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The Effect of Structural Damping on Binary Flutter

By

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Summary.—The paper describes an investigation of the effect of structural damping in the torsion mode on wing flutter with the object of finding circumstances in which damping reduces the flutter speed. The drop in flutter speed can be considerable (25 per cent) and extend to very large values of structural damping. The effect is most apparent when the relative density (wing to air) is high, and when the wing bending mode involves a relatively large aerodynamic stiffness.

The rate of decrease of flutter speed with damping in small, and for the amounts of damping normally encountered in practice the effect is unlikely to be important. Possible practical cases are, however, part-full under-wing fuel tanks, which can supply high structural damping, and large tailplane amplitudes in the wing torsion mode, which can supply considerable aerodynamic damping; in both cases the effect could be appreciably adverse.

1. *Introduction.*—It is well known that if a small amount of structural damping is added to one degree of freedom in a binary-flutter problem the critical flutter speed can be reduced thereby. If control-surface flutter is to be prevented by hydraulic damping rather than mass-balance, for example, the curve of flutter speed against damping often falls for a little way. The examples considered later are rather more academic than this, since they are concerned with flexure-torsion wing flutter where the deliberate variation of structural damping is not normally possible, but they are noteworthy because a very large value of the structural damping is necessary before the flutter speed starts to increase. Certain cases where the effect might be important are discussed in the conclusions to the paper; they include an under-wing fuel tank partly full, and the effect of tail aerodynamic damping on a wing mode.

The first example was discovered during the routine solution by means of the Royal Aircraft Establishment Flutter Simulator of a wing flutter problem on a particular aircraft. When operating the simulator it is often desirable to reduce the amplitude of the motion, and a common way of doing this is to introduce a large amount of structural damping into the direct term of one of the essential degrees of freedom. In the present case this was found to increase the oscillations, and subsequent investigation showed that the same effect occurred even in a binary calculation. In a larger calculation, such as the quinary from which this binary was taken, structural damping in one degree of freedom could have an adverse effect merely by suppressing a stabilizing degree of freedom; this explanation cannot hold for a binary in which each degree of freedom is essential to the flutter. The present paper describes this binary calculation, which was carried out on a desk machine, and some later investigations made to decide how general the effect is likely to be as well as to consider its characteristics in more detail. A criterion for the fall of flutter speed with damping is discussed and an example shows that the effect can occur in the bending mode as well as the torsion mode.

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2. *The Binary Calculations for Wing Flutter.*—The calculations discussed in this and the next Section are primarily of academic interest. The results have had no repercussions on the aircraft to which they relate, because a serious drop in flutter speed with structural damping only occurs at high altitude (the specific calculations quoted relate to 55,000 ft) where the flutter speed is in any case far outside the capabilities of the aircraft. This need not always be so, however, as an increase in altitude differs in only a minor way (due to the aerodynamic inertias) from an overall increase in the wing density and in this case the wing density was not particularly high. It is shown in a later section (Section 4.2) that the flutter speed of a *Meteor* wing carrying a heavy mass falls rapidly with structural damping at quite moderate heights, so that the problem itself may have practical importance in other examples.

In this case it is only in the torsional mode that an increase in structural damping causes a reduction in the flutter speed, so the direct torsional damping of the structure is the parameter which has been varied in the calculations. The more general implications are discussed in Section 6, but it is worth noting that the torsional mode, which involves shear distortion, is likely to contain more structural damping than the bending mode. Although the phenomenon was first noticed, in this particular case, on the R.A.E. Flutter Simulator during a larger calculation, the binaries of this Section were solved on a desk machine for greater precision.

The wing concerned possesses moderate sweepback and carries no large concentrated masses. The modes are antisymmetric resonance modes and possess no unusual features ; the nodal line in the torsion mode runs along the span at roughly the half-chord location.

2.1. *The Flutter Coefficients.*—The equations of motion at the flutter speed, appropriate to the generalised co-ordinates q_1 for bending and q_2 for torsion, are

$$\begin{bmatrix} 4400\lambda^2 + 210\lambda + 493 + 941y\,, & 17\lambda^2 - 21\lambda + 389 \\ 84\lambda^2 - 26\lambda - 826\,, & 718\lambda^2 + 86\lambda - 432 + 1100y \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0\,. \qquad .. \qquad (1)$$

Here $\lambda^2 = -\nu^2$

ν is the frequency parameter $\omega c_r/V$ based on a reference chord c_r

ω is the flutter frequency

V is the flutter speed

$y = (V_0/V)^2 = (1/v)^2$

V_0 is a reference speed.

This is the standard way of writing the flutter equations in Great Britain, and assumes that the aerodynamic coefficients are independent of frequency parameter. This assumption does not lead to serious error as a rule, particularly when the aspect ratio is not large[1]. Equation (1) is written in the form suitable for solution on a desk machine, for which purpose the determinant of the second-order matrix is equated to zero, but the coefficients have been derived from the values prepared for the simulator. To change the coefficients from one form of the equations to another[2] (*i.e.*, from being suitable to the simulator to being suitable for desk solution, and *vice versa*) is a simple matter and in the present case only involved a correction for the time constant of $\frac{1}{2}$ used in degree of freedom (1). The solution of equation (1) is usually carried out for y and λ only ; y gives the flutter speed (it is inversely proportional to the square of the flutter speed) and λ gives the frequency parameter, from which, the flutter speed being known, the flutter frequency can be found.

It may be noted that each degree of freedom contains positive aerodynamic damping (+ 210 and + 86 units respectively) and that the cross-dampings are small. The direct aerodynamic stiffness for bending (+ 493 units) is positive and of reasonable size, as befits the bending mode of a swept-back wing, while that for torsion (— 432 units) is typically negative. The direct inertias (aerodynamic and structural) are large and positive (+ 4400 and + 718 units) and so are the direct structural stiffnesses (+ 941 and + 1100 units), but the cross-inertias although small are not zero as they should be for true normal modes. The values quoted (+ 17 and + 84 units) are not important in the flutter and probably owe their existence (in part, at least) to the difference between the aerodynamic inertias at 55,000 ft and sea level.

The aerodynamic cross stiffnesses are typical for this type of flutter calculation, being large and positive (+ 389 units) for the bending force due to wing torsion, and large and negative (− 826 units) for the torsional moment due to wing bending. It is these terms which promote the flutter, and the reason for this is mathematically easy to see. Suppose the coupling coefficients were all zero, then the determinant of the direct terms only can be expanded as

$$p_0\lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4 = 0\,, \quad \ldots \quad (2)$$

where the coefficients p_r are independent of λ. Clearly equation (2) must represent stable motion since each equation of (1) is positively damped and there is no coupling. It follows from the well known stability criteria (*see*, for example, Ref. 3 or Ref. 4) that all the coefficients p_r must be positive*, and also that the determinant

$$\begin{vmatrix} p_1 & p_3 & 0 \\ p_0 & p_2 & p_4 \\ 0 & p_1 & p_3 \end{vmatrix} \equiv T_3 \equiv p_1p_2p_3 - p_0p_3^2 - p_1^2p_4 \quad \ldots \quad (3)$$

must be positive. The vanishing of the test function T_3 as y is reduced from infinity gives the lowest flutter speed. So far the motion must be stable at all speeds above zero* but we may now introduce the effect of the aerodynamic cross-stiffness coefficients. These coefficients can only affect p_4, and since their product is negative they must increase the algebraic value of p_4. In the expanded form of T_3, therefore, only the last term is affected by the aerodynamic couplings now introduced, and this term is changed in the sense of reducing stability.

Equating (3) to zero gives an equation that is quadratic in y, say,

$$y^2 + 2l_1y + l_2 = 0\,, \quad \ldots \quad (4)$$

where l_2 must be positive and l_1 must either be positive or numerically less than $\sqrt{l_2}$, since the equation (with zero coupling coefficients) has no positive real roots. The effect of gradually increasing the aerodynamic cross-stiffnesses is to make l_2 tend first to zero and then become negative, so that a real flutter speed exists. If l_1 is positive there will be no upper critical speed, and if l_1 is negative the upper critical speed will become infinite as l_2 passes through zero. For the values of c_{12} and c_{21} (the aerodynamic cross-stiffnesses) given by equation (1), it may be noted that the net value of p_4 is positive for all positive values of y so that no real divergence speed exists.

To conclude this diversion we may note that if the other couplings are zero (or negligible), aerodynamic cross-stiffnesses of the form shown in equation (1) will promote flutter at a speed which reduces with the magnitude of the coupling, and as there will usually be no upper critical speed the flutter is likely to be violent.

The flutter equations (1) contain no structural damping terms. In the present calculations a damping term has been introduced into the second degree of freedom of the form

$$\text{Torsional damping coefficient} = d_{22}\lambda\sqrt{y}\,. \quad \ldots \quad (5)$$

In this form the damping is of velocity type. It is introduced in this way because many of the calculations described in this paper were carried out on the simulator where damping can only be introduced in this form. The coefficient d_{22} has the additional use that it can be related directly to the degree of critical damping in the torsional mode, viz.,

$$(d_{22})_{\text{crit}} = 2\sqrt{(a_{22}e_{22})}\,, \quad \ldots \quad (6)$$

* Below the divergence speed at which p_4 vanishes. The existence of a finite divergence speed does not affect the present argument.

where a_{22} is the torsional inertia coefficient (718 units) and e_{22} is the elastic torsional stiffness coefficient (1100 units). In this example $(d_{22})_{crit}$ has the value 1780 units. The advantage of relating the damping to critical damping is that it gives a measure of the rate of decay of the motion at zero air speed, as determined in a ground resonance test. Specifically

$$\frac{d_{22}}{(d_{22})_{crit}} = \frac{1}{2\pi} \text{ (logarithmic decrement)} \quad \ldots \quad (7)$$

If desired, the equivalent phase lag (hysteresis) damping can be calculated in any particular case, the relation depending on the frequency. Hysteresis damping is usually written in terms of the coefficient g (in this case g_{22}) by replacing the pure stiffness term e_{22} by the term $e_{22}(1 + ig_{22})$. For this to be equivalent to the damping of equation (5), we must have

$$d_{22}\lambda\sqrt{y} = ie_{22}g_{22}y \,, \quad \ldots \quad (8)$$

i.e.,

$$g_{22} = \frac{d_{22}}{e_{22}}\frac{\omega c_r}{V_0} \,. \quad \ldots \quad (9)$$

The aerodynamic damping, b_{22}, is of velocity type, so that at a given speed it is directly interchangeable with the structural damping d_{22}, but not, however, with g_{22}.

2.2. *Variation of Flutter Speed with Structural Damping*.—The change of flutter speed with fractions of critical damping in the torsion mode is given in Table 1, and shown also in Fig. 1.

TABLE 1

d_{22} (per cent crit)	v (per cent)	$x = -\lambda^2 = \nu^2$	$\dfrac{\omega c_r}{V_0}$	g_{22}
0	100	0·53	0·70	0
10	91	0·49	0·61	0·099
30	81	0·53	0·57	0·277
60	79	0·52	0·55	0·534
100	81	0·49	0·54	0·874
200	96	0·37	0·56	1·81

It will be seen that the flutter speed falls immediately on introducing the structural damping, in contrast to the case of the *Meteor* with tip mass discussed later, and reaches a minimum over 20 per cent lower than the original speed. Even more surprising is the scale of the structural damping involved.

It is indeed a remarkable fact that the wing flutter speed with 100 per cent critical damping in the torsion mode should be 19 per cent less than for the undamped wing. This means that the torsional oscillations of the wing change from being dead beat at zero speed to a state of growing violently with time at a v. of 100 per cent, by which time the wing without structural damping has only just returned to its initial condition of undamped sinusoidal motion.

To examine this fact in more detail, the roots of equation (1) were obtained at speeds below the flutter speed with d_{22} having the two values of zero and critical damping. The solutions are given in Figs. 2 and 3. It can be seen from Fig. 2 (which shows the results for no structural damping) that one root becomes progressively more heavily damped while the other shows a maximum damping at about 75 per cent of the flutter speed, after which the rate of decay falls steeply. This is quite a typical result in itself, but what is less usual is that the higher-frequency root, which starts as the pure torsion mode at zero speed, is the one which becomes progressively more damped, whereas the lower frequency root leads to flutter. With critical damping in the torsional mode there is no speed for which the torsional root (*i.e.*, the root which defines pure torsion at zero speed) is oscillatory ; the bending root again leads to flutter after reaching a maximum rate of decay at about 60 per cent. of the flutter speed. It is shown in a later example that when the torsional root leads to flutter with zero structural damping in its mode, there is a change over and the flexural mode leads to flutter after a relatively small amount of structural damping has been introduced.

3. *Analysis of the Forces in the Binary of Section* 2.—The results given in the previous Section were sufficiently surprising to the authors for an investigation of the balance of forces to be undertaken. It is clear that the amount of energy extracted from the air stream must increase considerably as the structural damping in the torsion mode is increased ; the mechanism is of some interest.

3.1. *The Amplitude Ratio of the Two Co-ordinates at Flutter.*—In Fig. 4 the generalised co-ordinate q_2 is represented by a unit vector at the critical flutter condition. The corresponding vectors q_1 are drawn on the same diagram for several solutions of the flutter equations with different values of the torsional structural damping. The phase difference between the two co-ordinates is initially very small (*i.e.*, when $d_{22} = 0$) but becomes steadily larger as d_{22} is increased. This change is to be expected, since only by making the phase angle more favourable to flutter can more energy be extracted from the air stream. It is to be noted that the phase angle is initially very small and it is probably only in such circumstances that the flutter speed actually falls when structural damping is added. The modulus of q_1 also increases with torsional damping so that the motion as seen by an observer would change from being primarily torsional in character to being primarily bending in character.

3.2. *The Balance of Forces at the Flutter Speed.*—The balance of forces is indicated by the vector diagrams of Figs. 5 and 6. Fig. 5 refers to the second Lagrangian equation, *i.e.*, the equation of work done in a small torsional displacement,

$$(84\lambda^2 - 26\lambda - 826)q_1 + (718\lambda^2 + 86\lambda - 432 + 1100y + d_{22}\lambda\sqrt{y})q_2 = 0 \quad \ldots \quad (10)$$

The diagrams are drawn from the equation in this non-dimensional form, but for an equivalent displacement of q_2 in each case. To do this the quantity $1100yq_2$, which is proportional to the strain energy in the torsional mode and therefore to the torsional displacement, is equated to unity in each case. It is because the flutter equations, *e.g.*, equation (10), are divided through by V^2 that the speed factor y appears in this relation and q_2 itself is not constant throughout Fig. 5 ; this method adopted in drawing the diagrams of Fig. 5 retains the non-dimensional form of the equation but cancels the misleading effect of the division by V^2.

The vector diagram for $d_{22} = 0$ is shown in Fig. 5a. The length OA represents the term in phase with q_2 from the second bracket of equation (10). Thus

$$OA = (718\lambda^2 - 432 + 1100y)/1100y = 0\cdot32\,, \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

since $\lambda^2 = -0\cdot53$ (*see* Table 1) and $y = 1\cdot085$. The vector AB represents the damping term from the same bracket acting at 90 deg phase ; in this case since $d_{22} = 0$ the term is entirely aerodynamic ($= 86\lambda/1100y$). The vector OB represents the forces acting in the co-ordinate q_2 due to the motion of q_2 : the vector BO must therefore, for balance, represent the forces acting in the co-ordinate q_2 due to the motion of q_1. This is made up of the component BC in phase with q_1 and the small component CO at 90 deg phase. The value of BC is given by

$$\vec{BC} = (84\lambda^2 - 826)\vec{q_1}\,, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where $\lambda^2 = -0\cdot53$ as before and $\vec{q_1}$ has the appropriate value given by Fig. 4.

Figs. 5b, 5c and 5d show the corresponding diagrams with progressively increasing values of d_{22}, but although in Fig. 5d the value of d_{22} much exceeds critical damping for the torsion mode the flutter speed is still rather less than in Fig. 5a. In all these examples the vector OA is considerably greater than in Fig. 5a, partly because of the larger values of y (lower flutter speeds), particularly in Figs. 5b and 5c, and partly because of the less negative values of λ^2 (*see* Table 1) particularly in Fig. 5d. The direct damping vector (AB) increases rapidly with d_{22} to a value of nearly fifty times that of Fig. 5a by Fig. 5d. In the meanwhile, however, the vector q_1 has increased in magnitude and phase so that BC again almost closes the diagram in each case. It may be noted that the vector OA is positive throughout Fig. 5 because the flutter frequency is lower than the frequency of mode two taken alone, at the appropriate airspeed.

Fig. 6 gives the corresponding diagram for the first equation

$$(4400\lambda^2 + 210\lambda + 493 + 941y)q_1 + (17\lambda^2 - 21\lambda + 389)q_2 = 0 \quad \ldots \quad (13)$$

in which the bending displacement, proportional to $941yq_1$, has been made unity in each case. In these diagrams the vector OA, which again represents the direct in-phase term, is negative, *e.g.*,

$$OA = (4400\lambda^2 + 493 + 941y)/941y = -0{\cdot}54 \quad \ldots \quad (14)$$

for the condition $d_{22} = 0$ in Fig. 6a. This is so because the frequency of mode (1) taken alone, at the appropriate air speed, is below the flutter frequency. It is necessary that this mode should approach more and more closely to a resonant condition as the magnitude of $\vec{q}_2$ shrinks with increasing d_{22}, since otherwise the distance BO could not be spanned by the coupling terms. That this does in fact happen is indicated clearly by Fig. 6 as the point A steadily approaches the origin.

4. *Survey of some Recent Binary Wing Flutter Calculations.*—4.1. *Analysis of the Results.*—Following the detailed investigation described above, a survey was carried out of a number of binary wing flutter calculations to decide how common is the effect of reducing flutter speed with increasing torsional damping. This survey indicated that the effect is always much more pronounced at high altitude than at low, and that the occasions on which it is important at low altitude are very few. This clearly suggests that it is the initial values of the damping coefficients which have most significance in deciding whether or not the effect will exist. In particular the damping in the bending mode might be expected to be the more important since for quite large values of the structural damping in torsion (*see* Fig. 1 for example) the flutter speed is still falling.

Because of the importance of the aerodynamic dampings, the results of the survey are given in Table 2 together with the appropriate damping coefficients.

TABLE 2

	Type of aircraft	Height (ft)	$\dfrac{b_{11}v}{2\sqrt{(a_{11}e_{11})}}$	$\dfrac{b_{22}v}{2\sqrt{(a_{22}e_{22})}}$	$\dfrac{\delta v}{v}$ per 10% d_{22}
1	Bomber type, moderate sweepback, aspect ratio 5 to 6.	40,000	0·0498	0·123	−0·09
2		sea level	0·938	0·102	+0·283
		40,000	0·418	0·0454	0·273
3		40,000	0·443	0·193	0·01
3a		sea level	0·651	0·492	0·258
		40,000	0·239	0·180	0·0266
4		sea level	0·789	0·292	0·072
		40,000	0·340	0·126	+0·029
5	Delta aircraft, fighter size	sea level	0·164	0·152	−0·0064
		40,000	0·0797	0·0741	−0·0687
6		sea level	0·0939	0·0994	+0·014
		40,000	0·0469	0·0497	−0·0536
7	Straight wing fighter	sea level	0·101	0·0966	+0·192
		40,000	0·0464	0·0442	0·0279
7a		sea level	0·0344	0·0411	+0·0115
		40,000	0·0167	0·0199	−0·0741
8	Unswept bomber; high aspect ratio.	sea level	0·305	0·191	+0·083
		40,000	0·149	0·0937	+0·096
9	Hypothetical		0·00981	0·0944	−0·058

In the first column each different number refers to a different aircraft ; 3a and 7a refer to the same aircraft as 3 and 7 respectively, but in 3a the calculation is antisymmetric instead of symmetric, and in 7a the calculation covers a different loading condition. The fourth and fifth columns give the aerodynamic damping in the bending and torsional modes respectively. The damping is expressed as a fraction of critical damping at the flutter speed, and the large variation in this quantity is of some interest in itself. In those cases where the dampings are large, such as 2 and 4, a considerable phase difference will already be present at flutter so that little increase can be expected with the addition of structural damping, and hence the flutter speed must increase. The change in flutter speed caused by increasing d_{22} from zero to a value of 10 per cent critical damping is given as a fraction of the initial flutter speed in column 6. It can be seen that the only examples of a drop in flutter speed (with the exception of 5, in which a very small drop occurs at sea level, and 9 in which the height datum is arbitrary) occur at 40,000 ft. This suggests that the effect is unlikely to have great practical significance, but there is one exception, not given in Table 2, and that is the *Meteor* with a heavy wing tip mass which is treated separately in the next Section. Aircraft 7 and 8 of Table 2 both carry tip masses, and aircraft 4 carries a heavy mass in the outer half of the wing.

The change in flutter speed (column 6 of Table 2) is plotted in Fig. 7 against the damping values given by columns 4 and 5. There is clearly a general tendency for the low damping values to give the negative and low positive values of $\delta v/v$ but the scatter is considerable. In view of this an attempt has been made to deduce a theoretical expression for the condition that the flutter speed will initially fall with the addition of structural damping d_{22}. To simplify the analysis, the assumption has been made that the only coupling terms are the aerodynamic cross-stiffnesses. This is justified if the coefficients are similar to those of equation (1), but there are many exceptions to this, in particular any calculations in which the modes are far from normal (*e.g.*, assumed arbitrary modes), and also those cases in which the first mode involves no aerodynamic incidence change*.

With this approximation the condition for $\partial V/\partial d_{22}$ to be negative at $d_{22} = 0$ is that t_1 should be negative, where

$$t_1 = y\{b_{11}b_{22}(a_{11}e_{22} + a_{22}e_{11}) + 2a_{11}b_{22}{}^2e_{11}\} + \\ + b_{11}b_{22}(a_{11}c_{22} + a_{22}c_{11}) + 2a_{11}b_{22}{}^2c_{11} + c_{12}c_{21}\left(a_{11}{}^2\frac{b_{22}}{b_{11}} - a_{22}{}^2\frac{b_{11}}{b_{22}}\right) . \quad .. \quad (15)$$

The standard notation is used, so that a_{rs} is a typical inertia coefficient, b_{rs} a typical aerodynamic damping coefficient, c_{rs} a typical aerodynamic stiffness coefficient, and e_{rs} a typical structural stiffness coefficient.

The analysis leading to equation (15) is given in the Appendix, as is the extended form of t_1 when b_{21} is not zero.

The term involving y in t_1 is positive as long as the direct aerodynamic damping coefficients are positive, which they always are in practice at subsonic speeds. The term independent of y in t_1 involves the aerodynamic stiffness coefficients (the c's) and can therefore be negative, but it will be noticed that the first part of this term is identical with the term in y except that the aerodynamic stiffnesses replace the corresponding elastic stiffnesses. It follows that if the critical flutter speed is below the divergence speeds both of degree of freedom 1 by itself and of degree of freedom 2 by itself, then that part of t_1 which excludes the coupling coefficients must be positive. Accordingly t_1 can only be negative if

$$\frac{a_{11}}{b_{11}} > \frac{a_{22}}{b_{22}}, \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (16)$$

since the product $c_{12}c_{21}$ must be negative for a real flutter speed to exist.

* For example, the flutter of an unswept wing with the flexural and inertial axes coincident at the half-chord ; in this case a_{21} and c_{21} will be zero (or nearly so) but b_{21} will be relatively large and negative. An extension covering this case is given in the Appendix.

From the form of (15) and (16) we may summarise the rules for an initial reduction in flutter speed with structural damping as :

(1) The product $c_{12}c_{21}$ should be negative and large compared with the products of the direct dampings, b_{11}^2, $b_{11}b_{22}$ and b_{22}^2*

(2) If (1) is true, then the flutter speed will drop with increasing d_{22} if

$$\frac{a_{11}}{b_{11}} \gg \frac{a_{22}}{b_{22}}$$

and the flutter speed will drop with increasing d_{11} if

$$\frac{a_{22}}{b_{22}} \gg \frac{a_{11}}{b_{11}} .$$

The second part of (2) follows by interchanging the suffixes in the expression for t_1. The first rule is quite rough and assumes that the coefficients have been scaled to the same order of magnitude in each degree of freedom, and that the flutter-speed parameter is of the order of unity. The effect is, however, fairly clear, as shown by Fig. 8 in which $\delta v/v$ is plotted against the quantity $(-\, c_{12}c_{21}/b_{11}b_{22})$; the points follow the dotted curve reasonably well, and what scatter there is arised mainly from the different values of $(a_{11}/b_{11} - a_{22}/b_{22})$.

4.2. *The Meteor with Tip Mass.*—This case is especially interesting because of the very large value of the quantity $(-\, c_{12}c_{21}/b_{11}b_{22})$, which is, in fact, 139 units at 15,000 ft, the altitude at which most of the calculations have been carried out. This means that the term in $c_{12}c_{21}$ in the expression (15) for t_1 plays a dominating part. The coefficients at 15,000 ft are given by†

$$\Delta \equiv \begin{vmatrix} 1032\lambda^2 + 158\lambda + 104 + 1110y\,, & -\,100\lambda - 3544 \\ 8\lambda + 1110\,, & 616\lambda^2 + 90\lambda - 80 + 2660y \end{vmatrix} . \qquad \ldots \quad (17)$$

The last term of the expression for t_1 is

$$t_2 = c_{12}c_{21}\left(a_{11}{}^2\frac{b_{22}}{b_{11}} - a_{22}{}^2\frac{b_{11}}{b_{22}}\right) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

$$= -\,3{\cdot}934(0{\cdot}6067 - 0{\cdot}6662) \times 10^{12} \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

using the coefficients of equation (17). The quantity t_2 is clearly positive so that the flutter speed increases with the addition of d_{22}. Unless the speed is changing very rapidly, however, an increase in d_{22} will have the same general effect as an increase in b_{22} at a slightly different rate. Now, because of the small difference in equation (19), an increase in b_{22} soon makes t_2 negative, and because t_1 in this example depends principally on the value of t_2 it follows that t_1 also soon becomes negative. It is therefore to be expected that although the flutter speed increases with d_{22} initially it will soon start to fall as the controlling term t_2 effectively becomes negative on allowing for d_{22}. The graph of V against d_{22} therefore shows an initial increase followed by a prolonged decrease, as shown in Fig. 9, with a minimum flutter speed about 75 per cent of the basic value.

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* In wing flutter, the equivalent air-speed at the critical flutter condition does not vary much with height, and this speed (rather than the true speed) is obtained if the height variation is effected by multiplying the b's by $\sqrt{\sigma}$ and the $\bar{a}$'s by σ ; where σ is the air density ratio and $\bar{a}$ the aerodynamic inertia (the change in the $\bar{a}$'s is often neglected). In applying rule (1), therefore, $b\sqrt{\sigma}$ should be used rather than b itself if the altitude is other than sea level. This has been done in the text and in plotting Figs. 7 and 8.

† These coefficients correspond to $-\,c_{12}c_{21}/b_{11}b_{22} = 277$, because they have been corrected for time constants. Strictly, however, this quantity should be compared on the basis of coefficients scaled to give unit critical speed ($y = 1$) and frequency ($\lambda = i$). If this is done the ratio drops to 69. For the purpose of applying the rough rule of Section 4.1 it is good enough to take the coefficients directly as scaled for the simulator since the mean frequency (and the flutter speed) should be not very far from unity ; this gave 139 units as quoted in the text.

On the same Figure is plotted the effect of an increase in d_{11}. In this case the flutter speed falls immediately with increase in damping, as would be expected from the application of the rules given in the last Section, but the maximum drop in flutter speed is less with d_{11} than with d_{22}, the minimum speed being about 85 per cent of the value with no damping. By the time d_{11} has been increased to $2 \cdot 04$ times critical damping, the flutter speed has returned to its initial value, and thereafter rises steadily. For comparison the flutter speed is below its initial value when d_{22} lies between $0 \cdot 054$ and $3 \cdot 66$ times critical damping.

Although a large drop in flutter speed can be obtained by the introduction of structural damping either in mode 1 or in mode 2, the form of equation (15) shows that the two effects are not additive. The result of adding a constant proportion of critical damping in each degree of freedom is shown in Fig. 9, and it can be seen that the increase of flutter speed is quite steady.

The amplitude ratio (q_1/q_2) is plotted in Fig. 10 (and the amplitude ratio q_2/q_1 in Fig. 11) for those values of d_{22} which give the same flutter speed as when $d_{22} = 0$. The effect is much the same as for the example of Section 3, the initial phase difference between $- q_1$ and q_2 is about 10 deg and by the time d_{22} has increased to $3 \cdot 66$ times critical damping this phase angle has reached about 80 deg. The sign of the co-ordinate q_1 relative to q_2 is, of course, just a matter of the initial choice, and in the present example the choice made was clearly the opposite of that made for the example of Sections 2 and 3 (compare the sign of the aerodynamic cross-stiffnesses in equations 1 and 17). At the same time as the phase angle is increasing through 70 deg, the amplitude ratio increases about 16 times.

The converse, for increasing d_{11}, is shown in Fig. 11. Again the phase difference increases, to about 75 deg by the time the flutter speed has returned to its original value. At the same time the amplitude of the undamped mode (q_2 in this case) increases several times. The increase is not so marked as when d_{22} is increased, being about 5 times compared with 16. Another interesting effect is on the flutter frequency ; in the examples where d_{22} is increased this frequency falls steadily, but when d_{11} is increased the flutter frequency increases ; in general it appears that the flutter frequency tends towards the natural frequency of the mode whose amplitude is increasing.

As in the example of Section 2 the complex latent roots of the binary calculation have been evaluated at speeds below the critical flutter speed for $d_{11} = 0$ and $d_{22} = 0$, $0 \cdot 054$ and $3 \cdot 66$ times critical damping respectively ; these three conditions all give the same flutter speed. When $d_{22} = 0$ the torsion mode leads to flutter whilst the bending mode becomes progressively more damped, but between $d_{22} = 0$ and $d_{22} = 0 \cdot 054$ times critical damping the two change over as shown in Fig. 12b. For some intermediate damping ratio the behaviour would be such as to render flight flutter tests difficult to analyse, because of the rapid change in frequency of the two roots, and the sudden drop in damping to the flutter condition. For the largest value of d_{22}, as shown in Fig. 12c the torsional root is dead beat all the way.

5. *Binary Investigations of a Hypothetical Wing.*—In an attempt to understand what type of normal modes give rise to a form of flutter in which the critical speed falls with structural damping in one of the modes, some calculations were made in the reverse direction starting with a range of coefficients known to cause the required effect, and, assuming standard derivatives, working back to the modes. A certain amount of trial and error was involved, and the resulting modes for a rectangular wing do not look very plausible (*see* Fig. 13). The flutter equations are

$$\begin{bmatrix} 4000\lambda^2 + 26\lambda + 37 + 1000y\,, & 110\lambda + 183 \\ - 84\lambda - 225\,, & 250\lambda^2 + 63\lambda - 313 + 1000y \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0$$

and the resulting fall in flutter speed with d_{22} is indicated in Fig. 14.

If the flexural and inertial axes coincide for an unswept wing, then c_{21} is nearly zero but b_{21} will provide sufficient coupling to promote flutter if the combined axis is sufficiently far aft of the aerodynamic centre. The expression corresponding to t_1 for this case is given in the Appendix and it is only for most unlikely conditions that the value becomes negative. A particular example of an unswept wing with an extreme frequency ratio (favourable to negative t_1) has been worked out on the assumption of pure bending for mode one and pure torsion for mode two, but the flutter speed increases with d_{22} at all practical heights.

It is concluded from these investigations that the conditions in which the flutter speed may fall on the addition of structural damping are :

(1) The relative density (wing to air) must be high

(2) The wing normal modes must involve coupled flexure and torsion*, either

(i) through sweepback

(ii) through a wide separation between the flexural and inertial axes

(iii) through the presence of large concentrated masses with offset c.g.s.

All the examples of Table 2 in which $\delta v/v$ is negative satisfy both these conditions, although it is to be noted that deltas must be classed as swept-back aircraft for this purpose.

6. *Conclusions.*—The results quoted in the present paper show that :

(1) The binary wing flutter speed may fall on the addition of structural damping in either the torsional mode or the bending mode, but more often in the torsional mode.

(2) The effect, if it occurs at all, gets worse with height, and often only takes effect at heights for which wing flutter presents no problem.

(3) The drop in flutter speed may continue to very high values of the damping, and in some cases more than three times critical damping is necessary to raise the flutter speed above its initial value.

(4) The drop in flutter speed for practical values of purely structural damping is small.

(5) If damping is added in the same proportion in each mode the flutter speed always increases.

It can be concluded that in a routine wing flutter investigation the effect of structural damping is unlikely to be serious, but there are some circumstances in which damping is neglected, because it is thought to be beneficial, that could be dangerous. An example is that of a wing carrying under-wing fuel tanks partly full. It is usually considered that for the first few cycles of a flutter oscillation the damping effect of the fluid is small, and this is regarded as the dangerous period. When turbulence is set up in the fluid after a few cycles quite large values of structural damping in the wing torsion mode can occur (*see* Refs. 5 and 6, for example) ; if the flutter is of similar form to that of the *Meteor* with tip mass, for example, this, far from being beneficial, could have a serious adverse effect.

Another way in which one mode may have a large degree of damping associated with it is if the damping is aerodynamic, which would have the same basic effect as structural damping. Suppose the wing torsional frequency coincides with the tailplane bending resonance, for example. The tailplane effect might be neglected in flutter calculations by the argument that the wing torsion mode is supplied with a damper (aerodynamic in this case) which would be expected to be favourable ; again this could be dangerous.

Finally any device, or type of construction, intended to introduce a high degree of damping artificially as a wing flutter preventive must be carefully considered in relation to the particular application concerned.

* The torsion to be understood in an aerodynamic sense.

Should a wing torsion mode be associated with very heavy structural damping in practice, it might lead to many difficulties which could be serious if the wing flutter was of the type discussed in the present paper. In the first place the mode would be very difficult to excite and measure accurately in a ground resonance test. Similarly any flight vibration work would be difficult to carry out on such a mode. This would not be so serious in one sense because the heavily damped mode is not the root which leads to flutter in the examples considered here, but on the other hand the bending mode might not be suspected as a possible dangerous mode. Added to these difficulties is the fact that the approach to flutter is rapid as shown in Figs. 3 and 12c, for example.

Mention should perhaps be made of the fact that in the present paper the effect of the damping forces on the mode itself has been neglected. Clearly this assumption might be seriously wrong when the damping reaches the order of critical damping, but unless there is a marked discontinuity in the distribution of damping the qualitative effects given in the paper are unlikely to be affected. In any case, whatever the source of the damping, values greater than about a quarter of critical are unlikely to be reached, but this amount can still give a substantial drop in flutter speed.

LIST OF SYMBOLS

q_r		Generalized co-ordinate
ν		Frequency parameter $= \omega c_r/V$ based on a reference chord c_r
λ	$=$	$i\nu$
ω		Flutter frequency
V		Flutter speed
p_r		Coefficient of λ^n in the expansion of the flutter determinant (*see* equation (2))
T_3	$=$	The 3rd test function
l_1	$=$	The coefficient of $2y$ in T_3
l_2	$=$	The constant term in T_3
a_{rs}	$=$	$\hat{a}_{rs} + \bar{a}_{rs}$
$\hat{a}_{rs}$		Typical structural inertia coefficient
$\bar{a}_{rs}$		Typical aerodynamic inertia coefficient
b_{rs}		Typical aerodynamic damping coefficient
c_{rs}		Typical aerodynamic stiffness coefficient
e_{rs}		Typical structural stiffness coefficient
d_{rs}		Typical structural damping coefficient
g		Coefficient used in expressing hysteresis damping when the stiffness term e is replaced by $e(1 + ig)$
t_1		$t_1 < 0$ gives the condition that $(\partial V/\partial d_{22})_{d_{22}=0} < 0$, and is expressed in terms of a, b, c, e and y (*see* Appendix and equation (15) in text)
t_2		Principal negative term in the expression t_1 (*see* equation (16) in text)
σ		Air density ratio

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APPENDIX

Expressions for the Initial Variation of Flutter Speed with Damping in the Torsion Mode of a Wing Binary Calculation

In the first case considered the only coupling terms are assumed to be the aerodynamic cross-stiffnesses. The flutter determinant is then

$$\begin{vmatrix} a_{11}\lambda^2 + b_{11}\lambda + c_{11} + e_{11}y , & c_{12} \\ c_{21} , & a_{22}\lambda^2 + b_{22}\lambda + c_{22} + d_{22}\lambda\sqrt{y} + e_{22}y \end{vmatrix} . \qquad \text{(A.1)}$$

This is expanded to give

$$p_0\lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4$$

and the penultimate test function is given by

$$T_3 = p_1p_2p_3 - p_0p_3^2 - p_1^2p_4 . \qquad \text{(A.2)}$$

The lowest critical flutter speed occurs when T_3 passes through zero from positive to negative. If we consider the condition $T_3 = 0$ for $d_{22} = 0$ (the critical flutter condition for zero structural damping), then

$$\frac{\partial T_3}{\partial d_{22}} > 0 \qquad \text{(A.3)}$$

gives the condition that an increment in d_{22} will lead to a stable condition, *i.e.*, the flutter speed is rising with d_{22} in this case, and conversely.

With $d_{22} = 0$ we have

$$\begin{aligned} \frac{\partial T_3}{\partial d_{22}} = y^{1/2}b_{11}\Big[& y^2(a_{11}e_{22} - a_{22}e_{11})^2 + \\ & + y\Big\{2(a_{11}c_{22} - a_{22}c_{11})(a_{11}e_{22} - a_{22}e_{11}) + a_{11}b_{22}(3b_{22}e_{11} + 2b_{11}e_{22}) \\ & \quad + a_{22}b_{11}(2b_{22}e_{11} + b_{11}e_{22})\Big\} + \\ & + \Big\{(a_{22}c_{11} - a_{11}c_{22})^2 + 2c_{12}c_{21}a_{11}\left(a_{11}\frac{b_{22}}{b_{11}} + a_{22}\right) + \\ & \quad + a_{22}b_{11}(b_{11}c_{22} + 2b_{22}c_{11}) + a_{11}b_{22}(2b_{11}c_{22} + 3b_{22}c_{11})\Big\}\Big] \end{aligned} \qquad \text{(A.4)}$$

and also with $d_{22} = 0$ the condition $T_3 = 0$ gives, after dividing by $b_{11}b_{22}$,

$$\begin{aligned} & y^2(a_{11}e_{22} - a_{22}e_{11})^2 \\ & + y\Big\{2(a_{11}c_{22} - a_{22}c_{11})(a_{11}e_{22} - a_{22}e_{11}) + (a_{11}b_{22} + a_{22}b_{11})(b_{11}e_{22} + b_{22}e_{11})\Big\} + \\ & + \Big\{(a_{11}c_{22} - a_{22}c_{11})^2 + 2c_{12}c_{21}a_{11}\left(a_{22} + \frac{a_{11}}{2}\frac{b_{22}}{b_{11}}\right) + a_{22}b_{11}(b_{22}c_{11} + b_{11}c_{22}) + \\ & + a_{11}b_{22}(b_{11}c_{22} + b_{22}c_{11}) + \frac{a_{22}^2b_{11}c_{12}c_{21}}{b_{22}}\Big\} = 0 . \end{aligned} \qquad \text{(A.5)}$$

After dividing A_4 by $y^{1/2}b_{11}$ (which is essentially positive and cannot affect the sign of $\partial T_3/\partial d_{22}$) and subtracting A_5 we have

$$\begin{aligned} \frac{1}{b_{11}y^{1/2}}\frac{\partial T_3}{\partial d_{22}} = & \; y\Big\{b_{11}b_{22}(a_{11}e_{22} + a_{22}e_{11}) + 2a_{11}b_{22}^2e_{11}\Big\} + \\ & + \Big\{b_{11}b_{22}(a_{11}c_{22} + a_{22}c_{11}) + 2a_{11}b_{22}^2c_{11} + c_{12}c_{21}\left(a_{11}^2\frac{b_{22}}{b_{11}} - a_{22}^2\frac{b_{11}}{b_{22}}\right)\Big\} . \end{aligned} \qquad \text{(A.6)}$$

This equation is identical with equation (15) of the main text, where $t_1 = (\partial T_3/\partial d_{22})/b_{11}y^{1/2}$.

If the total stiffness at the flutter speed ($= c + ey$) is denoted by σ, equation (A.6) simplifies to

$$\frac{1}{b_{11}y^{1/2}}\frac{\partial T_3}{\partial d_{22}} = b_{11}b_{22}(a_{11}\sigma_{22} + a_{22}\sigma_{11}) + 2a_{11}b_{22}{}^2\sigma_{11} + c_{12}c_{21}\left(a_{11}{}^2\frac{b_{22}}{b_{11}} - a_{22}{}^2\frac{b_{11}}{b_{22}}\right) . \qquad \text{(A.7)}$$

If b_{21} is not zero, the additional term to be added to the right-hand side of equation (A.7) is

$$b_{21}c_{12}\left\{\frac{a_{22}}{b_{22}}(a_{11}\sigma_{22} - a_{22}\sigma_{11}) + \frac{a_{11}a_{22}b_{21}c_{12}}{b_{11}b_{22}}\right\} . \qquad \text{(A.8)}$$

In general c_{21} (equation (A.7)) and b_{21} (equation (A.8)) will be negative.

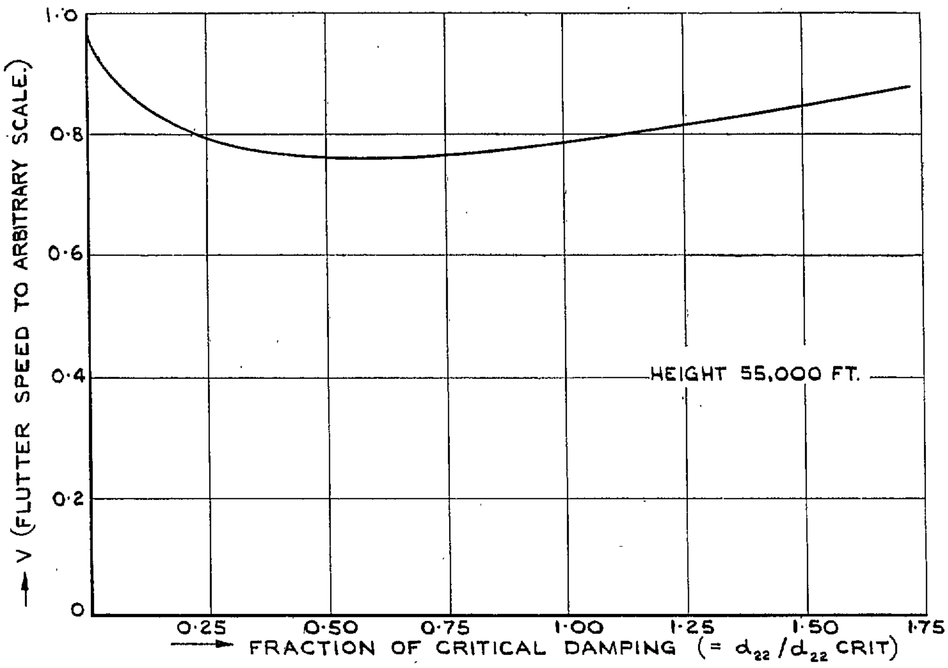


FIG. 1. An example of the variation of flutter speed with torsional damping.

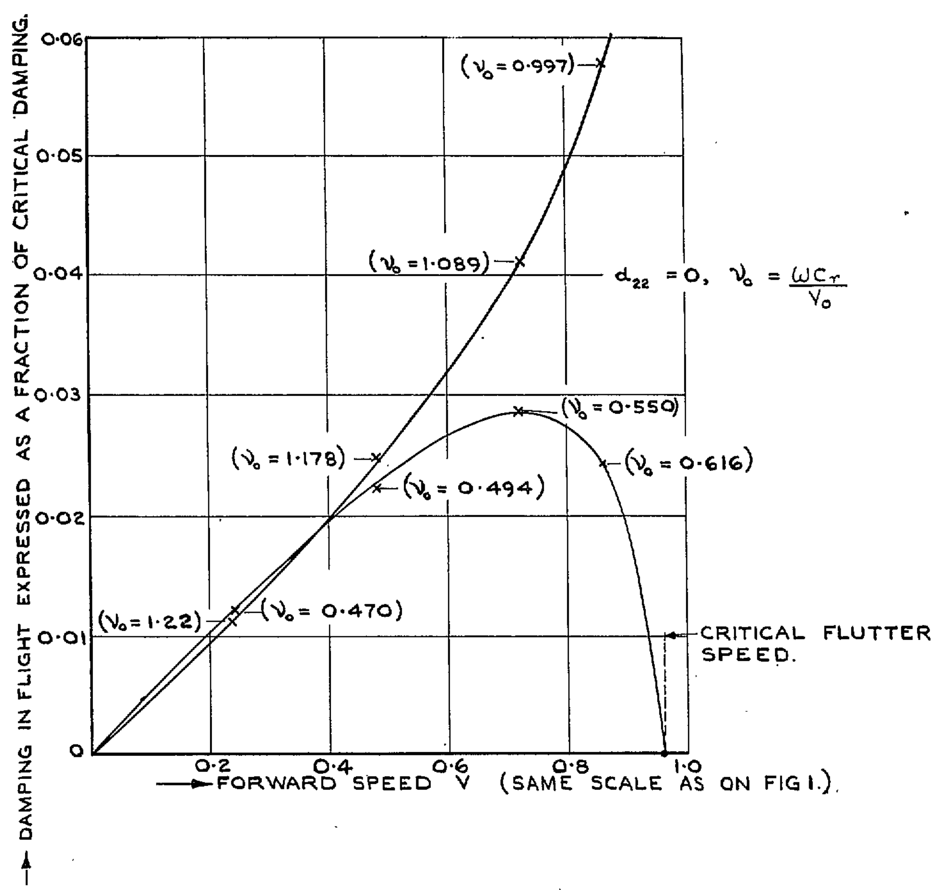


FIG. 2. The variation of flight damping with speed, with zero damping in the torsion mode.

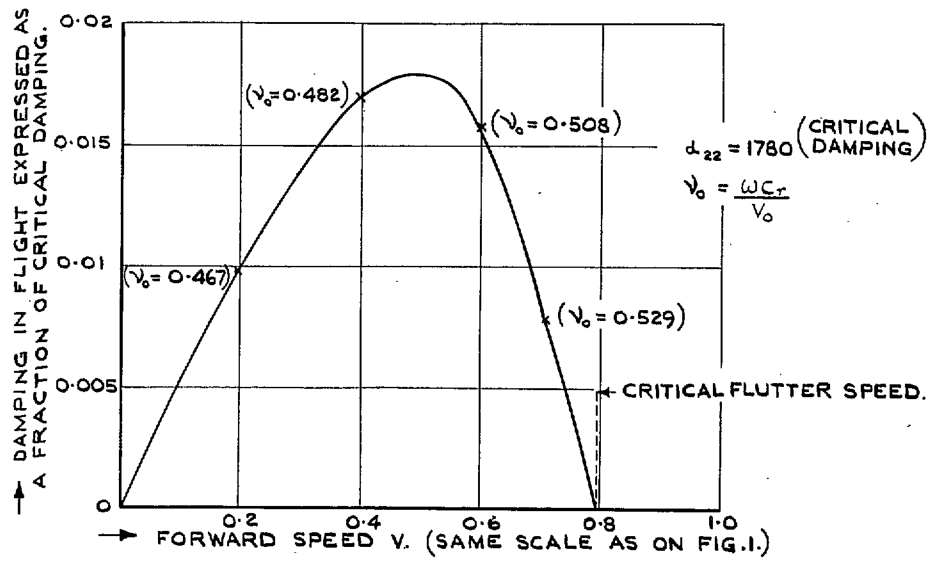


FIG. 3. The variation of flight damping with speed, with critical damping in the torsion mode.

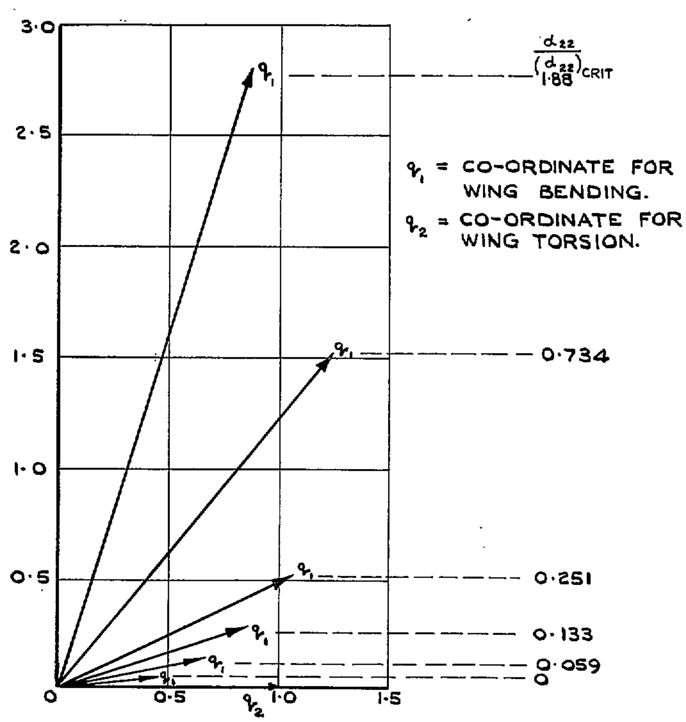
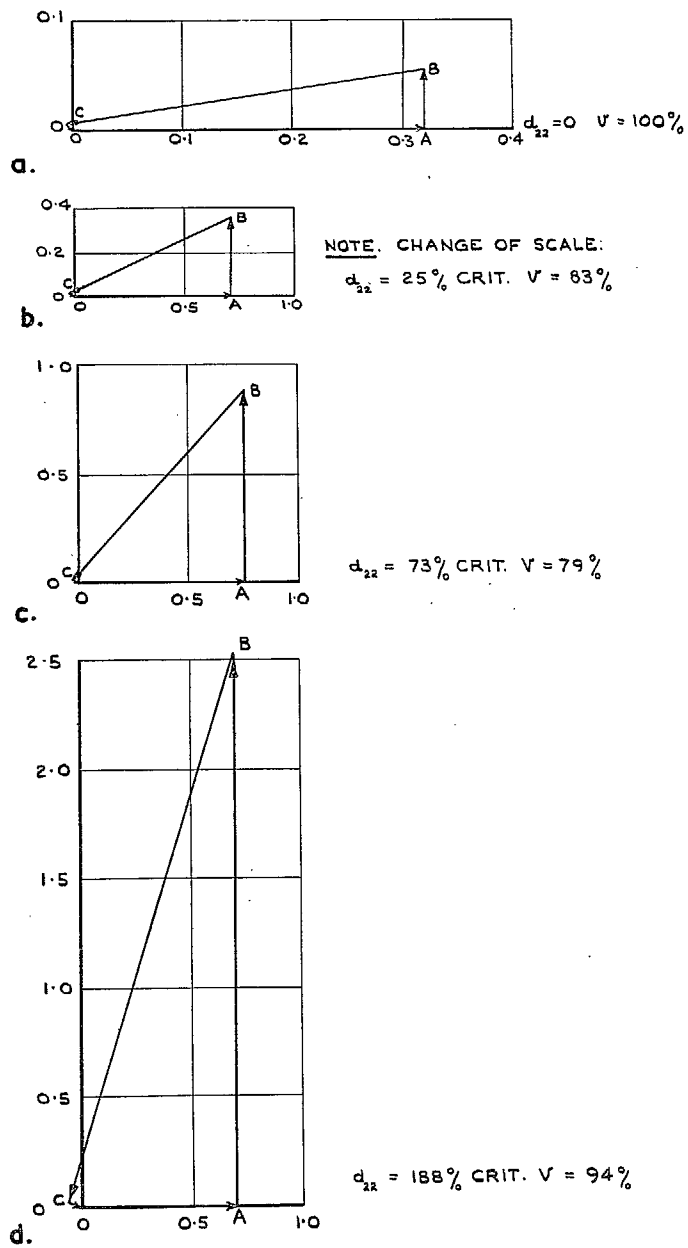
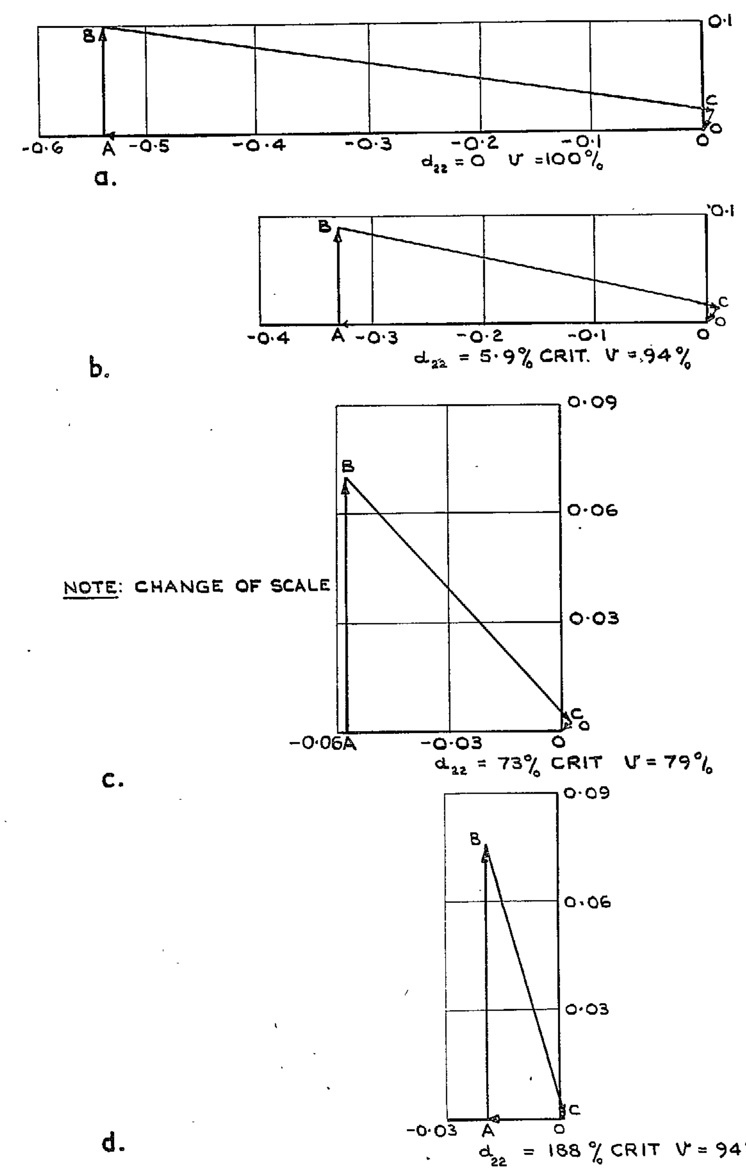


FIG. 4. Amplitude ratios for various values of torsional damping.



FIGS. 5a to 5d. Force diagrams for different amounts of damping (Torsional equation).



FIGS. 6a to 6d. Force diagrams for different amounts of damping (Bending equation).

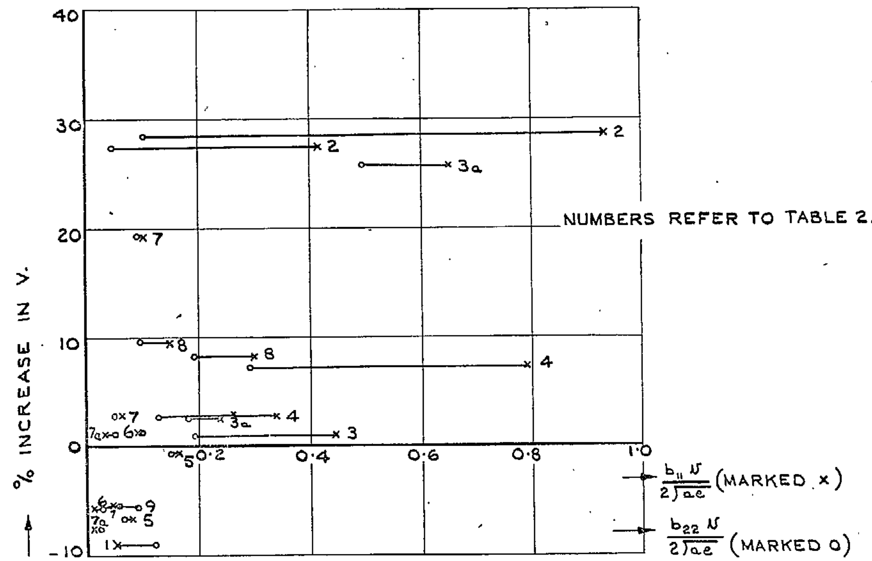


FIG. 7. Change in flutter speed for 10 per cent critical damping plotted against aerodynamic damping coefficients for several aircraft.

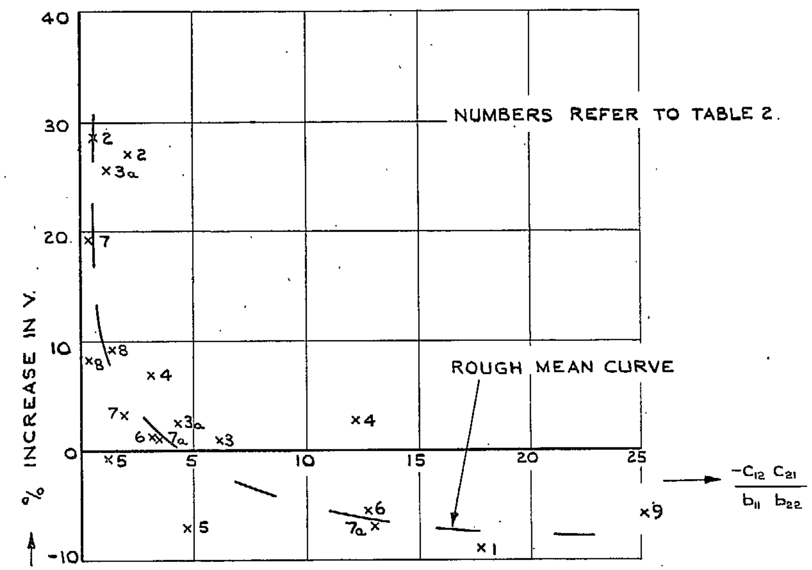


FIG. 8. Change in flutter speed for 10 per cent critical damping plotted against $(-\, c_{12}c_{21}/b_{11}b_{22})$.

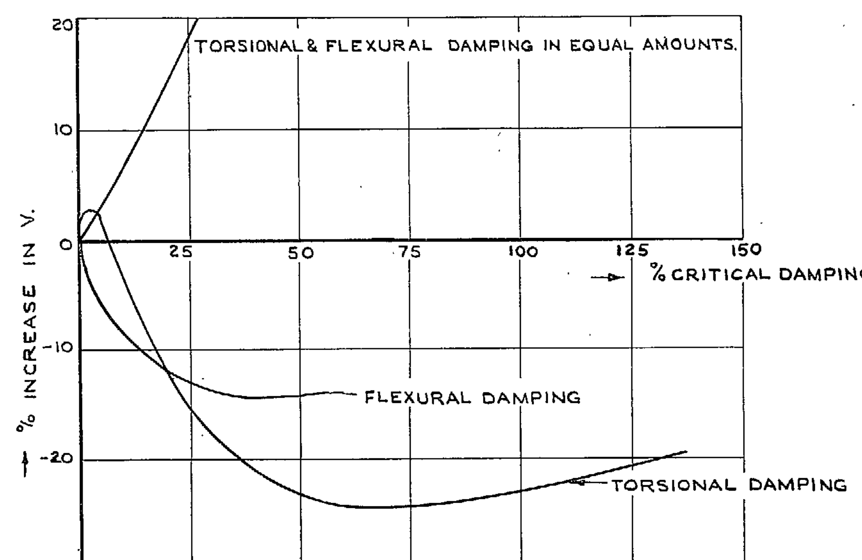


FIG. 9. Percentage increase in flutter speed for *Meteor* with tip mass plotted against structural damping.

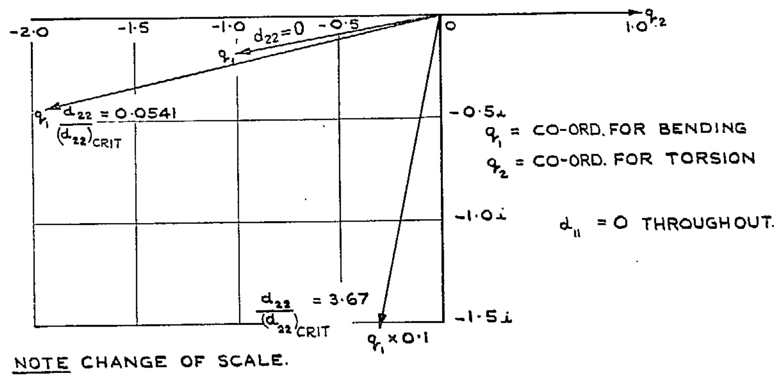


FIG. 10. Amplitude ratio (q_1/q_2) for various values of structural damping (*Meteor* with tip mass).

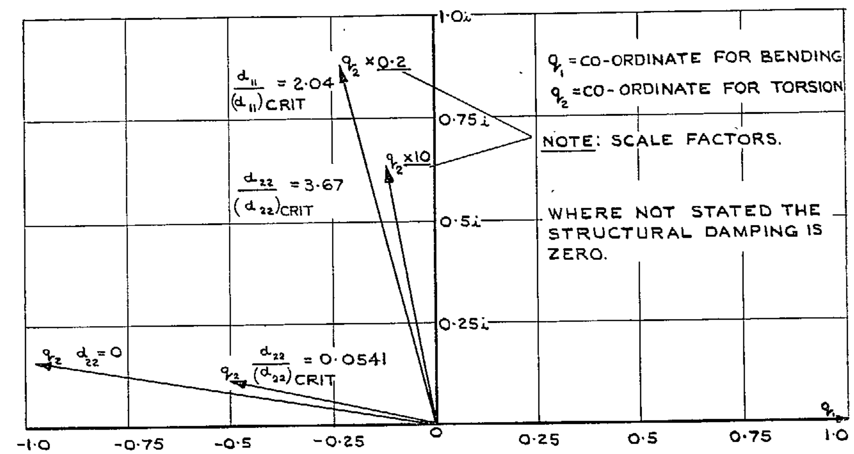


FIG. 11. Amplitude ratio (q_2/q_1) for various values of structural damping (*Meteor* with tip mass).

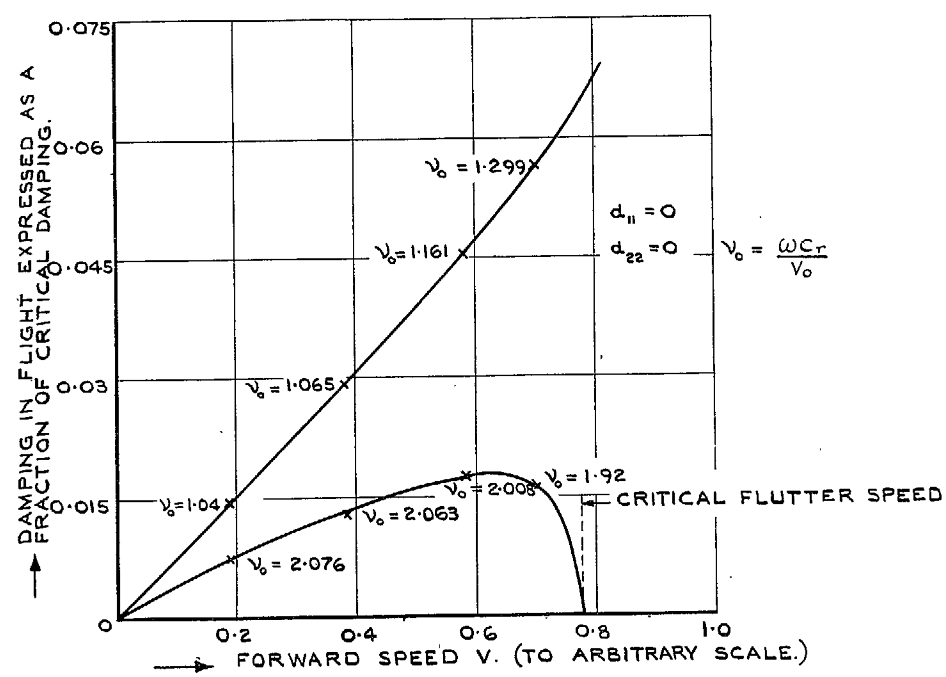


FIG. 12a. The variation of flight damping with speed for zero structural damping (*Meteor* with tip mass).

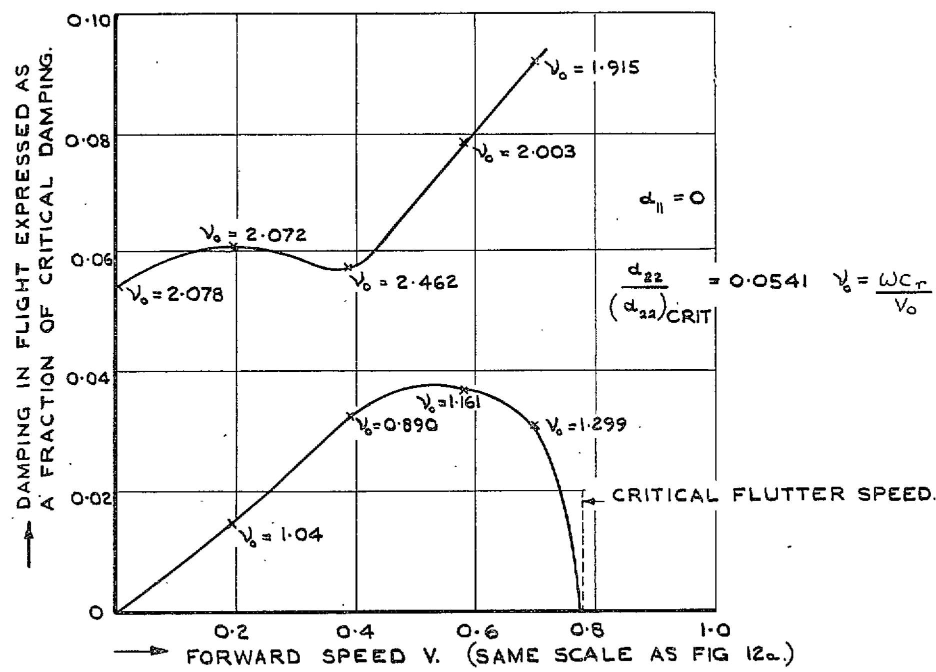


FIG. 12b. The variation of flight damping with speed for 5·4 per cent. critical structural damping (*Meteor* with tip mass).

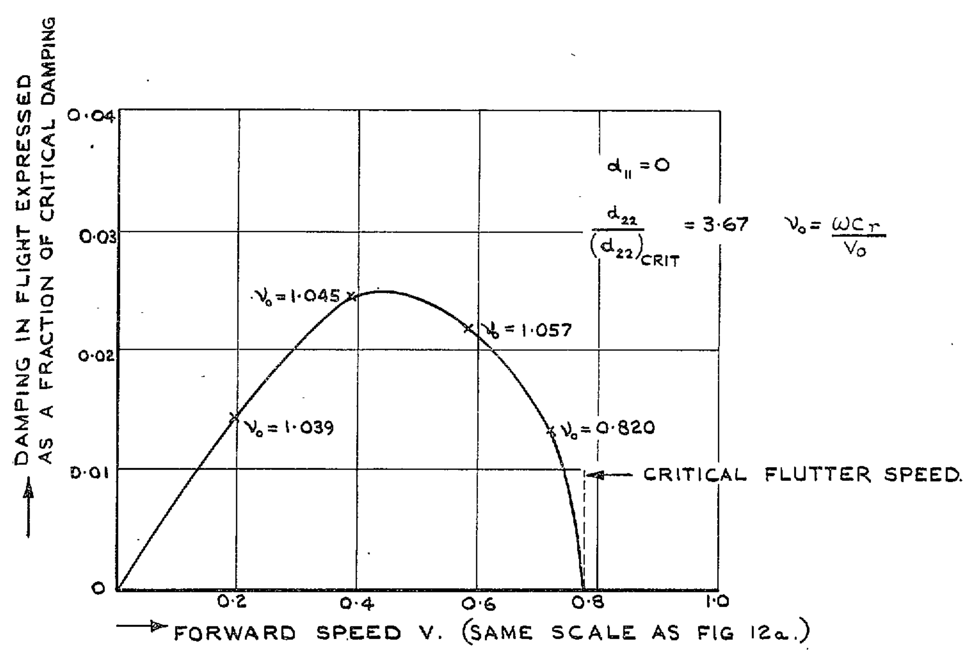


FIG. 12c. The variation of flight damping with speed, for 3·67 per cent critical structural damping (*Meteor* with tip mass).

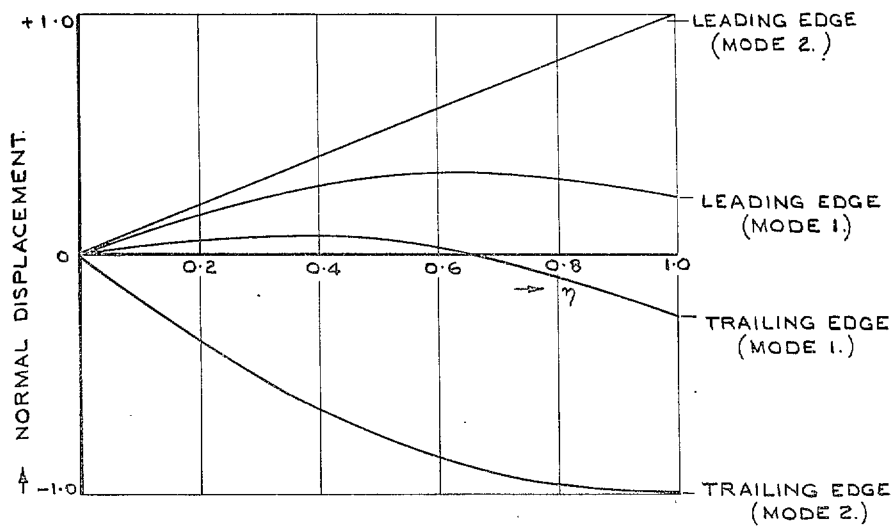


FIG. 13. Modes constructed to produce a fall in flutter speed when the structural damping was increased.

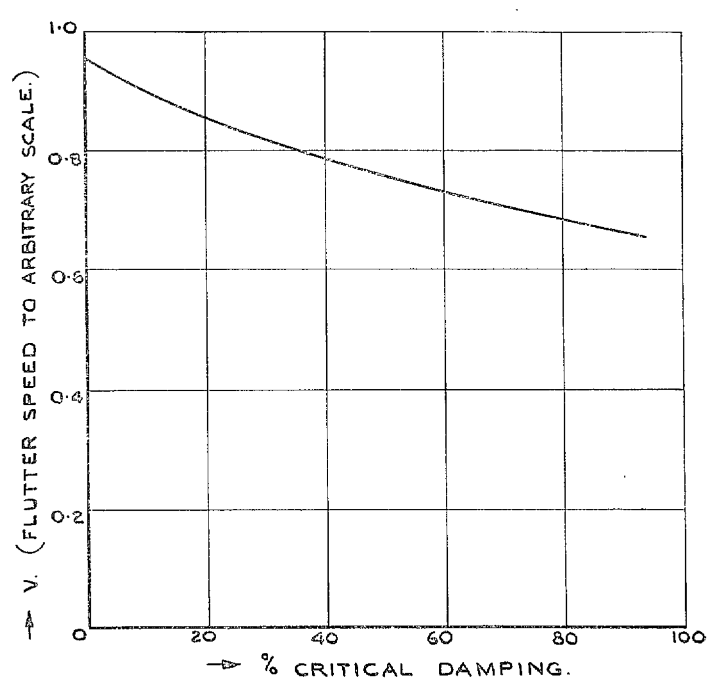


FIG. 14. Variation of flutter speed with torsional damping for a hypothetical wing having the modes of FIG. 13.

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