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Notes on the Induced Drag of a Wing-Tail Combination

By

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Summary.—An expression has been derived for the factor to be applied to ideal induced drag to allow for wing-tail interference. This factor is primarily dependant on the wing-tail lift and span ratios. It is of the order of 1.1 for a normal aircraft when the tailplane carries 10 per cent of the weight of the aircraft, and can reach unexpectedly large values at high speed.

Charts, generalised curves, and sufficient information are included to permit rapid evaluation of the factor for any particular case.

1. *Introduction.*—One of the largest sources of error in present methods of predicting aircraft drag lies in the ideal induced-drag multiplication factor. Flight measurements show that a factor between 0.9 and 1.4 is required, and the best estimate that can be made, since there is no real guide, is to assume a figure in the neighbourhood of 1.1 and 1.2. Under maximum range or maximum economic conditions, where induced drag is between 25 and 50 per cent of the total drag, an error of 0.1 in the factor results in an error of 2.5 to 5 per cent in total drag. On a long-range aircraft an error of this size may easily swallow a considerable portion of the payload.

It is generally accepted that this factor is due to a variety of effects, such as variation of profile drag with incidence, departure of wing lift distribution from elliptical, induced drag due to slipstream, etc. Whilst these effects are undoubtedly present a further suggestion is put forward here that part of this factor is due to wing-tailplane interference. Drag due to wing-tailplane interference can be predicted, and in the following pages this factor has been investigated and rough charts have been prepared to estimate it for any particular aircraft.

2. *Induced Drag Due to Wing-Tailplane Interference.*—The induced drag of a monoplane is generally expressed as a function of the span loading of the mainplane, and the effect of the tailplane neglected. Strictly it is a function of the span loading of both wing and tailplane and should be considered as a special type of biplane. This has been done in Appendix I, and the results are expressed in the form of a multiplication factor on the conventional expression for induced drag. The factor is given in equation (6) and plotted in Fig. 1 and is seen to depend on the wing-tail lift and span ratios. It would appear from this diagram that factors of the order of 1.1 are possible. However, further information on tailplane load is required before the diagram can be used.

* Received 5th October, 1946.

3. *Tailplane Lift*.—From the pitching-moment equation in the tail-off condition the tailplane lift required for longitudinal equilibrium can be obtained. This has been done in Appendix II and is expressed in equation (7) as x , the ratio to the main wing lift. x is seen to be dependent on aircraft speed and the distance of the C.G. aft of the tail-off neutral point. At low speed x is positive and there is a lift on the tailplane, at high speed x is negative and there is a download which may become quite large. In the region of $C_L = 0.2$ (depending on wing camber) the tailplane load becomes zero.

In evaluating $(h - h_0)$, the distance of the C.G. aft of the tail-off neutral point, tunnel tests or detailed calculations are required, since the combined effect of propellers, nacelles and body (particularly on multi-engined civil aircraft) is to move the neutral point well forward of the wing quarter-chord point.

An approximate method of arriving at the above distance, if the desired degree of static stability is known or if flight stability measurements (or tunnel in the tail-on condition) are available, is to subtract the effects of the tailplane. For this reason the shift of neutral point due to tailplane (mainly a function of geometry) has been evaluated (equation (9a)) and is plotted in Fig. 3. The above distance is then $(h - h_0) = h_{nT} - \text{static margin}$.

The following table has been drawn up to show how tailplane load varies with C_L , for the aircraft defined by the conditions of Fig. 4 with a static margin of $0.05\bar{c}$, and $C_{mo} = -0.03$. Here $(h - h_0) = 0.39\bar{c} - 0.05\bar{c} = 0.34\bar{c}$, so that

$$x = 0.3 \left[0.34 - \frac{0.03}{C_L} \right] \div \left[1 + \frac{.3 \times .03}{C_L} \right]$$

C_L	1.5	1.0	0.5	0.2	0.05
x	0.095	0.092	0.083	0.054	-0.066

x is very dependent on static margin and C_{mo}/C_L and may never go negative. With $C_{mo} = 0$, x is constant at 0.102 for the above conditions.

4. *The Induced Drag Factor 'R'*.— R is a function of C_L and is defined in Appendix I as the factor which the 'ideal induced drag' is multiplied by to include wing-tail interference. It is evaluated by solving equation (7) or (8) for x , and substituting for x in equation (6) or Fig. 1.

As an aid to the evaluation of R over a speed range Fig. 2 has been prepared. Two examples of the use of this chart are given, (a) at low speed $C_{mo} = -0.03$ and (b) at high speed $C_{mo} = -0.06$ for the aircraft defined by the conditions in Fig. 4. The sequence of operations, indicated by dotted lines, is

$$\frac{C_L}{1+x} \rightarrow C_{mo} \rightarrow (h - h_n) \rightarrow \frac{\bar{c}}{l} \rightarrow \frac{b_1}{b_2} \rightarrow R,$$

where x is initially taken as zero, and a further approximation, including x , made if necessary.

For (a) the tailplane lift is positive and $R = 1.065$, for (b) it is negative and $R = 1.47$.

Under optimum cruising conditions on any given aircraft, x is mainly a function of static margin. To illustrate this we can fix C_L at say 0.7 (for the aircraft of Fig. 4) and show how R varies with static margin. In this case

$$\begin{aligned} x &= 0.3 - 0.04 + 0.39 - (h - h_n) \div 1 + 0.3 \times 0.04 \\ &= 0.104 - (h - h_n) \times 0.3, \end{aligned}$$

and a static margin scale can be drawn in on Fig. 1. Since $(b_1/b_2)^2 = 9.5$, R is given directly by Fig. 1 and is tabulated below.

Static Margin	0	0.1 \bar{c}	0.2 \bar{c}	0.3 \bar{c}	0.4 \bar{c}	0.5 \bar{c}
R	1.08	1.04	1.02	1.0	1.0	1.02

5. *The Induced Drag Factor K.*—As opposed to R , K is the slope of the $C_{Di} - C_L^2$ curve at any point. It also varies with C_L and has been determined in Appendix III as a function of x .

In any particular case K is greater than R (except at negative tailplane loads where it can be less than 1.0, or for $C_{mo} = 0$). To illustrate this K and R are plotted against C_L^2 in Fig. 4.

Over the cruising C_L 's for maximum range, indicated by vertical boundaries in Fig. 4,

$$K = R \text{ for } C_{mo} = 0,$$

$$\text{and } R > K > R,$$

$$C_{mo} = 0 \quad C_{mo} = -0.03 \quad C_{mo} = -0.03$$

and all are of the order of 1.06. This suggests that the error under cruising conditions will be small if C_{mo} is neglected (particularly as the present trend on high-speed aircraft is toward a wing section of low C_{mo}). With this assumption generalised curves for K are drawn in Fig. 5. These curves show that K is roughly proportional to wing aspect ratio, and decreases quite rapidly as the aircraft stability rises, and that for a neutrally stable aircraft is of the order of 1.1 at $A = 10$.

6. *Comparison with Flight Measurements.*—Numerous flight measurements of K are available, but in very few cases has stability also been measured. Perhaps the best check available at the moment comes from AFEE* measurements on gliders in free flight, since the effects of propellers and slipstream are avoided.

Table 1 compares flight measurements of K with estimates of K and R . These estimates have been made for a range of static margins, since the static stability is not known (except that handling tests suggest that all the aircraft were stable). It is thought that the static margins were in the region of 0 to $0.2\bar{c}$, and if this was the case then agreement is reasonably good.

Summary and Conclusions.—Induced drag due to wing-tail interference is mainly dependent upon the relative lift and span of the tailplane. A tailplane load of 10 per cent of the all-up weight increases the induced drag by about 10 per cent. The tailplane load, however, is very dependent on aircraft design and should be worked out in detail for any particular case.

No direct comparison with measurement was available, but an indirect comparison with flight measurements on gliders suggests that the factor given by the above theory and generalised curves is of the right order.

Comparison with measurements on powered aircraft have not been made here since it is considered that the effects of slipstream should be included. Further investigation is required in this direction as it is thought that the associated downwash at the tailplane has a direct effect on, and will increase, the main factor in wing-tail interference drag: the backward inclination of the resultant force on the tailplane.

APPENDIX I

Induced Drag of Wing-Tailplane Combination

For a biplane the induced drag may be written

$$D_i = \frac{L_1^2}{b_1^2} + 2\sigma \frac{L_1 L_2}{b_1 b_2} + \frac{L_2^2}{b_2^2}, \quad \dots \quad (1)$$

where L = lift, b = span,
and suffix 1 and 2 refer to the two aerofoils respectively.

* Airborne Forces Experimental Establishment.

The wing and tail of a normal monoplane aircraft may be considered as a biplane of large stagger and small gap, and the above expression used. In this case, since the 'gap' is generally small and variable and the wing-tailplane span ratio of the order of 3 we may write

$$\sigma = \frac{b_2}{b_1}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

without much loss of accuracy. Substituting this in (1) gives

$$D_i \propto \frac{L_1^2}{b_1^2} + \frac{2L_1L_2}{b_1^2} + \frac{L_2^2}{b_2^2}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

where suffix 1 refers to the wing and 2 to the tailplane.

In current practice the total induced drag is written

$$D_i \propto K \frac{(L_1 + L_2)^2}{b_1^2}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (4)$$

where K is a constant between 0.9 and 1.4. Denoting by $(R - 1)$ that portion of K due to wing-tailplane interference, the corresponding expression to (3) is

$$D_i \propto R \frac{(L_1 + L_2)^2}{b_1^2}. \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

The constants of proportionality of expressions (3) and (5) are equal, so that

$$\begin{aligned} R &= 1 + \frac{L_2^2}{(L_1 + L_2)^2} \left(\frac{b_1^2}{b_2^2} - 1 \right), \\ &= 1 + \frac{x^2}{(1+x)^2} \left(\frac{b_1^2}{b_2^2} - 1 \right), \text{ where } L_2 = xL_1. \quad \dots \quad \dots \quad (6) \end{aligned}$$

R is plotted against x in Fig. 1. This figure suggests that factors of the order of 1.1 are obtainable for quite modest tailplane loads.

APPENDIX II

Tailplane Lift

Taking pitching moments about the aircraft neutral point in the tail-off condition, and in the absence of slipstream, gives the force to be provided by the tailplane for equilibrium as

$$L = \frac{\frac{1}{2}\rho v^2 s \bar{c}}{1} \left[C_{mo} + (h - h_0)C_{L1} \right],$$

or

$$x = \frac{\sigma}{l} \left[\frac{C_{mo}}{C_{L1}} + (h - h_0) \right] \div \left[1 - \frac{\bar{c}}{l} \frac{C_{mo}}{C_L} \right]. \quad \dots \quad \dots \quad \dots \quad (7)$$

Owing to the presence of nacelles, propellers and body, h_0 is further forward than the mean wing quarter-chord point, and in order to avoid the calculation of its position, if the static margin is known the above equation can be written as

$$x = \frac{\bar{c}}{l} \left[\frac{C_{mo}}{C_{L1}} + (h - h_n) - (h_0 - h_n) \right], \quad \dots \quad \dots \quad \dots \quad (8)$$

where h_n is the position of the longitudinal static stick-fixed neutral point of the complete aircraft. In this form $(h - h_n)$ is the stability margin, and $(h_0 - h_n)$ the shift of the neutral point due to the tailplane.

The shift of the neutral point due to the tailplane is mainly dependent on the aircraft geometry, *i.e.*,

$$(h_0 - h_n) = \eta \frac{a'_1}{a_1} T_v \left(1 - \frac{d\varepsilon}{d\alpha} \right), \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

$$= \eta \frac{1 + (2/A)}{1 + (2/A')} T_v \left(1 - \frac{\epsilon}{\epsilon_0} \frac{1}{1 + (A/2)} \right) \dots \dots \dots (9a)$$

(by substituting $a_1 = \frac{2\pi A}{2 + A}$ and $\frac{d\epsilon}{d\alpha} = \frac{\epsilon}{\epsilon_0} \frac{a_1}{\pi A}$, etc.).

Also

$$\begin{aligned} -\frac{\epsilon}{\epsilon_0} &= \frac{1}{2} \left(\frac{s}{s'} \right)^2 \left[1 + \sqrt{1 + \left(\frac{s'}{l} \right)^2} \right], \\ &= 0.811 \left(1 + \sqrt{1 + 0.157 \frac{(b_1)^2}{(l)^2}} \right) \text{ for elliptic loading,} \\ &\equiv 1.84 \text{ for } b_1/l = 2, \\ &\equiv 2.06 \text{ for } b_1/l = 3. \end{aligned}$$

Thus equation (8) becomes

$$\begin{aligned} x &= \frac{2}{A} \left[\frac{C_{mo}}{C_{L1}} + (h - h_n) + \frac{1 + (2/A)}{1 + (2/A')} T_v \left(1 - \frac{1.84}{1 + (A/2)} \right) \right] \text{ for } \frac{b_1}{l} = 2, \\ &= \frac{3}{A} \left[\frac{C_{mo}}{C_{L1}} + (h - h_n) + \frac{1 + (2/A)}{1 + (2/A')} T_v \left(1 - \frac{2.06}{1 + (A/2)} \right) \right] \text{ for } \frac{b_1}{l} = 3. \end{aligned} \quad (10)$$

x is now expressed in terms of geometry, static stability margin and C_{mo}/C_{L1} .

To demonstrate the effect of geometry and stability on the induced drag factor K a series of numerical values have been inserted in equations (10) and (6), and the results plotted in Fig. 5.

APPENDIX III

Slope of Induced Drag—(Lift Coefficient)² Curve

Equation (5) may be rewritten as

$$C_{Di} = R \frac{C_L^2}{\pi A},$$

where R is a function of lift coefficient.

Hence

$$\frac{dC_{Di}}{dC_L^2} = \frac{1}{\pi A} \left[R + C_L^2 \frac{dR}{dx} \cdot \frac{dx}{dC_L^2} \right] \dots \dots \dots (11)$$

From (6)

$$\frac{dR}{dx} = \left[\left(\frac{b_2}{b_1} \right)^2 - 1 \right] \frac{2x}{(1+x)^3} \dots \dots \dots (12)$$

From (7)

$$\frac{dx}{dC_L^2} = -\frac{1}{2} \frac{\bar{c} C_{mo}}{l C_L^3} (1+x) \left/ \left(1 - \frac{\bar{c}}{l} \cdot \frac{C_{mo}}{C_L} \right) \right. \dots \dots \dots (13)$$

This gives

$$\begin{aligned} \frac{dC_{Di}}{dC_L^2} &= \frac{1}{\pi A} \left[R - \left(\left(\frac{b_2}{b_1} \right)^2 - 1 \right) \frac{x}{(1+x)^2} \frac{\bar{c} C_{mo}}{l C_L} \left/ \left(1 - \frac{\bar{c} C_{mo}}{l C_L} \right) \right. \right], \dots \dots (14) \\ &= \frac{1}{\pi A} \left[R - \frac{(R-1)}{x} \cdot \frac{\bar{c}}{l} \cdot \frac{C_{mo}}{C_L} \left/ \left(1 - \frac{\bar{c} C_{mo}}{l C_L} \right) \right. \right]. \end{aligned}$$

If R had been a constant (say K) the corresponding expression would have been

$$\frac{dC_{Di}}{dC_L^2} = \frac{K}{\pi A}, \quad \dots \dots \dots \quad (15)$$

so that

$$K = R - \frac{(R-1)\bar{c}C_{mo}}{x l C_L} \left/ \left(1 - \frac{\bar{c}C_{mo}}{l C_L} \right) \right. \dots \dots \dots \quad (16)$$

This expression has been evaluated for a typical aircraft and is plotted in Fig. 4 as a function of lift coefficient. It is seen from Fig. 4 that

$$\begin{array}{cccc} R & = & K & > & R \\ C_{mo} = 0 & & C_{mo} = 0 & & C_{mo} = -0.03 & & C_{mo} = -0.03 \end{array}$$

but that over the cruising region indicated all are roughly equal. For cruising conditions therefore

$$\begin{array}{cc} K & \simeq R \\ C_{mo} = -0.03 & C_{mo} = 0. \end{array}$$

NOMENCLATURE

- L_1 Wing lift
- L_2 $xL_1 =$ tailplane lift
- b_1 Wing span
- b_2 Tailplane span
- \bar{c} Wing mean chord
- Δh_{nT} Shift of neutral point due to tailplane
- h C.G. position relative to leading edge standard mean chord
- h_0 Longitudinal static neutral point stick-fixed in tail-off condition
- h_n Ditto in tail-on condition
- C_{mo} Pitching moment about tail-off neutral point
- R Multiplication factor on ideal induced drag to include wing-tail interference
- K Multiplication factor on slope of $C_{Di} - C_L^2$ curve to include wing-tail interference
- l Tail moment arm
- $A =$ Wing aspect ratio
- A' Tail aspect ratio
- $T_v = \frac{s'l}{s\bar{c}} =$ tail volume
- S Wing area
- C_{Di} Induced drag coefficient $= R \cdot C_L^2 / \pi A$
- W $L_1 + L_2$

TABLE 1

Comparison with Flight Measurements (From AFEE/Res 8)

($C_{m0}/C_L = -0.05$ assumed)

Aircraft	Measured K	Estimated R (Figs. 1 and 3)						K (from Fig. 5)		
		Static Margin						Static Margin		
		0	0.1 \bar{z}	0.2 \bar{z}	0.3 \bar{z}	0.4 \bar{z}	0.5 \bar{z}	0	0.1 \bar{z}	0.52 \bar{z}
Hotspur II ..	1.05	1.05	1.02	1.01	1.01	1.02	1.07	1.07	1.03	1.07
Horsa	1.05	1.04	1.01	1.00	1.01	1.04	1.08	1.05	1.02	
Hamilcar ..	1.00	1.05	1.03	1.00	1.00	1.01	1.03	1.08	1.04	
Hadrian ..	1.05	1.03	1.00	1.00	1.04	1.12	1.25	1.07	1.01	
Hengist ..	1.10*	1.06	1.03	1.01	1.00	1.00	1.02	1.08	1.04	

(* Re-analysis of flight measurements.)

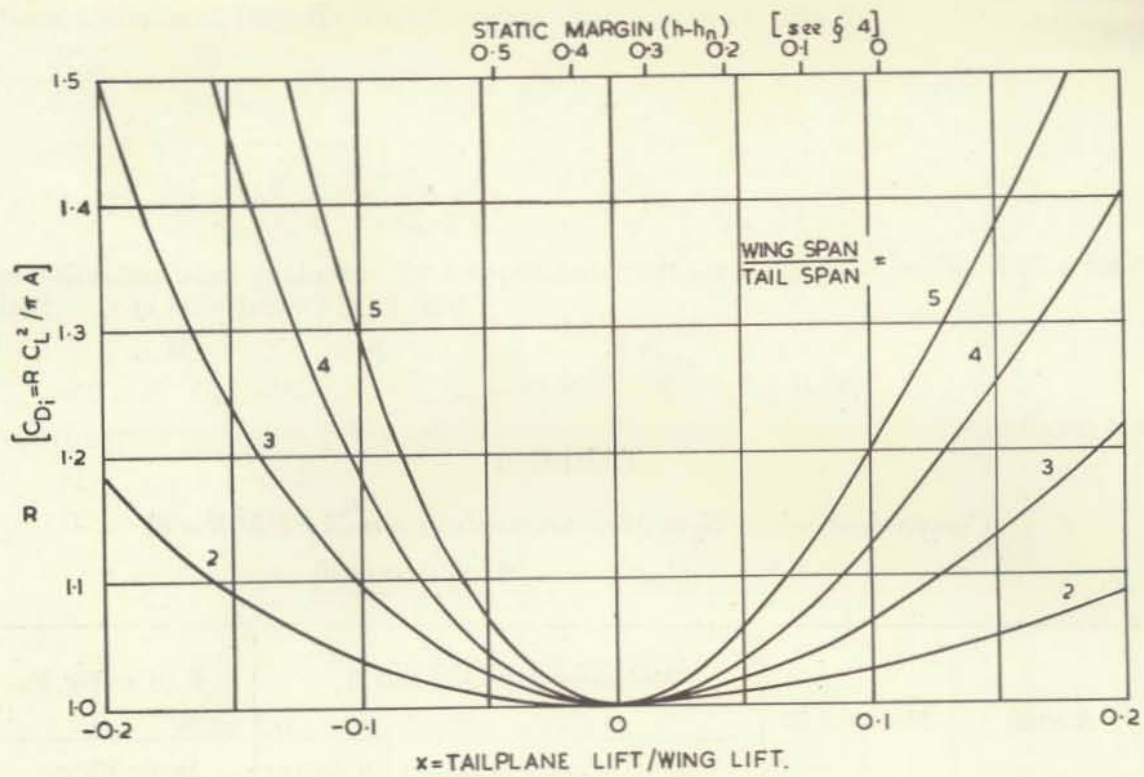


FIG. 1. Variation of induced-drag factor R with tailplane lift x .

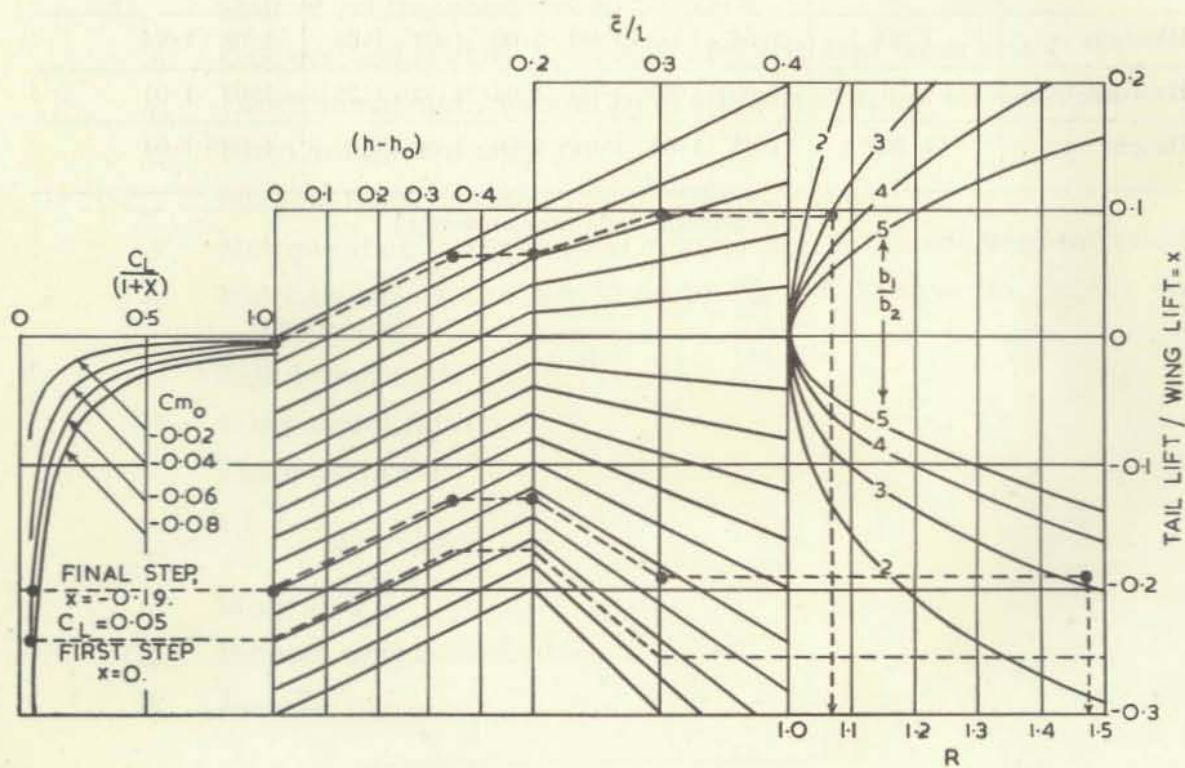


FIG. 2. Chart for determining induced-drag factor R ($C_{m_0} + R \frac{C_L^2}{\pi A}$, see section 4).

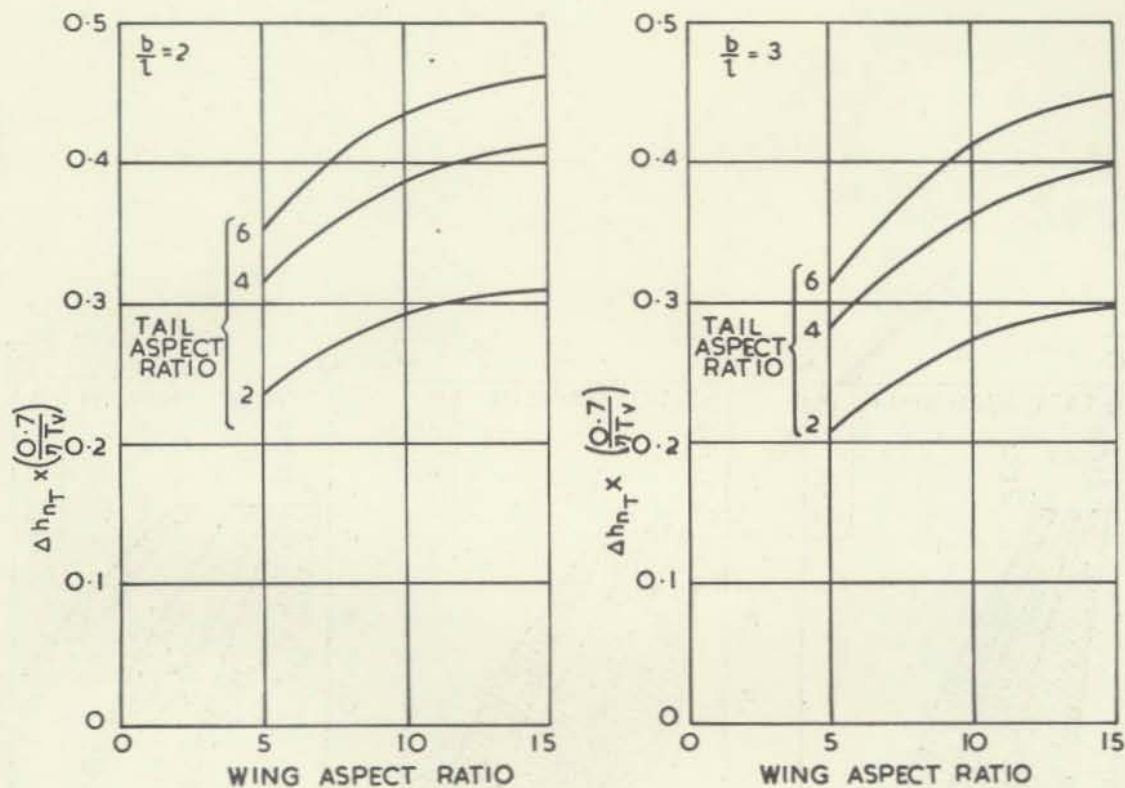


FIG. 3. Shift of longitudinal static stick-fixed neutral point due to tailplane.

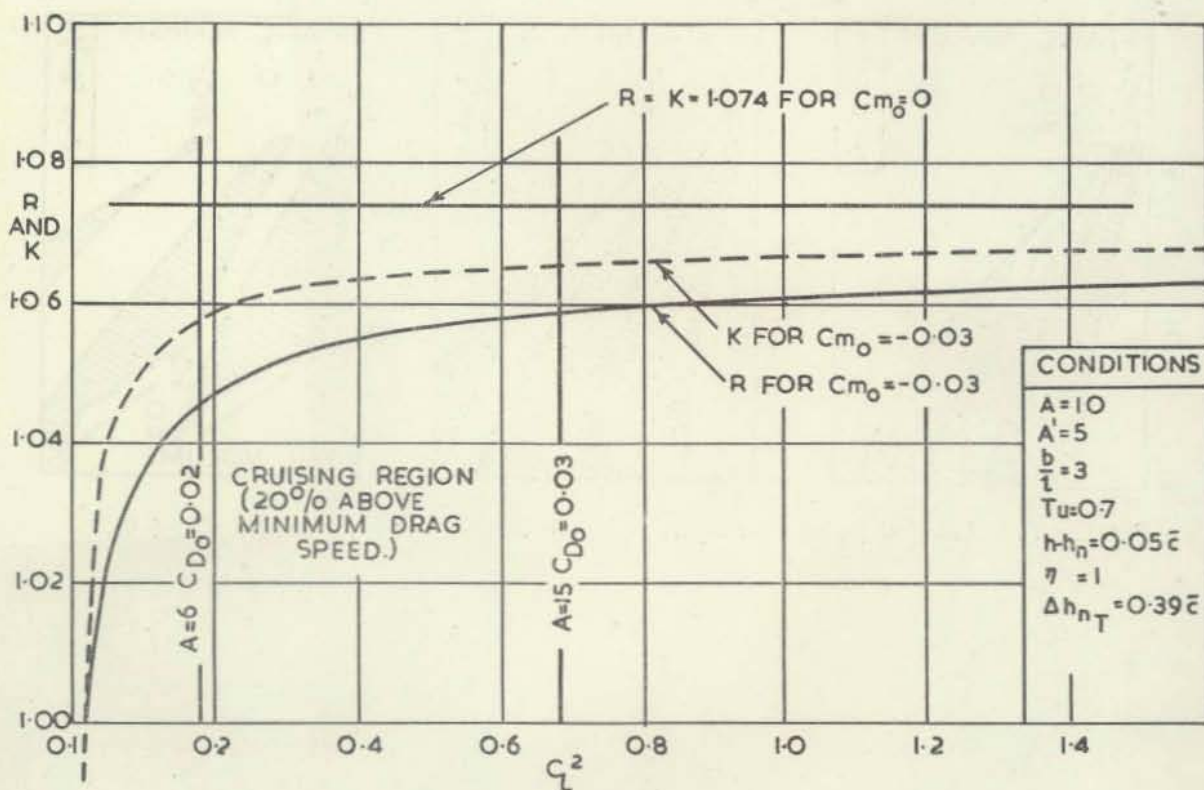


FIG. 4. Example of variation of R and K with C_L^2 .

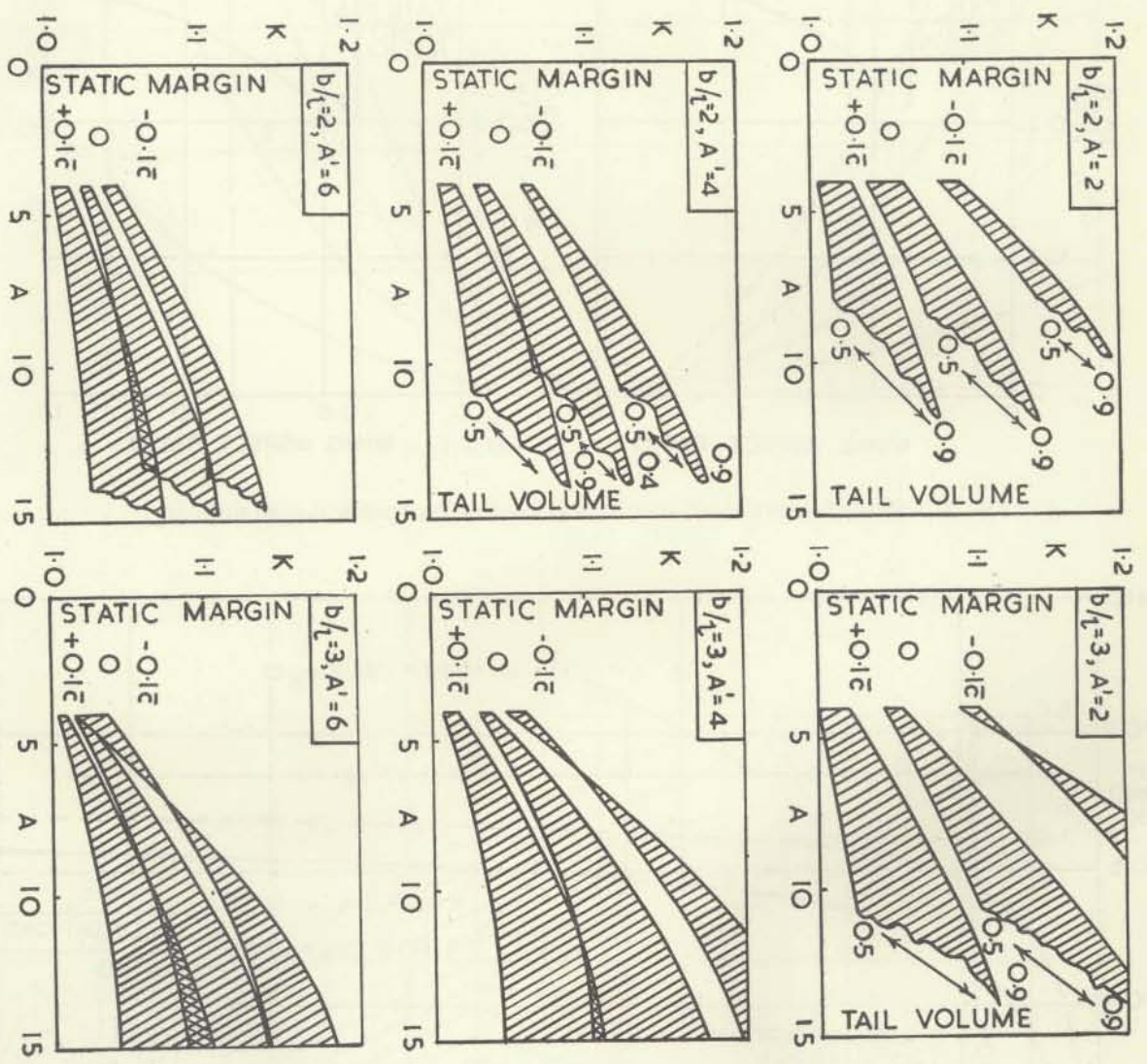


FIG. 5. Variation of K (or R) with aspect ratio, tail volume, and static margin ($C_{mo} + 0$).

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