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The Effect of Wing Torsion on Aileron-Tab Flutter

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Summary.—This paper gives the results of an investigation into the effects of wing torsion in a ternary-type spring-tab flutter. The results presented are for a series of calculations on an idealised system having degrees of freedom in wing pitch, rotation of the control surface and rotation of the tab; the pitching degree of freedom is spring restrained and is intended to represent wing torsion. The tab is restrained as a trim tab or pure aerodynamic servo tab, but the results are expected to apply qualitatively to a spring tab.

A comparison is made between these results and those of earlier investigations into a similar problem by Wittmeyer[1] and Wittmeyer and Templeton[2], where the main surface mode considered was one of vertical translation to represent wing bending. From this previous work criteria were suggested for the prevention of this type of flutter, and in an analogous manner criteria are now tentatively proposed for prevention of wing torsion-aileron-tab flutter.

1. *Introduction*.—Past investigations of spring-tab flutter have considered in the main a ternary-type flutter involving one wing mode, rotation of the aileron and rotation of the tab†. The wing mode usually considered was one of wing bending represented by vertical translation under a spring restraint. Little investigation of a general character has been made into the problem of spring-tab flutter where the wing mode involved is one of torsion. This paper gives the results of an investigation into this problem where pitching of the wing replaces vertical translation. The pitching degree of freedom is spring restrained and is intended to represent wing torsion. The tab is restrained as a trim tab (the representation being equally valid for a pure aerodynamic servo tab), but the results can be expected to apply qualitatively to a spring tab also.

The work of Wittmeyer and Templeton[1,2], in particular, provides an interesting comparison with this investigation. They dealt with the flutter of trimming and spring-tab systems and suggested criteria for the prevention of flutter of these systems. They pointed out[2] that, although their proposed criteria did not apply to systems which have considerable amounts of pitching in the wing mode, it might be that modifications to the constants k_1 to k_9 could be found to cover such cases. This possibility is investigated in the present paper.

2. *Description of the System*.—2.1. *The System and its Degrees of Freedom*.—The idealised wing-aileron-tab system which has been investigated (Fig. 1) is a rectangular wing of 6-ft chord fitted with a rectangular control surface of the same span s and having a chord of $1 \cdot 2$ ft. The tab, which is of span $\frac{1}{2}s$, occupies a cut out on the trailing edge of the aileron and has a chord of $0 \cdot 36$ ft. These relative dimensions for the chords of the various surfaces are considered to be reasonably typical of a modern aircraft.

* R.A.E. Report Struct. 203, received 9th October, 1956.

† For simplicity reference is made here and throughout this paper to the wing-aileron combination; analogous cases always exist for the elevator and rudder controls.

The system had the following elastic constraints:

(*a*) A constraint in respect of the uniform pitching of the wing

(*b*) A constraint in respect of the rotation of the aileron relative to the wing

(*c*) A constraint in respect of the rotation of the tab relative to the aileron.

The wing is fixed against vertical translation.

The degrees of freedom considered are:

(i) Pitching of the wing about an axis at 30 per cent chord aft of the leading edge

(ii) Rotation of the aileron relative to the wing

(iii) Rotation of the tab relative to the aileron.

2.2. *Structural Details.*—The relevant inertia data are as follows:

(*a*) Wing mass = 40 lb/ft span (including aileron and tab)

(*b*) Aileron mass = 10 lb/ft span (including tab)

(*c*) Tab mass = 1 lb/ft span.

The c.g. of the aileron is assumed to be on its hinge line throughout. The basic c.g. position of the tab (without mass-balance) is $\bar{x} = 0 \cdot 10c_t$ and tab mass-balance is added to vary the tab c.g. position.

The radii of gyration about the appropriate rotational axes for the wing, aileron and tab are 25 per cent of the appropriate chords.

Tab mass-balance is distributed along a line 1 in. forward of the tab hinge line.

With these structural details it can be verified that in the basic condition $(dm\bar{x})/I_c = 0 \cdot 0168$. The A.P. 970 criterion for prevention of tab flutter[5] is

$$\delta_1 \leqslant (dm\bar{x})/I_c \leqslant (1 - \delta_2)K - (N + 1)(I_t/I_c) ,$$

where d is distance between tab and aileron hinge lines

m is tab mass

$\bar{x}$ is the distance of tab centre of gravity aft of the tab hinge line

I_c is the aileron moment of inertia about its hinge line

I_t is the tab moment of inertia about its hinge line.

K is a factor depending on the ratio of tab to control surface chord

N is the follow-up ratio (zero for a trimming tab)

δ_1 and δ_2 are safety margins.

For the configuration used in the present investigation this criterion reduces to $0 \cdot 003 \leqslant (dm\bar{x})/I_c \leqslant 0 \cdot 0152$, so that the criterion is not met without additional tab mass-balance. The range of tab centre-of-gravity positions for which the criterion is met is

$$0 \cdot 0147c_t \leqslant \bar{x} \leqslant 0 \cdot 0857c_t .$$

2.3. *Structural Parameters Considered.*—The following parameters are varied in the flutter calculations:

(*a*) Tab mass-balance weight, in terms of which all the inertias are expressed

(*b*) The ratio of the uncoupled natural frequency of aileron rotation to that of wing pitch

(*c*) The ratio of the uncoupled natural frequency of tab rotation to that of wing pitch.

Throughout the calculations the c.g. of the aileron is assumed to be on its hinge line, implying that the aileron mass-balance is changed with the tab mass-balance, as it normally would be in practice. The range of values of the parameters considered are as described below.

(i) Tab mass-balance is varied such that the tab c.g. position ranges from $\bar{x} = 0 \cdot 10c_t$ (its basic value) to $\bar{x} = -0 \cdot 10c_t$, although a few cases are considered outside the latter limit.

(ii) Aileron natural frequency is varied between zero and a value $\sqrt{2}$ times that of the wing torsional frequency, which is kept constant throughout at 15 c.p.s.

(iii) Tab natural frequency is varied between zero and three times that of the wing torsional frequency.

3. *Calculations*.—Ternary flutter calculations were carried out as described by Templeton[3]. Solutions of the flutter determinant were obtained in the main using the Royal Aircraft Establishment Electronic Flutter Simulator.

The derivatives used in calculating the aerodynamic coefficients were those given by Minhinnick[4] for a Mach number of 0·7. It was found that the solutions of the flutter determinant fell into two groups which corresponded roughly with the values of the derived frequency parameter. These groups were (*a*) one corresponding to a frequency parameter of 1 and (*b*) one corresponding to a frequency parameter of 2 and over. The order of agreement between the assumed and the calculated values of frequency parameter in order for the flutter speed to have been considered as sufficiently accurate was taken to be similar to that given by Templeton[3]. This agreement was achieved without difficulty in the case of solutions corresponding to the lower frequency parameter. In the case of the higher frequency parameter solutions, however, this balancing was not possible. The control surface and tab derivatives are only tabulated for frequency parameters up to 1·4. In the calculations the values of the control surface and tab derivatives corresponding to this frequency parameter have been used for all higher values. This amounts to an assumption that these particular derivatives are tending to some asymptotic value at high frequency parameters, which is likely in practice. Where very low flutter speeds are found, however, the frequency parameters being correspondingly high, it should be remembered that the speeds are not accurate.

The elastic matrix corresponds to that for a trim tab or a pure aerodynamic servo tab, all the coupling terms being zero. Thus, if e_{11} is the elastic stiffness for the torsion mode, the matrix of the elastic coefficients can be written as

$$e_{rs} = e_{11} \begin{bmatrix} 1 & 0 & 0 \\ 0 & r_1 & 0 \\ 0 & 0 & r_2 \end{bmatrix} .$$

This notation allows variations in the control surface and tab natural frequencies relative to the wing to be made easily.

4. *Results*.—The results of the calculations are plotted in Figs. 2 to 6 as curves of critical flutter speed against tab c.g. position. For each figure the tab natural frequency is kept constant and the aileron frequency has the four values 0, 7·5, 15 and 21·2 c.p.s. as plotted on the diagrams a, b, c, d respectively. The values of the tab natural frequency in Figs. 2 to 6 respectively are 0, 9, 15, 30 and 45 c.p.s. In Fig. 7 the results of binary wing-aileron flutter calculations are plotted; the aileron c.g. is still kept on the hinge line. This was the main type of flutter experienced with the ternary system when the tab was in the overbalanced* condition. The wing torsional frequency corresponding to all the Figs. 2 to 7 is 15 c.p.s. The figures on each curve represent the frequency parameters at particular points.

* The terms ' overbalanced ' and ' underbalanced ' refer respectively to the c.g. position being in front of and behind the hinge line.

It is convenient to discuss the results in the separate groups according to the tab natural frequency considered. The following points then emerge.

4.1. *Results for High Values of the Tab Natural Frequency*, i.e., $f_\gamma = 30$ *or* 45 *c.p.s.* (*Figs.* 5 *and* 6).—(*a*) There is no flutter of the system with zero mass-balance ($\bar{x} = 0\cdot 10c_t$) whatever the value of the aileron natural frequency considered. When resonance occurs between the wing and aileron, for $f_\gamma = 30$ c.p.s. a region of low damping occurs at speeds slightly greater than 200 ft/sec. This indicates that the nose of the right-hand branch of the curve, which exists for other frequency-ratio conditions mentioned below, lies very near to this region. The low damping was not present for $f_\gamma = 45$ c.p.s. indicating that the nose of this branch has moved off to the right.

(*b*) For small amounts of tab mass-balance, such that $\bar{x}$ lies between $0\cdot 085c_t$ and $0\cdot 10c_t$, flutter still does not occur.

(*c*) If the tab is statically balanced then the flutter speed depends on the appropriate value of f_β. As f_β increases from zero the critical speed V_c decreases until a minimum value is reached when resonance occurs between the wing and aileron natural frequencies. For further increase of f_β there is a sharp increase in V_c, the rate of increase with f_β being greater than that of decrease on the other side of the resonance condition.

(*d*) When the tab is overbalanced, there is very little change in V_c from the statically balanced case for those values of f_β less than f_α. When resonance occurs there is a slight increase in V_c as mass-balance is increased. For a further increase in f_β the flutter speed continues to fall until the point is reached where the tab c.g. is as far forward of the hinge line as it was initially aft. In all cases the flutter speed for the heavily overbalanced tab has roughly the same value.

The largest range of tab c.g. positions for which there is no flutter ($0\cdot 021c_t \leqslant \bar{x} \leqslant 0\cdot 10c_t$) occurs when f_β is greater than f_α. For all practical purposes there is little change in flutter speed caused by the increase in tab natural frequency to 45 c.p.s. (Fig. 6) from 30 c.p.s. (Fig. 5).

4.2. *Results when Resonance Occurs Between the Tab and Wing Natural Frequencies* (*Fig.* 4).—(*a*) Irrespective of the aileron natural frequency, a right-hand branch of the flutter curve now exists and covers the condition of zero tab mass-balance ($\bar{x} = 0\cdot 10c_t$). This second branch of the curve extends to very low speeds for small values of mass-balance weight and in fact the zero critical speeds noticed by Wittmeyer[1] are again obtained here, although in this case they do not correspond to $f_\beta = f_\gamma = 0$. In the case where there is complete coincidence of frequencies, $f_\alpha = f_\beta = f_\gamma$, the calculated speed is not actually zero, which is unusual as a coincidence in frequencies often leads to zero flutter speeds. It is possible that structural damping may seriously influence these very low-speed flutter conditions, but this has not been investigated in the present work.

(*b*) The left-hand branch of the flutter now extends to just beyond $\bar{x} = 0\cdot 07c_t$ for low values of f_β and is very similar to the corresponding branch of Figs. 5 and 6. When the aileron frequency reaches the wing frequency, the two branches of the curve join up (*see* Fig. 4c), so that for a region $f_\alpha \simeq f_\beta \simeq f_\gamma$, flutter cannot be eliminated for any value of tab mass-balance. The lower critical speed is, moreover, very low throughout this region, which must evidently be avoided in practice. Finally, as f_β exceeds f_α, the two branches separate, and the left-hand branch rapidly recedes again with increasing f_β as in Figs. 5 and 6.

4.3. *Results for Tab Natural Frequency Less than that of Wing* (*Figs.* 3 *and* 2).—In this instance the same types of curve are obtained as those described in section 4.2 above. Some slight differences do occur, however, and these may be classified as follows: The right-hand branch, which in the previous instance had been noticed in connection with zero critical speeds, has now moved away from the horizontal axis. For low values of tab mass-balance and f_β less than f_α, the nose of this right-hand branch is narrower than that of the corresponding one of Fig. 4. For all the other combinations of tab mass-balance and aileron natural frequency the sets of curves 2, 3, 4 are closely similar. The reduction in f_γ has had little effect on the flutter characteristics of the system, and in particular it can be seen that the region in which the two branches join together is $f_\alpha \simeq f_\beta \simeq$ or $> f_\gamma$.

4.4. *The Binary Results.*—In Fig. 7 the results of the wing-aileron binary calculations are plotted; these calculations yield an approximation to the left-hand branch of the ternary flutter curve. For values of f_β less than f_α the lower critical speeds for the binary-type flutter are lower than those in the corresponding ternary case. The difference is greater when comparison is made with the corresponding ternary cases for the lower values of f_γ. Similarly, the upper critical speeds for the binary are lower than the ternary values; the effect of the tab being to increase the area in which flutter occurs. The shape of the curve is not greatly affected when $f_\beta = f_\alpha$ (there is, of course, no other curve which can join up with it), but for f_β greater than f_α it recedes so rapidly that no flutter remains when f_β has reached 21·2 c.p.s. The binary-type flutter corresponds most nearly to the ternary flutter with very high tab frequencies. Binary-type flutter occurs mainly in the region of tab overbalance and does not, as in the left-hand branch of the ternary flutter, extend over a fairly large part of the underbalanced region as well.

4.5. *The Flutter Free Regions.*—The flutter-free regions obtained for the frequency conditions considered are summarised in tabular form below, where the regions are defined by their limiting tab c.g. positions.

TABLE 1

$f_\alpha = 15$ c.p.s.

f_γ (c.p.s.)	f_β (c.p.s.)	Figure	Tab c.g. positions defining flutter-free regions		
			Region	Left-hand branch	Right-hand branch
0	0	2a	(d)	$> 0{\cdot}068c_t$	$< 0{\cdot}072c_t$
	7·5	2b		$> 0{\cdot}068c_t$	$< 0{\cdot}069c_t$
	15	2c		Continuous flutter region	
	21·2	2d		$> 0{\cdot}022c_t$	$< 0{\cdot}036c_t$
9	0	3a	(d)	$> 0{\cdot}071c_t$	$< 0{\cdot}075c_t$
	7·5	3b		$> 0{\cdot}069c_t$	$< 0{\cdot}085c_t$
	15	3c		Continuous flutter region	
	21·2	3d		$> 0{\cdot}02c_t$	$< 0{\cdot}027c_t$
15	0	4a	(d)	$> 0{\cdot}072c_t$	$< 0{\cdot}071c_t$
	7·5	4b		$> 0{\cdot}07c_t$	$< 0{\cdot}070c_t$
	15	4c		Continuous flutter region	
	21·2	4d		$> 0{\cdot}019c_t$	$< 0{\cdot}024c_t$
30	0	5a	(c)	$> 0{\cdot}071c_t$	No branch $< 0{\cdot}10c_t$
	7·5	5b		$> 0{\cdot}07c_t$	No branch $< 0{\cdot}10c_t$
	15	5c	(b)	$> 0{\cdot}055c_t$	*
	21·2	5d	(a)	$> 0{\cdot}017c_t$	No branch $< 0{\cdot}10c_t$
45	0	6a	(c)	$> 0{\cdot}079c_t$	No branch $< 0{\cdot}10c_t$
	7·5	6b		$> 0{\cdot}083c_t$	No branch $< 0{\cdot}10c_t$
	15	6c	(b)	$> 0{\cdot}06c_t$	No branch $< 0{\cdot}10c_t$
	21·2	6d	(a)	$> 0{\cdot}021c_t$	No branch $< 0{\cdot}10c_t$

* Low damping at $0{\cdot}10c_t$ at $V_c \simeq 200$ ft/sec.

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The flutter-free regions given by existing criteria* are, for comparison:

A.P. 970[5]	$0\cdot0147c_t < \bar{x} < 0\cdot0857c_t$	
Criterion I	$-0\cdot0773c_t < \bar{x} < 0\cdot0451c_t$	Wittmeyer and Templeton[2].
Criterion II	$0\cdot0046c_t < \bar{x} < 0\cdot0451c_t$	
Criterion III	$0\cdot0046c_t < \bar{x} < 0\cdot0517c_t$	

It can be seen from Table 1 that five general flutter-free regions are obtained, of varying degrees of importance, and these are considered below:

(*a*) To obtain freedom from flutter over the widest possible tab c.g. range it is necessary to achieve the frequency conditions of Figs. 5d and 6d, i.e., $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \geqslant \sqrt{2}$ (and possibly also $f_\gamma/f_\beta \geqslant \sqrt{2}$). If these frequency requirements are met, then the flutter-free region is defined by the relation $0\cdot021c_t \leqslant \bar{x} \leqslant 0\cdot10c_t$ (the original value).

(*b*) A second flutter-free region, depicted in Figs. 5c and 6c, covers a rather more restricted tab c.g. range. The frequency conditions to be satisfied are $f_\gamma/f_\alpha \geqslant 2$, $f_\beta \simeq f_\alpha$ and the flutter-free range is then defined by the relation $0\cdot06c_t \leqslant \bar{x} \leqslant 0\cdot10c_t$.

(*c*) The third region, Figs. 5a, 5b, and 6a, 6b, exists when the frequency conditions $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \leqslant 0\cdot5$ are satisfied; the flutter-free region is then defined by $0\cdot085c_t \leqslant \bar{x} \leqslant 0\cdot10c_t$. The range of tab c.g. positions here has been limited so that it only includes positions within the range of mass-balance considered. The right-hand bound of the flutter-free region is in fact much further to the right of the basic tab condition as isolated results have shown, the actual flutter-free region being much wider than that defined above.

(*d*) Other flutter regions can also be distinguished corresponding to Figs. 2, 3 and 4. These regions are so small that, when an appropriate safety margin is allowed in the form of a small shift of the limiting tab c.g. position into the stable region beyond the nose of the curve (the movement assumed here was $\bar{x} = 0\cdot005c_t$), they become non-existent or are negligibly small.

4.6. *Summary*.—The overall picture which emerges from these results is that the ternary spring-tab flutter involving a main surface torsion mode may best be prevented, or the possibility of it occurring be minimised under the following conditions:

(*a*) A high value of the aileron natural frequency, greater than that of wing torsion

(*b*) A high value of the tab natural frequency: twice that of the wing torsional frequency would seem to be a reasonable value

(*c*) Slightly less than static balance of the tab combined with at least static balance of the aileron.

Providing the tab is underbalanced or at most statically balanced the particular effects associated with the frequency conditions (*a*) and (*b*) above are as follows: For a particular value of f_γ the high value of f_β required by (*a*) will ensure that the lower critical value of V_c on the left-hand branch will be as high as possible. If f_β is now fixed at this high value and f_γ increased, the effect will be to increase the lower critical speed for the statically balanced tab but for slight underbalance of the tab the effect on flutter speed is small.

Flutter may also be prevented over more restricted ranges of tab c.g. position under different frequency conditions as we have seen in section 4.5, but these methods are secondary to that outlined at the beginning of this section.

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* These existing criteria, which do not relate specifically to wing torsion-aileron-tab flutter, are discussed later in section 5, but for convenience of comparison the flutter-free regions associated with them are stated immediately after Table 1.

The worst possible case from the flutter point of view occurs when there is resonance between all three of the natural frequencies. When $f_\beta \simeq f_\alpha$ it is impossible to obtain a flutter-free region for any value of $f_\gamma \leqslant f_\alpha$ for all values of the tab mass-balance weight considered. Dangerous flutter conditions are also evident for all frequency ratios when the tab is overbalanced.

5. *Comparison with Earlier Work.*—At present there are several criteria available by means of which a designer may estimate whether a projected spring-tab system will be free from flutter. These criteria fall into two groups depending on their derivation and the possible application of these or some modified form of them to the system studied here will be considered.

5.1. *The Wittmeyer-Templeton Criteria.*—In two previous reports by Wittmeyer[1] and Wittmeyer and Templeton[2] criteria are developed for the prevention of flutter of tab systems. The first work done by Wittmeyer was for a trim-tab system and flutter calculations were performed for the ternary system having degrees of freedom in wing vertical translation, aileron rotation and tab rotation. The wing vertical translation under a spring restraint was taken to represent wing bending. From these extensive calculations he was able to formulate three criteria for the prevention of flutter of this type of system. Wittmeyer and Templeton then developed the scope of the work and proposed criteria, similar to those mentioned above, for a generalised spring-tab system which they call the S-system. As special cases of this S-system there are the spring tab, trimming, servo, and geared tabs and, by appropriate modifications to the criteria developed for the S-system, results for the other systems were deduced.

The criteria applied to a trim-tab system such as that considered here are as follows:

Criterion I.—The following conditions are to be satisfied:

$$f_\gamma/f_\beta \geqslant 2k_1, \quad f_\gamma/f_z \geqslant 2k_2, \quad f_\beta/f_z \leqslant k_3 \text{ where } k_i = 1 \quad (i = 1, 2, 3)$$

$$\left.\begin{array}{l}(1)\ p_t \geqslant F_2(\mu, i_c, i_t, p_c, \bar{q}) + \delta_4 \\ (2)\ p_t \leqslant F_1\{\mu, i_c, (1+\bar{N})i_t + \alpha_{33}\bar{N}, p_c, \bar{q}\} - \delta_3\end{array}\right\},$$

the functions F_1 and F_2 being defined in list of symbols. From (1) and (2) the flutter-free region for this system is defined by the inequality $-0\cdot0773c_t \leqslant \bar{x} \leqslant 0\cdot0451c_t$.

Criterion II.—This results from the criterion above by omitting the second frequency condition when this cannot be met and replacing the inequality (1) by $p_t \geqslant k_6$ where the suggested value for k_6 is $0\cdot1$. The flutter-free region is now reduced to that defined by $0\cdot0046c_t \leqslant \bar{x} \leqslant 0\cdot0451c_t$, the greater restriction on the forward limit of the tab c.g. position being imposed to counteract the relaxation in frequency conditions.

Criterion III.—There is a further relaxation in the frequency conditions here, the only condition to be satisfied being $f_\gamma/f_\beta \geqslant 2k_1$. The limits of tab c.g. position for the flutter-free region are determined from two new inequalities:

$$m\bar{x} \geqslant 0\cdot4k_6\rho c_w c_t^2 qs$$

and

$$\frac{(1+\bar{N})I_t + (E_1 - E_2)c_w m\bar{x}}{I_c} \leqslant k_7 C p^{3/2},$$

where the suggested values for the constants are $k_1 = 1$, $k_6 = 0\cdot1$, $k_7 = 1$. The flutter-free region is then defined by $0\cdot0046c_t \leqslant \bar{x} \leqslant 0\cdot0517c_t$.

The range of validity of the criteria is $\mu \simeq 6$; $i_c = 1$ to $7\cdot78$; $p_c = 0$; $i_t = 1\cdot31$ to $13\cdot1$; $E = 0\cdot2$ to $0\cdot4$; $p = 0\cdot13$ to $0\cdot25$, and it is suggested that they may be extended to cover values of μ up to 50. All the values of the relevant parameters for the present system lie within the ranges recommended above except that for p which has the value $0\cdot3$.

We will now examine these criteria to see whether any simple modifications can be made to them so that they may be applied tentatively, as recommended flutter preventatives, to the case in which the main wing motion is one of wing torsion. The main frequency condition for prevention of flutter, common to all three flutter-free regions defined in section 4.5 is that $f_\gamma/f_\alpha \geqslant 2$. This frequency condition appears only in the first of the criteria mentioned above (on replacing f_z by f_α), to which we shall mainly confine our attention. The two other frequency conditions of Criterion I are satisfied in the flutter-free regions (*b*) and (*c*) of section 4.5 (the relation $f_\gamma/f_\beta \geqslant 2$ being implicit in the frequency conditions satisfied in these regions). The frequency conditions appropriate to the main flutter-free region do not, however, fully satisfy those of Criterion I.

The aim is to derive three modified criteria for the prevention of flutter from the original Criterion I which will cover the flutter-free regions (*a*), (*b*), (*c*) of section 4.5. It is evident that the first of these modified criteria involves a change in both the frequency and inertia parts of the Criterion I, whereas the second and third only need modifications to the inertia part.

First Modification of Criterion I.—Part of the largest flutter-free region found in the calculations (region (*a*)) is included in the tab c.g. range defined by the original Criterion I. By changing the values of the constants in this criterion it is possible to move the tab c.g. range defined by it in its entirety to the right so that the whole flutter-free region is included. Thus if in the inequalities (1) and (2) we give the constants the values $k_4 = 0\cdot64$, $k_5 = -0\cdot25$, $\delta_3 = 0$ and $\delta_4 = 0\cdot09$, the tab c.g. range defined is $0\cdot026c_t \leqslant \bar{x} \leqslant 0\cdot10c_t$ and this allows a safety margin of $\bar{x} = 0\cdot005c_t$ at the left-hand bound of the region. The right-hand bound is kept at the basic tab c.g. position ($\delta_3 = 0$) and this will represent a considerable safety margin as the right-hand branch of this flutter curve is well to the right of this point. The frequency conditions to be satisfied are $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \geqslant \sqrt{2}$. There is no corresponding frequency condition here to the first one given in the original criterion, due to the fact that the second frequency condition given here requires that $f_\beta > f_\alpha$ whereas previously $f_\beta < f_z$ was required.

Second Modification of Criterion I.—The tab c.g. range to be defined here is $0\cdot065c_t \leqslant \bar{x} \leqslant 0\cdot095c_t$, thus allowing a safety margin of $\bar{x} = 0\cdot005c_t$ at either end of the flutter-free region. This range does not coincide with that defined by the inequalities (1) and (2). It can, however, be defined by these inequalities if the values of the constants k_4 and k_5 in the functions F_1 and F_2 are given the values $0\cdot66$ and $-0\cdot47$ respectively and by taking the values of the safety margins δ_3 and δ_4 to be $0\cdot05$ and $0\cdot07$ respectively. The appropriate frequency conditions are $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \simeq 1$; the first frequency condition of Criterion I being implicit in these conditions.

Third Modification of Criterion I.—A reduction of the aileron frequency from the value appropriate to the second modified form of Criterion I but with the same tab frequencies results in a shift of the two branches of the flutter curve to the right (Figs. 5a and 5b, 6a and 6b). As we are arbitrarily limiting the investigation to tab c.g. positions forward of the basic position at $0\cdot10c_t$, this lowering of f_β reduces the size of the flutter-free region to the tab c.g. range $0\cdot090c_t \leqslant \bar{x} \leqslant 0\cdot10c_t$. This allows a safety margin of $0\cdot005c_t$ at the left-hand bound of the region; the right-hand bound is kept at the basic tab c.g. position for the same reason as that given in the first modified form of the criterion.

The flutter-free region can be defined using the original inequalities (1) and (2) but with the values of the constants k_4 and k_5 amended to $0\cdot64$ and $-0\cdot57$ respectively and the safety margins set at $\delta_4 = 0\cdot06$ and $\delta_3 = 0$. The corresponding frequency conditions are $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \leqslant 0\cdot5$, which again fully satisfy the frequency requirements of Criterion I.

It is quite possible that the relation $f_\gamma/f_\alpha \geqslant 2$ common to all the three flutter-free regions found is too conservative and that some smaller value of the ratio will ensure that flutter is prevented.

The Original Criteria II and III.—The range of values of tab c.g. positions defined in the three modified criteria are the only ones in which flutter can be prevented. These ranges could be defined in a different manner using the expressions of the original criteria II and III. The frequency conditions of II, however, do not ensure that $f_\gamma/f_\alpha \geqslant 2$ (f_α being substituted for f_z) and we have seen that only if this condition is satisfied will flutter be prevented, so that the criterion is not applicable to this system.

The only frequency condition required by Criterion III is $f_\gamma/f_\beta \geqslant 2$, and, as for Criterion II, this restriction will not be sufficient to prevent the wing torsion-aileron-tab-type flutter, so that the criterion is not applicable.

5.2. *The A.P. 970 Criterion*[5].—The criterion requires that the tab c.g. position shall be such that the following inequality is satisfied:

$$(3) \qquad \delta_1 \leqslant \frac{dm\bar{x}}{I_c} \leqslant (1 - \delta_2)K - (N + 1)\frac{I_t}{I_c},$$

where the symbols are defined in section 2.2. Further, it is recommended that the frequencies shall satisfy the inequalities (*a*) $f_\gamma/f_\beta \geqslant 2$. (*b*) $f_\gamma/f_z \geqslant 1\cdot5$. With these conditions satisfied flutter of the wing bending-aileron-tab type should be prevented. The limits on tab c.g. position are based on some theoretical considerations of binary aileron-tab flutter by Collar and Sharpe[6] and on the need to prevent the ternary type of flutter (with wing flexure) that occurs with an overbalanced tab. The recommended values for the safety margins are $\delta_1 = 0\cdot003$, $\delta_2 = 0\cdot2$.

As we have seen earlier the range of tab c.g. positions given by this criterion is $0\cdot0147c_t \leqslant \bar{x} \leqslant 0\cdot0857c_t$. If we replace f_z by f_α in the recommended frequency ratios it can be seen (Figs. 5a, 5b, 5c, 6a, 6b, 6c, and Table 1) that prevention of the torsion-type flutter by mass-balancing according to the criterion will not be possible. Flutter of this type can, however, be prevented over a similar range of tab c.g. positions for somewhat different frequency conditions. Thus, a flutter-free region (*a*) exists (*see* section 4.5) between the limits $0\cdot021c_t \leqslant \bar{x} \leqslant 0\cdot10c_t$ providing that $f_\gamma/f_\alpha \geqslant 2$ and $f_\beta/f_\alpha \geqslant \sqrt{2}$ and possibly $f_\gamma/f_\beta \geqslant \sqrt{2}$. This region is defined by the inequality (3) when the safety margins are given the values $\delta_1 = 0\cdot0047$, $\delta_2 = 0\cdot136$.

Similarly, the flutter-free regions (*b*) and (*c*) of section 4.5 can be defined using this inequality. The appropriate values of the safety margins and the corresponding frequency conditions to be satisfied within these regions to prevent flutter are

Region (*b*) $\qquad \delta_1 = 0\cdot0113, \quad \delta_2 = 0\cdot163, \quad f_\gamma/f_\alpha \geqslant 2, \quad f_\beta/f_\alpha \simeq 1$

Region (*c*) $\qquad \delta_1 = 0\cdot0154, \quad \delta_2 = 0\cdot136, \quad f_\gamma/f_\alpha \geqslant 2, \quad f_\beta/f_\alpha \leqslant 0\cdot5.$

These values of the safety margins δ_1 and δ_2 are chosen so that the margins of safety in terms of tab c.g. position are the same as those allowed in the corresponding cases in section 5.1.

It should be noticed that flutter of wing bending-aileron-tab and wing torsion-aileron-tab type can both be prevented over a common tab c.g. range in parts of the regions (*a*) and (*b*) of section 4.5 providing that the correct frequency conditions are satisfied. Thus, over the tab c.g. range $0\cdot026c_t \leqslant \bar{x} \leqslant 0\cdot0857c_t$* the bending-type flutter will be prevented providing $f_\gamma/f_z \geqslant 1\cdot5$, $f_\gamma/f_\beta \geqslant 2$ the torsion type when $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \geqslant \sqrt{2}$. For the smaller tab c.g. range $0\cdot065c_t \leqslant \bar{x} \leqslant 0\cdot0857c_t$* the bending-type flutter is prevented under the same frequency conditions but the prevention of the torsion-type flutter depends on the frequency ratios $f_\gamma/f_\alpha \geqslant 2, f_\beta/f_\alpha \simeq 1$ being satisfied. Implicit in these is the condition $f_\gamma/f_\beta \geqslant 2$ so that in this region the frequency ratio of tab to aileron required to prevent the bending-type flutter is sufficient to suppress the torsion type also.

6. *Conclusions*.—The results of these calculations show that it will be possible under certain conditions to prevent wing torsion-aileron-tab-type flutter for a trim tab or pure aerodynamic

* The safety margin allowed at the nose of the left-hand branch of the torsion-type flutter curve by both these tab c.g. ranges is $0\cdot005c_t$. By fixing the right-hand end of these ranges to coincide with that of the flutter-free region for the bending-type flutter, a bigger safety margin than has previously been used in this investigation has been allowed at the nose of the right-hand branch of the torsion-type flutter curve.

servo tab such as that considered here. The necessary conditions may be expressed qualitatively in three ways depending on the frequency and mass-balance conditions as:

(*a*) (i) A high value of the aileron natural frequency, greater than that of wing torsion

(ii) A high value of the tab natural frequency; twice that of the wing torsional frequency would seem to be a reasonable value

(iii) An underbalanced tab ($0 \cdot 026c_t \leqslant \bar{x} \leqslant 0 \cdot 10c_t$) combined with at least static balance of the aileron

or

(*b*) (i) An aileron frequency roughly equal to that of the wing torsion

(ii) A high tab frequency, twice that of wing torsion again seems appropriate

(iii) An underbalanced tab ($0 \cdot 065c_t \leqslant \bar{x} \leqslant 0 \cdot 095c_t$) together with static balance of the aileron

and

(*c*) (i) An aileron frequency lower than that of wing torsion

(ii) A high tab frequency, similar to that in (*a*) and (*b*)

(iii) An underbalanced tab $0 \cdot 090c_t \leqslant \bar{x} \leqslant 0 \cdot 10c_t$ and static balance of the aileron.

These results are also expected to apply qualitatively to a spring tab besides the trim and servo tabs for which they are derived. The form (*a*) may be of most use for a trim tab on a powered control and form (*c*) for an aerodynamic servo tab with a free control, or a spring tab.

It is possible from the results of these investigations to modify the existing criteria[1, 2] for prevention of wing flexure-aileron-tab flutter to provide corresponding criteria for the prevention of wing torsion-aileron-tab flutter of the present system. Three criteria are tentatively proposed, all derived from the Criterion I of Wittmeyer and Templeton[2].

In all cases the limiting tab c.g. positions are defined by the inequalities

$$p_t \geqslant F_2(\mu, i_c, i_t, p_c, \bar{q}) + \delta_4$$
$$p_t \leqslant F_1\{\mu, i_c, (1 + \bar{N})i_t + \alpha_{33}\bar{N}, p_c, \bar{q}\} - \delta_3 ,$$

with appropriate values for the safety margins δ_3 and δ_4 and the constants k_4 and k_5 involved in the expressions for F_1 and F_2. The three criteria are, corresponding to the cases *a*, *b* and *c* outlined earlier in this section:

(1) $p_t \geqslant F_2 + \delta_4$, $p_t \leqslant F_1 - \delta_3$ where $k_4 = 0 \cdot 64$, $k_5 = -0 \cdot 25$; $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \geqslant \sqrt{2}$, $\delta_3 = 0$, $\delta_4 = 0 \cdot 09$ — First modified form of Criterion I.

(2) $p_t \geqslant F_2 + \delta_4$, $p_t \leqslant F_1 - \delta_3$ where $k_4 = 0 \cdot 66$, $k_5 = -0 \cdot 47$; $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \simeq 1$, $\delta_3 = 0 \cdot 05$, $\delta_4 = 0 \cdot 07$ — Second modified form of Criterion I.

(3) $p_t \geqslant F_2 + \delta_1$, $p_t \leqslant F_1 - \delta_3$ where $k_4 = 0 \cdot 64$, $k_5 = -0 \cdot 57$; $f_\gamma/f_\alpha \geqslant 2$, $f_\beta/f_\alpha \leqslant 0 \cdot 5$, $\delta_3 = 0$, $\delta_4 = 0 \cdot 06$ — Third modified form of Criterion I.

It may well be that the requirement $f_\gamma/f_\alpha \geqslant 2$ of all these criteria is too stringent and that some value of the ratio in the range $1 < f_\gamma/f_\alpha < 2$ will be sufficient to ensure that flutter is prevented.

The flutter-free regions can also be defined in an analogous manner to that of A.P. 970[5] so that three requirements could be made having the same frequency conditions as those of Criteria (1) to (3) above but which have the tab c.g. ranges defined by the inequality

$$\delta_1 \leqslant \frac{dm\bar{x}}{I_c} \leqslant (1 - \delta_2)K - (N + 1)\frac{I_t}{I_c} .$$

The values given to δ_1 and δ_2 in the three cases will be:

(1) $\delta_1 = 0\cdot 0047, \quad \delta_2 = 0\cdot 136$

(2) $\delta_1 = 0\cdot 0113, \quad \delta_2 = 0\cdot 163$

(3) $\delta_1 = 0\cdot 0154, \quad \delta_2 = 0\cdot 136$

and the safety margins are then the same as those of the criteria in terms of tab c.g. position.

7. *Further Developments.*—It is proposed to investigate this type of flutter on a wind-tunnel model and this should provide more definite information on the general applicability of the criteria proposed and a check on the tentative values assigned to the constants k_4 and k_5 in them.

LIST OF SYMBOLS

(and Definitions of their Values)

$a_1 \; = \; 0\cdot 222 + 0\cdot 013 i_t + (-\,0\cdot 0145 + 0\cdot 00149 i_t) i_c$

$a_2 \; = \; 1\cdot 12 - 0\cdot 0267 i_t + (0\cdot 0365 - 0\cdot 001 i_t) i_c$

$a_3 \; = \; -\,0\cdot 164 - 0\cdot 0965 i_t + (0\cdot 0778 + 0\cdot 00489 i_t) i_c$

$$a_4 \; = \; \frac{i_t}{0\cdot 448 + 0\cdot 277 i_t} - 0\cdot 238 i_c$$

c_c Aileron chord

c_t Tab chord

c_w Wing chord

E_1 Ratio c_c/c_w

E_2 Ratio c_t/c_w

e_{ij} Non-dimensional elastic stiffness coefficients for the tab system

f_z Natural frequency of wing in vertical translation

f_α Natural frequency of wing pitch

f_β Natural frequency of aileron rotation

f_γ Natural frequency of tab rotation

$\left.\right\}$ For the tab system, including effect of virtual inertia

$$F_1 \; = \; \tfrac{3}{4}\frac{E_1}{(E_1 - E_2)}\left\{\frac{1}{6}\left(1 - 4\frac{E_2}{E_1}\right) i_t + a_3 - a_4(a_2 - \bar{q}) + k_4\sqrt{\left(\frac{a_2 - \bar{q}}{a_1 \bar{q}}\right)}\right\}$$

$$F_2 \; = \; a_3 - a_4(a_2 - \bar{q}) - k_5\sqrt{\left(\frac{a_2 - \bar{q}}{a_1 \bar{q}}\right)}$$

$i_c \; = \; 16 I_c/\pi\rho c_w s c_c^3$, non-dimensional inertia of the aileron

$i_t \; = \; 16 I_t/\pi\rho c_w s c_t^3 q$, non-dimensional inertia of the tab

$j \; = \; \sqrt{p}\{0\cdot 93 + 1\cdot 28\,(1\cdot 97 - E_1)(0\cdot 745 - p)\}$

k's		Constants occurring in tab criteria
m		Mass of the tab
N		Follow-up ratio
$\bar{N}$		Modified follow-up ratio
p		Ratio $E_2/E_1 = c_t/c_c$
p_c	=	$8m_c x_c/\pi\rho c_w c_c^2 s$, non-dimensional mass moment of aileron
p_t	=	$8m\bar{x}/\pi\rho c_w c_t^2 qs$, non-dimensional mass moment of tab
q		Ratio of tab span to control-surface span
$\bar{q}$	=	jq
r_1, r_2		Ratio of direct elastic stiffness coefficients e_{22}/e_{11} and e_{33}/e_{11}
s		Semi-span of main surface
$\bar{x}$		Distance of tab c.g. aft of tab hinge line
x_c		Distance of aileron c.g. aft of aileron hinge line
α		Angle of pitch of main surface about the 30 per cent chord line
α_{33}		Non-dimensional value of $a_{33} = a_{33}/\{(\pi/16)\rho c_w^4\, qsE_2^3\}$
β		Rotation of the aileron about its hinge relative to the wing
γ		Rotation of the tab about its hinge relative to the aileron
δ_1, δ_2		Safety margins occurring in the criterion of A.P. 970
δ_3, δ_4		Safety margins occurring in the criterion (1) of Ref. 2.

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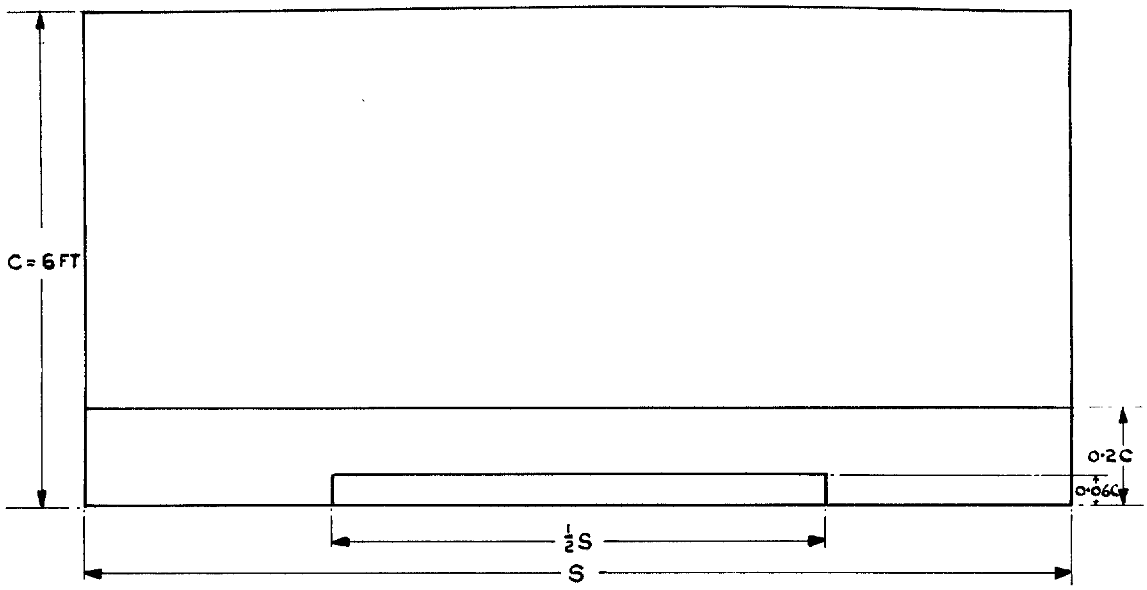


PLANFORM OF THE WING WITH CONTROL SURFACE AND TAB.

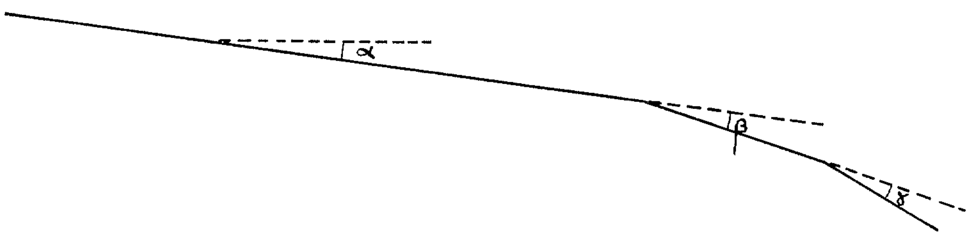
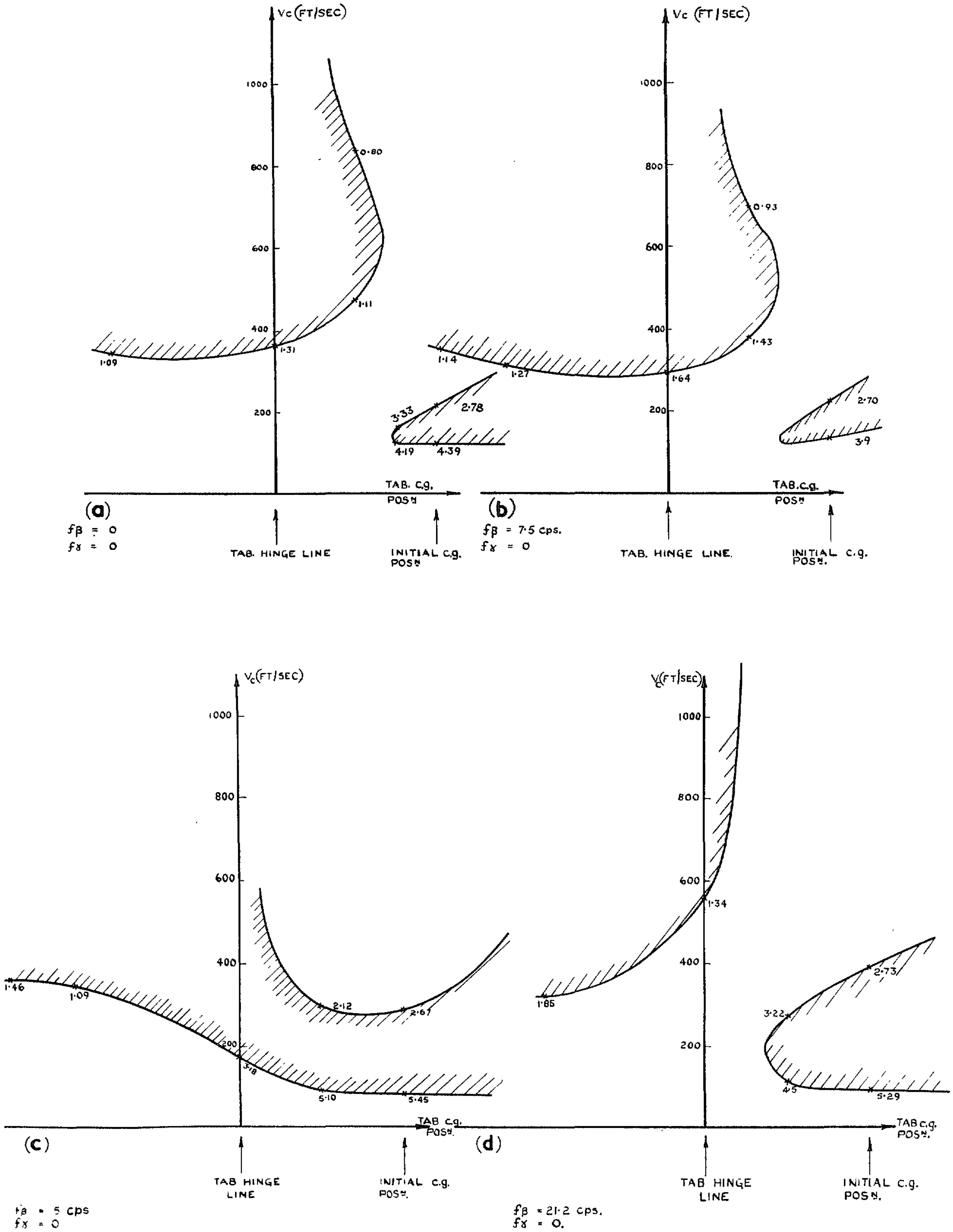
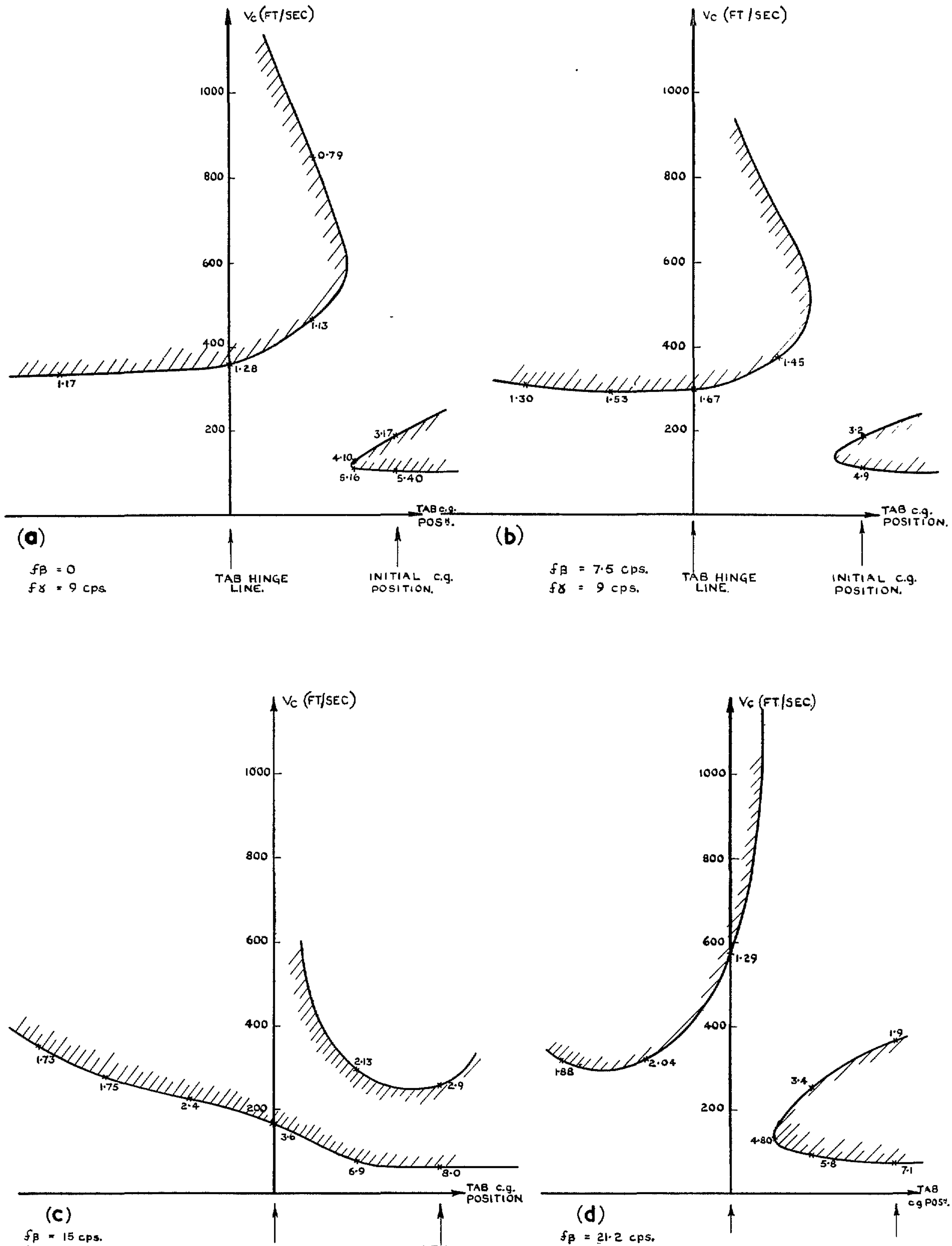


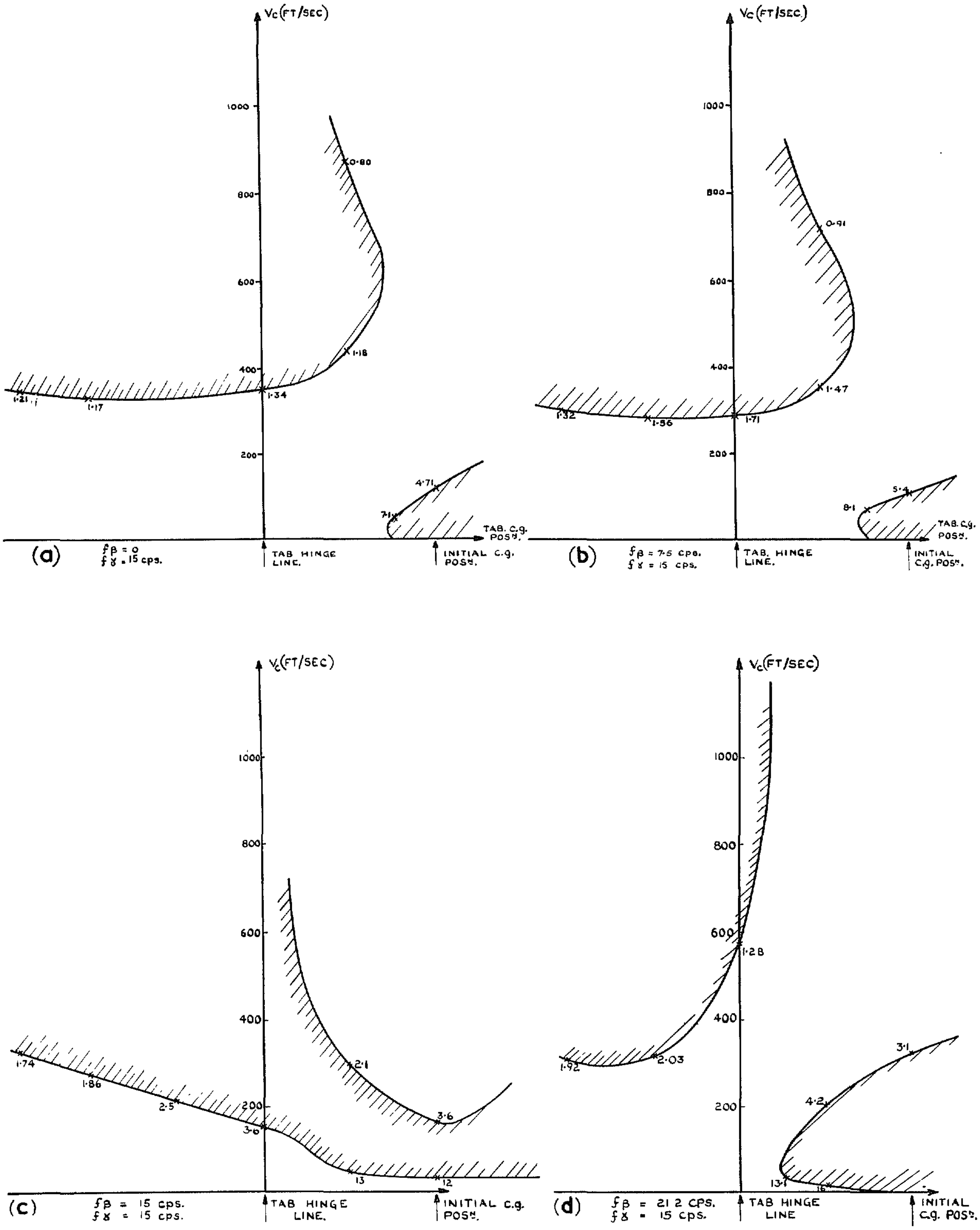
FIG. 1. Degrees of freedom of the system considered.



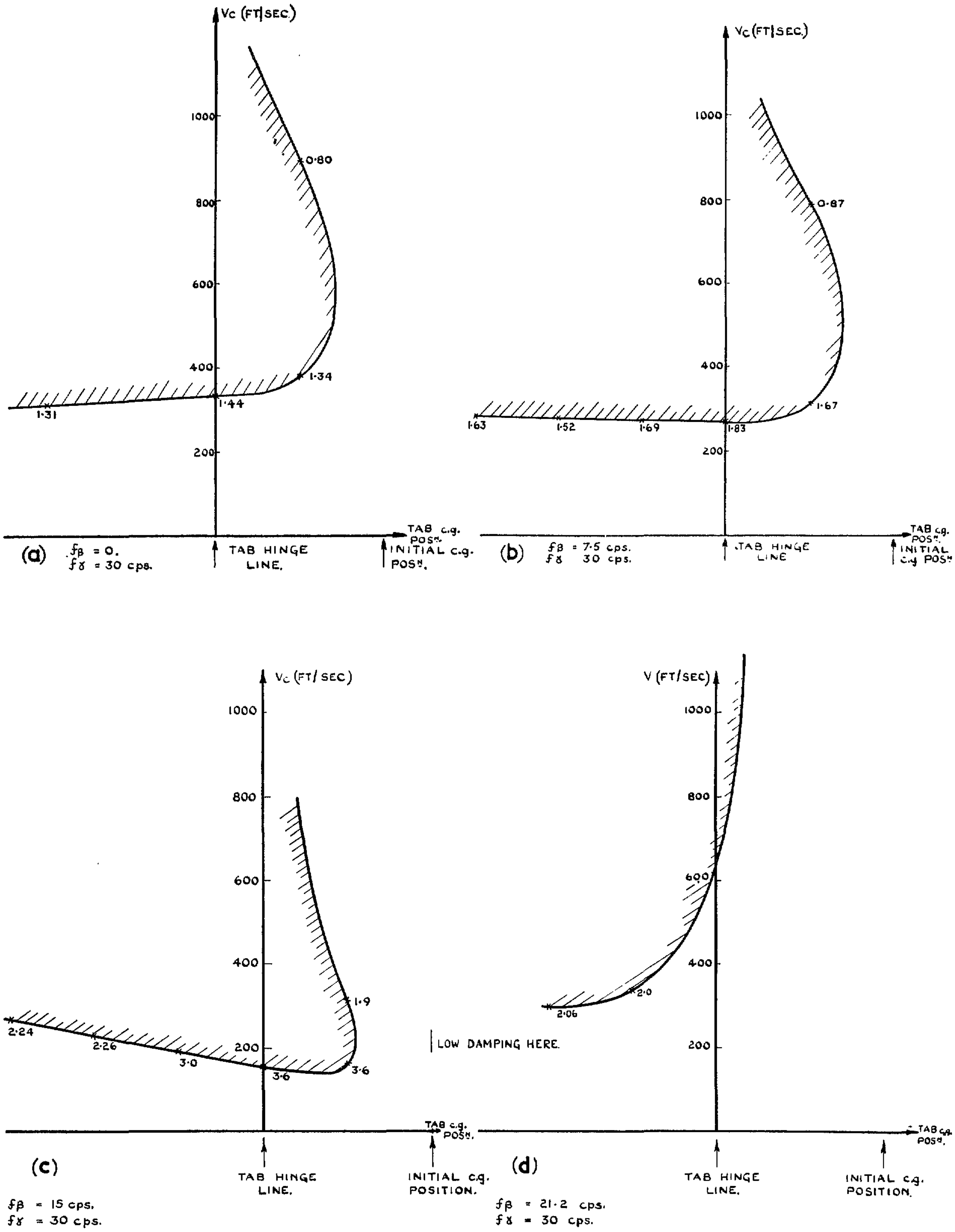
FIGS. 2a to 2d. Critical flutter speed vs. tab centre-of-gravity position.



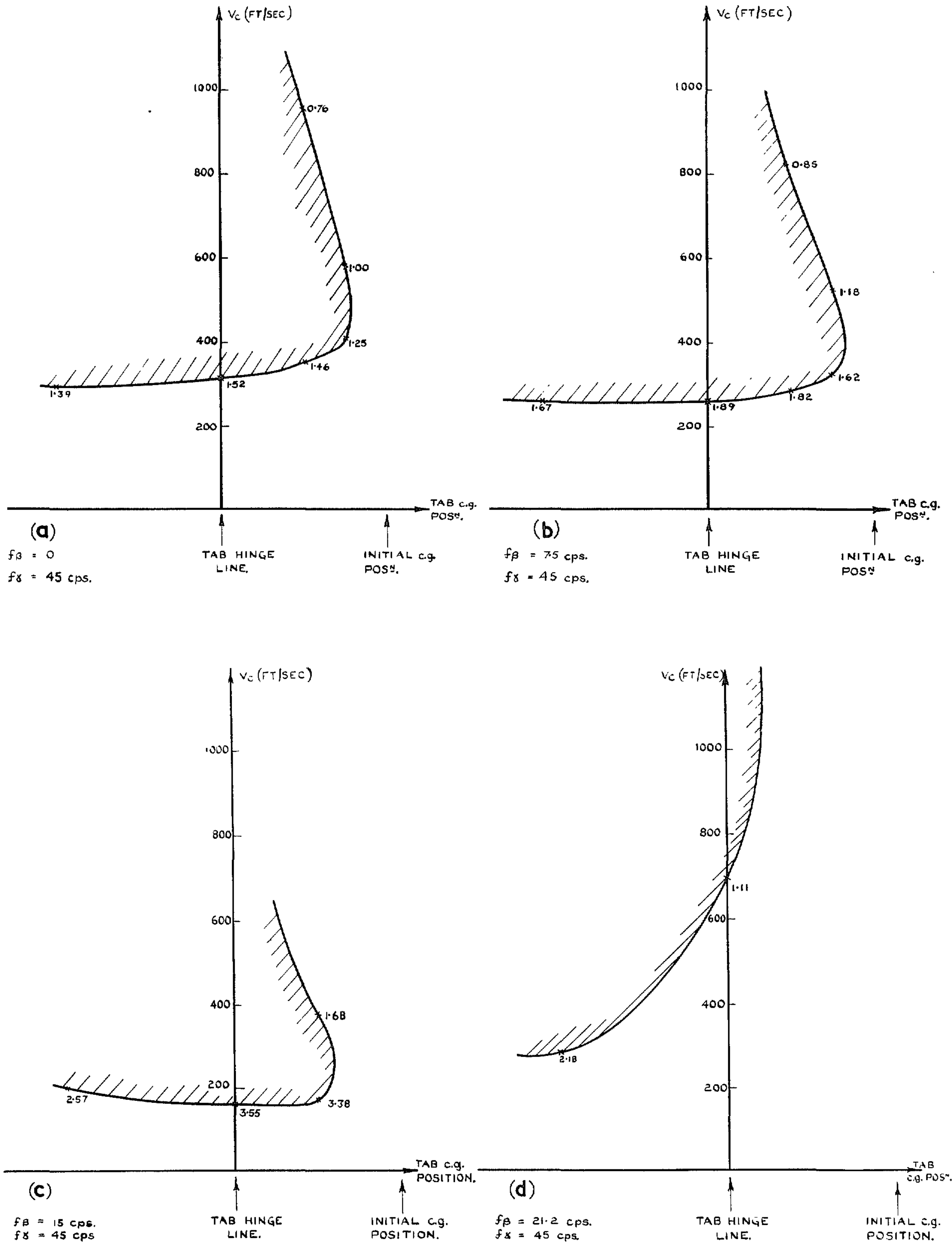
FIGS. 3a to 3d. Critical flutter speed vs. tab centre-of-gravity position.



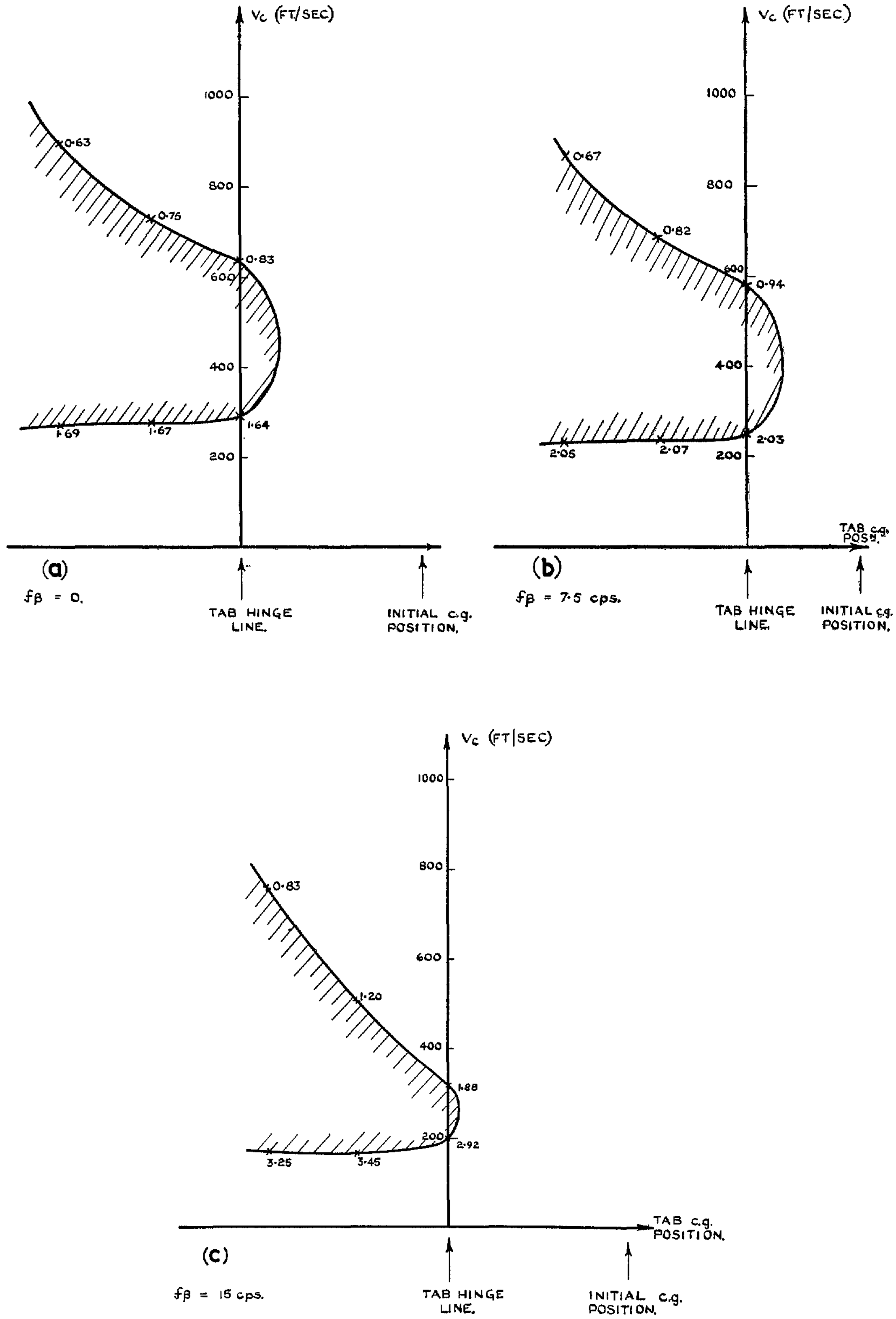
FIGS. 4a to 4d. Critical flutter speed vs. tab centre-of-gravity position.



FIGS. 5a to 5d. Critical flutter speed vs. tab centre-of-gravity position.



FIGS. 6a to 6d. Critical flutter speed vs. tab centre-of-gravity position.



FIGS. 7a to 7c. Critical flutter speed vs. tab centre-of-gravity position for binary wing aileron flutter.

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