R. & M. No. 3090
(18,798)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Pressure in a Two-Dimensional Static Hole at Low Reynolds Numbers

By
A. THOM and C. J. APELT
Department of Engineering Science, University of Oxford

LONDON: HER MAJESTY'S STATIONERY OFFICE
1958
FIVE SHILLINGS NET

The Pressure in a Two-Dimensional Static Hole at Low Reynolds Numbers

By

A. Thom and C. J. Apelt

Department of Engineering Science, University of Oxford

Reports and Memoranda No. 3090

February, 1957

Summary.—A description is given of an arithmetical method for obtaining solutions for steady incompressible viscous flow at low Reynolds numbers in the form of expansions in powers of the Reynolds numbers. The method has been used to find a solution for the flow past the mouth of a two-dimensional static hole. The pressure in the hole is determined and it is shown that the disturbance to the flow caused by the hole produces an error in the pressure recorded in the hole. The error is positive and if it is expressed in non-dimensional form, *i.e.*, (pressure error/$\frac{1}{2}\rho U^2$), its magnitude decreases with increasing Reynolds number for the range for which the solution is valid. The theoretical results are compared with experimental results obtained for the error in the pressure recorded by a circular static hole.

1. *Method*.—(*Note:* In what follows, for convenience, the usual convention is reversed and a dash indicates that a variable is dimensional while the absence of a dash means that a variable is in non-dimensional form).

The Navier-Stokes equations for the steady flow in two-dimensions of an incompressible viscous fluid may be written:

$$\nabla'^2\zeta' = \frac{1}{\nu}\left(\frac{\partial\psi'}{\partial x'}\frac{\partial\zeta'}{\partial y'} - \frac{\partial\psi'}{\partial y'}\frac{\partial\zeta'}{\partial x'}\right), \quad \ldots \quad (1)$$

$$\nabla'^2\psi' = \zeta', \quad \ldots \quad (2)$$

where

$$\nabla'^2 \equiv \left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}\right),$$

ν is the kinematic viscosity, ψ' the stream function defined by

$$u' = -\frac{\partial\psi'}{\partial y'}, \; v' = \frac{\partial\psi'}{\partial x'} \quad \ldots \quad (3)$$

and ζ' is the vorticity

$$\zeta' = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'}. \quad \ldots \quad (4)$$

(u', v') are the rectangular components of the velocity q' in the directions of the axes of x' and y' respectively.

The equations (1) and (2) may be rendered dimensionless by the substitutions

$$\left.\begin{aligned} x' &= Lx,\, y' = Ly,\, u' = Uu,\, v' = Uv,\, q' = Uq \\ \psi' &= UL\psi,\, \zeta' = \frac{U}{L}\zeta,\, p' = \tfrac{1}{2}\rho U^2 p \end{aligned}\right\}, \quad \ldots \quad (5)$$

where x, y, u, etc., are dimensionless variables corresponding to x', y', u', etc., U is a representative velocity and L a representative length. The equations then become

$$\nabla^2\zeta = R\left(\frac{\partial\psi}{\partial x}\frac{\partial\zeta}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\zeta}{\partial x}\right), \quad \ldots \quad (1a)$$

$$\nabla^2\psi = \zeta, \quad \ldots \quad (2a)$$

where $R = \frac{UL}{\nu}$, a Reynolds number.

It is proposed to expand ζ and ψ in the form

$$\left.\begin{aligned} \zeta &= \zeta_h + \delta_1 R + \delta_2 R^2 + \delta_3 R^3 + \ldots \\ \psi &= \psi_h + \Delta_1 R + \Delta_2 R^2 + \Delta_3 R^3 + \ldots \end{aligned}\right\}. \quad \ldots \quad (6)$$

Here ζ_h, ψ_h represent the values of ζ, ψ in the solution of $\nabla^4\psi = \nabla^2\zeta = 0$, and the δ's and Δ's are numerical coefficients and are functions of position. Substitution of these expansions in equations ($1a$ and $2a$) gives, on rearranging,

$$\left.\begin{aligned} &\nabla^2\zeta_h + R\nabla^2\delta_1 + R^2\nabla^2\delta_2 + R^3\nabla^2\delta_3 + \ldots \\ &\quad = R\left(\frac{\partial\psi_h}{\partial x}\frac{\partial\zeta_h}{\partial y} - \frac{\partial\psi_h}{\partial y}\frac{\partial\zeta_h}{\partial x}\right) \\ &\quad + R^2\left(\frac{\partial\psi_h}{\partial x}\frac{\partial\delta_1}{\partial y} - \frac{\partial\psi_h}{\partial y}\frac{\partial\delta_1}{\partial x} + \frac{\partial\Delta_1}{\partial x}\frac{\partial\zeta_h}{\partial y} - \frac{\partial\Delta_1}{\partial y}\frac{\partial\zeta_h}{\partial x}\right) \\ &\quad + R^3\left(\frac{\partial\psi_h}{\partial x}\frac{\partial\delta_2}{\partial y} - \frac{\partial\psi_h}{\partial y}\frac{\partial\delta_2}{\partial x} + \frac{\partial\Delta_1}{\partial x}\frac{\partial\delta_1}{\partial y} - \frac{\partial\Delta_1}{\partial y}\frac{\partial\delta_1}{\partial x}\right. \\ &\quad \left. + \frac{\partial\Delta_2}{\partial x}\frac{\partial\zeta_h}{\partial y} - \frac{\partial\Delta_2}{\partial y}\frac{\partial\zeta_h}{\partial x}\right) \\ &\quad + \ldots\ldots\ldots \\ &\text{and} \\ &\nabla^2\psi_h + R\nabla^2\Delta_1 + R^2\nabla^2\Delta_2 + R^3\nabla^2\Delta_3 + \ldots\ldots \\ &\quad = \zeta_h + \delta_1 R + \delta_2 R^2 + \delta_3 R^3 + \ldots\ldots \end{aligned}\right\} \quad \ldots \quad (7)$$

The coefficient of R^{n+1} in expansions (6) has no effect on the coefficient of R^n, and equation (7) may be separated into a series of pairs of equations:

$$\left.\begin{array}{l}
\left.\begin{array}{l}\nabla^2\zeta_h = 0\\ \nabla^2\psi_h = \zeta_h\end{array}\right\}, \quad \ldots \ldots \quad \text{(i)}\\[2ex]
\left.\begin{array}{l}R\nabla^2\delta_1 = R\left(\dfrac{\partial\psi_h}{\partial x}\dfrac{\partial\zeta_h}{\partial y} - \dfrac{\partial\psi_h}{\partial y}\dfrac{\partial\zeta_h}{\partial x}\right)\\ R\nabla^2\Delta_1 = R\delta_1\end{array}\right\}, \quad \ldots \ldots \quad \text{(ii)}\\[2ex]
\left.\begin{array}{l}R^2\nabla^2\delta_2 = R^2\left(\dfrac{\partial\psi_h}{\partial x}\dfrac{\partial\delta_1}{\partial y} - \dfrac{\partial\psi_h}{\partial y}\dfrac{\partial\delta_1}{\partial x} + \dfrac{\partial\Delta_1}{\partial x}\dfrac{\partial\zeta_h}{\partial y} - \dfrac{\partial\Delta_1}{\partial y}\dfrac{\partial\zeta_h}{\partial x}\right)\\ R^2\nabla^2\Delta_2 = R^2\delta_2\end{array}\right\} \quad \text{(iii)}\\[2ex]
\quad\text{and so on.}
\end{array}\right\} \quad \ldots \quad (8)$$

When ζ_h, ψ_h have been obtained as functions of position from equations (8(i)) it is possible to proceed to determine δ_1, Δ_1 as functions of position from equations (8(ii)). The process may be continued as far as is desired, since all the functions required for the solution of any pair of equations will have been determined from the solutions of the equations previous to it.

2. *Numerical Solutions.*—The equations (8) may be solved by a numerical process similar to that employed for the Navier-Stokes equations[1]. The continuous field of flow is replaced by a rectangular mesh and finite difference approximations to the equations are employed to calculate values of the functions ζ_h, δ_1, δ_2 ...., ψ_h, Δ_1, Δ_2 .... at the discrete points of the mesh. The finite difference approximations to equations (8) used in the problem described below were:

$$\left.\begin{array}{l}
\left.\begin{array}{l}\zeta_{h0} = \zeta_{hm}\\ \psi_{h0} = \psi_{hm} - \dfrac{n^2}{2}\zeta_{h0}\end{array}\right\}, \quad \ldots \ldots \quad \text{(i)}\\[2ex]
\left.\begin{array}{l}\delta_{10} = \delta_{1m} - \tfrac{1}{16}[(a-c)(B-D) + (b-d)(C-A)]\\ \Delta_{10} = \Delta_{1m} - \dfrac{n^2}{2}\delta_{10}\end{array}\right\}, \quad \ldots \quad \text{(ii)}\\[2ex]
\left.\begin{array}{l}\delta_{20} = \delta_{2m} - \tfrac{1}{16}[(\delta_{1A} - \delta_{1C})(B-D) + (\delta_{1B} - \delta_{1D})(C-A)\\ \qquad + (a-c)(\Delta_{1B} - \Delta_{1D}) + (b-d)(\Delta_{1C} - \Delta_{1A}]\\ \Delta_{20} = \Delta_{2m} - \dfrac{n^2}{2}\delta_{20}\end{array}\right\} \quad \ldots \quad \text{(iii)}\\[2ex]
\quad\text{etc.}
\end{array}\right\} \quad \ldots \quad (9)$$

where ζ_{h0} is the value of ζ_h at the centre of a square of side $2n$ recalculated from the corner values and ζ_{hm} is the mean of these corner values. The small letters represent ζ_h values and the capital letters ψ_h values at the mesh points as shown in Fig. 1. The method of solution is one of reiteration. Assumed values of ζ_h, ψ_h are placed at the mesh points and they are progressively improved at each point by use of equations (9(i)). When ζ_h, ψ_h have been determined in this way, to the desired degree of accuracy, the factor $\frac{1}{16}$ $[(a - c)(B - D) + (b - d)(C - A)]$ is calculated for each point and equations (9(ii)) are then used to obtain values of δ_1, Δ_1 in a similar fashion. The process can be continued to give as many terms as desired in the expansion of ζ and ψ; however, there is a practical limit to the number of terms which can be obtained because of the increasing complexity of the factor to be determined for insertion in the right-hand side of the first of each pair of equations.

The method described above has similarities to that used by Thom and Klanfer[2] to obtain a solution for the potential function in compressible flow in the form of an expansion in powers of the Mach number.

3. *Boundary Conditions.*—At a solid boundary ψ is usually known, but the value of ζ must be calculated from the pattern of flow in the vicinity of the boundary. The formula used here is that due to Woods[3]:

$$\zeta_E = \frac{3(\psi_F - \psi_E)}{m^2} - \frac{\zeta_F}{2}, \quad \ldots \quad (10)$$

where ζ_E, ψ_E are the values at E, a point on the boundary, and ζ_F, ψ_F the values at a point, F, in the flow distant m from E (Fig. 2). Substitution of the expansions (6) for ζ and ψ in equation (10) gives:

$$\begin{aligned} \zeta_{hE} + R\delta_{1E} + R^2\delta_{2E} + \ldots\ldots \\ = \frac{3}{m^2}(\psi_{hF} + R\Delta_{1F} + R^2\Delta_{2F} + \ldots\ldots - \psi_{hE}) \\ - \tfrac{1}{2}(\zeta_{hF} + R\delta_{1F} + R^2\delta_{2F} + \ldots\ldots) \end{aligned}$$

(Note that on the boundary $\psi_E = \psi_{hE}$).

The equation can be separated into

$$\left.\begin{aligned} \zeta_{hE} &= \frac{3}{m^2}(\psi_{hF} - \psi_{hE}) - \frac{1}{2}\zeta_{hF} \\ \delta_{1E} &= \frac{3}{m^2}\Delta_{1F} - \frac{1}{2}\delta_{1F} \\ &\text{etc.} \end{aligned}\right\}, \quad \ldots \quad (11)$$

which enables boundary values for each term in the expansion of ζ to be determined.

The validity of solutions obtained by the method described above is discussed in the Appendix.

4. *Viscous Flow Past a Two-dimensional Static Hole.*—The method described above was used to obtain a solution to the steady viscous flow past a two-dimensional static hole in the side of a channel, shown in Fig. 3. The representative velocity U of the substitution (5) above was taken as the centre-line velocity in the undisturbed flow and the representative length L as the width of the mouth of the static hole. The solution has been continued far enough to give the coefficients δ_2, Δ_2 of R^2 in the expansions of ζ and ψ and to give a reliable estimate of the magnitude of δ_3, Δ_3. Numerical values obtained for the first three terms are recorded for the part of the field near the static hole in Figs. 4, 5 and 6. The grid used had eight squares to the width of the slot. The sharp corner at the edge of the slot presents a difficulty and further subdivision of the mesh in the immediate neighbourhood of the sharp corner would be required to give the details

of the flow in that area. However, the advance from a coarser mesh (four squares across the mouth of the slot) to the present mesh made little difference to the solution except in the immediate vicinity of the corner and altered the magnitude of the integrals for pressure by less than 5 per cent. It is considered that further subdivision would have little effect on the solution, except at the corner itself.

The first term of the solution, which is in fact the solution for $\nabla^4\psi = 0$, gives a pattern which is symmetrical about the line BAB′ of Fig. 3. The next term, in R, destroys this symmetry, being itself anti-symmetrical about BAB′. The term in R^2 is symmetric about BAB′, that in R^3 anti-symmetric, and so on. The streamlines for the $\nabla^4\psi = 0$ solution are drawn in Fig. 7a. In Fig. 7b the dividing streamline across the mouth of the static hole is drawn for $R = 0$ and for $R = 5$, to illustrate the destruction of the symmetry about BAB′ when $R \neq 0$.

5. *Pressure in Static Hole.*—In Ref. 1, equations are obtained by integration of the Navier-Stokes equations along lines $x =$ constant, $y =$ constant, which enable the difference in pressure at points in the fluid to be calculated. The corresponding equations in non-dimensional form are

$$p_2 - p_1 = q_1^2 - q_2^2 + \frac{2}{R}\int_1^2 \frac{\partial \zeta}{\partial x}\,dy - 2\int_1^2 u\zeta\,dy \quad \ldots \quad (12)$$

for integration between points 1 and 2, on a line $x =$ constant, and

$$p_4 - p_3 = q_3^2 - q_4^2 - \frac{2}{R}\int_3^4 \frac{\partial \zeta}{\partial y}\,dx + 2\int_3^4 v\zeta\,dx \quad \ldots \quad (13)$$

for integration between points 3 and 4, on a line $y =$ constant. These equations were used to evaluate the pressure difference between points O and B (Fig. 3). The point O was taken at such distance from the slot (OA = 2L) that the flow there was practically the undisturbed flow. The result obtained by taking account of the first three terms of expansion (6) was

$$p_B - p_0 = \frac{-7\cdot 49}{R} + 0\cdot 187 - 0\cdot 00046R - 0\cdot 00144R^2 . \quad \ldots \quad (14)$$

If the slot had caused no disturbance of the flow:

$$(p_B - p_0)_{\text{undisturbed}} = (p_A - p_0)_{\text{undisturbed}} = OA\,\frac{\partial p}{\partial x} = -\frac{8\cdot 00}{R} .$$

Let $$(p_B - p_0) - (p_B - p_0)_{\text{undisturbed}} = \Delta p,$$

then Δp is the error in the pressure recorded in the static hole due to the disturbance caused in the flow by the hole itself.

$$\Delta p = \frac{0\cdot 51}{R} + 0\cdot 187 - 0\cdot 00046R - 0\cdot 00144R^2. \quad \ldots \quad (15)$$

Also

$$p_B - p_A \equiv \Delta' p = 0\cdot 082 - 0\cdot 00144R^2. \quad \ldots \quad (16)$$

It is of interest to note that if the integrations for pressure take account of only the first terms of expansions (6) (*i.e.*, the $\nabla^4\psi = 0$ solution), equations (15) and (16) become respectively,

$$\Delta p = \frac{0\cdot 51}{R} + 0\cdot 354 \ldots\ldots, \qquad (15a)$$

$$\Delta' p = 0\cdot 196, \qquad (16a)$$

which differ from the result obtained if, in equations (15) and (16), R is made negligibly small. This would seem to be an example of what Birkhoff[4] calls an 'asymptotic paradox'.

To the accuracy of the present solution the fluid is stationary in the lower half of the slot. Thus, the pressure has a constant value across the bottom of the slot, the value being given by equation (15).

6. *Effect of Depth of Slot.*—It was found that for a slot of half the depth of that drawn in Fig. 3, the magnitudes of Δp, $\Delta' p$ were not appreciably different from those given above. It is considered, however, that a further reduction in the depth of the slot would cause the magnitudes of Δp, $\Delta' p$ to be changed by an appreciable amount.

7. *Comparison of Results with Experimental Values.*—In Fig. 8 the value of Δp of equation (15) is plotted as log Δp against log R (curve 'a'). On the same diagram is plotted the function suggested by Ray[5] as giving the error in the pressure measured by a circular static hole:

$$F(R) = 0\cdot 58R^{-3/4}.$$

Ray's $F(R)$ is defined by

$$F(R) = \frac{\Delta p'}{\frac{1}{2}\rho\left(d'\,\dfrac{\partial u'}{\partial y'}\right)^2},$$

where d' is the diameter of the static hole. Since most of his experiments were made with hole diameters quite small compared to the half-width of the channel, $\{d'(\partial u'/\partial y')\}$ approximates to the value U, where U is the velocity of the flow at a distance d' away from the wall.

Hence

$$F(R) = \frac{\Delta p'}{\frac{1}{2}\rho U^2} \text{ approximately},$$

which is the same expression as was used to obtain Δp:

$$\Delta p = \frac{\Delta p'}{\frac{1}{2}\rho U^2}.$$

However, here U is the velocity at the centre-line of the two-dimensional channel, since the width of the slot in the solution of this paper is one half the width of the channel.

It would seem more reasonable to take

$$\Delta p_1 = \frac{\Delta p'}{\frac{1}{2}\rho U\left(d'\,\dfrac{\partial u'}{\partial y'}\right)},$$

which takes some account of the shape of the velocity profile near the static hole as well as of the velocity itself. If this latter form is used, $F(R)$ of Ref. 5 is altered relatively little, as shown in Fig. 8, but $\Delta p_1 = \frac{1}{2}\Delta p$. Log $(\Delta p/2)$ is plotted on Fig. 8 also, as curve 'b'.

In comparing the theoretical with the experimental results, account should be taken of the fact that the theoretical analysis is for a two-dimensional static hole. The value of Δp for the three-dimensional case of a circular hole could be expected to be smaller than in the two-dimensional case by a factor which might be expected to be approximately one half. These tentative three-dimensional values are shown as curve 'c', which is a plot of log $(\Delta p/4)$.

The curve 'c' is considerably removed from the extrapolated value of Ray's function, $F(R)$. However, it must be recorded that the theoretical results presented are for laminar flow and are considered to be valid only for $0 < R < 1$; Ray's results are almost entirely for turbulent flow and were obtained in the range $3 < R < 1000$. It is possible that as in the case of resistance to flow in a pipe there should be two different functions for Δp, one for laminar and one for turbulent flow, connected by a range of transition values.

LIST OF SYMBOLS

ψ	Stream function in viscous flow
ζ	Vorticity
ν	Kinematic viscosity
q	Local velocity of flow
u, v	Rectangular components of q in the direction of x and y respectively
∇^2	The Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$
R	Reynolds number
U	A representative velocity
L	A representative length
Δp	Error in the pressure recorded in a static hole due to the disturbance caused in the flow by the hole itself.

The addition of a dash to the symbol for a variable indicates that the variable is in dimensional form while the symbol without a dash represents a variable which is in non-dimensional form.

REFERENCES

No.	*Author*	*Title, etc.*
1	A. Thom	An investigation of fluid flow in two dimensions. R. & M. 1194. 1928.
2	A. Thom and L. Klanfer	The method of influence factors in arithmetical solutions of certain field problems. R. & M. 2440. November, 1947.
3	L. C. Woods	A note on the numerical solution of fourth order differential equations. *Aero. Quart. V*, Pt. 3. September, 1954.
4	G. Birkhoff	*Hydrodynamics.* Princeton University Press. 1950.
5	A. K. Ray	On the effect of orifice size on static pressure reading at different Reynolds numbers. *Ing.-Arch. 24.* 3. p. 171. 1956. Translated by Sylvia W. Skan, Aerodynamics Division, N.P.L. A.R.C. 18,829. November, 1956.

APPENDIX

Validity of Solutions

The Navier-Stokes equations for steady flow in two dimensions of an incompressible viscous fluid may be written (in non-dimensional form)

$$X - \frac{1}{2}\frac{\partial p}{\partial x} = -\frac{1}{R}\nabla^2 u + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}, \qquad (17)$$

$$Y - \frac{1}{2}\frac{\partial p}{\partial y} = -\frac{1}{R}\nabla^2 v + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}. \qquad (18)$$

Where X, Y, are in non-dimensional form the components of the external forces acting on unit mass of the fluid $\{X' = (U^2/L)\,X\}$. On eliminating p we obtain

$$\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x} = \frac{1}{R}\nabla^2\zeta - \left(\frac{\partial\psi}{\partial x}\frac{\partial\zeta}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\zeta}{\partial x}\right). \qquad (19)$$

In the solution presented in this paper it has been assumed that

$$\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x} = 0. \qquad (20)$$

The correctness of the solution obtained is determined by how accurately the values of ζ, ψ obtained fulfil this condition when substituted into equation (19). The solution has been obtained in the form (6):

$$\left.\begin{aligned}\zeta &= \zeta_h + \delta_1 R + \delta_2 R^2 + \delta_3 R^3 + \ldots\ldots \\ \psi &= \psi_h + \Delta_1 R + \Delta_2 R^2 + \Delta_3 R^3 + \ldots\ldots\end{aligned}\right\} \qquad (6)$$

Inspection of Figs. 4, 5 and 6 will show that everywhere δ_1 and δ_2 are respectively of the orders $\zeta_h/50$ and $\zeta_h/500$ or less, and that Δ_1 and Δ_2 are respectively of the orders $\psi_h/500$ and $\psi_h/5000$ or less. It is known also that δ_3, Δ_3 are of the order one-tenth δ_2, Δ_2 respectively. Consequently, for $R < 1$ the solution presented in Figs. 4, 5 and 6 should be a good approximation to the final solution for ζ and ψ.

At the same time the condition (20) above has been satisfied to the same degree of accuracy, for $(\partial X/\partial y - \partial Y/\partial x)$ differs from zero by an amount

$$\frac{1}{R}\nabla^2\zeta - \left(\frac{\partial\psi}{\partial x}\frac{\partial\zeta}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\zeta}{\partial x}\right),$$

which has a magnitude

$$R^2\left[\nabla^2\delta_3 - \left(\frac{\partial\psi_h}{\partial x}\frac{\partial\delta_2}{\partial y} - \frac{\partial\psi_h}{\partial y}\frac{\partial\delta_2}{\partial x} + \frac{\partial\Delta_1}{\partial x}\frac{\partial\delta_1}{\partial y} - \frac{\partial\Delta_1}{\partial y}\frac{\partial\delta_1}{\partial x} + \frac{\partial\Delta_2}{\partial x}\frac{\partial\zeta_h}{\partial y} - \frac{\partial\Delta_2}{\partial y}\frac{\partial\zeta_h}{\partial x}\right)\right]$$

for the solution taken as far as the terms in R^2. This is known to have a magnitude of the same order as that of δ_3, Δ_3. It is considered therefore that the solution of Figs. 4, 5 and 6 is valid for $R < 1$. It may be valid for somewhat higher values of R, but this could be determined only when more terms in the expansions of ζ, ψ are known.

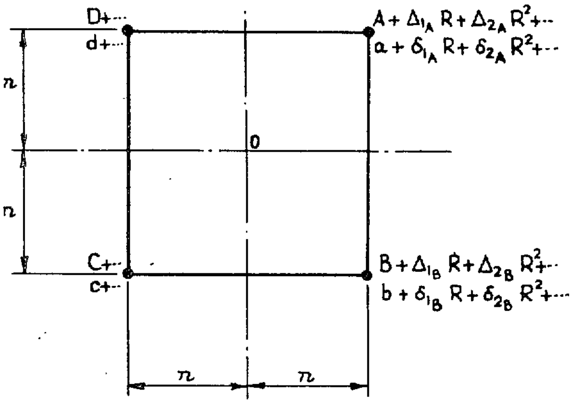


FIG. 1.

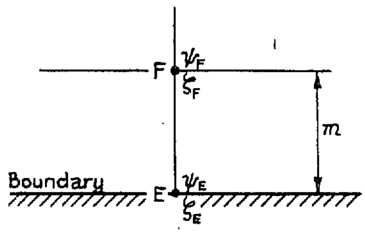


FIG. 2.

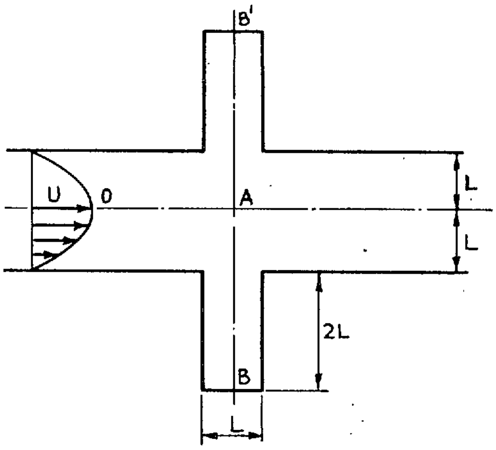
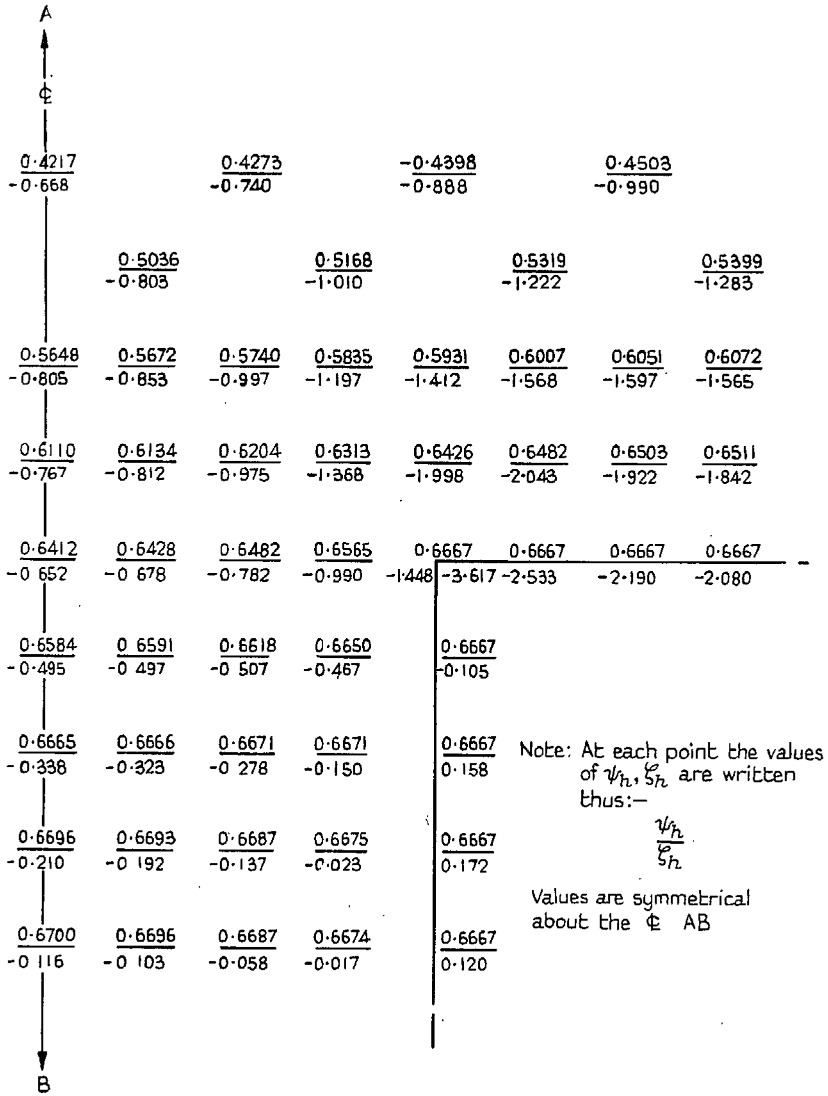
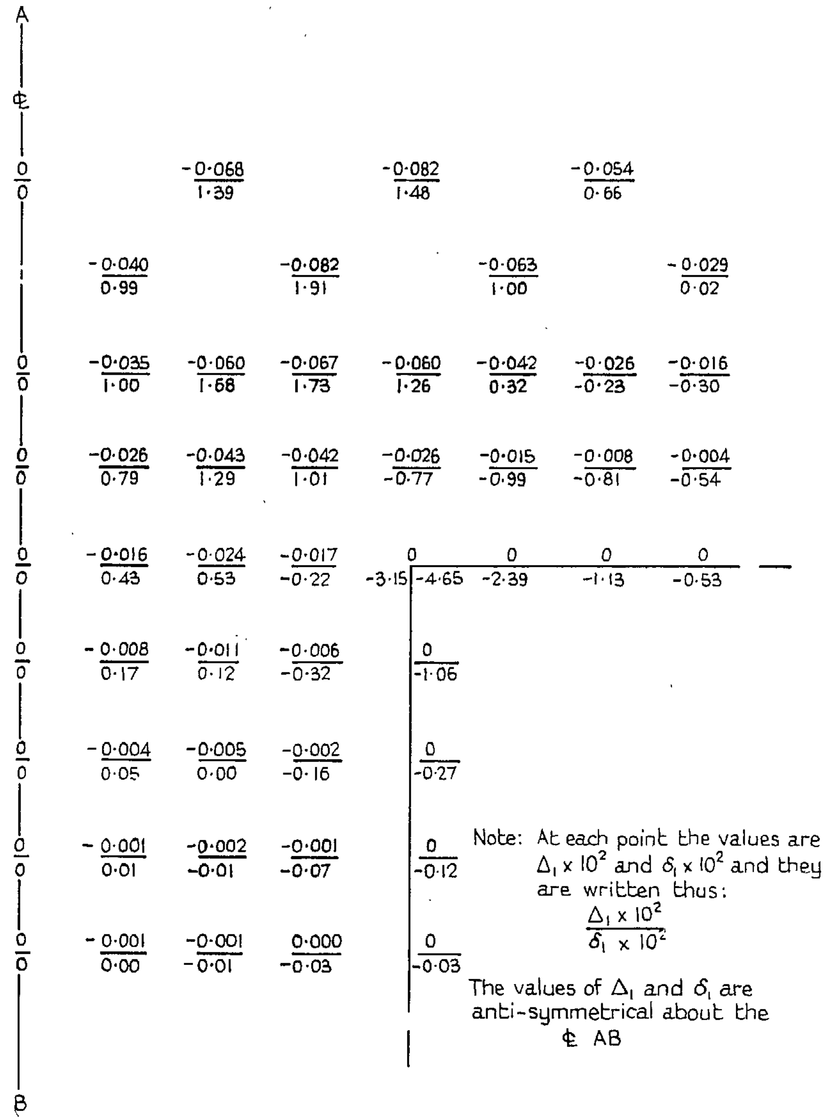


FIG. 3.



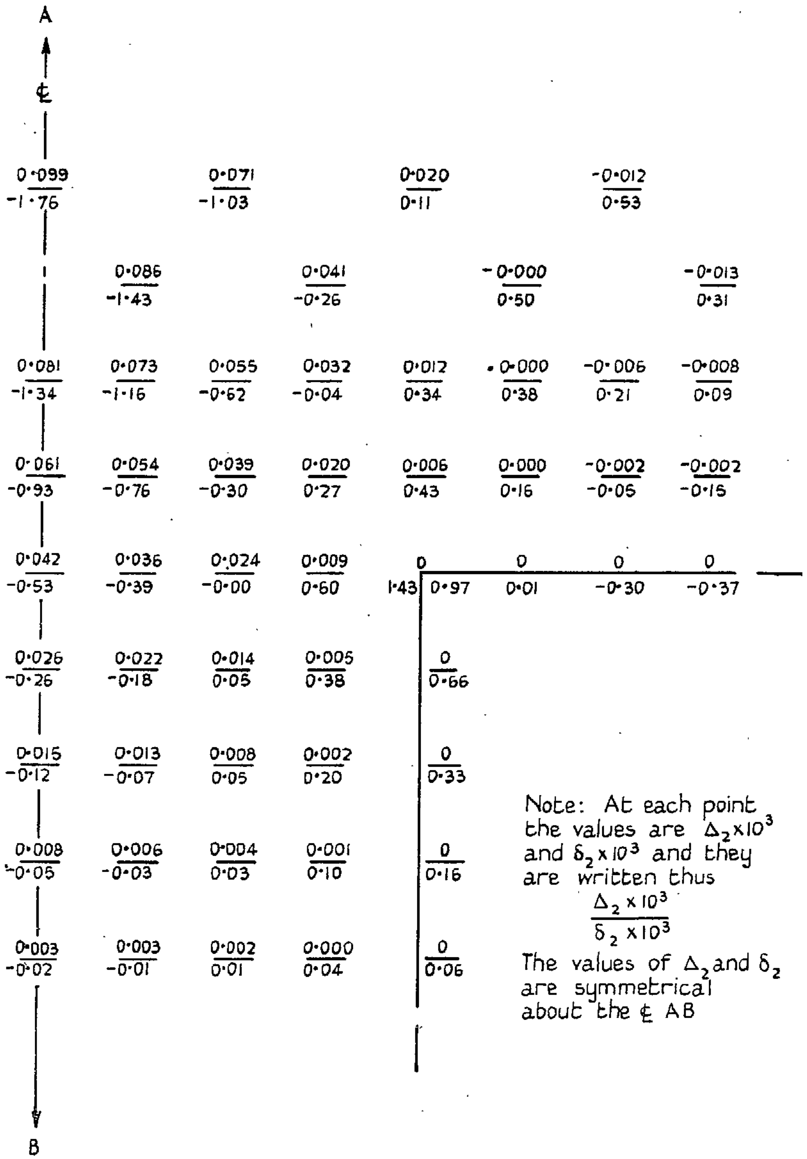
$(\psi_h,\ \xi_h)$

FIG. 4.



$(\Delta_1,\ \delta_1)$

FIG. 5.



(Δ_2, δ_2)

FIG. 6.

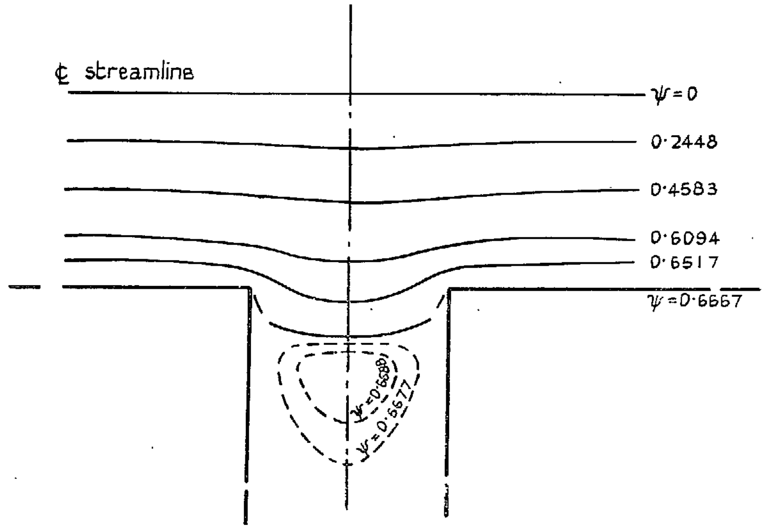


FIG. 7a.

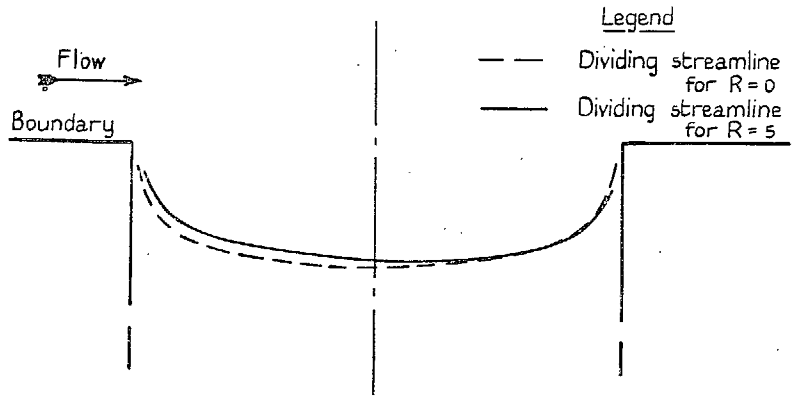


FIG. 7b.

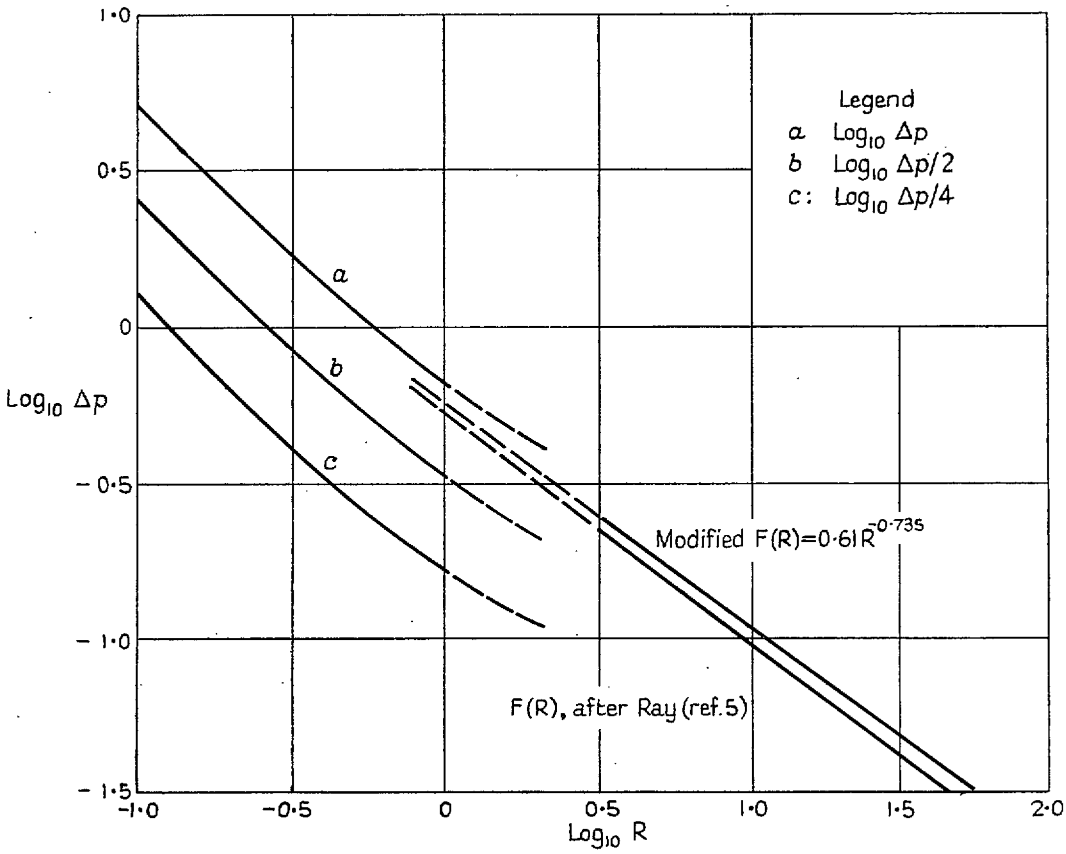


FIG. 8.

(43370) Wt. Wt.52 K7 4/58 F. M. & S. PRINTED IN GREAT BRITAIN

Publication of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (52*s.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (65*s.*)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control Structures, and a miscellaneous section. 50*s.* (52*s.*)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control Structures. 63*s.* (65*s.*)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s.* (77*s.*)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s.* 6*d.* (49*s.* 6*d.*)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80*s.* (82*s.*)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90*s.* (92*s.* 9*d.*)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84*s.* (86*s.* 6*d.*)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84*s.* (86*s.* 6*d.*)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130*s.* (132*s.* 9*d.*)
Vol. II. Aircraft, Airscrews, Controls. 130*s.* (132*s.* 9*d.*)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130*s.* (132*s.* 6*d.*)
Vol. IV. Stability, Structures, Wind Tunnels, Wind Tunnel Technique. 130*s.* (132*s.* 6*d.*)

Annual Reports of the Aeronautical Research Council—

1937 2*s.* (2*s.* 2*d.*) 1938 1*s.* 6*d.* (1*s.* 8*d.*) 1939–48 3*s.* (3*s.* 5*d.*)

Index to all Reports and Memoranda published in the Annual Technical Reports, and separately—

April, 1950 - - - - - R. & M. 2600 2*s.* 6*d.* (2*s.* 10*d.*)

Author Index to all Reports and Memoranda of the Aeronautical Research Council—

1909—January, 1954 R. & M. No. 2570 15*s.* (15*s.* 8*d.*)

Indexes to the Technical Reports of the Aeronautical Research Council—

December 1, 1936—June 30, 1939	R. & M. No. 1850	1*s.* 3*d.* (1*s.* 5*d.*)
July 1, 1939–June 30, 1945	R. & M. No. 1950	1*s.* (1*s.* 2*d.*)
July 1, 1945—June 30, 1946	R. & M. No. 2050	1*s.* (1*s.* 2*d.*)
July 1, 1946—December 31, 1946	R. & M. No. 2150	1*s.* 3*d.* (1*s.* 5*d.*)
January 1, 1947—June 30, 1947	R. & M. No. 2250	1*s.* 3*d.* (1*s.* 5*d.*)

Published Reports and Memoranda of the Aeronautical Research Council—

Between Nos. 2251–2349	R. & M. No. 2350	1*s.* 9*d.* (1*s.* 11*d.*)
Between Nos. 2351–2449	R. & M. No. 2450	2*s.* (2*s.* 2*d.*)
Between Nos. 2451–2549	R. & M. No. 2550	2*s.* 6*d.* (2*s.* 10*d.*)
Between Nos. 2551–2649	R. & M. No. 2650	2*s.* 6*d.* (2*s.* 10*d.*)
Between Nos. 2651–2749	R. & M. No. 2750	2*s.* 6*d.* (2*s.* 10*d.*)

Prices in brackets include postage

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London W.C.2; 423 Oxford Street, London W.1; 13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; 109 St. Mary Street, Cardiff; Tower Lane, Bristol, 1; 80 Chichester Street, Belfast, *or through any bookseller.*

S.O. Code No. 23-3090

R. & M. No. 3090