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COMMUNICATED BY THE DIRECTOR-GENERAL OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

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Summary.—Consideration of experimental results obtained with relatively large pitot-tubes in relatively thin laminar boundary layers on cones[1] and flat plates[3, 5] in supersonic wind tunnels and then analysed using standard pitot equations, shows that the most noticeable distortion of the velocity profile is the appearance of a peak near the outer edge of the boundary layer. The displacement of the main body of the profile, familiar in incompressible-flow tests[4], may be small in supersonic flow and difficult to detect when making measurements with small-diameter pitot-tubes.

Displacement and momentum thicknesses (δ^* and θ) calculated from pitot traverses will be in error because of this distortion and displacement. The simple correction factor obtained by Davies[1] is shown to correlate results obtained by Blue and Low[5] and indicates that if the ratio of tube diameter to boundary-layer thickness is less than $0 \cdot 2$, then measured values of δ^* and θ may be less than 4 per cent above their true values.

Additional profile distortion may occur if flattened pitot-tubes are used to measure boundary layers on slender bodies of revolution and Appendix II describes the method of manufacture of tapered quartz tips of circular cross-section, which have a better response rate than flattened tubes of the same internal height.

1. *Introduction.*—The pitot-tube offers one of the simplest and most widely used methods for measuring the characteristics of boundary layers, either in subsonic or in supersonic flow. Its main disadvantage is that it involves the insertion of a probe into the boundary layer. This is bound to cause some flow disturbance, so that *a priori* there is no reason to believe that the picture given by the pitot is a true representation of an undisturbed boundary-layer flow. One difficulty is that a disturbance may be propagated upstream in sufficient strength to modify the growth and characteristics of the layer ahead of the pitot-tube, so that the pressure at the mouth of the tube would not correspond to that in the same position of an undisturbed flow. However, it might seem reasonable to assume that the scale of such a disturbance would be related to the size of the probe relative to the local thickness of the boundary layer, so that comparison of measurements obtained with probes of various sizes might give an indication of the errors to be expected. Obviously the best check would be given by measurements of a standard boundary layer whose characteristics are known in advance.

A second difficulty, linked to some extent with the first, arises when analysing the pressures recorded by a pitot-tube in a boundary layer. The standard equations relating pitot pressure, static pressure and Mach number are based on the assumption that the flow is uniform across the mouth of the pitot-tube. Inside the boundary layer there can be strong transverse gradients of velocity and temperature, so that unless the pitot is so small that the whole of the flow affecting its reading is sensibly uniform, then experimental results may show apparent inconsistencies when reduced by the standard equations.

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Both of these arguments indicate that the pitot diameter should be very small by comparison with the thickness of the boundary layer. However, this requirement can lead to practical difficulties since, as the pitot size is reduced, its rate of response to pressure variation is lessened, which lengthens the time needed for a boundary-layer traverse and indeed may make it almost impossible to obtain true readings. Also, low Reynolds-number effects may alter the reading of a small pitot-tube so that the standard equations can no longer be applied.

Most subsonic wind tunnels are of a sufficient size to permit tests to be made with boundary layers of a thickness sufficient to permit an adequate compromise between these conflicting requirements. On the other hand, most supersonic wind tunnels are smaller in size and the problem can be acute. This is illustrated by Fig. 1 which gives theoretical estimates[10, 11] of the thickness of a laminar boundary layer along the length of a cone under some typical operating conditions in supersonic and hypersonic wind tunnels. Thus, for example, at $M = 2$ and 10 in. back from the tip, the boundary-layer thickness may be less than $0 \cdot 03$ in., so it is clear that accurate measurement of its velocity profile could be difficult.

In general, turbulent boundary layers will be thicker than their laminar counterparts so that if sufficiently accurate measurements can be made in the thin laminar layer near the tip or leading edge of a model, then no serious difficulties should be encountered in the thicker turbulent layer further back.

The present note summarises the results of some research on the laminar problem at the Royal Aircraft Establishment[1, 2]. It describes the various effects (real or apparent) of pitot size found in a series of measurements of laminar boundary layers in supersonic flow. These lead to errors in the values of displacement and momentum thickness calculated from the pitot measurements, but it seems that it may be possible to obtain the correct experimental values by applying a simple correction factor dependent only on the ratio of pitot diameter to boundary-layer thickness.

Ideally the experimental results should be compared with the theoretical results which are available for laminar boundary layers in supersonic flow (using these latter as a standard). However, the theories assume that there is no disturbance at the leading edge or tip, whereas in practice there is such a disturbance and it can have appreciable effect on the subsequent development of the laminar boundary layer[3]. Consideration of such effects is outside the scope of the present paper, which is more concerned with errors that can be associated directly with the pitot-tube. Hence the emphasis is on trends shown up by results obtained with pitot-tubes of different sizes rather than on comparisons with theoretical estimates. Even so, the presence of leading-edge effects can be an undesirable extra parameter, but the majority of the results in Ref. 1 were obtained from traverses on a cone for which the tip effect should be small[3]. The pitot-tubes used in the tests had tapered quartz tips[2], which show a faster response rate than comparable flattened hypodermic tubes and should therefore prove useful for boundary-layer traverses in all small supersonic wind tunnels. Their method of manufacture and the results of some tests of response rate are given in Appendix II.

2. *The Effects of Pitot Size on Measurements of Laminar Boundary Layers in Supersonic Flow.*— The majority of the results considered by Davies in Ref. 1 were obtained from a series of measurements of the laminar boundary layer on a cone of 20 deg included angle in a 5 in. $\times$ 5 in. supersonic wind tunnel operating at $M = 2 \cdot 45$, with a stagnation pressure of 1 Atm and a stagnation temperature of about 35 deg C. The pitot-tubes used in the tests had tapered quartz tips of circular cross-section[2], and the outside diameters at their orifices ranged from $0 \cdot 003$ in. to $0 \cdot 020$ in. The boundary-layer thicknesses at the measuring stations were between $0 \cdot 02$ in. and $0 \cdot 03$ in. and at each station a series of traverses was made with tubes of different sizes.

The results were analysed using the standard equations (Appendix I) to give velocity profiles and values for displacement and momentum thickness. Thus both real and apparent effects may be present in these quantities. The conclusions of the analysis are summarised below and comparisons are made with results from other sources.

2.1. *Effects on Velocity Profile.*—Davies' results[1] indicated that tube size may have three effects on velocity profile when plotted in the conventional form of u/u_1 against $y/x\sqrt{Re_x}$.

These are illustrated by the sketch of Fig. 2, where for clarity the distortions have been magnified and are:

(1) a displacement of the main body of the profile (which may be accompanied by variations in its slope)

(2) the appearance of a peak in the profile near the outer edge of the boundary layer

(3) a distortion of the profile near the wall.

Some or all of these effects appear to be common to all measurements of laminar boundary layers in supersonic flow and the results obtained in Ref. 1 are considered individually in the following sub-sections, where they are compared with those given by other authors.

2.1.1. *Displacement of the profile along the y-axis.*—This displacement is considered first, since it is a commonly quoted effect of pitot size in low-speed flow. For example, Young and Maas[4] took measurements with circular pitot-tubes in the wake of an aerofoil at a wind speed of 60 ft/sec and found that decreasing the tube diameter displaced the main body of the profile in a positive direction along the y-axis (*i.e.*, in the opposite direction to that shown in Fig. 2). Physically this could be interpreted to mean that the pitot, by virtue of its presence in a flow with a pressure gradient normal to its axis, measured a pressure displaced a distance d' from its geometrical centre towards the region of higher velocity. Young and Maas found an average experimental value of $0 \cdot 2$ for d'/d, where d is the diameter of the pitot-tube.

The significant feature of Davies' results[1] for the laminar boundary layer on a cone at $M = 2 \cdot 45$ is that any displacements were in the opposite direction to that found by Young and Maas, *i.e.*, towards the region of lower velocity as illustrated in Fig. 2.

The displacements were accompanied by some distortion of the profiles, so that d' was not constant across the layer, but it had a maximum around $u/u_1 = 0 \cdot 8$ where values of d'/d up to $0 \cdot 15$ were found. However, the diameter of the pitot-tubes ranged in size only from $0 \cdot 003$ in. to $0 \cdot 020$ in. and the majority of the measurements were taken with tubes whose diameters were less than $0 \cdot 016$ in., so that the values of d'/d could have been affected considerably by experimental inaccuracies. For this reason, too much reliance should not be placed on the numerical value given above. It is quoted mainly to illustrate that the displacements were small, and may be difficult to detect when making measurements with small-diameter pitot-tubes in supersonic wind tunnels.

In this respect, Bradfield, De Coursin and Blumer[3] give the result of a single check of the effect of pitot size, made in the course of their investigation of the effects of leading-edge bluntness at $M = 3 \cdot 05$. Two flattened tubes of heights $0 \cdot 0025$ in. and $0 \cdot 008$ in. were used in a boundary layer about $0 \cdot 050$-in. thick and the results show no displacement effect on the main body of the velocity profile (Blue and Low do not give velocity profiles obtained with tubes of different sizes in their report[5] on the effects of pitot size in supersonic flow, so no direct comparisons can be made).

From these considerations it would seem that this displacement (if present at all) may be difficult to detect when measuring boundary layers in supersonic flow with small-diameter pitot-tubes. The second and third effects mentioned at the beginning of the section can be much more apparent and will be considered below. However, the variations in displacement and momentum thicknesses found by Davies and considered in section 2.2 could not be explained solely by distortions near the wall and near the free stream, so that the displacement of the main body of his profiles (though small) must be considered as having had some influence (in this respect it may be noted that Blue and Low's results for the variation of momentum thickness agree well with that found by Davies).

2.1.2. *Profile peaking near the outer edge of the boundary layer.*—This effect is common to all the investigations in supersonic flow considered above[1, 3, 5] and the peak increases in height and extent as the pitot size is increased (Fig. 2). Fig. 3 gives the maximum values of u/u_1 measured by Davies[1] over a range of values of d/δ, where δ is the thickness of the boundary layer (an average experimental value of δ was used in this case). The scatter of the results made it impossible for them to be represented by a reliable mean curve and, as in Ref. 1, the broken line in Fig. 3 is included only for comparison.*

In practice this particular distortion is likely to have considerable effect on derived values of displacement and momentum thickness. It is an undesirable effect because there can be a considerable variation in profile from one tube size to another and also when determining displacement thickness (δ^x) and momentum thickness (θ) from the standard integral formulae (Appendix I or equations (1) and (2) of section 2.2 below), there is a choice of two upper limits of integration, the first corresponding to the (u/u_1) maximum at the distortion peak and the second to the free stream $(u/u_1) = 1 \cdot 0$. Table 1 gives an example quoted by Davies[1] which shows the magnitude of these effects.

TABLE 1

Pitot outside diameter d (in.)	0·010		0·005
Upper limit of integral	δ at $(u/u_1)_{max}$	δ at $(u/u_1) = 1 \cdot 0$	No peak discernible
δ^x (in.) θ (in.)	0·00885 0·00147	0·00850 0·00142	0·00689 0·00127

It is clear from the table that while there are small variations in δ^x and θ (approximately 4 per cent) introduced by the ambiguity of the upper limit of integration (for $d = 0 \cdot 010$ in.), there are much larger variations when comparison is made with results from a smaller tube ($d = 0 \cdot 005$ in.) producing less distortion in the velocity profile. Undoubtedly there may also be a contribution from a profile displacement of the type considered in section 2.1.1, but it is difficult to separate the effects, so that while there is some merit in extending the integration to the free-stream limit, it is apparent that it is more important to define an upper limit of tube size which will avoid or considerably reduce both types of distortion. Such a criterion is best obtained from the collected results for δ^x and θ in section 2.2 rather than from an arbitrary choice of limit from inspection of velocity profiles.

2.1.3. *Distortion of the profile near the wall.*—This effect also is common to Refs. 1, 3 and 5. As illustrated in Fig. 2 the profile bends away from the origin, giving velocities in excess of those expected (corresponding to an excess in pitot pressure) and the distortion increases as the tube size is decreased.

This may be a real effect caused by wall interference, but Davies[1] suggests that it may be largely an effect of low Reynolds number on pitot-tube reading or indeed that the pitot may not be operating under continuum flow conditions when near to the wall. Support for this suggestion is given by the fact that the distortion could largely be removed by correcting the measured pitot pressure in accordance with experimental results obtained by Kane and Maslach for rarefied gas flow at supersonic speeds[6].

* If p_m is the pressure measured by the pitot-tube, then from the Rayleigh pitot equation and assuming constant static pressure and constant total energy (as in Ref. 1), we have for $\gamma = 1 \cdot 4$:

$$\left(\frac{p_m}{p_{m1}}\right)_{max} - 1 \simeq 2(1 + 1/5\, M_1^2)\, \frac{M_1^2 - 1/2}{M_1^2 - 1/7} \left\{ \left(\frac{u}{u_1}\right)_{max} - 1 \right\},$$

so that the peaks in the original pitot pressure profile will be more marked than those in the derived velocity distribution.

Incompressible flow measurements[7, 8, 9] in pipes and channels have shown that for pitot Reynolds numbers $ud/v < 200$, the pitot-tube will measure pressures higher than would be expected from the Bernoulli relation between total head and static pressure, and will therefore indicate a velocity higher than the true velocity (this corresponds to a displacement of the effective centre of the tube towards the region of higher velocity).

Now in incompressible flow, the low Reynolds numbers are associated with low local velocities and their effects can be related to the classical theories for the flow of a viscous incompressible fluid. However, in supersonic flow, both in the free stream of a rarefied gas[6] and close to the surface of a solid body even when the free stream is of higher density, the pitot Reynolds number is low mainly because the density is low*. Hence in addition to ' viscous ' effects, the measured pitot pressure may also be affected by the fact that the local mean free path of the air particles may be comparable with the diameter of the tube, so that continuum flow conditions no longer apply.

Kane and Maslach[6] considered the interpretation of the impact pressure measured by pitots in a supersonic stream at low Reynolds numbers. The Mach numbers ranged from 2·3 to 3·8 and the tubes were submerged in the flow remote from any wall or surface, with no pressure or velocity gradients normal to the tube axis. It was assumed that the true Mach number was given by the Rayleigh pitot equation when tubes of different diameters recorded identical pressures under the same conditions. The results are given in Figs. 4 and 5 (taken from Ref. 6).

Fig. 4 plots pressure ratio p_m/p_t (where p_m is measured and p_t is Rayleigh pitot pressure) against pitot Reynolds number ($Re_d = ud/v$) as would be suggested by incompressible viscous flow considerations. The results show that application of the Rayleigh pitot equation to measured pitot pressure would give an overestimate of Mach number when $Re_d < 200$, which is the limit found in incompressible flow.

Fig. 5 plots the same pressure ratios against M/Re_d, which is related to the ratio of the mean free path to the diameter of the pitot-tube, and in this case the breakdown occurs when $\log M/Re_d > -1{\cdot}75$ or $M/Re_d > 0{\cdot}018$ (included on this graph is the range of values of M/Re_d covered by Stanton's incompressible pipe flow tests[8]. When recast on a pressure basis[1], Stanton's surface pitot results are within 0·01 per cent of unity).

Consideration of Figs. 4 and 5 shows that there is not yet sufficient evidence to decide between the parameters Re_d and M/Re_d, although the latter might seem more reasonable. When applied to the boundary-layer problem near the wall, the Mach number may in any case be small, so either graph may prove applicable (this point may be clarified when pitot traverses with tubes of different sizes in boundary layers in hypersonic wind tunnels become available, since much larger variations in density will then be encountered).

Meanwhile it is suggested that Figs. 4 and 5 may be used as a first approximation to a correction of the flow distortion near the wall.

In many practical cases this may not be necessary, since as Fig. 2 indicates, it may be possible to make a reasonable extrapolation of the main body of the velocity profile to the origin, and the ' curl-up ' can be neglected. This is particularly so for tests at low supersonic Mach numbers, when the drop in density through the boundary layer is relatively small, but as operating speeds increase, the drop in density can increase rapidly and if the low pitot Reynolds-number region spreads across an appreciable portion of the boundary layer, then the resulting ' distortion ' may not be overcome so easily.

This last point is illustrated by Fig. 6 (taken from Ref. 1) which shows that if $d/\delta < 0{\cdot}2$ (to reduce the profile peaking of section 2.1.2), then the length of run along a flat plate required if the pitot Reynolds number is to exceed 100 or 200 at various positions within the boundary

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* Since the static pressure across the boundary layer is constant, then by the equation of state and the fact that the local static temperature increases as the wall is approached, the local density must decrease from its free-stream value. Under zero heat-transfer conditions between solid and gas we have in the limit $\rho_1/\rho_w \simeq 1 + 0{\cdot}85\{(\gamma - 1)/2\} M_1^2$, where ρ_w is the density at the wall.

layer, increases rapidly with increase in free-stream Mach number. For example, taking the criterion $ud/v > 200$ when $y/\delta = 0\cdot 2$, then the required value of $u_1 x/v_1$ increases from about 1 million at $M_1 = 1\cdot 5$ to about 5 million at $M_1 = 3$. These values would be multiplied by 3 for the laminar boundary layer on a cone.

2.2. *Effects on Momentum and displacement Thicknesses.*—So far the various types of velocity profile distortion which accompany the measurement of relatively thin boundary layers with relatively large pitot-tubes have been discussed. Obviously these will have their effects on displacement thickness (δ^x) and momentum thickness (θ) calculated from the integrals

$$\delta^x = \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy \quad \ldots \quad (1)$$

and

$$\theta = \int_0^\delta \frac{\rho u}{\rho_1 u_1} \left(1 - \frac{u}{u_1}\right) dy\,. \quad \ldots \quad (2)$$

2.2.1. *Results from tests by Davies*[1].—Fig. 7 shows values obtained by Davies[1] from measurements with tubes of different diameters of the laminar boundary layer on a 20-deg cone at a distance of 3·8 in. from the tip. The local Mach number was 2·2 and the Reynolds number immediately outside the boundary layer was 1 million. In obtaining δ^x and θ, the integrals were taken to the values of y corresponding to $(u/u_1)_{\max}$ and the velocity profiles were faired into the origin, neglecting the ' curl-up ' discussed in section 2.1.3.

The boundary-layer thickness {to $(u/u_1)_{\max}$} was between 0·025 in. and 0·030 in. which was rather thin in view of possible experimental inaccuracies.

These inaccuracies are reflected in the large amount of scatter of the results (particularly for θ) in Fig. 7, so that from a straight plot of δ^x or θ against d/δ it was impossible to predict the true values which would be obtained with $d/\delta = 0$.

However, Davies found that it was possible to overcome this difficulty by plotting $\delta^x/(1 - d/\delta)$ or $\theta/(1 - d/\delta)$ against $1 - d/\delta$. Using a logarithmic plot it was possible to fair a straight line through the experimental results as shown in Fig. 8 and ' true values ' ($d/\delta = 0$) of $\delta_T{}^x = 0\cdot 0077$ and $\theta_T = 0\cdot 00140$ were obtained. These compare with theoretical values[10, 11] of 0·0093 and 0·00141 respectively. Thus there is a good agreement for θ_T, but not for $\delta_T{}^x$, but it is remarkable that the equations for the correction curves are identical, namely

$$\frac{\delta^x}{\delta_T{}^x} = \left(1 - \frac{d}{\delta}\right)^{-0\cdot 16} \quad \ldots \quad (3)$$

and

$$\frac{\theta}{\theta_T} = \left(1 - \frac{d}{\delta}\right)^{-0\cdot 16}. \quad \ldots \quad (4)$$

The curves given by equations (3) and (4) are superimposed as broken lines in Fig. 7, to show how they compare with the original unforced data.

Now equations (3) and (4) are based on only one set of test results and also the plots in Fig. 8 are relatively insensitive to changes in the index of $(1 - d/\delta)$, so that further evidence is required before they could be accepted as genuine correction functions.

2.2.2. *Results from tests by Blue and Low*[5].—In this respect further data can be obtained from the tests by Blue and Low[5] on the laminar boundary layer on a flat plate at $M = 3$. In their own analysis they compare momentum thicknesses obtained from traverses with flattened pitot-tubes of various tip heights with corresponding theoretical values and derive a correction function which is not comparable directly with equation (4) above. To do this, a fresh analysis has been made of their data for momentum thickness at a distance of 2·38 in. from the leading edge, with Reynolds numbers ranging from $0 \cdot 3$ to $0 \cdot 6$ million and tube heights from $0 \cdot 003$ to $0 \cdot 016$ in. Boundary-layer thicknesses were not available (except from profiles obtained with a tube of height $0 \cdot 008$ in.), so these were calculated from the formulae of Ref. 10, giving

$$\frac{\delta}{x} \sqrt{Re_x} = 11 \cdot 3 \qquad \ldots \qquad (5)$$

(assuming that the constant in incompressible flow is 6). This gave values of δ slightly beyond the peak in the profiles given by the $0 \cdot 008$-in. tube.

Following the lines of Davies' analysis[1] as described above, a 'true value' of θ was obtained which was 10 per cent. above the corresponding theoretical value for a flat plate. This factor of 10 per cent may not be unreasonable, since the results of Bradfield, De Coursin and Wheeler[3] for leading-edge effect indicate that the true measured value under Blue and Low's conditions (leading-edge thickness of $0 \cdot 001$ in.) could be up to 18 per cent above the theoretical value.

2.2.3. *Suggested design requirements.*—Accepting this value of θ_T, Fig. 9 compares the results of Davies[1] and of Blue and Low[5] by plotting

$$\frac{\theta}{\theta_T} \frac{1}{1 - (h/\delta)} \text{ against } \left(1 - \frac{h}{\delta}\right)$$

where h is either the outside height or the outside diameter at the mouth of the pitot tube.

The majority of the results are represented within $\pm$ 5 per cent (on this plotting basis) by the curve

$$\frac{\theta}{\theta_T} = \left(1 - \frac{h}{\delta}\right)^{-0 \cdot 18} \qquad \ldots \qquad (6)$$

and it is suggested that it may be applied with some degree of confidence to correct results obtained with relatively large pitot-tubes in relatively thin laminar boundary layers, provided conditions are not widely different from those of the test data on which it is based. From Davies' work[1] it seems that a similar correction can be applied to displacement thickness.

Reference back to Fig. 7 indicates the additional warning that equation (6) should be applied with caution if the value of d/δ is small. In this case it is more useful to derive confidence limits from equation (6). Thus if the experimental accuracy were perfect, then pitot-tubes whose values of h/δ were $0 \cdot 1$ and $0 \cdot 2$ would give values of θ (or δ^*) 2 and 4 per cent above the true value. Fig. 1 shows that these requirements should become easier to meet as operating Mach numbers increase and will be easier to meet under zero heat-transfer conditions than if heat is flowing from the air stream to the body. The latter condition is illustrated by the limiting case of surface temperature (T_w) equal to the static temperature outside the boundary layer (T_1).

However, it must be remembered that while it may be easier to avoid profile peaking in tests in hypersonic tunnels, the low Reynolds number effects (section 2.1.3) may become much more serious and equation (6) or Fig. 9 would not be applicable as a correction (unless by coincidence), since it has been derived from tests wherein profile peaking was the main distortion.

3. *The Effect of Pitot Shape and End Finish.*—The size-effect data considered in the previous section were obtained

(*a*) with circular-section pitot-tubes measuring the laminar boundary layer on a cone[1]

(*b*) with flattened pitot-tubes measuring the laminar boundary layer on a flat plate[3, 5].

The general order of agreement between the results was good.

Before the introduction of quartz-tipped pitot-tubes, the general practice was to reduce the orifices of stainless-steel hypodermic pitot-tubes from a circular section ($0 \cdot 020$ in. O.D.) to an elongated section (height $\simeq 0 \cdot 008$ in.) by a flattening process. No specific size or shape investigations have been made at the Royal Aircraft Establishment with these flattened tubes, but it has been found that whereas the use of flattened tubes for measuring laminar boundary layers on flat plates made little difference to the velocity profiles, the extension of their use to cone boundary layers resulted in additional distortion of the profile near the surface. This may be due to the fact that the mouth of the flattened tube does not follow the cone surface and hence may not receive similar filaments of boundary-layer flow across its width.

In view of this, the results in this paper may not be applicable to boundary-layer traverses made on slender bodies of revolution with flattened pitot-tubes. It would be preferable to use tubes of circular section.

The method of manufacture of the quartz-tipped pitot-tubes with circular cross-section[2] used in boundary-layer research at the R.A.E. is described in Appendix II and illustrated in Fig. 10. Fig. 11 shows end views of typical quartz and hypodermic pitot-tubes. In general it has been found possible to retain a round and true bore with the quartz tubes even when drawn down to 0·001 in. O.D.

To check the effects of end finish and degree of taper on accuracy of measurement, boundary-layer traverses were made on a flat plate with the four tubes shown in Fig. 12[2].

Fig. 12a shows a steeply tapered tube with a good polished end surface.

Fig. 12b shows a short tube with normal taper and highly polished end surface.

Fig. 12c shows a normal length tube with normal taper, but with a bad chip on the under side of the end surface. No attempt was made to polish this tube.

Finally, Fig. 12d shows a normal length tube with only a slight taper. The polishing was left unfinished and some chipping remains.

Comparison of the pitot pressure profiles[2] showed that taper had negligible effect, but that end finish was important. Tubes (c) and (d) would therefore have been rejected and as a result all the quartz-tipped pitot-tubes are examined under a microscope before being used in the wind tunnel and are accepted only if their end finish is similar to that of tubes (a) and (b).

4. *Conclusions.*—(*a*) At supersonic speeds the most noticeable distortion of the velocity profile arising from the use of relatively large pitot-tubes in relatively thin laminar boundary layers and then analysing the measurements with standard pitot equations is the profile peaking near the outer edge of the boundary layer, illustrated in Fig. 2.

(*b*) The distortion near the wall (Fig. 2) may probably be mainly a low Reynolds-number or low density effect on pitot-tube reading and this may have serious consequences in tests at hypersonic speeds (when the low density region may spread through an appreciable portion of the boundary layer).

(*c*) The displacement of the main body of the profile (effect 1 in Fig. 2), familiar in incompressible flow tests, may be small and difficult to detect when measuring boundary layers in supersonic flow with small-diameter pitot-tubes. Davies' results[1] indicate that it may be in the opposite direction to that found at low speeds, *i.e.*, towards the region of lower velocities.

(*d*) Displacement and momentum thicknesses calculated from pitot traverses reduced by the standard equations will be in error because of these distortions. A correction factor of the form derived by Davies[1] can also be applied to the results of Blue and Low[5] giving (Fig. 9) for both sets of results:

$$\frac{\theta}{\theta_T}\left(\text{or } \frac{\delta^*}{\delta_T}\right) = \left(1 - \frac{h}{\delta}\right)^{-0\cdot 18}$$

where θ and δ^* are the measured momentum and displacement thicknesses

θ_T and δ_T are the true values

h is the outside height or diameter at the mouth of the pitot-tube

δ is the boundary-layer thickness.

(*e*) The factor of conclusion (*d*) is derived from test results at $M = 2 \cdot 3$ and $3 \cdot 0$ and is unlikely to be applicable in hypersonic tunnels. (*Note* conclusions (*a*) and (*b*) above.)

(*f*) If $h/\delta = 0 \cdot 1$ and $0 \cdot 2$, then from conclusion (*d*) the measured values of θ and δ^x would be 2 and 4 per cent above their true values (assuming perfect experimental accuracy).

(*g*) Additional profile distortion near the wall (effect 3 in Fig. 2) may occur if flattened pitot tubes are used to measure boundary layers on slender bodies of revolution. The use of circular-section tubes is preferable in this case.

(*h*) Circular-section tubes with a better response rate than flattened tubes of the same internal height can be obtained by using tapered quartz tips (Appendix II). Normal variations in taper have negligible effect on measuring accuracy, but it is important to secure a good end finish by polishing (this last conclusion would apply equally well to flattened steel tubes).

LIST OF SYMBOLS

x	Distance along plate or cone from leading edge or tip, measured along the surface
y	Distance normal to the surface
p_0	Stagnation pressure
p_m	Measured pitot pressure
p	Static pressure
T	Static temperature
T_H	Total temperature
M	Local Mach number
u	Local velocity
u_1	Velocity just outside the boundary layer
d	Pitot outside diameter
d'	Displacement of the point of measured pressure from the pitot-tube centre
Re_x	Free-stream Reynolds number
$=$	$(u_1 x/\nu_1)$
Re_d	Local Reynolds number of pitot-tube
$=$	(ud/ν)
μ	Viscosity
ρ	Density
ν	Kinematic viscosity
$=$	(u/ρ)
δ^x	Displacement thickness
$=$	$\int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy$
θ	Momentum thickness
$=$	$\int_0^\delta \frac{\rho u}{\rho_1 u_1} \left(1 - \frac{u}{u_1}\right) dy$
δ_T^x, θ_T	True values of δ^x and θ
δ	Boundary-layer thickness
γ	Ratio of specific heats of air

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APPENDIX I

Equations Used in the Reduction of Pitot Traverse Measurements at Supersonic Speeds

1. *Relation between Pressure and Mach Number.*—The evaluation of the boundary-layer characteristics in supersonic flow requires direct or indirect measurement of the following physical quantities:

p_0	Stagnation or total pressure
p	Static pressure
T_H	Total temperature.

The total pressure p_0 and the static pressure p determine the local Mach number M, the relation between p_0, p and M being given by the well-known Bernoulli energy equation for isentropic flow:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)} . \qquad \text{(I.1)}$$

The normal boundary-layer equations indicate that the change in static pressure across the layer is of small order; it is therefore usually assumed constant across the layer and can be conveniently measured by wall static holes. The pitot pressure p_m is measured by small-diameter pitot-tubes[2]. For subsonic flow ($M < 1 \cdot 0$) the flow is brought to rest isentropically at the mouth of the tube and the measured pitot pressure p_m is equivalent to the stagnation pressure p_0. Equation (I.1) can therefore be used to relate the flow velocity with the measured pitot and static pressures.

When a pitot-tube is inserted into a supersonic stream the air is decelerated in two stages. The first is a non-isentropic compression through a detached shock wave in front of the tube, the second an isentropic compression from the shock wave to the mouth of the tube. Because the deceleration through the shock wave is accompanied by an increase in entropy the Bernoulli equation cannot apply (the tube reads something less than the total pressure of the flow). Instead the Rayleigh pitot equation is usually applied

$$\frac{p_m}{p} = \frac{\{\frac{1}{2}(\gamma + 1)M^2\}^{\gamma/(\gamma-1)}}{\left(\frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma - 1}{\gamma + 1}\right)^{1/(\gamma-1)}} \quad \text{(I.2)}$$

This is derived from the Rankine-Hugoniot relations across a normal shock wave

$$\frac{p_2}{p} = \frac{\gamma M_2^{\,2} - \frac{1}{2}(\gamma - 1)}{\frac{1}{2}(\gamma + 1)} \quad \text{(I.3)}$$

$$M_2^{\,2} = \frac{1 + \frac{1}{2}(\gamma - 1)M^2}{\gamma M^2 - \frac{1}{2}(\gamma - 1)} \quad \text{(I.4)}$$

(where p_2 and M_2 are the static pressure and Mach number respectively immediately behind the shock wave) and from the Bernoulli equation for the subsequent isentropic compression, which in this case gives

$$\frac{p_m}{p_2} = \left\{1 + \frac{\gamma - 1}{2} M_2^{\,2}\right\}^{\gamma/(\gamma-1)} . \quad \text{(I.5)}$$

For a strict application of either the Bernoulli or Rayleigh equations the flow pattern should be symmetrical. This is obviously not so in a boundary layer and unless the pitot is so small that the whole of the flow affecting its reading is sensibly uniform, experimental results may show apparent inconsistencies when reduced by equations (I.1) and (I.2). Some of the inconsistencies which have arisen in the course of boundary-layer traverses on flat plates and cones are considered in the main text.

2. *Relation between Mach Number, Velocity and Temperature.*—To determine the velocity distribution we have

$$M = \frac{u}{\sqrt{(\gamma RT)}} , \quad \text{(I.6)}$$

where R is the gas constant, so that local velocity can be obtained if the local Mach number and local static temperature are known. The former is obtained from equation (I.2) but the latter cannot be measured directly. It may be obtained indirectly by using an interferometer to measure the variation in density across the boundary layer and then applying the equation of state

$$p = \rho RT \quad \text{(I.7)}$$

which, since p is constant across the layer gives

$$\rho \sim \frac{1}{T} \quad \text{(I.8)}$$

and hence the temperatures can be determined.

An alternative is to assume that the theoretical relations between temperature and velocity will apply (*e.g.*, Ref. 10). It is useful to express equation (I.6) in the form

$$\begin{aligned} u &= M\sqrt{(\gamma RT)} \\ &= \frac{M\sqrt{(\gamma RT_H)}}{\sqrt{(T_H/T)}} \\ &= \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{1/2}}\sqrt{(\gamma RT_H)} \,. \qquad \text{(I.9)} \end{aligned}$$

The advantage of equation (I.9) is that if no heat is being transferred between the body and the air stream then T_H does not vary much across the boundary layer, and velocities are usually derived with the simple assumption that T_H is constant and equal to its free-stream value. The errors introduced by this assumption are small[5].

However, if heat is being transferred, then this approximation is not valid and the correct theoretical variations must be used. The approximate formulae of Ref. 10 may be useful in this respect.

3. *Displacement and Momentum Thicknesses.*—Displacement thickness (δ^x) is given by

$$\delta^x = \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy \qquad \text{(I.10)}$$

and momentum thickness (θ) by

$$\theta = \int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right) dy \,. \qquad \text{(I.11)}$$

The density variations across the boundary layer can be obtained from equation (I.8) and the fact that (as in equation (I.9))

$$T_H/T = 1 + \tfrac{1}{2}(\gamma - 1)M^2 \,. \qquad \text{(I.12)}$$

4. *Skin Friction.*—The momentum thickness is a measure of the loss of momentum between the leading edge or tip and the station under consideration. In the absence of pressure gradients this can be equated to the skin friction acting over the same distance and the following formulae are obtained for local skin-friction coefficient

$$c_f = \frac{\tau_0}{\frac{1}{2}\rho_1 {u_1}^2}$$

and mean skin-friction coefficient

$$C_F = \frac{\int \tau_0 \, ds}{\frac{1}{2}\rho_1 {u_1}^2 S}$$

(*a*) *Flat Plates*

$$c_f = 2\frac{d\theta}{dx}$$

$$C_F = 2\frac{\theta}{x}$$

(*b*) *Cones*

$$c_f = 2\frac{d\theta}{dx} + 2\frac{\theta}{x}$$

$$C_F = 4\frac{\theta}{x}$$

APPENDIX II

The Use of Quartz in the Manufacture of Small-Diameter Pitot-Tubes[2]

The relatively thin boundary layers encountered in wind-tunnel tests at supersonic speeds necessitate the development of pitot-tubes of extremely small dimensions. The most common method of doing this has been to flatten small stainless steel hypodermic tubing (original outside diameter : 0·020 in.) and then to hone this to an overall height of approximately 0·010 in. or less if desired.

Now as Bradfield and Yale[12] have stated, the optimum shape at the entry to a pitot-tube for a fast response rate is obtained if the bore expands immediately behind the mouth. The reason for this is that such a shape keeps the length of the smallest diameter passageway to a minimum (in this passageway the Reynolds number will be small because of its small height and the friction coefficient will be correspondingly high). Unfortunately the process of flattening tubes tends to produce a fairly long passageway with little expansion which together with the small inside diameter of the tubing used (usually about 0·010 in. for 0·020 in. outside-diameter tubing) causes a considerable restriction.

On the other hand the technique of drawing down quartz tube described below gives a pitot head with an entry much closer to the optimum shape and with a much improved response rate for a given tip diameter.

1. *Method of Manufacture.*—The method of manufacture of quartz pitot-tubes (shown in step by step stages in Fig. 10) is briefly as follows.

The centre portion of a piece of quartz* tubing approximately 3 in. to 4 in. long × 0·050 in. O.D.† (0·030 I.D.) is heated in an oxygen-coal gas flame. By pulling at each end the quartz flows and the centre portion of the tube is drawn down to the required diameter. The tube is removed from the flame and bent between the fingers, being brittle it breaks and two heads in the rough state are obtained. As in drawing glass, care must be taken to avoid a very fierce flame or the quartz will flow too easily and break off in the flame. In the early stage of development an air-gas flame was used but the heat was hardly sufficient to make the quartz flow.

By adjustment of flame, care in drawing down and in breaking, the heads can be made to any required diameter and degree of taper. Tubes drawn out in a staright line retain a perfectly round and true bore even when drawn down to 0·001 in. O.D.

The heads are now polished to obtain a smooth end surface in the plane normal to the axis (as in Fig. 12b). Care must be taken at this stage as the quartz chips easily (*cf.* Fig. 12c). Smooth gentle sweeps over a fine hone holding the head between the thumb and forefinger have yielded the best results and by this method the proportion of rejected tubes has been kept down to about 50 per cent.

After polishing, the head is gently heated in an air-gas flame and bent through 90 deg (Fig. 10) to form a total-head probe. It is then inserted in a piece of 2 mm stainless-steel hypodermic tubing (I.D. = 0·053 in.) and an air-tight seal is made between the quartz and the steel. As shown in Fig. 10, two methods have been used. The original method was to coat the quartz with platinum (liquid platinum process), copper-plate the platinum and solder into the 2 mm stainless-steel tube. This method was discontinued when it was found that the platinum was tending to peel off the quartz. The tubes are now sealed with a cold-setting Araldite. No trouble has been experienced with this seal but it requires 24 hours for setting‡ (liquid-platinum process gives an immediate seal).

A completed pitot-tube is shown as the final stage in Fig. 10.

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* Quartz is used in preference to glass as it is considerably stronger and was found to be easier to work with.
† Tubes of these dimensions may be obtained from the Thermal Syndicate, Wallsend-on-Tyne.
‡ Araldite with a shorter setting time (1 hour) is now available and would most likely be suitable for this process.

Checks are made during manufacture for trueness of bore, and before each test run, for smoothness of end surface. Fig. 11 shows a comparison of bore between two quartz tubes, a 0·020-in. O.D. hypodermic tube and a 0·020-in. O.D. hypodermic tube flattened to 0·010 in. overall height (the flattened tube is a very good example of its type).

2. *Response Rates for Tubes of Varying Diameter.*—2.1. *Test Rig.*—The response rates of a number of pitot-tubes (quartz and hypodermic) were obtained using the test rig shown in Fig. 13. The pitot-tube under test was inserted in one side of the test chamber and connected *via* an isolation valve and a T-piece to a mercury manometer. By closing the isolation valve and opening valve A (pressure) or B (vacuum) the pitot side could be set at any desired pressure. The test chamber was evacuated to a required pressure by a vacuum pump and maintained at this pressure by using the bleed valve. When conditions in the chamber and on the pitot side were steady, valve A (or B) was closed and the isolation valve opened. With a stop watch the rate at which the pitot manometer was brought to the test-chamber pressure was noted. The chamber was set at a low pressure as this simulated the actual tunnel running conditions.

2.2. *Response Rates.*—Five quartz pitot-tubes of inside tip diameters ranging from 0·004 in. to 0·013 in., but of approximately constant taper and length, and two hypodermic tubes (one a standard 0·020-in. O.D. tube and the other a flattened version) were tested. Table 2 gives the main dimensions of each tube. The length of connecting tube to the pitot manometer was fairly typical of that used in wind-tunnel tests and was kept constant.

Fig. 14 gives plots of the time which the pitot manometer took to reach within 5 per cent and 1 per cent of the test-chamber pressure. In Fig. 14a the pressure was rising inside the pitot-tube from 173 mm Hg abs. to a test chamber pressure of 381 mm Hg abs., while in Fig. 14b the pressure was falling from atmospheric to a test-chamber pressure of 36·5 mm Hg abs.

In both cases the quartz tubes show a marked superiority over the hypodermic tubes, for which the main explanation is probably the differences in diameter of the two types of tubes following the entry (*see* Table 2). It is also likely that the quartz tubes possess cleaner bores.

The smallest quartz tip tested had an I.D. of 0·0042 in. and an O.D. of 0·0078 in. and took 100 sec (for 1 per cent accuracy) to fall from atmospheric to 36·5 mm Hg abs. (Fig. 14b). This is regarded as the limiting size for a tube of the geometry detailed in Table 2 if a reasonable response rate is desired. The response rate could be improved if there was a steeper taper on the tip.

The response rate is also affected by the volume of the system, bore of connecting tubing, etc., so that improvements could be obtained by adjustment of these parameters. This was outside the scope of the present tests but a method for finding the optimum length and inside diameter of connecting tubing is given in Ref. 13.

Another possibility for improving the response rate (for a given tube size) might be to flatten the tips in addition to drawing them down (*Note* that in Fig. 14 the reduction in response rate in going from a circular to a flattened hypodermic tube is not as great as that experienced by the quartz tips over the same range of inside diameters, since the cross-section area at the mouth of a flattened tube is considerably greater than that of one of circular section of the same height).

3. *Additional Comments.*—It has been shown that very small pitot heads can be made by drawing down quartz tubing and the examples considered have shown very satisfactory response rates. Because of their taper and smooth finish the quartz tips gave faster response rates than those obtained with hypodermic tubes of the same orifice height.

A quartz tip cannot be as robust as a steel tip and more care is required when handling them but they have stood up to quite arduous conditions inside a wind tunnel and their ease of manufacture compensates for any fragility (more breakages have occurred outside than inside the wind tunnel). Finally, any blockage by foreign matter is easily detected since the tube is transparent.

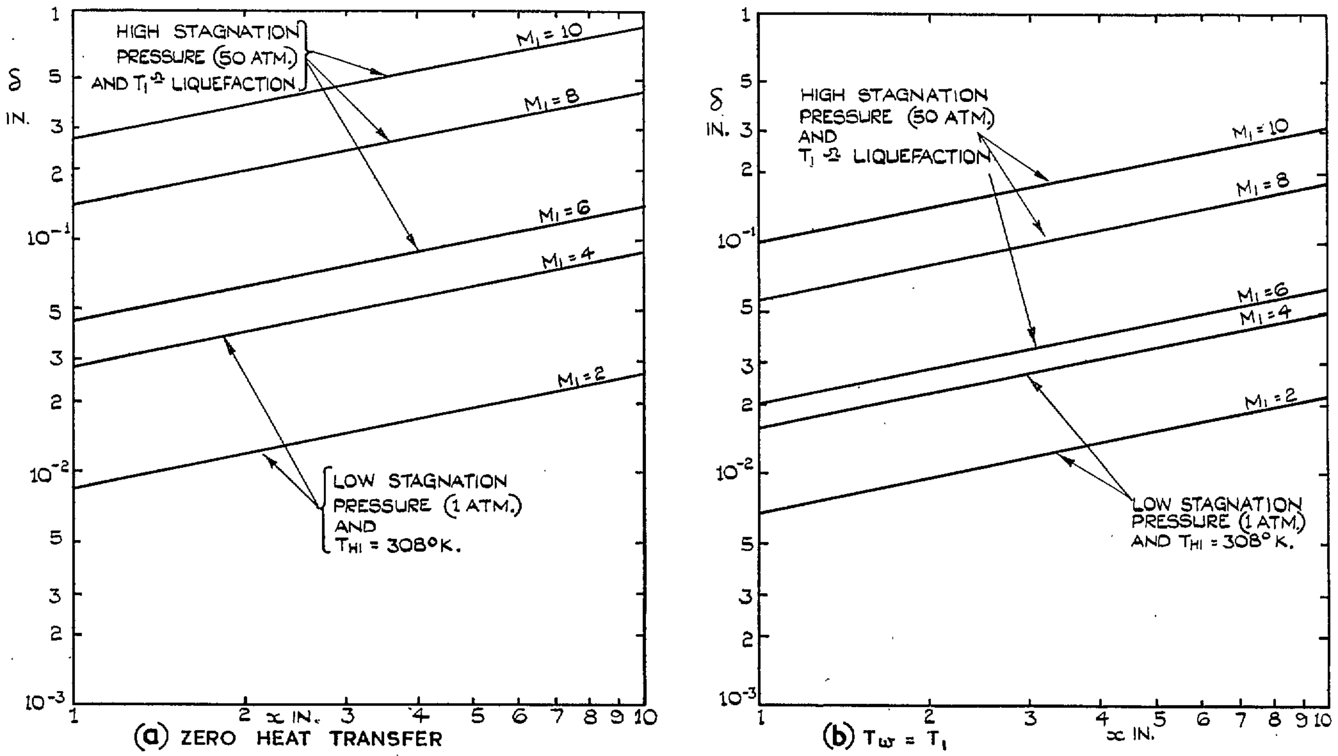
TABLE 2

Dimensions of Tubes Used in Response-Rate Tests

Type	Outside diameter (in.)	Inside diameter (in.)	Taper (Total) (angle)	Bore and length of connecting tubing (in.)	Bore of manometer tube* (in.)	Inside diameter and length of fine-bore tubing (in.)
Quartz	0·0078	0·0042	5° 50′	0·125 × 110·0	0·28	0·030 × 1·2
Quartz	0·009	0·006	5° 40′	0·125 × 110·0	0·28	0·030 × 1·2
Quartz	0·014	0·0072	5° 14′	0·125 × 110·0	0·28	0·030 × 1·2
Quartz	0·0168	0·0108	5° 42′	0·125 × 110·0	0·28	0·030 × 1·2
Quartz	0·0204	0·0126	6° 20′	0·125 × 110·0	0·28	0·030 × 1·2
Hypodermic ..	0·020	0·012	None	0·125 × 110·0	0·28	0·012 × 1·0
Hypodermic flattened†	0·008 × 0·0234	0·0032 × 0·0137		0·125 × 110·0	0·28	0·012 × 1·0

* For length of manometer tube in system *see* Fig. 13.

† This tube is similar to that shown in Fig. 11.



FIGS. 1a and 1b. Theoretical estimates of laminar boundary-layer thickness on cones in supersonic and hypersonic wind tunnels.

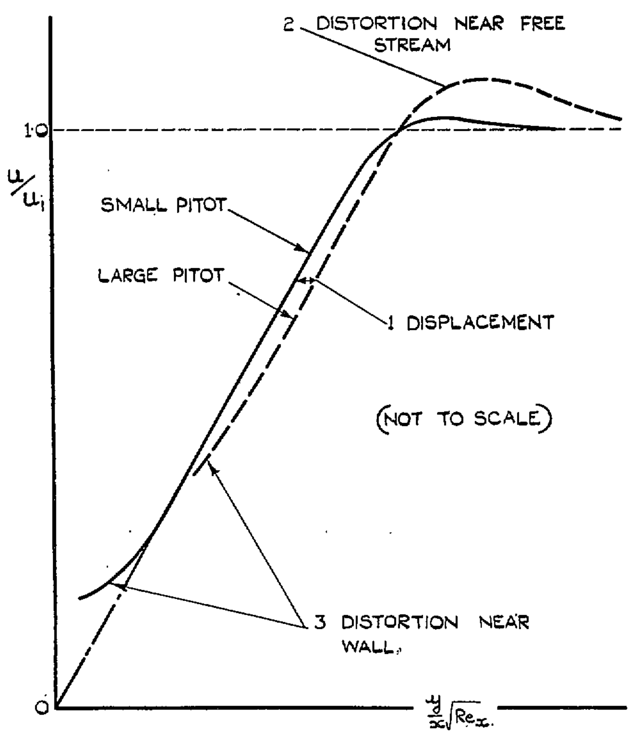


FIG. 2. Effect of pitot size on velocity profile of laminar boundary layer, as given by Ref. 1.

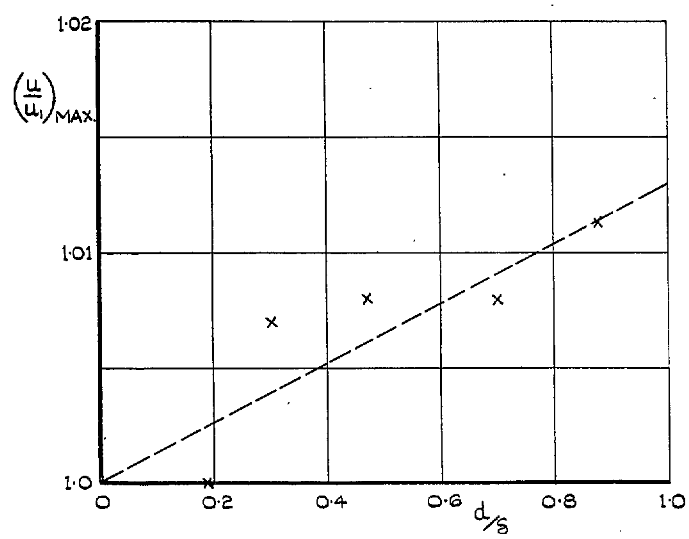


FIG. 3. Effect of pitot size on maximum (peak) velocity. Laminar boundary layer (d = outside diameter of pitot-tube; δ = thickness of boundary layer).

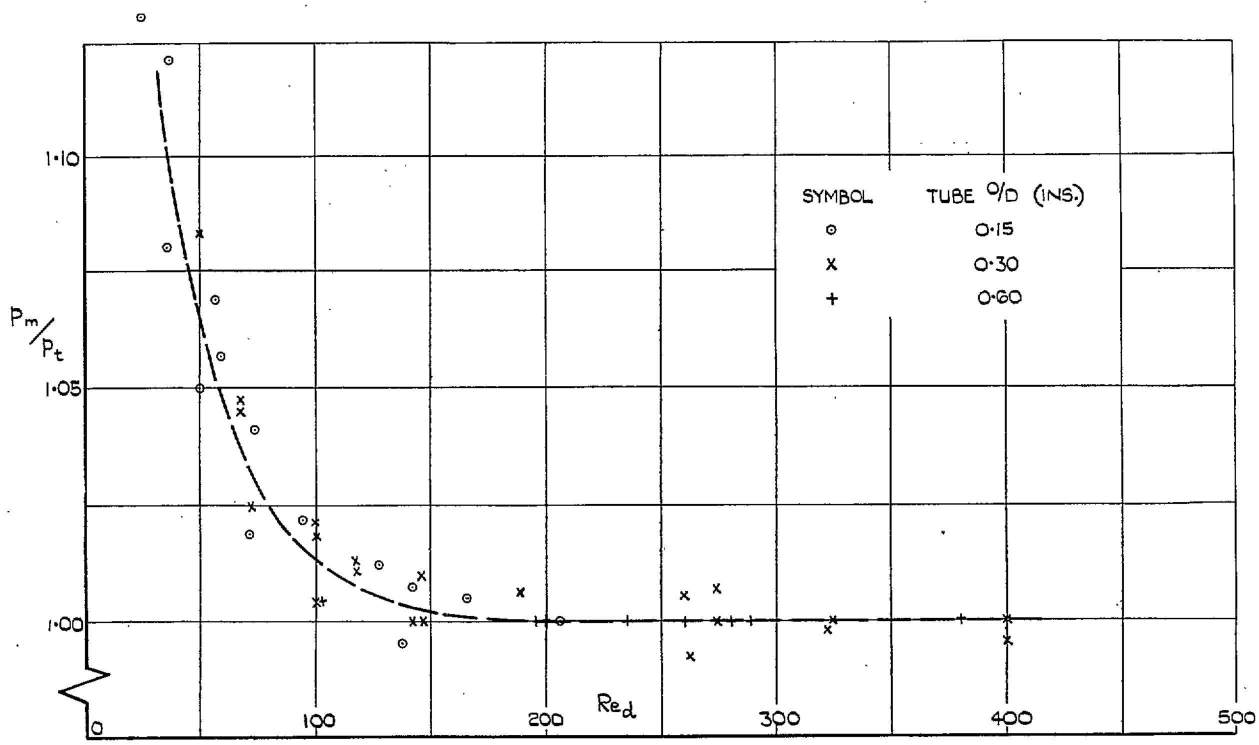


FIG. 4. Variation of p_m (measured)/p_t (theoretical) with Reynolds number (ud/ν) (Kane and Maslach's results, Ref. 6).

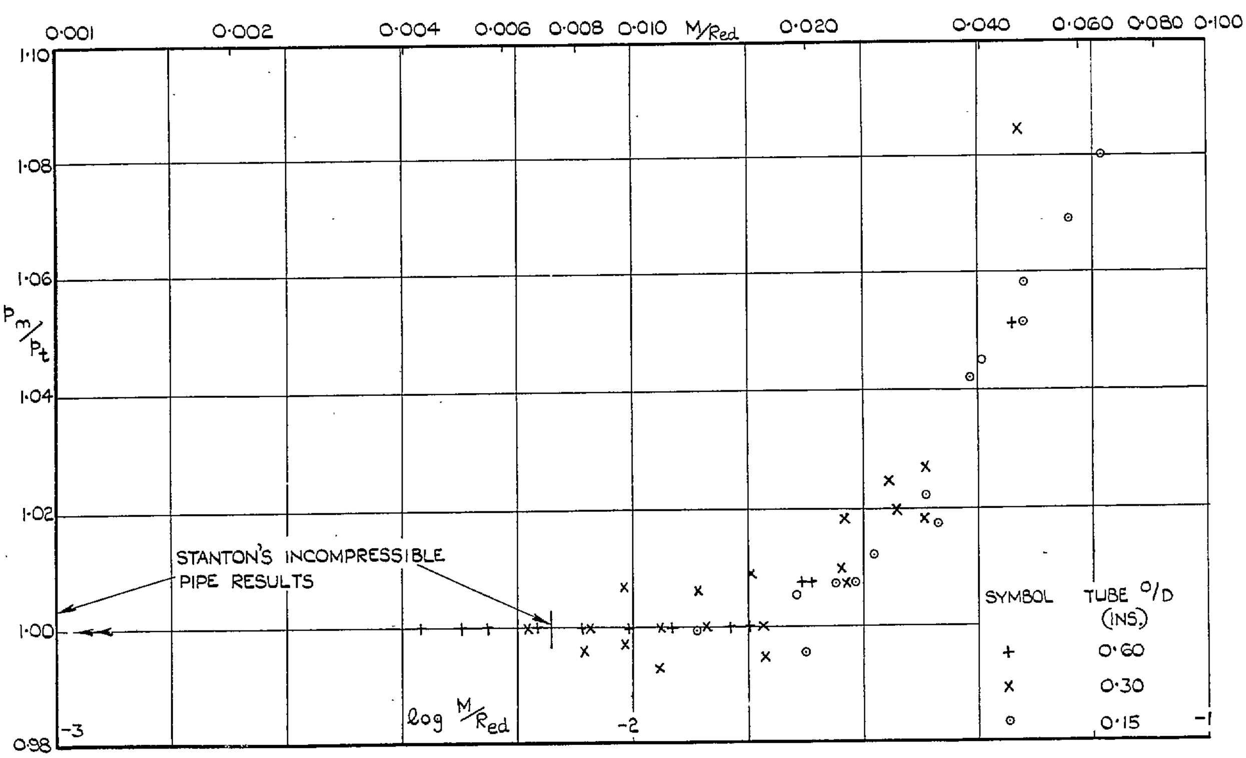
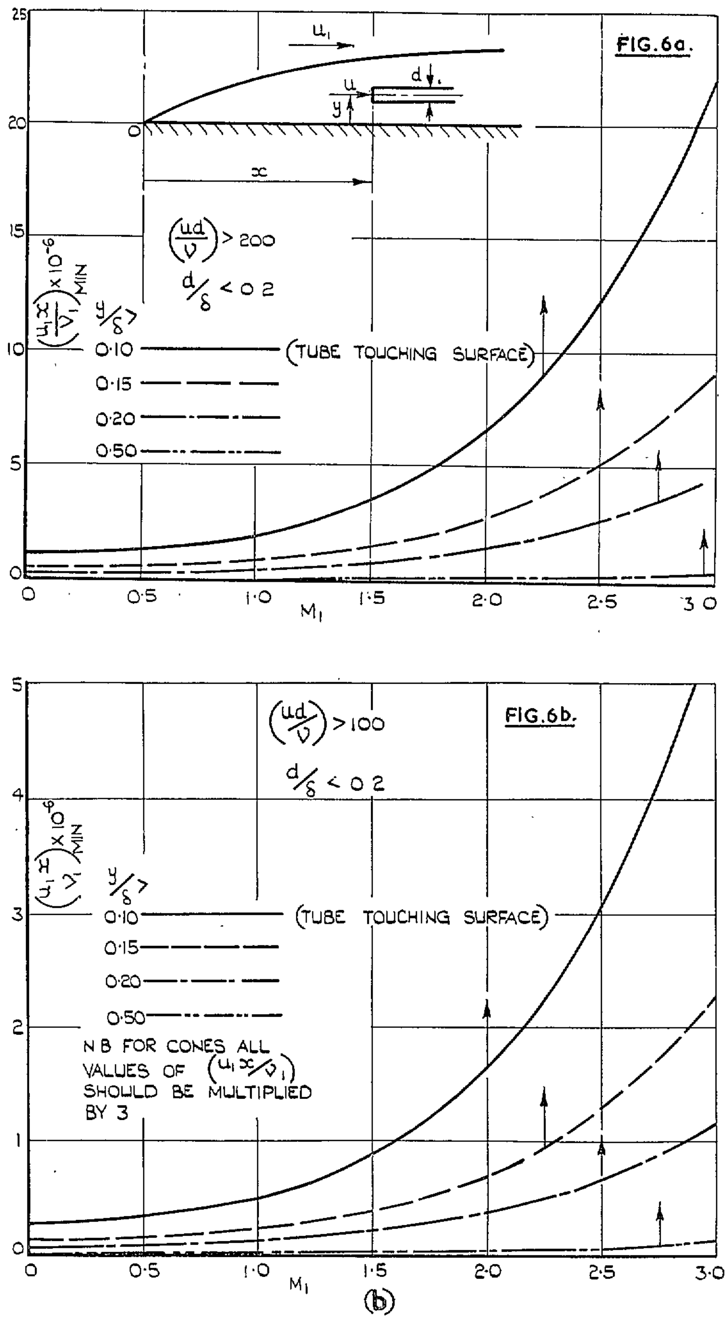


FIG. 5. Variation of p_m (measured)/p_t (theoretical) with M/Re_d $(2 \cdot 3 < M < 3 \cdot 6)$ $(25 < Re_d < 804)$ (Kane and Maslach's results, Ref. 6).



FIGS. 6a and 6b. Variation of minimum Reynolds number $(u_1x/\nu_1)_{\min}$ with Mach number M_1 to satisfy pitot criteria $(ud/\nu) > 200 : 100$ and $d/\delta < 0{\cdot}2$ for various y/δ (Flat-plate laminar boundary layer).

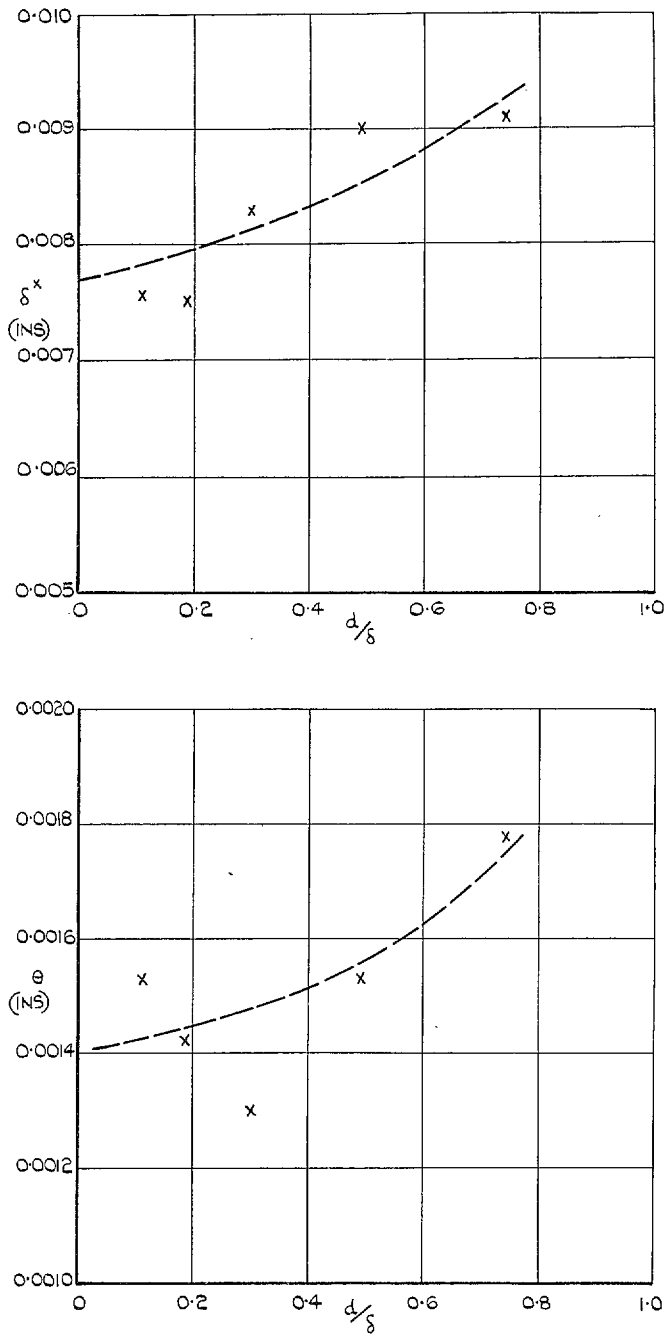


FIG. 7. Variation of displacement and momentum thicknesses (δ^x and θ) with pitot diameter/boundary-layer thickness (d/δ) (Laminar boundary layer. 20-deg cone. $x = 3 \cdot 8$ in. $M_1 = 2 \cdot 2$).

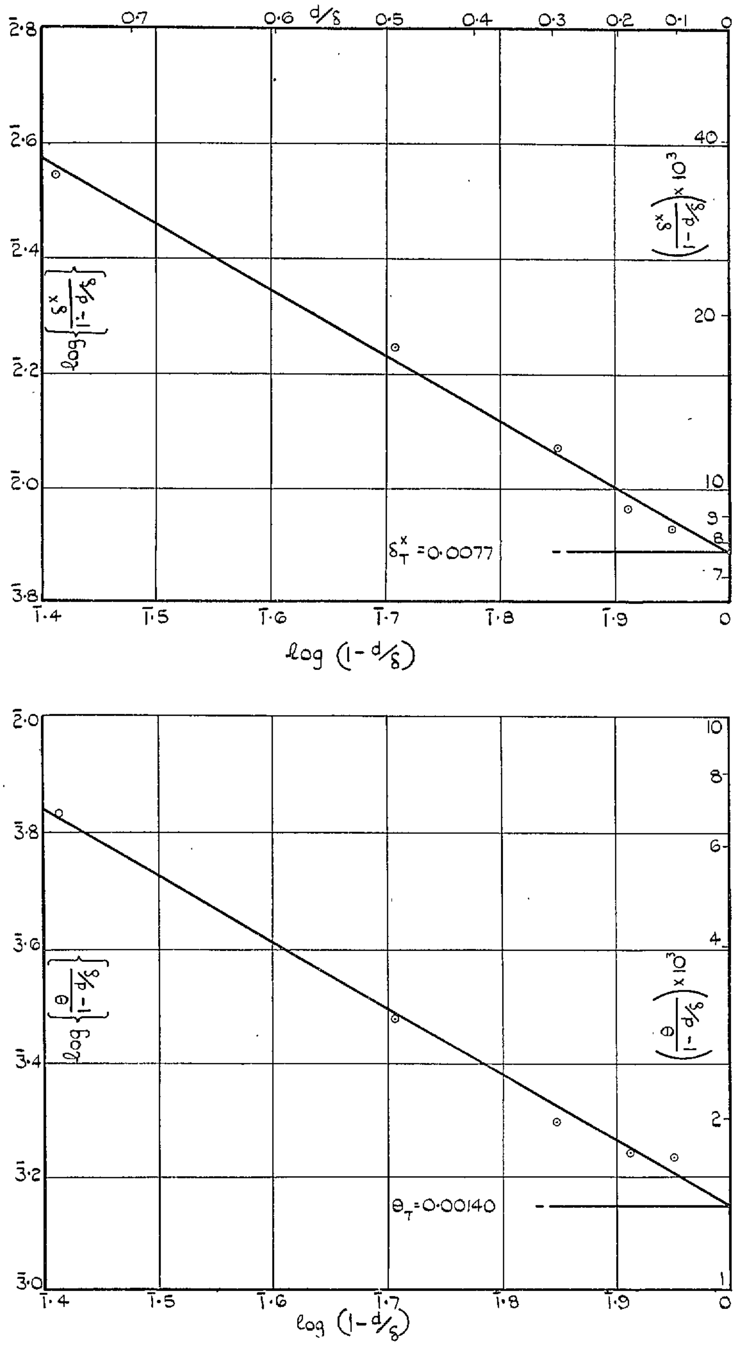


FIG. 8. Variation of displacement and momentum thicknesses (δ^x and θ) with size parameter $(1 - d/\delta)$(20-deg cone. $x = 3 \cdot 8$ in. $M_1 = 2 \cdot 2$).

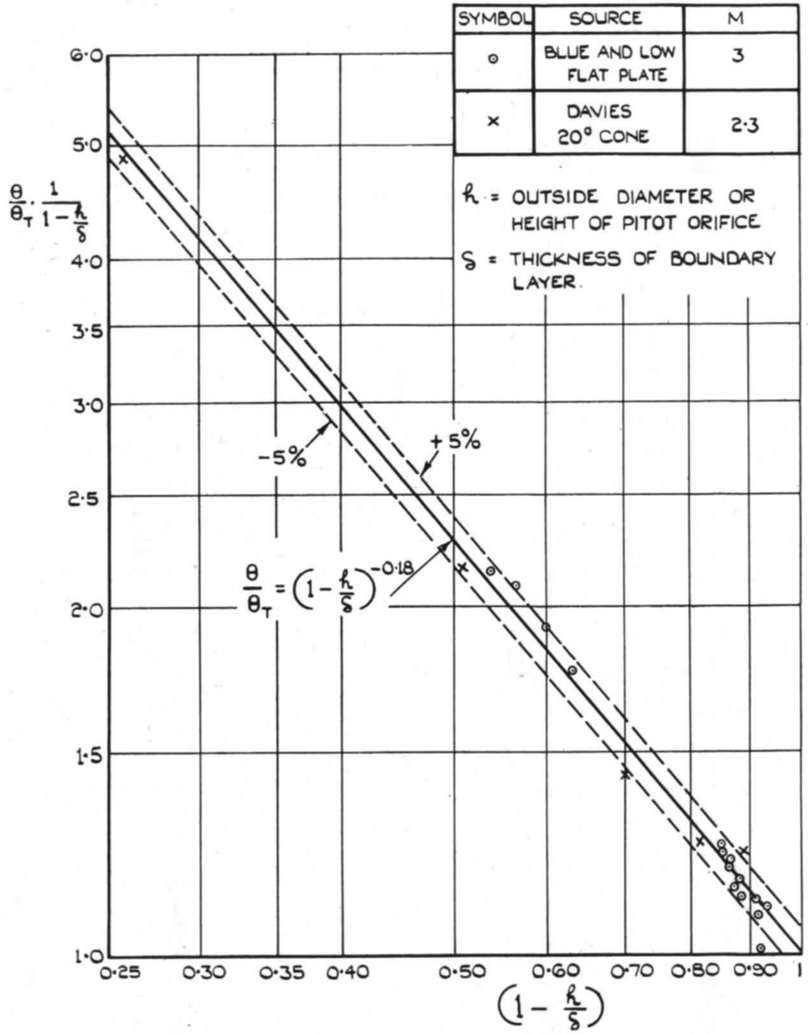


FIG. 9. Effect of pitot size on measured momentum thickness of laminar boundary layer (θ_T = True value).

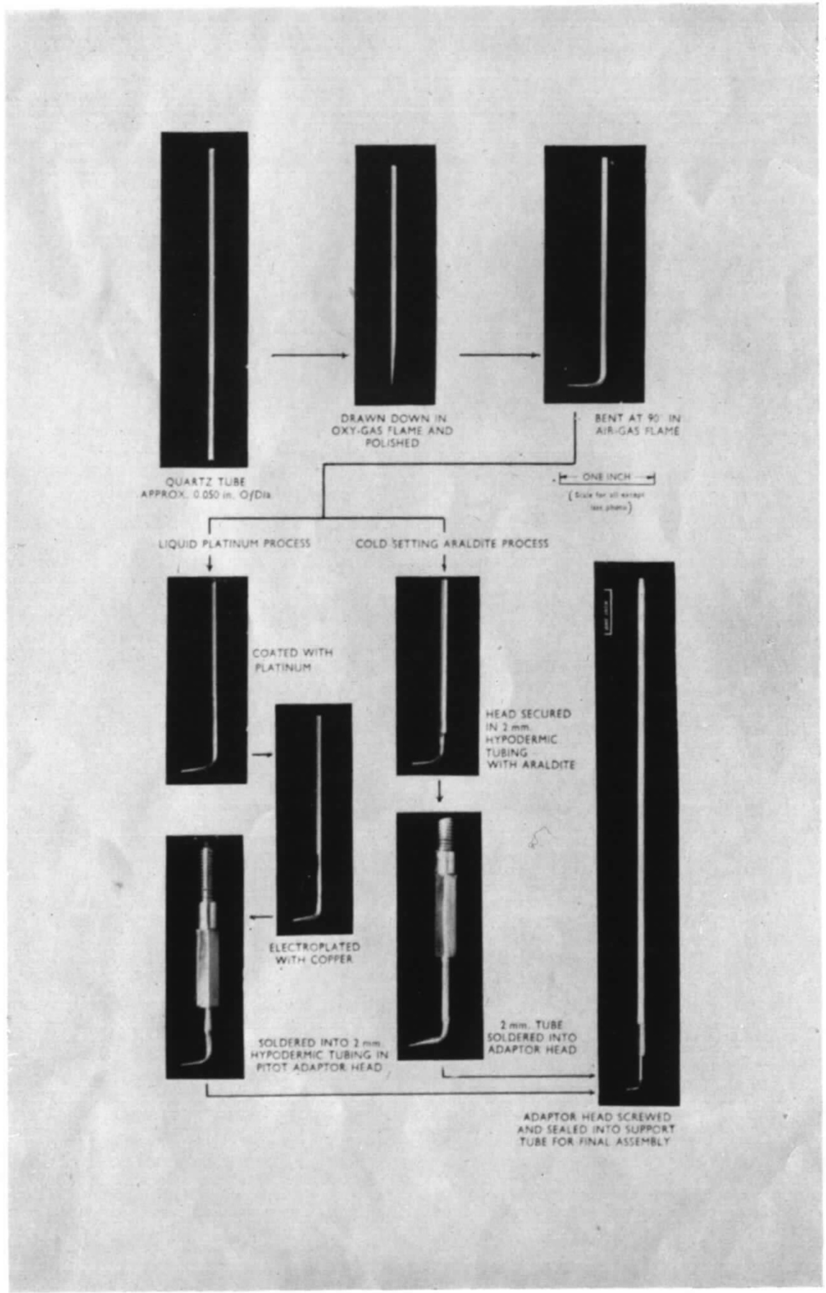


FIG. 10. Manufacture of quartz pitot heads.

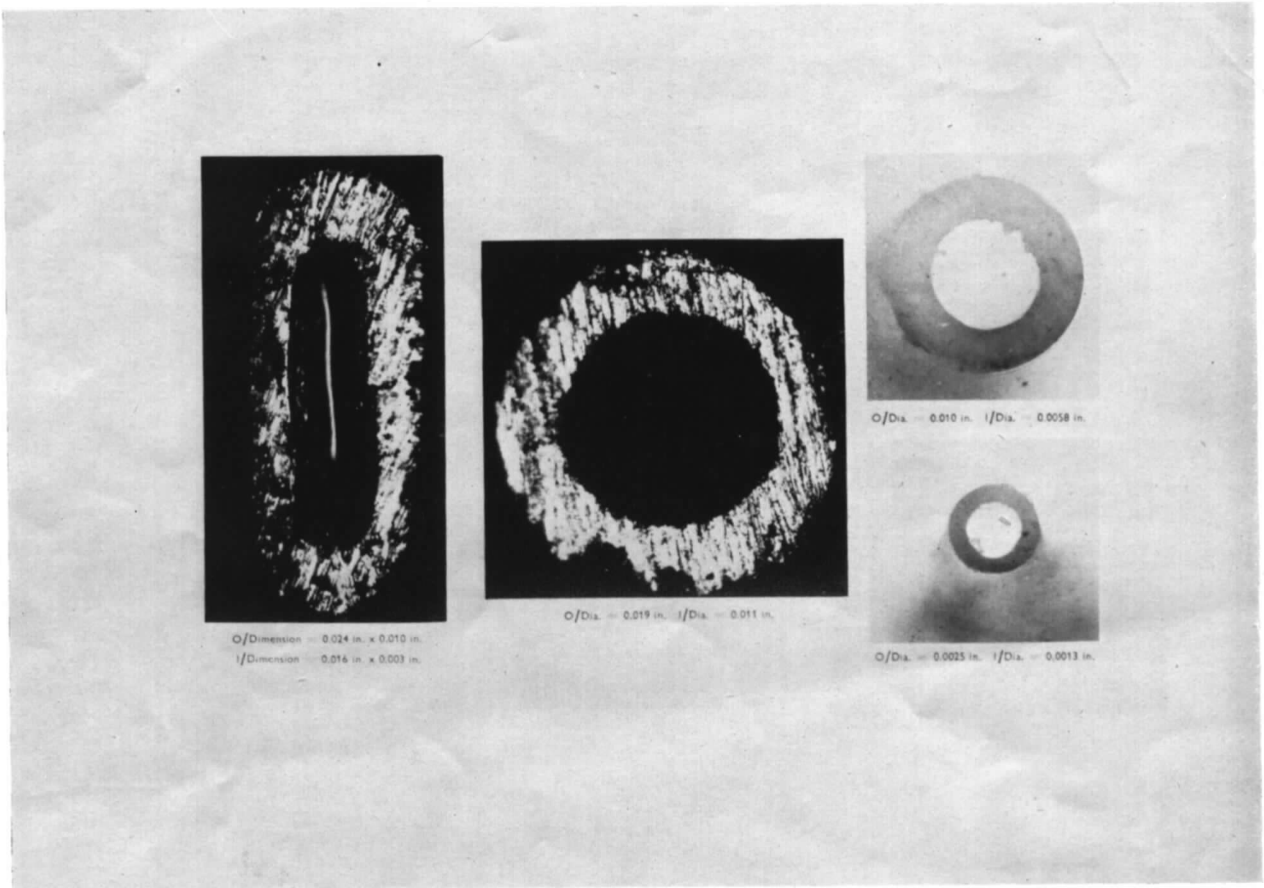
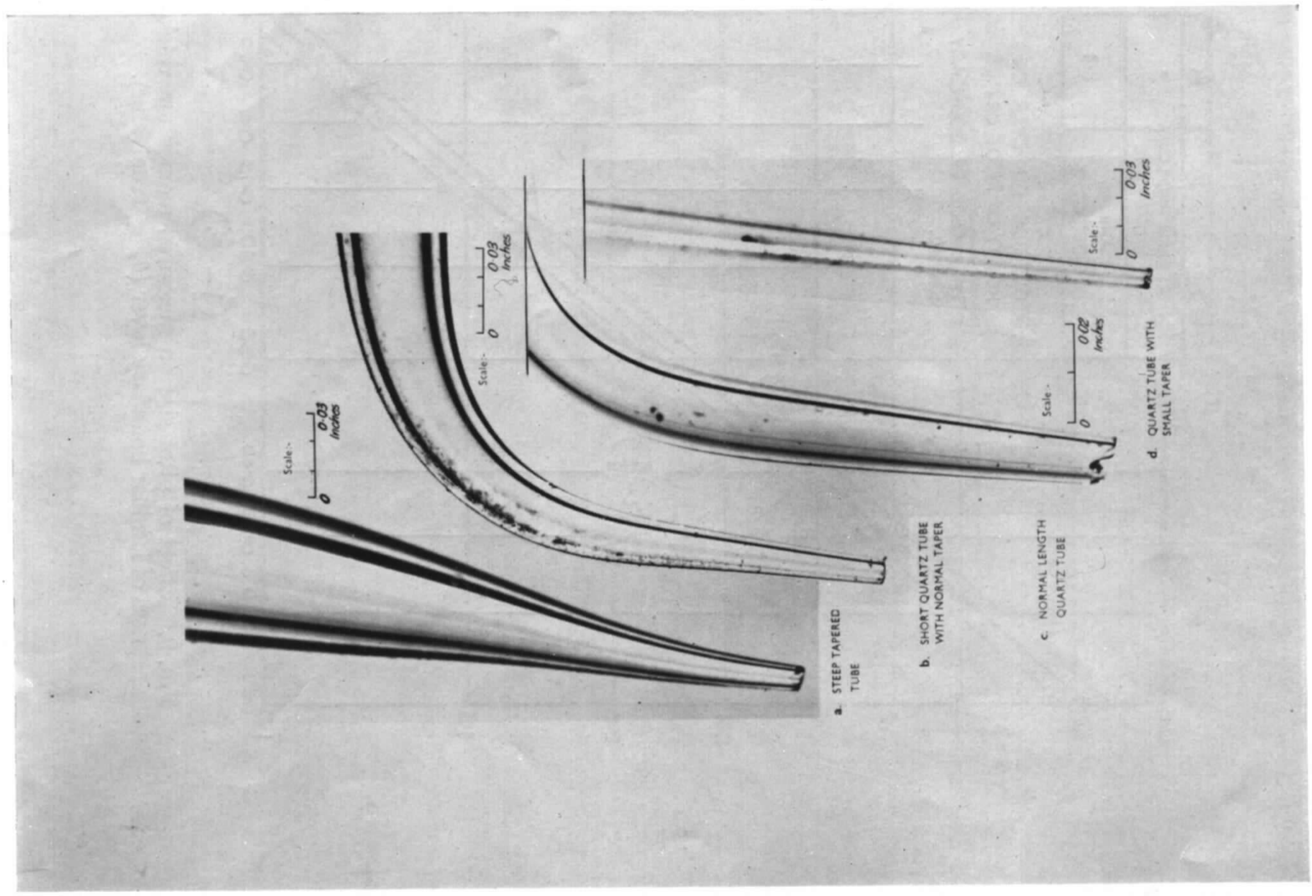


FIG. 11. End-on views of quartz and stainless-steel tubes of comparable dimensions.



FIGS. 12a to 12d. Quartz tips used in tests of measuring accuracy.

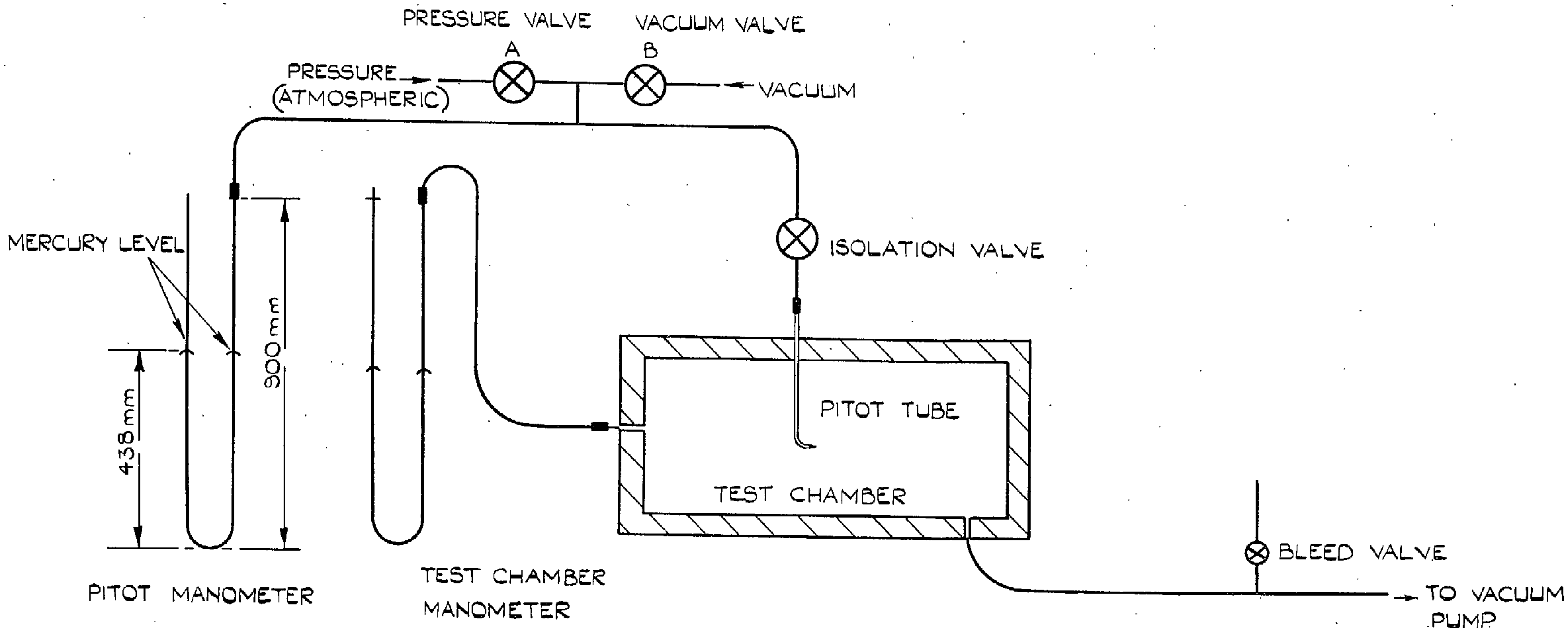
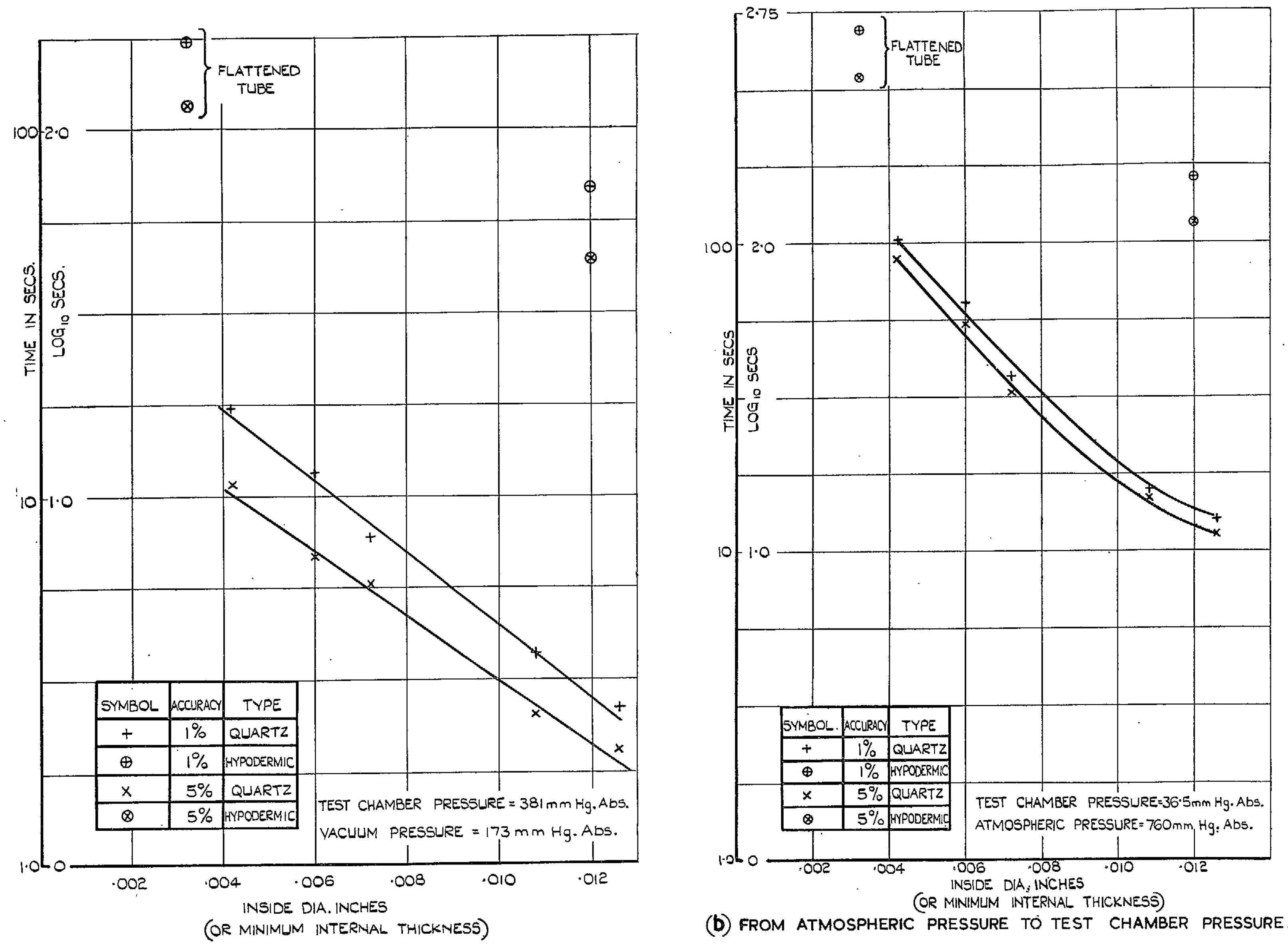


FIG. 13. Test rig for pitot response rate (Line drawing).

71243) Wt. 52/8942 K.7 12/57 Hw.



(a) FROM A VACUUM TO TEST CHAMBER PRESSURE

(b) FROM ATMOSPHERIC PRESSURE TO TEST CHAMBER PRESSURE.

FIGS. 14a and 14b. Response rates for quartz and hypodermic tubes (Times required to reach within 5 per cent and 1 per cent of test-chamber pressure).

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