R. & M. No. 3122
(17,524, 19,881)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Formation of Regions of Separated Flow on Wing Surfaces

Part I

Low-Speed Tests on a Two-Dimensional Unswept Wing with a 10 per cent Thick RAE 101 Section

Part II

Laminar-Separation Bubbles and the Mechanism of the Leading-Edge Stall

By

L. F. CRABTREE

LONDON: HER MAJESTY'S STATIONERY OFFICE
1959
TEN SHILLINGS NET

The Formation of Regions of Separated Flow on Wing Surfaces

Part I

Low-Speed Tests on a Two-Dimensional Unswept Wing with a 10 per cent Thick RAE 101 Section

Part II

Laminar-Separation Bubbles and the Mechanism of the Leading-Edge Stall

By

L. F. Crabtree

Communicated by the Director-General of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 3122*

July, 1957

PART I

Summary.—Tests of a two-dimensional straight wing with a 10 per cent thick RAE 101 section have been made in a low-speed wind tunnel to check the validity of a criterion suggested by Owen and Klanfer for the type of bubble which will be formed when a laminar boundary layer separates from the surface of an aerofoil.

The results confirm this hypothesis and show that if the boundary-layer Reynolds number based on displacement thickness at separation, calculated from an observed pressure distribution, is greater than 450 a short bubble is formed, and for $(R_{\delta_1})_S$ less than 400 a long bubble is formed. For values of $(R_{\delta_1})_S$ within the range 400 to 450 it is uncertain which type of bubble will occur. A method is given, based on these results, for predicting the type of bubble formed on a two-dimensional unswept wing of arbitary section shape for a given incidence and Reynolds number.

A brief discussion of the physical structure of bubbles is given, and the more important problems yet to be solved are indicated.

A hypothesis is put forward to explain the phenomenon of the 'leading-edge stall' of moderately thin aerofoil sections, and some remarks are added on the scale effect on the maximum lift attained by aerofoils which experience this type of stall.

1. *Introduction*.—Aerofoil sections may be divided into three categories depending on the type of stall:

(*a*) Trailing-edge stall, with the separation point of the turbulent boundary layer moving forward from the trailing edge as the incidence increases

(*b*) Leading-edge stall, caused by an abrupt separation of the flow near the leading edge without subsequent reattachment.

(*c*) Thin-aerofoil stall with laminar separation near the leading edge and turbulent re-attachment at a point which moves progressively rearward with increasing incidence.

* Part I.—RAE Report Aero. 2528, received 31st March, 1955.
Part II.—RAE Report Aero. 2578, received 23rd January, 1958.

This report is concerned only with types (*b*) and (*c*). In such cases the laminar boundary layer may separate from the upper surface of the aerofoil at incidence near the leading edge and reattach to the surface further aft. At low speeds this reattachment is due to turbulent mixing in the initially laminar separated viscous shear layer. This separated layer may still be considered as a boundary layer in the conventional way except that it is formed along a free surface and not a solid wall. Corresponding to (*b*) and (*c*) above, two distinct types of such a bubble of separated flow have been observed, the short bubble for which the ratio of chordwise length of separated flow to the boundary-layer displacement thickness at the separation point, $l/(\delta_1)_S$, is of the order of 10^2, and the long bubble where $l/(\delta_1)_S$ is of the order of 10^3 to 10^4.

It should be noted that such bubbles are not restricted to thin aerofoils. They have also been observed well aft on sections of up to 20 per cent thickness/chord ratio, but only at such low Reynolds numbers that transition to turbulence does not occur at a station ahead of the laminar separation point. Experiments on the interaction of shock waves with the laminar boundary layer on a flat plate have also exhibited a bubble or dead-air region at the leading edge, but the reattachment mechanism is different in this case. It is connected with a supersonic expansion round the bubble which tends to force the flow back to the surface, and it is then possible for the reattached boundary layer to remain in a laminar state.

Owen and Klanfer[1] have suggested a criterion to determine which type of bubble will form on a two-dimensional unswept wing in a particular case: if the boundary layer Reynolds number based on the displacement thickness and the velocity at the edge of the boundary layer at the separation point, $(R_{\delta 1})_S = (V\delta_1/\nu)_S$ is greater or less than a certain critical value then the bubble will be short or long respectively. By analysis of some previous two-dimensional tests in which bubbles had been observed, Owen and Klanfer concluded that this critical value of $(R_{\delta 1})_S$ was about 400 to 500.

The development of the boundary layer was calculated by Thwaites's method applied to the observed pressure distributions. This method gives

$$\left(\frac{\delta_2}{c}\right)^2 R = 0\cdot 45\left(\frac{V}{V_0}\right)^{-6}\int_0^{s/c}\left(\frac{V}{V_0}\right)^5 d\left(\frac{s}{c}\right) = g\left(\frac{s}{c}\right), \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where s is distance along the aerofoil surface from the stagnation point, δ_2 is the momentum thickness and R is the free-stream Reynolds number $(V_0 c/\nu)$ based on the aerofoil chord, c.

Empirically, at separation,

$$\frac{\delta_1}{\delta_2} = 3\cdot 7\,, \quad \ldots \quad (2)$$

so that

$$(R_{\delta 1})_S = \left(\frac{V\delta_1}{\nu}\right)_S = \left(\frac{V}{V_0}\right)_S\left(\frac{\delta_1}{c}\right)_S R \quad \ldots \quad (3)$$

and therefore

$$(R_{\delta 1})_S = 3\cdot 7\,(g_S)^{1/2}\left(\frac{V}{V_0}\right)_S R^{1/2}\,, \quad \ldots \quad (4)$$

the subscript $_S$ indicating conditions at the separation point.

Taken together with equation (4), the criterion suggested by Owen and Klanfer implies that either type of bubble may be formed on an aerofoil at a given incidence depending on the free-stream Reynolds number, since from equation (4) $(R_{\delta 1})_S \propto R^{1/2}$. Although McCullough and Gault[2] have indicated that the same aerofoil could exhibit long or short bubbles over different incidence ranges at a constant R, none of the data used by Owen and Klanfer for their analysis covered both types of bubble on the same section. Thus the purpose of the tests reported here, as well as to verify the suggested criterion for a two-dimensional unswept wing, was to investigate in particular whether both types of bubble could be obtained on the same model at the same incidence according to Reynolds number. Further tests are planned to find a corresponding criterion for an infinite sheared wing, but this paper deals only with tests on a two-dimensional straight wing of RAE 101 section with a thickness/chord ratio of 10 per cent, and a 30-in. chord.

The presence of a short bubble was noted during previous tests on this model[3] and some preliminary calculations based on the suggested criterion produced the following Table:

$\frac{t}{c}$	C_L	$(R_{\delta 1})_s$	$R \times 10^{-6}$ to give $(R_{\delta 1})_s = 500$
0·10	0·8	$0 \cdot 390 R^{1/2}$	1·64
	0·6	$0 \cdot 466 R^{1/2}$	1·15
	0·4	Laminar separation occurs far back	
0·06	0·8	$0 \cdot 325 R^{1/2}$	2·37
	0·6	$0 \cdot 367 R^{1/2}$	1·85
	0·4	$0 \cdot 540 R^{1/2}$	0·85

From the results for the 10 per cent thick section it appears that a long bubble would be found only at incidences approaching that corresponding to maximum lift, if the Reynolds number based on the chord was above about $1 \cdot 6 \times 10^6$ (100 ft/sec for a 2·5-ft chord length). At moderate incidences of about 6 to 9 deg, a very low tunnel speed would be required if a long bubble were to be formed near the leading edge, and at lower angles the laminar-separation point is well aft. However, since the model was in existence and being used for other experiments[4], it was decided to carry out an investigation of bubble formation at the same time, although this 10 per cent thick section was not ideally suited for all the purposes of the test programme.

Tests of the 6 per cent thick wing are planned shortly, as also are tests on two 'infinite' wings sheared by 45 and 60 deg respectively. It is also hoped to carry out an investigation of the leading-edge bubble at high subsonic Mach number in a small tunnel at the Royal Aircraft Establishment.

2. *Details of Model and Tests.*—The tests were done in the No. 2, 11½-ft Tunnel at the R.A.E. in December, 1953. The model, that used for the tests reported in Refs. 3 and 4, was a two-dimensional straight wing of 2½-ft chord spanning the 8½-ft vertical dimension of the tunnel. The wing was made of wood with a Tufnol trailing edge, and the actual profile at the test section was accurate to within 0·01 in., *i.e.*, $0 \cdot 0003c$. Pressure measurements on the wing surface were made by means of two rows of flush holes 4 in. apart, at the centre of the span. The holes were of $\frac{1}{32}$-in. diameter and their chordwise positions on the two rows were staggered. Thus on the original wing, 26 chordwise measuring points were available on each surface without overcrowding the pressure holes. To assist in locating the suction peak on the upper surface of the wing when at incidence, and in determining the position and extent of any bubbles, an extra orifice was provided for the present tests. This pressure hole was located at a chordwise position of $x/c = 0 \cdot 002$.

Pressures were read on a multitube manometer, and in some of the low-speed tests on a Prandtl micromanometer, which has provision for damping any fluctuations in pressure. This proved useful in taking surface-pressure measurements in the region covered by a long bubble, where such fluctuations occurred.

The incidence was measured by the movement of a light beam which was reflected from a mirror countersunk in the wing surface well clear of the pressure holes, as in the previous tests on this model. The incidence could be read to about 0·01 deg. The usual tunnel correction was applied to the incidence; this was small and of the order of $0 \cdot 17 C_L$ deg.

3. *Results and Discussion.*—The first object of the tests was to show that two regimes of flow, namely, long and short bubbles of separated flow, could exist on the same model depending on the angle of incidence and the free-stream Reynolds number. Accordingly, the technique was

to increase the tunnel speed to about 120 ft/sec with the model at zero incidence. The angle of incidence was then increased at constant tunnel speed to a value just below that corresponding to maximum lift, *i.e.*, to about 10 deg. The pressure distribution over the aerofoil surface was read from the multitube manometers. The free-stream Reynolds number was decreased in stages by reducing the tunnel speed, and pressure distributions were obtained at each stage.

On approaching a certain main-stream velocity from above, the suction peak on the upper surface collapsed and a long bubble was formed.

Typical velocity distributions near the leading edge of the aerofoil, obtained as $V/V_0 = \sqrt{(1 - C_p)}$ by means of this technique, at three different Reynolds numbers and at a constant angle of incidence of 9·8 deg are shown in Fig. 1. This shows that there is in fact a small range of free-stream Reynolds number over which either a short bubble or a long bubble is possible.

Several sets of pressure distributions at different angles of incidence and over a range of Reynolds number for each angle were obtained, each such range including the Reynolds number for transition from a short to a long bubble.

The observed pressure distributions were corrected in the usual way for the effects of tunnel constraint, and were used to calculate the development of the laminar boundary layer from the stagnation point to separation by Thwaites's method. This method predicts the location of the laminar separation point with good accuracy, particularly in the steep adverse pressure gradients aft of peak suction which are typical of thin aerofoils at incidence. The agreement of the calculated separation point with experiment is illustrated in Fig. 1. Calculated boundary-layer characteristics such as displacement thickness are expected to be more accurate than measured values in the vicinity of the leading edge, since the boundary layer is so thin there.

The boundary-layer Reynolds number based on displacement thickness at separation, $(R_{\delta 1})_S$, was calculated for all the measured pressure distributions and the results are given in Table 1, together with the non-dimensional quantities $l/(\delta_1)_S$ and $V_S l/\nu$, where l represents the length of aerofoil surface in the chordwise direction which is covered by the bubble of separated flow. It is rather difficult to define this length with any exactness, but it has been very roughly estimated in each case from the plotted velocity distributions by assuming reattachment to have taken place when the observed velocity distribution merges into the theoretical curve for the same value of C_L.

Fig. 2 shows how sharply defined is the transition from one type of bubble to the other by plotting $V_S l/\nu$ against $(R_{\delta 1})_S$ on semi-logarithmic graph paper. The calculated values of $(R_{\delta 1})_S$ in those cases where the change from a short to a long bubble was imminent were within the range 400 to 450 (cases 2, 5 and 6 of Table 1). If the theoretical inviscid pressure distributions are used as a basis of the boundary-layer calculations, however, the range is different and much wider, being from 450 to 550, as shown in Table 1. A prediction method for general use is described in the Appendix, whereby the type of bubble on any aerofoil at a given incidence and Reynolds number can be found. Obviously this method must use the latter range of values for the critical $(R_{\delta 1})_S$, since experimental pressure distributions will not in general be available for an arbitrary aerofoil section.

A different technique was used to investigate the variation in length of a long bubble with incidence at a constant Reynolds number. The lowest incidence at which a long bubble was observed at practicable values of main-stream dynamic pressure was 7·6 deg. In this case the tunnel speed was 24·5 ft/sec, giving a free-stream Reynolds number of $0 \cdot 4 \times 10^6$. That this is so much lower than the value estimated in the preliminary calculations (*see* Section 1), is due to the decrease in circulation caused by the thick turbulent boundary layer over the rear of the section. The effect of this on the pressure distribution near the leading edge is such as to delay the formation of a long bubble at a given incidence to a lower free-stream Reynolds number. A bubble increases the thickness of the downstream layer, which is also increased in any case as the Reynolds number is decreased. With such a low main-stream velocity it was necessary to use a micromanometer to obtain sufficient accuracy in the surface-pressure measurements, and due to pressure fluctuations inside the bubble it was difficult to make the observations over the bubble region even with full damping on the instrument. Fig. 3 shows how the long bubble in this case develops with incidence at a constant free-stream Reynolds number.

It will be noted from Fig. 1 that whereas the theoretical pressure distribution is little affected by the presence of a short bubble, there are large changes in the $V(s/c)$ distribution when a long bubble is formed, although there is not necessarily a decrease in circulation. Thus whilst the criterion holds also for transition from a long bubble to a short bubble there is an appreciable hysteresis in the free-stream speed at which the change takes place for a given incidence. The large changes in pressure distribution due to a long bubble, illustrated in Fig. 1, emphasize the need for a calculation method by which such effects may be predicted, although a start in this direction has been made[5].

The tests of the long-bubble regime at a constant incidence with varying free-stream velocity demonstrated very clearly the changes in length of such a bubble with Reynolds number. With the pressure orifices connected to a multitube manometer, the tunnel speed was decreased at a fixed incidence until the original short bubble changed to the long type. This change as observed on the manometer was quite dramatic; the general level of liquid in the tubes connected to orifices on the upper surface near the leading edge dropped, showing a decreased suction peak, and several of the tubes showed the same level of liquid, these being connected to surface holes under the constant-pressure part of the bubble. As the speed was decreased the liquid in tubes connected to holes further aft ' popped ' up one by one, showing how the constant-pressure part, and thus the overall length of bubble increased with decreasing Reynolds number. The general level of liquid in the manometer tubes corresponding to the forward part of the wing upper surface dropped noticeably as the Reynolds number was decreased in this way, showing the decrease in circulation, which in this case accompanied the increasing length of bubble.

However, as has been stated, this 10 per cent thick section is not ideal for studying the long-bubble phenomenon, being in fact too thick, since very low free-stream Reynolds numbers are necessary to ensure the formation of a long bubble at incidences much below that corresponding to maximum lift in the short-bubble regime. The tests which are planned on a 6 per cent thick RAE 101 section should prove more suitable in this respect.

Recent tests at Bristol University reported by Black and Hunt[6] provide further data on the formation of regions of separated flow on aerofoil surfaces. It is hoped to analyse these tests and to include the results in a later part of this report on the R.A.E. programme. Black and Hunt also used the lampblack-paraffin technique for flow visualisation, and this yielded results which were interpreted in a very different way from those of their boundary-layer traverses at incidences of 10 deg and above. This emphasizes how much care is needed in interpreting the patterns produced by oil-flow and other visualisation techniques. It is quite possible for the airflow over a wing to give two distinct patterns using the oil-flow technique, depending on whether the oil is applied liberally or sparingly. The balance between viscous, inertia, and surface-tension forces may well be different in the two cases, and thus lead to marked differences in the observed oil pattern.

4. *The Physical Structure of Bubbles.*—When the laminar boundary layer separates from the surface of an aerofoil the Kirchoff theory of discontinuous motions would indicate an infinite wake with the separated viscous layer forming along a free streamline. As was emphasized in Section 1, the boundary-layer conception may be applied to this layer except, of course, that the lower limit is not formed by a solid wall. On the other hand it might be argued that the boundary layer should become turbulent almost immediately upon separation. Experiment has shown that the actual state of affairs is usually intermediate between these two extreme models, and the phenomenon is controlled by the Reynolds number or scale of the motion, transition of the laminar shear layer at the ' surface of discontinuity ' or free surface playing an important role. At low free-stream Reynolds numbers (such that $(R_{\delta 1})_S$ is less than the critical value separating the two types of bubble), a long bubble will be formed on a thin aerofoil at incidence and obviously about 10^3 or more boundary-layer thicknesses are required before transition to turbulence occurs

in the separated layer. As the free-stream Reynolds number, R, is increased, so the chordwise length covered by the bubble decreases, simply because transition to turbulence in the separated layer occurs sooner for a higher Reynolds number. As R increases so does $(R_{\delta 1})_S$, since it is proportional to $R^{1/2}$, so that at some stage $(R_{\delta 1})_S$ reaches the critical value and the long bubble suddenly changes to a short bubble. As R is further increased the pressure distribution approximates more and more closely to the theoretical inviscid distribution (for infinite R) with its higher suction peak and larger pressure gradients. The separation point therefore moves forward with increasing R, and due to the steeper adverse pressure gradients the same pressure recovery can be obtained within a shorter chordwise length. Thus, due to the lower pressure inside the bubble and the more highly curved flow nearer the leading edge, the short bubble also contracts with increasing R. However, at some stage R will be high enough to cause transition to turbulence ahead of the laminar separation point, so that the bubble disappears altogether.

Some experimental evidence supporting these ideas may be found in the work of Bursnall and Loftin[7] and of McCullough and Gault[2]. The former is an investigation of a laminar separation bubble occurring well aft on a low-drag section of 18 per cent thickness/chord ratio at zero lift. In this case the lower limit of the separated boundary layer, defined as the line of zero velocity, may be fairly well represented by the tangent to the aerofoil surface at the separation point, up to the point where transition to turbulence occurs in the separated flow. This is not true for long and short leading-edge bubbles on thin aerofoils, however, as is shown by the results of Ref. 2. Here the lower limit of the separated layer exhibits a pronounced curvature away from the tangent line and towards the aerofoil surface, although the tangent line is followed for a few boundary-layer thicknesses downstream of the laminar separation point. This situation, in the case of a short bubble, is illustrated by Fig. 4 in which the ' boundary-layer ' profiles in the region of a bubble on the surface of a NACA 63-009 section at 4 deg incidence have been replotted from Ref. 2 to a much larger scale. This bending back of the separated layer toward the surface is presumably due to the fairly high curvature of the potential flow near the leading edge of a thin aerofoil at incidence, and an attempt is being made to prove that the phenomenon exists in the general case, including the effects of viscosity.

Theoretically, knowing the location of the laminar-separation point (which can be predicted fairly accurately by Thwaites's method as has already been emphasized), it should be possible to calculate the free streamline which is formed downstream of separation by Kirchoff's theory of discontinuous motions, modified so as to take into account the lower pressure in the bubble (*see* Ref. 5). Since the aerofoil surface is curved, however, this calculation may be rather laborious as it involves the conformal transformation of the aerofoil shape into a slit, before the Schwarz-Christoffel theorem can be applied. Once available, the solution for the free streamline could be inserted into Stewartson's results[8] to give the solution for the separated laminar-boundary layer.

A knowledge of the stability of a separated layer is then required, and it appears that such a study could be based on existing literature on the stability of the laminar boundary layer between two parallel streams[9 to 12]. However, any developments of the newer theories of the nature of transition to turbulence in the boundary layer which are beginning to emerge[13, 14] may require this approach to be modified.

The fact that the reattached flow behind a bubble at a supersonic leading edge may remain laminar, as mentioned in Section 1, may be explained by Lin's result[11] that, for Mach numbers above 1·7, the laminar flow in the upper of two parallel streams (the velocity of the lower being zero) is completely stable to infinitesimal disturbances.

5. *The Leading-Edge Stall.*—The type of stall associated with a short bubble, usually referred to as the ' leading-edge stall ' occurs when, as the incidence is increased, the bubble after first gradually contracting suddenly ' bursts ' and fails to reattach to the wing surface. These tests have shown how a long bubble grows with incidence at a constant free-stream Reynolds number,

and this flow regime is that associated with the 'thin-aerofoil stall'. Maximum lift occurs when the long bubble extends over most of the chord; the lift curve peak is well rounded. It seems likely that the chordwise length of separated flow in this case is related to the pressure recovery which is required to take place through turbulent mixing as the separated layer reattaches to the wing surface. Thus the criterion for the bursting of a short bubble at a given free-stream Reynolds number may be simply that the change to a long bubble occurs at such an incidence that the necessary pressure recovery (*i.e.*, necessary to bring the pressure back to its value in the absence of the bubble), cannot be attained in the reattachment process, and the bubble extends out into the wake behind the aerofoil. The pressure gradient aft of the peak-suction point, which becomes steeper with incidence, is actually the controlling factor which decides whether this pressure recovery can be made within the aerofoil chord length. The stall observed by Brebner and Bagley[3] on the same model as was used in the present tests, was due to the transition from a short bubble to a long bubble. The resulting long bubble, however, extended over most of the chord length and there was an abrupt loss of lift at this point. The limit of the short bubble regime in fact corresponded to maximum lift at their test Reynolds number of $1 \cdot 6 \times 10^6$. This result therefore lends support to our hypothesis of the leading-edge stall, and furthermore, the ideas put forward above may be used to explain qualitatively the observed scale effect on the maximum lift of aerofoils in this category.

We have seen how a short bubble contracts in length as the Reynolds number is increased at a fixed incidence. The reattached turbulent boundary layer is thinner the shorter the bubble, so that there is a smaller loss in circulation (*i.e.*, lift) due to boundary layer at higher Reynolds numbers. This only partially accounts for the observed increase in $C_{L\,\max}$ with Reynolds number. As the incidence is increased at a fixed Reynolds number, the laminar-separation point moves forward, and $(R_{\delta 1})_S$ decreases, in spite of the fact that V_S increases, as illustrated by the Table in Section 1. Eventually, $(R_{\delta 1})_S$ falls below the critical value for transition to a long bubble. This is essentially the mechanism of the leading-edge stall outlined above. The greatest increase of $C_{L\,\max}$ is shown by aerofoils that begin with a long bubble at low Reynolds numbers and switch over to short bubbles, whereby the lift is considerably increased.

Since $(R_{\delta 1})_S$ is proportional to the square root of the free-stream Reynolds number, any increase in R means that a higher incidence will be reached before $(R_{\delta 1})_S$ falls below the critical value. Thus, as the Reynolds number is increased, an increase in $C_{L\,\max}$ is to be expected.

In some cases the leading-edge stall may be due to another cause. If the adverse pressure gradient aft of peak suction is very steep (as, for instance, in the case of fairly sharp-nosed aerofoils or behind the kink of certain aerofoils with drooped leading edges), it may be impossible to obtain the required pressure recovery *via* the short-bubble phenomenon. Thus the short bubble in such a case may burst, causing the stall, although $(R_{\delta 1})_S$ is still greater than the critical value. Experimental data are now available for such cases (*see* Part II).

6. *Conclusions.*—The results of these tests verify the criterion suggested by Owen and Klanfer as to the type of bubble formed when a laminar boundary layer separates from the surface of a two-dimensional unswept wing: when the boundary-layer Reynolds number based on the displacement thickness at the separation point, calculated from an observed pressure distribution, is greater than 400 to 450, a short bubble is formed with the ratio of bubble length to displacement thickness at separation, $l/(\delta_1)_S$ being of the order of 100. When $(R_{\delta 1})_S$ is less than 400 to 450, a long bubble of separated flow results with $l/(\delta_1)_S$ of the order of 10^3 to 10^4. It is shown that, corresponding to the range of 400 to 450 in $(R_{\delta 1})_S$, there is a range of free-stream Reynolds number over which it is uncertain which type of bubble will be formed.

If the boundary-layer development is calculated from the theoretical inviscid pressure distribution, the critical value of $(R_{\delta 1})_S$ lies within the range 450 to 550, and this result can be used to predict the type of bubble formation to be expected on an arbitrary aerofoil section under two-dimensional unswept conditions.

It is suggested that this criterion is directly related to the phenomenon of the leading-edge stall of moderately thin sections, say 9 to 12 per cent thick. Here a short bubble exists near the leading edge at incidences below the stall, and maximum lift is reached when the bubble bursts, producing the distinctive sharp peak in the lift curve. The bursting of a short bubble in many cases probably coincides with the abrupt transition to a long bubble according to the criterion of Owen and Klanfer, but the resulting long bubble extends into the wake behind the aerofoil. In other cases, the adverse pressure gradient aft of peak suction may be too steep to allow the necessary pressure recovery to be made by means of the short-bubble reattachment process, so that the bubble bursts, although $(R_{\delta 1})_S$ is greater than the critical value.

A hypothesis is also put forward to explain the scale effect on the maximum lift of aerofoils which experience the leading-edge stall, and there is a short discussion of the physical structure of bubbles of separated flow together with an indication of the most profitable lines of attack for future research.

An appendix gives in detail a method of predicting the type of bubble formation to be expected on an arbitrary aerofoil section where pressure distribution measurements are not available.

PART II

Summary.—The characteristics of laminar-separation bubbles on thin and moderately thin aerofoil sections at low speeds are discussed and explained. Further experimental confirmation of Owen's criterion for distinguishing between long and short bubbles is presented. However, despite the fact that Owen's criterion distinguishes between what may be termed the static states of the two kinds of bubble, the results of this investigation also show that there is no universal value of the boundary-layer Reynolds number at separation, $(R_{\delta 1})_S$, at which short bubbles break down into long bubbles, thus while $(R_{\delta 1})_S$ is always less than about 500 when a long bubble is present (*e.g.*, just after the short bubble has burst), the value of $(R_{\delta 1})_S$ for a short bubble which is on the point of bursting may be appreciably more than 500. Analysis of the pressure recovery by turbulent mixing in short bubbles shows that there may be a maximum possible value of σ, the pressure recovery factor, and that once this is reached the flow breaks down into a long bubble. The mechanism of the leading-edge stall can be explained in this way.

A possible reason for the partial success of Owen's criterion is found in the condition for the growth of turbulent spots in the process of transition which was discovered by Schubauer and Klebanoff.

These considerations fill in some of the main gaps in previously published pictures of laminar-separation bubbles so that a reasonably complete and coherent explanation of the characteristics of such flows is now possible.

At the same time this picture of bubble flows enables a more satisfactory explanation to be given of the scale effect on maximum lift and stalling characteristics of those aerofoils which experience such bubble separations.

---

7. *Introduction.*—The pressure distribution over a thin or moderately thin aerofoil (less than 12 per cent thickness/chord ratio, say) at incidence shows a pronounced suction peak near the leading edge with a subsequent steep adverse pressure gradient. This may cause the laminar boundary layer to separate or break away from the upper surface of the aerofoil just behind the suction peak. Below the incidence corresponding to maximum lift the flow reattaches to the surface further downstream as a turbulent boundary layer, and a ' bubble ' is formed on the aerofoil surface. In transonic and supersonic flow the boundary layer may reattach to the surface almost immediately whilst still retaining its laminar nature, as in the process of ' transonic reattachment '.

The present report is mainly concerned with the low-speed case, and the problem is to determine the flow pattern as it is affected by a laminar separation near the leading edge, and to calculate the resulting aerodynamic forces and moments. A similar problem arises with shock-induced separations and this is being studied elsewhere, notably in the Aerodynamics Division of the National Physical Laboratory[18].

The simple two-dimensional case to which we restrict ourselves here is one where we know what the result of a laminar separation will be, namely, a bubble. In other cases where the flow is three-dimensional the result may be either a bubble or a vortex sheet lying above the wing surface (with no bubble) or possibly a combination of both. Thus an important problem is the formulation of a criterion to determine the type of flow arising after a three-dimensional separation (*see* Ref. 19).

In the case of a singular separation on a two-dimensional unswept wing two different types of bubble are possible. Previously they have been distinguished mainly by their length and are therefore called short and long bubbles. A criterion based on the boundary-layer Reynolds number at separation, $(R_{\delta 1})_S$, has been suggested by Owen and Klanfer[1] in an attempt to make a more basic distinction between the two types of bubble. The criterion has been confirmed by wind-tunnel tests[20], and further experimental results[21, 22, 23] have been analysed to produce Table 1. These results show that although Owen's criterion successfully distinguishes between the two types of bubble (provided observed pressure distributions are used in the calculations of boundary-layer development), there is no universal value of $(R_{\delta 1})_S$ at which all short bubbles break down to form long bubbles. Thus while $(R_{\delta 1})_S$ is always less than about 500 whenever a long bubble is present, Table 2 shows that in the case of those short bubbles, where breakdown is imminent, the value of $(R_{\delta 1})_S$ may even be as high as 1,000. Therefore the method given in the appendix for predicting the type of bubble formation from theoretical pressure distributions using Owen's criterion is not strictly correct. However, the results obtained by Schubauer and Klebanoff[24] in their investigations into the mechanism of transition in the boundary layer on a flat plate provide an explanation of the significance of the Reynolds number in Owen's criterion. This is not the complete story therefore; to fill in the picture, it is necessary to study the turbulent mixing region in order to find what pressure recovery may be obtained in this process.

The general problem of the variation in bubble formation with thickness/chord ratio, incidence and Reynolds number is studied in this report, and various theories which have been proposed to explain the bursting of a short bubble (of which the leading-edge stall is a particular case), are critically examined. In this connection careful measurements of the extent of separated flow in short bubbles together with the corresponding pressure distributions have recently become available[23, 26]. Analysis of these results shows that there is a maximum possible value of the pressure recovery factor

$$\sigma = \frac{p_R - p_S}{\frac{1}{2}\rho V_S^2} = \frac{(C_{pR} - C_{pS})}{(1 - C_{pS})}$$

in the free turbulent-mixing process preceding reattachment in a short bubble (subscripts $_R$ and $_S$ refer to conditions at reattachment and separation respectively and C_p is the usual pressure coefficient, $(p - p_0)/(\frac{1}{2}\rho V_0^2)$. It is suggested that this is the cause of the breakdown of a short bubble as either the incidence is increased or the Reynolds number decreased. The relationship with Reynolds number arises because the latter directly controls the length of laminar separated shear layer, *i.e.*, the distance required to develop the turbulent-mixing process is dependent on Reynolds number.

A more complete picture of the phenomenon of laminar-separation bubbles is therefore obtained, and this enables us to explain, for instance, the scale effect on stalling characteristics of thin or moderately thin aerofoils.

8. *Laminar-Separation Bubbles.*—On two-dimensional wings the bubble resulting from a laminar separation near the nose may be one of two distinct types:

(i) A short bubble whose chordwise extent is of the order of 1 per cent of the chord length (say, about 100 times the boundary-layer displacement thickness at separation, $(\delta_1)_S$), which contracts as the incidence is increased. The presence of a short bubble has little effect on the pressure distribution and high peak suctions can be maintained on a thin aerofoil at incidence. As the incidence is increased the peak suction continues to rise even though a short bubble is still present.

or

(ii) A long bubble which is about 2 or 3 per cent of the chord length on formation at low incidences, and grows rapidly with increasing incidence until the separated layer fails to reattach to the aerofoil surface. The bubble then extends out into the wake. In the case of a long bubble the ratio of bubble length to $(\delta_1)_S$ is of the order of 10^4 or more. The pressure distribution when a long bubble is present is very different from that in inviscid flow, and the peak suction near the leading edge collapses. As incidence is increased and the bubble grows, so the suction over the leading edge continues to fall gradually.

It is preferable to distinguish between the two types of bubble on the basis of their different effects on the pressure distribution rather than on their different lengths, since misunderstandings have arisen in attempts to utilise a bent flat plate in experiments on bubble formation. In the tests of Maekawa and Atsumi[27], for instance, where separation is forced at the sharp bend in the plate, unless a high suction can be maintained in the front part of the bubble, a true short bubble cannot be obtained. In any case there is no guarantee that the behaviour of the separated layer will be exactly equivalent to that on an aerofoil.

From the analysis of a large number of experimental observations of bubbles on different aerofoils, Owen and Klanfer[1] were able to suggest a more fundamental criterion to distinguish between the two types. This is based on the calculated boundary-layer Reynolds number at the separation point,

$$(R_{\delta 1})_S = \left(\frac{V\delta_1}{\nu}\right)_S ,$$

where V is the velocity at the edge of the boundary layer and δ_1 is the displacement thickness. The latter is obtained from calculations of the development of the laminar boundary layer by Thwaites's method, which also predicts the position of separation. For a short bubble $(R_{\delta 1})_S$ was always found to be greater than about 500, whereas for a long bubble $(R_{\delta 1})_S < 500$. Now since $(R_{\delta 1})_S$ is proportional to $R^{1/2}$, where R is the free-stream Reynolds number (*see* Appendix), it was deduced that both types of bubble may occur on the same aerofoil depending on the value of R. Tests on a 10 per cent t/c RAE 101 section[20] have borne out this prediction and confirmed Owen's criterion remarkably well, at least for this one section and a rather limited range of Reynolds number.

Further experimental confirmation of Owen's criterion is given in Table 1 which was obtained by analysis of the most recently available tests. These now show that although $(R_{\delta 1})_S > 500$ for short bubbles and $(R_{\delta 1})_S < 500$ when a long bubble is present (based on observed pressure distributions), there is not a universal critical value of $(R_{\delta 1})_S$ for the breakdown of a short bubble. Nevertheless, some of these tests[23, 26] have yielded evidence on which to base a new theory of the mechanism by which a short bubble bursts; this will be discussed in Section 9.

The influence of the type of bubble formation on the stalling characteristics (*i.e.*, the behaviour at maximum lift) of thin aerofoils is now fairly clear and has been described in a classic report by McCullough and Gault[2]. Characteristic lift curves have been replotted from their work in Fig. 5; curve (a) is for a typical rear-stalling aerofoil section (NACA 63_3–018) and is shown for comparison. This type of stall is caused by a turbulent boundary-layer separation spreading forward rapidly but smoothly from the trailing edge with increasing incidence, and hence the lift curve has a well-rounded peak.

Curve (b) illustrates a typical leading-edge stall (the section in this case being NACA 63–009), which is caused by the bursting of a short bubble into a very long one with a marked loss of lift. The phenomenon of the leading-edge stall will be discussed more fully in Section 9.

When a short bubble is present the lift curve is almost the same as if there were no leading-edge separation, because the short bubble has such a small effect on the pressure distribution. If a long bubble is formed at an incidence well below that for maximum lift, the pressure distribution is radically changed but the lift is only slightly smaller than it would have been had the short bubble been maintained. This is the case for curve (c) (NACA 64A006 section), and the slight kink marks the change from a short to a long bubble. Thereafter with increasing incidence the long bubble grows smoothly until maximum lift is attained and the bubble finally extends out into the wake. Curve (c) therefore illustrates a typical 'thin aerofoil' type of stall.

The leading-edge and thin aerofoil types of stall may be subject to a large scale effect; this will be discussed in Section 10.

For a complete understanding it is necessary to have a clear picture of the main physical features of bubble flows. Certain of these are treated more fully in Ref. 28 where the emphasis is on long bubbles; many features are common to both long and short bubbles, however. Thus the flow pattern around an aerofoil with a laminar-separation bubble near the leading edge consists of three regions as sketched in Fig. 6. These are:

(1) The external flow which may be considered as inviscid and treated as a potential flow (but the boundary of which is not a solid surface)

(2) That part of the viscous layer surrounding the body which consists of main-stream air. This is composed of the shear layer outside the dividing streamline, together with the region of turbulent mixing at the rear end of the bubble as well as the wake

(3) Another part of the viscous region which consists of air carried away with the body, *i.e.*, enclosed in the bubble.

Experiments on bubbles (either short bubbles or long bubbles which cover no more than about half the chord length), have demonstrated the following main features:

(1) The pressure over the front part is nearly constant (both along the bubble surface and the aerofoil surface), and the separated boundary layer remains laminar over the length at constant pressure. This implies that the air inside the front part of the bubble is very nearly at rest and the shear layer is quite thin and without turbulent mixing. The boundary of the external stream (*i.e.*, the displacement boundary of the bubble) is curved in a certain manner depending on the level of constant pressure.

(2) There is a pronounced region of pressure rise (jointly along the boundary of the external stream and along the dividing streamline as well as along the aerofoil surface). The beginning of this pressure rise has been demonstrated[23] to coincide with the first appearance of fully developed turbulence in the shear layer. This implies that the boundary of the external stream must straighten out and that there is intense turbulent mixing within the constrained stream tube of region (2) above. An eddy is formed inside the rear part of the bubble; this is necessary for reasons of continuity of mass flow anyway[28].

(3) The pressure over the aerofoil surface behind the bubble (provided the bubble is not too long), is approximately that which would occur if the bubble were not present, *i.e.*, as calculated for the external stream around the aerofoil without a bubble. This implies that the boundary of the viscous region aft of the bubble is not very much different from and roughly parallel to the aerofoil surface. The effect of the bubble is, of course, to thicken the reattached boundary layer, but it is still thin enough for the usual boundary-layer assumptions to be approximately true.

All this requires various matching conditions of the constituent parts of bubble flow outlined above, and all of these regions must be considered if a complete and satisfactory explanation of the phenomenon of laminar-separation bubbles is to be obtained. For instance, the pressure rise in the free stream over the rear part of the bubble must match the pressure recovery experienced by the shear layer after transition to turbulence, and both must be compatible with the turbulent-mixing process of reattachment. Also, conditions in the boundary layer at separation and along the initial separated shear layer must affect the behaviour at transition and the process of reattachment.

So far, Ref. 28 considered mainly the part of region (2) where mixing occurs. An analysis was made of the pressure recovery in a bubble as it is related to the turbulent mixing occurring in the rear part of the bubble. This work fits in with Maskell's model of a long-bubble flow[33] in which only the external stream was considered.

The viscous flow inside the bubble has not yet been considered in detail nor has the question of the stability of the separated shear layer and the mechanism of transition to turbulence in this layer. The problem is to explain the ability of the shear layer to remain laminar for some distance after separation and then begin to 'mix', and also to explain why this mixing occurs shortly after separation in one case and a comparatively long way downstream in another case. In short, the problem is: why are two completely different types of bubble possible ?

The main gaps in previously published pictures of bubble flows can be filled in by

(1) the proposed hypothesis concerning the mechanism of the bursting of a short bubble

(2) the results of the Schubauer-Klebanoff investigation into the mechanism of transition.

These points will be explained in the following Section, after a critical discussion of various theories of the mechanism of the leading-edge stall or abrupt change from a short to a long bubble.

9. *Mechanism of the Leading-Edge Stall.*—The leading-edge stall occurs when the short bubble suddenly bursts with an abrupt loss of lift, thus giving the characteristic sharp peak to the lift curve. The breakdown of the short bubble in a leading-edge stall is another example of the change to a long bubble as explained in Ref. 20, but the aerofoil incidence is so high that the resulting long bubble fails to reattach onto the aerofoil surface or at least does so only very near to the trailing edge.

Wallis[21] and others at the Australian A.R.L. have found that the reattached turbulent boundary layer just downstream of a short bubble on a NACA 64A006 section exhibits a peak in the growth of the shape parameter H (the ratio of displacement and momentum thicknesses). There is therefore a region of incipient turbulent separation just aft of the short bubble and Wallis suggests that this provides the mechanism of the leading-edge stall. As the incidence is increased the peak value of H approaches the value associated with turbulent separation; Wallis postulates, however, that the flow pattern involving a short bubble, a short length of attached turbulent boundary layer and then complete turbulent separation, is unstable, so that the flow breaks away completely at the laminar separation point and a long bubble is formed.

However, the region just downstream of a short bubble is initially a transition from the mixing region to an ordinary turbulent boundary layer and the velocity profiles in this region are not of the conventional type. In such circumstances it is difficult to know what interpretation is to be placed on the values of H measured in this region. In any case the example discussed by McCullough and Gault[2] (NACA 63–009 section) does not support this theory, since no region of incipient turbulent separation just aft of the reattachment of a short bubble was revealed; nor did McGregor[26] find this tendency to separate in the reattached turbulent layer on a Piercy aerofoil. Thus the result observed by Wallis on one particular aerofoil cannot be generalized to give an explanation of the mechanism of the leading-edge stall.

Another theory of this mechanism has been proposed by McGregor[26], who examined the balance of energy of the standing eddy in the rear part of a short bubble. The bursting of the short bubble is said to occur at the stage where the supply of kinetic energy from the separated boundary layer is insufficient to maintain the eddy against the dissipation of energy by diffusion so that it disintegrates. It is probably true that the circulatory flow in the rear part of the bubble must be maintained in equilibrium in some such fashion as McGregor suggests. However, as emphasized in the previous Section, a complete picture can only be obtained by considering together all the constituent parts of the bubble flow; McGregor's theory does not supply this demand because it considers only one part of the problem and cannot therefore ensure that all the different regions are compatible with each other. Generally, it would seem that the theory might be able to explain the bursting of short bubbles as a phenomenon where equilibrium is seriously upset, but for short bubbles away from the bursting point and for long bubbles, the eddy always seems to accommodate itself somehow. It is not hopeful, therefore, to start building up a theory from this end. This approach may be necessary when the bubble reaches up to the trailing edge or extends beyond it so that two eddies are formed, and also when the problem of buffeting is being considered.

All the available experimental evidence serves to confirm Owen's criterion, namely, that when a long bubble is present $(R_{\delta 1})_S$ is always less than about 500, and for short bubbles $(R_{\delta 1})_S$ is greater than 500. However, a short bubble flow does not always break down into a long bubble pattern at the same value of $(R_{\delta 1})_S$. The recent results obtained by Schubauer and Klebanoff offer an explanation of why Owen's criterion does distinguish between the two types of bubble, but the criterion does not explain the abrupt breakdown of the short-bubble regime. Careful measurements of the extent of the separated region in short bubbles, covering a wide range of incidence and Reynolds number on different aerofoils, reported in Refs. 23 and 26, have enabled the pressure recovery factor

$$\sigma = (C_{pR} - C_{pS})/(1 - C_{pS})$$

in the reattachment process to be calculated much more accurately than had been possible before. The results of this analysis are plotted in Figs. 7 and 8, and show that as the short bubble approaches the bursting point, either by increasing the aerofoil incidence or by decreasing the Reynolds number at constant incidence, the pressure recovery in the turbulent mixing region (Fig. 3 of Ref. 28) tends to a maximum value of about $\sigma = 0 \cdot 35$ in all cases. Such a possibility was predicted in Ref. 20. Now the pressure recovery in the flow through a pipe after a sudden enlargement (Borda-Carnot shock), has a maximum theoretically possible value of $\sigma = 0 \cdot 5$ (for an area ratio of 2), so that it seems physically reasonable for the turbulent-mixing process causing reattachment in a bubble to provide only a certain maximum pressure recovery. This suggests a possible mechanism of the leading-edge stall.

Conditions in the boundary layer at separation, which may be defined by $(R_{\delta 1})_S$, determine the length of laminar separated layer in the front or constant-pressure part of the bubble, and this in turn determines the amount of pressure recovery necessary for reattachment as a turbulent boundary layer to occur. After reattachment the pressure distribution returns to that found for attached flow, or very nearly does so. If this necessary value of σ is less than the suggested maximum possible, then the high suction in the front part of the bubble can be maintained and the bubble will be of the short type. As the incidence is increased, the inviscid pressure distribution shows higher suction peaks and subsequently steeper adverse pressure gradients. Although the short bubble contracts somewhat with incidence, the pressure recovery necessary for reattachment still rises, until eventually the maximum possible value of σ is reached. The process is illustrated in Fig. 19.

Similarly, with decreasing Reynolds number at a fixed incidence the value of σ rises (as shown in Fig. 10), since $(R_{\delta 1})_S$, which is proportional to $R^{1/2}$, falls and the length of laminar flow in the separated shear layer increases. Thus, although the suction peak is slightly lower for a lower Reynolds number (there is a greater loss of circulation due to the thicker boundary layer), the pressure recovery necessary for reattachment may eventually reach the maximum possible value.

In both cases the flow breaks down completely and the peak suction collapses. The value of $(R_{\delta 1})_S$ also falls to a much lower level, so that a greater length of laminar separated layer is obtained. With the completely redistributed pressures over the aerofoil surface it may be possible for the flow to reattach much further aft with a pressure recovery of $\sigma = 0 \cdot 35$, say, and thus form a long bubble. Although it is difficult to define accurately or measure the reattachment point in a long bubble, all the experimental evidence[28] shows that the value of σ is of this order, provided the bubble does not extend over much more than about half the chord and divergence of the trailing-edge pressure has not occurred[18].

A possible explanation of the hysteresis effect with varying Reynolds number which has been observed on thin aerofoils (*see*, for example, Ref. 22), may now be given. Assume that an aerofoil exhibits a short bubble in a test at high Reynolds number. Upon decreasing R a long bubble is formed at some stage according to the process outlined above. The pressure distribution is now much different and the value of $(R_{\delta 1})_S$ is much lower. As R is now increased again, the bubble length becomes gradually smaller until initial conditions are eventually reached, but this may require a value of R higher than that at the start of the test. The hysteresis effect with varying incidence but at constant R might be explained in a similar fashion. In this case the change back to a short bubble takes place at a much lower (decreasing) incidence than that at which the short bubble collapsed on increasing incidence.

The success achieved by the use of Owen's criterion in distinguishing between the two types of bubble may be connected with the results obtained by Schubauer and Klebanoff[24] in their investigation of the mechanism of transition in the boundary layer on a flat plate. This would also fill in the main gap in an overall picture of bubble flows, namely, the reason for the disparate lengths of laminar separated flow in the two different types of bubble. It is now known that transition in a laminar boundary layer can be caused by turbulent spots which, once formed, may grow as they move downstream with the fluid. The initial cause of such turbulent spots is not yet fully established, although Dryden[25] states that Görtler's theory is probably the most nearly correct. This is that flow curvature may generate three-dimensional vortices of the type observed in flow over concave surfaces for which Görtler himself gave the theory. However that may be, research at the National Bureau of Standards indicates that turbulent spots in the boundary layer on a flat plate do not grow if the Reynolds number based on displacement thickness, $R_{\delta 1} = V\delta_1/\nu$, is less than 450, where V is the velocity at the edge of the boundary layer. Thus if $R_{\delta 1} > 450$ at the laminar separation point on a thin aerofoil at incidence, we would expect transition to turbulence to occur shortly after separation. Turbulent mixing then sets in, the flow returns to the surface as a turbulent boundary layer and a short bubble is formed. The sequence of events in the formation of a long bubble, according to the mechanism suggested above, could be as follows:

(1) A short bubble is no longer possible due to the maximum possible value of σ having been exceeded.

(2) The peak suction collapses and as a result the value of $(R_{\delta 1})_S$ falls to below 450 say.

(3) Thus turbulent spots are unable to grow and the separated shear layer remains laminar. Now in his experimental investigation of short bubbles McGregor[26] found only a very small growth in the displacement thickness of the separated shear layer along the outer edge of the bubble from separation up to the onset of turbulence. This is likely to be qualitatively true in the case of long bubbles also.

(4) Eventually, however, $R_{\delta 1}$ will become greater than 450 so that turbulent spots can now grow, and finally turbulent mixing occurs.

(5) Now the pressure distribution is such that reattachment can occur with a small enough value of the pressure recovery factor, σ.

Thus all three regions have accommodated themselves again so that a possible flow results.

We must bear in mind that the condition for growth of turbulent spots was formulated for an attached boundary layer; the situation may be different for a shear layer away from the solid surface. The problem of the stability of laminar wakes is being studied by McKoen[29] and Curle[30], for instance, and their results should later be applied to our particular problem.

The characteristic differences in the behaviour of short and long bubbles with variations of Reynolds number and incidence may be explained on this basis. Firstly, both types of bubble contract with increasing Reynolds number at constant incidence. Since $(R_{\delta 1})_S$ is proportional to $R^{1/2}$, it is clear that a shorter length of laminar flow in the separated layer is to be expected for a higher free-stream Reynolds number, and so the bubble itself becomes shorter. The effect on short bubbles is seen in Figs. 10 and 11.

Secondly, short bubbles contract with increasing incidence at a fixed Reynolds number as illustrated in Fig. 9. The external flow is predominantly responsible for this phenomenon; as the incidence rises so does the peak suction on a thin aerofoil, *i.e.*, the level of suction in the front half of a short bubble rises rapidly. Thus a more highly curved flow around the bubble is necessary to maintain this higher suction. Some of McGregor's results on a Piercy aerofoil (approx. 10 per cent thick) illustrate this trend. The mechanism could be a secondary factor contributing to the contraction of a short bubble as R is increased. There is less loss of lift due to boundary-layer effects at higher Reynolds numbers, and so the peak suction is higher. The level of pressure in the front part of the bubble is lower, the curvature of the flow is higher and therefore the bubble is shorter.

On the other hand, long bubbles expand with increasing incidence at a constant Reynolds number. This is again mainly the effect of the external stream; in fact, this result comes out from an application of Maskell's hodograph method[33]. It might also have been expected from simple geometrical considerations, since to a first approximation the height of the bubble dividing streamline above the aerofoil surface is proportional to α. Assuming a constant angle of spread of the turbulent-mixing region, it follows that the length of a long bubble increases with incidence. In addition, $(R_{\delta 1})_S$ falls with increasing incidence in the long-bubble regime, so that a longer length of laminar separated layer is possible. The whole process is probably iterative until equilibrium is established between the chordwise pressure distribution and the boundary-layer development, *i.e.*, between the suction in the forward part of the bubble, the shape of the displacement surface, the length of laminar separated layer and the value of $(R_{\delta 1})_S$ (which defines the stability of the latter).

10. *Scale Effect on Maximum Lift and Stalling Characteristics.*—The stalling characteristics of an aerofoil are here defined as the behaviour at or near the maximum-lift condition. On the basis of the arguments of the preceding Sections, the effect of Reynolds-number variation on maximum-lift and stalling characteristics of aerofoil sections up to about 12 per cent t/c ratio at low speed may be explained. Possible lift curves for such an aerofoil at three different Reynolds numbers are sketched in Fig. 13. For the lowest value of R the curve (c) shows a kink at a certain (fairly low) incidence representing the change from a short to a long bubble. This curve corresponds to curve (c) of Fig. 5 which was discussed in Section 8. An increase in R shortens the length of laminar separated layer over the front part of the bubble so that the pressure recovery ratio σ is reduced. Thus a higher incidence may be attained before the maximum possible value of σ is reached and the bubble bursts into the long type. On the other hand, the thinner boundary layer at a higher Reynolds number means that there is less loss of lift due to viscous effects, so that at a given incidence the suction peak is higher and the following adverse pressure gradient is steeper. Thus σ will be higher on this score, but it may be expected to be only a secondary effect in most cases.

If the Reynolds number is sufficiently high, the change in bubble formation may be postponed to such a high incidence that the resulting long bubble covers most of the chord or even extends out into the wake, so that maximum lift has been attained. Thus curve (b) illustrates a typical leading-edge stall and corresponds to curve (b) of Fig. 5.

The curve (a) representing the highest free-stream Reynolds number shows a typical rear-stalling behaviour. For this case R may be so high that transition to turbulence occurs in the boundary layer ahead of the laminar-separation point. Also, some cambered aerofoils, whilst exhibiting a short bubble near the leading edge at incidence, stall in this manner due to a turbulent separation spreading forward rapidly from the trailing edge.

It must be remembered that the curves of Fig. 13 represent the same aerofoil at different Reynolds numbers. It is still possible for a thin aerofoil with leading-edge stalling behaviour to attain a higher $C_{L\max}$ than a thicker aerofoil with a rear-stalling behaviour. In Fig. 13, however, the rear-stall pattern produces the highest $C_{L\max}$.

At a given incidence below the stall the difference in lift coefficient, C_L, between the long-bubble flow of curve (c) and the flows at higher Reynolds numbers may be explained, according to Maskell[34], almost entirely by a consideration of the wake circulation. Whatever the type of stall, the flow patterns should eventually be similar on the same aerofoil, and so we expect all three lift curves to run together beyond the incidences for maximum lift at each Reynolds number. Similar ideas to these have been arrived at independently by McGregor[26].

The possibility of curve (c), the trailing-edge stall, being obtained, may be estimated in some cases. Available evidence[31] indicates that if $(R_{\delta 1})_S$ is less than about 2,700, transition will not occur forward of the laminar separation point. The development of a turbulent boundary layer may be calculated fairly accurately, and an estimate made of whether turbulent separation occurs ahead of the trailing edge. This is no longer true if a short bubble is present; the difficulty lies in our inability to choose accurately enough the values of the boundary-layer parameters (such as H) at reattachment, expecially for higher incidences. In any case, roughness of the aerofoil surface or free-stream turbulence may vitiate the calculations. In fact, artificial roughness may be a way of delaying the formation of a long bubble by decreasing the stability of the boundary layer at the laminar-separation point, or even by inducing transition ahead of the latter. For instance, Wallis[21] has demonstrated that blowing out of small holes just behind the leading edge on the lower surface is an effective way of inhibiting the leading-edge stall. Wallis then found that a turbulent separation spreading forward rapidly from the trailing edge (on a NACA 64A006 section) curtailed much further increase in maximum lift. This was delayed in turn by blowing out of holes on the upper surface just behind the leading edge, so that energy was fed into the turbulent boundary layer to enable it the better to withstand the severe adverse pressure gradients.

This is yet another way of preventing long-bubble formation, *i.e.*, by feeding energy into the mixing region prior to reattachment. Thus there would now be no attempt to directly influence conditions upstream of separation and a bubble would still be formed. Then tangential blowing out of holes in the upper surface just behind the separation point allows a greater pressure recovery in the reattachment process by intensifying the turbulent mixing. High suctions can be maintained around the leading edge, and a simple example serves to show that this could be an appreciable effect. If the pressure recovery factor σ is increased from $0 \cdot 35$ to $0 \cdot 6$, say, then assuming that C_p at reattachment remains constant at $-1 \cdot 0$, C_p at separation falls from -2 to -4. Such effects have been observed by Williams at the N.P.L. and it is hoped to analyse the results of these experiments in a later report.

Again, at higher Reynolds numbers (if obtained by increased speeds), the effect of compressibility may have to be considered. At high subsonic Mach numbers the long bubble on a thin aerofoil at incidence is inhibited by the process of transonic reattachment. The possibility of this occurring in a given case may be estimated, however, since the peak suction coefficient, and thus the maximum local Mach number, will be known at any particular incidence and free-stream Mach number.

Finally, for sharp-nosed sections, since separation is forced at the leading edge where there is theoretically an infinite velocity in inviscid flow, we would expect a thin aerofoil type of stall with a long bubble at low speeds, *i.e.*, at Mach numbers below that for transonic reattachment.

11. *Conclusions.*—Further experimental evidence has been presented which again confirms Owen's criterion as a useful practical rule for distinguishing between long and short bubbles of separated flow. However, these results show that contrary to previous suggestions there is no universal critical value of $(R_{\delta 1})_S$ for the breakdown of a short bubble. As soon as a short bubble has burst the value of $(R_{\delta 1})_S$ falls to below about 500, although immediately prior to breakdown $(R_{\delta 1})_S$ may be much greater than 500.

Careful measurements of the extent of separated flow in some of these experiments have enabled fairly accurate estimates to be made of the pressure recovery in the free turbulent-mixing region as the short bubble reattaches. These show that there seems to be a maximum possible pressure recovery that can be obtained in this process and that once this is exceeded this particular flow is no longer possible and the bubble bursts. Due to the consequent redistribution of the loading on the aerofoil, the flow is able to reattach as a long bubble.

The partial success of Owen's criterion, or rather the significance of the Reynolds number in this criterion might be explained by the condition discovered by Schubauer and Klebanoff for the growth of turbulent spots in a boundary layer, which is now believed to be part of the mechanism of transition. However, the applicability of this theory to a separated shear layer away from the aerofoil surface remains to be proved.

Some of the main gaps in previously published pictures of laminar separation bubbles can be filled in as a result of these considerations. Details of the viscous flow inside the bubble (including the standing eddy at the rear of the bubble) need much closer examination, but even so a reasonably coherent picture of the flow patterns involving bubbles has been given. From this it is possible to explain the scale effect on maximum lift and stalling characteristics of aerofoils experiencing such bubble separations.

It is, however, still not possible to predict the complete lift curve, expecially $C_{L\max}$; this is mainly due to our lack of knowledge of the actual internal flow of a bubble, but also more information is needed about the pressure recovery, σ, in the reattachment process.

Only the low-speed case of two-dimensional straight wings has been treated here, so that an important problem which remains is the study of three-dimensional separations.

REFERENCES

No.	*Author*	*Title, etc.*
1	P. R. Owen and L. Klanfer.. ..	On the laminar boundary-layer separation from the leading edge of a thin aerofoil. C.P. 220. October, 1953.
2	G. B. McCullough and D. E. Gault	Examples of three representative types of aerofoil-section stall at low speed. N.A.C.A. Tech. Note 2502. September, 1951.
3	G. G. Brebner and J. A. Bagley ..	Pressure and boundary-layer measurements on a two-dimensional wing at low speed. R. & M. 2886. February, 1952.
4	L. F. Crabtree and J. A. Bagley ..	Brief tests of forward-mounted flaps on an aerofoil at high incidences. R.A.E. Tech. Note Aero. 2311. A.R.C. 17,400. August, 1954.
5	L. C. Woods	Theory of aerofoils on which occur bubbles of stationary air. R. & M. 3049. November, 1953.
6	J. Black and R. D. Hunt	Some measurements of leading-edge separation bubbles on a 10 per cent thick symmetrical aerofoil. *J. R. Aer. Soc.* Vol. 57. p. 819. 1953.
7	W. J. Bursnall and L. K. Loftin, Jr.	Experimental investigation of localized regions of laminar boundary-layer separation. N.A.C.A. Tech. Note 2338. April, 1951.
8	K. Stewartson	On the flow downstream of separation in an incompressible fluid. *Proc. Camb. Phil. Soc.* Vol. 49. p. 561. 1953.
9	W. Wasow	The complete asymptotic theory of a fourth-order differential equation of hydrodynamics. *Ann. Math.* Vol. 49. p. 852. 1948.
10	Martin Lessen	On the stability of the free laminar boundary layer between parallel streams. N.A.C.A. Tech. Note 1929. August, 1949.
11	C. C. Lin	On the stability of the laminar mixing region between two parallel streams in a gas. N.A.C.A. Tech. Note 2887. January, 1953.
12	R. C. Lock	Hydrodynamic stability of the flow in the laminar boundary layer between parallel streams. *Proc. Camb. Phil. Soc.* Vol. 50. p. 105. 1954.
13	T. Theodorsen	Mechanism of turbulence. Proceedings Second Midwestern Conference on Fluid Mechanics, 1952. Also A.R.C. 16,615. March, 1954.
14	Max. M. Munk	On the mechanism of turbulent fluid motion. Proceedings Second Midwestern Conference on Fluid Mechanics, 1952. Also A.R.C. 16,616. March, 1954.
15	B. Thwaites	Approximate calculation of the laminar boundary layer. *Aero. Quart.* Vol. 1. pp. 245 to 280. 1949.
16	Th. von Kármán and C. B. Millikan	On the theory of laminar boundary layers involving separation. N.A.C.A. Report 504. 1934.
17	J. Weber	The calculation of the pressure distribution over the surface of two-dimensional and swept wings with symmetrical aerofoil sections. R. & M. 2918. July, 1953.
18	H. H. Pearcy	The occurrence and development of boundary-layer separations at high incidences and high speeds. A.R.C. 17,901. September, 1955.
19	E. C. Maskell	Flow separation in three dimensions. R.A.E. Report Aero. 2565. A.R.C. 18,063. November, 1955.
20	L. F. Crabtree	The formation of regions of separated flow on wing surfaces. Part I: Low-speed tests on a two-dimensional unswept wing with a 10 per cent thick RAE 101 section. November, 1954. First part of this R. & M.

No.	Author	Title, etc.
21	R. A. Wallis	Experiments with air jets to control the nose stall on a 3-ft chord NACA 64A006 aerofoil. A.R.L. (Australia) Aerodynamics Note 139. September, 1954. A.R.C. 17,311. January, 1955.
22	G. B. McCullough	The effect of Reynolds number on the stalling characteristics and pressure distributions of four moderately thin airfoil sections. N.A.C.A. Tech. Note 3524. November, 1955.
23	D. E. Gault	An experimental investigation of regions of separated laminar flow. N.A.C.A. Tech. Note 3505. September, 1955.
24	G. B. Schubauer and P. S. Klebanoff	Contributions to the mechanics of boundary-layer transition. N.A.C.A. Tech. Note 3489. September, 1955. Also Proceedings of Symposium on Boundary-Layer Effects in Aerodynamics, N.P.L. March, 1955.
25	H. L. Dryden..	Recent investigations of the problem of transition. *Z.F.W.* Vol. 4. pp. 89 to 95. March/April, 1956.
26	I. McGregor	Regions of localised boundary-layer separation and their role in the nose stalling of aerofoils. Ph.D. Thesis, Queen Mary College, London, E.1. June, 1954.
27	T. Maekawa and S. Atsumi ..	Transition caused by the laminar-flow separation. N.A.C.A. Tech. Memo. 1352. 1952. (Translated from *J. Soc. App. Mech.*, Japan. Vol. 1. No. 6. pp. 187 to 192. November, 1949.)
28	J. F. Norbury and L. F. Crabtree ..	A simplified model of the incompressible flow past two-dimensional aerofoils with a long-bubble type of flow separation. R.A.E. Tech. Note Aero. 2352. A.R.C. 17,945. June, 1955.
29	C. H. McKoen	On the stability of a laminar wake. C.P. 303. October, 1955.
30	N. Curle	Hydrodynamic stability of laminar wakes. A.R.C. 18,275. March, 1956.
31	L. F. Crabtree	Prediction of transition in the boundary layer on an aerofoil. *J. R. Ae. Soc.* Vol. 62, p. 525. 1958.
32	N. Curle and S.W. Skan	Approximate methods for predicting properties of laminar boundary layers. *Aero. Quart.* Vol. VIII. Part 3. August, 1957.
33	E. C. Maskell	A theoretical treatment of the thin-aerofoil stall. (R.A.E. Report to be published.)
34	E. C. Maskell	Wake circulation and its influence on lift in two-dimensional compressible flow. (R.A.E. Report to be published.)

(75238) B*

APPENDIX

A Method for Predicting the Type of Bubble Separation on a Two-Dimensional Unswept Wing of Arbitrary Section Shape

Thwaites[15] has given a formula for the momentum thickness, δ_2, of the laminar boundary layer on an aerofoil surface as follows:

$$R\left(\frac{\delta_2}{c}\right)^2 = 0\cdot 45\left(\frac{V}{V_0}\right)^{-6}\int_0^{s/c}\left(\frac{V}{V_0}\right)^5 d\left(\frac{s}{c}\right) = g\left(\frac{s}{c}\right), \qquad (1)$$

where

$$R = \frac{V_0 c}{\nu}.$$

Hence

$$\frac{\delta_2}{c} = \frac{1}{R^{1/2}}\left\{g\left(\frac{s}{c}\right)\right\}^{1/2} \qquad (2)$$

and the pressure-gradient parameter is given by

$$m = -\frac{\delta_2{}^2}{\nu}\frac{dV}{ds} = -g\left(\frac{s}{c}\right)\frac{d\left(\dfrac{V}{V_0}\right)}{d\left(\dfrac{s}{c}\right)}. \qquad (3)$$

Separation of the laminar boundary layer is taken to occur when $m = 0\cdot 082$. Empirically it is found that at separation,

$$H(m) = \frac{\delta_1}{\delta_2} = 3\cdot 7\,, \qquad (4)$$

so that knowing the momentum thickness of the boundary layer at the separation point the displacement thickness is easily obtained, *i.e.*,

$$\left(\frac{\delta_1}{c}\right)_S = \frac{3\cdot 7}{R^{1/2}}(g_S)^{1/2}. \qquad (5)$$

Hence

$$(R_{\delta 1})_S = \left(\frac{V\delta_1}{\nu}\right)_S = \left(\frac{V}{V_0}\right)_S\left(\frac{\delta_1}{c}\right)_S R$$

$$= 3\cdot 7\left(\frac{V}{V_0}\right)_S (g_S)^{1/2} R^{1/2}. \qquad (6)$$

A simpler but more approximate method of predicting the laminar separation point is due to von Kármán and Millikan[16]. This method predicts separation at the point where the velocity over the surface of the aerofoil has fallen to $0\cdot 89$ of the maximum velocity. Some further remarks on this method have been made by Owen and Klanfer.[1]

The procedure for predicting the type of bubble separation on a two-dimensional unswept wing is thus as follows:

(1) Calculate the theoretical inviscid velocity distribution, V/V_0, as a function of chordwise position x/c, e.g., by the method of Ref. 17.

(2) Calculate the distance along the aerofoil surface from the leading edge, s/c, from the given section ordinates, as a function of x/c.

(3) Plot V/V_0 against s/c and determine the stagnation point, which then becomes the zero of the s/c scale.

(4) Tabulate V/V_0 against s/c from the stagnation point ($s/c = 0$), round the leading edge and along the upper surface to a short distance behind the peak-suction point. The values are read off from the curve of (3), and at least two decimal places in V/V_0 are required.

(5) Calculate $\int_0^{s/c} \left(\frac{V}{V_0}\right)^5 d\left(\frac{s}{c}\right)$ from $s/c = 0$, and hence $g(s/c)$ and m according to equations (1) and (3), up to the point where $m = 0 \cdot 082$.

(6) This is the laminar separation point, at which $R_{\delta 1}$ is obtained from equation (6), in terms of $R^{1/2}$.

Now the position of laminar separation is independent of Reynolds number (neglecting the loss in circulation due to the boundary layer, which affects the position of separation through its effect on the pressure distribution round the aerofoil). Thus the calculation so far is independent of R, and for a given incidence δ_2/c, δ_1/c and $(R_{\delta 1})_S$ can be calculated for any value of the free-stream Reynolds number.

The experimental results discussed in Part I show that for $(R_{\delta 1})_S > 550$ a short bubble will be formed, and for $(R_{\delta 1})_S < 450$ a long bubble. For $450 < (R_{\delta 1})_S < 550$, either type of bubble is possible.

However, the later results and analysis of Part II show that for a short bubble which is on the point of bursting, the value of $(R_{\delta 1})_S$ may be appreciably more than 500. The most that can be said, therefore, is that $(R_{\delta 1})_S$ is always less than about 500 when a long bubble is present.

TABLE 1

Values of Boundary-Layer Reynolds Number at Separation and Bubble Length on an RAE 101 Section of 10 per cent t/c Ratio

Case Number	$R \times 10^{-6}$	$\frac{V_s}{V_\infty}$	$\frac{l}{(\delta_1)_s}$	$\frac{V_s l}{\nu} \times 10^{-3}$	$(R_{\delta_1})_s$ (Observed pressure distribution)	$(R_{\delta_1})_s$ (Theoretical pressure distribution)
1	1·566	2·80		35	464	
2	1·215	2·83		34	412	472*
3	1·308	1·92	2500	750	302	
4	1·934	3·08	90	48	475	
5	1·710	3·03	70	52	428	565*
6	1·503	2·93	80	53	431	530*
7	1·247	1·725	2000	645	342	
8	1·164	2·67	60	31	493	
9	1·885	2·89	70	44	607	
10	1·177	2·685	70	38	454	
11	1·918	3·00	80	46	503	
12	0·40	2·01	400	80	210	
13	0·40	1·67	800	147	192	
14	0·40	1·58	1500	215	168	

* = Imminent change to long bubble.

TABLE 2

Experimental Confirmation of Owen's Criterion

Aerofoil	Test $\alpha°$	$R \times 10^{-6}$	$(R_{\delta 1})_s$	Type of bubble
RAE 101 6 per cent t/c (Unpublished tests)	2		1050	Short
	3		540	Short
	4		460	Short
	5	2·4	460	Short
	5·7		420	Short
	5·8		420	Short*
	5·8		300	Long
NACA 64A006 (Ref. 7)	7·2	1·9	460	Short*
	8·2		350	Long
NACA 0006 (Ref. 8)	6·0	2·71	430	Short*
	6·5		310	Long
	6·0	4·58	660	Short
	6·5		630	Short*
	7·0		360	Long
NACA 0007 (Ref. 8)	7·5	2·0	625	Short*
	7·75		350	Long
	7·5	4·0	790	Short
	8·0		780	Short*
	8·25		350	Long
	7·5	6·0	1030	Short
	8·0		1015	Short*
	8·25		435	Long
NACA 0007·5 (Ref. 8)	8·0	2·0	700	Short*
	9·0	4·0	690	Short*
	9·5	6·0	830	Short*
NACA 0008 (Ref. 8)	9·0	6·0	1030	Short*
NACA 0010 (mod) (Ref. 9)	4	1·5	800	Short
	8		740	Short
	12		760	Short*
	4	3·0	1040	Short
	8		1050	Short
	12		1170	Short
	4	6·0	1500	Short
	8		1440	Short
	12		1740	Short

An asterisk denotes conditions at or very close to maximum lift at that particular Reynolds number.

All the values of $(R_{\delta 1})_s$ quoted were obtained by calculations of the laminar boundary-layer development by Thwaites's method based on observed pressure distributions. The modified form of Thwaites's method recommended by Curle[18] (*i.e.*, separation at $m = 0 \cdot 090$ with $H = 3 \cdot 55$) should have very little effect on the values of $(R_{\delta 1})_s$ quoted above.

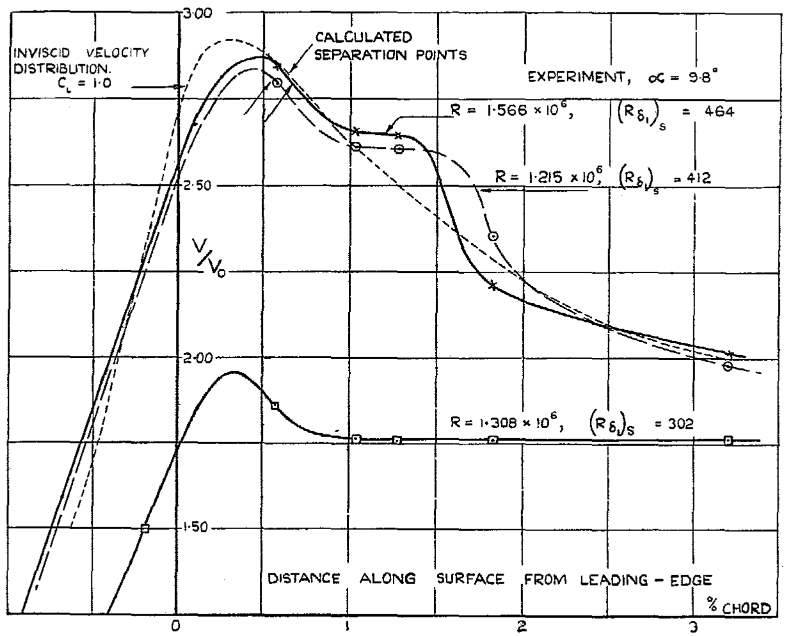

FIG. 1. Transition from short to long bubble (RAE 101; $t/c = 0 \cdot 10$).

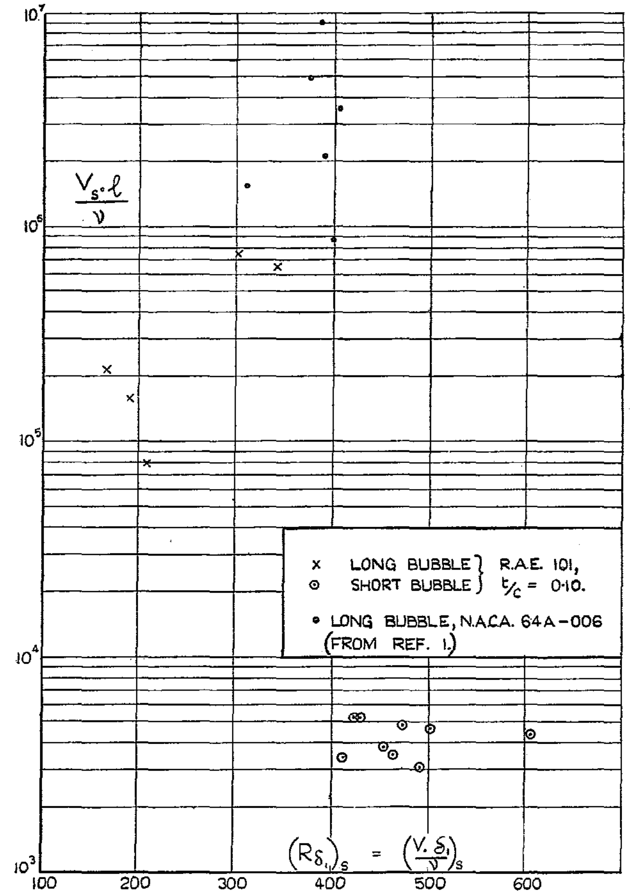

FIG. 2. The two regimes of separation bubbles on aerofoils.

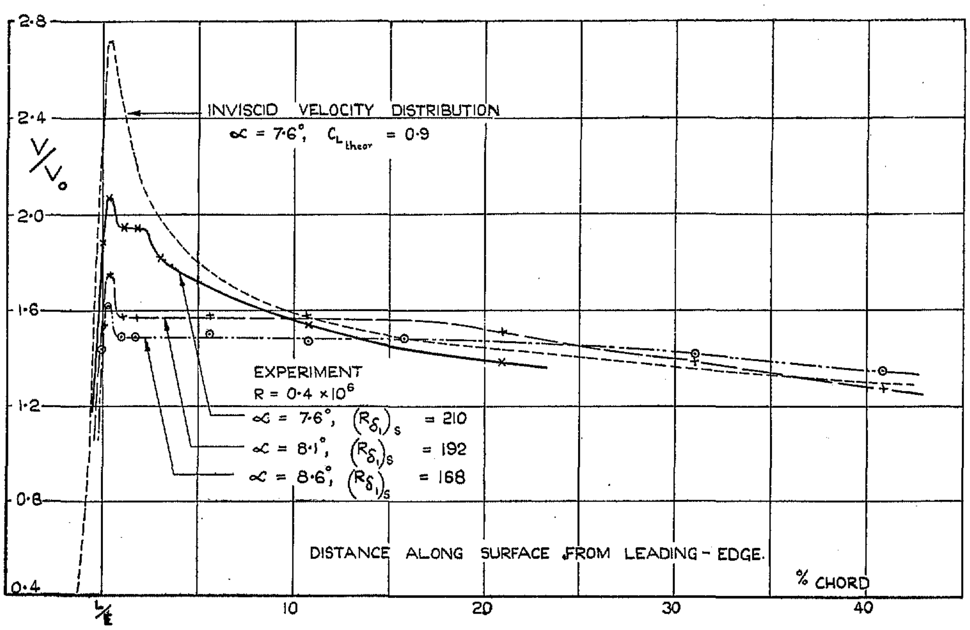

FIG. 3. Growth of long bubble with incidence (RAE 101 section; $t/c = 0·10$).

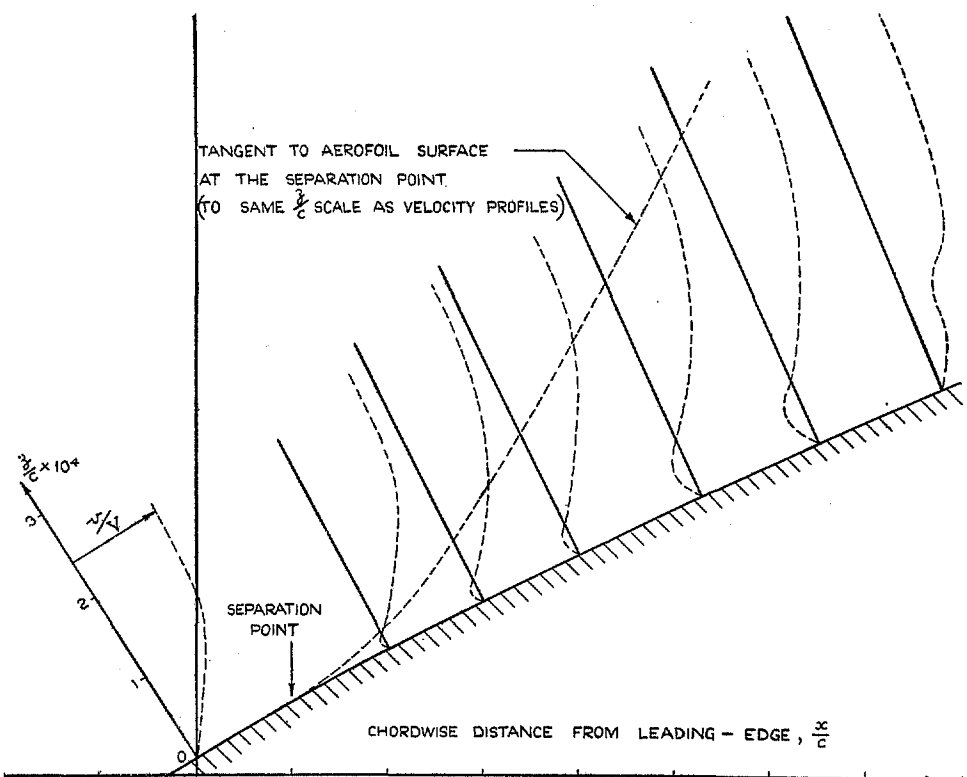

FIG. 4. Velocity profiles in a typical short bubble (NACA 63–009 section; $\alpha = 4$ deg.—From Ref. 2).

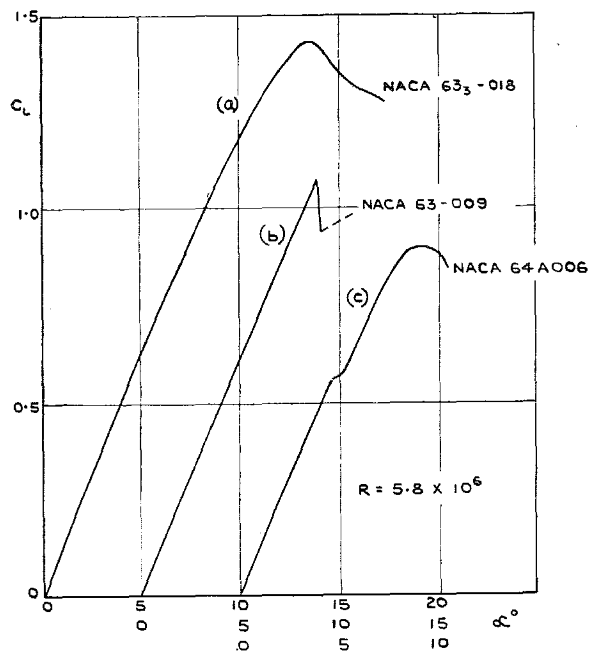

FIG. 5. Lift curves illustrating three types of aerofoil-section stall (From Ref. 2).

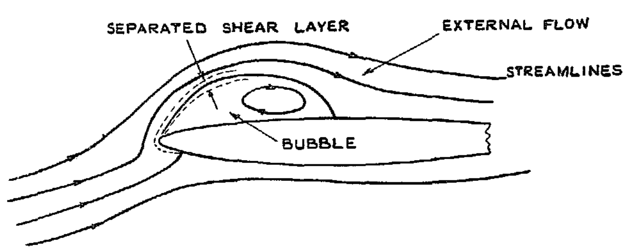

FIG. 6. Bubble formation on a thin aerofoil at incidence (Schematic).

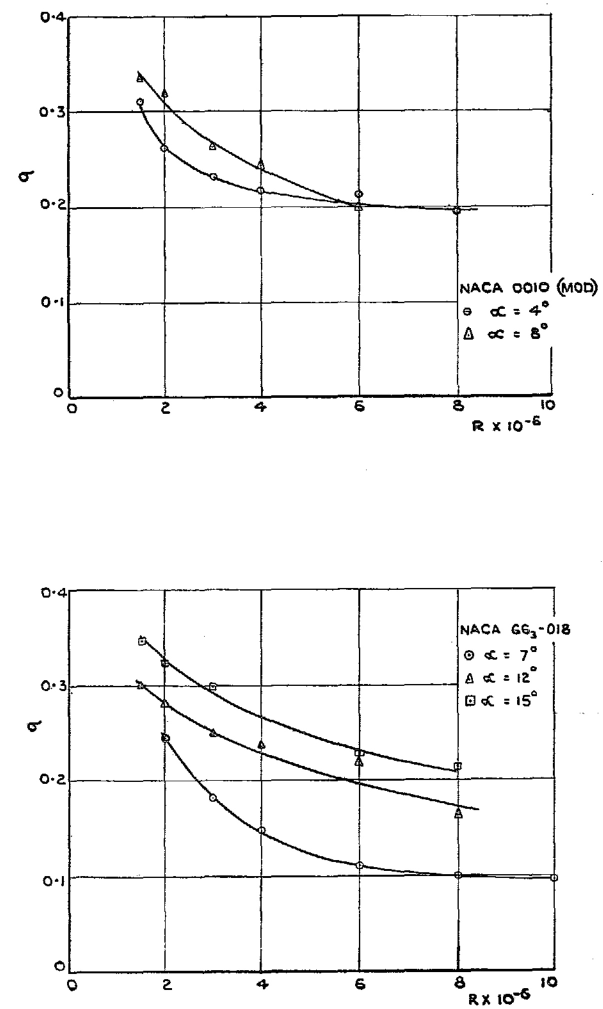

FIG. 7. Variation of pressure-recovery factor σ in short bubbles (From Ref. 23).

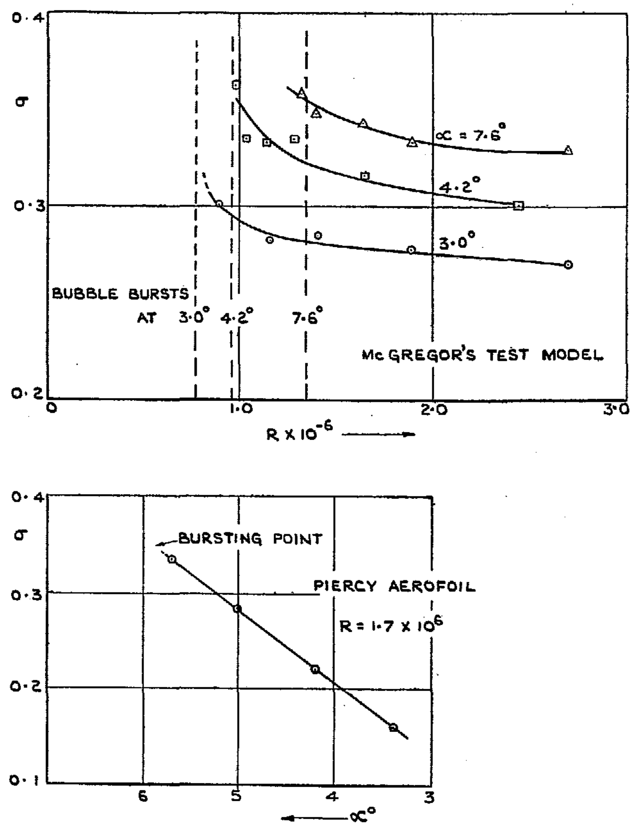

FIG. 8. Variation of pressure-recovery factor σ in short bubbles (From Ref. 26).

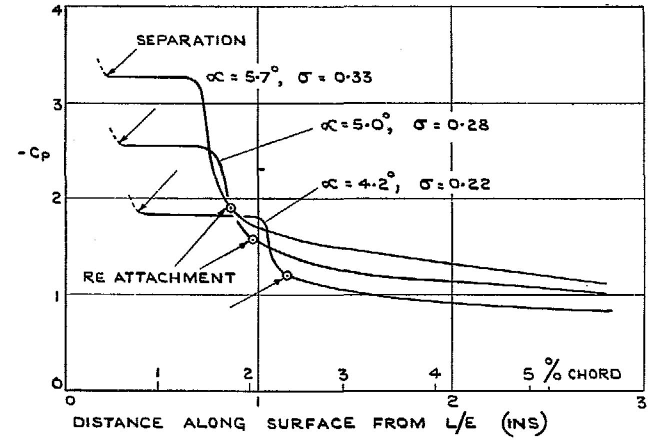

FIG. 9. Pressure distribution near leading edge of 4-ft chord Piercy aerofoil; $R = 1 \cdot 7 \times 10^6$.—Ref. 26.

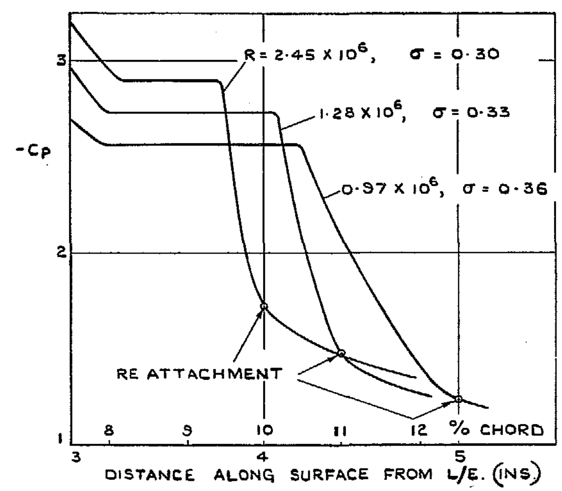

FIG. 10. Pressure distribution near leading edge of 40-in. chord special test model at $\alpha = 4 \cdot 2$ deg (Ref. 26).

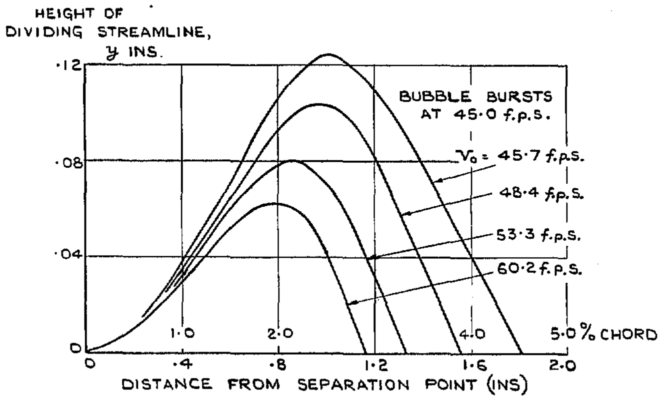

FIG. 11. Variation of bubble shape with Reynolds number on 40-in. chord test model at 4·2-deg. incidence (From Ref. 26.)

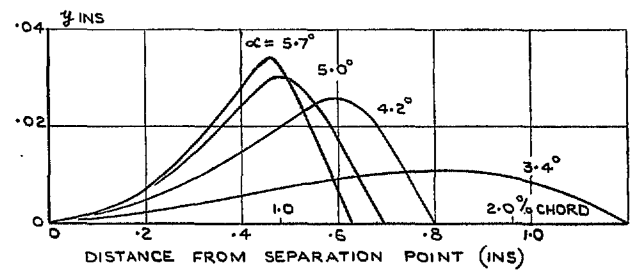

FIG. 12. Variation of bubble shape with incidence on a 4-ft chord Piercy aerofoil (Approx. 10 per cent thick) at $R = 1 \cdot 7 \times 10^6$ (From Ref. 26).

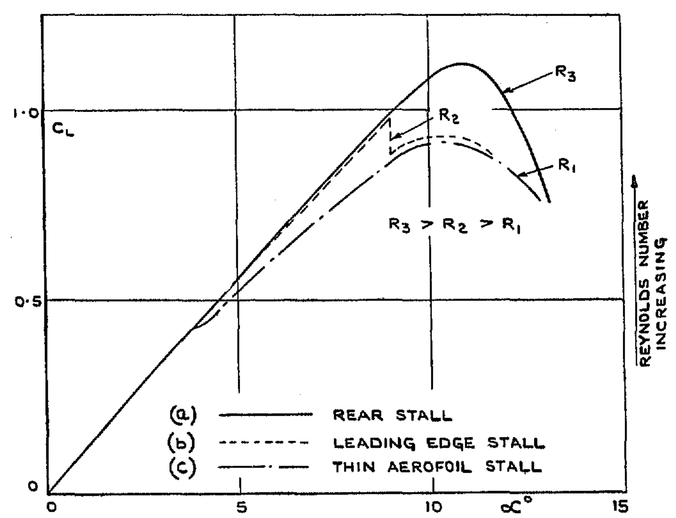

FIG. 13. Possible scale effect on maximum lift of a moderately thin aerofoil section.

(75238) Wt. 53/2293 K. 7 7/59 Hw.

PRINTED IN GREAT BRITAIN

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s*. (52*s*.).
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s*. (65*s*.)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s*. (52*s*.)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 63*s*. (65*s*.)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s*. (77*s*.)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s*. 6*d*. (49*s*. 6*d*.)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80*s*. (82*s*.)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90*s*. (92*s*. 9*d*.)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84*s*. (86*s*. 6*d*.)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84*s*. (86*s*. 6*d*.)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130*s*. (132*s*. 9*d*.)
Vol. II. Aircraft, Airscrews, Controls. 130*s*. (132*s*. 9*d*.)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130*s*. (132*s*. 6*d*.)
Vol. IV. Stability, Structures, Wind Tunnels, Wind Tunnel Technique. 130*s*. (132*s*. 6*d*.)

Annual Reports of the Aeronautical Research Council—

1937 2*s*. (2*s*. 2*d*.) 1938 1*s*. 6*d*. (1*s*. 8*d*.) 1939–48 3*s*. (3*s*. 5*d*.)

Index to all Reports and Memoranda published in the Annual Technical Reports, and separately—

April, 1950 - - - - R. & M. 2600 2*s*. 6*d*. (2*s*. 10*d*.)

Author Index to all Reports and Memoranda of the Aeronautical Research Council—

1909—January, 1954 R. & M. No. 2570 15*s*. (15*s*. 8*d*.)

Indexes to the Technical Reports of the Aeronautical Research Council—

December 1, 1936—June 30, 1939	R. & M. No. 1850	1*s*. 3*d*. (1*s*. 5*d*.)
July 1, 1939—June 30, 1945	R. & M. No. 1950	1*s*. (1*s*. 2*d*.)
July 1, 1945—June 30, 1946	R. & M. No. 2050	1*s*. (1*s*. 2*d*.)
July 1, 1946—December 31, 1946	R. & M. No. 2150	1*s*. 3*d*. (1*s*. 5*d*.)
January 1, 1947—June 30, 1947	R. & M. No. 2250	1*s*. 3*d*. (1*s*. 5*d*.)

Published Reports and Memoranda of the Aeronautical Research Council—

Between Nos. 2251–2349	R. & M. No. 2350	1*s*. 9*d*. (1*s*. 11*d*.)
Between Nos. 2351–2449	R. & M. No. 2450	2*s*. (2*s*. 2*d*.)
Between Nos. 2451–2549	R. & M. No. 2550	2*s*. 6*d*. (2*s*. 10*d*.)
Between Nos. 2551–2649	R. & M. No. 2650	2*s*. 6*d*. (2*s*. 10*d*.)
Between Nos. 2651–2749	R. & M. No. 2750	2*s*. 6*d*. (2*s*. 10*d*.)

Prices in brackets include postage

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London W.C.2; 423 Oxford Street, London W.1; 13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; 109 St. Mary Street, Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast, *or through any bookseller.*

S.O. Code No. 23-3122

R. & M. No. 312