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Wave Theory of the Mach-Zehnder Interferometer

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Summary. A theory of the Mach-Zehnder interferometer is developed using the approach of investigating the history of a wave front produced by an element of an extended light source. Account is taken of the effects of imperfections of the optical components including all the first-order aberrations of the collimating and collecting lenses or mirrors and of the camera lens. The theory can also be applied to other two-beam interferometers.

1. *Introduction.* The Mach-Zehnder interferometer is one of a large class of instruments in which two-beam interference is produced by division of amplitude. Its principle of operation is simple and can be stated very briefly, if the components of the instrument are assumed to be optically perfect. Referring to Fig. 1, S, a point source of monochromatic light in the focal plane of a perfect collimating lens L produces a parallel beam of light which meets a semi-reflecting mirror M_1. This mirror divides the amplitude of the incident light equally between one transmitted and one reflected beam, which are in turn reflected by the fully reflecting mirrors M_4 and M_3, respectively, and reunited by the second semi-reflecting mirror M_2. Since under the idealised conditions assumed the two semi-reflecting mirrors have zero thickness, observable interference would take place beyond M_2 wherever the two beams overlap*. The fringe pattern would depend only on the relative inclination of the two beams; any desired fringe spacing and direction could be obtained by rotating any one of the mirrors about two mutually perpendicular axes in its own plane. Therefore, the theory of the ideal instrument (with a point source of monochromatic light) consists merely of the relation between the displacements of one of the mirrors and the corresponding fringe spacing and direction.

Complications arise when one attempts to take into account departures of a real instrument from the ideal model. It is convenient to classify these departures under the following headings:

(*a*) Light source

(i) Finite band width

(ii) Finite size

* We shall not be concerned, in this paper, with the other pair of beams (shown dotted in Fig. 1) also produced by M_2. In this pair, one beam has been reflected once and the other three times; in consequence there is a difference of phase and polarisation between them.

(*b*) Mirrors (and other glass plates such as wind tunnel windows)

(i) Finite thickness of semi-reflecting mirrors

(ii) Differences of thickness and refractive index between the two semi-reflecting mirrors

(iii) Non-uniform thickness ('wedge shaped' plates)

(iv) Surface irregularities

(*c*) Aberrations of lenses

(i) Collimating lens (or mirror)

(ii) Collecting lens (or mirror) and camera lens.

The effects of imperfections of mirrors and lenses are linked with those of the light source in the sense that their importance depends on their effects being different for different points (and wavelengths) of the light source. With a point source of monochromatic light, the only effect of mirror and lens imperfections would be a distortion of the fringe pattern. On the other hand, the spectral characteristics and the band width of the source would have very important effects even in the absence of mirror and lens imperfections. For, every element of the source produces a fringe pattern of its own; these patterns overlap and their combined effect depends on the spectrum and the dimensions of the source, as well as on the arrangement of the mirrors. This has the following most important consequences: first, observable interference will be produced only provided the difference in the optical path length along the two routes between the source and a point on the screen is not too great, and second, the fringes are localised, *i.e.*, there is an optimum position for observing interference where the contrast of the fringes is highest; to this corresponds an optimum adjustment of the mirrors which minimizes the effects of the size and the spectrum of the source.

The fact that the fringe contrast is governed by the characteristics of the source, the adjustment of the mirrors and the quality of the optical components stresses the importance of an adequate and yet reasonably simple theory which takes into account all the essential features of the instrument. The need for such a theory arises both in the design of a new instrument and in the efficient operation of an existing one. In the past, many extravagant notions as to the cost of construction and complexity of adjustment of a Mach-Żehnder interferometer have been widely entertained, largely because of the lack of detailed understanding of the optics of the instrument.

The early theoretical work on the subject, reviewed by Tanner (1956), was unnecessarily restricted to various special cases and many of the results were wrong. The first attempt to give a comprehensive theory was made by Winkler (1947), who considered the aspects listed above under the headings (*a*) and (*b*). In common with his predecessors, Winkler used laborious ray tracing methods leading to complicated geometrical arguments, some of which the present writer must confess to having failed to follow. The aspects of the theory considered by Winkler and the earlier writers were put on a sound basis by Tanner who, in his 1956 paper, rederived the results obtained previously, correcting those that were in error (in particular, Winkler's conclusions on the effects of the finite and non-uniform thickness of the semi-reflecting mirrors). Instead of using the conventional ray tracing methods, Tanner expressed the optical path difference between the two beams, at a point on the screen, in terms of the separation of the two images of the point as seen by an observer situated between the collimating lens and the first mirror. This ingenious device leads to a valuable simplification of the theory; it has, however, obvious didactic drawbacks.

The purpose of the present paper is to put forward an alternative theory based on the physically direct approach of investigating the history of a wave-front produced by a point of the (extended) light source. The use of elementary vector notation considerably simplifies the analysis and helps to clarify the physical significance of the equations derived. In Section 2 is given the theory of the ideal interferometer with an extended light source, whilst Section 3 deals with the effects of imperfections of mirrors and windows. Section 4 is devoted to what the writer believes to be the first complete account of the effects of first-order aberrations of the collimating, collecting and camera lenses. Tanner (1956) considered only the effects of the spherical and of the longitudinal chromatic aberrations of the collimating lens. In interferometers with a large field of view (greater than, say, 10 cm square), it may be necessary to use spherical mirrors rather than lenses (as good lenses of such dimensions are very costly) and the effects of oblique monochromatic aberrations (coma, astigmatism, curvature and distortion) may become important.

2. *Ideal Interferometer with Extended Light Source.* Any point of an extended light source may be regarded as the origin of spherical wave-fronts*. If the point is situated in the focal plane of a perfect collimating lens (or mirror) L (Fig. 1), the lens transforms the spherical front F_s into a plane front F_p. The plane front is divided by the first semi-reflecting mirror into two coherent wave fronts; of these one is transmitted by M_1, reflected by M_4 and M_2 and emerges as F' in Fig. 1, the other front is reflected by M_1 and M_3, transmitted by M_2 and emerges as F. Displacements between the corresponding points of F and F' determine the interference pattern on the screen, as will be shown below. No observable interference can take place between fronts originating from different points of the source.

Suppose now that the mirrors are so adjusted that F and F' coincide. Then, an observer looking into M_2 would not be conscious of the particular arrangement of the mirrors. Beyond M_2 he would see M_3 together with the images of the mirrors M_4 and M_1 and the image of the lens L (Fig. 2); the coincident wave-fronts would appear to have travelled from O, the centre of the collimating lens, with their centres moving along the straight line joining O to S, the point in the source from which the wave-fronts originated. This 'linear' representation of the interferometer is optically completely equivalent to the actual arrangement.

Consider now a point S_0 of the source which may be regarded as the 'centre' of the source (it need not be its geometric centre, but merely a convenient reference point). Then, F_0 is a wave-front originating from S_0 and we take the path of propagation of its centre (*i.e.*, its central ray) as the optical axis of the interferometer and as the z axis of a cartesian co-ordinate system, origin at O, the y axis perpendicular to the plane of the centres of the mirrors†, positive upwards, with the x axis completing the right-handed system; the centres of the mirrors are the points of intersection of the central ray of F_0 with the mirror planes.

Suppose M_2 is rotated through a small angle α_{m2} about an axis in the plane of M_2 passing through its centre. Then, from the laws of reflection it follows that the wave-front F' reflected by M_2 will be rotated about the same axis through twice the angle; this may be represented by the vector

* The term 'wave' is used here in the sense of a train of surfaces of constant phase which are orthogonal to and propagate along the rays of geometrical optics. This view takes no account of diffraction which is of no significance in our problem (*cf.* Hopkins, 1950).

† It is not necessary for the centres of the mirrors to be co-planar, but it is usually convenient to arrange them so and such arrangement will be assumed throughout the paper.

$\boldsymbol{\alpha}_2$ ($= 2\boldsymbol{\alpha}_{m2}$), Fig. 2. Similarly, a small translation τ_{m2} in the direction of the mirror unit normal $\mathbf{m}_2$ will result in the wave-front translation $\boldsymbol{\tau}_2 = 2\boldsymbol{\tau}_{m2}$ ($= 2\boldsymbol{\tau}_{m2}\, \mathbf{m}_2$).

Consider a point P′ on F', position vector $\mathbf{p}$ (Fig. 3). The displacement of P′ due to the rotation $\boldsymbol{\alpha}_2$ and translation $\boldsymbol{\tau}_2$ is, to first order in α and τ,

$$\mathbf{d}_2 = \boldsymbol{\alpha}_2 \wedge (\mathbf{p} - z_2\mathbf{k}) + \boldsymbol{\tau}_2$$

where $\mathbf{k}$ is the unit vector in the z direction. Since all the displacements are assumed to be small, and their squares and higher-order powers are neglected, they are additive. Further, since we are only concerned with the relative displacements of the wave-fronts, the displacement of one of the fronts is equivalent to a displacement of the other front of equal magnitude but opposite direction. Thus, for convenience, we may regard F as the 'undisturbed' front and F' as being affected by adjustments of all the mirrors (with the signs of $\boldsymbol{\alpha}_{m1}$, $\boldsymbol{\alpha}_{m3}$, $\boldsymbol{\tau}_{m1}$ and $\boldsymbol{\tau}_{m3}$ reversed). The combined effect of all the mirrors is then a displacement of F' given by

$$\mathbf{D} = \mathbf{A} \wedge \mathbf{p} + \mathbf{T} - \sum_{i} z_i \boldsymbol{\alpha}_i \wedge \mathbf{k}; \qquad i = 1, 2, 3, 4\,, \tag{2.1}$$

where

$$\mathbf{A} = \Sigma \boldsymbol{\alpha}_i = 2\,\Sigma(-1)^i\, \boldsymbol{\alpha}_{mi}\,,$$
$$\mathbf{T} = \Sigma \boldsymbol{\tau}_i = 2\,\Sigma(-1)^i\, \boldsymbol{\tau}_{mi}\,.$$

If $\mathbf{n}$ is the unit normal at C, the centre of F, $\mathbf{n}'$, the corresponding unit normal of F' is

$$\mathbf{n}' = \mathbf{n} + \mathbf{A} \wedge \mathbf{n}\,. \tag{2.2}$$

Then, referring to Fig. 4, if P is a point (on the screen S) whose position vector is $\mathbf{p}$, the path difference l at P is*

$$l = PR$$
$$= \mathbf{D} \,.\, \mathbf{n}'$$

or, by equation (2.2),

$$l = \mathbf{D} \,.\, \mathbf{n} + 0(A^2 p,\, AT)\,. \tag{2.3}$$

Now, $\mathbf{D}$ can be shown to be also the displacement between the two images of P in M_3, M_1 and in M_2, M_4, respectively, due to the image rotation $\mathbf{A}$ and translation $\mathbf{T}$. Interpreted this way our expression for l is precisely equivalent to Tanner's.

To investigate the dependance of l on source size it is convenient to write $\mathbf{n}$ as

$$\mathbf{n} = \mathbf{k} \cos \omega + \mathbf{s} \sin \omega\,, \tag{2.4}$$

where ω is the angle between $\mathbf{n}$ and the z axis and $\mathbf{s}$ is the unit vector in the direction of projection onto a $z =$ const plane of S_0S, the line joining the source point S to the centre of the source (*see* Fig. 2). Further, if C_s is the point of intersection of the screen with the z axis, or the 'centre' of the screen, position vector $z_s\mathbf{k}$, equation (2.1) may be written

$$\mathbf{D} = \mathbf{A} \wedge \mathbf{r} + \mathbf{T} + \mathbf{T}_s\,, \tag{2.5}$$

where $\mathbf{r} = \mathbf{p} - z_s\mathbf{k}$, is the position vector of the point P relative to C_s, $\mathbf{A} \wedge \mathbf{r}$ is the rotational displacement of the point P on F due to the rotation $\mathbf{A}$ with the axis of rotation passing through the centre of the screen C_s, and $\mathbf{T}_s = \Sigma\,(z_s - z_i)\, \boldsymbol{\alpha}_i \wedge \mathbf{k}$, is the translation of C_s due to the fact that, in general, the effective total rotation $\mathbf{A}$ is not about an axis passing through C_s (Note that $\mathbf{T}_s$ has no z component).

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* Positive value of l implies that F lags behind F', i.e., $l =$ path length in beam $LM_1M_3M_2$ − path length in beam $LM_1M_4M_2$.

With these changes of notation, equation (2.3) becomes

$$l = (\mathbf{A}\wedge\mathbf{r} + \mathbf{T} + \mathbf{T}_s) \cdot (\mathbf{k} \cos \omega + \mathbf{s} \sin \omega) \,. \tag{2.6}$$

To make l independent of the source size to first order in ω ('optimum adjustment'), we must make $\mathbf{D} \cdot \mathbf{s} = 0$, i.e., there must be no lateral displacement of the wave fronts. This requires:

(1) $\mathbf{T} \cdot \mathbf{s} = 0$, *i.e.*, translation in the z direction only.

(2) $\mathbf{T}_s = 0$, *i.e.*, mirror rotations must be such that the effective total rotation is about the centre of the screen; this determines the mirror rotations necessary to 'focus' the fringes onto the screen.

(3) $\mathbf{s} \cdot \mathbf{A}\wedge\mathbf{r} = 0$; for this triple scalar product to vanish we can either:

(i) make $\mathbf{s}$, $\mathbf{A}$, and $\mathbf{r}$ co-planar, *i.e.*, the screen must be the $z = z_s$ plane and $\mathbf{A}$ must have no z component, or

(ii) $A_z = 0$, $\mathbf{s}$ parallel to $\mathbf{A}$, *i.e.*, the source must be a line (or narrow slit) parallel to $\mathbf{A}$.

With a non-parallelogram interferometer it is always possible to make $A_z = 0$, independently of the other components of $\mathbf{A}$. In a parallelogram interferometer all $\mathbf{m}_i$ are equal, which results in A_z being proportional to A_x. Then, for a given source size, an A_x rotation inevitably involves some loss of fringe contrast. The effect of A_z may be minimised by inclining the screen so that it contains $\mathbf{A}$ (then $\mathbf{s} \cdot \mathbf{A}\wedge\mathbf{r} = 0$ where $\mathbf{r}$ is parallel to $\mathbf{A}$), but this is seldom practicable.

The theory of this Section can be applied to other two-beam interferometers. For instance, Fig. 5 shows the Twyman-Green modification of the Michelson interferometer and its linear representation. Here, the fringes are always localised in the plane coincident with the images of the mirrors 2 and 3; also, $\mathbf{A}$ can have no z component, so that fringe contrast is, to first order, independent of the source size.

3. *Effects of Finite Thickness and Imperfections of Glass Plates.* 3.1. *Flat Plates of Uniform Thickness and Refractive Index.* Consider a plane wave-front F which is transmitted by a flat glass plate of uniform thickness t and refractive index μ (Fig. 6). F is incident at an angle β on the plate whose unit normal is $\mathbf{m}$, so that $\mathbf{m} \cdot \mathbf{n} = \cos \beta$, where as before $\mathbf{n}$ is the unit normal to the wave-front. Refraction in the plate produces two effects (ignoring changes of phase and polarisation).

(i) Increase of optical path of amount $tF(u, \mu)$, where $F(u, \mu) = \sqrt{\{(\mu^2 - 1) + u^2\}} - u$; $u = \cos \beta = \mathbf{m} \cdot \mathbf{n}$. This corresponds to a backward displacement of F along its normal equal to $-\ tF\mathbf{n}$.

(ii) Transverse shift of F of amount $-\ tF_u \sin \beta$ in the direction normal to $\mathbf{n}$ and in the plane of $\mathbf{n}$ and $\mathbf{m}$, *i.e.*, in the direction of $(\mathbf{m} - u\mathbf{n})$. This corresponds to a displacement $-\ t(\mathbf{m} - u\mathbf{n})F_u$, where $F_u = \partial F/\partial u$.

Let the primed quantities refer to the corresponding plate through which the wave-front F' is transmitted. Then, the change of the optical path difference is

$$\delta l = tF(u, \mu) - t'F'(u', \mu') \,. \tag{3.1.1}$$

Suppose that t, μ and u of the two plates differ slightly*, so that we may write

$$t' = t + \delta t, \mu' = \mu + \delta\mu, u' = u + \delta u .$$

Now, $\delta u = \delta(\mathbf{m} \,.\, \mathbf{n}) = \mathbf{n} \,.\, \delta\mathbf{m} + \mathbf{m} \,.\, \delta\mathbf{n}$. Of these, $\delta\mathbf{n}$ is produced either by mirror rotations or by the presence of a disturbance in one of the beams; in either case its effect is qualitatively similar to that of $\delta\mathbf{m}$, the change of incidence due to a slight departure from the parallelogram arrangement. Here, we shall consider only the effects of $\delta\mathbf{m}$, so that (3.1.1) becomes

$$-\frac{\delta l}{t} = F(u, \mu)\frac{\delta t}{t} + F_\mu(u, \mu)\,\delta\mu + F_u(u, \mu)(\mathbf{n} \,.\, \delta\mathbf{m}) . \tag{3.1.2}$$

This depends on the position of the source point, since $u = \mathbf{m} \,.\, \mathbf{n}$ and

$$\begin{aligned} \mathbf{n} &= \mathbf{k} \cos\omega + \mathbf{s} \sin\omega \\ &= \mathbf{k} + (\omega\mathbf{s} - \tfrac{1}{2}\omega^2\mathbf{k}) + 0(\omega^3) \\ &= \mathbf{k} + \delta_\omega\mathbf{n} , \end{aligned} \tag{3.1.3}$$

so that

$$u = u_0 + \mathbf{m} \,.\, \delta_\omega\mathbf{n} = u_0 + [\omega(\mathbf{m} \,.\, \mathbf{s}) - \tfrac{1}{2}\omega^2 u_0] ,$$

the suffix $_0$ indicating that the quantity concerned refers to the wave fronts from the centre of the light source. Thus, to second order in the source size, any function $G(u, \mu)$ may be written in the form

$$\begin{aligned} G(u, \mu) = G(u_0, \mu) &+ G_u(u_0, \mu)\,[\omega(\mathbf{m} \,.\, \mathbf{s}) - \tfrac{1}{2}\omega^2 u_0] \\ &+ \tfrac{1}{2}G_{uu}(u_0, \mu)\omega^2 (\mathbf{m} \,.\, \mathbf{s})^2 \end{aligned}$$

or

$$G = G_0 + \omega(\mathbf{m} \,.\, \mathbf{s})G_{0u} + \tfrac{1}{2}\omega^2[(\mathbf{m} \,.\, \mathbf{s})^2 G_{0uu} - u_0 G_{0u}] . \tag{3.1.4}$$

Difference of thickness.

By equations (3.1.2) and (3.1.4), the path difference produced by a difference of thickness is obtained by identifying G with F. Then, using (2.3) and (3.1.3), the path difference on the screen is

$$\begin{aligned} l = (\mathbf{D} - \mathbf{k}F_0\,\delta t) \,.\, \mathbf{k} &+ \omega(\mathbf{D} - \mathbf{m}F_{0u}\,\delta t) \,.\, \mathbf{s} \\ &- \tfrac{1}{2}\omega^2\,[(\mathbf{D} - \mathbf{m}F_{0u}\,\delta t) \,.\, \mathbf{k} + (\mathbf{m} \,.\, \mathbf{s})^2 F_{0uu}\,\delta t] . \end{aligned} \tag{3.1.5}$$

l may be made independent of δt to first order in ω by an adjustment of the mirrors giving a wave-front displacement $\delta\mathbf{D}$ such that

$$\delta\mathbf{D} = [\mathbf{k}F_0 + (\mathbf{m} - u_0\mathbf{k})F_{0u}]\,\delta t . \tag{3.1.6}$$

Then

$$l = l_i - \tfrac{1}{2}\omega^2[F_0 - u_0F_{0u} + (\mathbf{m} \,.\, \mathbf{s})^2 F_{0uu}]\,\delta t , \tag{3.1.7}$$

where l_i is the path difference of the ideal interferometer.

Note that $\delta\mathbf{D}$ is equal to minus the relative displacement (longitudinal and transverse) between the two wave-fronts from the centre of the source, due to the difference of thickness of the two plates.

Effect of dispersion.

It can be shown (*see*, for example, Tanner, 1956) that as a result of dispersion caused by the different thicknesses of glass in the two beams, fringes produced by the wavelengths of the source

* In Section 2 we shall be concerned with parallelogram interferometers only.

near λ will reinforce not when $l = 0$ but when $l = -\lambda F_\lambda\,\delta t$. Therefore, to keep the fringe of highest contrast in the middle of the screen requires a wave-front translation $\delta\mathbf{T} = -\lambda F_{0\lambda}\,\mathbf{k}\,\delta t$ whose effect is to introduce the term $-\lambda F_{0\lambda}$ into the square bracket of equation (3.1.7).

Difference of refractive index.

The effects of differences of μ of the two plates may be found simply by replacing in equations (3.1.5) to (3.1.7) δt, F, F_u and F_{uu} by $t\,\delta\mu$, F_μ, $F_{\mu u}$ and $F_{\mu uu}$ respectively.

Difference of incidence.

Identifying G in equation (3.1.4) with F_u and substituting for F_u and $(\mathbf{n}\,.\,\delta\mathbf{m})$ in equation (3.1.2), we find that the compensating wave-front displacement

$$\delta\mathbf{D} = t[F_{0u}\,\delta\mathbf{m} + (\mathbf{m} - u_0\mathbf{k})\,F_{0uu}\,(\mathbf{k}\,.\,\delta\mathbf{m})] \tag{3.1.8}$$

makes l independent of $\delta\mathbf{m}$ to first order in ω, leaving a quadratic dependence on ω:

$$l = l_i - \tfrac{1}{2}\omega^2 t\{[F_{0uuu}(\mathbf{s}\,.\,\mathbf{m})^2 - u_0F_{0uu}]\,\mathbf{k}\,.\,\delta\mathbf{m} + 2(\mathbf{s}\,.\,\mathbf{m})\,F_{0uu}(\mathbf{s}\,.\,\delta\mathbf{m})\}\,. \tag{3.1.9}$$

At zero incidence (*i.e.*, windows normal to the optical axis), $\mathbf{m} = \mathbf{k}$, so that $\mathbf{s}\,.\,\mathbf{m} = 0$ and since $\delta\mathbf{m}$ must be perpendicular to $\mathbf{m}$, $\mathbf{k}\,.\,\delta\mathbf{m} = 0$ and the quadratic term in ω vanishes, making l independent of difference of incidence to second order in source size.

3.2. *Wedge-shaped Plates.* A glass plate with plane sides which include a small angle ϕ is equivalent to a plate of uniform thickness equal to the thickness at the centre of the real plate, plus a thin wedge or prism of angle ϕ whose apex lies along the constant-thickness line passing through the centre of the plate (one half of the prism must, of course, be regarded as a 'negative' prism). From the law of refraction it follows that the effect of such a prism (Fig. 7) is to rotate the incident wave-front through an angle $\alpha_w = \phi\,F(u, \mu)/u$ about the apex of the wedge. This rotation can be represented by the vector

$$\boldsymbol{\alpha}_w = \phi\frac{F}{u}\mathbf{w}\,, \tag{3.2.1}$$

where $\mathbf{w}$ is the unit vector along the apex of the wedge. This displaces the emergent wave front by the amount

$$\delta_w\mathbf{D} = \boldsymbol{\alpha}_w\wedge(\mathbf{p} - z_i\mathbf{k})\,, \tag{3.2.2}$$

(z_i being the co-ordinate of the centre of the plate in question) and the corresponding path difference is

$$\delta_w l = \phi\frac{F}{u}\mathbf{w}\wedge(\mathbf{p} - z_i\mathbf{k})\,.\,[\mathbf{k} + (\omega\mathbf{s} - \tfrac{1}{2}\omega^2\mathbf{k})]\,.$$

Expanding F/u in powers of ω [*cf.* equation (3.1.4)], we have, to second order in ω,

$$\begin{aligned}\delta_w l = {} & \alpha_w^0[wpk] + \omega\{\alpha_w'[wpk] + \alpha_w^0[w(p - z_ik)s]\} \\ & + \tfrac{1}{2}\omega^2\{(\alpha_w'' - \alpha_w^0)\,[wpk] + 2\alpha_w'[w(p - z_ik)s]\}\,,\end{aligned} \tag{3.2.3}$$

where $[abc]$ denotes the triple scalar product of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ and

$$\left.\begin{aligned}\alpha_w^0 &= \frac{\phi}{u_0}F_0,\ \alpha_w' = \frac{\phi}{u_0}(F_{0u} - F_0/u_0)\,\mathbf{m}\,.\,\mathbf{s} \\ \alpha_w'' &= \frac{\phi}{u_0}\{[1 + (\mathbf{m}\,.\,\mathbf{s})^2/u_0^2](F_0 - u_0F_{0u}) + (\mathbf{m}\,.\,\mathbf{s})^2\,F_{0uu}\}\end{aligned}\right\}. \tag{3.2.4}$$

Mirrors.

To first order in ω we may write

$$l = [\mathbf{D} + \alpha_w^0(\mathbf{w}\wedge\mathbf{p})] \,.\, \mathbf{k} + \omega[\mathbf{D} + \alpha_w^0\, \mathbf{w}\wedge(\mathbf{p} - z_i\mathbf{k})] \,.\, \mathbf{s} + \omega\, \alpha_w'[wpk] \,. \tag{3.2.5}$$

The term independent of ω may be eliminated by a mirror rotation producing the displacement

$$\delta\mathbf{D} = -\,\alpha_w^0\, \mathbf{w}\wedge(\mathbf{p} - z_i\mathbf{k})\,,$$

leaving a first-order dependence of l on ω:

$$l = l_i + \omega\alpha_w'\, [wpk] \,. \tag{3.2.6}$$

Windows.

At zero incidence, $u_0 = 1$, $\alpha_w^0 = (\mu - 1)\phi$, $\alpha_w' = 0$, $\alpha_w'' = \dfrac{\mu^2 - 1}{\mu}\,\phi$, hence,

$$\begin{aligned} l = {} & [\mathbf{D} + (\mu - 1)\phi(\mathbf{w}\wedge\mathbf{p})] \,.\, \mathbf{k} + \omega[\mathbf{D} + (\mu - 1)\phi\mathbf{w}\wedge(\mathbf{p} - z_i\mathbf{k})] \,.\, \mathbf{s} \\ & - \tfrac{1}{2}\omega^2\, [\mathbf{D} - \frac{\mu - 1}{\mu}\,\phi(\mathbf{w}\wedge\mathbf{p})] \,.\, \mathbf{k} \,. \end{aligned}$$

To eliminate the zeroth and first-order terms in ω would require a wave-front rotation $-\,(\mu - 1)\phi$ about the axis $\mathbf{w}$ through the centre of the wedge-shaped plate. In a parallelogram interferometer this is possible only if the wedge apex is parallel to the y axis. For, a wave-front rotation about the x axis necessarily involves a proportional rotation about the z axis ($A_z = A_x \tan\beta_0$). Thus, the ω term can be eliminated only when $\mathbf{w} = \mathbf{j}$. Otherwise, the compensating wave-front displacement

$$\delta\mathbf{D} = -\,(\mu - 1)\phi(\mathbf{w} + \mathbf{k}w_x \tan\beta_0) \wedge (\mathbf{p} - z_i\mathbf{k})$$

again leaves a first-order dependence of l on ω:

$$l = l_i + \omega(\mu - 1)\phi w_x \tan\beta_0\, [spk] \,. \tag{3.2.7}$$

For direct comparison of the theory of this Section with the corresponding results of Tanner we note that if the x and y co-ordinates of the source point S are x', y' and f is the focal length of the collimating lens,

$$\omega\mathbf{s} = \left(-\frac{x'}{f}\mathbf{i} - \frac{y'}{f}\mathbf{j}\right)[1 + 0(\epsilon^2)]\,,$$

where ϵ is the angle between the axis of the lens and the z axis. Further, the mirror unit normals

$$\mathbf{m}_i = -\,\mathbf{i}\sin\beta_0 + \mathbf{j}\cos\beta_0\,,$$

where our β_0 corresponds to Tanner's β. With these substitutions our results can be shown to be equivalent to Tanner's, thus providing a complete confirmation of Tanner's conclusions, including those that differ from Winkler's.

4. *Effects of Lens Aberrations.* 4.1. *Introduction.* Consider a lens (or a system of lenses in contact) so thin that its two principal planes may be assumed to coincide (Fig. 8). Let F' be a spherical wave-front emanating from an object point P′. If the lens were perfect, F' would be transformed into the spherical front F_p converging towards the Gaussian image point P. A real lens transforms the spherical front F' into a wave-front F_a which, in general, is not spherical but suffers from aberrations. The measure of the aberration at a point on the wave-front is the perpendicular

distance at that point, that is, the path difference, between F_a and F_p, which will be denoted here by W (positive when F_a is lagging behind F_p). What part of F_a reaches the image point is determined by the position of the diaphragm, or stop, D; the ray $P'Q_PQ_DP$ is the principal ray for the object point P′, *i.e.*, the ray from P′ which passes through the point of intersection of the plane of the diaphragm with the axis of the lens.

For a detailed account of the wave theory of aberrations the reader is referred to the book by Hopkins (1950). For our purpose it will suffice to note that the first order, or Seidel, monochromatic aberrations of the wave front may be expressed as a power series

$$W = \Sigma\, w_{lmn}\,, \tag{4.1.1}$$

where the general term is of the form

$$\frac{w_{lmn}}{f} = C_{(2l+m,\,2n,\,m)}(\eta/f)^{2l+m}(\rho/f)^{2n}\left(\frac{\rho\cos\phi}{f}\right)^{m}\,;\; 2\,(l+m+n)' = 4\,.$$

η, ρ and ϕ are defined* in Fig. 8, and the C coefficients depend on the characteristics of the lens and on the image and stop positions relative to the lens.

To (4.1.1) should be added the longitudinal and transverse chromatic aberration terms; then (4.1.1) becomes

$$\frac{W}{f} = C_{040}\theta^4 + C_{121}\sigma\theta^2\psi + C_{202}\sigma^2\psi^2 + C_{220}\sigma^2\theta^2 + C_{301}\sigma^3\psi + C_{lc}\theta^2 + C_{tc}\sigma\psi\,, \tag{4.1.2}$$

where

$\sigma = \mu/f$, $\theta = \rho/f$, $\psi = \theta\cos\phi = \boldsymbol{\rho}\,.\,\boldsymbol{\xi}/f$ (*see* Fig. 8). The various terms of (4.1.2) represent, respectively, spherical aberration, coma, astigmatism, curvature, distortion, longitudinal and transverse chromatic aberrations.

4.2. *Collimating Lens or Mirror.* In the case of the collimating lens, the effective stop is the lens itself. Therefore, the principal ray from any source point passes through the centre of the lens (which is also the origin of our co-ordinate system). Further, since a wave-front emerging from a perfect collimating lens would be plane, ρ and ϕ may be taken to be the polar co-ordinates of a point on the wave-front, not only when the front is at the lens, but also anywhere beyond the lens.

Referring to Fig. 9, consider the situation at a point P on the screen (which is taken to be normal to the z axis); as in Section 2, F is the undisturbed front and F' is the coherent front which has been displaced by mirror adjustments. In the absence of aberrations, the path difference at P would be $l_i = PR\ (= \mathbf{D}\,.\,\mathbf{n})$. With aberrations, the path difference is

$$\begin{aligned} l &= PS - PQ \\ &= PS - W_P \\ &\doteq PU - W_P \text{ (neglecting terms of the same order as the higher-order aberrations)} \\ &= l_i + W_R - W_P\,. \end{aligned}$$

* There are several possible ways of defining η, the only essential requirement being that η should provide a measure of the obliquity of the principal ray.

Therefore, since $\mathbf{D}$ is small,

$$\left.\begin{aligned} l &= l_i + \mathbf{D} \,.\, \nabla W \\ &= \mathbf{D} \,.\, (\mathbf{n} + \nabla W) \end{aligned}\right\} . \tag{4.2.1}$$

It can be shown that ∇W, the gradient of W, is given by

$$f\nabla W = W_\theta \frac{\boldsymbol{\rho}}{\rho} + W_\psi \boldsymbol{\xi}\,, \tag{4.2.2}$$

where $\boldsymbol{\xi}$ is the unit vector defined in Fig. 8, and the suffices on W indicate partial differentiation.

Now, let the unit normal to the lens surface, at the centre of the lens (Fig. 2) be

$$\mathbf{l} = \epsilon\mathbf{t} + \mathbf{k} + 0(\epsilon^2)\,,$$

where ϵ is the small angle that the axis of the lens makes with the z axis. For wave-fronts originating from the centre of the source (suffix $_0$),

$$\boldsymbol{\rho} = \boldsymbol{\rho}_0 = \mathbf{r}\,;\ \boldsymbol{\eta} = \boldsymbol{\eta}_0 = \epsilon f\mathbf{t},\ \boldsymbol{\xi} = \boldsymbol{\xi}_0 = \mathbf{t}\,,$$

so that

$$\left.\begin{aligned} \sigma_0 = \epsilon,\ \theta_0 = \frac{r}{f},\ \psi_0 = \frac{\mathbf{r}\,.\,\mathbf{t}}{f},\ {}_0W = W(\sigma_0, \theta_0, \psi_0) \\ f\nabla_0 W = {}_0W_\theta \frac{\mathbf{r}}{r} + {}_0W_\psi\, \mathbf{t} \end{aligned}\right\} . \tag{4.2.3}$$

For wave-fronts originating from other points of the source we may write to the first order in $\bar{\omega} = \omega/\epsilon$

$$\nabla W = \nabla_0 W + \bar{\omega} \left(\frac{\partial \nabla W}{\partial \bar{\omega}}\right)_{\bar{\omega}=0} . \tag{4.2.4}$$

The contribution of aberrations to l, independent of the source size, is $\delta_a^0 l = \mathbf{D} \,.\, \nabla_0 W$. When the interferometer is in the optimum adjustment (*cf.* Section 2), $\mathbf{D}(= \mathbf{A}\wedge\mathbf{r} + \mathbf{T})$ is in the z direction only and $\delta_a^0 l = 0$, for $\nabla_0 W$ has no z component. If $\mathbf{A}$ has a z component, then

$$\delta_a^0 l = A_z\, {}_0W_\psi \frac{[krt]}{f} . \tag{4.2.5}$$

The effects of the size of the source depend on the value of $(\partial\nabla W/\partial\bar{\omega})_{\bar{\omega}=0}$, which can be shown to be given by

$$\begin{aligned} f\left(\frac{\partial \nabla W}{\partial \bar{\omega}}\right)_{\bar{\omega}=0} &= (\mathbf{t}\,.\,\mathbf{s})[\epsilon f \nabla_0 W_\sigma - (\epsilon z_s + \mathbf{r}\,.\,\mathbf{t})\nabla_0 W_\psi] \\ &\quad + (\mathbf{r}\,.\,\mathbf{s}) \left[\nabla_0 W_\psi - \frac{\epsilon z_s}{r} \nabla_0 W_\theta\right] \\ &\quad + \left[\frac{\epsilon z_s}{r} \left(\frac{\mathbf{r}\,.\,\mathbf{s}}{r}\right) {}_0W_\theta\right] \frac{\mathbf{r}}{r} - (\mathbf{t}\,.\,\mathbf{s})\, {}_0W_\psi\, \mathbf{t} \\ &\quad + \left({}_0W_\psi - \frac{\epsilon z_s}{r}\, {}_0W_\theta\right) \mathbf{s} \end{aligned} \tag{4.2.6}$$

(where terms of the order of r/z_s times those displayed have been neglected). This also has no z component and contributes nothing to l if $\mathbf{D}$ is in the z direction only. When $A_z \neq 0$, its contribution is

$$\delta_a' l = \frac{\omega}{\epsilon} A_z \left\{ \left[(\mathbf{t} \cdot \mathbf{s}) \left(\epsilon \, {}_0W_{\sigma\psi} - \frac{\epsilon z_s + \mathbf{r} \cdot \mathbf{t}}{f} \, {}_0W_{\psi\psi} \right) + \frac{\mathbf{r} \cdot \mathbf{s}}{f} \left({}_0W_{\psi\psi} - \frac{\epsilon z_s}{r} \, {}_0W_{\psi\theta} \right) - (\mathbf{t} \cdot \mathbf{s}) {}_0W_{\psi} \right] \frac{[krt]}{f} + \left({}_0W_{\psi} - \frac{\epsilon z_s}{r} \, {}_0W_{\theta} \right) \frac{[krs]}{f} \right\} . \tag{4.2.7}$$

Lens.

When the collimating element is a lens, there is no reason, apart from the possibility of a slight misalignment, why the centre of the source should not coincide with the focal point of the lens. Then, $\epsilon = 0$ and the only terms in (4.1.2) are the two axial aberrations, the spherical and the longitudinal chromatic aberrations, i.e., W depends on θ only. Since $W_\psi = 0$, $\delta_a^0 l = 0$ whether or not $A_z = 0$, which means that the fringe pattern is not affected by the quality of the collimating lens (this, of course, excludes local defects of the lens which would lead to fringe deformation in the presence of A_z).

When $\epsilon = 0$, the expansion on which equation (4.2.7) is based is not valid, but the corresponding result for $\epsilon = 0$ may be obtained formally from (4.2.7) by retaining only the θ derivative of W. Then,

$$\delta_a' l = - \omega A_z \frac{z_s}{r} \, {}_0W_\theta \frac{[krs]}{f} , \tag{4.2.8}$$

where

$$\frac{{}_0W_\theta}{f} = 4C_{040} \left(\frac{r}{f} \right)^3 + 2C_{lc} \frac{r}{f} .$$

For an equiconvex collimating lens of refractive index μ (at the mean wavelength of the source), the spherical aberration coefficient is (*see* for example, Hopkins, Ch. IX)

$$C_{040} = \frac{1}{32} \left[\left(3 + \frac{2}{\mu} \right) + \frac{\mu^2}{(\mu - 1)^2} \right]$$
$$\simeq 0 \cdot 4, \text{ taking } \mu = 1 \cdot 5 ,$$

whilst the longitudinal chromatic aberration coefficient is

$$C_{lc} = \frac{1}{2} \frac{\delta \mu}{\mu - 1} \left(= \frac{1}{2} \frac{1}{V} \right) ,$$

where $\delta\mu$ is the change of μ over the range of wavelengths of the source. For many glasses, $V \simeq 50$ with a 'white' light source, so that $C_{lc} \simeq 0 \cdot 01$, and ${}_0W_\theta(r)/f \simeq 1 \cdot 7 \, (r/f)^3 + 0 \cdot 02 \, (r/f)$. If $2b$ is the aperture of the collimating lens, the maximum value of $\delta_a' l$ is, clearly

$$(\delta_a' l)_m = \omega A_z z_s \frac{{}_0W_\theta(b)}{f} .$$

As was shown in Section 2, an A_z rotation produces a linear dependence of the path difference on the source size even in an ideal interferometer with a perfect collimating lens; the maximum value of the corresponding additional path difference l' is $(l')_m = \omega A_z b$.

Therefore

$$\frac{(\delta_a' l)_m}{(l')_m} = \frac{z_s}{f}\frac{f}{b}\frac{{}_0W_\theta(b)}{f}$$

$$\simeq \frac{z_s}{f}\left[1\cdot 7\left(\frac{b}{f}\right)^2 + 0\cdot 02\right].$$

Now, $z_s/f = 0(1)$, so that even for an $f/3$ lens ($b/f = 1/6$) this ratio is less than about $0\cdot 07$. This shows that the aberrations of a simple collimating lens with aperture as large as $f/3$ are of little significance.

Mirror.

When the collimating element is a spherical mirror, ϵ, the angle between the axis of the mirror and the optical axis of the interferometer cannot be less than $\frac{1}{2}a$, if a is the angular aperture of the mirror; in practice it is likely that $\epsilon \simeq a$. Then, as is clear from equations (4.2.5), (4.2.3) and (4.1.2) (with the chromatic aberration terms absent), there is a distortion of the fringe pattern which depends on the coma, astigmatism and distortion coefficients of the mirror. In this paper we shall only estimate the order of magnitude of the effects of mirror aberrations. A fuller discussion, including details of derivation of the equations quoted here without proof will be given elsewhere.

In terms of a and ϵ, ${}_0W_\psi = 0[(a + \epsilon)^3 f]$, so that by equation (4.2.5) the fringe number error introduced by the aberrations is

$$\delta N = \frac{\delta_a^0 l}{\lambda} = 0\left[\frac{A_z}{\lambda}(a + \epsilon)^3\, af\right].$$

In a 'rectangular' interferometer ($\beta_0 = 45$ deg) with N fringes parallel to the x axis in the field of view

$$A_z = A_x = \frac{N\lambda}{fa}.$$

Therefore

$$\frac{\delta N}{N} = 0(a + \epsilon)^3.$$

Taking $\epsilon = a$, the fringe number error will not exceed a few per cent if $a < 1/6$, say, *i.e.*, if the aperture of the mirror is not more than $f/6$. An $f/3$ collimating mirror would produce fringe number errors of the order of 10 per cent; a large offset angle ϵ (≈ 20 deg) with a small aperture mirror would have similar effects.

Turning now to the effects of source size, we note that all the terms within the curly brackets of equation (4.2.7) are $0[(a + \epsilon)^3 f]$. Therefore, taking $\epsilon = a$,

$$\delta_a' l = 0(\omega z_s A_z a^2),$$

whilst for a collimating lens with $\epsilon = 0$, the corresponding term is $0(\omega z_s A_z a^3)$. Thus an $f/6$ collimating mirror would produce roughly similar loss of fringe contrast as an $f/3$ simple equiconvex lens.

4.3. *Collecting Lens (or Mirror) and Camera Lens.*

Perfect lenses.

Referring to Fig. 10, consider the system formed by the collecting lens (or mirror) L_1, whose axis makes a small angle ϵ_1 with the z axis, and the camera lens L_2, whose axis coincides with the z axis. The image plane of this system of two lenses is the screen Sc. The plane I is the conjugate plane of Sc in L_1 and L_2, *i.e.*, the object plane of L_1 and L_2. If L_1 and L_2 form a perfect optical system, the plane I is mapped onto Sc without additional optical path differences being introduced in the process. The interference pattern observed on the screen is then, apart from a magnification factor, the (virtual) interference pattern in the plane I.

If $\mathbf{l}_1 = \epsilon_1 \mathbf{t}_1 + \mathbf{k} + 0\,(\epsilon_1^2)$ is the unit normal to the plane of L_1, $\mathbf{l}_I$, the unit normal to the plane of interference (plane I) can be shown to be

$$\mathbf{l}_I = \epsilon_I \mathbf{t}_1 + \mathbf{k}\,,$$

where

$$\epsilon_I = \left(1 - \frac{1}{m_1}\right) \epsilon_1$$

and $m_1 = - f_1/u_1$ is the (transverse) magnification factor of L_1 (u and v are, respectively, the distances of the centres of the object and image planes from the corresponding focal points of the lens (*cf.* Fig. 10), so that the perfect thin lens equation is $uv = f^2$). Thus, an inclination of the axis of the perfect collecting lens relative to the axis of the interferometer is equivalent in effect to a tilting of the virtual plane at which interference is observed.

Let $\boldsymbol{\pi}$ be the position vector of a point on the screen relative to the centre of the screen Cs (zs, 0, 0). Then, the ideal path difference is

$$l_i = \left\{\frac{1}{m} \mathbf{A} \wedge [\boldsymbol{\pi} - (\boldsymbol{\pi} \,.\, \mathbf{t}_1)\epsilon_I] + \mathbf{T} + \mathbf{T}_I\right\} .\, (\mathbf{k} \cos \omega + \mathbf{s} \sin \omega)\,, \tag{4.3.1}$$

where $m = m_1\, m_2 = f_1/u_1 .\, f_2/u_2$ is the combined magnification factor of L_1 and L_2 and

$$\mathbf{T}_I = \Sigma\,(z_I - z_i)\, \boldsymbol{\alpha}_i \wedge \mathbf{k}\,.$$

Imperfect collecting lens (or mirror).

To investigate the effects of aberrations on the mapping of the virtual interference plane onto the screen, it is convenient to regard, in accordance with Huygens principle, any point P_I on I as the origin of a spherical wave-front which in the absence of aberrations would converge towards P_s, the image point of P_I on Sc. We then calculate the difference, due to aberrations, between the optical paths along a ray leaving P_I in the direction of $\mathbf{n}$ (the ray P_IQ in Fig. 10) and the coherent ray disturbed by the mirror adjustments, leaving P_I in the direction of $\mathbf{n}'$ ($= \mathbf{n} + \mathbf{A} \wedge \mathbf{n}$) (the ray P_IQ' in Fig. 10). The exit stop of L_1 is formed by the image of the source, whose centre lies at S_0' on the z axis and not on the axis of L_1. Therefore, it is necessary to construct the auxiliary principal ray $P_IQ_PF_1'$ which leaves P_I in the direction of $\mathbf{l}_1$. It can be shown that the change of path difference due to aberrations of L_1 is, to first order in ϵ_1,

$$\delta_a l\ (= \delta_a^0 l + \delta_a' l) = \delta\boldsymbol{\rho} \,.\, \nabla W\,, \tag{4.3.2}$$

where W is the aberration function of L_1 expressed in the form of equation (4.1.2)

$$f_1\nabla = \frac{\boldsymbol{\rho}}{\rho}\frac{\partial}{\partial\theta} + \frac{\boldsymbol{\eta}}{\eta}\frac{\partial}{\partial\psi}$$

$$\boldsymbol{\rho} = \overrightarrow{Q_PQ},\ \delta\boldsymbol{\rho} = \overrightarrow{QQ'}\ \text{(Fig. 10)}$$

$$\boldsymbol{\eta} = \overrightarrow{O_{L1}Q_P};\ \eta = O_{L1}Q_P = O_IP_I[1 + 0(\epsilon_1^2)]$$

$$\theta = \rho/f_1$$

$$\psi = \frac{\boldsymbol{\rho}\cdot\boldsymbol{\eta}}{f_1\eta}$$

[Note than $\rho = 0(\epsilon_1 d_1)$, $\delta\rho = 0(Ad_1)$ (*cf.* Fig. 10].

In what follows we shall be concerned with only the x and y components of $\mathbf{A}$ and it will be found convenient to define A_q and $\mathbf{q}$ such that $\mathbf{A} = A_q\mathbf{q} + A_z\mathbf{k}$. Calculating the various quantities in equation (4.3.2) to first order in ω/ϵ_1 yields*

$$\delta_a^0 l = \frac{d_1}{f_1}A_qV \tag{4.3.3}$$

$$\delta_a' l = \frac{\omega}{\epsilon_1}A_q\left(\frac{d_1}{f_1}\right)^2\left\{\epsilon_1\left[\frac{\mathbf{s}\cdot\boldsymbol{\eta}}{\eta}V_\psi - (\mathbf{s}\cdot\mathbf{t}_1)V_\theta\right]\right.$$

$$\left. + \left[[qks] - (\mathbf{t}_1\cdot\mathbf{s})\,[qkt_1]\right]\frac{f_1\,{}_0W_\theta}{d_1}\right\}, \tag{4.3.4}$$

where

$$V = \frac{[qk\eta]}{\eta}\,{}_0W_\psi - [qkt_1]\,{}_0W_\theta\,, \tag{4.3.5}$$

$${}_0W = W(\sigma, \theta_0, \psi_0),\ \text{(the form of } W \text{ is still given by (4.1.2))}$$

$$\sigma = \eta/f_1$$

$$\theta_0 = \rho_0/f_1 = \frac{\epsilon_1 d_1}{f_1}$$

$$\psi_0 = -\frac{\epsilon_1 d_1\eta}{f_1}(\mathbf{r}\cdot\mathbf{t}_1 + \epsilon_1 d_1)$$

and $\boldsymbol{\eta}$ may be approximated by $\boldsymbol{\eta} = \mathbf{r} + \epsilon_1 d_1\mathbf{t}_1$, where $\mathbf{r}$ is the two-dimensional position vector of the point P_I shown in Fig. 10.

Mirror.

In the case of a collecting mirror, of angular aperture a_1, V involves all the monochromatic aberrations and is $0[(a_1 + \epsilon_1)^3 f_1]$, since $d_1/f_1 = 0(1)$. Therefore, the relative fringe number error is

$$\frac{\delta N}{N} = 0\left[\frac{(a_1 + \epsilon_1)^3}{a_1}\right]$$

* In deriving (4.3.4) two terms (proportional to A_z) whose order is η/f_1 times those displayed have been neglected.

(Note that for the collimating mirror this was only $0[(a + \epsilon)^3]$. Thus, the aperture of the collecting mirror should not exceed $f_1/10$ and ϵ_1 should be kept as small as possible. With $\epsilon_1 = a_1$, the terms in the curly bracket of equation (4.3.4) are $0(f_1 a_1^3)$, so that as for the collimating mirror $\delta'_a l = 0(\omega f_1 A a_1^2)$ and this will not have any significant effects on fringe contrast if the aperture of the collecting mirror is small enough to avoid appreciable fringe distortion.

Lens.

When the collecting element is a lens, we take $\epsilon_1 = 0$. Then, ρ and $\delta\rho$ are both $0(Ad_1)$, so that θ and ψ are both $0(A)$. Since we have neglected terms $0(A)^2$ throughout the analysis, in equations (4.3.3) and (4.3.4) we need only retain the derivatives of ${}_0W$ which are of zeroth order in A (*i.e.*, zeroth order in θ and ψ). As a result, these equations become

$$\delta^0_a l = \frac{d_1}{f_1} A_q \frac{[qkr]}{r} {}_0W_\psi \tag{4.3.6}$$

$$\delta'_a l = \omega A_q \left(\frac{d_1}{f_1}\right)^2 \left\{ [qks] \, {}_0W_{\theta\theta} + \frac{\mathbf{r} \cdot \mathbf{s}}{r} \frac{[qkr]}{r} \, {}_0W_{\psi\psi} \right\}, \tag{4.3.7}$$

where ${}_0W_\psi$ involves only distortion and transverse chromatism,

${}_0W_{\theta\theta}$ involves only curvature and longitudinal chromatism,

${}_0W_{\psi\psi}$ involves only astigmatism,

the two chromatic aberrations being of little importance except, perhaps, with white light fringes.

Assuming that the collecting lens has not been specially designed, the orders of magnitude of its $\delta^0_a l$ and $\delta'_a l$ will be the same as for a spherical mirror of equal aperture, and the same gloss applies.

Imperfect camera lens.

The equations expressing the effects of aberrations of the camera lens L_2 may be obtained directly from those for the collecting lens if d_1 is replaced by $-d_2/m_1$ and f_1 is replaced by f_2, where $d_2 = u_2 + f_2$ (*see* Fig. 10) and $m_1 = -f_1/u_1$ is the (transverse) magnification factor of L_1.

It is usually necessary and convenient to arrange the lenses so that L_2 is near the image of the light source formed by L_1 in its focal plane. This has two advantages:

(i) L_2 need only have a small aperture ($f_2/40$ is not uncommon),

(ii) When the 'undisturbed' image of the centre of the source coincides with the centre of L_2, the principal ray of L_2 (the ray $P_I Q_0 P'_I$ in Fig. 10) passes through the centre of L_2; then, the distortion and the transverse chromatic aberration of the lens are zero.

NOTATION

Latin symbols

a	Angular aperture of lens or mirror
$\left.\begin{matrix}\mathbf{A} \\ = (A_x, A_y, A_z) \\ = (A_q, A_z)\end{matrix}\right\}$	Vector specifying resultant rotation of wave-front
C	Aberration coefficients [*see* equation (4.1.2)]
$\mathbf{D}$	Vector displacement between corresponding points on coherent wave-fronts [equation (2.1)]
$d\,(= u + f)$	Object distance for a lens or mirror (Fig. 10)
f	Focal length of lens or mirror
$F\,(u, \mu)$	$\sqrt{\{(\mu^2 - 1) + u^2\}} - u$
F_u, F_μ, etc.	Partial derivatives of F
F, F'	Disturbed and undisturbed wave-fronts
$G\,(u, \mu)$	Arbitrary function of u and μ
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors in x, y, z directions, respectively
l	Optical path difference
l_i	Optical path difference for ideal interferometer
$\mathbf{l} = \epsilon\mathbf{t} + \mathbf{k}$	Unit normal at centre of lens or mirror (Figs. 2 and 10)
$\mathbf{l}_I = \epsilon_I\mathbf{t}_1 + \mathbf{k}$	Unit normal at centre of virtual interference plane (Fig. 10)
$\mathbf{m}_i$	Unit normal to mirror M_i (Fig. 2)
$m = -f/u$	Transverse magnification factor
$\left.\begin{matrix}\mathbf{n} \\ = \mathbf{k}\,cos\,\omega + \mathbf{s}\,sin\,\omega\end{matrix}\right\}$	Unit normal at centre of undisturbed wave-front
N	Fringe number
$\mathbf{p}$	Position vector of a point on wave-front or screen
$\mathbf{q}$	unit vector defined in Section 4.3
$\mathbf{r}$	$\mathbf{p} - z\mathbf{k}$
$\mathbf{s}$	Unit vector, defined in Section 2 and Fig. 2, specifying position of source point
t	Thickness of glass plate
$\mathbf{t}$	Unit vector in tangential plane of lens and normal to lens axis (Section 4)
$\mathbf{T}, \mathbf{T}_s, \mathbf{T}_I$	Translational components of $\mathbf{D}$ (Sections 2 and 4, equation (2.1))
u	$= \mathbf{m} \cdot \mathbf{n} = \cos\beta$ (Section 3)
u	Distance between centre of object plane and corresponding focal point (Section 4, Fig. 10)
v	Distance between centre of image plane and corresponding focal point (Section 4, Fig. 10)

V	Quantity related to W and defined in equation (4.3.5)
$\mathbf{w}$	Unit vector along wedge apex (Section 3.2, Fig. 7)
W	Aberration function (Section 4)
W_θ, W_σ, W_ψ	Partial derivatives of W
x, y, z	Co-ordinates in cartesian system of axes defined in Section 2
z_i, z_s, z_I	Co-ordinates of centres of mirrors, screen and virtual interference plane, respectively

Greek symbols

$\boldsymbol{\alpha}_i$	Vector specifying rotation of wave-front due to rotation of mirror M_1 (Section 2)
$\boldsymbol{\alpha}_w = \phi \dfrac{F}{u} \mathbf{w} \, (= \alpha_w \mathbf{w})$	Rotation of wave-front produced by refraction in a wedge-shaped plate
α_w^0, α_w', α_w''	Quantities related to α_w and defined by equation (3.2.4)
β	Angle of incidence of undisturbed wave-front on mirror or plate (*i.e.*, angle between $\mathbf{m}$ and $\mathbf{n}$)
δ	Denotes increment of quantity it precedes
ϵ	Angle between lens axis and z axis
$\boldsymbol{\eta}$	Object height vector defined in Section 4 and Figs. 8 and 10
θ	ρ/f
λ	Wavelength of light
μ	Refractive index
$\boldsymbol{\xi}$	Unit vector defined in Fig. 8
$\boldsymbol{\pi}$	Position vector of a point on the screen relative to the centre of the screen (Section 4.3 and Fig. 10)
$\boldsymbol{\rho}$	Vector defined in Section 4 and Figs. 8 and 10
σ	η/f
$\boldsymbol{\tau}_i$	Vector specifying translation of wave-front due to translation of mirror M_i (Section 2)
ϕ	Wedge angle (Section 3)
ϕ	Angle defined in Sections 4.1 and 4.2 and in Fig. 8
ψ	$= \theta \cos \phi = \dfrac{\boldsymbol{\rho} \cdot \boldsymbol{\xi}}{f}$ in Sections 4.1 and 4.2 $= \dfrac{\boldsymbol{\rho}}{f_1} \cdot \dfrac{\boldsymbol{\eta}}{\eta}$ in Section 4.3
ω	Angle between central ray of wave-front and z axis, *i.e.*, angle between $\mathbf{n}$ and $\mathbf{k}$
$\bar{\omega}$	ω/ϵ

Suffices

0	Denotes quantities related to wave-fronts from the centre of the light source (point S_0 in Fig. 2)
$1, 2$	Denote, respectively, quantities associated with lenses L_1 and L_2
s, I	Refer to the screen and the virtual interference plane, respectively
$i = 1, 2, 3, 4$	Identifies quantities associated with mirrors M_i, also used as a summation suffix
Prime ($'$) is used	(i) to distinguish between disturbed and undisturbed wave-fronts
	(ii) to denote, in Section 4, those contributions of aberrations to the optical path difference which are proportional to ω.

Vector notation

$.$	Scalar multiplication
$\wedge$	Vector multiplication
$[abc] \equiv \mathbf{a}\wedge\mathbf{b}\,.\,\mathbf{c}$	Triple scalar product of **a**, **b** and **c**

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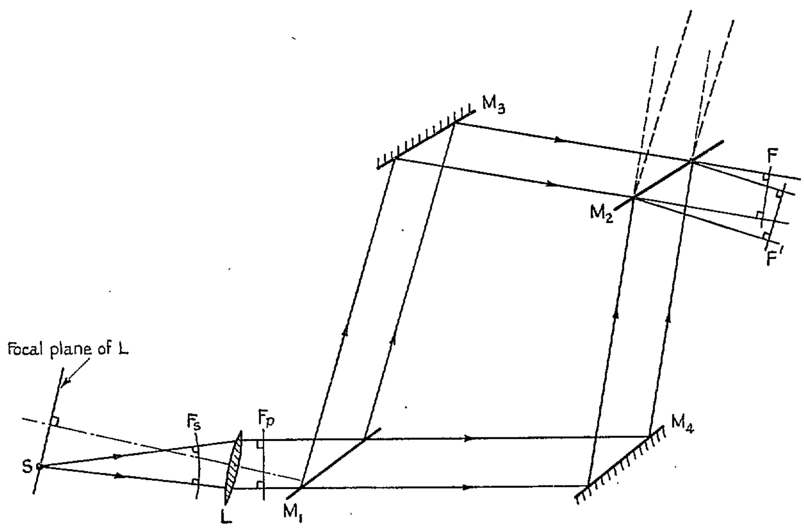


FIG. 1. Lay-out of an ideal two-beam interferometer.

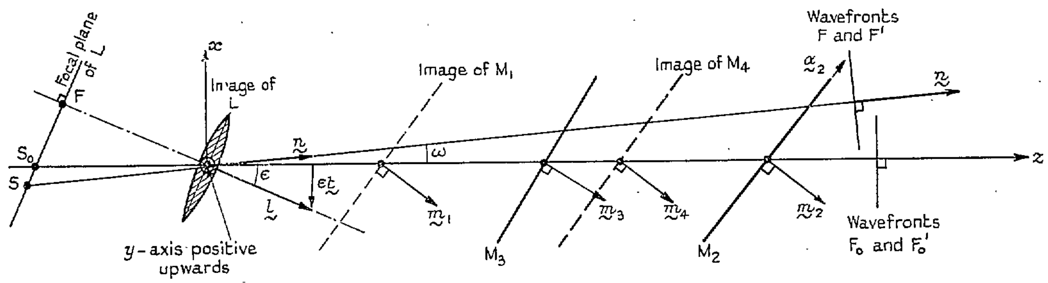
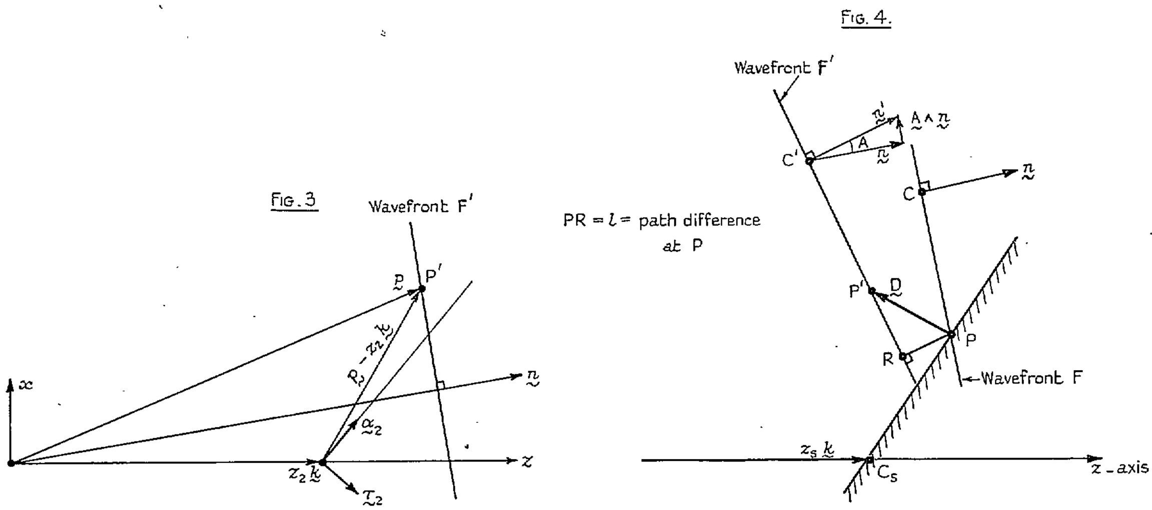


FIG. 2. Linear representation of the ideal interferometer.



FIGS. 3 and 4. Illustrating notation of Section 2.

Note.—Symbols with a wavy line underneath correspond to bold type in the text.

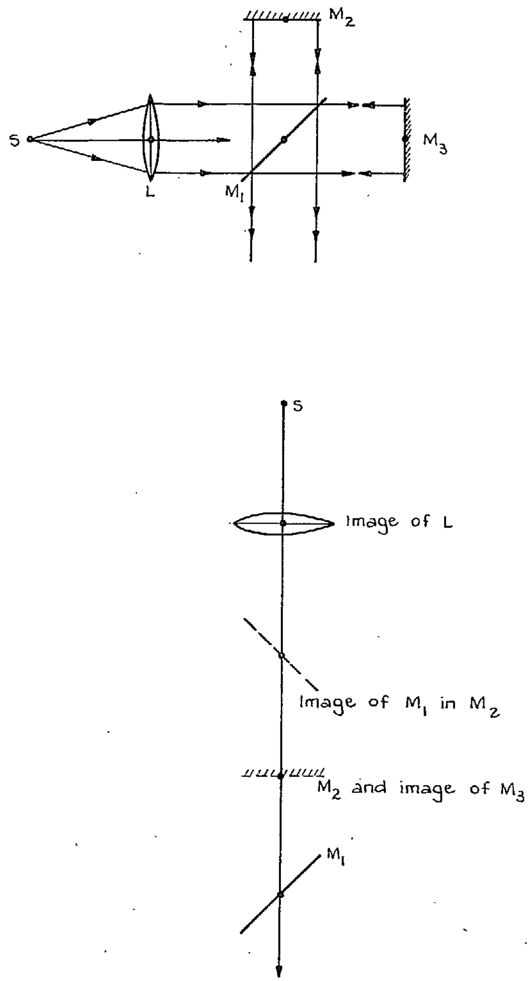


FIG. 5. Ideal Twyman-Green interferometer and its linear representation.

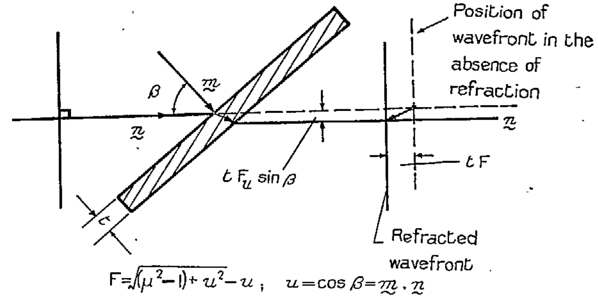


FIG. 6. Refraction of a plane wave-front.

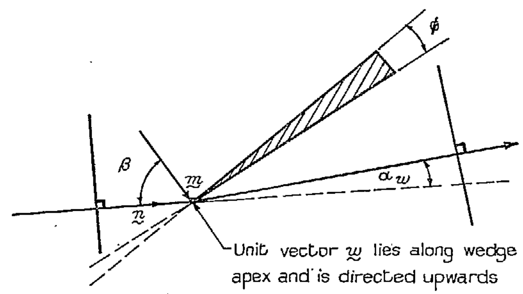
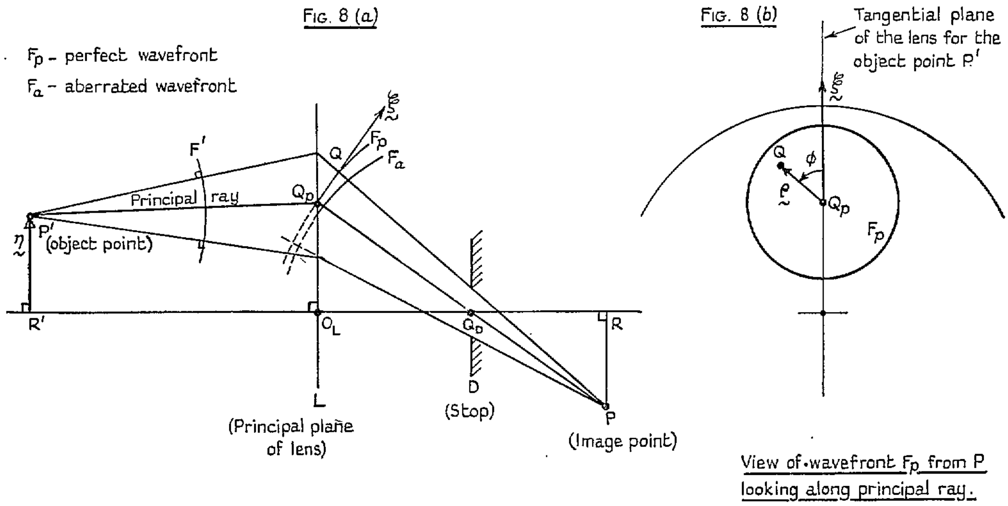


FIG. 7. Thin prism representing effect of refraction in a wedge-shaped plate.



N.B. In Fig. 8 (a) the plane of the paper contains the axis of the lens and the principal ray from P' and is the "tangential plane" of the lens for the object point P'.

FIGS. 8a and 8b. Wave-front aberration produced by a thin lens.

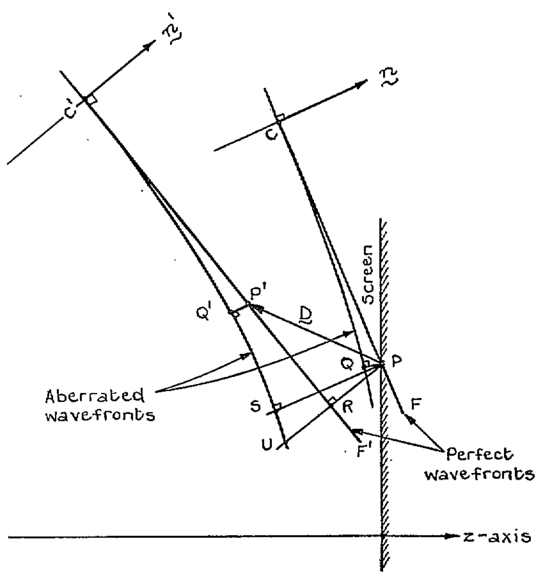
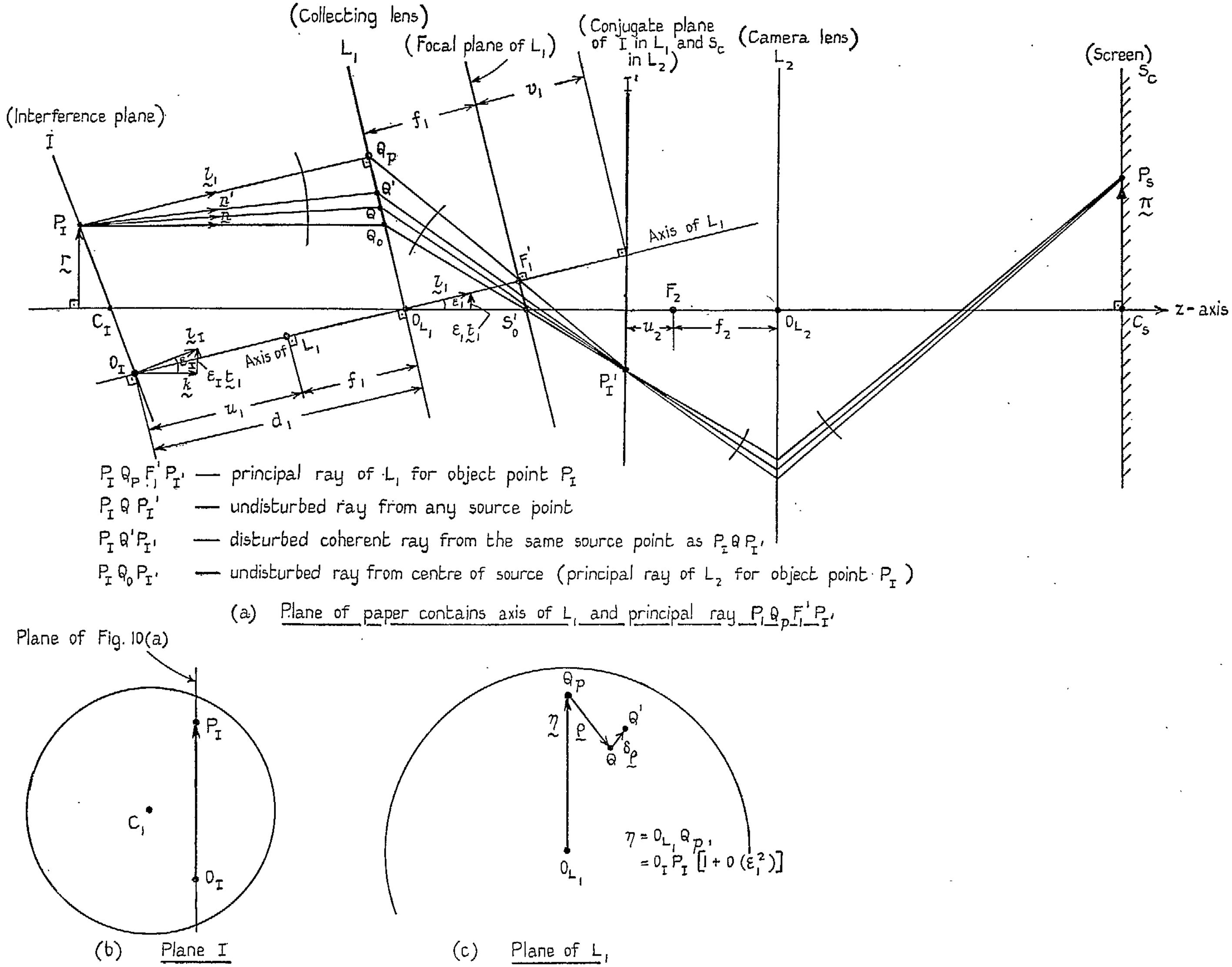


FIG. 9. Effect of wave-front aberration on path difference.

(78998) Wt. 54/8210 K.5 3/61 Hw.



FIGS. 10a to 10c. Collecting and camera lens system.

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