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The Determination of Aerodynamic Coefficients from Flutter Test Data

By

W. G. Molyneux, B.Sc.

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SUMMARY

A technique is described that enables the oscillatory aerodynamic coefficients for an aerofoil to be determined from data measured in flutter tests on the aerofoil. The method is attractive in that it dispenses with the excitation equipment that is usually required for oscillatory force measurements.

Preliminary measurements have been made in a low speed tunnel on two rigid rectangular wings, and the results show that the technique is worth developing.

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1 Introduction

Various schemes are in use for the measurement of the aerodynamic forces on oscillating aerofoils[1,2,3] but, in the main, these schemes require some mechanism, generally complicated and expensive, for controlled excitation of the aerofoil. A technique requiring no excitation mechanism is that of measuring the decay form of the oscillations from a spring mounted model when initially disturbed[4], but this technique is, in general, only used for obtaining isolated coefficients and not for obtaining the complete set of coefficients that are required for flutter work.

Another technique that requires no excitation mechanism is that of allowing the aerofoil to flutter and measuring the flutter characteristics. From this information, together with the structural data for the aerofoil, the aerodynamic coefficients for the particular degrees of freedom of the aerofoil can be obtained. A basic assumption for the technique is that the change of the aerodynamic coefficients with frequency parameter is negligible for the limited range of variation associated with the flutter conditions investigated. This assumption would appear to be justified, at least for rigid, aerofoils of finite aspect ratio in subsonic flow, on the basis of available data[1,2,3,4,5]. At transonic speeds and supersonic speeds involving mixed flow conditions it seems probable that the coefficients are more sensitive to variations of frequency parameter.

To illustrate the technique the required equations are derived for an aerofoil oscillating with two degrees of freedom. A simple rig has been constructed to enable measurements to be made on two rigid rectangular wings oscillating in two modes (pitch about two axes), and values for the oscillatory aerodynamic coefficients have been obtained. A comparison of these coefficients with calculated values indicates that the method is worth developing.

2 Equations for a system with two degrees of freedom

The flutter equations of motion for a system with two degrees of freedom may be written[6]:-

$$\{-A_{11}\,\omega^2 + i(D_{11}+B_{11}\,V)\,\omega + C_{11}\,V^2 + E_{11}\}\,q_1 + \{-A_{12}\,\omega^2 + i\,B_{12}\,V\omega + C_{12}\,V^2\}\,q_2 = 0$$

$$\{-A_{21}\,\omega^2 + i\,B_{21}\,V\omega + C_{21}\,V^2\}\,q_1 + \{-A_{22}\,\omega^2 + i(D_{22}+B_{22}V)\,\omega + C_{22}\,V^2 + E_{22}\}\,q_2 = 0 \quad (1)$$

where the A's, D's and E's are the structural coefficients of inertia (including aerodynamic inertia), damping and stiffness, the B's and C's are the required aerodynamic coefficients, V is the flutter speed, ω is the flutter frequency and q_1 and q_2 are the generalised co-ordinates. It is assumed in equations (1) that there are no cross structural dampings or stiffnesses. At flutter we may replace q_1 by $e^{i\omega t}$ and q_2 by $Ke^{i(\omega t-\psi)}$ where $K = \left|\dfrac{q_2}{q_1}\right|$,

ψ = phase angle, q_1 leading q_2.

On expanding equations (1) and equating real and imaginary parts to zero we obtain the following four equations.

Equating real terms to zero

$$- (A_{11} + K_1 A_{12}) \omega^2 + K_2 B_{12} V\omega + (C_{11} + K_1 C_{12}) V^2 + E_{11} = 0 \qquad (2)$$

$$- (A_{21} + K_1 A_{22}) \omega^2 + K_2 D_{22} \omega + K_2 B_{22} V\omega + (C_{21} + K_1 C_{22}) V^2 = 0 \qquad (3)$$

Equating imaginary terms to zero

$$K_2 A_{12} \omega^2 + D_{11} \omega + (B_{11} + K_1 B_{12}) V\omega - K_2 C_{12} V^2 = 0 \qquad (4)$$

$$k_2 A_{22} \omega^2 + K_1 D_{22} \omega + (B_{21} + K_1 B_{22}) V\omega - K_2 C_{22} V^2 - K_2 E_{22} = 0 \qquad (5)$$

where $K_1 = K \cos\psi$, $K_2 = K \sin\psi$.

Equations (2) and (4) contain the four aerodynamic coefficients C_{11}, C_{12}, B_{11}, B_{12} and equations (3) and (5) contain the four aerodynamic coefficients C_{12}, C_{22}, B_{12}, B_{22}. To obtain a complete solution for the aerodynamic coefficients, in terms of the structural coefficients and measured data from flutter tests, two sets of such equations are required. These can be obtained from flutter tests for two different conditions for the aerofoil, for example for two values of the stiffness E_{11}. Different flutter conditions will, in general lead to flutter at different frequency parameters, and the assumption must be made that the aerodynamic coefficients are not appreciably effected by the change of frequency parameter.

If the aerofoil is flexible and the flutter involves modes of distortion of the aerofoil then a measurement of the flutter mode is required, in addition to other flutter test measurements, to enable the equations to be resolved (the structural and aerodynamic coefficients will involve the mode). However, a satisfactory technique for measurement of the flutter mode has not yet been developed. If the aerofoil is rigid with flexibilities provided at the root then the modes are known and the equations can be solved. The technique is, therefore, primarily applicable to rigid aerofoils with root flexibilities - a limitation that also applies to forced excitation methods for aerodynamic force measurements. Measurements that are available for rigid aerofoils[1,2,3,4] indicate that the rate of variation of aerodynamic coefficients with frequency parameter is generally small, at least for finite aspect ratio wings. An allowable variation of about 40% in frequency parameter would seem reasonable for frequency parameters greater than 0.1.

3 Measurements on two rigid rectangular wings with freedoms in pitch about the leading edge and about an upstream axis

To obtain an indication of the practicability of the technique a simple rig was constructed to enable measurements to be made on two rigid rectangular wings (one of aspect ratio 3.70 and the other of aspect ratio 2.47) with freedoms in pitch about two axes. For these motions the coefficients of equations (1) to (5) are as defined in the notation list.

3.1 Details of the test rig

The rig is shown diagrammatically in Fig.1. It consisted of a light, stiff frame hinged to a rigid supporting table by cross springs at the upstream end, and with the wing attachment supported on a ball bearing hinge at the other end. The wing attachment enabled the wing to be supported with its leading edge coincident with the ball bearing axis. Stiffness in pitch about the leading edge was provided by coil springs between the wing attachment and the support frame, and the pitch amplitude was indicated by measuring the strain (using strain gauges) in two cantilever strips displaced by motion of the wing about the leading edge. Similar coil springs and cantilever strips were used between the frame and the supporting table to provide stiffness and to indicate displacement about the upstream axis. The latter springs could be changed to provide two different flutter test conditions. The rig should have been rigid for motions other than pitch about the prescribed axes, but in fact the stiffness against rolling motion of the wing was not very great.

The supporting table had a flat top through which the wing protruded, with a sufficient gap around the wing to allow for flutter oscillations. This gap was covered by an end plate attached to the wing and oscillating with it. The gap between end plate and table top was about 0.1 inch.

3.2 Method of test

The tests were made in the R.A.E. 5 ft diameter open jet tunnel. The wing was mounted vertically in the tunnel with the centre line of the table top some 8 inches from the edge of the jet boundary (Fig.1). The strain gauges of the cantilever strips were connected to form two bridge circuits, and the outputs of these circuits were connected to a double beam oscilloscope, one to each amplifier. The tunnel speed was then increased until flutter occurred, and when a stable flutter condition was obtained measurements were made of flutter speed and flutter frequency and a photographic record was made of the oscilloscope traces. Measurements were made on each of the two wings for two different stiffnesses in pitch about the upstream axis.

Following the flutter tests, measurements were made in still air to obtain the inertia and stiffness coefficients for the wings. In addition records were made of the oscilloscope trace of the decay of oscillation in one degree of freedom with the other fixed, to enable logarithmic decrements for structural damping to be obtained.

3.3 Analysis of results

The photographic records were enlarged to facilitate analysis. In general the order of accuracy obtained for the various measurements was as follows:-

Flutter speed measured to within $\pm 0.5\%$

Flutter frequency measured to within -2%

Amplitude ratio measured to within $\pm 2\%$

Phase angle measured to within $\pm 1^\circ$

Structural inertias measured to within $\pm 3\%$

Structural stiffnesses measured to within $\pm 3\%$.

It should be noted that the flutter speed was the mean tunnel speed with no account for irregularities in the flow that may have exceeded ±0.5% of the mean speed. Further, the flutter frequency was the mean frequency of oscillation over an interval of 10 seconds, and amplitude ratio and phase angle were also average values measured over about seven cycles of the flutter oscillation. The structural coefficients were those for the rigid structure and took no account of unwanted flexibilities that were present in the rig.

The structural data for the wings, and the flutter test measurements are given in Table I. From this data the aerodynamic coefficients for the two wings have been derived (Table II). The aerodynamic coefficients have also been expressed as equivalent constant strip derivatives (Table III) that are chosen to be constant over the span and which when integrated over the span in the appropriate mode of oscillation give the correct aerodynamic force in that mode; i.e.

$$B_{11} = \rho sc \{h^2 \ell_{\dot{z}} + hc(\ell_{\dot{\alpha}} - m_{\dot{z}}) - c^2 m_{\dot{\alpha}}\} \;;\quad C_{11} = \rho s \{h^2 \ell_z + hc(\ell_\alpha - m_z) - c^2 m_\alpha\}$$

$$B_{12} = \rho sc^2 (h\ell_{\dot{\alpha}} - c\, m_{\dot{\alpha}}) \;;\quad C_{12} = \rho sc (h\, \ell_\alpha - c\, m_\alpha)$$

$$B_{21} = -\rho sc^2 (h\, m_{\dot{z}} + c\, m_{\dot{\alpha}}) \;;\quad C_{21} = -\rho sc (h\, m_z + c\, m_\alpha)$$

$$B_{22} = -\rho sc^3 m_{\dot{\alpha}} \;;\quad C_{22} = -\rho sc^2 m_\alpha$$

in terms of wing leading edge derivatives.

4 Comparison of measured and calculated values

The measured values of the equivalent constant strip derivatives are compared with calculated values in Table III. The calculated derivatives are average values for the same range of frequency parameter as those measured, obtained from the results of Lawrence and Gerber[5] for wings of low aspect ratio. It can be seen that the agreement between the measured and calculated values of the derivatives is poor, though, in general, the measured derivative values are of the same order as those calculated. It is not possible at present to determine the extent to which these differences are due to insufficient accuracy in the measuring technique or limitations in the theory. Certainly the flutter testing and measuring technique could have been improved - in particular the lack of rolling stiffness in the rig must have led to inaccuracies - and with such improvements reliable derivative values should be obtainable. However, on the basis of the results obtained here it is thought that the technique is worth developing.

5 Further developments

It seems probable that the technique can be used to obtain control surface and tab coefficients, in addition to those for the main aerofoil. These coefficients are difficult to obtain by other techniques because of the small forces involved.

As mentioned in Section 2, the difficulty of mode measurement for flutter cases involving a combination of flexible wing modes makes this method of derivative measurement inapplicable. However, the problem is less difficult where the flutter is a combination of a single flexible wing mode with a rigid body mode, since the modes can easily be separated. A rig that permits wing flutter in modes of wing bending and uniform pitch is in use at the R.A.E. at present, for flutter tests at supersonic speeds, and it might prove possible to apply this technique here.

6 Conclusions

The technique for obtaining aerodynamic coefficients from measured flutter test data is simple and only comparatively inexpensive equipment is required. It has been applied to obtain the coefficients for two rectangular unswept wings using measuring equipment of no great standard of accuracy, and promising results have been obtained. With refinements in the methods of measurement reliable values of the aerodynamic coefficients required for flutter work should be obtainable.

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$\ell_{\dot\alpha}$, ℓ_α — non-dimensional equivalent strip derivative for lift $\begin{Bmatrix}\text{damping}\\ \text{stiffness}\end{Bmatrix}$ at wing leading edge due to pitch about leading edge

$m_{\dot z}$, m_z — non-dimensional equivalent strip derivative for pitching moment $\begin{Bmatrix}\text{damping}\\ \text{stiffness}\end{Bmatrix}$ about wing leading edge due to vertical translation

$m_{\dot\alpha}$, m_α — non-dimensional equivalent strip derivative for pitching moment $\begin{Bmatrix}\text{damping}\\ \text{stiffness}\end{Bmatrix}$ about wing leading edge due to pitch about wing leading edge

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TABLE I

Structural data and flutter test measurements

	Wing 1. Aspect ratio 3.70		Wing 2. Aspect ratio 2.47	
Wing chord, c	0.5 ft		0.5 ft	
Wing length (root to tip), s	0.925 ft		0.617 ft	
Wing aspect ratio $\frac{2s}{c}$	3.70		2.47	
Distance between pitch axes	1.196 ft		1.196 ft	
Air density, ρ	0.002378 slugs/ft^3		0.002378 slugs/ft^3	
$A_{12} = A_{21}$	0.00618 slugs ft^2		0.00598 slugs ft^2	
A_{22}	0.00141 slugs ft^2		0.00143 slugs ft^2	
D_{11}	0.0005 lb ft sec		0.0005 lb ft sec	
D_{22}	0.003 lb ft sec		0.003 lb ft sec	
E_{22}	1.64 lb ft/rad		1.64 lb ft/rad	
E_{11}	49.5 lb ft/rad	69.8 lb ft/rad	49.5 lb ft/rad	69.8 lb ft/rad
A_{11}	0.0845 slugs ft^2	0.0854 slugs ft^2	0.0825 slugs ft^2	0.0836 slugs ft^2
K	4.21	8.76	5.97	12.57
V	89.6 ft/sec	83.5 ft/sec	113.8 ft/sec	105.8 ft/sec
ψ	44.6°	55.0°	43.2°	60.2°
ω	37.0 rads/sec	41.1 rads/sec	37.4 rads/sec	41.4 rads/sec
$\nu = \frac{\omega c}{V}$	0.21	0.25	0.16	0.20

TABLE II

Derived Aerodynamic Coefficients

	Wing 1 Aspect Ratio 3.70	Wing 2 Aspect Ratio 2.47
B_{11}	0.0054 lb sec^2	0.0040 lb sec^2
B_{12}	0.00099 "	0.00072 "
B_{21}	0.00054 "	0.00033 "
B_{22}	0.00034 "	0.00021 "
C_{11}	0.0035 lb sec^2/ft	0.0016 lb sec^2/ft
C_{12}	0.0022 "	0.0012 "
C_{21}	0.00051 "	0.00040 "
C_{22}	0.00026 "	0.00014 "

TABLE III

Measured and calculated values for equivalent strip derivatives referred to wing leading edge

	Wing 1 Aspect Ratio 3.70		Wing 2 Aspect Ratio 2.47	
	Range of frequency parameter 0.21 - 0.25		Range of frequency parameter 0.16 - 0.20	
	Measured	Calculated	Measured	Calculated
$\ell_{\dot z}$	2.23	1.67	2.60	1.38
$\ell_{\dot\alpha}$	0.98	1.36	1.16	1.38
$m_{\dot z}$	-0.30	-0.41	-0.28	-0.31
$m_{\dot\alpha}$	-1.24	-0.66	-1.14	-0.65
ℓ_z	0.16	-0.005	-0.067	-0.04
ℓ_α	1.47	1.70	1.21	1.35
m_z	-0.19	0.005	-0.29	0.005
m_α	-0.47	-0.40	-0.38	-0.30

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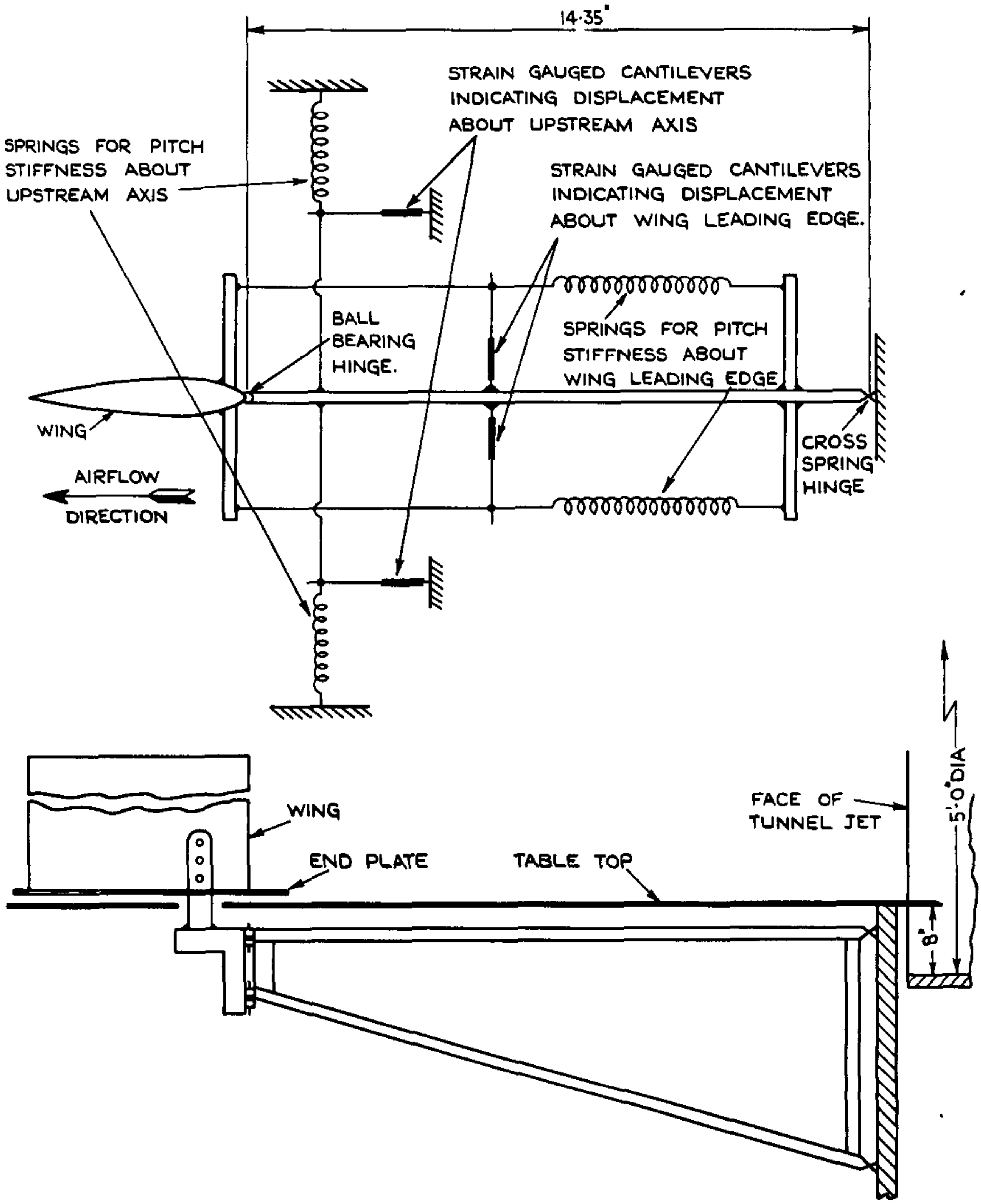

FIG. 1. DIAGRAMMATIC LAYOUT OF FLUTTER TEST RIG.

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