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Limitations of Use of Busemann's Second-order Supersonic Aerofoil Theory

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Summary.—The author has found the Busemann theory very rapid in use for the determination of C_L, C_D and C_m, and this report will enable the exact scope of its use to be determined.

It has been tacitly assumed in the past that Busemann's second-order theory of aerofoils at supersonic speeds was subject to the same limitations of wedge angle as the exact theory given by Lighthill[1] and others, namely, the wedge angle at which the bow wave detaches.

The range of angles for which Busemann's theory gives a pressure coefficient in error by less than 1 per cent is shown to be smaller than the angle range for the shock wave to be attached. There is also a limit to the application of Busemann's method to angles of expansion as well as to angles of compression, unlike the exact theory, which can be extended to expansive angles of the order of one right-angle without breaking down, in fact far beyond the useful range.

The limits of angle given for the use of Busemann's theory are conservative, since they give the pressures to 1 per cent, and the force coefficients will be more accurately determined since the errors tend to cancel out when integrating pressures to obtain forces.

1. *Introduction*.—Busemann assumed[2] that the pressure coefficient C_p on a two-dimensional supersonic wing surface can be expanded in the form

$$C_p = \frac{p_1 - p_0}{\frac{1}{2}\rho\, v^2} = C_1\phi + C_2\phi^2 + C_3\phi^3 + \text{etc.}, \qquad (1)$$

and showed that for the case of expansive flow (the Prandtl-Meyer expansion)

$$C_1 = 2\,(M^2 - 1)^{-1/2}, \qquad (2)$$

$$C_2 = \tfrac{1}{2}\{\,(M^2 - 2)^2 + 1\cdot 4M^4\}\,(M^2 - 1)^{-2}, \qquad (3)$$

$$\left.\begin{aligned} C_3 &= \frac{0\cdot 4M^8 - 1\cdot 813M^6 + 4M^4 - 2M^2 + 1\cdot 3333}{(M^2 - 1)^{3\cdot 5}} \\ b_3 &= \frac{0\cdot 36M^8 - 1\cdot 493M^6 + 3\cdot 6M^4 - 2M^2 + 1\cdot 3333}{(M^2 - 1)^{3\cdot 5}} \end{aligned}\right\}, \qquad (4)$$

ϕ being the angle in radians of the element of surface to the free stream at infinity, negative for expansive flow, and taking the ratio of the specific heats for air to be $1\cdot 4$.

A similar type of series can be applied to calculate the increase of pressure associated with an inclined shock wave. In this case ϕ is positive, and it can be shown that while the C_1 and C_2 terms of equation (1) are also valid in this case, the third and higher order terms are different and are denoted by b_3 and b_4. Thus the C_1 and C_2 terms are independent of the shock-wave pattern, and this leads to a second-order theory in which the shape of the aerofoil ahead and behind the point in question has no influence on the pressure at that point. More generally it may be shown that the first two terms of the series apply to any aerofoil with sharp leading and trailing edges, provided that the maximum inclination of the surface to the free stream does not exceed a certain limit, which depends on the stream Mach number.

If the third or higher order terms are to be considered, a special allowance will have to be made for the particular shock wave strengths involved.

2. *Calculation of Criterion for* 1 *per cent Error in Busemann's Theory*.—It was decided to allow an error Δp in p_1 of not more than 1 per cent of p_1. This is a fairly strict criterion, since the errors tend to cancel out when computing forces and moments.

If we take the error in using second-order theory as represented by the $C_3\phi^3$ term alone (neglecting fourth and higher orders), then we have

$$100\, C_3\,\phi^3 = \frac{1}{0\cdot 7M^2} + C_1\phi + C_2\phi^2 \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

as the criterion for 1 per cent error in pressure, remembering that

$$\frac{p_1 - p_0}{\frac{1}{2}\rho v^2} = \frac{p_1 - p_0}{p_0\,\frac{1}{2}\gamma M^2}\,.$$

This criterion is only true for expansive flow, and the limitation for 1 per cent accuracy in the case of compressive flow is considered in section 4 below.

The curve of ϕ against Mach number calculated from equation (5) is shown in Fig. 3.

3. *Limitation for Expansive Flow*.—As shown by Prandtl and Meyer, a supersonic flow of initial Mach number between 1 and 2 may be turned expansively through an angle as great as 90 deg. or more. Lighthill's theory[1] allows for this expansion exactly; moreover, this limit to surface angles in the expansive direction is unlikely to affect any practical design of wing section. Therefore, it is somewhat surprising to find a limit to expansive flow, as there is no shock wave to become detached in this case. This limitation arises solely from neglect of the third and higher order terms of equation (1).

In particular, if we use only the first two terms of the Busemann series, negative values of ϕ (expansive flow) of about $-$ 20 to $-$ 23 deg give a minimum value of $C_p = -\,0\cdot 75\, C_1^2/C_2$ occurring at $\phi = -\,\frac{1}{2}C_1/C_2$ radians. This can be seen by differentiating the first two terms of equation (1) with respect to ϕ, and equating to zero. This apparent recompression in expansive flow is due entirely to neglecting higher order terms; presumably the negative $C_3\phi^3$ term which was neglected becomes greater than the positive $C_2\phi^2$ term causing the apparent recompression. This effect is illustrated in Fig. 1, and the limiting value of ϕ ($= -\,\frac{1}{2}C_1/C_2$ radians) is given in Fig. 3.

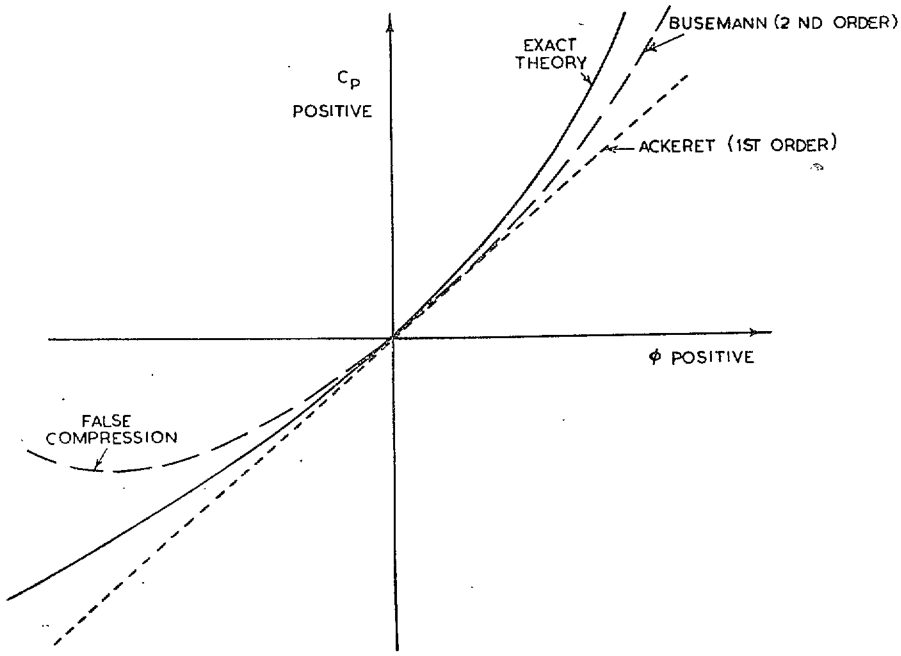


FIG. 1. False Compression Effect in Second-order Theory.

4. *Angular Limitation for Compressive Flow.*—Exact knowledge of supersonic aerofoil theory is at present limited to those cases where the bow wave is attached to the sharp leading edge of the aerofoil. This limits the maximum angle between the leading edge and the flow at infinity to a definite value, beyond which the exact theory does not apply. This limiting angle is a function of Mach number, and is also plotted in Fig. 3.

The pressure on a wedge of given angle at a given Mach number can be calculated from the Rankine-Hugoniot relations, and curves of these results are given in Fig. 4. These curves hold good as long as the flow does not pass through a second shock.

The curves calculated on Busemann's second-order theory are shown dotted in Fig. 4.

The difference between the two curves represents the effect of the $C_3\,\phi^3$ term plus the third-order effect of the bow wave. The Mach number at which this difference amounts to 1 per cent was determined, and the results are shown in Fig. 3.

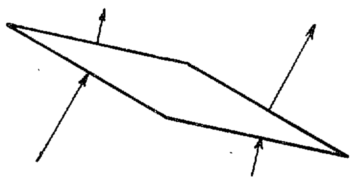
5. *Error to be Expected when Calculating Force Coefficients by Busemann's Method.*—If the pressure distribution is known to within 1 per cent, in general the resultant force on the aerofoil should also be known to the order of 1 per cent. The optimum supersonic aerofoil section has both lateral and fore-and-aft symmetries. For such a shape, every element of surface at a distance x from the leading edge on the top surface will have its counterpart distance x forward of the trailing edge on the lower surface.

The first-order effects on this pair of elements will be both upward or both downward. Integration of these first-order terms will yield a lift and drag force, but no moment about the mid-chord point. The second-order forces on this pair of surface elements will give equal and opposite forces, *i.e.* a couple without lift or drag.

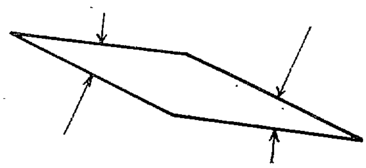
Thus we see why Ackeret's first-order theory is good for lift and drag, but theory is poor for pitching moment, while second-order theory gives a fair approximation to moment also.

The third-order forces, usually neglected in force calculations, will be of the same sign as the first-order forces, but unequal on the two components of the pair of surfaces. The more intense shock wave ahead of the lower surface will increase pressures over the whole lower surface relative to the upper surface, resulting in extra lift and drag at high incidences, over and above the simple Ackeret values. This causes the well-known increase of $dC_L/d\alpha$ with α, which is so unlike subsonic results, and which is not given by Ackeret's or Busemann's theories.

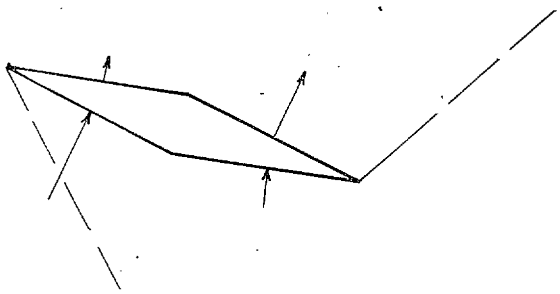
The curves of limiting angle given in Fig. 3 are of general application to two-dimensional problems, but it



(*a*) First-order pressures on aerofoil.



(*b*) Second-order pressures on aerofoil.



(*c*) Third-order pressures on aerofoil including shock wave effects. Sign and magnitude of pressure are indicated by arrows.

FIG. 2.

is useful to consider the case of the double wedge (rhombus) and the bi-convex sections separately. If maximum wing thickness $= t$ and chord $= c$, then the semi-angle of the double wedge will be $\tan^{-1}(t/c)$ and that of the bi-convex $\tan^{-1} 2(t/c)$. Since for thin wings $\tan x = x$, we may take the limitations on wedge angle for a 10 per cent double wedge as applicable to a 5 per cent bi-convex aerofoil.

For any given Mach number $\phi_{\max}$ is known, and may be sub-divided into wedge semi-angle (t/c) and incidence α, so that

$$\phi_{\max} = (t/c)_{\max} + \alpha,$$

or

$$(t/c)_{\max} = \phi_{\max} - \alpha. \quad \ldots \quad (6)$$

This gives the maximum incidence at which this aerofoil may be used in order that the theory shall apply.

Figs. 5 and 6 have been plotted showing the maximum incidences for which Busemann's theory applies to certain aerofoils. Fig. 5 shows the curves appropriate to Busemann's theory, and Fig. 6 shows the curves appropriate to Lighthill's exact theory. Figs. 7 and 8 are similar, but show the maximum values of C_L to which the theories apply.

6. *Busemann Force Coefficients.*—For convenience of reference the Busemann force coefficients[3] are given below, for a closed aerofoil contour with its chord line joining leading edge and trailing edge $C_{M,1/2}$ is the moment coefficient about the half chord.

	First-order Terms	*Second-order Terms*
C_L	$= 2C_1 \alpha + C_1 (I_{1L} - I_{1U})$*#	$+ C_2(I_{2L} - I_{2U})$*
C_D	$= C_1(I_{2U} + I_{2L}) + 2C_1 \alpha^2 + 2C_1 \alpha (I_{1L} - I_{1U})$*#	$+ C_2(I_{3U} + I_{3L})$# $+ 3C_2(I_{2L} - I_{2U})$* α
$C_{M,1/2}$	$= C_1(I_{4U} - I_{4L})$*	$- 2C_2(I_{4U} + I_{4L}) \alpha + C_2(I_{5U} - I_{5L})$*#

where

$$I_1 = \int \theta \, dx; \quad I_2 = \int \theta^2 \, dx; \quad I_3 = \int \theta^3 \, dx; \quad I_4 = \int \theta x \, dx; \quad I_5 = \int \theta^2 x \, dx$$

and θ is the angle between aerofoil surface and aerofoil chord, and the origin of x is taken at the half chord. Terms marked * vanish for zero camber; terms marked # vanish for fore-and-aft symmetry, leaving only the unmarked terms for aerofoils with double symmetry. Suffices U and L refer to upper and lower surfaces. Note that $(- I_{4U} - I_{4L})$ is the cross-sectional area of aerofoil.

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No.	*Author*	*Title, etc.*
1	M. J. Lighthill	The Conditions Behind the Trailing Edge of the Supersonic Aerofoil. R. & M. 1930. January, 1944.
2	A. Busemann	Aerodynamic Lift at Supersonic Speeds. (Lecture given at the Fifth Volta Conference at Rome.) (*L.F.F.* Vol. 12, No. 6, 3.10.35.) Translated by W. J. Stern, A.R.C.S. A.R.C. 2844.
3	C. N. H. Lock	Examples of the Application of Busemann's Formula to Evaluate the Aerodynamic Force Coefficients on Supersonic Aerofoils. R. & M. 2101. September, 1944.

TABLE OF BUSEMANN COEFFICIENTS

M	C_1	C_2	C_3 (Isentropic)	b_3 (One Shock)
1·1	4·364	30·32	568·98	544·4
1·2	3·015	8·307	54·034	53·22
1·3	2·408	4·300	14·247	14·53
1·4	2·041	2·919	5·801	6·128
1·5	1·789	2·288	3·059	3·331
1·6	1·601	1·950	1·937	2·153
1·7	1·455	1·748	1·4109	1·583
1·8	1·336	1·618	1·1444	1·280
1·9	1·238	1·529	1·0050	1·111
2·0	1·155	1·467	0·9341	1·0161
2·2	1·021	1·386	0·8946	0·9394
2·4	0·9167	1·337	0·91921	0·9356
2·5	0·8728	1·320	0·94322	0·9476
2·6	0·8333	1·306	0·97189	0·9654
2·8	0·7647	1·284	1·0382	1·013
3·0	0·7071	1·269	1·1116	1·069
3·5	0·5963	1·245	1·3090	1·231
4·0	0·5164	1·232	1·5132	1·405
4·5	0·4559	1·224	1·7191	1·584
5·0	0·4082	1·219	1·9250	1·764
∞	0	1·2	∞	∞

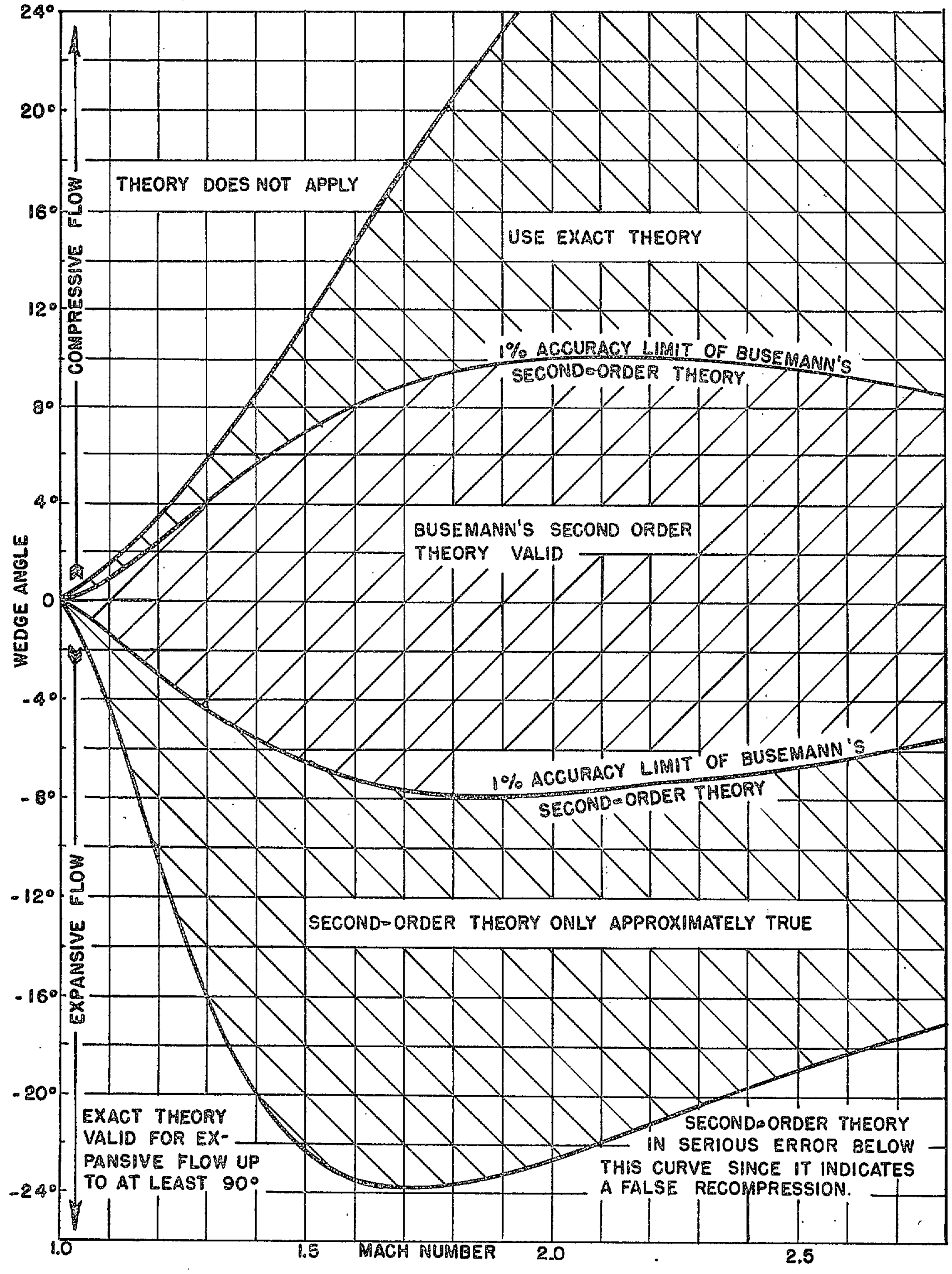

FIG. 3. Limiting Wedge Angles as a Function of Mach Number.

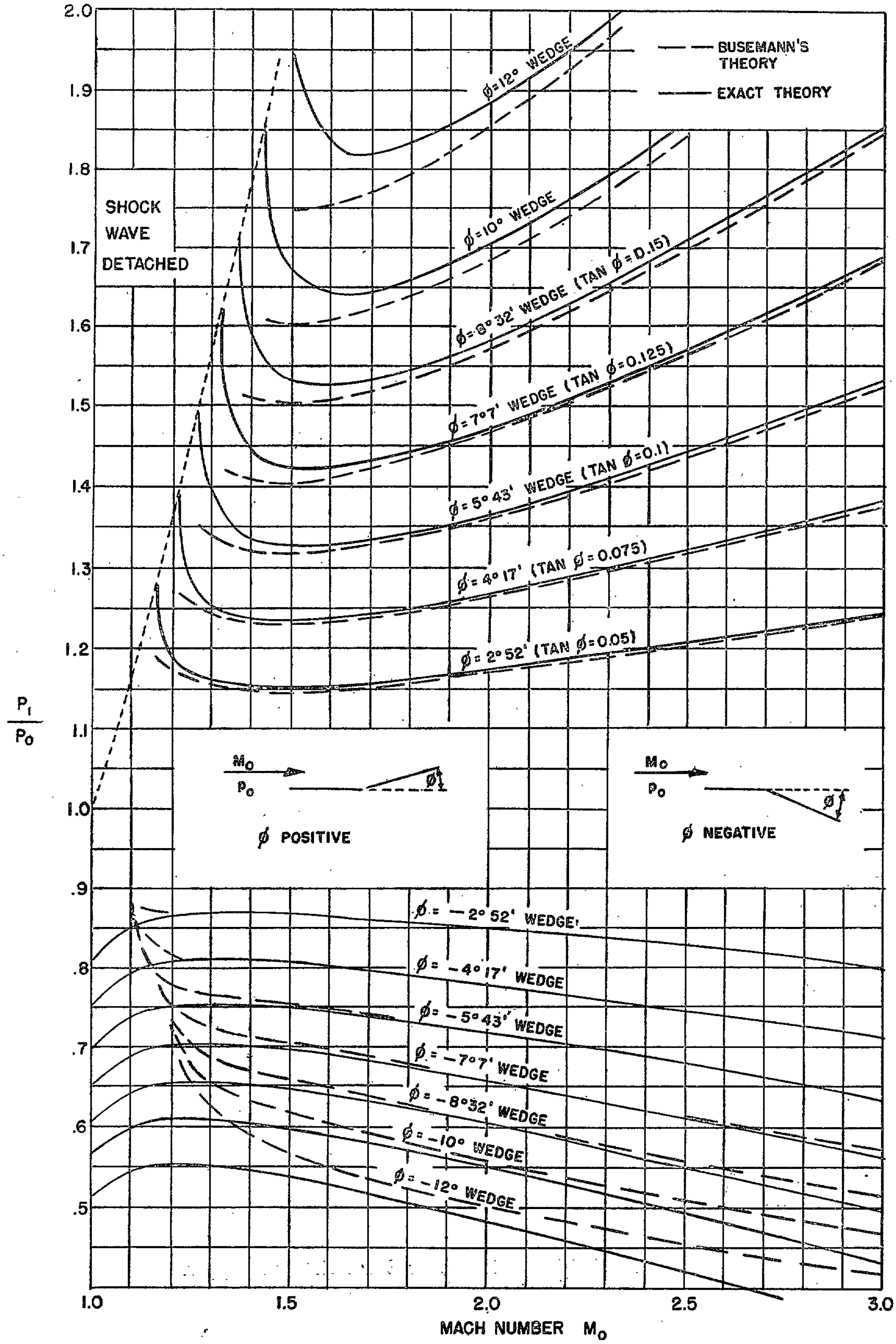

FIG. 4. Comparison of Exact Theory with Busemann's Second-order Theory for Two-dimensional Flow.

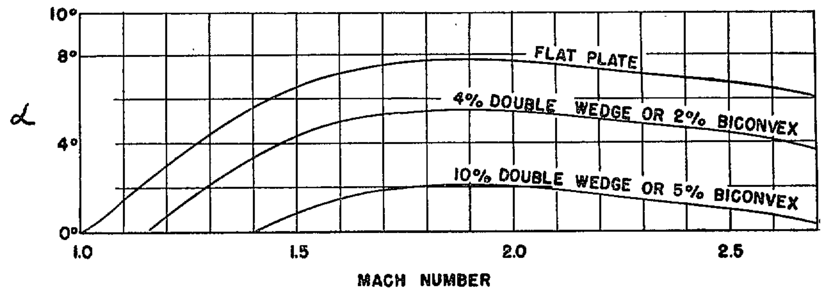

FIG. 5. Limiting Incidence for Busemann's Second-order Theory to be 1 per cent in Error on Pressure.

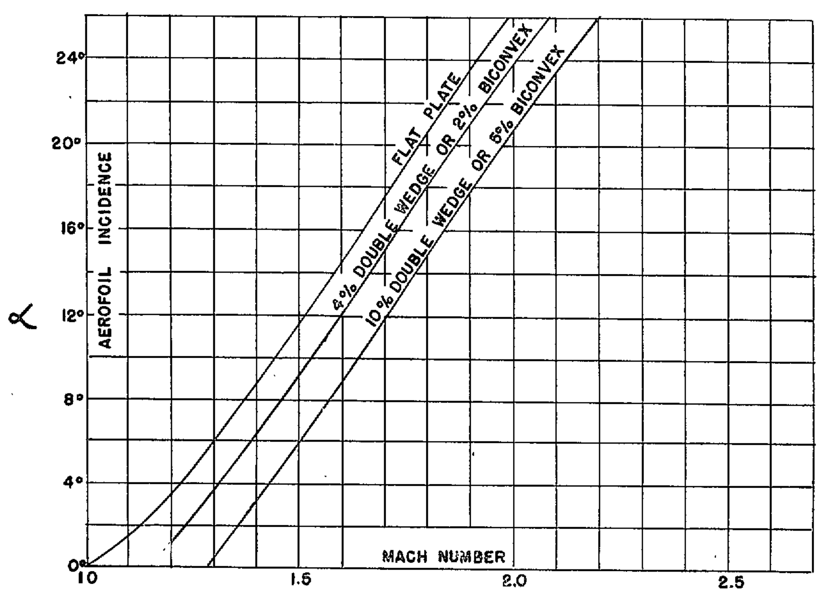

FIG. 6. Limiting Incidence for Exact Theory to Apply.

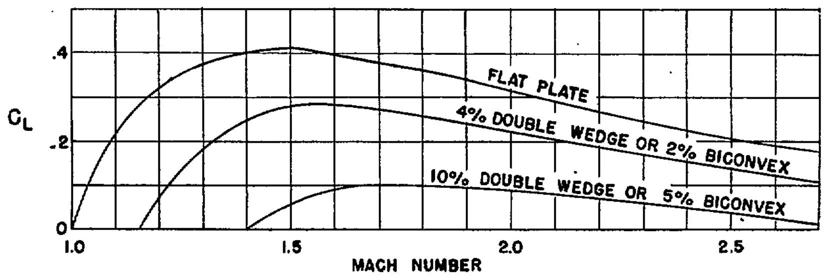

FIG. 7. Limiting C_L for Busemann's Second-order Theory to be ·1 per cent in Error on Pressure.

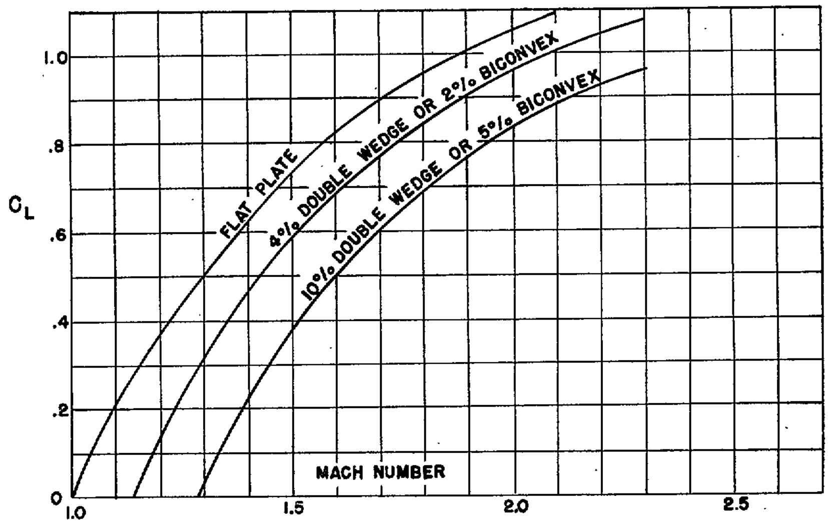

FIG. 8. Limiting C_L for Exact Wing Theory to Apply.

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