C}\right), \qquad (10)$$

$$i_A^\circ = i_A \left(1 - \frac{i_E^2}{i_A i_C}\right). \qquad (11)$$

The corrections to these effective coefficients are essentially of a small order and for most practical cases :

$$\frac{\dot{p}_0}{\xi} \simeq V_i^2 \frac{0\cdot 1532}{W/S\,.\,b} \frac{l_\xi}{i_A}. \qquad (12)$$

2.4. *Aircraft Response to Sudden Aileron Application.*—Calculations of aircraft response to control application require strictly the solution of equations (1). Such solutions (Refs. 4 and 5) are algebraically very cumbersome and are therefore unsuitable for the discussion of the effects of individual parameters. They take into account the spiral mode and the lateral oscillation which are both unavoidably excited by control disturbance. A useful approximation for the roll subsidence as the fundamental motion involved can, however, be obtained by considering an exponential motion

$$p(t) = P(1 - e^{t/t_\xi}) \qquad (13)$$

which must satisfy the boundary condition derived in Sections 2.3 and 2.4 :

$$\left.\begin{aligned} p(t = 0) &= 0 \qquad & p(t = \infty) &= p_\infty \\ \dot{p}(t = 0) &= \dot{p}_0 \qquad & \dot{p}(t = \infty) &= 0 \end{aligned}\right\} \qquad (14)$$

With these conditions, $P = p_\infty$ and $t_\xi = p_\infty/\dot{p}_0$. Substituting for p_∞ and $\dot{p}_0$ the expressions derived in equations (4) and (9) :

$$t_\xi \simeq -\,13\cdot 05 \frac{W/S}{V_i\sqrt{\sigma}} \frac{i_A^\circ}{l_p^*} \frac{l_\xi^*}{l_\xi^\circ}. \qquad (15)$$

t_ξ is a rough approximation for the damping time of the roll subsidence motion and it is suggested that it is defined with regard to aileron response as the 'response time'. It will be shown later in this report to be the dominating criterion for the response characteristics of an aircraft.

The aircraft response in angle of bank, ϕ, and acceleration in roll, $\dot{p}$, are obtained from equation (13) by integration and differentiation respectively :

$$\phi(t) = p_\infty\{t - t_\xi(1 - e^{-t/t_\xi})\}, \qquad (16)$$

$$p(t) = p_\infty\{1 - e^{-t/t_\xi}\}, \qquad (17)$$

$$\dot{p} = \frac{dp}{dt}(t) = \frac{p_\infty}{t_\xi} e^{-t/t_\xi}. \qquad (18)$$

Equations (16) and (18) have been computed for a representative range of $0 < t_\xi < 3 \cdot 0$ sec and plotted against time in Figs. 5 and 6.

As indicated by equation (16) and illustrated in Fig. 5, t_ξ is determined by the intersection of the asymptote to the steady-state rate of roll with the time abscissa. This suggests that for small values of t_ξ, this parameter may be interpreted as a response time lag which will be apparent as such to the pilot. For larger t_ξ, however, it is unlikely to be perceived as a time lag because it becomes of the same order as the total time normally taken by aileron manoeuvres.

2.5. *Aileron movements required for given aircraft manoeuvres.*—The Idealized rolling-motion equation (13) is represented by the differential equation :

$$l_p{}^*\hat{t}_p + l_\xi{}^*\xi\mu_2 - i_A{}^\circ\hat{t}^2 \frac{l_\xi{}^*}{l_\xi{}^\circ}\frac{dp}{dt} = 0 . \qquad (19)$$

Substituting p_∞ and t_ξ according to equations (4) and (9) equation (19) may be written :

$$\xi = \frac{1}{(p_\infty/\xi)}\left(p - t_\xi \frac{dp}{dt}\right), \qquad (20)$$

where (p_∞/ξ) is the steady aileron effectiveness (equation (4)). Thus an equation for the aileron movement $\xi(t)$ required for a given bank manoeuvre $p(t)$ is obtained.

A simple bank manoeuvre as illustrated in Fig. 7 is, for instance, given by

$$\phi(t) = \Delta\phi \left\{\frac{t}{t_m} - \frac{1}{\pi}\sin\left(\pi\frac{t}{t_m}\right)\cos\left(\pi\frac{t}{t_m}\right)\right\}, \qquad (21)$$

where

$\Delta\phi$ = total change in angle of bank.

t_m = duration of manoeuvre.

Differentiating equation (21) gives $p = d\phi/dt$ and $dp/dt = d^2\phi/dt^2$. Substituting these expressions for p and dp/dt in equation (20),

$$\frac{\xi(t)}{\Delta\phi(\xi/p_\infty)} = \frac{2}{t_m}\left\{\sin^2\left(\pi\frac{t}{t_m}\right) - 2\pi\frac{t_\xi}{t_m}\sin\left(\pi\frac{t}{t_m}\right)\cos\left(2\pi\frac{t}{t_m}\right)\right\}. \qquad (22)$$

Equation (22) has been computed for a representative range of the ratio $0 < t_\xi/t_m < 1 \cdot 0$ and plotted against t/t_m in Fig. 8.

A similar manoeuvre where constant rate of roll is maintained during $\frac{1}{4}t_m < t < \frac{3}{4}t_m$ (Fig. 9) has also been considered and results are plotted in Fig. 10.

Figs. 8 and 10 show that the character of the stick movement required to perform a given manoeuvre is fundamentally determined by the ratio of the response time, t_ξ, of the aircraft to the duration of the manoeuvre, t_m. With negligible t_ξ (or for very large t_m) the pilot will only have to apply a smooth control movement in proportion to the instantaneous rate of roll, *i.e.*, he operates the ailerons as a rate control.

With increasing t_ξ/t_m an increasingly complex control operation becomes necessary with counter control during the second half of the manoeuvre. Since large t_ξ indicates the predominance of inertia in roll over damping in the dynamic of the aircraft movement, the pilot has to use control mainly to accelerate and later decelerate the rolling motion. In this case ailerons will be essentially acceleration controls displaying the difficulties discussed in a recent paper on 'The human operator of control mechanism'[6].

If the pilot fails to apply the appropriate check movement and thus to decelerate the aircraft, it will roll on by virtue of its inertia and overshoot the anticipated angle of bank. The manoeuvre has to be repeated with reversed sign and this may go on as an oscillatory motion which the pilot may find difficult to recognise as induced by himself.

If in such a case the ailerons are very powerful, the duration of the manoeuvre will be shorter, i.e., t_m will be small, t_ξ/t_m will increase further and handling becomes even more difficult.

3. *Numerical Examples*.—3.1. *Effect of Positive n_p*.—It has been shown in Section 2.2 that large positive n_p may substantially reduce the effective damping in roll, l_p^*, of an aircraft. This will be illustrated here by considering the Fairey F.D. 1, an advanced research aircraft which with respect to lateral control is one of the most extreme examples on which flight evidence is available (Fig. 10). In addition to its present configuration two hypothetical modifications will be considered :

(i) Horizontal tailplane removed.

(ii) Horizontal tailplane removed and fin area reduced to leave $n_v = 0 \cdot 025$.

The relevant aerodynamic derivatives for the three versions are given in Table 1 together with corresponding values of the effective damping in roll derivative, l_p^*. The latter is also plotted against C_L in Fig. 12. For all three versions, l_p^* is considerably smaller than the already small basic l_p, in particular at low C_L. The aircraft with the reduced fin would actually lose roll damping completely for $C_L < 0 \cdot 15$, i.e., in this condition it would become unstable in roll.

Analysis of the lateral stability of version (ii) reveals, moreover, a complete change of the type of aircraft motions present with this configuration. The roots of the lateral frequency equation for this aircraft are plotted in Fig. 13. For $C_L > 0 \cdot 6$ the usual modes of motion are obtained, one oscillatory motion and two subsidences, the spiral mode and the roll subsidence. All motions are damped (real roots negative). The same modes appear also for $C_L < 0 \cdot 02$, both aperiodic modes being, however, undamped. Within the range $0 \cdot 02 < C_L < 0 \cdot 6$ the aircraft will have two oscillatory modes of motion, one having a relatively long period, similar to the phugoid in longitudinal stability.

The roll subsidence assumed in Section 2 would correspond to a root

$$\lambda = l_p^*/i_A{}^\circ . \qquad \ldots \ldots \ldots \ldots \ldots \ldots \quad (23)$$

This term has been plotted for comparison in Fig. 13 and it is seen to be in moderate agreement with the corresponding root obtained from the complete solution. Between $0 \cdot 02 < C_L < 0 \cdot 6$, the λ obtained from the approximation equation (23) requires, however, another interpretation, since within this range it represents apparently the damping of an oscillatory motion. The occurrence of this mode appears to be produced by the unusually large positive n_p, as is also apparent from the results of Ref. 7.

3.2. *Comparison of Various Aircraft*.—A number of aircraft will now be compared with respect to their aileron-response characteristics. The examples are chosen to represent recent trends in high-speed aircraft designs :

Supermarine *Spitfire*

Vampire, Mk. 5

Avro 707

Boulton Paul P. 111

Fairey F.D. 1.

The geometric, aerodynamic and inertia data for these aircraft are given in Table 2 and corresponding general-arrangement drawings in Figs. 14 and 15. Using equations (4), (12) and (15) the roll-response parameters have been estimated and the results are given at the bottom of Table 2.

The steady response in roll is seen to reach quite excessive values for the delta aircraft (p_∞/ξ up to 260 deg/sec per degree aileron) when compared with more conventional values such as those obtained, *e.g.*, for a *Vampire* ($p_\infty/\xi \leqslant 20$ deg/sec). The largest values were obtained for the aircraft with the smallest span, the Fairey F.D. 1. Both the Boulton Paul P. 111 and the Fairey F.D. 1 were fitted with elevons. Such surfaces are designed to provide for the combined requirements of pitch and lateral control and for this type of aircraft both are large. Operating on a short arm the elevator must be relatively more powerful and the aileron power will be designed by cross-wind requirements resulting from the large l_v values associated with sweep at approach incidence. These requirements result in a control surface that is then extremely powerful in particular for lateral control at high speeds.

In order to provide satisfactory lateral trimming, for instance, one would expect the control surface to be trimmable to a degree which maintains residual rolling of the aircraft within one or two deg/sec. To have achieved this on, for example, the Fairey aircraft would have required elevons to be controllable to say 1/200 deg, calling for an exceptional degree of engineering refinement of the control circuit to exclude backlash and friction. Even if trim to 1/50 deg control deflection could have been achieved, the associated residual rate of roll would still have been $2 \cdot 6$ deg/sec at 450 kt at sea level.

With regard to acceleration in roll per degree aileron there is less difference between the various designs, the delta aircraft again having greater accelerations.

The latter results would suggest satisfactory response for the delta aircraft but, as shown in Sections 2.4 and 2.5, the response characteristics of an aircraft are basically determined by the response time, t_ξ, and not by the initial acceleration alone. Table 2 shows that the delta designs have much greater values of t_ξ throughout, when compared with the two straight-wing fighters. The Fairey F.D. 1 represents the most extreme case with t_ξ exceeding 3 sec for low V_i at high altitude.

Pilots' comments indicated undesirable lateral-control characteristics with both the Boulton Paul and the Fairey aircraft, criticism being more severe in the latter case. Unfortunately in neither case was it possible to exclude with certainty other factors contributing to these handling defects, but if they can be attributed solely to the roll-response lag as expressed by the time constant t_ξ it would appear from the numerical values given in Table 2 that the borderline of acceptability lies somewhere between $1 < t_\xi < 2$ sec. It should be noted that such a criterion must necessarily be a function of aircraft size (or possibly of the potential rolling power of the aircraft), as it is known that many large aircraft fly very satisfactorily with aileron-response lags of the order of 2 sec or more. However, as no critical cases are reported for this class of aircraft it is at present impossible to define a response criterion for anything but the type of aircraft considered in this report.

Recently the author and others had the opportunity of a practical demonstration of the problem of aileron response on a flight simulator which was set up to represent alternatively the lateral control and response characteristics of a *Vampire* and a Fairey F.D. 1. The experiment showed convincingly the deterioration in lateral control when the aileron-response time lag t_ξ approached values typical for the Fairey aircraft.

4. *Aileron Control of the Lateral Oscillation.*—The lateral oscillation of swept-wing aircraft is known to be a motion predominantly in roll (dutch-rolling) and pilots will consequently attempt to control this motion with the ailerons. This method is known to fail with delta aircraft, where aileron control appears in fact to stimulate rolling rather than suppress it. A simplified analysis of the phenomena involved will be given in the following Section.

4.1. *Free Lateral Oscillation.*—If the period and damping of a lateral oscillation as illustrated in Fig. 16 are known, the dutch-roll ratio, *i.e.*, the ratio between the amplitudes in roll and those in yaw, Φ/Ψ, can be obtained by treating the problem as that of finding the response of the aircraft in roll to a known forcing directional oscillation. This is shown in Appendix I and the results are presented in Fig. 17 where the dutch-roll ratio is given in terms of

$$P = \Phi/\Psi \left(\frac{\pi b \sqrt{\sigma}}{T_\psi V_i} \frac{l_p}{l_v}\right), \quad \ldots \quad (24)$$

from which Φ/Ψ is readily obtained. The roll amplitudes during the lateral oscillation are a function of the response parameter

$$t_\phi = -\frac{W/S}{g\rho_0 V_i}\frac{1}{\sqrt{\sigma}}\frac{i_A}{l_p}, \quad \ldots \quad (25)$$

which is closely related to the aileron-response time t_ξ (equation (15)). t_ϕ and t_ξ are in fact identical when the secondary effects on l_p and i_A are negligible.

The phase angle, ε_ϕ, of angle of bank against angle of yaw can also be obtained from Appendix I if so desired.

For the case illustrated in Fig. 18 it has been assumed that $T_\psi V_i = 1000$ ft, $t_\phi V_i = 200$ ft, $b = 50$ ft, and $l_v/l_p = 0\cdot12$. For $t_\phi/T_\psi = 0\cdot2$ Fig. 17 gives $P = 0\cdot665$ and from equation (24) the dutch-roll ratio can be calculated as $\Phi/\Psi = 1\cdot6$ for sea level. The phase angle between roll and yaw has been computed from equation (A.9) in Appendix I as $\varepsilon_p = 141$ deg.

4.2. *Lateral Oscillation with Aileron Control*.—It is now assumed that the pilot applies aileron in proportion to angle of bank, *i.e.*,

$$\xi = G\phi . \quad \ldots \quad (26)$$

Control of this type is, for instance, obtained if the pilot holds the stick vertical in space during the lateral oscillation of the aircraft. G would then be the gearing between aileron deflection and the angular stick movement. This type of aileron movement can be represented by an aircraft derivative

$$l_\phi = \frac{\partial C_l}{\partial \phi} = l_\xi G . \quad \ldots \quad (27)$$

The rolling response of an aircraft with this term in operation is treated in Appendix II in a manner analogous to that applied to the free lateral oscillation in Appendix I. The results are presented diagrammatically in Figs. 18 and 19, showing the amplification of the amplitudes in roll due to aileron control, which is represented by the parameter

$$\left(\frac{l_\xi}{l_p} G \frac{T_\psi V_i}{\pi b\sqrt{\sigma}}\right) .$$

Fig. 18 shows clearly that the effect of aileron control on the rolling amplitudes during the lateral oscillation is largely governed by t_ϕ. With small t_ϕ, the application of aileron will essentially reduce the amount of roll, this reduction becoming more effective with increasing aileron gearing. With $t_\phi/T_\psi \leqslant 0\cdot2$ there is a considerable range of aileron gearings where lateral control amplifies the amplitudes in roll. For large t_ϕ this region extends to a wide range of gearings and the maximum amplification reaches excessive values.

This applies even more to the case when the lateral oscillation is damped as can be seen from Fig. 19, where the maximum amplifications obtainable for each t_ϕ/T_ψ are plotted with aircraft damping (δ) as parameter.

It can be concluded that the pilot will only succeed in reducing the dutch-rolling amplitudes with the ailerons if the aircraft has a small t_ϕ, i.e., with the type of aircraft which has also small aileron-response time t_ξ. On an aircraft with large t_ϕ, however, pilot's control is likely to stimulate rolling rather than to suppress it, as is suggested by pilot's complaints on delta aircraft. Even if a practicable aileron control allows an amount of aileron 'gearing' beyond the resonance region (in Fig. 18) where roll would be actually reduced, it is difficult to conceive that pilots will apply such control movements, if the aircraft responds so violently to a lesser amount of aileron control.

It must be emphasized that in the above analysis aileron control is assumed not to alter the stability (damping and period) of the aircraft oscillation. This assumption will, however, not hold. It is in fact conceivable that, in particular cases, aileron control will improve the damping of the oscillation and will then be considered more useful than the above conclusions alone suggest.

4.3. *Numerical Examples*.—The two extremes of the aircraft considered earlier in this report may be chosen as examples, the *Spitfire* and the Fairey F.D. 1. The relevant aerodynamic data, applicable to flight at sea level, for these aircraft are given in Table 3. It is assumed that the lateral oscillation of both aircraft is damped with $\delta = 1 \cdot 0$ and that this is not altered by aileron control.

From Fig. 17 the dutch-roll ratio Φ/Ψ of the uncontrolled oscillation can be obtained for the corresponding values of t_ϕ/T_ψ, the results are given in Table 4. It is seen that the ratio of the amplitudes in roll to those in yaw present during the lateral oscillation of the Fairey F.D. 1 would be approximately three times that for the *Spitfire*. This increase in the dutch-roll ratio in itself is generally considered undesirable, but an inevitable feature of the modern swept-wing aircraft.

From Fig. 19 the maximum amplification that aileron control in proportion to bank angle can produce for each of these aircraft can be easily read and the results are given in Table 4. With the *Spitfire* even the most adverse application of aileron can only increase the rolling amplitudes in a dutch-roll oscillation by approximately 15 per cent. This would hardly be noticed by the pilot. On the other hand the already rather large rolling amplitude of the Fairey aircraft can be easily induced by the pilot to quite alarming magnitudes. It is obvious that the pilot will be well advised to use the ailerons as little as possible for the control of the lateral oscillation in an aircraft with such response characteristics.

5. *Design Recommendations*.—From the foregoing analysis of the various aspects of aileron control of small-aspect-ratio aircraft two main criteria have emerged which may be used as a guide in design. These are :

(i) Reasonable aileron power in terms of rate of roll per deg aileron deflection (possibly $p_\infty/\xi < 50$)

(ii) Aileron response time $t_\xi < 1 \cdot 0$ sec.

In order to keep within these acceptable limits the following design features should be considered :

(*a*) Small wing span makes aircraft sensitive to aileron control and the choice of an extremely short span may make it very difficult to achieve satisfactory lateral response.

(*b*) Damping in roll is basically a function of the geometry of the wing. For the delta plan-form l_p is practically proportional to the aspect ratio. If low aspect ratio occurs together with small span artificial damping in roll may become desirable to supplement the inherently low roll damping of such wings.

(*c*) Inertia in roll is the principal factor determining t_ξ. It should therefore be kept small by the avoidance of external stores (such as tip tanks, armament, etc.) on the outer portions of the wing.

(*d*) Ailerons should generally be small. In order to reduce the demands for crosswind take-off and landing l_v should be reduced to a minimum by the possible use of wing anhedral. This will also be beneficial for the stability of the lateral oscillation.

If backlash within the control circuit cannot be reduced substantially, the aileron should be designed to operate over a large angular range. Thus a small control surface with large deflections appears preferable to the reverse case, when at high speeds the control may tend to operate substantially within the backlash range.

(*e*) Elevons will be basically dimensioned for longitudinal control and are thus apt to be too powerful as ailerons. Separate controls should therefore be recommended.

(*f*) Control circuit.—Backlash and similarly friction within the control circuit may seriously impair trimming and accurate lateral control of small-span aircraft and will need special attention in the design.

In order to achieve satisfactory control characteristics at both ends of the speed range a number of refinements in the circuit may be considered :

(i) Non-linear gearing between stick and control so as to obtain relatively large stick movements for the small deflections used in high-speed flight.

(ii) Variable gearing for different speed ranges.

(iii) Split ailerons where only part of the total surface is used in high-speed flight.

6. *Conclusions.*—It has been shown that the aileron-response characteristics of an aircraft tend to become unsatisfactory if its wing has small span, small aspect ratio, larger taper ratio and sweep. The small delta aircraft combines all these features and in consequence the steady response in roll of such designs is seen to reach values of the order of 250 deg/sec rate of roll per degree aileron deflection. This high lateral sensitivity is aggravated by the reduction of the effective damping in roll due to the contribution of the high fin to n_p. This configuration is typical of the tailless design and actual instability of the roll subsidence motion may result.

The large ratio between inertia in roll and damping in roll typical of the delta design affects the aircraft response to aileron control in a sense that gives the aileron the character of an 'acceleration control' instead of the familiar 'rate control'. This is shown to result in increasingly difficult lateral control, the pilot becoming practically unable to maintain level flight.

The same characteristic renders the ailerons useless or even dangerous for the control of lateral oscillations.

It should be noted that the above phenomena occur not only with the delta aircraft but should generally be suspected with low-aspect-ratio wings, in particular, if the design features large inertia in roll.

To achieve acceptable aileron response on a small-aspect-ratio design the following recommendations have been made :

(*a*) Inertia in roll should be kept to a minimum. External stores on wing tips are undesirable.

(*b*) Reduce l_v to a minimum by the use of wing anhedral. This eases low-speed lateral-control requirements and is beneficial to the damping of the lateral oscillation.

(*c*) Restrict aileron size.

(*d*) The use of non-linear or variable aileron gearing and split ailerons to provide satisfactory control characteristics over the speed range is suggested.

(*e*) If acceptable lateral-control characteristics can not be achieved by these means artificial damping in roll should be considered.

LIST OF SYMBOLS

A		Inertia in roll
AR		Aspect ratio
b		Wing span
C		Inertia in yaw
C_L		Lift coefficient
E	$=$	$(C - A) \sin \varepsilon$ (Product of inertia)
i_A	$=$	$\frac{A}{m}\left(\frac{2}{b}\right)^2$ (Inertia in roll coefficient)
i_A°		*See* equation (11)
i_C	$=$	$\frac{C}{m}\left(\frac{2}{b}\right)^2$ (Inertia in yaw coefficient)
i_E	$=$	$\frac{E}{m}\left(\frac{2}{b}\right)^2$ (Product of inertia coefficient)
l_p		Damping in roll derivative
l_p^*		Effective l_p (equation (5))
l_r	$=$	$\frac{\partial C_l}{\partial \frac{rb}{2V}}$ Aircraft derivatives
l_v	$=$	$\frac{\partial C_l}{\partial \beta}$ Aircraft derivatives
l_ξ	$=$	$\frac{\partial C_l}{d\xi}$ Aircraft derivatives
l_ξ^*		Effective l_ξ (equation (6))
l_ξ°		Equation (10)
m		Aircraft mass
n_p	$=$	$\frac{\partial C_n}{\partial \frac{pb}{2V}}$ Aircraft derivatives
n_r	$=$	$\frac{\partial C_n}{\partial \frac{rb}{2V}}$ Aircraft derivatives
n_v	$=$	$\frac{\partial C_n}{\partial \beta}$ Aircraft derivatives
n_ξ	$=$	$\frac{\partial C_n}{\partial \xi}$ Aircraft derivatives
p	$=$	$d\phi/dt$ (Rate of roll)
p_∞		Steady rate of roll
$\dot{p}_0$		Initial acceleration in roll
r	$=$	$d\psi/dt$ (Rate of yaw)

S		Wing area
t_m		Duration of manoeuvre
t_ξ		Aileron-response time (equation (15))
t_ϕ		Roll response parameter (equation (25))
T_ψ		Period of the lateral oscillation
V		True speed
V_i		Indicated speed
W		Aircraft weight
y_v	=	$\frac{1}{2}\frac{\partial C_Y}{\partial \beta}$ (Side-force derivative)
β		Angle of sideslip
δ		Logarithmic decrement of the lateral oscillation
λ		Root of the frequency equation
Λ		Angle of sweep
ξ		Aileron angle
ρ		Air density
σ	=	ρ/ρ_0 (Relative density)
ϕ		Angle of bank
ψ		Angle of yaw

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APPENDIX I

Roll Response in Free Lateral Oscillations

Considering two degrees of freedom, yawing, ψ, and rolling, ϕ, only :

$$\psi = -\beta \quad \ldots \qquad \text{(A.1)}$$

The differential equation of rolling moments is

$$-\mu_2 l_v \psi + l_r \hat{t}\frac{d\psi}{dt} + i_E \hat{t}^2 \frac{d^2\psi}{dt^2} + l_p \hat{t}\frac{d\phi}{dt} - i_A \hat{t}^2 \frac{d^2\phi}{dt^2} = 0 \quad \ldots \qquad \text{(A.2)}$$

Assuming the aircraft performs a lateral oscillation in yaw

$$\psi = \Psi\, \mathrm{e}^{(\lambda + i\omega)t}, \qquad \text{(A.3)}$$

there will be a corresponding roll response

$$\phi = \Phi\, \mathrm{e}^{(\lambda + i\omega)t}. \qquad \text{(A.4)}$$

Substituting these solutions for ψ and ϕ equation (A.2) is reduced to an algebraic equation, which can be solved for Φ/Ψ. This will be a complex expression giving the amplitude ratio and phase relationship between rolling and yawing during the lateral oscillation.

It can be shown that for the type of aircraft under discussion (small-span delta), l_r and i_E are negligible, so that equation (A.2) can be simplified to :

$$\mathrm{e}^{(\lambda + i\omega)t}\left[-\mu_2 l_v \Psi + \Phi\{l_p \hat{t}(\lambda + i\omega) - \hat{t}^2 i_A (\lambda + i\omega)^2\}\right] = 0 \quad \ldots \qquad \text{(A.5)}$$

This equation gives the solution for the ratio of Φ/Ψ, i.e., the dutch-roll ratio

$$\frac{\Phi/\Psi}{\left\{\dfrac{T_\psi V_i}{b\pi\sqrt{\sigma}}\dfrac{l_v}{l_\psi}\right\}} = -\frac{2\pi\dfrac{t_\phi}{T_\psi}\left(1 - \left(\dfrac{\delta}{\pi}\right)^2 + \dfrac{\delta}{\pi} + i\left(1 - 4\pi\dfrac{t_\phi}{T_\psi}\dfrac{\delta}{\pi}\right)\right.}{\left\{2\pi\dfrac{t_\phi}{T_p}\left(1 - \left(\dfrac{\delta}{\pi}\right)^2\right) + \dfrac{\delta}{\pi}\right\}^2 + \left\{1 - 4\pi\dfrac{t_\phi}{T_\psi}\dfrac{\delta}{\pi}\right\}^2}, \qquad \text{(A.6)}$$

with

$$\delta = -2\pi\frac{\lambda}{\omega} = \text{logarithmic decrement of the lateral oscillation}$$

$$T_\psi = \frac{2\pi}{\omega} = \text{period of the lateral oscillation}$$

$$t_\phi = -\frac{i_A}{l_p}\frac{W/S}{g\rho_0 V\sqrt{\sigma}} = \text{roll response parameter.}$$

Equation (A.6) is a complex expression. If one puts

$$\frac{\Phi/\Psi}{\left\{\dfrac{T_\psi V_i}{b\pi\sqrt{\sigma}}\dfrac{l_v}{l_p}\right\}} = P_R + iP_I, \qquad \text{(A.7)}$$

the amplitude ratio is given as the modulus

$$\frac{\Phi/\Psi}{\left\{\dfrac{T_\psi V_i}{b\pi\sqrt{\sigma}}\dfrac{l_v}{l_p}\right\}} = \sqrt{(P_R{}^2 + P_I{}^2)} \qquad \text{(A.8)}$$

and the phase angle between angle of bank, ϕ, and angle of yaw, ψ, by

$$\tan \varepsilon_\phi = \frac{P_I}{P_R}. \qquad \text{(A.9)}$$

APPENDIX II

Lateral Oscillation with Aileron Control

With aileron control in proportion to angle of bank, ϕ, a term $\mu_2 l_\phi \phi$ will be added to the left-hand side of equation (A.2) and consequently

$$- l_v \mu_2 \Psi + \Phi \{l_\phi \mu_2 + l_p \hat{t}(\lambda + i\omega) - i_A \hat{t}^2(\lambda + i\omega)^2\} = 0 . \qquad \text{(B.1)}$$

Solved for Φ/Ψ, the roll response of an aircraft with aileron control is then given as

$$\frac{\Phi/\Psi}{\left\{\dfrac{l_v}{l_p}\dfrac{T_\psi V_i}{b\pi\sqrt{\sigma}}\right\}} = - \frac{\left\{2\pi \dfrac{t_\phi}{T_\psi}\left(1 - \left(\dfrac{\delta}{\pi}\right)^2\right) + \dfrac{\delta}{\pi} - L_\phi\right\} + i\left\{1 - 4\pi \dfrac{t_\phi}{T_\psi}\dfrac{\delta}{\pi}\right\}}{\left\{2\pi \dfrac{t_\phi}{T_\psi}\left(1 - \left(\dfrac{\delta}{\pi}\right)^2\right) + \dfrac{\delta}{\pi} - L_\phi\right\}^2 + \left\{1 - 4\pi \dfrac{t_\phi}{T_\psi}\dfrac{\delta}{\pi}\right\}^2}, \qquad \text{(B.2)}$$

with

$$L_\phi = \frac{l_\phi}{l_p}\frac{T_\psi V_i}{b\pi\sqrt{\sigma}} \qquad \text{(B.3)}$$

as the parameter representing aileron control.

Computing equation (B.2) for various L_ϕ and comparing these with the corresponding values for $L_\phi = 0$, i.e., with equation (A.6) for the amplitudes in roll of the uncontrolled lateral oscillation the ratio,

$$(\Phi)_{L\phi}/(\Phi)_{L\phi} = 0$$

is obtained showing the amplification of the rolling amplitudes due to aileron control.

TABLE 1

Aerodynamic Data and Effective Damping in Roll for Three Variations of the Fairey F.D. 1 Aircraft

(a) Standard aircraft

C_L	0	0·2	0·4	0·6	0·8
l_p	−0·250	−0·233	−0·223	−0·217	−0·212
$\frac{n_p}{l_p}$	−0·561	−0·463	−0·350	−0·221	−0·083
$\frac{l_v}{n_v}$	−1·24	−1·13	−1·293	−1·437	−1·353
$\left(1 - \frac{n_p}{l_p}\frac{l_v}{n_v}\right)$	+0·304	+0·477	+0·547	+0·683	+0·888
l_p^*	−0·076	−0·111	−0·122	−0·148	−0·188
(b) Horizontal tailplane removed					
l_p	−0·235	−0·224	−0·218	−0·214	−0·210
$\frac{n_p}{l_p}$	−0·374	−0·299	−0·216	−0·119	−0·021
$\frac{l_v}{n_v}$	−1·49	−1·72	−2·09	−2·72	−1·49
$\left(1 - \frac{n_p}{l_p}\frac{l_v}{n_v}\right)$	+0·443	+0·486	+0·548	+0·676	+0·969
l_p^*	−0·104	−0·109	−0·119	−0·145	−0·204
(c) Horizontal tailplane removed, fin reduced					
l_p	−0·230	−0·222	−0·216	−0·211	−0·208
$\frac{n_p}{l_p}$	−0·335	−0·261	−0·176	−0·095	0
$\frac{l_v}{n_v}$	−3·75	−3·50	−3·97	−6·20	−2·60
$\left(1 - \frac{n_p}{l_p}\frac{l_v}{n_v}\right)$	−0·257	+0·085	+0·301	+0·413	+1·0
l_p^*	+0·059	−0·019	−0·065	−0·087	−0·208

TABLE 2

Comparison of Aileron Performance of some Fighter and Research Aircraft

Aircraft	*Spitfire*		*Vampire* 5		Avro 707		BP Delta P. 111		Fairey F.D. 1	
W/S	27·5		32·0		26·3		35·7		44·0	
b	36·87		38·0		33·0		29·5		19·54	
$A.R.$	5·67		4·78		3·02		3·0		2·45	
i_A	0·0522		0·092		0·111		0·0734		0·107	
V_i (kt)	100	300	150	450	150	450	150	450	150	450
C_L	0·80	0·090	0·42	0·046	0·34	0·038	0·46	0·052	0·54	0·060
l_ξ	−0·0865		−0·100		−0·175		−0·190		−0·180	
l_p	−0·360		−0·340		−0·205	−0·204	−0·210	−0·230	−0·220	−0·245
l_p^*	−0·360		−0·353	−0·340	−0·199	−0·200	−0·150	−0·120	−0·140	−0·106
$-(p_\infty/\xi)$ Sea Level	2·2	6·5	3·78	11·75	12·9	40·3	21·7	81·5	33·3	132·0
$-(p_\infty/\xi)$ 40,000	3·13[1]	9·2	7·56	23·5	25·8	80·6	43·4	163·0	66·6	264·0
$-(p_0/\xi)$	7·15	64·4	8·8	79·1	16·4	148	24·1	217·0	19·2	173·0
t_ξ (sec) Sea Level	0·31	0·10	0·43	0·148	0·785	0·253	0·90	0·376	1·735	0·762
t_ξ (sec) 40,000	0·44[1]	0·14[1]	0·86	0·296	1·574	0·506	1·80	0·752	3·47	1·524

[1] Figures refer to 22,000 ft altitude.

TABLE 3

Lateral-Response Parameters for the Spitfire and the Fairey F.D. 1 at Sea Level

Aircraft	V_i (kt)	t_ξ	t_ϕ	T_ψ	$\frac{t_\phi}{T_\psi}$	$\frac{l_v}{l_p}$
Spitfire ..	100	0·31	0·31	4·5	0·0690	0·1855
	300	0·10	0·10	1·5	0·0665	0·189
Fairey ..	150	1·735	1·105	2·41	0·460	0·098
	450	0·762	0·33	1·07	0·309	0·058

TABLE 4

Dutch-Rolling Characteristics of the Spitfire and the Fairey F.D. 1 With and Without Aileron Control

Aircraft	V_i (kt)	Free oscillation			With aileron control	
		P	$\frac{l_v}{l_p} \frac{T_\psi V_i}{b\pi}$	$\frac{\phi}{\psi}$	$\frac{\phi_G}{\phi_F}$	$\frac{\phi}{\psi}$
Spitfire ..	100	0·955	1·217	1·162	1·15	1·34
	300	0·945	1·236	1·167	1·13	1·32
Fairey ..	150	0·332	9·75	3·235	∞	∞
	450	0·480	7·65	3·760	4·6	16·9

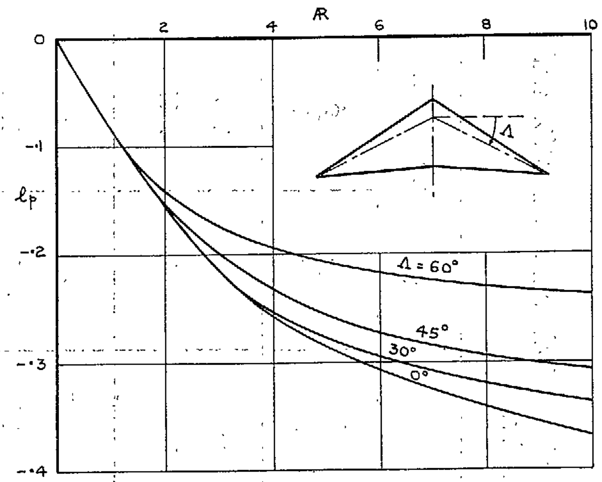


FIG. 1. Change of damping in roll (l_p) with aspect ratio and sweep of a wing with taper ratio $\lambda = 0$ (From Ref. 1).

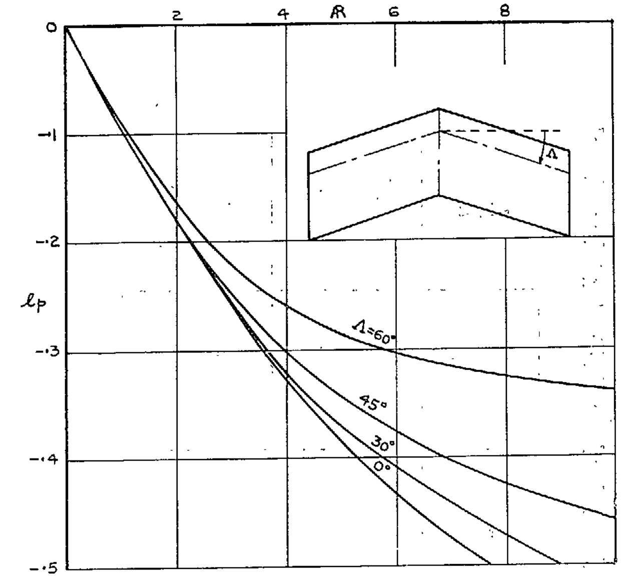


FIG. 3. Change of damping in roll (l_p) with aspect ratio and sweep of a wing with $\lambda = 1 \cdot 0$.

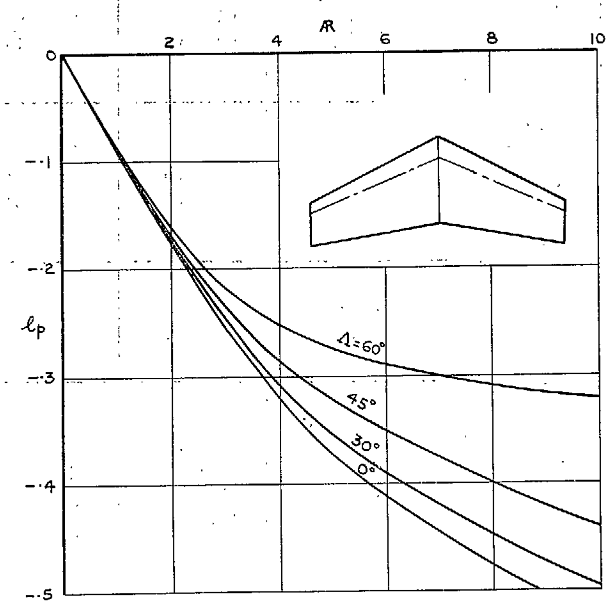


FIG. 2. Change of damping in roll with aspect ratio and sweep of a wing with $\lambda = 0 \cdot 5$.

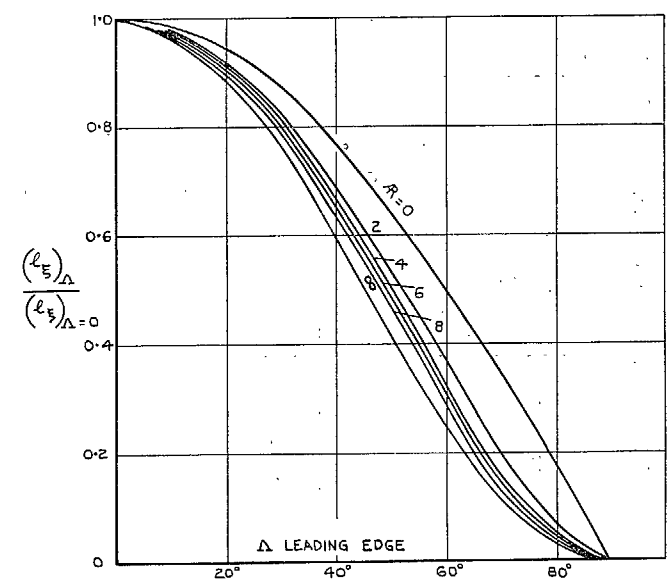


FIG. 4. Effect of sweep on aileron power l_ξ (Ref. 2).

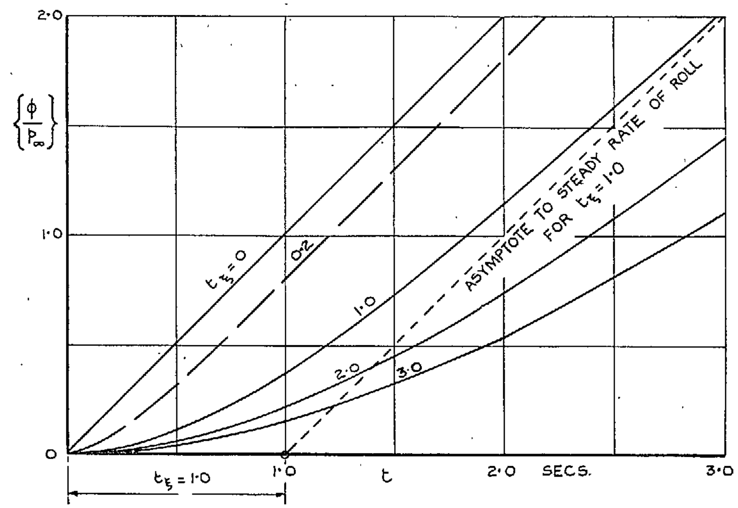


FIG. 5. Response in roll to sudden aileron deflection at $t = 0$ of aircraft with various time constants, t_ξ.

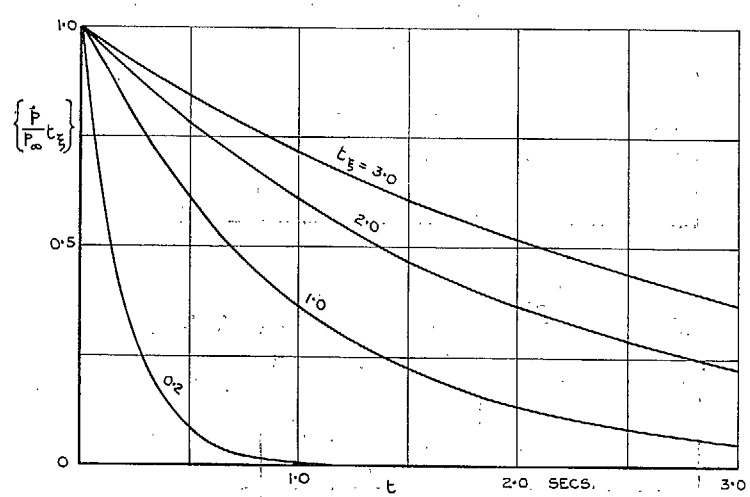


FIG. 6. Aircraft acceleration in roll ($\dot{p}$) in response to sudden aileron deflection at $t = 0$ for aircraft with various time constants, t_ξ.

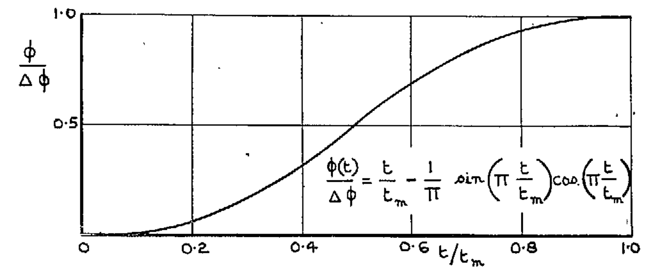


FIG. 7. Manoeuvre in roll considered in Section 3.2.

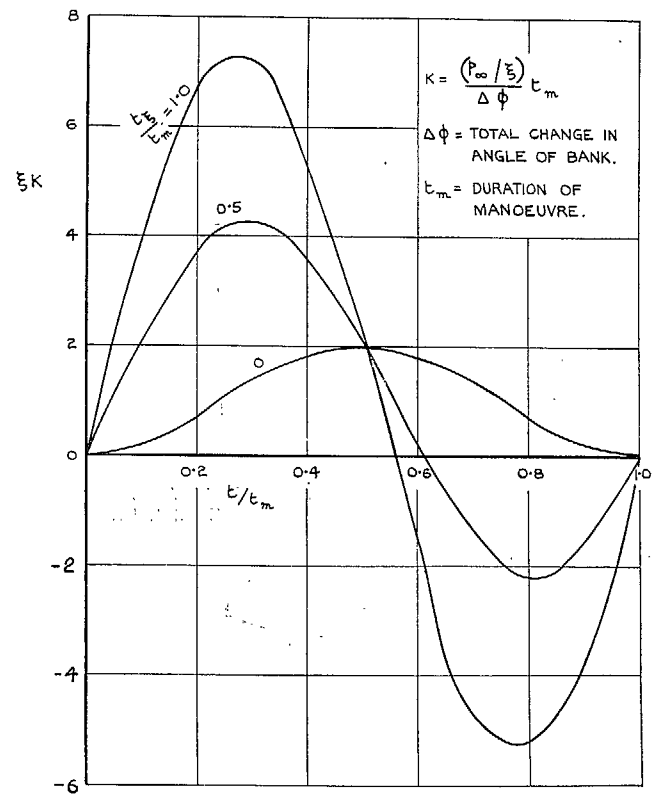


FIG. 8. Aileron movements required to perform the manoeuvre in roll illustrated in FIG. 7.

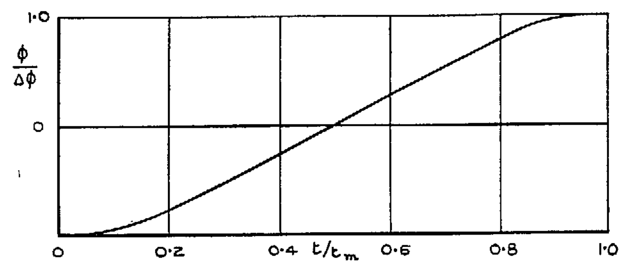


FIG. 9. Manoeuvre in roll with a period with constant rate of roll ($\bar{p}$).

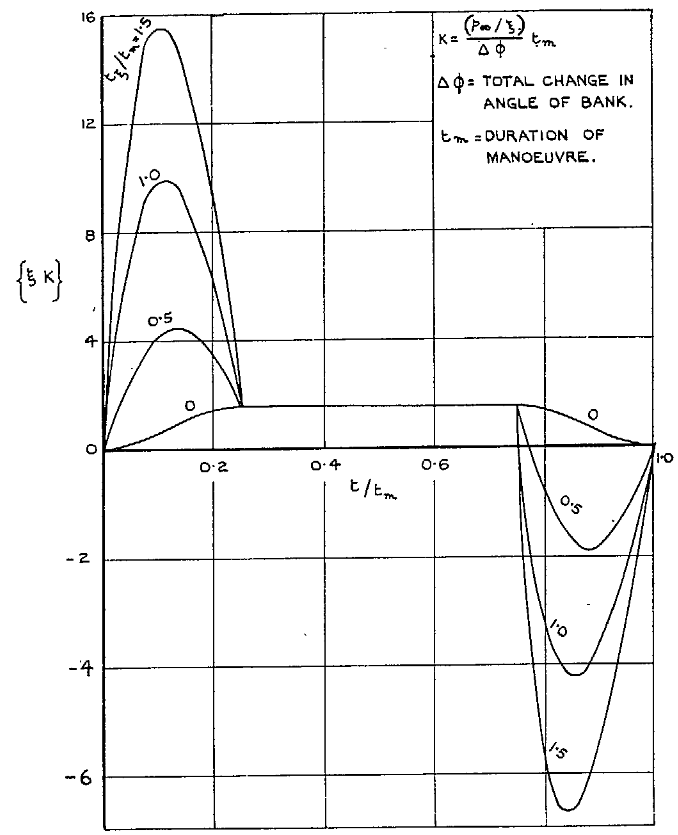


FIG. 10. Aileron movements required to perform the manoeuvre in roll illustrated in FIG. 9.

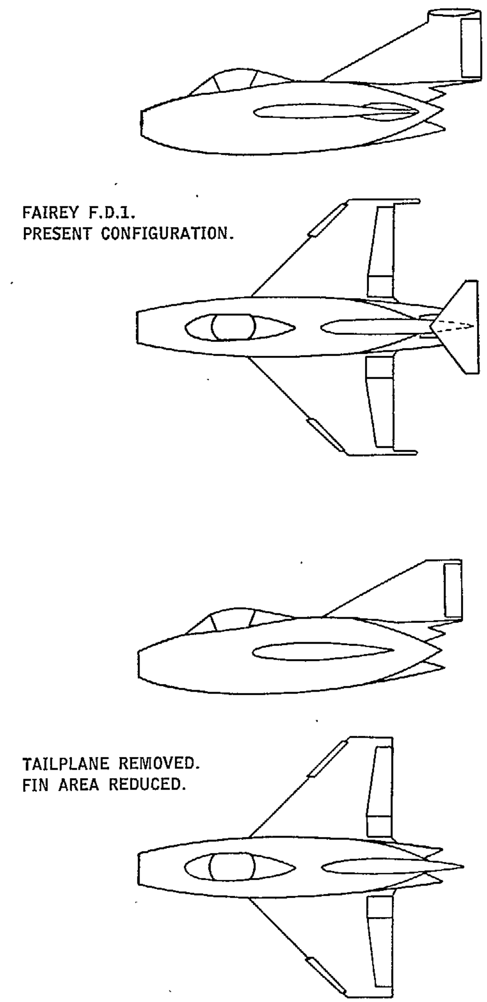


FIG. 11. General arrangement of the Fairey F.D.1 in its standard configuration and with modified tailplane.

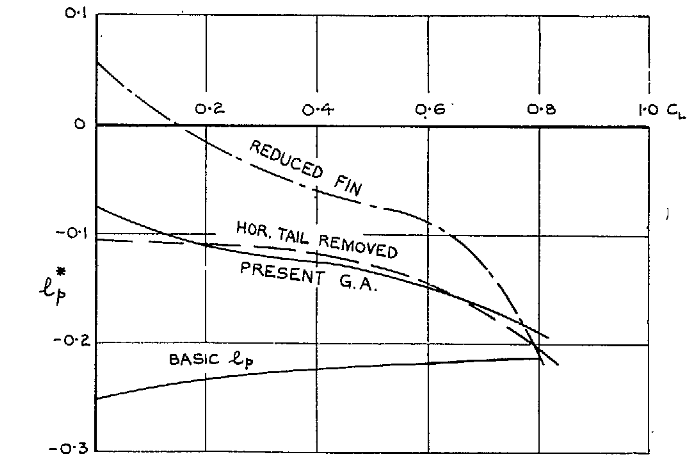


FIG. 12. Effective damping in roll (l_p*) of the Fairey F.D.1.

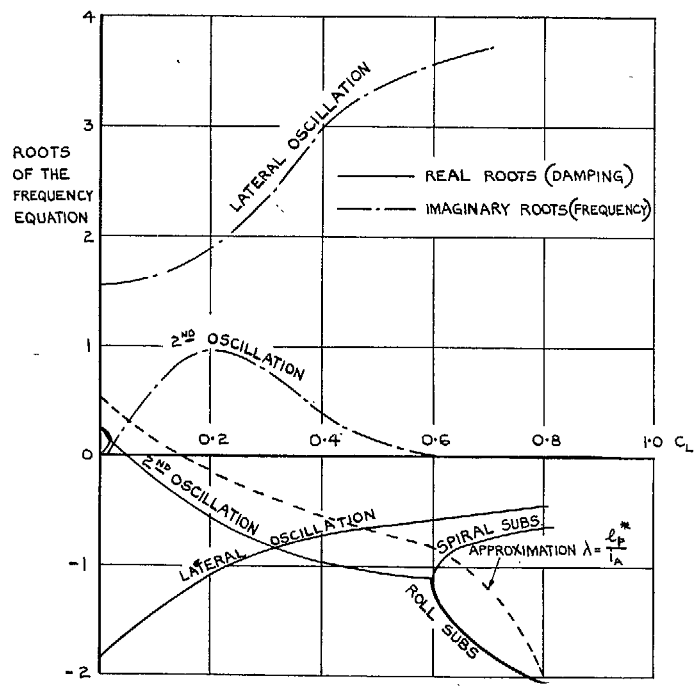


FIG. 13. Lateral stability roots of the Fairey Delta aircraft with reduced fin area.

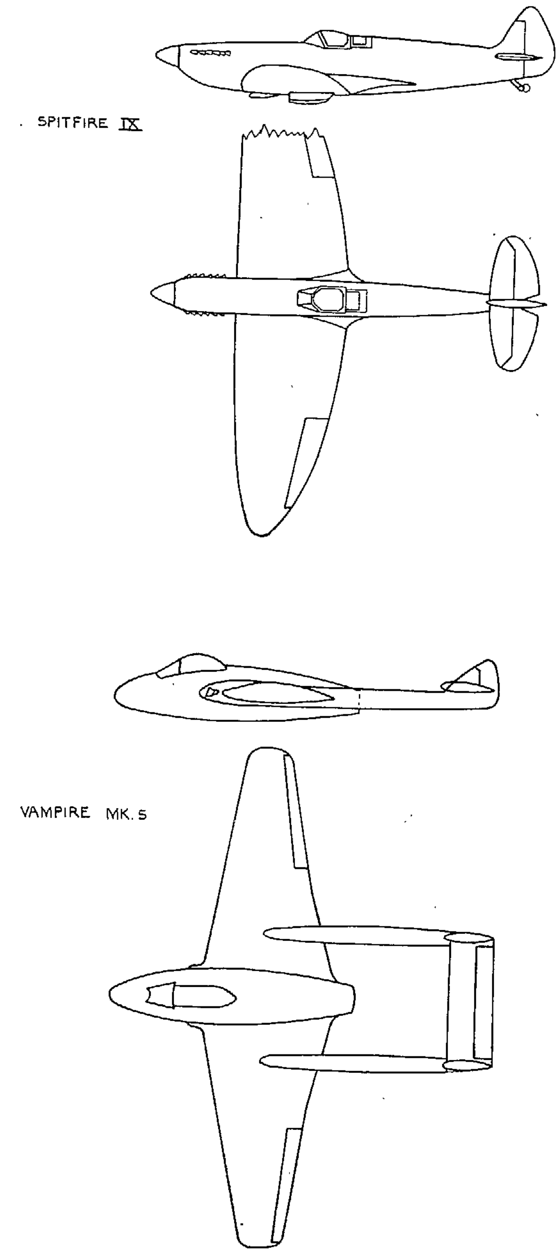


FIG. 14. General arrangement of the *Spitfire* Mk. IX and the *Vampire* Mk. 5.

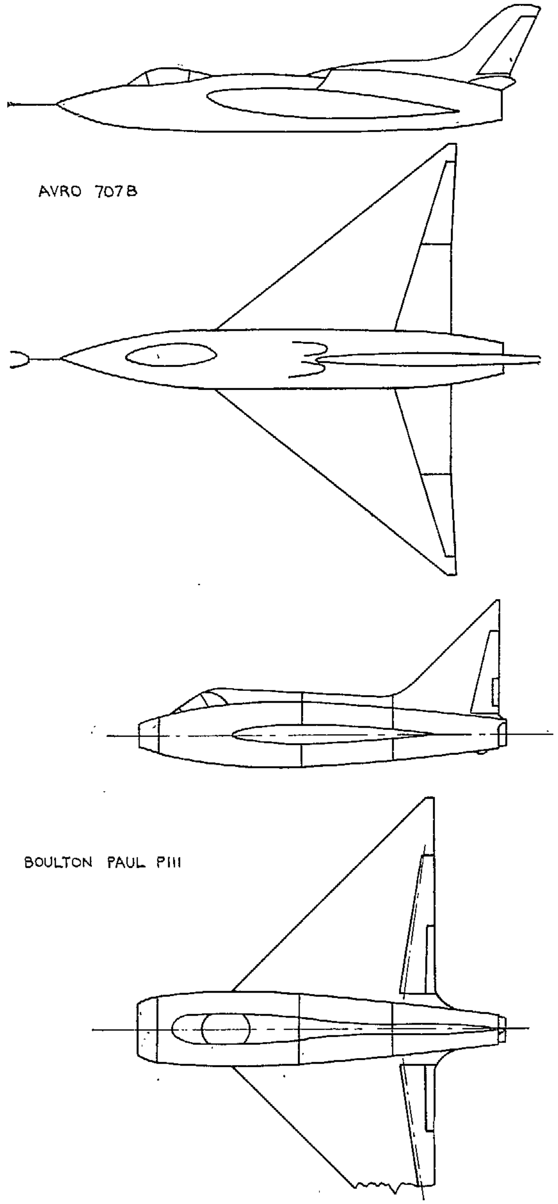


FIG. 15. General arrangement of the Avro 707B and the Boulton Paul P. 111.

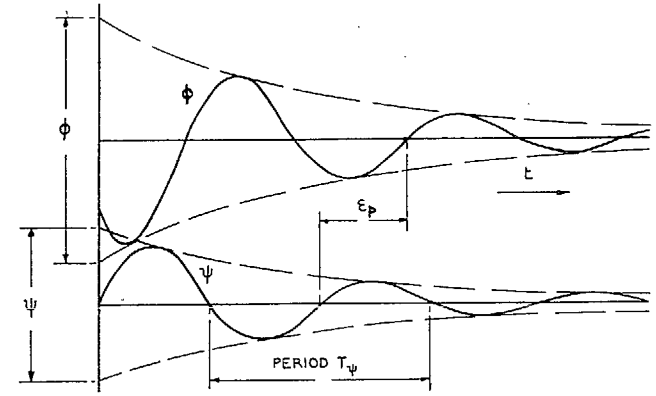


FIG. 16. Time history of the lateral oscillation of an aircraft with $\delta = 1 \cdot 0$.

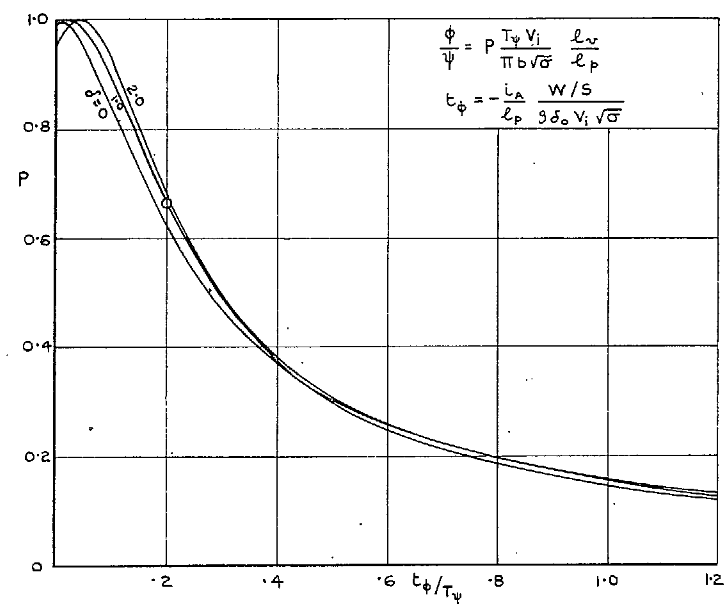
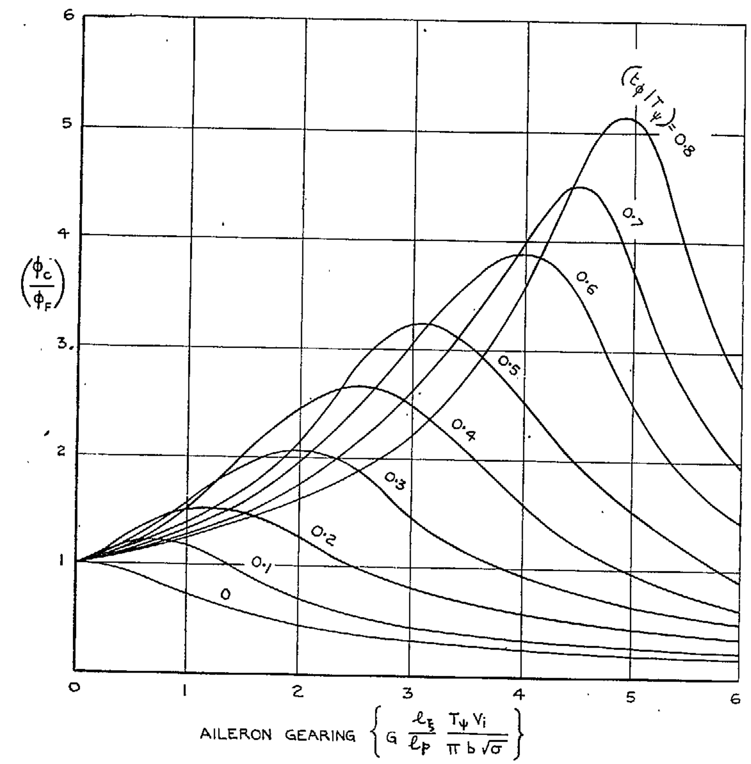


FIG. 17. Diagram for the determination of the dutch-roll ratio ϕ/ψ.

(80316) Wt. 61/991 K.7 12/60 Hw.



ϕ_c = ROLL AMPLITUDE WITH AILERON CONTROL.
ϕ_F = ROLL AMPLITUDE OF THE FREE OSCILLATION.
G = AILERON DEFLECTION : ANGLE OF BANK.

FIG. 18. Effect of aileron control on the amplitudes in roll of the lateral oscillation of an aircraft ($\delta = 0$).

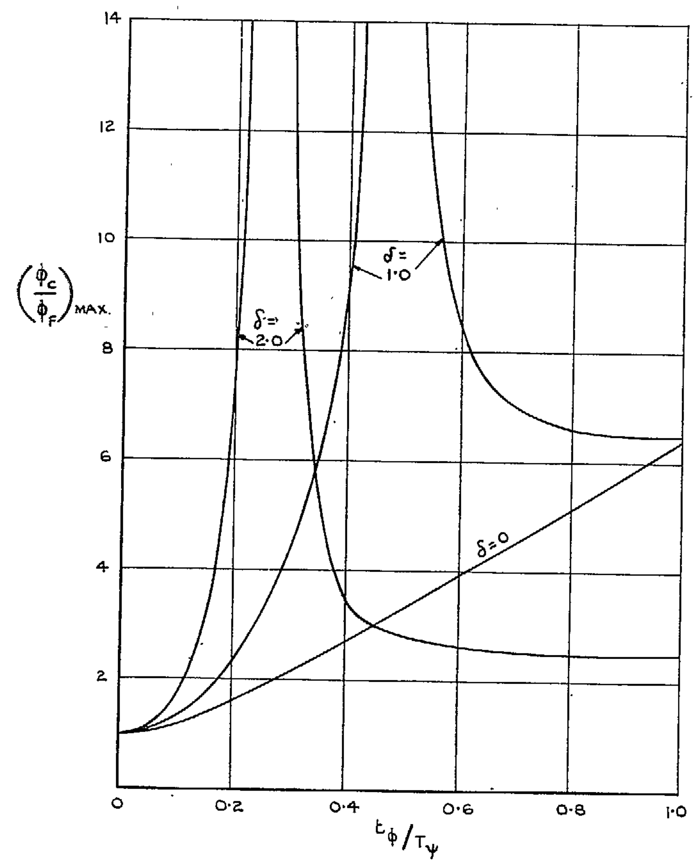


FIG. 19. Effect of t_ϕ on the maximum possible amplification of the rolling amplitudes with aileron control.

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