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Parts I and II

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Summary.—This report, which is presented in two parts, develops an approximate method of estimating the effect of structural deformability on the manoeuvre point of an aircraft. The introduction outlines the scope of the complete work in relation to the work of Lyon and Ripley (R. & M. 2331 and 2415).

Part I opens with a detailed discussion of the structural deformability of wings, unswept and swept, and proceeds on the basis of certain aerodynamic and structural approximations to derive relatively simple formulae for the calculation of the shift of manoeuvre point due to elastic camber, elastic wash-out (wing torsion and bending, and the effect of fuselage interference) and the direct effect of wing bending (which changes moment arms) on pitching moment. A summary and discussion of some comparative calculations of the effect of elastic wash-out, using the present method and that proposed by Lyon (R. & M. 2331) are included. They demonstrate the dangerously large shifts of manoeuvre point which may arise from elastic wash-out with swept wings and show that while the present method is somewhat less accurate than that of Lyon, it has the important advantage of being far less laborious in application.

Part II examines the effects of fuselage and tailplane deformability, and at the same time investigates the effect of wing deformability (including root-region deformability) on the fuselage and tailplane contributions to manoeuvring stability. Bending of the fuselage, torsion of the (unswept) tailplane and deformability of the tailplane attachment are the main fuselage and tailplane effects considered, and among the subsidiary effects examined is that of engine nacelles situated in the wing.

A simple procedure for numerical calculation of the fuselage and tailplane contributions to manoeuvre-point shift is set out and illustrated by a worked example, which demonstrates how elastic attachments of wing and tailplane may be used to augment the effect of the tailplane in counteracting the destabilizing effect of wing and fuselage.

A simple description of the method of analysis used in Part II, together with typical results obtained from it, is given in section 12.

Introduction.—Following the publication in 1946 of R. & M. 2331[1], theoretical investigation at the Royal Aircraft Establishment into the effects of aero-elastic distortion on the stability and control of aircraft was temporarily suspended. R. & M. 2331[1] had outlined a possible method of estimating the effect of aero-elastic distortion of swept-back wings on stability and control derivatives, so that in conjunction with R. & M. 2415[2], it provided a basis for investigating within the general framework of R. & M. 2027[3], the overall effect on stability and control, of the aero-elastic distortion of the complete aircraft structure.

In Germany, Fingado had written a paper[4] on the effect of aero-elastic distortion on manoeuvre point, and on coming to England he expanded and extended the work to incorporate new ideas

* R.A.E. Report Aero. 2320, received 3rd August, 1949. R.A.E. Report Aero. 2362.

and subsequent experience gained while working with Deutsche Versuchsanstalt für Luftfahrt and at the R.A.E. in co-operation with German and British aircraft firms. Much of the later work was carried out in collaboration with Taylor.

Fingado's approach to the subject differs considerably from that developed by Lyon and Ripley[2] and implied, though not developed, by Lyon[1]. The British method investigates effects of deformability on stability characteristics such as manoeuvre margin by considering the effects of such deformability on the values of the generalised stability derivatives A, A_1, etc., while Fingado considers directly the effect of deformability on the pitching moment of the aircraft. Furthermore, while Lyon and Ripley leave their results in terms of general stiffnesses of the various aircraft components involved, Fingado substitutes representative values for the stiffnesses, arrived at from considerations of strength and stiffness requirements*. This enables him to reduce the final result for the shift of manoeuvre point to a summation of terms involving only the non-dimensional dynamic pressure number Q and certain ' construction figures ' which are functions of aircraft geometry, wing structural layout and limiting design conditions (ultimate load factor and maximum permissible dynamic pressure). In this form, the result is very convenient for the rapid computation of the effects of varying such purely geometrical parameters as wing aspect ratio and sweep angle and such structural parameters as the position of the wing flexural axis.

In order to avoid undue complication, it has been necessary to restrict the investigation to wings of constant chord and to make the assumption of a uniform basic lift distribution. The method cannot therefore be expected to yield results of great accuracy for highly tapered wings or for heavily swept wings of small aspect ratio, for which the basic lift distribution is far from uniform. It has been used as the basis of an *ab initio* investigation into the optimum layout (from the point of view of aero-elastic distortion) of a fast subsonic long-range aircraft and it would appear to be for this type of investigation where general qualitative, rather than quantitatively accurate, results are required, that the method is most useful. Where more accurate results are required for some specific design, it would seem desirable to revert to the methods of Lyon and Ripley. Allowance may then be made for the effects of wing taper, and the correct basic lift distribution may be determined from lifting-plane methods.

When the various sections of the work had been completed by the respective authors, it became evident that considerable modifications to the lay-out of the report as originally planned, were desirable in order to improve the exposition and to render possible more effective comparison of the method with that of Lyon and Ripley. The work of rearrangement and general editing was undertaken by Taylor.

The report is presented in two parts, the first of which deals with the effect on manoeuvre point of wing deformability, including the effects of sweep. The second deals, in the main, with the effects of deformability of the fuselage and of an unswept tail unit, but at the same time applies some results of Part I to determine the effect of wing deformability on the fuselage and tailplane contributions to stability†.

Although the derivation of the formulae is somewhat long and complicated, the method is quite simple and rapid in application. For the numerical calculation of the various effects, routine procedures are suggested in sections 9 and 17 of the report, and the reader who is interested in the numerical application of the method, rather than in its theoretical background, need do little more than study these sections in conjunction with Tables 1 and 2.

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* These and other structural assumptions throughout the report have been fully discussed with Structures Department, R.A.E.

† Footnote (1956) : Originally the authors intended to devote a third part of the report to a consideration of the effects of swept tail-units and to a detailed analysis of the effect of an elastic attachment of tailplane to fuselage, but circumstances conspired to prevent the completion of this part of the work. However, the first two parts, here presented, are self-contained and together constitute a complete method for estimating the effect of elastic deformability on the manoeuvre point of an aircraft with swept wings and unswept tailplane.

PART I

The Effect of Wing Deformability

1. *The Elastic Deformability of the Aircraft Structure with Respect to Symmetrical Loads.*—To investigate the effects of elastic deformability of the aircraft structure, it is first necessary to examine the character and range of this deformability. As the aircraft is assumed to be symmetrical and it is intended to consider only a symmetrical flight condition (the pull-out), it will be necessary only to consider the deformability with respect to symmetrical loads. The most important deformations affecting the manoeuvre point are bending of the fuselage, bending and torsion of the tailplane, angular deflections of the tailplane due to deformability of its attachment to the fuselage, and twisting and bending of the wing. The fuselage and tailplane effects are considered in Part II of the report, attention being concentrated in the present part on the wing effects.

1.1. *The Deformability of the Wing Structure.*—For simplicity we consider only constant-chord wings, assumed in the first instance to be unswept. The deformability of a wing can, with sufficient accuracy in most cases, be divided into two parts, *viz.*, bending of the 'elastic axis'* and twist about that axis. We consider first the twist. Here we shall assume the wing to be so constructed that the flanges take only a very small part of the strain energy due to torque, as is the case with most wings, but not, for instance, with shell† wings.

Then a torque T about the elastic axis causes an angle of twist (or negative wash-out) per unit length.

$$\frac{d\theta}{d|y|} = \frac{T}{GJ},$$

where G denotes the shear modulus and GJ the torsional rigidity. At, and in the neighbourhood of, places where the twist is disturbed (for instance by cut-outs in the skin or by prevention of warping of cross-section planes) it is generally possible to specify an equivalent GJ which does not exactly equal the real GJ, but gives—for all loadings of practical interest—a sufficiently accurate ratio between torque and twist per unit length. To derive a representative value for the torsional rigidity GJ, we start from the 'aileron rolling effectiveness reduction factor at maximum permissible dynamic pressure' which we shall denote by F_ξ^*. With well-designed wings this reduction factor usually defines the minimum required torsional rigidity. It is given by

$$F_\xi^* \simeq 1 - \frac{q^*}{q_{c\xi}},$$

where q^* is the maximum permissible dynamic pressure and $q_{c\xi}$ is the 'critical' dynamic pressure at which the factor becomes zero (*see*, for instance, Ref. 6). For an unswept wing of constant chord and constant torsional rigidity (which, from the viewpoint of the weight of the torsion box required for a given $q_{c\xi}$, approximates to the best distribution for a constant-chord wing) and average aileron/span ratio, the critical dynamic pressure is approximately

$$q_{c\xi} = \frac{4 \cdot 30}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{GJ}{S^2},$$

* Any slender prismatic beam encased perpendicular to its generators has an 'elastic axis', characterised as follows: any system of arbitrarily distributed forces perpendicular to, and acting on that axis, causes a pure bending deformation (a parallel displacement of all sections lying perpendicular to the generators), and any system of arbitrarily distributed couples about that axis causes a pure twist about the axis. If the beam is not exactly prismatic, or is otherwise encased, there generally exists no elastic axis in the sense of that strict definition, but in many cases it is sufficiently accurate to calculate the deformations by assuming the existence of such an axis. (For details, *see* for instance Ref. 5.)

† The term 'shell wing' signifies a wing with no concentrated spar flanges, for which both bending and torsional stiffness are provided by skin and stiffeners.

where c_ξ/c denotes the mean aileron/chord ratio. (This equation is a somewhat extended form of an equation given by Roxbee Cox[6].) Using this formula in the above equation for F_ξ^* and introducing a ' dynamic pressure number ' Q defined as the ratio of dynamic pressure $q = \frac{1}{2}\rho V^2$ to wing loading, we get

$$GJ = \frac{\left(1 - \frac{c_\xi}{c}\right)^2}{4\cdot30} \frac{Q^*}{1 - F_\xi^*} WS \,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

The two equations from which we have deduced equation (1) are correct only on the assumption that air-compressibility effects are negligible up to the critical dynamic pressure, which for many aircraft types is not true. Equation (1) itself, however, assumes only that these effects are negligible up to the maximum dynamic pressure, which even today* is still true for many aeroplane types. Since, for some aircraft, the necessary torsional rigidity is determined not by aileron effectiveness, but for instance by flutter considerations, we shall introduce a ' torsional rigidity factor ' F_τ so that (1) becomes

$$GJ = \frac{\left(1 - \frac{c_\xi}{c}\right)^2}{4\cdot30} \frac{Q^*}{1 - F_\xi^*} WSF_\tau \,.$$

It is easily established that if the elastic axis is swept back by an angle ϕ, the wash-out† per unit span‡ due to torsion is still given by T/GJ where T is the true torque about the elastic axis and GJ is the torsional rigidity of sections perpendicular to that axis. Comparing swept-back and unswept wings of the same chord‡ and span, it is clear that for the same lift distribution,

$$T_\phi = T_{\phi=0} \cos\phi \,,$$

and hence to obtain the same wash-out per unit span due to torsion with a given lift distribution, we must have

$$(GJ)_\phi = (GJ)_{\phi=0} \cos\phi \,.$$

For a wing of infinite bending stiffness, neglecting the effect of sweep on the values of the aerodynamic coefficients, this would lead to the equation

$$GJ = \frac{\left(1 - \frac{c_\xi}{c}\right)^2}{4\cdot30} \frac{Q^*}{(1 - F_\xi^*)} WS \cos\phi$$

as defining the torsional rigidity required from aileron-reversal considerations. Introducing again a torsional rigidity factor F_τ, we adopt the equation

$$GJ = \frac{\left(1 - \frac{c_\xi}{c}\right)^2}{4\cdot30} \frac{Q^*}{(1 - F_\xi^*)} WSF_\tau \cos\phi \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1a)$$

for the general case of a swept wing of finite bending stiffness, leaving discussion of its accuracy to section 4. When the wing is subject to aerodynamic or inertia loads, its torsional flexibility will give rise to elastic wash-out, the magnitude and sign of which will be investigated in section 3.

Bending of the elastic axis can be split up into two components respectively in the directions of the two principal axes of inertia of the wing structure resisting bending. For an unswept wing with conventional structural lay-out, the longitudinal principal axis of inertia practically coincides

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* Footnote (1956) : It should be borne in mind that this was written in 1949.

† *i.e.*, the spanwise change in angle of attack of chordwise sections.

‡ Throughout this report, the chord of a swept wing is measured parallel to the plane of symmetry, and the ' chordwise ' direction is defined accordingly. Span is measured perpendicular to the plane of symmetry.

with the zero lift direction. We shall first investigate the deflection curve which results from bending about this axis. Ultimately, we require the derivative of the deformation with respect to load factor, and here we may calculate this directly without first considering the deformability (in this case the bending rigidity EI) because, in the case of the spars, as opposed to the torsion box, the manoeuvring loads are critical. At the dynamic pressure for which we are investigating the deflections, we shall assume the spar flanges to behave as follows: they remain unstrained throughout their length at zero load factor, and at the proof load factor the stresses are everywhere equal to the proof stress. (The first assumption is often not fulfilled, because it requires among other things, a wing shape without wash-out and with a profile having zero moment at zero lift. The second assumption similarly does not always hold, but since they simplify the investigation considerably, both assumptions will, at this stage, be considered to hold true.)

With the foregoing assumptions, the difference between the strains of upper and lower flanges is proportional to load factor, and at the proof load factor, is equal to the difference between the proof tensile strain u_{pt} and the proof compressive strain u_{pc}. The proof strain for tension is equal to the proof tensile stress p_t divided by the elasticity modulus E; for all materials used in practice this ratio is approximately $(p_t/f_t)(1/150)$, where f_t is the ultimate tensile stress. The proof compressive strain of centrally loaded, initially uncurved structural members equals $(p_c/f_t)(1/150)$. Here p_c/f_t depends fundamentally on instabilities arising at the higher compressive stresses, which depend on the materials and type of construction. In the case of metal-covered wings with spars (as distinct from shell wings, for instance), these instabilities are, in general, relatively unimportant because of the large mutual effects of the structural members in supporting one another. Therefore, for the compression flanges of a metal wing we can, in most cases, assume the proof compressive strain u_{pc} to be approximately equal and opposite to the proof tensile strain u_{pt} of the same material, that is to say, equal to $-(p_t/f_t)(1/150)$. The proof curvature

$$\left(\frac{1}{R}\right)_P = \frac{u_{pt} - u_{pc}}{\text{beam height } (\simeq \text{ wing thickness})}$$

is therefore equal to $\dfrac{2}{150}\dfrac{1}{\text{wing thickness}}\dfrac{p_t}{f_t}$. If, for simplicity, the wing thickness is assumed constant in spanwise direction, the curvature will also be constant. Furthermore, its sign will be the same over the whole span. Thus for constant-chord wings the curvature, expressed non-dimensionally in terms of semi-span s, will be

$$\frac{d^2\left(\dfrac{z}{s}\right)}{d\left(\dfrac{y}{s}\right)^2} = -\frac{1}{150}\frac{p_t}{f_t}\frac{A}{\tau}, \text{ for the proof load factor}$$

$$= -\frac{1}{150}\frac{p_t}{f_t}\frac{n}{n_p}\frac{A}{\tau},$$

$$= -\frac{1}{150}\frac{n}{n_u}\frac{A}{\tau}, \text{ for an arbitrary load factor, where } A$$

denotes aspect ratio, τ the thickness/chord ratio of the wing profile, and n_u the ultimate or breaking load factor. Later, we shall need the derivative of the curvature with respect to load factor; it is given by

$$\frac{d}{dn}\left[\frac{d^2\left(\dfrac{z}{s}\right)}{d\left(\dfrac{y}{s}\right)^2}\right] = -\frac{1}{150}\frac{A}{n_u\tau}.$$

By introducing a ' bending rigidity factor ' F_b, similar to the ' torsional rigidity factor ' above, we get

$$\frac{d}{dn}\left[\frac{d^2\left(\frac{z}{s}\right)}{d\left(\frac{y}{s}\right)^2}\right] = -\frac{1}{150}\frac{A}{n_u \tau F_b}. \qquad (2)$$

When we come to consider swept wings, it will be more convenient to consider this result in the form:

$$\text{curvature of elastic axis} = -\frac{n}{n_u}\frac{1}{75\tau c F_b} \qquad (2a)$$

(independent of sweep angle).

It should be observed that, because the mutual interactions of deformations and load distribution vary with dynamic pressure, equations (2) and (2a) will hold exactly only for one selected dynamic pressure. It will subsequently appear (*see* section 3.1) that in the cases for which it is legitimate to use these equations, the chosen dynamic pressure should be the maximum permissible for the aircraft under consideration. For lower dynamic pressures, the equations will be only approximately true.

The stiffness in bending about the other principal axis of inertia must be large enough to prevent excessive strains in the leading- or trailing-edge members of the wing. The maximum bending moment about this axis occurs when the highest load factor is combined with the highest lift coefficient: it can be shown to be about one-quarter as large as the largest bending moment about the longitudinal principal axis. Hence the ratio between the section bending moduli must be at least one-quarter, and the ratio between the moments of inertia at least $1/4\tau$, the moment of inertia about the vertical principal axis being at least $1/4\tau$ times as large as that about the longitudinal one. In practice it is larger, being for tubular-spar wings about twice, and for single-spar wings of conventional construction about three times as large* as the value calculated here as a minimum. The authors cannot yet quote a corresponding figure for a two-spar wing but it would certainly be larger than three.

When a wing is subject to aerodynamic or inertia loads it is apparent that its bending deformability will have no effect on elastic wash-out as long as the elastic axis is unswept. When, however, that axis is swept, bending deformability will contribute to the elastic wash-out as discussed in section 3. A further deformability which is important because it gives rise to an elastic camber of the wing (*see* section 3) is the bending deformability of the wing ribs. The derivative of the curvature of the ribs with respect to load factor, depends very much on the general structural lay-out and in particular on the number and position of the spar webs, the distance between the ribs which have bending stiffness, and the lay-out of the regions between them. With two or more spars, the deflection curve of a rib will generally have two or more points of inflection and therefore the effects will be smaller then with single-spar wings. For single-spar wings with narrow rib spacing, a rough estimate—similar to that for the spar above—gives for ribs with no excessive strength:

$$\frac{d}{dn}\left(\frac{1}{R}\right)_{\text{rib}} = \frac{-1}{150 n_u c\tau},$$

it being assumed that the ultimate compressive stress is half the ultimate tensile stress, due to column failure and to the neighbouring skin being effective only in tension. Most aircraft have much stiffer ribs, the ' rib-stiffness factor ' $F_{r.b.}$ being usually in the region of 5. Introducing this factor we get

$$\frac{d}{dn}\left(\frac{1}{R}\right)_{\text{rib}} = \frac{-1}{150 n_u c\tau F_{r.b.}}. \qquad (3)$$

* These figures were supplied by the firms Blohm and Voss, and Messerschmitt, respectively.

In addition to rib bending, variation in load factor may give rise to changes in aileron hinge moments which result in an automatic deflection of the ailerons working against their flexible control circuits. Similar effects do not, in general, arise with flaps, since they are usually very rigidly mounted with a self-locking device. Before proceeding to estimate the elastic wash-out and the overall elastic camber of a wing under the loads arising from a pull-out, we shall consider in some detail the deformability of swept wings.

1.2. *The Deformability of Swept Wings.*—The deformability of swept wings depends to a large extent on the details of their structural lay-out. Since wide variations of detail design are possible, especially near the root, it is possible, by suitable or unsuitable choice of lay-out, to improve or worsen the effect considerably. At the same time, it is clearly impossible to derive a formula of general validity, and accordingly we shall deal with the deformability of swept wings by investigating a number of typical structural lay-outs, restricting the investigation to the most usual and convenient case where the sweep angle is constant along the semi-span.

The investigation is most simple for those types for which the chord of the load-carrying structure—apart from the ribs and perhaps from a negligibly thin skin—near the wing root is so small compared with the other linear dimensions of the wing, that the effects of the oblique encasement are negligible (Fig. 1). Structure (a.1) differs from (a.2), and (b.1) from (b.2) only by a small locally concentrated disturbance at the root. Structures (a.2) and (b.2) are easily dealt with. The former has an elastic axis in the strict definition of the second footnote in section 1.1 and does not offer any difficulties. With structure (b.2), sections perpendicular to the direction in which an elastic axis could lie, change their shape with deformation of the spar; therefore wing (b.2) has no elastic axis in the strict sense of the above-mentioned footnote. Since, however, the elastic behaviour of sections lying in the direction of *flight* is the most important feature from the aerodynamic point of view, wing (b.2) is very convenient for our later considerations and we can consider its spar to be a kind of elastic axis in a somewhat looser sense.

The next structural lay-out to be dealt with is the single-spar lay-out with a torsion box of appreciable chord ratio. We consider wing (a.1) of Fig. 1, but with a skin, which may contribute to torsional but not to bending stiffness, in front of the spar. The skin is assumed to extend only from the wing tip to rib c, which is itself assumed to be completely rigid. Then for any arbitrary load, the deformation can be compounded of two contributions, each of which can be simply calculated. These are the deformation of the beam in the root region of the wing, and the deformation of the outer region incorporating the torsion box. For each of these contributions there exists an elastic axis in the exact sense, but for the combined effect there is no such axis.

At a given effective wing lift coefficient, the contribution of the root region has a far greater influence on the effect of deformation on angle of attack of fuselage and tail unit than it has on the aerodynamic pitching moment of the wing. If it is not required to investigate the effect of root region deformability on wing pitching moment at all, we can substitute for the deformability of the root region, a ball-joint with springs at a kind of elastic point of the root region. Such an equivalent system promises to be very helpful in the consideration of aero-elastic problems of swept wings.

Structural lay-outs having a larger chord ratio for the load-carrying structure, including lay-outs with more than one spar can similarly be treated by using such an equivalent system. This can be chosen so as to give the correct effects of the deformation upon the angles of attack of fuselage and tail unit at a given effective wing lift coefficient. On the other hand, it completely neglects the effect of root-region deformation on aerodynamic wing pitching moment at that lift coefficient. This effect is negligible in comparison with the effect of deformation of the outer region on wing pitching moment, if the span of the root region is negligible in comparison with wing span, that is to say, if the chord ratio of the load-carrying structure near the root, multiplied by the sweep angle, is sufficiently small compared with the half aspect ratio. This condition is satisfied for most swept wing lay-outs except possibly for delta wings and it is therefore reasonable to use the equivalent system here where it has the important advantage of decreasing the number of variables. Accordingly, we neglect the effect of root-region deformability on wing pitching moment and consider such deformability to influence the aircraft manoeuvre point only by virtue of its effect on the angles of attack of the fuselage and tail unit. The magnitude of this effect will be investigated in Part II of the report.

Having examined in some detail the character and range of the wing deformability, we are now in a position to proceed with the development of the formula giving the effect of this deformability on manoeuvre point. At the outset, however, we shall, in order to simplify the investigation and to present the results more clearly, make some restrictive assumptions. The importance of some of these will be examined in section 7.

2. *Initial Assumptions and Approximations.*—The wing is assumed to be a cantilever one with no engine nacelles and to have constant chord and constant sweep angle ϕ. Dihedral effects are neglected since they are small. Furthermore, the wing is assumed to have no wash-out for zero external load. The deformability assumptions are taken from section 1.1, the elastic axis being assumed to lie at a constant percentage of the chord and hence to have the constant sweep angle ϕ. Flap and aileron angles are assumed equal at all spanwise locations, to those of the unloaded, and therefore undeformed, aircraft, and to be constant along the span. All deflections caused by shear, and minor effects on load distribution such as are caused by non-linearity of the stress/strain relationship, are neglected.

The wing outside the root region is assumed to have an aerodynamic axis in the exact sense, the axis lying at a constant chordwise position and hence having, like the elastic axis, a constant sweep angle ϕ. (It should be noted that with tapered wings, the sweep angles of elastic and aerodynamic axes are not, in general, equal and it is important to distinguish between them. The analysis of the present report is, however, restricted to untapered wings.)

In general it will not be legitimate to neglect aerodynamic compressibility effects. In most cases, however, allowance can be made for these effects by adopting suitable values for the aerodynamic coefficients, and no further discussion of this point is called for here.

The spanwise distribution of lift corresponding to a uniform angle of attack distribution is assumed rectangular, and the rotary damping of the wing is neglected (*see* section 7). The weight outside the fuselage is assumed to be distributed with constant weight per unit span along an inertia axis at a constant chordwise position.

The tail load is assumed to be negligible in comparison with the lift on the wing which is therefore equated to the factored weight of the aircraft.

3. *The Calculation of Elastic Camber and Elastic Wash-out.*—In general, wing deformability gives rise to camber of chordwise wing profiles. With unswept wings, this elastic camber is due entirely to rib bending. The radius of curvature of the rib camber-line is given by equation (3):

$$\frac{d}{dn}\left(\frac{1}{R}\right)_{\text{rib}} = \frac{-1}{150 n_u \tau c F_{r.b.}} .$$

From simple geometrical considerations (*see* Fig. 2a), the camber ratio γ is equal to $(c/8)(1/R)_{\text{rib}}$ and so equation (3) gives for unswept wings:

$$\frac{d\gamma}{dn} = -\frac{1}{1200 n_u \tau F_{r.b.}} . \qquad \text{(4)}$$

The elastic camber of swept wings depends on the direction of the ribs. If they lie in the direction of flight (*cf.* Fig. 1, structure b) we may assume chordwise sections of the wing to suffer no deformation due to wing-spar bending and then, as with unswept wings, the elastic camber of chordwise wing profiles is due solely to rib bending and is given by equation (4).

If the ribs lie perpendicular to the spar of a single-spar wing (*cf.* Fig. 1, structure a), contributions to elastic camber arise both from rib bending and from wing-spar bending which in this case, changes the shape of chordwise sections.

The radius of curvature of the rib camber-line is again given by equation (3) but the corresponding contribution to the camber of a chordwise section is now given (*see* Fig. 2b) by:

$$\frac{d\gamma}{dn} = \left(\frac{c}{8}\right) \frac{d}{dn} \left(\frac{1}{R}\right)_{\text{rib}} \cos^2 \phi = - \frac{\cos^2 \phi}{1200 n_u \tau F_{r.b.}} .$$

The additional curvature of chordwise camber-lines due to spar bending can be shown by the geometrical considerations illustrated in Fig. 3 to be $\sin^2 \phi$ times the curvature of the elastic axis whose value at load factor n, given by equation (2a) is:

$$\frac{-n}{75 n_u \tau c F_b} .$$

The additional curvature is thus

$$\frac{-n \sin^2 \phi}{75 n_u \tau c F_b} ,$$

and the corresponding increment in camber ratio is

$$\frac{-n \sin^2 \phi}{75 n_u \tau c F_b} \times \frac{c}{8} = \frac{-n \sin^2 \phi}{600 F_b n_u \tau} .$$

The combined effect on camber ratio, of rib and spar bending, in the case of swept wings with ribs perpendicular to the spar, is thus given by:

$$-\frac{d\gamma}{dn} = \frac{1}{n_u \tau} \left[\frac{\cos^2 \phi}{1200 F_{r.b.}} + \frac{\sin^2 \phi}{600 F_b} \right] . \quad \ldots \quad (5)$$

For unswept wings and swept wings with ribs in the direction of flight:

$$-\frac{d\gamma}{dn} = \frac{1}{1200 n_u \tau F_{r.b.}} . \quad \ldots \quad (4)$$

We now calculate the additional wash-out due to torsion caused by a pull-out.

A pull-out causes in the wing, at a distance $|y|$ from the plane of symmetry a torque T about the elastic axis (positive if it reduces the wash-out) whose value, estimated without regard to the effect of deformation on lift distribution, is given by:

$$\left.\frac{\partial T}{\partial n}\right|_V = \frac{W}{2} c \left(\frac{e_F + \Delta e_F}{c} + \frac{r}{c} \frac{W_w}{W} \right) \left(1 - \frac{|y|}{s} \right) \cos \phi .$$

(Throughout this report the symbol $\left(\left.\frac{\partial}{\partial}\right|_{V, \text{etc.}} \right)$ is used to denote partial differentiation with V, etc., held constant.) In the above equation:

- e_F denotes the chordwise distance of the elastic axis behind the line of aerodynamic centres of the undeformed wing profiles,
- Δe_F the forward displacement of the aerodynamic centre of the wing profile due to elastic camber (the value of this displacement will be derived later in equation (26)),
- r the chordwise distance of the inertia axis of the wing behind the elastic axis, and
- W_w the weight of the wing.

In conjunction with equation (1a) this gives as the effect of load factor on negative wash-out per unit span:

$$\left.\frac{\partial}{\partial n}\right|_V \left[\frac{d\alpha}{d \frac{|y|}{s}} \right]_\tau = \frac{s}{GJ} \left.\frac{\partial T}{\partial n}\right|_V = \frac{4 \cdot 30}{4} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{1}{Q^* F_\tau} \left\{ \frac{e_F + \Delta e_F}{c} + \frac{r}{c} \frac{W_w}{W} \right\} \left(1 - \frac{y}{s} \right) .$$

Suffix $_\tau$ denotes an effect due solely to torsion, with bending stiffness assumed infinite. Integrating with respect to $|y|/s$, we obtain the derivative with respect to load factor n, at constant speed, of the local geometric angle of attack arising from wing torsion:

$$\left[\frac{\partial \alpha}{\partial n}\bigg|_V\right]_\tau = \frac{4 \cdot 30}{4} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{1}{Q^* F_\tau} \left\{\frac{(e_F + \Delta e_F)}{c} + \frac{r}{c}\frac{W_w}{W}\right\} \left[\frac{|y|}{s} - \frac{1}{2}\left(\frac{y}{s}\right)^2\right] + K_1 . \quad .. \quad (6)$$

The integration constant K_1 is determined by the condition that $\frac{\partial \bar{C}_L}{\partial n}\Big|_V$ must equal $1/Q$.

If $$\frac{\partial \alpha}{\partial n}\bigg|_V = K_2 + K_3 \frac{|y|}{s} + K_4 \left(\frac{y}{s}\right)^2 ,$$

we have for a wing of constant chord and aspect ratio about 7,

$$\frac{\partial \bar{C}_L}{\partial n}\bigg|_V = \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{\alpha \neq f(y)} (K_2 + 0 \cdot 455 K_3 + 0 \cdot 285 K_4)$$

which may be verified by calculating two lift distributions. Small changes of aspect ratio have a negligible effect on these figures. In our case we have, from equation (6):

$$K_3 = \frac{4 \cdot 30}{4} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{1}{Q^* F_\tau} \left\{\frac{(e_F + \Delta e_F)}{c} + \frac{r}{c}\frac{W_w}{W}\right\}$$

and

$$K_4 = \frac{- K_3}{2} .$$

To satisfy the condition $\frac{\partial \bar{C}_L}{\partial n}\Big|_V = \frac{1}{Q}$, the integration constant of equation (6) must be:

$$K_1 = \frac{1}{Q}\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{\alpha \neq f(y)} - \frac{4 \cdot 30}{4}\left(0 \cdot 455 - \frac{0 \cdot 285}{2}\right) \frac{1 - F_\xi^*}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{1}{Q^* F_\tau} \left\{\frac{(e_F + \Delta e_F)}{c} + \frac{r}{c}\frac{W_w}{W}\right\}$$

$$= \left[\frac{\partial \alpha}{\partial n}\bigg|_V\right]_r - \frac{4 \cdot 30}{4} \times 0 \cdot 3125 \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{1}{Q^* F_\tau} \left\{\frac{(e_F + \Delta e_F)}{c} + \frac{r}{c}\frac{W_w}{W}\right\}$$

where the suffix $_r$ means 'completely rigid'. Using this integration constant in equation (6) we have:

$$\left[\frac{\partial \alpha}{\partial n}\bigg|_V\right]_\tau = \left[\frac{\partial \alpha}{\partial n}\bigg|_V\right]_r + \frac{4 \cdot 30}{4} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \frac{1}{Q^* F_\tau} \left\{\frac{(e_F + \Delta e_F)}{c} + \frac{r}{c}\frac{W_w}{W}\right\} \left[\frac{|y|}{s} - \frac{1}{2}\left(\frac{y}{s}\right)^2 - 0 \cdot 3125\right] \quad (7)$$

where the second term is due to the twist.

In the case of swept wings, the bending deformability gives rise to a second contribution to elastic wash-out. In estimating this contribution we shall first use the bending deformability assumptions of section 1.1. It should be noted that in general, because of the variation with speed, of the relative effect of deformation on lift distribution, the assumption of constant curvature for the flexural axis at a given load factor can only hold true for one speed—say the limiting diving speed. It will appear later (*see* section 3.1) that it is legitimate to assume constant curvature for the flexural axis at the limiting diving speed, only in the case of swept-forward wings and of wings with a small amount of sweepback. For such wings the formula for elastic wash-out due to bending, which we now develop, will be exact at the limiting speed, but will become somewhat inaccurate at lower speeds. For wings with sweepback so large that the bending effect appreciably exceeds the torsion effect, the formula is basically unsound and must be abandoned; a more accurate solution for such wings, based on the revised bending deformability assumptions of section 4 will be given in section 4.1.

In considering bending, we must assume the wing to be loaded on the elastic axis and we further assume the wing to have a rigid root region and attachment. Then, in accordance with the definition of elastic axis, sections of the wing perpendicular to that axis are deflected parallel to themselves (without rotation). From geometrical considerations illustrated in Fig. 3 it will be seen that this deflection causes an elastic wash-out per unit span of:

$$-\left[\frac{\partial \alpha}{\partial |y|}\right]_b = -\tan\phi \times (\text{curvature of elastic axis})\,.$$

By introducing equation (2a) into this formula, integrating with respect to $|y|$ and determining the integration constant as for equation (6) we get:

$$-\left[\frac{\partial \alpha}{\partial n}\bigg|_V\right]_b = \frac{1}{150}\,\frac{A \tan\phi}{n_u \tau F_b}\left(\frac{|y|}{s} - 0{\cdot}455\right)\,. \qquad (8)$$

The deformability of root region and wing attachment as derived above, has no appreciable effect on the elastic wash-out. Equations (7) and (8) together give the following relation for the elastic wash-out where the effect of the deformation on lift distribution is still neglected:

$$-\Delta\left(\frac{\partial \alpha}{\partial n}\bigg|_V\right) = -D_6\left[\frac{|y|}{s} - \frac{1}{2}\left(\frac{y}{s}\right)^2 - 0{\cdot}3125\right] + D_1\left[\frac{|y|}{s} - 0{\cdot}455\right]$$

where

$$D_6 = \frac{1{\cdot}075\,(1 - F_\xi^*)}{Q^* F_\tau \left(1 - \frac{c_\xi}{c}\right)^2}\left(\frac{e_F + \Delta e_F}{c} + \frac{r}{c}\,\frac{W_w}{W}\right) \qquad (9)$$

and

$$D_1 = \frac{A \tan\phi}{150 n_u \tau F_b}$$

represent composite construction figures.

In the special case of $y = 0$, which will be needed in Part II when considering the effect of deformability on fuselage angle of attack, equation (9) gives:

$$\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) = -0{\cdot}3125 D_6 + 0{\cdot}455 D_1\,. \qquad (10)$$

To illustrate the order of magnitude of the elastic wash-out as given by equation (9) we express $\Delta\left(\frac{\partial \alpha}{\partial n}\Big|_V\right)$ at the wing tip as a fraction of the value of $\frac{\partial \alpha}{\partial n}\Big|_V$ for a completely rigid aircraft:

$$\frac{\Delta\left(\frac{\partial \alpha}{\partial n}\Big|_V\right)_{y=s}}{\left(\frac{\partial \alpha}{\partial n}\Big|_V\right)_r} = Q\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_A (0{\cdot}1875 D_6 - 0{\cdot}545 D_1)\,.$$

Fig. 4 shows the curves of $\dfrac{\Delta\left(\frac{\partial \alpha}{\partial n}\Big|_V\right)_{y=s}}{\left(\frac{\partial \alpha}{\partial n}\Big|_V\right)_r}$ against ϕ calculated from this formula for a typical fighter with relevant data as given in the figure. The ratio is observed to be positive (corresponding to wash-in at the tip) and of fairly small magnitude, for unswept wings. It varies considerably with angle of sweep, decreasing rapidly with increasing sweep angle and becoming negative at a fairly small positive angle. Further, at the maximum permissible dynamic pressure, its absolute value is, in general, considerably larger than at the highest dynamic pressure possible in steady horizontal flight. At the maximum permissible dynamic pressure, the geometrical angle of attack at the wing tip decreases at 45 deg sweepback by about one half, and at 25 deg sweepforward increases by a half—that is to say to one and a half times—its original value. These values of the ratio are quite important since they imply a very considerable effect of elastic wash-out on the absolute magnitude of the angle of attack of the wing tip. Under the maximum normal acceleration ($n = 8$) for all dynamic pressures, there is a reduction of nearly 4 deg for 45 deg sweepback, while for 25 deg sweepforward, there is an increase of about 3 deg.

If the relative elastic wash-out occurring in a pull-out is not small compared with unity it has a considerable effect on manoeuvring stability, as well as on the maximum lift coefficient attainable in the pull-out and on the stresses in the pull-out. Here we shall be concerned only with the effect on manoeuvring stability. Before proceeding, however, we consider the circumstances in which, as suggested earlier, the formula just developed (equation (9)) breaks down.

3.1. *The Validity of the Bending Assumptions as Applied to Swept-back Wings.*—In developing equation (2) it was assumed that at the dynamic pressure for which the deflections were being investigated (normally a high value), the spar flanges would, over their complete span, be unloaded at zero load factor and would have the proof stress at the proof load factor. This implied that at the proof load factor, the curvature was at all points of the span equal to the proof curvature, and that in the special case considered, of a wing of constant chord and thickness, the curvature was constant (in magnitude and sign).

For swept wings, bending deformation produces positive or negative wash-out according as the wing is swept-back or swept-forward; in both cases, the corresponding torsional deformations usually produce a negative wash-out. From Fig. 4, it is evident that the net wash-out due to bending and torsion is likely to be zero at a quite small value of sweepback; for larger angles of sweepback, the net wash-out will be positive, while for smaller angles of sweepback and for all angles of sweepforward, the net wash-out will be negative.

When the net wash-out is negative (sweepforward and small sweepback) there will be an increase in lift coefficient from root to tip and it will follow that for a given load factor, the bending moments on the wing increase as the dynamic pressure increases, so that the critical bending moments occur at the maximum permissible dynamic pressure. In such cases, the assumptions leading to equations (2) and (2a) are valid if they are associated with the maximum permissible dynamic pressure. The equations are then exact for that dynamic pressure, but hold only approximately at lower dynamic pressures.

When the net wash-out is positive (at the larger angles of sweepback) there is a reduction in lift coefficient from root to tip and it will follow that for a given load factor, the bending moments on the wing increase as the dynamic pressure decreases. Accordingly, if the spars are assumed to have the proof curvature at a high dynamic pressure, they will have excessive curvature at lower dynamic pressures. If, on the other hand, the proof curvature were to be associated with a low dynamic pressure, then at high dynamic pressure the reduction due to elastic deformation in angle of attack towards the tip would, in relation to the mean angle of attack, be relatively greater than at low dynamic pressure. Then in some cases, the values of the wash-out calculated from equation (9) might be such as to indicate negative angles of attack (and hence negative loading) towards the tip, at high dynamic pressure. Thus, far from having the constant curvature assumed in equations (2) and (2a), the spars would actually appear, from the calculations, to have a reversal of curvature towards the tip.

It is apparent from these considerations of extreme cases, that in the general case of swept-back wings for which the effects of bending are large in relation to those of torsion, equation (9) is likely to lead to very inaccurate results. For the bending deformability of such wings we now make new assumptions which, while still of an arbitrary nature, are nevertheless more logical than the earlier ones. At the same time we shall take full account of the interaction of deformation and lift distribution, and thus obtain formulae of equal validity at all points of the speed range.

4. *Revised Bending Deformability Assumptions for Swept-back Wings with Bending Effect Large in Relation to Torsion Effect.*—From the preceding section it is clear that for swept-back wings, with bending effect large compared with torsion effect, the assumed bending strength should be related to the bending moments occurring during a pull-out at low, rather than at high dynamic pressure. At the lowest dynamic pressures, deformability effects are negligible and we may assume a distribution of bending stiffness appropriate to the bending moments of an absolutely rigid wing.

The general mode of elastic wash-out is unknown; it depends on load distribution and *vice versa*. The exact solution of the problem would involve a complicated differential equation but we shall simplify the solution* by assuming the wash-out, or more conveniently, the spanwise increment of the lift coefficient, to result from the superposition of two terms of arbitrarily chosen form with unknown coefficients D and H. Two expressions can then be obtained for the incidence of the deformed wing, at a general spanwise location specified by a parameter ζ. By equating the two expressions for two arbitrarily chosen values of ζ, a pair of equations in D and H is obtained from which the values of these unknown coefficients may be determined.

Closer approximations would necessitate the assumption of more then two modes of wash-out with correspondingly more equations in the unknown coefficients, arising from the fulfilment of conditions at additional points of the span. In the limit, fulfilment of the conditions at all points of the span would result in an infinite number of equations.

Choice of the two arbitrary modes of wash-out will be guided by the consideration that the modes should be sufficiently dissimilar and that the corresponding deflection curve should have zero slope at the plane of symmetry. Linear and quadratic modes will in fact be assumed.

We consider as before, an aircraft of weight W with a wing of constant chord c, semi-span s, aspect ratio $A = 2s/c$ and constant thickness τc, swept back by an angle ϕ. The wing weight W_w is assumed to be distributed uniformly in the spanwise direction.

If the wing is regarded as an absolutely rigid beam we have, for bending about an axis perpendicular to the elastic axis, under load factor n:

$$B_r = n\frac{(W - W_w)s}{4 \cos \phi}\left[1 - \frac{|y|}{s}\right]^2 \quad \ldots \quad (11)$$

where B_r is the bending moment (positive for upward load) about the specified axis at a perpendicular distance $|y|$ from the plane of symmetry. We now assume a distribution of bending stiffness such that, under the ultimate bending moments appropriate to the absolutely rigid wing, the curvature of the elastic axis is everywhere equal to the ultimate curvature. We thus have:

$$EI = B_{ru}R_u \quad \ldots \quad (12)$$

where I is the moment of inertia of a cross-section of the equivalent beam taken perpendicular to the elastic axis, and R_u is the ultimate radius of curvature. From section 1.1 we have

$$\frac{1}{R_u} = \frac{1}{75\tau c}. \quad \ldots \quad (13)$$

* For full details refer to Appendix II.

Hence, from equations (11), (12) and (13):

$$(EI)\cos\phi = \frac{n_u(W - W_w)s \times 75\tau c}{4}\left(1 - \frac{|y|}{s}\right)^2$$

$$= (EI_0)\cos\phi\left(1 - \frac{|y|}{s}\right)^2,$$

where I_0 is the moment of inertia of the section corresponding to $y = 0$. Introducing a bending stiffness factor F_b as before, we have:

$$(EI)\cos\phi = (EI_0)\cos\phi\left(1 - \frac{|y|}{s}\right)^2 = \frac{F_b n_u(W - W_w)s \times 75\tau c}{4}\left(1 - \frac{|y|}{s}\right)^2 . \quad .. \quad (14)$$

For torsional rigidity we retain the previous expression:

$$GJ = \frac{\left(1 - \frac{c_\xi}{c}\right)^2}{4\cdot 30}\,\frac{Q^*}{1 - F_\xi^*}\,WSF_\tau\cos\phi . \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1a)$$

It should be pointed out at this stage, that when the angle of sweep is large, the value of the torsional rigidity required for adequate aileron effectiveness, given by equation (1a) with $F_\tau = 1$, is very inaccurate*.

Since sweep reduces the aerodynamic coefficients (by some fractional power of $\cos\phi$, we may assume), the equation in question somewhat overestimates the required torsional rigidity in the case of infinite bending stiffness, but it is correct in representing it as a symmetrical function of sweep angle with its maximum corresponding to the unswept wing. At a given sweep angle, the required torsional rigidity varies considerably with bending stiffness. It increases with decreasing bending stiffness in the case of swept-back wings and decreases with decreasing bending stiffness for swept-forward wings. With a reasonable bending stiffness, the combined effect gives a relatively small dependence on sweep for sweepback, and a much larger dependence for sweepforward. The torsional rigidity required with large sweepback will usually be somewhat greater than for the unswept wing while with large sweepforward, it will be much smaller than in the unswept case. For any particular aircraft type these effects can be considered by calculating the GJ required for aileron effectiveness on the lines of Ref. 7 and introducing a corresponding value for F_τ in the formula of the present report. In some cases it may be more economical from the weight viewpoint, or even necessary for the prevention of aileron reversal, to increase the bending stiffness above that required for strength.

It will be useful to translate the rigidity assumptions of equations (1a) and (14) in terms of the non-dimensional stiffness parameters M_θ and L_ϕ of Ref. 7, which are defined by the equations:

$$M_\theta = \frac{m_\theta}{qc^2 s}\ ; \quad L_\phi = \frac{l_\phi}{qcs^2},$$

where m_θ is the wing torsional stiffness (applied moment per unit twist about the flexural axis) measured at a convenient reference section, and l_ϕ is the wing stiffness in bending about an axis perpendicular to the flexural axis, defined by

$$l_\phi = \frac{Pl^2}{\delta}$$

where δ is the deflection produced by a load P acting at a reference station at distance l from the wing root measured along the flexural axis.

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* For details, *see* Ref. 7.

For the illustrative examples of Ref. 7, the reference sections for both m_θ and l_ϕ are taken at $0{\cdot}8s$ (the mid-aileron section for the wings considered). For considerations of flutter, the reference section for l_ϕ is generally taken at $0{\cdot}7s$.

We now derive expressions for M_θ and L_ϕ corresponding to the rigidity assumptions given by equations (1a) and (14) and appropriate to the critical dynamic pressure for aileron reversal, *i.e.*,

$$q = q_{c\xi} = \frac{q^*}{(1 - F_\xi^*)} \cdot$$

If distance along the flexural axis from the root is denoted by $y' = y/\cos\phi$, and if the reference section is taken at $y' = y_R' = y_R/\cos\phi$ we have:

$$\frac{1}{m_\theta} = \int_0^{y_R'} \frac{1}{GJ}\, dy' = \int_0^{y_R} \frac{1}{GJ} \frac{dy}{\cos\phi}$$

$$= \frac{y_R}{GJ\cos\phi},$$

or putting $\quad y = 0{\cdot}8s$,

$$m_\theta = \frac{GJ\cos\phi}{0{\cdot}8s} .$$

Substituting for GJ from equation (1a) we obtain:

$$(M_\theta)_{q=q_{c\xi}} = \frac{m_\theta(1 - F_\xi^*)}{q^* c^2 s} = 1{\cdot}16\left(1 - \frac{c_\xi}{c}\right)^2 F_\tau \cos^2\phi . \quad \ldots \quad (15)$$

To obtain the corresponding expression for L_ϕ, we first find the deflection δ_R produced by a load P applied at a point of the flexural axis at distance $\zeta_R s'$ from the root, with a stiffness distribution EI' given by equations (12) and (14). It is

$$\delta_R = \frac{8Ps'^3\vartheta}{75F_b n_u \tau WS\left(1 - \dfrac{W_w}{W}\right)\cos\phi}$$

where $$\vartheta = \int_0^{\zeta_R}\int_0^{\zeta} \frac{(\zeta_R - \zeta)}{(1 - \zeta)^2}\, d\zeta\, d\zeta .$$

Then by definition,

$$l_\phi = \frac{P(\zeta_R s')^2}{\delta_R}$$

$$= \frac{75F_b n_u \tau WS\left(1 - \dfrac{W_w}{W}\right)\cos^2\phi\,\zeta_R^2}{8\vartheta s} .$$

If $\zeta_R = 0{\cdot}8$, we have $\vartheta = 0{\cdot}3162$ and then

$$(L_\phi)_{q=q_{c\xi}} = \frac{l_\phi(1 - F_\xi^*)}{q^* cs^2} = 152\frac{n_u \tau F_b(1 - F_\xi^*)\left(1 - \dfrac{W_w}{W}\right)\cos^2\phi}{AQ^*} . \quad \ldots \quad (16)$$

It may be noted that if the reference section for bending stiffness is taken at $0{\cdot}7s$, the numerical coefficient in the last equation becomes 196. These expressions for M_θ and L_ϕ may be used in conjunction with the curves of Ref. 7 to check whether the assumed stiffnesses are, in any particular case, satisfactory from the point of view of aileron reversal. Furthermore, they enable us to calculate appropriate values of F_b and F_τ in the case of a specific design for which stiffnesses have been measured by the standard British method.

4.1. *The Calculation of Elastic Wash-out for Such Wings.*—It is shown in Appendix II that with the rigidity assumptions of section 4 (equations (14) and (1a)), the elastic wash-out is given by:

$$-\Delta\left(\frac{\partial\alpha}{\partial n}\bigg|_V\right) = -\left\{\frac{dD}{dn}(\zeta - 0\cdot455) + \frac{dH}{dn}(\zeta^2 - 0\cdot285)\right\}\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m} \quad \ldots \quad \ldots \quad (17)$$

where ζ is written for $|y|/s$. The change in angle of attack of the fuselage, with rigid root region and attachment is given by:

$$\Delta\left(\frac{\partial\alpha_{y=0}}{\partial n}\bigg|_V\right) = -\left(0\cdot455\frac{dD}{dn} + 0\cdot285\frac{dH}{dn}\right)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

and the relative elastic wash-out at the tip by:

$$\frac{-\Delta\left(\frac{\partial\alpha}{\partial n}\bigg|_V\right)_{y=s}}{\left(\frac{\partial\alpha}{\partial n}\bigg|_V\right)_r} = -Q\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_A\left[0\cdot545\frac{dD}{dn} + 0\cdot715\frac{dH}{dn}\right]\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m}.$$

The value of the latter expression at $Q = Q^*$ has been calculated for the typical fighter of Fig. 4 with $\phi = 45$ deg, and is indicated on the figure. It it seen to be considerably less than the value calculated with the earlier formula.

The quantities dD/dn and dH/dn in equations (17) and (18) are very complicated and their calculation is dealt with fully in the appendix. It is shown there that:

$$\frac{dD}{dn} = \frac{\mathscr{U} - \mathscr{F}\mathscr{V}}{\mathscr{P}}\ ; \qquad \frac{dH}{dn} = \frac{-\mathscr{V} + \mathscr{G}\mathscr{U}}{\mathscr{Q}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

where

$$\left.\begin{aligned}\mathscr{U} &= 0\cdot5[(B_1 + B_2) - \tan\phi(B_3 - B_4)]\\ \mathscr{V} &= 0\cdot375(B_1 + B_2) - 0\cdot25\tan\phi(B_3 - B_4)\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

and $\mathscr{F}$, $\mathscr{G}$, $\mathscr{P}$ and $\mathscr{Q}$ are functions of the two variables

$$QB_1 = \frac{Q}{Q^*}\frac{1\cdot075\,e_F'/c}{F_\tau}\frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2}\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m}$$

and

$$\tan\phi\,QB_3 = \frac{\tan\phi\,QA\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m}}{75n_u\tau F_b\left(1 - \frac{W_w}{W}\right)},$$

which directly involve the dynamic pressure number Q and the geometrical and structural features of the wing. The quantities $(B_1 + B_2)$ and $\{\tan\phi\,(B_3 - B_4)\}$ on which $\mathscr{U}$ and $\mathscr{V}$ depend are given by:

$$(B_1 + B_2) = \frac{1\cdot075}{F_\tau Q^*}\left(\frac{e_F'}{c} + \frac{W_w}{W}\frac{r}{c}\right)\frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2}\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m} = D_6\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m} \quad \ldots \quad \ldots \quad (21)$$

$$\tan\phi\,(B_3 - B_4) = \frac{A\tan\phi\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m}}{75n_u\tau F_b} = 2D_1\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

and apart from the secondary effect of Q on $(B_1 + B_2)$ through the factor $e_F' = e_F + \Delta e_F$, depend solely on the geometrical and structural features of the wing.

Some graphical aids to the calculation of dD/dn and dH/dn are discussed in section 2 of Appendix II.

Comparison between equations (17) and (9) shows that irrespective of the angle of sweep, we may express the wash-out in the form:

$$\Delta\left(\frac{\partial \alpha}{\partial n}\bigg|_V\right) = D_7(\zeta - 0 \cdot 455) + D_8(\zeta^2 - 0 \cdot 285) \quad \ldots \quad (23)$$

where for wings of large* sweepback

$$D_7 = \left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m} \frac{dD}{dn} ; \qquad D_8 = \left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m} \frac{dH}{dn} \quad \ldots \quad (24a)$$

and for other* wings:

$$D_7 = D_6 - D_1 ; \qquad D_8 = -\frac{D_6}{2} . \quad \ldots \quad (24b)$$

5. *The Effect of Deformability on the Pitching Moment of the Wing.*—The position of the aircraft manoeuvre point depends directly on the quantity $\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V$, *i.e.*, the derivative with respect to lift coefficient, at constant speed, of the pitching-moment coefficient about a given centre of gravity.

We now consider the increment in $\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V$ due to wing deformability. The pitching-moment coefficient C_M of the complete aircraft may be expressed as:

$$C_M = C_{MW} + C_{MF} + C_{MT} ,$$

where suffixes $_W$, $_F$ and $_T$ denote respectively contributions from the wing, fuselage and tail unit. In virtue of its effect on the angle of attack of the fuselage, wing deformability affects C_{MF} and C_{MT} as well as C_{MW} so that if Δ_W is used to denote an increment due to wing deformability we shall have:

$$\Delta_W\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right) = \Delta_W\left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V\right) + \Delta_W\left(\frac{\partial C_{MF}}{\partial \bar{C}_L}\bigg|_V\right) + \Delta_W\left(\frac{\partial C_{MT}}{\partial \bar{C}_L}\bigg|_V\right) .$$

The last two terms are most conveniently dealt with in conjunction with the effects of fuselage and tail-unit deformability and are therefore considered in Part II of the report. We now consider $\Delta_W\left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V\right)$ which will be due partly to a change in the pitching moment of the inertia forces acting on the wing and partly to a change in the aerodynamic pitching moment.

5.1. *The Effect on the Pitching Moment of the Inertia Forces.*—If we relate the pitching moment to a point of the fuselage near the wing root, all bending deformations of the wing result in a change in the contribution of the inertia forces of the wing to the pitching moment, because the moment arms of these forces are affected by bending, An approximate estimate of the effect can be made without much difficulty when the bending deformability assumptions are the relatively simple ones of section 1.1. For then, since we have assumed the wing weight to be uniformly distributed along the span, it will follow that its centre of gravity is displaced by one-third of the displacement of the wing tip. The displacement of the centre of gravity will have two components corresponding respectively to bending about the two principal axes of

* The precise ranges of validity of the two sets of formulae are investigated in section 7.

inertia. The longitudinal principal axis may be assumed to lie in the no-lift plane* of the half-wing and at right-angles to the elastic axis. The downward displacement of the wing tip due to bending about this axis is readily derived from equation (2a) as:

$$-\frac{1}{300}\frac{n}{n_u}\frac{A}{\tau F_b}\frac{s}{\cos^2\phi} = -\frac{D_1 A Q \bar{C}_L}{2\sin 2\phi}$$

where D_1 is given by equation (9). The forward inertia force, whose moment arm is changed by one-third of this amount is equal to the product of the wing weight W_W and the load factor in the direction of the longitudinal principal axis which (*see* Fig. 5) is approximately

$$Q[C_{D\,0} - \bar{C}_L^2(\partial\alpha/\partial C_L)_\infty] ,$$

where $C_{D\,0}$ is the drag coefficient for the complete aircraft at zero lift.

The vertical principal axis is normal to the longitudinal axis and therefore, with our previous assumption, normal to the wing no-lift plane, and the ratio of the moment about this axis to that about the longitudinal axis is (*see* Fig. 5):

$$\left[\frac{\left(C_{D\,P} - \frac{W_w}{W} C_{D\,0}\right)}{\bar{C}_L\left(1 - \frac{W_w}{W}\right)} - \bar{C}_L\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty\right]\cos\phi ,$$

where $C_{D\,P}$ is the wing profile drag coefficient. If the bending rigidities about the vertical and longitudinal principal axes are denoted by $(EI)_v$ and $(EI)_h$ respectively, the ratio of the curvatures and of the normal deflections is:

$$\frac{(EI)_h}{(EI)_v}\left[\frac{\left(C_{D\,P} - \frac{W_w}{W} C_{D\,0}\right)}{\bar{C}_L\left(1 - \frac{W_w}{W}\right)} - \bar{C}_L\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty\right]\cos\phi .$$

It follows (*see* Fig. 5) that the ratio of the rearward deflection of the wing weight c.g. parallel to flight direction (seen from above) to the downward deflection is given by:

$$-\frac{(EI)_h}{(EI)_v}\left[\frac{\left(C_{D\,P} - \frac{W_w}{W} C_{D\,0}\right)}{\bar{C}_L\left(1 - \frac{W_w}{W}\right)} - \bar{C}_L\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty\right]\cos^2\phi .$$

The downward inertia force whose arm is changed is approximately nW_w. The change in pitching-moment coefficient due to the change of moment arms of the inertia forces is now readily established as:

$$\begin{aligned}\Delta_1(C_{M\,W}) &= \frac{1}{WQc}\,\Delta_1(M_W)\\ &= \frac{D_1 AQ}{6\sin 2\phi}\frac{W_w}{W}\left[\left\{ - C_{D\,0}\bar{C}_L + \bar{C}_L^3\left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_\infty\right\}\right.\\ &\quad\left. + \frac{(EI)_h\cos^2\phi}{(EI)_v}\left\{\frac{\bar{C}_L C_{D\,P} - \frac{W_w}{W}\bar{C}_L C_{D\,0}}{1 - \frac{W_w}{W}} - \bar{C}_L^3\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty\right\}\right] . \qquad (25)\end{aligned}$$

* We are considering a constant-chord wing of constant section, and if the effects of twist on moment arms are neglected, the no-lift lines of all sections of the half-wing may be considered coplanar.

5.2. *The Effect on the Pitching Moment of the Aerodynamic Forces.*—Elastic camber and wash-out affect the aerodynamic forces acting on the wing and hence also the pitching moment of those forces. Furthermore, the pitching moment is directly affected by a change, due to bending, in the arms of the aerodynamic forces. We deal first with the effect of camber.

For a conventional wing section with a camber-line of constant curvature, the pitching-moment coefficient relative to the aerodynamic centre is about $-(2 \cdot 5\gamma/\beta)$ where γ denotes the camber ratio and β the reciprocal of the Prandtl-Glauert factor for the normal component of velocity. From equations (6) and (7), it follows that the change due to elastic camber, in the section pitching-moment coefficient about the aerodynamic centre is, for unswept wings and swept wings with ribs in flight direction,

$$\Delta_2 C_{MW} = \frac{2 \cdot 5}{\beta} \frac{n}{1200 n_u \tau F_{r.b.}}$$

and for swept wings with ribs perpendicular to the elastic axis,

$$\Delta_2 C_{MW} = \frac{2 \cdot 5}{\beta} \frac{n}{n_u} \frac{1}{\tau} \left[\frac{\cos^2 \phi}{1200 F_{r.b.}} + \frac{\sin^2 \phi}{600 F_b}\right].$$

Making the substitution $n = \bar{C}_L Q$ and differentiating with respect to $\bar{C}_L$ at constant Q (and hence constant V) we obtain:

$$\Delta_2\left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V\right) = D_4 Q$$

where, for unswept wings and swept wings with ribs in flight direction:

$$D_4 = \frac{1}{480 \beta n_u \tau F_{r.b.}} \qquad \qquad \ldots \quad (26)$$

and for swept wings with ribs perpendicular to the elastic axis:

$$D_4 = \frac{1}{480 \beta n_u \tau}\left[\frac{\cos^2 \phi}{F_{r.b.}} + \frac{2 \sin^2 \phi}{F_b}\right]$$

The quantity $\Delta_2\left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V\right)$ is a measure of the forward displacement, expressed as a fraction of the chord, of the section aerodynamic centre, due to deformability. Elastic camber is thus responsible for a forward displacement Δe_F of the section aerodynamic centre given by:

$$\frac{\Delta e_F}{c} = D_4 Q \,. \qquad \ldots \quad (26a)$$

This affects the pitching moment of the wing not only directly, but also indirectly, by influencing the torque. This indirect effect is taken account of by the term $\Delta e_F/c$ introduced into equations (7) and (9).

To calculate the effect of elastic wash-out on pitching moment and manoeuvre point we employ a modified strip method, the theoretical basis of which is discussed in Appendix I. Using the approximate result given by equation (14) of that appendix we assume the contribution $(\Delta_3 M_W)$ to wing pitching moment, due to elastic wash-out to be given by:

$$\Delta_3 M_W = -2q\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \tan \phi \int_0^s \Delta\alpha c\left(y - \frac{s}{2}\right) dy$$

where, for the time being, we assume $m \simeq 3 + \tan \phi$. Hence

$$\Delta_3 C_{MW} = -\tfrac{1}{2} A \tan \phi \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \int_0^1 \Delta\alpha(\zeta - 0 \cdot 5)\, d\zeta ,$$

and since $\dfrac{\partial}{\partial \bar{C}_L} = Q \dfrac{\partial}{\partial n}$,

$$\Delta_3 \left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V \right) = -\tfrac{1}{2} AQ \tan\phi \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m} \int_0^1 \Delta \left(\frac{\partial \alpha}{\partial n}\bigg|_V \right) (\zeta - 0\cdot 5)\, d\zeta$$

$$= -\tfrac{1}{2} AQ \tan\phi \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m} \left\{ D_7 \int_0^1 (\zeta - 0\cdot 455)(\zeta - 0\cdot 5)\, d\zeta + D_8 \int_0^1 (\zeta^2 - 0\cdot 285)(\zeta - 0\cdot 5)\, d\zeta \right\}$$

from equation (23)

$$= -0\cdot 04165 AQ \tan\phi \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m} (D_7 + D_8) .$$

From equations (24a) and (24b), we have for wings of large sweepback

$$D_7 + D_8 = \left(\frac{dD}{dn} + \frac{dH}{dn} \right) \left(\frac{\partial \alpha}{\partial \bar{C}_L} \right)_{A/m}$$

and for other wings,

$$D_7 + D_8 = \frac{D_6}{2} - D_1$$

so that for wings of large sweepback,

$$\Delta_3 \left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V \right) = -0\cdot 04165 AQ \tan\phi \left\{ \frac{dD}{dn} + \frac{dH}{dn} \right\} \quad \ldots \quad (27a)$$

and for other wings,

$$\Delta_3 \left(\frac{\partial C_{MW}}{\partial \bar{C}_L}\bigg|_V \right) = D_2 Q (-0\cdot 02083 D_6 + 0\cdot 04165 D_1)$$

where $$D_2 = A \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m} \tan\phi . \quad \ldots \quad (27b)$$

In addition to the effect arising from elastic wash-out, bending exerts a direct effect on the pitching moment of the aerodynamic forces, comparable with that investigated in section 5.1 for the inertia forces. As derived there for the case governed by the bending deformability assumptions of section 1.1, the mean bending deflections of the wing, expressed as fractions of the chord are:

upward: $\dfrac{D_1 AQ\bar{C}_L}{6 \sin 2\phi}$ (where D_1 is given by equation (9))

rearward: $$\frac{(EI)_h}{(EI)_v} \frac{D_1 AQ}{6 \sin 2\phi} \left[\frac{C_{DP} - \dfrac{W_w}{W} C_{D0}}{1 - \dfrac{W_w}{W}} - \bar{C}_L^2 \left(\frac{\partial \alpha}{\partial \bar{C}_L} \right)_\infty \right] \cos^2\phi .$$

The coefficients of the aerodynamic forces whose moment arms are changed by these deflections are respectively:

rearward: $\simeq C_{DP} - \bar{C}_L^2 \left(\dfrac{\partial \alpha}{\partial \bar{C}_L} \right)_\infty$

upward: $\simeq \bar{C}_L$.

The contribution to the pitching-moment coefficient of the wing is given by:

$$\Delta_4(C_{MW}) = \frac{D_1 A Q}{6 \sin 2\phi}\Bigg[C_{DP}\bar{C}_L - \bar{C}_L{}^3\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty - \frac{(EI)_h}{(EI)_v}\left\{\frac{C_{DP}\bar{C}_L - \frac{W_w}{W}C_{D0}\bar{C}_L}{1 - \frac{W_w}{W}} - \bar{C}_L{}^3\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty\right\}\Bigg]\cos^2\phi\,.$$

Combining this result with equation (25) and differentiating with respect to $\bar{C}_L$ at constant V (and hence constant Q), we obtain*:

$$\left.\begin{aligned}&\Delta_{1+4}\left(\left.\frac{\partial C_{MW}}{\partial \bar{C}_L}\right|_V\right) = D_1 D_5 Q\,,\\ &\text{where}\\ &D_5 = \frac{A}{6\sin 2\phi}\left[1 - \frac{(EI)_h}{(EI)_v}\cos^2\phi\right]\left[C_{DP} - \frac{W_w}{W}C_{D0} - 3\left(1 - \frac{W_w}{W}\right)\frac{n^2}{Q^2}\left(\frac{\partial\alpha}{\partial C_L}\right)_\infty\right]\end{aligned}\right\}\,. \qquad (28)$$

5.2.1. *The direct effect of bending on the pitching moment of the aerodynamic and inertia forces for wings of large sweepback.*—Substitution of typical values in equations (28) indicates that this effect is relatively small. It has therefore not been thought worth while working out a formula for the more complicated case governed by the assumptions of section 4. The absolute magnitudes of $\Delta_1(C_{MW})$ and $\Delta_4(C_{MW})$ in this case will certainly be less than the values given by the formulae of sections 5.1 and 5.2 so that it is possible to set an upper limit to the contribution $\Delta_{1+4}\left(\left.\frac{\partial C_{MW}}{\partial \bar{C}_L}\right|_V\right)$. This should suffice for all practical considerations.

5.3. *The Pitching Moment of the Wing in the Presence of a Fuselage.*—When the middle portion of a plain wing is replaced by part of a fuselage, the whole shape of the lift distribution is altered, and for a given lift coefficient the wing bending moment is changed without any considerable effect on angle of attack. The increase in bending moment is approximately proportional to

$$\underset{\text{with no fuselage}}{(C_L)_{y=0}}\, c b_{F\max}{}^2 q\,,$$

where $b_{F\max}$ denotes the maximum width of the fuselage. The factor of proportionality C_F is of the order of magnitude 0·1, being positive for low-wing and negative for mid-wing lay-outs†. This should be sufficiently accurate since the effect in question is comparatively small. The effect of deformability on the increase in bending moment is given by:

$$\Delta_5 B = C_F\left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/m}\bar{C}_L Q \Delta\left(\left.\frac{\partial\alpha_{y=0}}{\partial n}\right|_V\right) c b_{F\max}{}^2 q\,.$$

Multiplying the bending moment by $-2\tan\phi$ to get the pitching moment (*see* Appendix I) we obtain the effect on pitching-moment coefficient as:

$$\left.\begin{aligned}&\Delta_5\left(\left.\frac{\partial C_{MW}}{\partial \bar{C}_L}\right|_V\right) = -D_3 Q \Delta\left(\left.\frac{\partial\alpha_{y=0}}{\partial n}\right|_V\right)\\ &\text{where}\\ &D_3 = 2C_F\frac{b_{F\max}{}^2}{S}\left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/m}\tan\phi\end{aligned}\right\} \qquad (29)$$

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* C_{DP} is assumed independent of $\bar{C}_L$.

† *See* Ref. 11.

5.4. *The Resultant Pitching Moment of the Wing.*—The overall effect of elastic deformability on the derivative of the wing pitching-moment coefficient, with respect to lift coefficient at constant speed and constant elevator angle is given by:

$$\Delta_W\left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V,\eta}\right) = \sum_{r=1}^{5} \Delta_r \left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V}\right). \qquad (30)$$

For easy reference, the details of the various contributions are summarized in Table 1. The expressions involve several intermediate expressions, *viz.*, D_1, D_2, . . . D_6, $\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right)$, $\frac{dD}{dn}$, $\frac{dH}{dn}$ which are set out in full in Table 2. They depend either directly or indirectly on the geometry and structural details of the wing and partly also on the condition of flight. D_1, D_2, D_3 and D_4 are pure 'construction figures' in that they are independent of flight conditions if we neglect Mach number and Reynolds number effects. The other quantities depend directly on flight conditions. The primary effect of flight condition on $\Delta_W\left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V,\eta}\right)$ is given by the direct factor Q occurring in each of the quantities Δ_2, Δ_3, Δ_{1+4} and Δ_5.

6. *The Effect on the Manoeuvre Point.*—The value of $\Delta_W\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right)$ is independent of the choice of reference point for pitching moments, which may therefore be taken at the centre of gravity of the complete aircraft. Now if C_M is referred to the aircraft c.g. the quantity $-\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}$ is the 'manoeuvre margin, elevator fixed', that is, the distance of the c.g. ahead of the 'manoeuvre point, elevator fixed' expressed as a fraction of the wing mean chord (*see* Ref. 3). If the c.g. is at the manoeuvre point of the datum (absolutely rigid) aircraft we have:

$$\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right)_r = 0. \qquad (31)$$

Then for the aircraft with deformable wing, but otherwise rigid, we have:

$$\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta} = \left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right)_r + \Delta_W\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right) = \Delta_W\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right)$$

in virtue of equation (31). The manoeuvre margin is thus $-\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta} = -\Delta_W\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right)$ compared with the zero margin for the datum aircraft. It follows that there is a forward displacement of manoeuvre point, elevator fixed, due to wing deformability given by:

$$\Delta_W\left(\frac{x_{m.p.}}{c}\right) = \Delta_W\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\eta}\right). \qquad (32)$$

7. *The Respective Ranges of Validity of the Two Formulae for Elastic Wash-out.* (*Equations* (9) *and* (17).)—Equation (17) was developed using the bending deformability assumptions of section 4, which were introduced to cover the case where, at the maximum permissible dynamic pressure number Q^*, the combined effects of bending and torsion produce a positive wash-out. It will be valid, therefore, only if, for any given positive load factor n, at the dynamic pressure number Q^*, the angle of attack at the tip (specified by $\zeta = 1$) is greater for the rigid wing than for the flexible wing. Under subsonic conditions*, the effect of torsion is, in all practical cases, to increase the incidence from root to tip, while bending increases or decreases the incidence

* In this report, the term 'subsonic conditions' is applied to all flight conditions up to which the major portion of the wing has remained free from shock waves. In the case of swept wings, the flight speed may already be supersonic.

according as the wing is swept-forward or swept-back. We may therefore, at once conclude that equation (17) is not applicable to a swept-forward wing. Further, unless the torsional stiffness of the wing is infinite, there will be a range of angles of sweepback for which the effect of torsion on the incidence at the tip is greater than the effect of bending; over this range equation (17) will not apply.

Confining our attentions now to swept-back wings we may develop a criterion for testing the validity of equation (17) in any particular case.

7.1. *The Validity Criterion.*—Throughout this paragraph, suffices $_r$ and $_f$ are used to denote the rigid and flexible wings respectively while asterisked quantities are appropriate to $Q = Q^*$.

For the rigid wing at dynamic pressure Q^* we have, from equations (5) and (6) of Appendix II:

$$(\alpha_r)_{\zeta=1}{}^* = \bar{\alpha} = \frac{n}{Q^*}\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_A ,$$

and for the flexible wing, using equations (7) and (8)

$$(a_f)_{\zeta=1}{}^* = \alpha_0{}^* + (D^* + H^*)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m} .$$

From equations (6) and (10)

$$\alpha_0{}^* = \frac{n}{Q^*}\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_A - (0{\cdot}455D^* + 0{\cdot}285H^*)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m}$$

and hence

$$(\alpha_f)_{\zeta=1}{}^* = (0{\cdot}545D^* + 0{\cdot}715H^*)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m} + \frac{n}{Q^*}\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_A .$$

Thus

$$(\alpha_r - \alpha_f)_{\zeta=1}{}^* = -(0{\cdot}545D^* + 0{\cdot}715H^*)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m} \quad \ldots \quad (33)$$

and equation (17) will be valid only if:

$$(\alpha_r - \alpha_f)_{\zeta=1}{}^* > 0 ,$$

i.e., if

$$0{\cdot}545D^* + 0{\cdot}715H^* < 0 .$$

Now D^* and H^* are linear functions of n which vanish for $n = 0$, so that dD^*/dn and dH^*/dn will be of the same sign as D^* and H^* respectively for positive n. The inequality may therefore be written in the form:

$$0{\cdot}545\frac{dD^*}{dn} + 0{\cdot}715\frac{dH^*}{dn} < 0 .$$

This is shown in the appendix to lead to the condition:

$$\mathscr{U}^* - K\mathscr{V}^* < 0 , \quad \ldots \quad (34)$$

where

$$K = \frac{0{\cdot}2378 - 0{\cdot}0239Q^*B_1 + 0{\cdot}0225Q^*B_3 \tan\phi}{0{\cdot}3094 - 0{\cdot}0169Q^*B_1 + 0{\cdot}0143Q^*B_3 \tan\phi} . \quad \ldots \quad (35)$$

For fixed values of the construction figures B_1, B_2, B_3 and B_4 and of the maximum dynamic pressure number Q^*, the minimum value of ϕ for which equation (17) is valid will correspond to the positive root of the equation:

$$\mathscr{U}^* = K\mathscr{V}^*$$

which is a quadratic in $\tan \phi$. Substituting for $\mathscr{U}^*$ and $\mathscr{V}^*$ from equations (20) and for K from equation (35), and simplifying we arrive at the equation :

$$\tan^2 \phi + \left\{0 \cdot 8487 \frac{(B_1^* + B_2)}{(B_3 - B_4)} + \frac{(62 \cdot 63 - 1 \cdot 628 Q^* B^*_1)}{Q^* B_3}\right\} \tan \phi$$

$$- \left\{\frac{(43 \cdot 09 + 0 \cdot 3355 Q^* B_1^*)}{Q^* B_3} \frac{(B_1^* + B_2)}{(B_3 - B_4)}\right\} = 0 \qquad \ldots \quad \ldots \quad \ldots \quad (36)$$

to determine the critical value of ϕ.

It may be noted that B_1^*, B_2, B_3, B_4 are actually functions of ϕ in virtue of their dependence on the factor $(\partial \bar{C}_L/\partial \alpha)_{A/m}$ which varies with ϕ. The critical value of ϕ will, however, usually be quite small, and since $(\partial \bar{C}_L/\partial \alpha)_{A/m}$ varies very little for small values of ϕ, it will be sufficiently accurate to use the value corresponding to $\phi = 0$ in calculating B_1^*, etc., for equation (36).

For values of ϕ less than the critical, including negative values (sweepforward) the bending assumptions of section 1.1 are valid, and equation (9) gives the elastic wash-out with an accuracy which, at the limiting speed, depends only on the effect (usually small) of neglecting the interaction between deformation and lift distribution in calculating the torsion effect. At other speeds, the accuracy of the estimated bending contribution is also dependent on this neglected interaction (*see* section 8 (*c*)). As ϕ decreases to zero and thence to negative values, the relative elastic wash-in at the tip increases (*see* section 3 and Fig. 4), and accordingly, the effect of deformation on lift distribution increases. Thus, while the error in equation (9) should be small at the limiting speed, for all angles of sweep less than the critical, it may become large for large angles of sweepforward at low speed.

8. *A Review of the Assumptions and Approximations.*—(*a*) The assumption that the basic lift distribution is uniform is very inaccurate for wings of small aspect ratio and large sweep angle. It will be shown in section 11.1, however, that the bending-stiffness assumptions are such as to compensate for this inaccuracy as regards the bending term, thus considerably reducing the overall error. Although recent advances in lifting-plane theory make possible an accurate determination of the lift distribution in any particular case, it is clearly impossible to derive generalised formulae of the type developed in this report, which would take account of the variation of lift distribution with wing plan-form.

(*b*) The variation of the rotary damping coefficient m_q of the wing due to distortion has been neglected and we now consider to what extent this is justifiable.

For a rigid aircraft, the rotary damping of the wing gives a rearward (stabilizing) shift of manoeuvre point of magnitude $- m_q/\mu$, where $m_q = (1/\rho S c^2 V)(\partial M/\partial \dot{\theta})$ and $\mu = W/g\rho S c$, $\dot{\theta}$ being the angular velocity in pitch. Approximate calculations for constant-chord wings indicate that $- m_q$ is of the order $+ 0 \cdot 2$ for unswept wings and that it increases with both aspect ratio and sweep angle so that for aspect ratio 6 and sweep angle 45 deg, it is of the order $1 \cdot 0$.

In practice, μ is never likely to be much less than 10 so that the effect of the basic m_q on the manoeuvre point of unswept wings is unlikely to exceed 2 per cent of the chord, and the change in m_q due to distortion will cause a shift of manoeuvre point of less than this amount With a wing of aspect ratio 6 and sweepback angle of 45 deg, the basic m_q gives a rearward shift of manoeuvre point of the order $(1/\mu) \times c$. For a small heavily loaded fighter giving a minimum μ (at sea-level) of the order 100, the shift would be of the order $0 \cdot 01c$ while for a very large aircraft for which μ might be as low as 10, the shift would be of the order $0 \cdot 10c$.

Approximate calculations indicate that at high dynamic pressures, distortion will considerably reduce the value of $-m_q$ and may, in extreme cases where the bending stiffness is low (corresponding to a low value of the ultimate load factor) largely eliminate the stabilizing effect of wing damping. Thus the forward shift of manoeuvre point due to the effect of distortion on m_q for the wing plan-form considered may be expected to vary from less than $0 \cdot 01c$ for $\mu = 100$ to about $0 \cdot 07c$–$0 \cdot 08c$ for $\mu = 10$. It should be noted that, in general, this contribution to the total shift is relatively small in comparison with the shift due to the elastic wash-out arising from the basic lift distribution.

It may be concluded from the foregoing that, having regard to the general degree of accuracy which it is expected to achieve with the methods of this report, it is justifiable to neglect the variation of wing m_q due to distortion except for large aircraft with wings of high aspect ratio, large sweep angle and a low ultimate load factor. For such aircraft operating at high speed, it may be anticipated that distortion will, to a considerable extent, eliminate the appreciable stabilizing effect of wing damping which is obtained at low speed. If the effect of elastic wash-out is estimated by a method more exact than the present, then the effect of distortion on m_q should be included.

(*c*) Allowance is made for the effect of deformation on lift distribution in developing the expression for elastic wash-out applicable to wings of large sweepback. In deriving the other expression for elastic wash-out, the contribution due to torsion has been calculated without regard to the effect of deformation on lift distribution, while the formula for the bending contribution is exact for the limiting speed but becomes progressively less accurate at lower speeds, due to the variation with speed of the relative effect of deformation on lift distribution. The net error in the calculated wash-out should be small at the limiting speed, but at low speed might become large for wings with large sweepforward (bending contribution large).

(*d*) In accordance with the procedure usually adopted in stability theory, the tail load has been neglected in comparison with the lift on the wing. On modern aircraft, for which the ratio of tail area to wing area is relatively large, this assumption may lead to errors of just appreciable magnitude (*see* section 3 of Ref. 3).

(*e*) Only wings of constant chord have been considered because of the excessive degree of complication involved in attempting to include the effects of taper in general formulae of the type here developed. With tapered wings, it is necessary to distinguish between the sweep angle of the elastic axis and that of the axis of aerodynamic centres; the difference is especially important for wings of small aspect ratio.

(*f*) It has been assumed that no engine nacelles are installed in the wing. An approximate estimate of the effect of an engine nacelle, given in Part II of the report, shows that the effect is favourable and may be appreciable in magnitude.

(*g*) Of the remaining assumptions, those of constant sweep angle and of constant torsional rigidity are the most important, but detailed discussion of them lies outside the scope of this report.

9. *Suggested Procedure to be Adopted in Numerical Calculations.*—(*a*) If the wing is swept-back, calculate the construction figures B_1^*, B_2, B_3 and B_4 set out in Table 2 and also the quantities:

(i) $\mathscr{U}^* = 0 \cdot 5[(B_1^* + B_2) - \tan \phi (B_3 - B_4)]$

(ii) $\mathscr{V}^* = 0 \cdot 375(B_1^* + B_2) - 0 \cdot 25 \tan \phi (B_3 - B_4)$

(iii) $Q^* B_1^*$ and $\tan \phi \, Q^* B_3$.

From Fig. 12, read off the value of K appropriate to (iii). Calculate the quantity $(\mathscr{U}^* - K\mathscr{V}^*)$.

(*b*) *If* $\mathscr{U}^* - K\mathscr{V}^*$ *is negative :*

(i) Calculate the quantities QB_1, $\tan\phi\, QB_3$, $\mathscr{U}$ and $\mathscr{V}$ appropriate to the dynamic pressure Q under consideration

(ii) Obtain values of $\mathscr{F}$, $\mathscr{G}$, $\mathscr{P}$ and $\mathscr{Q}$ by interpolation in the graphs of Figs. 6 to 9

(iii) Obtain values of $dD/dn = (\mathscr{U} - \mathscr{F}\mathscr{V})/\mathscr{P}$ and $dH/dn = (-\mathscr{V} + \mathscr{G}\mathscr{U})/\mathscr{Q}$ from the charts of Figs. 10 and 11 or by direct calculation

(iv) Calculate the construction figures D_1, D_3, D_4 and D_5 set out in Table 2.

(*c*) *If* $\mathscr{U}^* - K\mathscr{V}^*$ *is positive :*

Calculate the construction figures $D_1, \ldots D_6$ set out in Table 2.

(*d*) Calculate from the formulae set out in Table 1, the values of the separate contributions to $\Delta_W\left(\left.\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\right|_{V,n}\right)$ and hence determine the forward shift of manoeuvre point due to the effect of wing deformability on wing pitching moment from the equation:

$$\Delta_W\left(\frac{x_{m.p.}}{c}\right) = \Delta_W\left(\left.\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\right|_{V,n}\right) = \sum_{r=1}^{5} \Delta_r\left(\left.\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\right|_{V}\right).$$

(*e*) For the estimation of the effects of fuselage and tail unit, the change in angle of attack of the fuselage due to wing deformability is required. This can be calculated from the equations:

$$\Delta\left(\left.\frac{\partial \alpha_{y=0}}{\partial n}\right|_{V}\right) = \begin{cases} -\left(0\cdot 455\,\dfrac{dD}{dn} + 0\cdot 285\,\dfrac{dH}{dn}\right)\left(\dfrac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m}, & \text{if } \mathscr{U}^* - K\mathscr{V}^* < 0 \\ -\,0\cdot 3125 D_6 + 0\cdot 455 D_1\,, & \text{if } \mathscr{U}^* - K\mathscr{V}^* > 0\,. \end{cases}$$

10. *Consideration of Some Limiting Cases for Swept-back Wings.*—10.1. *The Swept-back Wing of Infinite Torsional Rigidity.*—This case is interesting in that we are able to express the quantities $\Delta_3\left(\left.\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\right|_{V}\right)$ and $\Delta\left(\left.\frac{\partial \alpha_{y=0}}{\partial n}\right|_{V}\right)$ as functions of the single parameter $(QB_3 \tan\phi)$ and to present the results in a pair of curves. Further, by considering what happens when the bending stiffness tends to zero, we are able to obtain an indication of the probable accuracy of the method.

For the wing of infinite torsional rigidity $B_1 = B_2 = 0$, equations (20) reduce to:

$$\left.\begin{aligned} \mathscr{U} &= -\,0\cdot 5 \tan\phi (B_3 - B_4) = -\,0\cdot 5 B_3 \tan\phi \left(1 - \frac{W_w}{W}\right) \\ \mathscr{V} &= -\,0\cdot 25 \tan\phi (B_3 - B_4) = -\,0\cdot 25 B_3 \tan\phi \left(1 - \frac{W_w}{W}\right) \end{aligned}\right\} \quad \ldots \quad (37)$$

and equations (22) of Appendix II to

$$\left.\begin{aligned} \mathscr{W} &= 1 + 0\cdot 1891 QB_3 \tan\phi \\ \mathscr{X} &= 0\cdot 5 + 0\cdot 0738 QB_3 \tan\phi \\ \mathscr{Y} &= 1 + 0\cdot 2186 QB_3 \tan\phi \\ \mathscr{Z} &= 0\cdot 25 + 0\cdot 0781 QB_3 \tan\phi \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Writing $\chi = QB_3 \tan \phi$ and substituting for $\mathscr{U}$, $\mathscr{V}$, $\mathscr{W}$, $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ from (37) and (38) in equations (23) and (24) of Appendix II we obtain, after some algebraic simplification:

$$\frac{dD}{dn} = -\frac{\left(1 - \frac{W_w}{W}\right)}{Q} \frac{\chi(0 \cdot 0624\chi + 0 \cdot 500)}{1 + 0 \cdot 2309\chi + 0 \cdot 00546\chi^2}$$

$$\frac{dD}{dn} = \frac{\left(1 - \frac{W_w}{W}\right)}{Q} \frac{0 \cdot 04152\chi^2}{1 + 0 \cdot 2309\chi + 0 \cdot 00546\chi^2}$$

$$\frac{dD}{dn} + \frac{dH}{dn} = -\frac{\left(1 - \frac{W_w}{W}\right)}{Q} \frac{\chi(0 \cdot 02088\chi + 0 \cdot 500)}{1 + 0 \cdot 2309\chi + 0 \cdot 00546\chi^2}$$

$$0 \cdot 455 \frac{dD}{dn} + 0 \cdot 285 \frac{dH}{dn} = -\frac{\left(1 - \frac{W_w}{W}\right)}{Q} \frac{\chi(0 \cdot 01656\chi + 0 \cdot 2275)}{1 + 0 \cdot 2309\chi + 0 \cdot 00546\chi^2}.$$

Hence, from equations (27a) and (18):

$$\frac{\Delta_3 \left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)}{A \tan \phi \left(1 - \frac{W_w}{W}\right)} = \frac{\chi(0 \cdot 0008697\chi + 0 \cdot 020825)}{1 + 0 \cdot 2309\chi + 0 \cdot 00546\chi^2} \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

$$\frac{Q}{\left(1 - \frac{W_w}{W}\right)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m}} \Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) = \frac{\chi(0 \cdot 01656\chi + 0 \cdot 2275)}{1 + 0 \cdot 2309\chi + 0 \cdot 00546\chi^2}. \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

These quantities are plotted against $\chi = QB_3 \tan \phi$ in Figs. 13 (curve (a)) and 14 (curve (a)) respectively.

10.2. *The Swept-back Wing of Zero Bending Stiffness.*—This case has no practical significance but is of interest in providing some check on the accuracy of the results obtained by using the formulae developed for wings of large sweepback.

As the bending stiffness tends to zero, B_3 and hence χ tends to infinity, in which case the expressions on the right-hand side of equations (39) and (40) tend respectively to $0 \cdot 1593$ and $3 \cdot 033$. Thus, for the case of a swept-back wing of infinite torsional rigidity and zero bending stiffness, the formulae give:

$$\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right) = 0 \cdot 1593 A \tan \phi \left(1 - \frac{W_w}{W}\right) \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

$$\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) = \frac{3 \cdot 033\left(1 - \frac{W_w}{W}\right)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m}}{Q}. \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

Provided that QB_1 remains finite, the limiting values of dD/dn and dH/dn as given by equations (19) when B_3 tends to infinity, are respectively

$$\frac{-\,11 \cdot 43\left(1 - \frac{W_w}{W}\right)}{Q} \quad \text{and} \quad \frac{7 \cdot 61\left(1 - \frac{W_w}{W}\right)}{Q}.$$

The corresponding values of $\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)$ and $\Delta\left(\frac{\partial \alpha_{y=0}}{\partial \bar{C}_L}\bigg|_V\right)$ derived from equations (27a) and (18) are then identical with those given by equations (41) and (42) which are thus applicable for swept-back wings of zero bending stiffness with any positive degree of torsional rigidity.

A near approximation to the correct value of $\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)$ in this special case can be arrived at as follows.

For an absolutely rigid wing, the lift distribution would, depending on aspect ratio, lie between a rectangular distribution corresponding to infinite aspect ratio and an elliptic distribution corresponding to small* aspect ratio. The manoeuvre point for wing alone would accordingly be located in the plane of symmetry at a fore-and-aft location corresponding to the quarter-chord point of a chord situated somewhere between $0{\cdot}5s$ and $0{\cdot}424s$ from the plane of symmetry.

In the non-rigid case with zero bending stiffness, only that portion of the lift which balances the wing weight can be supported by the portions of the wing outboard of the root, and the remaining lift must be assumed concentrated on a narrow strip, symmetrically disposed about the centre-line. The bending line of the wing must deflect so as to counteract the torsional deflections and maintain the uniform distribution of lift which is necessary to balance the uniform distribution of weight and ensure that the bending moment is zero at all points of the wing span. If the torsional rigidity is infinite, there are no torsional deflections and the bending line must remain straight, except in the immediate vicinity of the root, where large curvature is necessary to provide an abrupt change in angle of attack between the outer portions of the wing and the narrow central strip.

The manoeuvre point corresponding to the first part of the lift is unaffected by deformability. For the remaining part of the lift, the manoeuvre point may be assumed to coincide with the quarter-chord† point of the centre-line chord so that, in relation to its position for the absolutely rigid wing, there is a forward shift of an amount lying between

$$0{\cdot}5s \tan\phi = 0{\cdot}25Ac \tan\phi$$

and

$$0{\cdot}424s \tan\phi = 0{\cdot}212Ac \tan\phi \,.$$

Combining the two lift distributions we get:

$$0{\cdot}212A \tan\phi \left(1 - \frac{W_w}{W}\right) \leqslant \left[\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)\right]_{\text{true}} \leqslant 0{\cdot}25A \tan\phi \left(1 - \frac{W_w}{W}\right), \qquad \ldots \quad \ldots \quad (43)$$

the actual value of $\left[\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)\right]_{\text{true}}$ depending on aspect ratio. Thus from equation (41) and the inequalities (43) we see that for the limiting case of zero bending stiffness, the formula for the wing of infinite torsional rigidity gives†:

$$0{\cdot}637 \leqslant \frac{\left[\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)\right]}{\left[\Delta_3\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_V\right)\right]_{\text{true}}} \leqslant 0{\cdot}751 \,. \qquad \ldots \quad \ldots \quad \ldots \quad (44)$$

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* The actual value depends on the sweep angle; for example, for a constant-chord wing of sweepback 45 deg it is about 2.

† With swept-back wings, the local aerodynamic centre at the root lies aft of the quarter-chord point, and the forward shifts of manoeuvre point are less than those quoted, so that the accuracy of the formula is somewhat greater than the inequalities (44) would indicate.

To assess the accuracy of the formula when the bending stiffness is large, we consider the slope at $\chi = 0$ of the curve (a) in Fig. 13. This is readily deduced from equation (39) to be 0·020825. With the original bending assumptions of section 1.1 the shift of manoeuvre point due to elastic twist in the case of infinite torsional rigidity ($D_6 = 0$) can be derived from equation (27b) and Table 2 in the form:

$$\frac{\Delta_3\left(\left.\frac{\partial C_M}{\partial \bar{C}_L}\right|_V\right)}{\Delta \tan \phi\left(1 - \frac{W_w}{W}\right)} = 0\cdot020825\{\tan \phi \; QB_3\} \; . \quad .. \quad .. \quad .. \quad .. \qquad (45)$$

This linear function of $\chi = QB_3 \tan \phi$ is plotted as curve (b) of Fig. 13 and it will be observed that curves (a) and (b) have the same slope, *viz.*, 0·020825 at $\chi = 0$. Now in the neighbourhood of $\chi = 0$, where the bending stiffness is very high, elastic deformations will be so small as to have negligible effect on the lift distribution and accordingly the assumptions used in deriving equation (27b) are valid in this region. Curves (a) and (b) therefore both have the correct slope at $\chi = 0$.

It is thus established that the correct curve in Fig. 13 should be tangential to curve (b) at $\chi = 0$, and at $100/\chi = 0$ should have an ordinate lying between 0·25 and 0·212, according to aspect ratio. Furthermore, it is reasonable to assume the general shape of the curve to be that of curve (a). Curves (c) and (d) of Fig. 13 have been drawn to satisfy the above conditions, and are considered to give a good approximation to the true shift of manoeuvre point for wings of infinite and small aspect ratio respectively.

From equation (40), the slope at $\chi = 0$ of the curve (*a*) in Fig. 14 is seen to be 0·2275. With the original bending assumptions we have from equation (10) and Table 2, putting $D_6 = 0$:

$$\frac{Q}{\left(1 - \frac{W_w}{W}\right)\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_{A/m}} \Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\right) = 0\cdot2275 \left\{QB_3 \tan \phi\right\} . \quad .. \quad .. \quad .. \quad .. \qquad (46)$$

This linear function of $\chi = QB_3 \tan \phi$ is plotted as curve (b) of Fig. 14 and it will be observed that curves (a) and (b) have the same (correct) slope, *viz.*, 0·2275, at $\chi = 0$.

If we approach the limiting condition of the wing of zero bending stiffness by considering a narrow central strip of wing of area dS supporting a load $n(W - W_w)$, with the factored wing weight nW_w supported by the remaining area $S\{1 - (dS/S)\}$, it is evident that as dS tends to zero (corresponding to zero bending stiffness) the limiting value of $\Delta \; (\partial \alpha_{y=0}/\partial n)$ derived from consideration of the outer wing area (which tends to S) will be finite, while the limiting value of $\Delta(\partial \alpha_F/\partial n)$ corresponding to the central strip whose area tends to zero, will be infinite. In the limiting condition there would thus be an infinite discontinuity in angle of attack on either side fo the wing centre-line for any non-zero value of the load factor n.

The foregoing does not appear to provide a basis for modifying curve (a) of Fig. 14. It seems reasonable to assume, however, that over the practical range of values of χ (up to $\chi = 25$ say) the order of accuracy of curve (a) of Fig. 14 should be about the same as that of curve (a) in Fig. 13.

11. *Comparative Calculations of the Effect of Elastic Wash-out by the Methods of Ref.* 1 *and of the Present Report.*—Of the various contributions to the shift of manoeuvre point of the wing alone, only that due to elastic wash-out is likely ever to become dangerously large and to necessitate special counteracting measures. It has therefore been considered desirable to obtain an indication of the probable accuracy of the present method of estimating elastic wash-out and its effect on manoeuvre point, by carrying out comparative calculations for a particular example, employing

in turn the methods of Ref. 1 and of the present report. In applying the latter method, the theoretical value $(\partial \bar{C}_L/\partial\alpha)_{A/3}$ for unswept wings was first used for the effective local lift slope corresponding to a wash-out. After comparison with the results obtained by the method of Ref. 1, this was modified to $(\partial \bar{C}_L/\partial\alpha)_{A/4}$ for the particular wing under consideration and to $(\partial \bar{C}_L/\partial\alpha)_{A/m}$, $(m \doteq 3 + \tan\phi)$ for the general case. (For details, *see* section 11.1 and Appendix I.)

The data assumed and a summary of the calculations are set out in Appendix II. It was thought desirable to consider a wing of fairly high aspect ratio and large sweepback angle in order that the effect in question should be large; the combination of $A = 6$; $\phi = 45$ deg, was selected because for this plan-form, the basic lift distribution, calculated by a lifting-plane method, was available in Ref. 9, while the incremental lift distribution due to wash-out could be readily estimated by the method of Ref. 10.

Bending and torsional stiffnesses were assumed in accordance with equations (14) and (1a): putting $F_b = F_\tau = 1$. It was realised that in the case of a particular aeroplane, the stiffness distributions would almost certainly differ from those assumed, so that for the application of the method of this report, it would be necessary to estimate approximately the values of F_b and F_τ which would make the distributions given by equations (14) and (1a) equivalent to the actual distributions, which could themselves be used when applying the method of Ref. 1. There seemed no point, however, in introducing artificial discrepancies between the distributions assumed for the two methods in this example, which thus serves purely to indicate the extent to which the inaccuracies of the *aerodynamic* assumptions affect the estimations of the shift of manoeuvre point due to elastic wash-out by the method of this report.

The principal results for the maximum diving speed of the aeroplane ($q = 1070$ lb/ft², $Q = 21\cdot4$) are summarized in the following table and in Figs. 15 to 20.

TABLE 3

	Method of Ref. 1 (Lyon)	Present method using $\left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/3}$	Present method using $\left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/4}$
$\dfrac{\left(\frac{\partial \bar{C}_L}{\partial\alpha_F}\right)}{\left(\frac{\partial \bar{C}_L}{\partial\alpha_F}\right)_r}$	0·743	0·700	0·723
Forward shift of manoeuvre point due to elastic wash-out $\Delta\frac{x_{m.p.}}{c}$	0·133	0·172	0·155
Increase in fuselage angle of attack $\Delta\left(\frac{\partial\alpha_{y=0}}{\partial n}\Big\|_V\right)$	0·0048	0·00597	0·00533
$\Delta\alpha_{y=0}$ for $n = 8$	2·20 deg	2·74 deg	2·44 deg

11.1. *Discussion of Results.*—The method of the present report as originally applied appears to overestimate the shift of manoeuvre point by approximately 30 per cent and the increase in fuselage angle of attack by nearly 25 per cent. This can be accounted for by two facts:

(*a*) Due to the assumption of uniform basic lift distribution right out to the tip, the elastic wash-out is overestimated by about 12 per cent over the inboard 70 per cent of the semi-span and by an increasing amount over the remaining part of the span (*see* Fig. 19).

[It should be observed, however, that if the bending-stiffness distribution of the wing considered were to correspond, not to a uniform basic lift distribution, but to the actual lift distribution (as it would, presumably, in practice) then the elastic wash-out as calculated by the method of Ref. 1 would be greater than that indicated in Fig. 19. It should, in fact, be more nearly equal to that calculated by the present method, which would remain unaltered since the bending-stiffness distribution to be used in this case is implicit in the method and is, in fact, that originally assumed.

This result is to be expected since the *combination* of lift and bending-stiffness distributions used in the present method is such as to give a uniform spanwise stress distribution at low speed (the stress being equal to the proof stress at proof load factor). The bending deflection curve at low speed is accordingly independent of the *individual* lift and stiffness distributions and should be identical with that derived using Ref. 1. At high speed the result is affected by the interaction of the deformations and lift distribution and thus depends on the assumed lift distributions for ' wash-out without lift ' which are different in the two cases.

Calculations show that in these circumstances, the present method underestimates the wash-out at maximum speed by amounts which vary from less than 6 per cent over the inboard 70 per cent of the span to a maximum of 12 per cent at the tip.]

(*b*) The approximate strip-method which is used to estimate the aerodynamic effects of elastic wash-out, and which in the first instance, assumes a mean effective local lift slope equal to $(\partial \bar{C}_L/\partial \alpha)_{A/3}$, leads to a gross overestimate of the lift increment due to wash-out over the outer 20 per cent of the semi-span, where the lift increment should actually decrease rapidly to zero at the tip itself (*see* Fig. 20). In order to assess the magnitude of this effect, an estimate has been made of the mean effective lift slope which, used in conjunction with the mode of wash-out calculated by the method of Ref. 1, gives the correct value for the shift of manoeuvre point as calculated by that method. This was found to be 1·97 as compared with the value of 2·30 of $(\partial \bar{C}_L/\partial \alpha)_{A/3}$.

The calculations using the method of this report have been repeated employing this reduced value in place of $(\partial \bar{C}_L/\partial \alpha)_{A/3}$. The resulting change in the incremental lift distribution due to distortion is shown in Fig. 20 and the revised values of $[(\partial \bar{C}_L/\partial \alpha_F)/(\partial \bar{C}_L/\partial \alpha_F)_r]$, $\Delta(x_{m.p.}/c)$ and $\Delta \alpha_{y=0}$ are given in Table 3. It will be noted that the initial differences from the values obtained by the other method have been approximately halved; the residual differences are due to (*a*) above.

The reduced value of 1·97 for the mean effective life slope is found, by interpolation in the results of Ref. 9, to correspond to an aspect ratio of approximately $A/4$. This result is used in Appendix I as the basis for a proposal that, in general, the mean effective lift slope corresponding to a wash-out should be taken to be $(\partial \bar{C}_L/\partial \alpha)_{A/m}$, with m given approximately by $m = 3 + \tan \phi$. This should reduce very considerably the errors arising from the aerodynamic approximations involved in the calculation of the effect of elastic wash-out on manoeuvre point by the present method. Although the method, as applied to wings of constant chord, may remain somewhat less accurate than that of Ref. 1, it has the important compensating advantage (apparent from the summary of calculations set out in Appendix III) of affording a great saving in time, by eliminating most of the laborious computational work involved in the latter method. If the present method is applied to wings with taper, or to wings having stiffness distributions other then those assumed, there will of course, be some further loss of accuracy.

The present method is, therefore, particularly to be recommended for investigations in which great accuracy is not the first essential, but which require a rapid assessment of the effect of varying selected design parameters.

TABLE 1

The Contributions to $\Delta_W \left(\frac{\partial C_{MW}}{\partial \bar{C}_L} \Big|_{V,n} \right)$

Increment	Due to :	Given by equation number	Formula (a) Criterion $\mathscr{U}^* - K\mathscr{V}^* < 0$	Formula (b) Criterion $\mathscr{U}^* - K\mathscr{V}^* > 0$
$\Delta_2 \left(\frac{\partial C_{MW}}{\partial \bar{C}_L} \Big|_V \right)$	Elastic camber arising from rib bending and spar bending	(26)	$D_4 Q$	$D_4 Q$
$\Delta_3 \left(\frac{\partial C_{MW}}{\partial \bar{C}_L} \Big|_V \right)$	Elastic wash-out due to wing torsion and bending	(*a*) (27(a)) (*b*) (27(b))	$-0{\cdot}04165 A \tan \phi \left\{ \frac{dD}{dn} + \frac{dH}{dn} \right\} Q$	$D_2 Q(-0{\cdot}02083 D_6 + 0{\cdot}04165 D_1)$
$\Delta_{1+4} \left(\frac{\partial C_{MW}}{\partial \bar{C}_L} \Big|_V \right)$	Direct effect of bending on the pitching moment of the inertia and aerodynamic forces acting on the wing	(*b*) (28)	*see* section 4.2.1	$D_1 D_5 Q$
$\Delta_5 \left(\frac{\partial C_{MW}}{\partial \bar{C}_L} \Big|_V \right)$	Fuselage interference in connection with wash-out	(*a*) (29) and (18) (*b*) (29) and (10)	$D_3 Q \left(0{\cdot}455 \frac{dD}{dn} + 0{\cdot}285 \frac{dH}{dn} \right) \left(\frac{\partial \alpha}{\partial \bar{C}_L} \right)_{A/m}$	$D_3 Q(0{\cdot}3125 D_6 - 0{\cdot}455 D_1)$

Note : Expressions for the construction figures $D_1, \ldots D_6$ are given in Table 2. The derivation of the quantities dD/dn, dH/dn is given in Appendix II ; the construction figures involved, $B_1, \ldots B_4$, are given in Table 2.

TABLE 2

The Construction Figures $D_1, \ldots D_6$ and $B_1, \ldots B_4$ Required in the Calculation of $\Delta_W \left(\left. \frac{\partial C_M}{\partial \bar{C}_L} \right|_{V,n} \right)$

Fig.	Formula	Equation in text
D_1	$\frac{A \tan \phi}{150 n_u \tau F_b}$	(9)
D_2	$A \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m} \tan \phi$	(27b)
D_3	$2 \cdot C_F \frac{b_{F\,\max}{}^2}{S} \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m} \tan \phi$	(29)
D_4	$\frac{1}{480 \beta n_u \tau F_{r.b.}}$ (wings with ribs in flight direction) $\frac{1}{480 \beta n_u \tau} \left[\frac{\cos^2 \phi}{F_{r.b.}} + \frac{2 \sin^2 \phi}{F_b} \right]$ (wings with ribs perpendicular to elastic axis)	(26)
D_5	$\frac{A}{6 \sin 2\phi} \left[1 - \frac{(EI)_h}{(EI)_v} \cos^2 \phi \right] \left[C_{D\,P} - \frac{W_w}{W} C_{D\,0} - 3 \left(1 - \frac{W_w}{W} \right) \frac{n^2}{Q^2} \left(\frac{\partial \alpha}{\partial C_L} \right)_\infty \right]$	(28)
D_6	$\frac{1 \cdot 075}{Q^* F_\tau} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c} \right)^2} \left(\frac{e_F + \Delta e_F}{c} + \frac{r}{c} \frac{W_w}{W} \right)$ where $\frac{\Delta e_F}{c} = D_4 Q$	(9) (26a)
B_1	$\frac{1 \cdot 075}{Q^* F_\tau} \frac{(e_F + \Delta e_F)}{c} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c} \right)^2} \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m}$	(15) Appendix II
B_2	$\frac{1 \cdot 075}{Q^* F_\tau} \frac{r}{c} \frac{W_w}{W} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c} \right)^2} \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m}$	(16) Appendix II
B_3	$\frac{A \left(\frac{\partial \bar{C}_L}{\partial \alpha} \right)_{A/m}}{75 n_u \tau F_b \left(1 - \frac{W_w}{W} \right)}$	(17) Appendix II
B_4	$\frac{W_w}{W} B_3$	

Note: (1) B_1^* is the value of B_1 when $\Delta e_F / c = D_4 Q^*$.
(2) $m \simeq 3 + \tan \phi$.

PART II

The Effect of Deformability of the Fuselage and of an Unswept Tailplane

12. *A General Survey of the Effects to be Considered.*—To investigate the effect on manoeuvre point of deformability in any part of the aircraft structure, we have to compare the pitching moment of the actual (deformable) aircraft with that of the corresponding completely rigid aircraft, at the same speed and load factor, which implies the same wing lift coefficient*, and hence the same angle of attack of the wing mean no-lift line. The most important effect of fuselage and tailplane deformability will be to change the angle of attack (and hence the pitching moment about the aircraft c.g.) of the tailplane. This angle of attack will also be influenced by deformability of the wing and of the wing-fuselage attachment. Since the geometrical relationships between the angles of attack of the tailplane, fuselage and wing of a deformable aircraft are somewhat complicated, we shall, at the outset, need to define them precisely.

12.1. *The Geometry of a Deformable Fuselage and Tailplane in Relation to a Deformable Wing.*—First we consider a side elevation of the hypothetical, completely rigid aircraft (*see* Fig. 21a). It will clarify the argument, without fundamentally affecting it, if we make certain simplifying assumptions. We will assume the undeformed wing to be of constant chord and constant symmetrical section, and to have no wash-out. Then the camber-line of the undeformed root section will coincide with its chord line RR′ which will also be the no-lift line of the section and will be parallel to the wing mean no-lift direction.

Let C be the three-quarter chord point of the section; then since the zero-lift direction of a section coincides approximately with the tangent to the camber-line at the three-quarter chord point, C may be taken as a wing-root reference point and CR, the no-lift direction, as a wing-root datum line. We will now suppose the wing elastic axis to intersect RR′ in a point E, and assume for bending considerations that the fuselage is encastré at a section through E, which may be adopted as a convenient fuselage reference point. Then if F is the point of the fuselage coinciding with C, EF (regarded as the tangent to the camber-line at E) may be taken as a fuselage datum line, coincident with the wing root datum line. The inclination of EF ≡ CR to the wind direction will give both the mean angle of attack $\bar{\alpha}_r$ of the rigid wing, and the angle of attack α_{F_r} of the rigid fuselage, corresponding to flight at a given speed V and load factor n. The root section of the fuselage may be assumed to lie in the plane through E to which EF is normal, and if this plane is represented in side elevation by N′EN, then (EF, EN) represents a pair of rectangular axes to which deflections of the fuselage and tailplane may conveniently be referred. In particular, the angular setting η_{TL} of an arbitrary chordwise section of the tailplane will be measured relative to EF.

Fig. 21b presents the corresponding picture for the actual deformable aircraft at the same speed V and load factor n. The wing mean no-lift line must still lie at an angle of attack $\bar{\alpha}_r$ to the wind direction, but owing to the effect of elastic camber the wing-root section will have changed its shape—the camber-line having become curved—while due to elastic wash-out, the wing-root datum (no-lift direction) as given by the tangent to the new camber-line at C, will have changed its direction relative to the mean no-lift line by an angle $\Delta_1\alpha_F$. If the wing-fuselage attachment is completely rigid, the fuselage datum will lie along EF_1, the tangent to the camber-line at E, and will be inclined at an angle $\Delta_2\alpha_F$ to the wing-root datum; the fuselage-root section will be represented by EN_1 normal to EF_1. In the more general case of a deformable attachment (*see* section 14.3) the frame of reference (EF_1, EN_1) will have moved through a further angle $\Delta_3\alpha_F$ into the position (EF, EN). The angle of attack of the fuselage datum to the free stream will now be given by:

$$\alpha_F = \alpha_{F_r} + \Delta_1\alpha_F + \Delta_2\alpha_F + \Delta_3\alpha_F .$$

Deformability of the fuselage, the tailplane, and the tailplane-fuselage attachment will have changed the setting (relative to the fuselage datum EF) of the arbitrary chordwise section of the tailplane by an amount $\Delta\eta_{TL}$.

* Here we are ignoring the tail lift in comparison with the wing lift.

With the fuselage datum and its relation to the wing-root datum and wing no-lift direction thus clearly defined, we may proceed to discuss in general qualitative terms the effects on manoeuvre point of wing, fuselage and tailplane deformability (including fuselage-wing and tailplane-fuselage attachment deformabilities), before developing in detail a method of determining quantitatively the effects in question.

12.2. *Qualitative Survey*.—To focus ideas clearly, we may consider an aircraft with wings having a sufficient degree of sweepback to ensure that the effect of bending deformability on local angle of attack exceeds that of torsional deformability. Then, as may be deduced from Part I, there will be a forward shift of the manoeuvre point of the wing alone, due to wing deformability. At the same time, for a given speed and positive increment of load factor, the wing-root section will acquire an additional negative camber, and an increased angle of attack, as compared with the corresponding rigid wing. Then, as explained in section 12.1 and Fig. 21, if we assume for the moment that the wing-fuselage attachment is completely rigid, the angle of attack of the fuselage datum will be increased by an amount $(\Delta_1\alpha_F + \Delta_2\alpha_F)$, and consequently the (generally) destabilizing pitching moment of the forces acting on the fuselage will be increased, thus giving a forward shift of manoeuvre point.

If the fuselage, tailplane, and tailplane-fuselage attachment were completely rigid, the setting of the tailplane relative to the wing mean no-lift direction would also increase by the amount $(\Delta_1\alpha_F + \Delta_2\alpha_F)$, and its angle of attack would increase by some fraction of this amount, depending on the additional downwash at the tail due to the 'wash-out without lift' distribution of wing angle of attack. Accordingly, the stabilizing effect of the tailplane would be increased, and the manoeuvre point shifted rearward, thus offsetting, at least partially, the destabilizing effect of the wing alone.

In the practical case, where the fuselage and tailplane (unswept) are not completely rigid, the bending of the fuselage, and the torsion of the tailplane, will give rise respectively to negative and positive* increments of mean tailplane setting, with corresponding negative and positive increments of stability.

For a high-speed aircraft with highly swept wings of moderately large aspect ratio ($A \simeq 6$), the net increase in stabilizing tailplane effect, due to wing, fuselage and tailplane deformability will probably not be sufficient to offset the combined destabilizing effects of wing and fuselage. Thus in an illustrative example worked out in section 17.2 for an aircraft of the plan-form illustrated in Fig. 23c, the various shifts of manoeuvre point at the maximum permissible dynamic pressure were as given in the following table:

Shift due to	Forward shift	Rearward shift
Effect of wing deformability on manoeuvre point of wing alone ..	$0 \cdot 155c$	—
Effect of wing deformability on fuselage angle of attack	$0 \cdot 050c$	—
Effect of wing, fuselage, and tailplane deformability on tailplane contribution	—	$0 \cdot 095c$

The net forward shift of manoeuvre point was thus $0 \cdot 11c$.

12.2.1. *The effect of elastic attachments of the wing and the tailplane to the fuselage*.—Any inherent deformability in the attachments of the wing and the tailplane to the fuselage may have important effects on the manoeuvre point. At the same time, by appropriate design of the attachments in question, deformability may deliberately be introduced as a means of adjusting the stability of the aircraft.

* Assuming the aerodynamic axis of the tailplane to lie ahead of its elastic axis.

In analysing the effect of an elastic attachment, it is convenient (*see* 14.3) to substitute for the actual deformability, a ball-joint, located at a point of the wing (or tailplane) root region, which may be regarded as the elastic axis of the attachment, together with a torsional spring restraining the relative angular deflection of the wing (or tailplane) and the fuselage. The direction of the relative deflection will depend on whether the axis of the attachment is located ahead of, or behind the point of application of the resultant aerodynamic and inertia force acting on the surface due to change of load factor.

In the case of an elastic wing attachment, the deformability will give rise to a third increment $\Delta_3\alpha_F$ in the fuselage angle of attack. If the increment $\Delta_3\alpha_F$ corresponding to a positive increment of load factor is positive, both the stabilizing fuselage effect and the stabilizing tailplane effect will be increased, so that the resultant effect may be either stabilizing or destabilizing. Opposite effects will, of course, result from arranging the attachment to give a negative $\Delta_3\alpha_F$.

In section 14.3, rough limits for the quantity $\Delta_3\alpha_F$ are determined from considerations of flutter and of the additional weight of the elastic attachment. For the example of section 17.2, the maximum effects on manoeuvre point were estimated to be given in the following table:

Sign of $\Delta_3\alpha_F$ for positive increment of load factor	Maximum increment in rearward (stabilizing) shift of manoeuvre point due to:		
	The fuselage	The tailplane	Fuselage and tailplane
Positive	$-0{\cdot}165c$	$0{\cdot}256c$	$0{\cdot}091c$
Negative	$0{\cdot}165c$	$-0{\cdot}256c$	$-0{\cdot}091c$

It will be observed that the net forward shift of manoeuvre point ($0{\cdot}11c$) due to deformability of the wing, fuselage and tailplane at the maximum permissible dynamic pressure, could be practically eliminated by a suitable elastic attachment of the wing.

The effect of an elastic tailplane attachment will be considered in detail only for the special case where the axis of the attachment is located at the elevator neutral point, so that the effect of the attachment deformability is to increase the angle of attack, and hence the stabilizing moment, of the tailplane, when the aircraft is subjected to a positive increment of load factor. Limits are set to what may be achieved in this way, by considerations of tail unit flutter, and of elevator effectiveness. Only the latter aspect has been considered in the present work (*see* section 13.3), and on the assumption that the minimum permissible elevator effectiveness factor at the maximum dynamic pressure is $0{\cdot}2$, the maximum stabilizing effect of elastically mounting the tailplane of the aircraft considered in the example of section 17.2 was estimated as $0{\cdot}40c$ if the wing attachment was rigid, or $0{\cdot}68c$ if it had the maximum permissible deformability. In either case, the overall effect of aircraft structural deformability at the maximum dynamic pressure, would be a large rearward (stabilizing) shift of manoeuvre point.

It is hoped that this survey of the effects to be considered, will help the reader to perceive the ends to which the inevitably tedious algebra of the succeeding pages is directed. The results of the analysis provide the basis of the routine procedure, described in section 17.1, and illustrated by a worked example in section 17.2, which may be adopted in numerical calculations.

13. *The Deformability of the Fuselage and Tailplane Structures*.—The three main effects arising in a consideration of fuselage and tailplane deformability, as affecting manoeuvre point, are:

(*a*) bending of the fuselage aft of the wing

(*b*) torsion of the tailplane*

(*c*) the angular deflection of the tailplane arising from the deformability of its attachment to the fuselage.

* With swept tailplanes, bending of the tailplane would also have to be considered.

These three deformations together produce a change $\Delta\eta_{TL}$ in the setting η_{TL} of each chordwise section of the tailplane, relative to the fuselage datum defined in section 12, and illustrated in Fig. 21. In general, $\Delta\eta_{TL}$ will vary across the span, but aerodynamically, the effect may be assumed equivalent to a rigid-body rotation of the tailplane through an angle $\Delta\eta_T$ which may be assumed, with sufficient accuracy for our purpose, to be equal to the mean value of $\Delta\eta_{TL}$ defined by:

$$\Delta\eta_T = \frac{1}{s_T}\int_0^{s_T} \Delta\eta_{TL}\, dy \,,$$

where s_T is the semi-span of the tailplane which will be assumed (*see* section 13.2) to be of constant chord.

$\Delta\eta_T$ depends on the flight conditions, as specified by the non-dimensional dynamic pressure number $Q = \left(\frac{\text{dynamic pressure } (q)}{\text{wing loading}}\right)$ and the load factor n. We may write (*see* Ref. 12):

$$\Delta\eta_T = -\, w_\alpha\bar{\alpha}_T Q - w_\eta\bar{\eta} Q + w_m n \,, \qquad \qquad (47)$$

where the first two terms are due to the aerodynamic forces arising respectively from a mean tailplane angle of attack $\bar{\alpha}_T$ and mean elevator deflection $\bar{\eta}$, while the last term is due to the inertia forces acting downward at the tail-end of the aircraft. The coefficients w_α, w_η and w_m may be referred to as ' deformability coefficients '; w_α and w_η are each due partly to the deformability of the fuselage, and partly to that of the tailplane and its attachment, while w_m is due almost entirely to deformability of the fuselage and of the fuselage-tailplane attachment, since the elastic and inertia axes of the tailplane will be nearly coincident.

13.1. *The Bending Deformability of the Fuselage.*—The contributions to the deformability coefficients arising from fuselage deformability are due to vertical bending of the rear fuselage. It should be noted that the bending deflections may be accompanied by a bodily rotation of the rear fuselage in the vertical plane, due to shear in the neighbourhood of the wing attachment (*see* Fig. 22). This rotation which, as observed in Ref. 2, section 3.30, may increase the change in tailplane angle by as much as 50 per cent, does not, however, affect the deformability coefficients as defined above. It may be regarded as a contribution to the deformability of the wing-fuselage attachment which is discussed in section 14.3.

To derive an expression for the vertical bending stiffness of the fuselage, we shall first assume the structure to be just strong enough to withstand the ultimate design loads arrived at from strength considerations. It should, however, be borne in mind that a minimum value for fuselage stiffness is set by the stiffness criterion of Ref. 13 (a), paragraph 7*. In many cases, the criterion will be automatically satisfied if the fuselage has adequate strength, but in other cases, particularly where the aircraft has a very slender fuselage, combined with a large maximum dynamic pressure number Q^*, and a high Mach number, it may be necessary to provide a reserve of strength in order to satisfy the stiffness criterion (*see* section 13.1.1).

For an approximate numerical calculation, we consider the fuselage to be replaced by a beam of constant depth h, encased at the section through the elastic centre of the wing-root section, normal to the fuselage datum (*see* section 12). We suppose the beam to be loaded with a force P_η acting at a point to which we shall refer as the ' elevator neutral point '†. P_η is assumed to be sufficiently small to avoid straining the beam beyond the elastic limit. We suppose the

* *Footnote* (1956): Certain Ministry of Supply Design Requirements were 'built in' to the method of this report. These requirements were those currently appearing in A.P. 970 when the report was written (1949–50). In the meantime, various amendments have been introduced into A.P. 970, but it has not been thought worthwhile to revise the text to take account of this. Accordingly the details given against item 13 in the list of references are those appropriate to the year 1950. However, a footnote has been added to the list to indicate where the corresponding, but not necessarily identical, requirements are now set out in A.P. 970.

† *i.e.*, the centre of pressure of the additional aerodynamic loading on the tailplane resulting from a change of elevator angle at constant tailplane angle of attack.

elevator neutral point to be at a distance l_η from the aircraft centre of gravity, and the latter point to be a distance l_G aft of the section at which the beam is assumed to be encased. Write

$$l_\eta' = l_\eta + l_G .$$

Let $\pm P_{Tp}$ denote the design tail loads corresponding to proof load factor (equal down- and up-loads being assumed for simplicity). Now suppose the flanges of the beam were designed so that when the load P_η acting at the elevator neutral point was equal to $\pm P_{Tp}$, the stresses in the compression flange were everywhere equal to the proof stress. Then if B_F denoted the bending moment at any section, we should have:

$$\left.\begin{aligned} p_c &= \frac{B_F h}{2I_F} \\ \text{and} \qquad \frac{B_F}{I_F} &= E\frac{d^2z}{dx^2} \end{aligned}\right\} , \qquad \ldots \qquad (48)$$

where p_c is the proof compressive stress for the material of the flanges ($\leqslant p_t$, the proof tensile stress),

I_F is the moment of inertia of a cross-section of the beam,

d^2z/dx^2 is the curvature of the beam,

and E is Young's Modulus.

It would follow that the change in tailplane setting due to a downward tail load P_{Tp} was:

$$(\Delta\eta_T)_F = \left(\frac{dz}{dx}\right)_{x=l_\eta'} = \frac{B_F}{I_F}\frac{1}{E}l_\eta' = \frac{2p_c}{E}\frac{l_\eta'}{h} ,$$

and for a tail load P_η, measured positively downwards:

$$(\Delta\eta_T)_F = \frac{p_c}{E}\frac{2l_\eta'}{h}\frac{P_\eta}{P_{Tp}} ;$$

or since

$$\frac{p_c}{P_{Tp}} = \frac{f_c}{P_{Tu}} ,$$

where f_c is the ultimate compressive stress and P_{Tu} the ultimate tail load,

$$(\Delta\eta_T)_F = \frac{f_c}{E}\frac{2l_\eta'}{h}\frac{P_\eta}{P_{Tu}} .$$

If now, we introduce a fuselage rigidity factor $F_F(\geqslant 1)$ to cover cases where there is a reserve of bending strength, we have:

$$\left(\frac{\partial \eta_T}{\partial P_\eta}\right)_F = \frac{1}{F_F}\frac{f_c}{E}\frac{2l_\eta'}{h}\frac{1}{|P_{Tu}|}$$

or

$$\left.\begin{aligned} \left(\frac{\partial \eta_T}{\partial P_\eta}\right)_F &= K_F\frac{2l_\eta'}{h}\frac{1}{|P_{Tu}|} \\ \text{where} \qquad K_F &= \frac{1}{F_F}\frac{f_c}{E} \end{aligned}\right\} \qquad \ldots \qquad (49)$$

If the depth h, of the fuselage structure taking bending, is not constant along its length, we may replace h in the above formula by a mean value $\bar{h}$ defined by:

$$\frac{1}{\bar{h}} = \int_0^1 \frac{1}{h(x)}\, d\left(\frac{x}{l_\eta'}\right) ,$$

where x is the distance along the fuselage datum measured from the wing attachment.

The distribution of bending rigidity (EI_F) is given by equation (48), with the rigidity factor F_F incorporated, as:

$$EI_F = F_F \frac{E}{p_c} \frac{B_F h}{2} = F_F \frac{E}{p_c} \frac{P_{Tp}(l_\eta' - x)h}{2}$$

$$= F_F \frac{E}{f_c} \frac{P_{Tu}(l_\eta' - x)h}{2} . \quad \ldots \quad (50)$$

We now consider the beam of depth h, with stiffness distribution corresponding to (50), under a load $P_\alpha \leqslant P_{Tp}$ (measured positively downwards) acting at the tailplane neutral point, whose distance from the aircraft centre of gravity is denoted by l_α. Writing

$$l_\alpha' = l_\alpha + l_G ,$$

we readily derive the curvature of the beam in the form:

$$\frac{d^2z}{dx^2} = \frac{P_\alpha(l_\alpha' - x)}{EI_F} = \frac{P_\alpha}{P_{Tu}} \frac{l_\alpha' - x}{l_\eta' - x} \frac{2}{h} \frac{f_c}{E} \frac{1}{F_F},$$

and integrating with respect to x between 0 and l_α', and then differentiating with respect to P_α, we obtain the result:

$$\left(\frac{\partial \eta_T}{\partial P_\alpha}\right)_F = \frac{\partial}{\partial P_\alpha} \left(\frac{dz}{dx}\right)_{x=l_\alpha'} = \frac{1}{F_F} \frac{1}{|P_{Tu}|} \frac{f_c}{E} \frac{2l_\alpha'}{h} \int_0^1 \frac{1-\zeta}{a-\zeta} d\zeta ,$$

where $\zeta = x/l_\alpha'$ and $a = l_\eta'/l_\alpha'$. Evaluation of the integral leads to the equation:

$$\left.\begin{aligned} \left(\frac{\partial \eta_T}{\partial P_\alpha}\right)_F &= K_F \frac{2K_\alpha l_\alpha'}{h} \frac{1}{|P_{Tu}|} \\ \text{where} \quad K_\alpha &= 1 - \frac{\delta l}{l_\alpha'} \log_e \left(\frac{l_\eta'}{\delta l}\right) \\ \text{and} \quad \delta l &= l_\eta' - l_\alpha' \end{aligned}\right\} . \quad \ldots \quad (51)$$

The centre of gravity of the rear end of the fuselage, including the complete tail unit, will usually lie somewhere between the points of application of the forces P_α and P_η. Hence if P_m denotes the inertia force acting at the tail, $(\partial\eta_T/\partial P_m)_F$ will lie between $(\partial\eta_T/\partial P_\alpha)_F$ and $(\partial\eta_T/\partial P_\eta)_F$, and as a reasonable approximation, we may assume:

$$\left(\frac{\partial \eta_T}{\partial P_m}\right)_F = K_F \frac{2l_\alpha'}{h} \frac{1}{P_{Tu}} . \quad \ldots \quad (52)$$

13.1.1. *The fuselage rigidity factor* F_F.—The factor F_F must clearly not be less than unity; in addition it must be sufficiently large to ensure that the stiffness criterion of Ref. 13(*a*), paragraph 7 is satisfied. With the parabolic bending mode assumed in section 13.1, the fuselage vertical stiffness F_{f1} of Ref. 13(*a*) is given approximately by:

$$F_{f1} = \frac{\partial(P_\eta l_\eta')}{\partial(\frac{1}{2}\eta_T)} = F_F \frac{E}{f_c} |P_{Tu}| h$$

using equations (49), and the minimum value which will satisfy the criterion is given approximately by the equation:

$$\frac{S}{S_T} \frac{\beta_d}{Q_d} \left(\frac{F_{f1}}{W l_\eta'}\right) = 12{\cdot}1$$

where Q_d is the dynamic pressure number and $1/\beta_d = 1/\sqrt{\{1 - (\text{Mach number})^2\}}$ is the corresponding value of the Prandtl-Glauert factor appropriate to the design diving speed at a height of 10,000 feet,

S, S_T are respectively wing and tailplane areas,

and W is the all-up weight of the aircraft.

It follows that F_F must not be less than:

$$F_{F0} = 12 \cdot 1 \frac{f_c}{E} \frac{S_T}{S} \times \frac{l_\eta'}{h} \frac{W}{|P_{Tu}|} \frac{Q_d}{\beta_d}, \qquad (53)$$

and hence we must have:

$$F_F \geqslant 1 \cdot 0 \text{ and } F_F \geqslant F_{F0}. \qquad (54)$$

13.2. *The Deformability of the Tailplane.*—Deformation of the tailplane comprises bending and twist. Bending of the tailplane affects its lift, and in consequence the aircraft pitching moment, only if the tailplane is swept. It is proposed to consider in this part of the report, only unswept tailplanes, and accordingly, we shall be concerned here, only with torsional deformability. In the interests of simplicity, we shall assume* that bending and torsion are taken by different structural parts, and in particular that torsional stiffness derives entirely from a torsion box in the rear part of the tailplane formed by the skin and a vertical web. We further assume that the tailplane is cylindrical (so that tailplane chord, elevator chord and aerofoil section are all constant across the span), and that it has only one spar, lying at a distance e_T behind the aerodynamic axis. The elastic axis is assumed to coincide with the spar.

We shall base our torsional rigidity assumptions on the stiffness criterion of Ref. 13(*a*), paragraph 2. In addition to satisfying this criterion, the distribution of skin thickness must be such as to ensure that when the tailplane is subjected to the ultimate design torque T_{Tu}, the skin shear stresses nowhere exceed the permissible design value. At the same time, considerations of the minimum gauge which is practicable from the manufacturing point of view, will set a lower limit to skin thickness. However a general numerical investigation, supported by current statistical evidence suggests that the stiffness criterion will be the critical factor in the case of most aircraft for which the manoeuvre point is seriously affected by structural deformability (that is to say in the case of very fast aircraft of all sizes, and in the case of very large aircraft of moderately high speed)†. Accordingly, we now consider what torsional rigidity is required to satisfy the above-mentioned criterion. We shall assume the skin thickness and hence the section torsional rigidity to be constant across the span, since, with such a distribution, the criterion can be met with a minimum weight of torsion box.

The minimum torsional stiffness T_τ measured at $0 \cdot 8 s_T$, which is required to satisfy the criterion is readily shown to be given non-dimensionally by:

$$\frac{(T_\tau)_{\min}}{W c_T} = \mathscr{C}_1 \left(\frac{S_T}{S}\right) \frac{Q_d}{\beta_d}$$

where

$$\mathscr{C}_1 = \begin{cases} 0 \cdot 545 \text{ for tailplanes without end-fins} \\ 0 \cdot 852 \text{ for tailplanes with end-fins} \end{cases}$$

and the minimum value of the section torsional rigidity GJ_T which is related to T_τ by the equation:

$$\frac{1}{T_\tau} = \int_0^{0 \cdot 8 s_T} \frac{1}{GJ_T} \, dy_T = \frac{0 \cdot 8 s_T}{GJ_T}$$

* The assumption is not strictly valid in the case of tailplanes of shell design, or in the case of tailplanes with two spars, for which some torsional stiffness is supplied by differential bending.

† In particular this is likely to apply to all tailed designs with highly swept wings (for which the overall effects of deformability are most serious) since the design speeds of such aircraft may be assumed to be very high.

is then given by:

$$\left.\begin{aligned} \frac{(GJ_T)_{\min}}{WS_T} &= \mathscr{C}_2 \left(\frac{S_T}{S}\right)\frac{Q_d}{\beta_d} \\ \text{where} \qquad \mathscr{C}_2 &= \begin{cases} 0\cdot 218 \text{ for tailplanes without end-fins} \\ 0\cdot 341 \text{ for tailplanes with end-fins} \end{cases} \end{aligned}\right\} \quad \ldots \quad (55)$$

In the above equations,

c_T, s_T, S_T denote respectively the chord, semi-span and area of the tailplane,

W is the aircraft weight in the flight condition under consideration,

S is the wing area,

Q_d is the dynamic pressure number, and

$1/\beta_d = 1/\sqrt{\{1 - (\text{Mach number})^2\}}$ the corresponding value of the Prandtl-Glauert factor, appropriate to the design diving speed at a height of 10,000 feet.

We now consider the tailplane under an arbitrary tail load P_T. If, for simplicity, we assume that the basic spanwise distribution of load is rectangular, with a constant chordwise position for the local centre of pressure, and if we neglect the effect of deformability on the distribution, the torque at a section distant y_T from the plane of symmetry will be given by $T_T\{1 - (y_T/s_T)\}$ where T_T denotes the root torque. The twist θ_T at this section will be given by:

$$\theta_T = \int_0^{y_T} \frac{T_T}{GJ_T}\left(1 - \frac{y_T}{s_T}\right) dy_T = \frac{T_T}{GJ_T}\left[y_T - \frac{y_T^2}{2s_T}\right]$$

if GJ_T is constant. We shall assume that the change in zero-lift direction, or mean tailplane setting $(\Delta\eta_T)_T$ due to tailplane twist is equal to the mean value of θ_T given by:

$$\bar{\theta}_T = \frac{T_T}{GJ_T}\frac{1}{s_T}\int_0^{s_T}\left(y_T - \frac{y_T^2}{2s_T}\right) dy_T = \frac{T_T}{GJ_T}\frac{s_T}{3}.$$

Thus we have

$$\left(\frac{\partial \eta_T}{\partial T_T}\right)_T = \frac{s_T}{3GJ_T} \quad \ldots \quad (56)$$

where for tailplanes just satisfying the stiffness criterion, GJ_T is equal to $(GJ_T)_{\min}$, as given by equation (55). More generally, introducing a torsional rigidity factor $F_\tau'(\geqslant 1)$ to cover cases with a reserve of stiffness, we have:

$$\left(\frac{\partial \eta_T}{\partial T_T}\right)_T = \frac{1}{F_\tau'}\frac{s_T}{3(GJ_T)_{\min}}. \quad \ldots \quad (57)$$

Now, denoting by $w_{\alpha 0}$, $w_{\eta 0}$ and $w_{m 0}$, the values of the deformability coefficients for an unswept tailplane with the structural lay-out described above, and with a completely rigid attachment to the fuselage, we have (from equation (47)) :

$$\left.\begin{aligned} w_{\alpha 0} &= -\frac{1}{Q}\left(\frac{\partial \eta_T}{\partial \alpha_T}\right)_0 = -\frac{1}{Q}\left[\left(\frac{\partial \eta_T}{\partial P_\alpha}\right)_F \frac{\partial P_\alpha}{\partial \alpha_T} + \left(\frac{\partial \eta_T}{\partial T_{T\alpha}}\right)_T \frac{\partial T_{T\alpha}}{\partial \alpha_T}\right] \\ w_{\eta 0} &= -\frac{1}{Q}\left(\frac{\partial \eta_T}{\partial \eta}\right)_0 = -\frac{1}{Q}\left[\left(\frac{\partial \eta_T}{\partial P_\eta}\right)_F \frac{\partial P_\eta}{\partial \eta} + \left(\frac{\partial \eta_T}{\partial T_{T\eta}}\right)_T \frac{\partial T_{T\eta}}{\partial \eta}\right] \\ w_{m 0} &= \left(\frac{\partial \eta_T}{\partial n}\right)_0 = \left(\frac{\partial \eta_T}{\partial P_m}\right)_F \frac{\partial P_m}{\partial n} + \left(\frac{\partial \eta_T}{\partial T_{Tm}}\right)_T \frac{\partial T_{Tm}}{\partial n} \end{aligned}\right\} \quad \ldots \quad (58)$$

where all the derivatives of η_T with respect to forces and torques have been derived above, but the derivatives of the forces and torques with respect to α_T, η and n remain to be considered. The derivatives of the forces are readily deduced to be:

$$\left.\begin{aligned}\frac{\partial P_\alpha}{\partial \alpha_T} &= -Q\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r \frac{S_T}{S} W \\ \frac{\partial P_\eta}{\partial \eta} &= -Q\left(\frac{\partial C_{LT}}{\partial \eta}\right)_r \frac{S_T}{S} W \\ \frac{\partial P_m}{\partial n} &= W_f\end{aligned}\right\} \quad \ldots \quad (59)$$

where the suffix $_r$ denotes 'completely rigid' and W_f denotes the weight of the rear end of the fuselage, including the complete tail unit. The torques $T_{T\alpha}$ and $T_{T\eta}$ are obtained from the corresponding forces P_α and P_η by multiplying $-P_\alpha/2$ and $-P_\eta/2$ by their appropriate moment arms about the elastic axis of the tailplane; these are respectively e_T and $-(\delta l - e_T)$. Hence

$$\left.\begin{aligned}\frac{\partial T_{T\alpha}}{\partial \alpha_T} &= \frac{Q}{2}\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r \frac{S_T}{S} We_T \\ \text{and} \qquad \frac{\partial T_{T\eta}}{\partial \eta} &= -\frac{Q}{2}\left(\frac{\partial C_{LT}}{\partial \eta}\right)_r \frac{S_T}{S} W(\delta l - e_T)\end{aligned}\right\} \quad \ldots \quad (60)$$

We shall neglect the torque T_{Tm} corresponding to P_m since in most cases the moment arm about the elastic axis will be very small. Thus

$$\frac{\partial T_{Tm}}{\partial n} = 0 \,. \quad \ldots \quad (61)$$

Using equations (49) to (52), (55) and (57) to (61), we obtain the following approximate equations for the deformability coefficients of the rigidly mounted tailplane:

$$\left.\begin{aligned}w_{\alpha 0} &\simeq \left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r \left[K_F \frac{2K_\alpha l_\alpha'}{h} \frac{W}{|P_{Tu}|} \frac{S_T}{S} - \mathscr{C} \frac{e_T}{c_T} \frac{\beta_d}{Q_d}\right] \\ w_{\eta 0} &\simeq \left(\frac{\partial C_{LT}}{\partial \eta}\right)_r \left[K_F \frac{2l_\eta'}{h} \frac{W}{|P_{Tu}|} \frac{S_T}{S} + \mathscr{C} \frac{\delta l - e_T}{c_T} \frac{\beta_d}{Q_d}\right] \\ w_{m 0} &\simeq K_F \frac{2l_\alpha'}{h} \frac{W_f}{W} \frac{W}{|P_{Tu}|} \\ \text{where} \qquad \mathscr{C} &= \frac{1}{12F_\tau'\mathscr{C}_2} = \begin{cases}\dfrac{0\cdot 382}{F_\tau'} & \text{for tailplanes without end-fins} \\ \dfrac{0\cdot 244}{F_\tau'} & \text{for tailplanes with end-fins}\end{cases}\end{aligned}\right\} \quad \ldots \quad (62)$$

13.3. *The Deformability of the Tailplane-Fuselage Attachment.*—The deformability of the tailplane-fuselage attachment gives rise to additional terms Δw_α, etc., in the formulae for the deformability coefficients. The attachment may be considered to be replaced by a hinge, representing the elastic axis of the attachment, about which the complete tailplane is assumed to rotate relative to the fuselage, under the constraint of a torsional spring of stiffness λ. Let e_A denote the distance of the elastic axis of the attachment behind the axis of aerodynamic centres of the tailplane (considered here as completely rigid) and assume the inertia and elastic axes of the tailplane to coincide at a distance behind the aerodynamic axis, to be denoted by e_T. Then we have:

$$\left.\begin{aligned} \Delta w_\alpha &= -\frac{e_A}{c_T}\left(\frac{\partial C_{L\,T}}{\partial \alpha_T}\right)_r \frac{S_T}{S}\frac{Wc_T}{\lambda} \\ \Delta w_\eta &= \frac{(\delta l - e_A)}{c_T}\left(\frac{\partial C_{L\,T}}{\partial \eta}\right)_r \frac{S_T}{S}\frac{Wc_T}{\lambda} \\ \Delta w_m &= \left(\frac{e_T}{c_T} - \frac{e_A}{c_T}\right)\frac{W_T}{W}\frac{Wc_T}{\lambda} \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \ldots \quad (63)$$

where W_T denotes the weight of the tailplane, including end-plate fins and rudders, if any.

The overall deformability coefficients of an unswept tailplane with elastic attachment are now given by:

$$\left.\begin{aligned} w_\alpha &= w_{\alpha\,0} + \Delta w_\alpha \\ w_\eta &= w_{\eta\,0} + \Delta w_\eta \\ w_m &= w_{m\,0} + \Delta w_m \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \ldots \quad (64)$$

The effect of the elastic attachment is seen to depend on two parameters, namely e_A/c_T and Wc_T/λ, defining respectively the position of the elastic axis of the attachment, and the flexibility of the hypothetical torsional spring. The latter can, in general, be varied by the designer within wide limits, the upper limit being set by considerations of elevator effectiveness and of tail unit flutter. For the present we consider only the problem of elevator effectiveness.

We may define an elevator effectiveness factor F_η (a function of dynamic pressure q, comparable with the rolling effectiveness factor F_ξ of Part I) by the equation:

$$F_\eta = 1 - \frac{q}{q_{c\eta}}$$

where $q_{c\,\eta}$ denotes the critical dynamic pressure with respect to elevator reversal. It is, of course, essential to avoid elevator reversal within the flight speed range, so that $F_\eta{}^* = (F_\eta)_{q=q^*}$ must be positive; in fact, general strength considerations for the tail unit will impose a minimum positive value for $F_\eta{}^*$, say $0{\cdot}2$. We now derive a formula connecting the effectiveness factor $F_\eta{}^*$ at $q = q^*$ with the deformability coefficients w_α and w_η.

The condition for zero elevator effectiveness on pitching moment about the aircraft c.g. is clearly:

$$\frac{\partial \bar{\alpha}_T}{\partial \bar{\eta}} = -\frac{l_\eta}{l_\alpha}\frac{\left(\dfrac{\partial C_{L\,T}}{\partial \eta}\right)_r}{\left(\dfrac{\partial C_{L\,T}}{\partial \alpha_T}\right)_r} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (65)$$

Now, if the elevator effectiveness is zero, the angle of attack of the wing α is unaffected by $\bar{\eta}$, and the change in tailplane angle of attack $\bar{\alpha}_T$ due to a change in $\bar{\eta}$ will be equal to the change in tailplane setting η_T, so that we must have:

$$\frac{\partial \bar{\alpha}_T}{\partial \bar{\eta}} = \frac{\partial \eta_T}{\partial \bar{\eta}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (66)$$

and then, from equation (47):

$$\frac{\partial \bar{\alpha}_T}{\partial \bar{\eta}} = \frac{-w_\eta}{\dfrac{1}{Q} + w_\alpha} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (67)$$

Now at $Q = Q_{c\eta} = Q^*/(1 - F_\eta^*)$, equations (65) and (67) are both valid, and it therefore follows that:

$$\frac{1 - F_\eta^*}{Q^*} = w_\eta \frac{\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r}{\left(\frac{\partial C_{LT}}{\partial \eta}\right)_r} \frac{l_\alpha}{l_\eta} - w_\alpha \,. \qquad \text{(68)}$$

The elevator effectiveness factor $F_{\eta 0}^*$ corresponding to a rigid tailplane-fuselage attachment is given by:

$$\frac{1 - F_{\eta 0}^*}{Q^*} = w_{\eta 0} \frac{\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r}{\left(\frac{\partial C_{LT}}{\partial \eta}\right)_r} \frac{l_\alpha}{l_\eta} - w_{\alpha 0} \qquad \text{(69)}$$

where $w_{\alpha 0}$ and $w_{\eta 0}$ are given by equations (62). From equations (68), (69), (64) and (63) we now obtain the relationship between the elevator effectiveness factor F_η^* at the maximum permissible dynamic pressure, and the flexibility of the tailplane-fuselage attachment, in the form:

$$\frac{Wc_T}{\lambda} = \frac{\{F_{\eta 0}^* - F_\eta^*\}\frac{1}{Q^*}}{\frac{S_T}{S}\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r \left(1 - \frac{l_\alpha}{l_\eta}\right)\frac{l_\alpha + e_A}{c_T}} \,. \qquad \text{(70)}$$

The maximum permissible value of Wc_T/λ (corresponding to the minimum permissible torsional stiffness of the attachment) associated with a given value of e_A/c_T is obtained from equation (70) by substituting for F_η^* its minimum permissible value. The corresponding w_α is the minimum (or maximum negative) permissible from the point of view of elevator effectiveness, and its value depends to a large extent on the position of the elastic axis of the attachment. In the special case† where the axis coincides with the elevator neutral axis ($e_A = l_\eta - l_\alpha$), the deformability of the attachment does not affect w_η, so that $w_\eta = w_{\eta 0}$ and the minimum permissible w_α can be calculated from equation (68).

If we write

$$\bar{\alpha}_T = \varkappa + \Delta\eta_T \,,$$

where $\varkappa$ denotes the angle of attack which the tailplane would have if the aerodynamic and inertia forces acting on it had no effect on its incidence, then equation (47) gives:

$$\Delta\eta_T = \frac{- w_\alpha \varkappa Q - w_\eta \bar{\eta} Q + w_m n}{1 + w_\alpha Q} \,.$$

From this it will be seen that tailplane divergence will occur if $w_\alpha \leqslant - 1/Q$, so that the minimum permissible value of w_α from the point of view of tailplane divergence is $- 1/Q^*$. From equation (68) it is clear that the minimum permissible value of w_α from considerations of elevator effectiveness is certainly greater than $- 1/Q^*$ if $w_\eta \geqslant 0$ so that tailplane divergence sets the limit to w_α only if w_η is negative which, in practice, can be so only if the tailplane is swept forward.

13.4. *Calculation of the Deformability Coefficients : Some Numerical Approximations.*—Calculation of the basic coefficients $w_{\alpha 0}$, etc., from equations (62) involves an estimation of the quantity $K_F = (1/F_F)(f_c/E)$ (equation (49)) and of the ultimate design tail load P_{Tu} (including the inertia forces on the rear end of the fuselage).

† The general case was to have been considered in more detail in the proposed Part III of this report which was, however, never completed.

For an existing design it will normally be possible to calculate K_F by substituting in equation (49), values of the various parameters appropriate to the specific design, while the ultimate tail load P_{Tu} will be known from routine strength calculations. For rough estimates in the early stages of a design, or for general investigations involving a qualitative assessment of the effects of varying specified design parameters, we may derive approximations for the quantities in question.

13.4.1. *The quantity K_F.*—

$$K_F = \frac{1}{F_F}\frac{f_c}{E} = \frac{1}{F_F}\frac{f_t}{E}\frac{f_c}{f_t}.$$

For materials likely to be used in practice, f_t/E is approximately equal to 1/150, while f_c depends on the instabilities (such as column failure and plate buckling) occurring in the compression members of the structure at the higher compressive stresses. For the plating and stringers of a shell fuselage, and similarly for the struts of a framework fuselage, all of which are supported only weakly (if at all) against such instabilities, $f_c/f_t \simeq 0 \cdot 5$. Hence for a rough approximation, we may take

$$K_F \simeq \frac{1}{300F_F}, \qquad \qquad (71)$$

and we thus have, from equation (49):

$$\left(\frac{\partial \eta_T}{\partial P_\eta}\right)_F \simeq \frac{1}{150F_F}\frac{l_\eta'}{h}\frac{1}{P_{Tu}}. \qquad \qquad (49a)$$

Similarly equations (51) and (52) give respectively:

$$\left(\frac{\partial \eta_T}{\partial P_\alpha}\right)_F \simeq \frac{1}{150F_F}\frac{K_\alpha l_\alpha'}{h}\frac{1}{P_{Tu}} \qquad \qquad (51a)$$

and

$$\left(\frac{\partial \eta_T}{\partial P_m}\right)_F \simeq \frac{1}{150F_F}\frac{l_\alpha'}{h}\frac{1}{P_{Tu}}, \qquad \qquad (52a)$$

where $F_F \geqslant 1 \cdot 0$ and $F_F \geqslant F_{F0}$; (54)

and from equation (53):

$$F_{F0} = 0 \cdot 0403 \frac{S_T}{S}\frac{l_\eta'}{h}\frac{W}{P_{Tu}}\frac{Q_d}{\beta_d}. \qquad \qquad (53a)$$

13.4.2. *The ultimate design tail load P_{Tu}.*—For aircraft stressing purposes, the ultimate tail load for a specific design must be determined from a consideration of the various flight conditions detailed in Ref. 13 (*b*), and since it is impossible to generalise as to which condition will be critical, it is not possible to give an approximating formula applicable to all designs. The reader who is familiar with the requirements of Ref. 13 (*b*), and experienced in their application, will readily derive for himself an approximate value, appropriate to any design, actual or hypothetical, which is under consideration. For the non-specialist in such matters, approximate formulae are developed in Appendix IV which should be applicable in a good proportion of cases.

13.5. *The Effect of Wing Sweep on the Deformability Coefficients.*—The expressions for $w_{\alpha 0}$ and $w_{\eta 0}$ established in equations (62) hold independently of the angle of sweep of the wing. It is, however, interesting to consider how the values of these coefficients will be affected when, for a given design, the sweep of the wing is varied, while maintaining unchanged all the other main characteristics. The only factors in the equations for $w_{\alpha 0}$ and $w_{\eta 0}$ that will be affected are $l_\alpha' = l_\alpha + l_G$ and $l_\eta' = l_\eta + l_G$. It may be assumed that the distance between the tail unit and the aircraft centre of gravity is not appreciably altered, and further that the wing aerodynamic centre remains fixed relative to the centre of gravity. Then l_α and l_η remain unchanged and we are concerned only with the change in l_G (*i.e.*, the distance of the centre of gravity aft of the encastré section of the fuselage). If we assume the aerodynamic centre of a half-wing of constant chord to lie at the quarter-chord point of the section $0 \cdot 45 s_T$ from the root, it will follow that for a constant-chord wing of sweep angle ϕ, the value of l_G will be about $0 \cdot 45 s \tan \phi$ greater than with the unswept wing, and the values of $w_{\alpha 0}$ and $w_{\eta 0}$ will be increased or decreased according as ϕ is positive (sweepback) or negative (sweepforward).

The third deformability coefficient w_{m0} will be similarly affected, but in this case an additional effect of sweep arises when the sweep angle is positive; for then the inertia forces of the middle part of the fuselage, including those of all equipment, useful load and, in some cases, power plant contained therein, bend the rear fuselage and therefore affect the angle of attack of the tail unit. We now make a rough estimate of this effect. Denote the weight in question by W_c and consider it concentrated at a fixed fuselage location, which may reasonably be defined as lying in the encastré section of the fuselage when the wing is unswept. When the wing is swept back at an angle ϕ, the weight W_c will lie at a distance of approximately $0 \cdot 45 s \tan \phi$ aft of the encastré section, and under load factor n there will be an inertia force nW_c producing, over a length of fuselage $0 \cdot 45 s \tan \phi$, a bending moment decreasing linearly from $0 \cdot 45 s \, nW_c \tan \phi$ at the encastré section to zero at the assumed location of W_c. For our present purpose, it will be good enough to assume the bending rigidity $(EI_F)_0$ of the relevant portion of fuselage to be constant and equal to its value at the encastré section. Then from equation (50), (putting $x = 0$), we have:

$$(EI_F)_0 = \frac{E}{2f_c} P_{Tu} l_\eta' h F_F$$

$$\simeq 150 P_{Tu} l_\eta' h F_F \, ,$$

using the approximation of section 13.4.1.

The effect in question thus gives an additional fuselage curvature which, with our assumptions, decreases linearly over the length $0 \cdot 45 s \tan \phi$ from the value $\dfrac{nW_c s \tan \phi}{F_F P_{Tu} h l_\eta'} \, 0 \cdot 9 \left(\dfrac{f_c}{E}\right)$ to zero. This gives an additional $\Delta \eta_T$ equal to:

$$\frac{0 \cdot 45 s \tan \phi}{2} \, \frac{0 \cdot 9 n W_c s \tan \phi \left(\dfrac{f_c}{E}\right)}{F_F P_{Tu} h l_\eta'}$$

$$= 0 \cdot 2025 \left(\frac{f_c}{E}\right) \frac{n s^2 \tan^2 \phi}{l_\eta' h} \, \frac{W_c}{P_{Tu}} \, \frac{1}{F_F}$$

$$\simeq \frac{1}{1500} \, \frac{n s^2 \tan^2 \phi}{l_\eta' h} \, \frac{W_c}{P_{Tu}} \, \frac{1}{F_F}$$

corresponding to an increase in w_m of amount

$$\Delta_2 w_m \simeq \frac{s^2 \tan^2 \phi}{1500 l_\eta' h} \, \frac{W_c}{P_{Tu}} \, \frac{1}{F_F} \, .$$

With negative (forward) sweep, there is no such effect on w_m, because the additional fuselage bending now occurs outside the region between wing and tail unit. To cover both swept-back and swept-forward wings in a single formula, we may write

$$\Delta_2 w_m \simeq \frac{1}{1500} \, \frac{s^2 \tan^2 \phi}{l_\eta' h} \, \frac{W_c}{W} \, \frac{W}{P_{Tu}} \, \frac{1}{F_F} \, \frac{\left(1 + \dfrac{\phi}{|\phi|}\right)}{2} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (72)$$

where the numerical factor must be regarded as very approximate.

It may be noted that the bending relief provided by the additional inertia forces considered above, should make it possible to reduce the bending rigidity EI_F of the fuselage and still maintain adequate strength.

14. *The Effect of Wing Deformability and Wing-Fuselage Attachment Deformability on the Angles of Attack of the Fuselage and Tailplane.*—In section 13 we were concerned only with the effect of fuselage and tailplane deformability on the setting of the tailplane relative to the fuselage datum defined in section 12. The contributions of fuselage and tailplane to the aircraft pitching moment at a given speed and load factor depend, however, on their respective angles of attack which, as already pointed out in section 12, are influenced by deformability of the wing, and of the wing-fuselage attachment. The change in the angles of attack, arising from this source, is measured by the angular deflection of the fuselage datum relative to the wing mean no-lift-line. There are, in general, three contributions to this relative deflection, arising respectively from:

(*a*) elastic wash-out of the wing

(*b*) elastic camber of the wing

(*c*) deformability of the wing-fuselage attachment.

The first of these produces an angular deflection of the wing-root datum relative to the mean no-lift line, while (*b*) and (*c*) together produce an angular deflection of the fuselage datum relative to the wing-root datum (*see* Fig. 21b). We now consider the three contributions in turn.

14.1. *The Effect of Elastic Wash-out of the Wing.*—The change in angle of attack of the wing-root section due to elastic wash-out has been investigated in Part I, where the result is given by equation (23), putting $\zeta = 0$†, in the form:

$$\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) = -0\cdot455D_7 - 0\cdot285D_8 \qquad (73)$$

Now if α_F denotes the angle of attack of the fuselage relative to the free stream, then the change $(\Delta_1\alpha_F)$ in α_F due to elastic wash-out will be equal to the change in $\alpha_{y=0}$, so that we have:

$$\Delta_1\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) = \Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) = -0\cdot455D_7 - 0\cdot285D_8 \qquad (73a)$$

14.2. *The Effect of Elastic Camber of the Wing.*—The elastic camber of the wing, arising from rib-bending and spar-bending has been investigated in Part I, from equations (4), (5) and (26) of which we may deduce that the camber ratio γ is given by:

$$-\frac{d\gamma}{dn} \simeq \frac{\beta}{2\cdot5}D_4$$

where, for constant-chord wings with ribs perpendicular to the spar,

$$D_4 = \frac{1}{480\beta n_u \tau}\left[\frac{\cos^2\phi}{F_{r.b.}} + \frac{2\sin^2\phi}{F_b}\right]$$

and for constant chord wings with ribs in the flight direction, . Equation (26)

$$D_4 = \frac{1}{480\beta n_u \tau F_{r.b.}}$$

† This neglects the effect of fuselage width, assuming, in effect, that the wing root coincides with the centre-line of the fuselage.

The curvature of a wing chord was shown in Fig. 2(a) to be given by:

$$\frac{1}{R} = \frac{8\gamma}{c},$$

positive radius of curvature R corresponding to positive camber, which is induced by negative lift.

In accordance with the assumptions of section 12, illustrated by Fig. 21b, the deflection of the fuselage datum relative to the wing-root datum is equal to the angle between the tangents to the camber-line at E (elastic centre) and C (three-quarter chord point). Hence the contribution to $\Delta\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$ due to elastic camber is given by:

$$\Delta_2\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) = \Delta_1\left[\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right] = -\left(0\cdot5 - \frac{e_F}{c}\right)c\frac{d}{dn}\left(\frac{1}{R}\right)_{\text{rib}}$$

$$= -8\left(0\cdot5 - \frac{e_F}{c}\right)\frac{d\gamma}{dn},$$

where e_F denotes the chordwise distance of the wing elastic axis aft of its axis of aerodynamic centres; or substituting for $d\gamma/dn$:

$$\Delta_2\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) = 3\cdot2\beta D_4\left(0\cdot5 - \frac{e_F}{c}\right). \qquad \text{(74)}$$

14.3. *The Effect of Deformability in the Region of the Wing-Fuselage Attachment.*—Some reference was made to this effect in section 1.2 of Part I when discussing the general question of the deformability of the root region of the wing. Numerical assessment of the effect was, however, deferred to the present part of the report. As suggested in Part I the effect of wing root-region deformability on the angle of attack of the fuselage can most conveniently be estimated by assuming that for the actual deformability is substituted a ball-joint, located at a point of the root region which can be regarded as the elastic axis of the attachment, together with a torsional spring restraining the relative angular deflection of wing and fuselage. For our present purpose, we do not need complete data for this equivalent system ; all that is required is the corresponding increment $\Delta_3\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$ in the rate of change of angle of attack of the fuselage, with load factor. The approximate limits within which this quantity must lie can readily be estimated, in the case of an unswept wing, by consideration of this purely torsional elastic attachment of the wing to the fuselage.

If it is stipulated that deformability of the attachment shall not reduce the critical dynamic pressure for wing flutter by more than 20 per cent (an arbitrarily chosen, but reasonable figure†) the maximum torsional deformability due to the attachment is given roughly by:

$$\left|\frac{d\psi}{d\mathscr{M}}\right| = \frac{0\cdot2}{0\cdot8}\frac{0\cdot7s}{GJ}, \qquad \text{(75)‡}$$

where ψ is the angular deflection of the wing-root section relative to the fuselage, due to an applied torque $\mathscr{M}$,

s is the wing semi-span,

and GJ is the section torsional rigidity of the wing (assumed constant).

† We are considering here the maximum amount of flexibility that may deliberately be introduced into the attachment as a device for adjusting the aircraft stability. The inherent flexibility of a nominally rigid attachment would normally be much less than this. In all cases, the flexibility of the attachment should be taken into account when assessing the flutter speed which should still exceed the design diving speed by the currently accepted margin.

‡ Equation (75) is derived from the assumption that (flutter speed)^2 is proportional to the wing torsional stiffness (applied moment per unit twist), measured at an appropriate reference section, here taken to be $0\cdot7s$ (*cf.* wing torsional stiffness criterion of Ref. 13 (*c*)). The relationship is applied in turn to the case with, and the case without, an elastic attachment.

Now $d\psi/dn = (d\psi/d\mathscr{M})(d\mathscr{M}/dn)$ is, like $d\mathscr{M}/dn$, proportional to the distance between the axis of the attachment and the point of application of the resultant aerodynamic and inertia force due to change of load factor. This distance may be varied by changing the position of the attachment axis, but an upper limit is imposed by the fact that the weight of the attachment structure will also increase in some manner with increasing distance. For, under proof load factor, the elastic energy to be stored in the attachment structure is:

$$\tfrac{1}{2}(\mathscr{M} \times \psi)_{n=n_p} = \tfrac{1}{2}\left|\frac{d\psi}{d\mathscr{M}}\right|(\mathscr{M}^2)_{n=n_p} = \tfrac{1}{2}\left|\frac{d\psi}{d\mathscr{M}}\right|\left(\frac{d\mathscr{M}}{dn}\right)^2 n_p^2 ,$$

while the energy which can be stored per unit attachment weight, with proof stresses in the attachment structure is† $p^2/2E\sigma$, where σ denotes the weight per unit volume of the material, p its proof stress, and E Young's Modulus. The weight penalty is therefore:

$$\Delta W = \tfrac{1}{2}\left|\frac{d\psi}{d\mathscr{M}}\right|\left(\frac{d\mathscr{M}}{dn}\right)^2 2E\sigma\,\frac{n_p^2}{p^2} ,$$

or since

$$\frac{n_p^2}{p^2} = \frac{n_u^2}{f^2} ,$$

$$\Delta W = \left|\frac{d\psi}{d\mathscr{M}}\right|\left(\frac{d\mathscr{M}}{dn}\right)^2 n_u^2\,\frac{E\sigma}{f^2} .$$

For materials of practical importance

$$\frac{f^2}{E\sigma} = \left(\frac{f}{E}\right)^2\frac{E}{\sigma} \simeq \left(\frac{1}{150}\right)^2\frac{E}{\sigma} \simeq 380 \text{ ft.}$$

Using this value in the last equation, eliminating $d\mathscr{M}/dn$ by means of the relation

$$d\psi/dn = (d\psi/d\mathscr{M})(d\mathscr{M}/dn) ,$$

and substituting for $|d\psi/d\mathscr{M}|$ from equation (75) with GJ as given by equation (1) of Part I, we obtain:

$$\frac{\Delta W}{W} = \frac{c}{143 \text{ ft}}\,\frac{n_u^2 Q^* F_\tau\left(1 - \frac{c_\xi}{c}\right)^2}{(1 - F_\xi^*)}\left(\frac{d\psi}{dn}\right)^2$$

where c is measured in feet.

If $\dfrac{\Delta W}{W} = 0\cdot01$ is taken as a reasonable maximum value, we obtain:

$$\left[\Delta_2\left\{\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right\}\right]^2 = \left(\frac{d\psi}{dn}\right)^2 \leqslant \frac{1\cdot43 \text{ ft}}{c}\,\frac{1}{n_u^2 Q^* F_\tau}\,\frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} .$$

The deformability of the wing attachment is thus estimated to lie between two values of opposite sign but equal magnitude, *i.e.*,

$$\pm\frac{1}{n_u\left(1 - \frac{c_\xi}{c}\right)}\sqrt{\left(\frac{1\cdot43 \text{ ft}}{c}\,\frac{1 - F_\xi^*}{F_\tau Q^*}\right)} .$$

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† *See*, for instance, '*An Introduction to the Theory of Elasticity*' by R. V. Southwell (Oxford, 1936), p. 34.

It should be noted that the effect in question may be larger with small than with large aircraft. By introducing a 'root-region stiffness factor' $F_{r.r.} (|F_{r.r.}| \geqslant 1)$ we obtain:

$$\Delta_3\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) = \Delta_2\left[\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right] = \frac{1}{F_{r.r.} n_u \left(1 - \frac{c_\xi}{c}\right)} \sqrt{\left(\frac{1 \cdot 43 \text{ ft}}{c} \frac{(1 - F_\xi^*)}{F_\tau Q^*}\right)} . \quad (76)$$

Combining equations (74) and (76), we have :

$$(\Delta_2 + \Delta_3)\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) = \Delta\left[\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right] = (\Delta_1 + \Delta_2)\left[\frac{\partial(\alpha_F - \alpha_{y=0}}{\partial n}\bigg|_V\right]$$

$$= 3 \cdot 2\beta D_4\left(0 \cdot 5 - \frac{e_F}{c}\right) + \frac{1}{F_{r.r.} n_u \left(1 - \frac{c_\xi}{c}\right)} \sqrt{\left(\frac{1 \cdot 43 \text{ ft}}{c} \frac{(1 - F_\xi^*)}{F_\tau Q^*}\right)} \ldots \quad (77)$$

A further deformability effect associated with the region of the wing-fuselage attachment has already been alluded to in section 13.1. This is the angular displacement of the rear fuselage due to shear in the vicinity of the wing attachment. The effect which may, in some cases, be quite large, will vary greatly according to the structural lay-out of the region concerned, and no attempt will be made to establish a formula for it. It should be noted that the effect should be allowed for, in estimating the vertical stiffness of the fuselage, and that an upper limit to the permissible magnitude of the effect is therefore imposed by the stiffness criterion of Ref. 13 (*a*), paragraph 7.

15. *The Effect of Fuselage and Tailplane Deformability on the Manoeuvre Point.*—As was seen in section 6 of Part I the forward shift of manoeuvre point (elevator fixed) due to deformability is given by:

$$\frac{\Delta x_{m.p.}}{c} = \Delta\left(\frac{\partial C_M}{\partial C_L}\bigg|_{V,\eta}\right) \quad .. \quad .. \quad .. \quad .. \quad .. \quad (78)$$

where C_M is the pitching-moment coefficient for the complete aircraft about its centre of gravity. To estimate the contribution of fuselage and tailplane deformability to the shift, we must therefore estimate the effect of such deformability on the pitching moments of the forces acting on the fuselage and tailplane. As we have already seen, the angles of attack of these components, during a pull-out at constant speed with elevator fixed, are affected by wing deformability. Thus in comparing the pitching moments of the forces acting on the fuselage and tailplane of a deformable aircraft with those for the corresponding rigid aircraft, we shall be taking account not only of the deformability of the fuselage and tailplane themselves, but also of this indirect effect of wing deformability. The direct effect of wing deformability on wing pitching moment is, of course, a separate matter, already dealt with in Part I.

15.1. *The Effect of Deformability on the Pitching Moment of the Aerodynamic Forces Acting on :*

15.1.1. *The fuselage.*—In estimating the pitching moment of the aerodynamic forces acting on the fuselage, we shall first neglect the effect of fuselage bending, which will be briefly dealt with in section 16.1. As an approximate expression for the fuselage moment coefficient of a rigid aircraft, referred to the wing area and wing chord, we may assume (*see* Ref. 11):

$$C_{MF} \simeq \frac{\pi}{2} \frac{\int_0^{l_F} \alpha_F' b_F^2 \, dx}{Sc} = \alpha_F \frac{\frac{\pi}{2}\int_0^{l_F} \frac{\partial \alpha_F'}{\partial \alpha_F} b_F^2 \, dx}{Sc} ,$$

where α_F' is the local angle of attack of the fuselage axis at a section, distant x from the nose, where the fuselage width is b_F; the integration being performed over the total length l_F of the fuselage. If α_F' were unaffected by the wing's presence, and thus equal to the angle of attack relative to the free stream, we should have $\partial\alpha_F'/\partial\alpha_F = 1$ and hence

$$C_{MF} \simeq 2\alpha_F \frac{\frac{\pi}{4}\int_0^{l_F} b_F^2\,dx}{Sc},$$

where $\frac{\pi}{4}\int_0^{l_F} b_F^2\,dx$ is the volume of the body of revolution based on the plan of the fuselage. Defining a factor F_I by the equation

$$F_I = \frac{\int_0^{l_F} \frac{\partial\alpha_F'}{\partial\alpha_F} b_F^2\,dx}{\int_0^{l_F} b_F^2\,dx},$$

we may write in general,

$$\left.\frac{\partial C_{MF}}{\partial\alpha_F}\right|_V \simeq 2F_I \frac{\frac{\pi}{4}\int_0^{l_F} b_F^2\,dx}{Sc},$$

where the value of F_I is $1\cdot0$ if the increment in α_F has no influence on the wing's velocity field, but is otherwise, as yet, undetermined.

The derivative of C_{MF} with respect to lift coefficient is now given by:

$$\left.\frac{\partial C_{MF}}{\partial\bar{C}_L}\right|_V = Q\left.\frac{\partial C_{MF}}{\partial n}\right|_V = Q\left(\left.\frac{\partial C_{MF}}{\partial\alpha_F}\right|_V\right)\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right)$$

$$= 2Q\frac{\frac{\pi}{4}\int_0^{l_F} b_F^2\,dx}{Sc}\left(F_I\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right).$$

Of the quantities occurring in this formula, only $\left(F_I\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right)$ is affected by the elastic deformations, and from sections 14.1, 14.2 and 14.3 we have:

$$\Delta\left(F_I\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) = F_{I1}\,\Delta_1\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) + F_{I2}\,\Delta_2\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) + F_{I3}\,\Delta_3\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right),$$

where the three terms are given, apart from the factors F_I, by equations (73a), (74) and (76). The quantities $\Delta_2\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right)$ and $\Delta_3\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right)$ correspond respectively to elastic camber of the wing, and deformability of the wing attachment, which have little effect on the induced velocity field of the wing, and accordingly we may assume that $F_{I2} = F_{I3} = 1\cdot0$. The quantity $\Delta_1\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right)$ arises from elastic wash-out of the wing which affects its lift distribution, and consequently the velocity field induced by that distribution. Thus in general $F_{I1} \neq 1\cdot0$. We may now write

$$\Delta\left(F_I\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) = F_{I1}\,\Delta_1\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) + \Delta_2\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) + \Delta_3\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right), \quad \ldots \quad (79)$$

and then the increment in $\left.\frac{\partial C_{MF}}{\partial\bar{C}_L}\right|_V$ due to deformability is given by:

$$\Delta\left(\left.\frac{\partial C_{MF}}{\partial\bar{C}_L}\right|_V\right) = D_9 Q\,\Delta\left(F_I\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) \quad \ldots \quad (80)$$

where

$$D_9 = 2\frac{\frac{\pi}{4}\int_0^{l_F} b_F^2\, dx}{Sc} \quad \ldots \quad (80a)$$

The factor F_{I1}.—The factor F_{I1} is defined by the equation:

$$\left(\int_0^{l_F} b_F^2\, dx\right) F_{I1} = \int_0^{l_F} \frac{\partial \alpha_F'}{\partial \alpha_F}\, b_F^2\, dx$$

$$= \int_0^{l_F} \left\{1 - \frac{\partial\, \varepsilon(x)}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L, V}\right\} b_F^2\, dx \quad \ldots \quad (81)$$

where $\varepsilon(x)$ is the distribution of downwash along the body axis due to the wing. (Ahead of the wing, $\varepsilon(x)$ will, of course, be negative corresponding to upwash.) The quantity $\frac{\partial\, \varepsilon(x)}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L, V}$ corresponds to a ' wash-out without lift ' distribution of wing angle of attack. The problem of evaluating the downwash field for a swept wing with arbitrary angle of attack distribution can be solved by one of several lifting-plane methods of which those of Schlichting (Ref. 10) and Multhopp (Ref. 14) should ultimately be the simplest to apply. When complete details of these methods are available†, it will be a fairly simple matter to calculate the function $\left\{1 - \frac{\partial\, \varepsilon(x)}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L, V}\right\}$ and hence to evaluate the integral (71), for any specific case. Alternatively, by performing a series of calculations with systematic variations of the main geometric parameters involved, it should be possible to devise charts for the rapid assessment of the factor F_{I1}.

It may be remarked that since the fuselage contribution to manoeuvre-point shift is relatively small, it is not usually essential that F_{I1} should be estimated with great accuracy. The value of the factor has therefore been estimated for a few wing-fuselage arrangements, using the method of Schlichting (Ref. 10) to determine the downwash distribution, and from the results plotted in Fig. 26, the reader should be able to obtain a fair idea of its value for other arrangements.

Appendix V gives details of the calculations which relate to a fuselage in the form of an ellipsoid of revolution of axis ratio 7 : 1, associated with constant-chord wings of aspect ratios 3 and 6, and sweepback 0 deg and 45 deg. All the wings were assumed to have the same area, the span of the wings with aspect ratio 6 being taken equal to the length of the fuselage. With the wings at a conventional location on the fuselage (mean quarter-chord points at $0{\cdot}4 l_F$ from nose) the values of F_{I1} are as given in the following table.

Typical values of the factor F_{I1} (see Fig. 26)

Sweepback of wing (deg)	Aspect ratio of wing	
	3	6
0	0·34	0·45
45	0·24	0·45

† *Footnote* (1956): Certain difficulties which arose in connection with Schlichting's method were never completely resolved and Multhopp's method is, therefore, to be preferred when an accurate solution is required.

15.1.2. *The tail unit.*—We consider the aircraft flying at constant airspeed, and investigate the derivative with respect to lift coefficient, of the pitching moment of the aerodynamic forces acting on the horizontal tail unit.

With the mean effective elevator angle assumed fixed, we may write

$$\left.\frac{\partial C_{MT}}{\partial \bar{C}_L}\right|_{V,\bar{\eta}} = \frac{-S_T l_\alpha}{Sc}\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r \left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V,\bar{\eta}} \qquad (82)$$

where only $\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V,\eta}$ contains deformability effects. The mean angle of attack of the tailplane, $\bar{\alpha}_T$ may be written

$$\bar{\alpha}_T = \alpha_F + (\eta_T)_0 + \Delta\eta_T - \bar{\varepsilon} + \alpha_\delta \qquad (83)$$

where $(\eta_T)_0$ denotes the tailplane setting for the unloaded, and therefore undeformed, aircraft; $\Delta\eta_T$ is the change in setting due to deformability (given by equation (47)), $\bar{\varepsilon}$ denotes the mean angle of downwash over the tailplane span, and α_δ the additional angle of attack due to angular velocity in pitch. It is readily shown that:

$$\left(\left.\frac{\partial \alpha_\delta}{\partial \bar{C}_L}\right|_V\right) = \frac{1}{2\mu_1}$$

where $\mu_1 \equiv \frac{W}{g\rho S l_\alpha}$ is the 'aircraft relative density'. Using equations (83) and (47), and noting that $\left.\frac{\partial n}{\partial \bar{C}_L}\right|_V = Q$, we obtain:

$$\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V,\bar{\eta}} = \frac{1}{1 + w_\alpha Q}\left[\left.\frac{\partial \alpha_F}{\partial \bar{C}_L}\right|_V - \left.\frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\right|_V + \frac{1}{2\mu_1} + w_m Q\right]. \qquad (84)$$

In this equation, w_α and w_m depend directly on the bending deformability of the fuselage and the torsional deformability of the tailplane and its attachment, while $\left.\frac{\partial \alpha_F}{\partial \bar{C}_L}\right|_V$ and $\left.\frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\right|_V$ depend indirectly on the deformabilities of the wing and its attachment to the fuselage. Equation (84) may be rewritten

$$\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V,\bar{\eta}} = \frac{1}{1 + w_\alpha Q}\left[\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)_r + \Delta\left(\left.\frac{\partial \alpha_F}{\partial \bar{C}_L}\right|_V - \left.\frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\right|_V\right) + w_m Q\right]$$

where

$$\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)_r = \left[\left.\frac{\partial(\alpha_F - \bar{\varepsilon})}{\partial \bar{C}_L}\right|_V\right]_r + \frac{1}{2\mu_1}. \qquad (85)$$

Comparing with the value $\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V,\bar{\eta}}\right)_r$ for the completely rigid aircraft, we obtain as the increment $\Delta\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)$ due to deformability:

$$\begin{aligned}\Delta\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right) &= \left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V - \left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)_r \\ &= \frac{1}{1 + w_\alpha Q}\left[\Delta\left(\left.\frac{\partial \alpha_F}{\partial \bar{C}_L}\right|_V - \left.\frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\right|_V\right) + w_m Q\right] + \left[\frac{1}{1 + w_\alpha Q} - 1\right]\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)_r \\ &= \frac{1}{1 + w_\alpha Q}\left[\Delta\left(\left.\frac{\partial \alpha_F}{\partial \bar{C}_L}\right|_V - \left.\frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\right|_V\right) + w_m Q - w_\alpha Q\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)_r\right]. \qquad (86)\end{aligned}$$

We may introduce some previous results into this equation. Thus

$$\Delta\left(\frac{\partial \alpha_F}{\partial \bar{C}_L}\bigg|_V\right) = \frac{\partial n}{\partial \bar{C}_L}\bigg|_V \Delta\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$$

$$= Q\left[\Delta_1\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) + \Delta_2\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) + \Delta_3\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)\right]$$

$$= Q\left[\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) + \Delta\left\{\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right\}\right] \quad \ldots \quad (87)$$

from equations (73a) and (77). Deformability affects the downwash angle in virtue of the elastic wash-out of the wing which it produces. We denote the ratio between mean additional downwash and additional angle of attack of the wing at $y = 0$ by $\frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L,V}$; clearly this ratio cannot exceed unity. We may write

$$\Delta\left(\frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\bigg|_V\right) = Q\,\frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L,V} \Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right). \quad \ldots \quad (88)$$

Combining results (87) and (88) we have:

$$\Delta\left(\frac{\partial \alpha_F}{\partial \bar{C}_L}\bigg|_V - \frac{\partial \bar{\varepsilon}}{\partial \bar{C}_L}\bigg|_V\right) = Q\left[\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right)\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L,V}\right) + \Delta\left\{\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right\}\right],$$

and introducing this into equation (86) we obtain:

$$\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right) = \frac{Q}{1 + w_\alpha Q}\left[\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right)\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{C_L,V}\right) + \Delta\left\{\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right\}\right.$$
$$\left. + w_m - w_\alpha\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)_r\right]. \quad \ldots \quad (89)$$

From equation (82) the increase in $\frac{\partial C_{MT}}{\partial \bar{C}_L}\bigg|_V$ corresponding to this increase $\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)$ in $\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)$ is given by:

$$\left.\begin{aligned} \Delta\left(\frac{\partial C_{MT}}{\partial \bar{C}_L}\bigg|_V\right) &= D_{10}\Delta\left(\frac{\partial \alpha_T}{\partial \bar{C}_L}\bigg|_V\right) \\ \text{where} \qquad D_{10} &= -\frac{S_T l_\alpha}{Sc}\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r \end{aligned}\right\} \quad \ldots \quad (90)$$

To calculate $\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)$ by equation (89), we must first calculate w_α and w_m using equations (62), (63) and (64); if P_{Tu} is not known at the outset, an approximate value may be determined in the manner set out in Appendix IV. The quantity $\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)_r$ is given by equation (85), $\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right)$ by equation (73a) and $\Delta\left\{\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right\}$ by equation (77).

To evaluate the factor $\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L,V}\right)$ we must know the spanwise distribution of downwash at the tailplane due to the 'wash-out without lift' distribution of the wing. This, like the distribution of downwash along the fuselage axis (*see* section 15.1.1), may be determined, for a particular case, by a lifting-plane method such as that of Schlichting (Ref. 10) or Multhopp (Ref. 14), and in due course it should be possible to prepare charts for the determination of

$\partial \bar{\varepsilon}/\partial \alpha_{y=0}$ as a function of wing and tailplane geometry, for typical distributions of wing wash-out. Meanwhile, to give the reader an indication of the value of the factor in question, the spanwise distributions of downwash behind the four wings used in the investigation of the factor F_{I1} (section 15.1.1), have been determined by Schlichting's method and the results plotted in Fig. 25. Some details of the calculations are given in Appendix V. The wings were assumed all to have the same area S and the downwash was evaluated in the lifting plane at the same distance $1 \cdot 02\sqrt{S}$ behind the wing mean quarter-chord point in each case. Fig. 25 indicates that for unswept tailplanes of span equal to one-third of the wing span, the values of the factor $\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L,V}\right)$ would be approximately as given in the following table:

Typical values of the factor $\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L,V}\right)$ *for* $\frac{s_T}{s} = \frac{1}{3}$

Sweepback of wing (deg)	Aspect ratio of wing	
	3	6
0	0·43	0·56
45	0·51	0·64

15.1.3. *The fuselage and tail unit combined.*—Combining equations (80) and (90), we have:

$$\Delta\left(\frac{\partial C_{M\,F+T}}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right) = D_9 Q\, \Delta\left(F_I \frac{\partial \alpha_F}{\partial n}\bigg|_V\right) + D_{10}\, \Delta\left(\frac{\partial \alpha_T}{\partial \bar{C}_L}\bigg|_V\right) \qquad (91)$$

representing the overall effect of aircraft structural deformability on the derivative of the pitching-moment coefficient of the aerodynamic forces acting on the fuselage and tail unit.

15.1.4. *The complete aircraft.*—The overall effect of aircraft structural deformability on the pitching-moment coefficient derivative of the complete aircraft is obtained by adding to the last result, the effect of deformability on the derivative of the wing pitching-moment coefficient. The method of estimating the latter effect, denoted by $\Delta\left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right)$ is described in Part I. Thus we have for the complete aircraft:

$$\Delta\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right) = \Delta\left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right) + \Delta\left(\frac{\partial C_{M\,F+T}}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right)$$

$$= \Delta\left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right) + D_9 Q\, \Delta\left(F_I \frac{\partial \alpha_F}{\partial n}\bigg|_V\right) = D_{10}\, \Delta\left(\frac{\partial \alpha_T}{\partial \bar{C}_L}\bigg|_V\right) \qquad (92)$$

where $\Delta\left(\frac{\partial C_{M\,W}}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right)$ can be calculated from Part I, and where

$$D_9 = 2\frac{\frac{\pi}{4}\int_0^{l_F} b_F^2\, dx}{Sc},$$

$$D_{10} = -\frac{S_T l_\alpha}{Sc}\left(\frac{\partial C_{L\,T}}{\partial \alpha_T}\right)_r,$$

$\Delta\left(F_I \frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$ is deduced from equations (79), (73a), (74), (76) and (81), and $\Delta\left(\frac{\partial \alpha_T}{\partial \bar{C}_L}\bigg|_V\right)$ from equation (89). The forward shift of manoeuvre point due to deformability (in terms of wing chord) is, by equation (78), equal to the value of $\Delta\left(\frac{\partial C_M}{\partial \bar{C}_L}\bigg|_{V,\bar{\eta}}\right)$ given by equation (92).

16. *Discussion of Some Hitherto Neglected Effects.*—The main analysis has, at some points, been simplified by neglecting certain effects, and in reviewing these, we now consider the orders of magnitude of the more important ones.

16.1. *The Effect of Fuselage Bending on the Pitching Moment of the Fuselage.*—In estimating the effect of deformability on the pitching moment of the fuselage (section 15.1.1) the bending deformability of the fuselage itself was neglected. In assessing the magnitude of this neglected effect, we consider in turn the deformability of the parts of the fuselage lying respectively in front of, and behind the wing.

16.1.1. *Bending of the front part of the fuselage.*—In a pull-out under load factor n, the front part of the fuselage will bend downwards under the dominating influence of the downward inertia forces. For the purposes of a rough numerical calculation of the maximum effect, we may assume the relevant portion of the fuselage to have no reserve of strength, and replace the actual bending structure by a beam of constant effective depth h, with flanges designed so as to have the proof compressive strain at the proof load factor for the wing. Then the curvature of the centre-line will be given by:

$$\left(\frac{1}{R}\right)_F = \frac{2f_c}{E}\frac{1}{h}\frac{n}{n_u}.$$

Now if for our present purpose, the forward fuselage is assumed to be built in at the wing-root quarter-chord point and x is the distance measured forward from this station, the increment in local angle of attack of the fuselage is given by

$$\Delta_1\alpha_F(x) = -\int_0^x\left(\frac{1}{R}\right)_F dx = -\frac{x}{h}\frac{2f_c}{E}\frac{n}{n_u}, \qquad (x > 0)$$

or using the approximation

$$\frac{2f_c}{E} \simeq \frac{1}{150},$$

$$\Delta_1\alpha_F(x) = -\frac{x}{h}\frac{1}{150}\frac{n}{n_u} \qquad (x > 0).$$

The resulting change in fuselage moment is approximately equal (*see* Ref. 11) to:

$$-\frac{\pi}{2}q\int_0^{l_N}\frac{d}{dx}(\Delta_1\alpha_F b_F^2)x\,dx$$

$$= \frac{\pi}{2}q\int_0^{l_N}\Delta_1\alpha_F b_F^2\,dx$$

if $b_F = 0$ at the nose of the body. Substituting for $\Delta_1\alpha_F$ we have

$$\Delta_1 M_F \simeq -0\cdot 01\frac{1}{h}\frac{n}{n_u}q\int_0^{l_N}xb_F^2\,dx$$

where l_N denotes the length of the front part of the fuselage, which for simplicity, we will assume to be in the form of half an ellipsoid of revolution. Then we may write

$$\left(\frac{b_F}{b_{F\max}}\right)^2 = 1 - \left(\frac{x}{l_N}\right)^2,$$

in which case

$$\Delta_1 M_F \simeq -0\cdot 0025\frac{b_{F\max}^2 l_N^2}{h}\frac{n}{n_u}q$$

and

$$\Delta_1\left(\frac{\partial C_{MF}}{\partial \bar{C}_L}\bigg|_V\right) = Q\left\{\frac{\partial}{\partial n}\left(\frac{\Delta_1 M_F}{Scq}\right)\bigg|_V\right\} = -0\cdot 0025\frac{b_{F\max}^2 l_N^2}{Sch}\frac{Q}{n_u}$$

$$= -0\cdot 0025\left(\frac{b_{F\max}}{c}\right)^2\left(\frac{l_N}{c}\right)^2\left(\frac{c}{h}\right)\left(\frac{Q}{n_u}\right)\frac{1}{A}. \qquad \ldots \quad (93)$$

This corresponds to a rearward (favourable) displacement of the manoeuvre point, the approximate magnitude of which we now investigate.

Typical values for $b_{F\,\max}/c$ and c/h, associated with an average aspect ratio, say $A = 5$, would be $0 \cdot 6$ and $1 \cdot 75$ respectively, while Q/n_u will seldom exceed $5 \cdot 0$, so that the numerical value of the quantity $\Delta_1\left(\left.\frac{\partial C_{MF}}{\partial \bar{C}_L}\right|_V\right)$ is not likely to exceed $0 \cdot 0016\ (l_N/c)^2$. The value of the parameter l_N/c will be of the order $1 \cdot 5$ for an unswept wing ($A \simeq 5$) at a conventional location on the fuselage; this value would give a rearward shift of manoeuvre point of $0 \cdot 36$ per cent of the wing chord. l_N/c will decrease as the wing is swept back, and increase as the wing is swept forward. As a maximum value we may assume $2 \cdot 5$, which gives a maximum rearward shift of manoeuvre point of 1 per cent of the wing chord.

16.1.2. *Bending of the rear part of the fuselage.*—A contribution to fuselage moment arises also from the bending deflection of the rear part of the fuselage. We may write

$$\Delta_2\left(\left.\frac{\partial C_{MF}}{\partial \bar{C}_L}\right|_{V,\bar{\eta}}\right) = \left.\frac{\partial(\Delta_2 C_{MF})}{\partial(\Delta_2 \eta_T)}\right|_{V,\bar{\eta}} \left.\frac{\partial(\Delta_2 \eta_T)}{\partial \bar{C}_L}\right|_V = \left.\frac{\partial(\Delta_2 C_{MF})}{\partial(\Delta_2 \eta_T)}\right|_{V,\bar{\eta}} \left.\frac{\partial(\Delta_2 \bar{\alpha}_T)}{\partial \bar{C}_L}\right|_V$$

where $(\Delta_2 \eta_T)$ and $(\Delta_2 \bar{\alpha}_T)$ denote respectively the equal changes of tail setting and tailplane angle of attack due to bending of the rear fuselage. Now

$$\left.\frac{\partial(\Delta_2 \bar{\alpha}_T)}{\partial \bar{C}_L}\right|_V = \Delta_F\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right),$$

and from equation (89), putting terms due to wing-deformability and wing attachment-deformability equal to zero, and neglecting w_m we have:

$$\Delta_F\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right) = \frac{-\,(w_\alpha)_F Q}{1 + (w_\alpha)_F Q}\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)_r$$

where $(w_\alpha)_F$ denotes the value of the deformability coefficient corresponding to a completely rigid tailplane and attachment.

With the assumptions of section 13.1, the fuselage bending line is approximately parabolic, and hence if we now measure x forward from the elastic centre of the wing-root section, we have

$$\Delta_2 \alpha_F \simeq \frac{-\,x}{l_\alpha'}(\Delta_2 \eta_T), \qquad (x < 0)$$

and the corresponding change of fuselage moment is given by

$$\Delta_2 M_F = \frac{\pi}{2} q \int_{-l_R}^{0} \Delta_2 \alpha_F b_F^2\, dx = \frac{\pi}{2} q \frac{\Delta_2 \eta_T}{l_\alpha'} \int_0^{-l_R} x b_F^2\, dx$$

where l_R denotes the length of the rear fuselage and may be assumed approximately equal to $1 \cdot 1 l_\alpha'$. Hence

$$\left.\frac{\partial(\Delta_2 C_{MF})}{\partial(\Delta_2 \eta_T)}\right|_{V,\bar{\eta}} = \frac{\pi}{2} \frac{1}{S c l_\alpha'} \int_0^{-l_R} x b_F^2\, dx\,.$$

As a shape for the rear fuselage, we assume for simplicity a cone of length l_R, so that

$$b_F = b_{F\,\max}\left(1 + \frac{x}{l_R}\right).$$

Then

$$\left.\frac{\partial(\Delta_2 C_{MF})}{\partial(\Delta_2 \eta_T)}\right|_{V,\bar{\eta}} \simeq 0\cdot 158 \frac{b_{F\max}{}^2 l_\alpha'}{Ac^3}$$

and

$$\Delta_2\left(\left.\frac{\partial C_{MF}}{\partial \bar{C}_L}\right|_{V,\bar{\eta}}\right) \simeq \frac{-0\cdot 158(w_\alpha)_F Q}{1+(w_\alpha)_F Q}\left(\frac{b_{F\max}}{c}\right)^2\left(\frac{l_\alpha'}{c}\right)\frac{1}{A}\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V}\right)_r . \qquad \ldots \quad \ldots \quad \ldots \quad (94)$$

For an aircraft with unswept wing of aspect ratio $A = 5$ we may take as typical values at $Q = Q^*$:

$$\frac{(w_\alpha)_F Q}{1+(w_\alpha)_F Q} = 0\cdot 25; \quad \frac{b_{F\max}}{c} = 0\cdot 6; \quad \frac{l_\alpha'}{c} = 2\cdot 8; \quad \left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_{V}\right)_r = 0\cdot 125 .$$

Then from equation (94)

$$\Delta_2\left(\left.\frac{\partial C_{MF}}{\partial \bar{C}_L}\right|_{V,\eta}\right) \simeq -0\cdot 001 .$$

corresponding to a rearward displacement of manoeuvre point by $0\cdot 1$ per cent of the wing chord. The value of l_α'/c would be somewhat higher if the wing were swept-back and lower if it were swept-forward; the rearward shifts of manoeuvre point would be correspondingly greater or smaller.

16.1.3. *Combined effect of front and rear fuselage bending.*—From the examples considered in sections 16.1.1 and 16.1.2, the combined effect would appear to be very small for aircraft with unswept or swept-back wings, the net rearward shift of manoeuvre point due to the effect of the bending on fuselage moment being not more than about $0\cdot 5$ per cent of the wing chord. With swept-forward wings the net rearward shift might be as much as 1 per cent of the wing chord.

16.2. *The Effect of Engine Nacelles in the Wing.*—When considering the effect of elastic wash-out of the wing in Part I, it was assumed that no engine nacelles were installed in the wing. If, in fact, the aircraft under consideration has its power plants installed in nacelles ahead of the wing, they will, in a pull-out at load factor n, give an additional torque in the inner part of the wing. In making an approximate numerical assessment of the effect, we may consider the additional torque to be due to the weight of those parts of the installation lying ahead of the wing leading edge, the residual power-plant weight being included in the estimate of the wing weight parameter W_w (*see* Part I).

For a typical twin piston-engined aircraft with unswept wing, we may assume the weight ahead of the leading edge to be one-fifth of the all-up weight, and its centre of gravity to lie at a distance $0\cdot 75c$ ahead of the wing elastic axis, so that the additional torque is $-0\cdot 075nWc$ for each side of the wing. If the wing torsional stiffness is assumed to be in accordance with equation (1) of Part I, the additional angle of twist per unit length would be:

$$\Delta\left(\frac{d\alpha}{d|y|}\right) = \frac{-0\cdot 161n(1-F_\xi^*)}{\left(1-\frac{c_\xi}{c}\right)^2 Q^* s F_\tau} .$$

In practice, the torsional stiffness of the inner part of the wing will be about three times the mean torsional stiffness so that if, as typical values, we substitute $(1-F_\xi^*) = 0\cdot 8$, $c_\xi/c = 0\cdot 25$, $Q^* = 15$ and $F_\tau = 1\cdot 0$, we obtain as the additional angle of wash-out per unit length

$$-\Delta\left(\frac{da}{d|y|}\right) \simeq \frac{0\cdot 0051n}{s} .$$

If the distance between the two nacelle centre-lines is equal to one-third of the wing span, the angle of attack of the fuselage will, due to the effect in question, be larger than that of the wing at the nacelle centre-line by approximately $0\cdot 0017n$ radian $\simeq 0\cdot 1\ n$ degree.

In making a rough estimate of the effect of this additional elastic wash-out on the manoeuvre point, we may neglect the effect on wing pitching moment and on downwash. Then we obtain from equation (91), as the effect of power-plant moments:

$$\Delta_{\text{nac}}\left(\frac{x_{m.p.}}{c}\right) = D_9 Q \Delta_{\text{nac}}\left(\left.\frac{\partial \alpha_F}{\partial n}\right|_V\right) + D_{10}\Delta_{\text{nac}}\left(\left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)$$

$$= Q\Delta_{\text{nac}}\left(\left.\frac{\partial \alpha_F}{\partial n}\right|_V\right)\left\{D_9 + \frac{1}{1+w_\alpha Q}D_{10}\right\} \text{ from equation (84)}$$

$$= 0\cdot 0017 Q\left\{D_9 + \frac{1}{1+w_\alpha Q}D_{10}\right\}$$

with D_9, D_{10} given by equations (80a) and (90). Representative values for these two coefficients would be $D_9 = 0\cdot 2$, $D_{10} = -1\cdot 8$, while for a rigidly attached tailplane, w_α would be of the order $0\cdot 01$, so that at the maximum dynamic pressure ($Q = 15$) we should have:

$$\Delta_{\text{nac}}\left(\frac{x_{m.p.}}{c}\right) = 0\cdot 0255(0\cdot 2 - 1\cdot 565) \simeq -0\cdot 035\,.$$

The effect is thus seen to be favourable, and of perceptible magnitude.

It is not proposed to investigate in detail the corresponding effect for the case of an aircraft with swept wings. It may be observed, however, that an aircraft with heavily swept wings will normally have jet power plants which are relatively lighter than piston-engine installations and moreover, will have much smaller moment arms relative to the wing elastic axis, so that the torsion effect will be much smaller. Furthermore, the additional bending moment due to the power units will now decrease the wash-out, thus tending to neutralize the torsion effect.

16.3. *An effect of Deformability on the Manoeuvre Point with Elevator Free.*—Attention has so far been confined to the effects of deformability on manoeuvre point with elevator fixed, and it is not our intention to enter into a detailed discussion of the 'elevator free' case. Nevertheless there is, in this connection, one effect which should not be overlooked, since by accident or design, it may exert an unfavourable or a favourable influence on stability.

The effect in question arises when the elevator has some form of local aerodynamic balance. To simplify discussion, we may assume that the elevator itself is perfectly rigid, and furthermore that its $b_1 \left(= \left.\frac{\partial C_H}{\partial \alpha_T}\right|_{\bar{\eta}}\right)$ would be zero if the tailplane were completely rigid. Since the tailplane is not in fact rigid, but twists nose-up under an up-load, and *vice versa*, the value of $\left.\frac{\partial \eta_T}{\partial \bar{C}_L}\right|_V$ $\left(\text{and hence of } \left.\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\right|_V\right)$ will be larger over the outer part of the tailplane span than over the inner. Consequently, for a given C_{LT}, corresponding to a given mean tailplane setting and tailplane twist, there will be a contribution to elevator hinge moment due to tailplane angle of attack, whose sign will depend on whether the local aerodynamic balance is situated inboard or outboard. With a horn balance, for instance, the hinge moment will tend to increase the elevator angle, and hence to produce a rearward shift of manoeuvre point, elevator free.

17. *Procedure to be Adopted in Numerical Calculations.*—To complete Part II of the report we now summarize the procedure to be adopted in numerical calculations of the fuselage and tailplane effects, illustrating it by a worked example, which will also serve to indicate the order of magnitude of the effects, and how they are made up.

17.1. *Procedure.*—I. *Fuselage Contribution.*—

(*a*) Calculate the following contributions to $\Delta\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$:

Contribution	Due to	Equation of text	Remarks
$\Delta_1\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$	Elastic wash-out of wing	(73a)	For calculation of D_7, D_8 *see* Part I
$\Delta_2\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$	Elastic camber of wing	(74)	Small compared with $\Delta_1\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$
$\Delta_3\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$	Deformability of wing attachment	(76)	$F_{r.r.} = \pm 1$ gives maximum permissible positive and negative values of this contribution which may, to a large extent, be controlled by designer

(*b*) Estimate the value of the factor F_{I1}, either directly on the lines of Appendix V, or by reference to Fig. 26.

(*c*) Calculate the quantity $\Delta\left(F_I \frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$ from equation (79).

(*d*) Calculate the fuselage construction figure:

$$D_9 = 2\frac{\frac{\pi}{4}\int_0^{l_F} b_F^2\,dx}{Sc}\,.$$

(*e*) Calculate

$$\Delta\left(\frac{\partial C_{MF}}{\partial \bar{C}_L}\bigg|_V\right) = D_9 Q \Delta\left(F_I \frac{\partial \alpha_F}{\partial n}\bigg|_V\right) \text{ (equation (80)).}$$

This gives the fuselage contribution to the forward shift of manoeuvre point.

II. *Tailplane Contribution.*—

(*a*) In the first instance, neglect any deformability of the tailplane attachment, and calculate the deformability coefficients $w_\alpha = w_{\alpha 0}$ and $w_m = w_{m0}$ from equations (62), using equation (51) to estimate K_α, and equations (49), (53) and (54) to estimate K_F. In the absence of more precise information, the approximation $f_c/E = 1/300$ may be used (*see* section 13.4.1).

If the ultimate tail load P_{Tu} (which, it should be noted, should include the relieving inertia load due to the tail-end weight W_f) is unknown, an approximate value may be estimated using Appendix IV.

(*b*) Calculate $\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)_r$ from equation (85) and obtain values of:

$$\Delta\left(\frac{\partial \alpha_{y=0}}{\partial n}\bigg|_V\right) = \Delta_1\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) \text{ and } \Delta\left\{\frac{\partial(\alpha_F - \alpha_{y=0})}{\partial n}\bigg|_V\right\} = \Delta_2\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right) + \Delta_3\left(\frac{\partial \alpha_F}{\partial n}\bigg|_V\right)$$

from the preceding fuselage calculations.

(*c*) Estimate the downwash factor $\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L, V}\right)$ appropriate to a tailplane of the given span, either directly on the lines of Appendix V, or by reference to Fig. 15.

(*d*) Calculate $\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)$ from equation (89).

(*e*) Calculate the tailplane construction figure :

$$D_{10} = -\frac{S_T l_\alpha}{Sc}\left(\frac{\partial C_{LT}}{\partial \alpha_T}\right)_r .$$

(*f*) Calculate $\Delta\left(\frac{\partial C_{MT}}{\partial \bar{C}_L}\bigg|_V\right) = D_{10}\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)$ (equation (90)).

This gives the tailplane contribution to the forward shift of manoeuvre point, and will normally be negative.

(*g*) *Maximum effect of a deformable tailplane attachment with axis coinciding with the elevator neutral point.*—

(i) Calculate $w_\eta = w_{\eta 0}$ from equation (62), and substitute in equation (68), with $F_\eta{}^*$ equal to its minimum permissible value, to obtain the minimum permissible value of $w_\alpha, = (w_\alpha)_{\min}$.

(ii) Calculate from equation (69), the elevator effectiveness factor $F_{\eta 0}{}^*$ corresponding to a rigid tailplane attachment, and then calculate the flexibility of the attachment corresponding to $w_\alpha = (w_\alpha)_{\min}$ from equation (70).

(iii) Calculate Δw_m corresponding to the elastic attachment from equation (63) and hence obtain $w_m = w_{m0} + \Delta w_m$ (equation (64)).

(iv) Re-calculate the quantity $\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)$, using the new values of w_α and w_m, and substitute in equation (90) to obtain the tailplane contribution to shift of manoeuvre point including the effect of the elastic attachment.

III. *Complete Aircraft.*—The forward shift of manoeuvre point for the complete aircraft is obtained by adding the fuselage and tailplane contributions as calculated above, to the shift for wing alone, as calculated by Part I of the report.

17.2. *Illustrative Examples.*—An aircraft with the following geometry, illustrated by Fig. 23c, has been considered:

Wing :

Constant-chord, $A = 6$, $\phi = 45$ deg.

Other wing particulars, and overall aircraft particulars, as specified in the example of Appendix III.

Fuselage :

Ellipsoid of revolution ; axis ratio = 7 : 1.

Length l_F = wing span.

Wing mean quarter-chord point at $0\cdot 4 l_F$ from fuselage nose (Root quarter-chord point at $0\cdot 15 l_F$).

Tailplane :

Constant-chord, unswept, without end fins,

$$A_T = 4\cdot 0; \quad \frac{s_T}{s} = \frac{1}{3}; \quad \frac{c_T}{c} = \frac{1}{2}; \quad \frac{S_T}{S} = \frac{1}{6}.$$

Distance from mean quarter-chord point of wing to quarter-chord point of tailplane $= 2\cdot 5c\ (= 1\cdot 02\sqrt{S})$.

Elastic axis at $0\cdot 4c_T$; $e_T/c_T = 0\cdot 15$.

Additional data, assumed or derived :

Aircraft c.g. (aftmost position):

$0 \cdot 4c$ behind leading edge of mean chord.

Therefore $\quad l_\alpha = (2 \cdot 5 - 0 \cdot 15)c = 2 \cdot 35c.$

$l_G =$ distance of c.g. aft of root of wing elastic axis

$= 1 \cdot 5c\,.$

Therefore $\quad l_\alpha' = l_\alpha + l_G = 3 \cdot 85c.$

Aerodynamic centre of wing + fuselage (rigid) :

$0 \cdot 25c$ behind leading edge of mean chord.

Mean depth of fuselage structure, effective in bending:

$$h = \frac{l_\alpha'}{6}\,.$$

Distance between tailplane and elevator neutral points:

$$\delta l = 0 \cdot 25 c_T = 0 \cdot 125c\,.$$

Therefore $\quad l_\eta' = l_\alpha' + \delta l = 3 \cdot 975c\,; \quad \frac{l_\eta'}{h} = 6 \cdot 2\,.$

Tail volume ratio $\dfrac{S_T l_\alpha}{S\bar{c}} = 0 \cdot 392\,.$

$$\left(\frac{\partial C_{L\,T}}{\partial \alpha_T}\right)_r = 3 \cdot 5\,; \qquad \left(\frac{\partial C_{L\,T}}{\partial \eta}\right)_r = 2 \cdot 1$$

$$\frac{W_f}{W} = 0 \cdot 05\,; \qquad \frac{W_T}{W} = 0 \cdot 015$$

$$\left[\frac{\partial(\alpha_F - \bar{\varepsilon})}{\partial \bar{C}_L}\bigg|_V\right]_r = 0 \cdot 2\,.$$

Aircraft relative density (at sea-level):

$$\mu_1 = \frac{W}{g\rho_0 S l_\alpha} = 43\,.$$

$C_{L\,\max}$ (plain wing) $= 1 \cdot 4\,.$

Wing ribs in flight direction.

Wing rib bending factor $F_{r.b.} = 2 \cdot 5\,.$

Wing and tailplane torsional rigidity factors $F_\tau,\ F_\tau' = 1 \cdot 0\,.$

Pitching moment of inertia of aircraft, less tail end of fuselage $= i_B' \dfrac{W}{g} l_\alpha^2$ with $i_B' = 0 \cdot 075.$

For the purposes of this example, assume:

$$\frac{Q_d}{\beta_d} = \frac{Q^*}{\beta^*}_{\text{sea-level}} = \frac{21 \cdot 4}{\sqrt{\{1 - (0 \cdot 85)^2\}}} = 40 \cdot 6\,.$$

Required to calculate :

Shift of manoeuvre point at $Q = Q^*$, $M = 0 \cdot 85$ (sea-level conditions).

Summary of calculations :

Following the procedure laid down in section 17.1, the following quantities have been calculated:

I. *Fuselage.*—

(*a*) $\Delta_1\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) = 0\cdot00533$. (*See* Appendix III, section 2.2.1.)

$\Delta_2\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) = 0\cdot00065$.

$\Delta_3\left(\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) = \frac{0\cdot0101}{F_{r.r.}}$ where $|F_{r.r.}| \geqslant 1\cdot0$.

(*b*) $F_{I1} = 0\cdot45$ (from Fig. 26).

(*c*) $\Delta\left(F_I\left.\frac{\partial\alpha_F}{\partial n}\right|_V\right) = (0\cdot45 \times 0\cdot00533) + 0\cdot00065 + \frac{0\cdot0101}{F_{r.r}}$.

$= 0\cdot0024 + 0\cdot00065 + \frac{0\cdot0101}{F_{r.r.}}$.

(*d*) $D_9 = 0\cdot77$; $D_9Q = 16\cdot48$.

(*e*) $\Delta\left(\left.\frac{\partial C_{MF}}{\partial \bar{C}_L}\right|_V\right) = \Delta\left(\frac{x_{m.p.}}{c}\right)$

$= 0\cdot0395$ (contribution due to wing elastic wash-out)

$+ 0\cdot0107$ (contribution due to wing elastic camber)

$+ \frac{0\cdot165}{F_{r.r.}}$ (contribution due to elastic wing attachment)

$= 0\cdot0502 + \frac{0\cdot165}{F_{r.r.}}$.

Forward shift of manoeuvre point with rigid wing attachment ($F_{r.r.} \to \infty$) $= 0\cdot050c$.

Limiting values of forward shift, with elastic wing attachment corresponding to $F_{r.r.} = \pm 1$:

$$= \begin{array}{l} 0\cdot215c \text{ for } F_{r.r.} = 1 \\ -\,0\cdot115c \text{ for } F_{r.r.} = -\,1\,. \end{array}$$

II. *Tailplane.*—

(*a*) (i) *Estimation of ultimate tail load P_{Tu} by method of Appendix IV :*

We have $V_1{}' = 0\cdot8V'^*$; $Q_1 = 0\cdot64Q^*$

$$Q_2 = \frac{n_1}{C_{L\max}} = \frac{8}{1\cdot4} = 5\cdot71 = 0\cdot267Q^*$$

$$V_2{}' = \sqrt{(0\cdot267)}V'^* = 0\cdot517V'^*\,.$$

From equation (1) $K = 0\cdot102$ radn/sec^2 .

From equation (7) $-\frac{P_a(V_1{}')}{n_1W} = 0\cdot0045$.

From equation (9) $\frac{|P_{Tu}|}{12W} = 0\cdot75\left\{\frac{l_m(Q_1)}{2\cdot35c + l_m(Q_1)} - 0\cdot0455\right\}$;

or $= 0\cdot8955\left\{\frac{l_m(Q_2)}{2\cdot35c + l_m(Q_2)} - 0\cdot0455\right\} + 0\cdot00247$;

whichever is the greater.

Values of $l_m(Q)$ for $Q = Q_1, Q_2$:

$$l_m(Q) = l_m(0) + \text{forward shift of manoeuvre point of wing plus fuselage due to deformability at dynamic pressure number } Q,$$

where $$l_m(0) = (0 \cdot 4 - 0 \cdot 25)c = 0 \cdot 15c$$

The forward shifts due to deformability have been estimated by the methods of Part I and the present report as follows (wing elastic camber and attachment deformability neglected):

Contribution due to	$Q = Q_1$	$Q = Q_2$
Wing	$0 \cdot 123c$	$0 \cdot 053c$
Fuselage	$0 \cdot 033c$	$0 \cdot 015c$
Total	$0 \cdot 156c$	$0 \cdot 068c$

Hence we have:

$$l_m(Q_1) = (0 \cdot 150 + 0 \cdot 156)c = 0 \cdot 306c$$

$$l_m(Q_2) = (0 \cdot 150 + 0 \cdot 068)c = 0 \cdot 218c\,,$$

and it follows that:

$$\frac{|P_{Tu}|}{12W} = 0 \cdot 0521 \text{ (the second expressions above being equal to } 0 \cdot 0379)\,,$$

or $$\frac{|P_{Tu}|}{W} = 0 \cdot 625\,.$$

(ii) *Estimation of K_α and K_F:*

From equation (51) $K_\alpha = 0 \cdot 888$.

From equation (49) $K_F \simeq \dfrac{1}{300F_F}$. $\left(\dfrac{f_c}{E} \simeq \dfrac{1}{300}\right)$

From equation (53) $F_{F0} = 2 \cdot 705$.

Therefore

From equation (54) $F_F \geqslant 2 \cdot 705$. We take $\underline{F_F = 2 \cdot 705}$.

(Note that for the aircraft under consideration, the stiffness of the fuselage is determined by the criterion of Ref. 13 (a), and not by strength considerations.)

Hence $$K_F = \frac{1}{300 \times 2 \cdot 705}\,.$$

(iii) From equations (62):

$$w_\alpha = w_{\alpha 0} = 0 \cdot 0073\,; \quad w_m = w_{m0} = 0 \cdot 00118\,.$$

(*b*) From equation (85):

$$\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right)_r = 0 \cdot 2116\,.$$

From fuselage calculations:

$$\Delta \left(\frac{\partial \alpha_{y=0}}{\partial n} \bigg|_V \right) = \Delta_1 \left(\frac{\partial \alpha_F}{\partial n} \bigg|_V \right) = 0 \cdot 00533 \, ,$$

$$\Delta \left\{ \frac{\partial (\alpha_F - \alpha_{y=0})}{\partial n} \bigg|_V \right\} = 0 \cdot 00065 + \frac{0 \cdot 0101}{F_{r.r.}} \, .$$

(*c*) From Fig. 25:

$$\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}} \bigg|_{\bar{C}_L, V} \right) \simeq 0 \cdot 65 \, .$$

(*d*) From equation (89):

$$\Delta \left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L} \bigg|_V \right) = \frac{21 \cdot 4}{1 \cdot 156} \left[0 \cdot 00346 + \left\{ 0 \cdot 00065 + \frac{0 \cdot 0101}{F_{r.r.}} \right\} - 0 \cdot 00036 \right]$$

$$= 0 \cdot 0695 + \frac{0 \cdot 1872}{F_{r.r.}} \, .$$

(*e*) Construction figure $D_{10} = - 0 \cdot 392 \times 3 \cdot 5 = - 1 \cdot 372$.

(*f*)
$$\Delta \left(\frac{\partial C_{MT}}{\partial \bar{C}_L} \bigg|_V \right) = - 1 \cdot 372 \left\{ 0 \cdot 0695 + \frac{0 \cdot 187}{F_{r.r.}} \right\}$$

$$= - \left\{ 0 \cdot 095 + \frac{0 \cdot 256}{F_{r.r.}} \right\} .$$

Hence the tailplane contribution to rearward shift of manoeuvre point is $0 \cdot 095c$ if wing attachment is rigid, while the limiting values of the contribution, with elastic wing attachment, corresponding to $F_{r.r.} = \pm 1$ are:

$0 \cdot 351c$ rearward shift for $F_{r.r.} = + 1$

and $0 \cdot 161c$ forward shift for $F_{r.r.} = - 1$.

Combined effect of fuselage and rigidly mounted tailplane :

Resultant rearward shifts of manoeuvre point due to fuselage and rigidly mounted tailplane are:

Wing attachment	Resultant rearward shift of m.p.
Rigid	$(0 \cdot 095 - 0 \cdot 050)c = \underline{0 \cdot 045c}$
Elastic; $F_{r.r.} = + 1$	$(0 \cdot 351 - 0 \cdot 215)c = \underline{0 \cdot 136c}$
Elastic; $F_{r.r.} = - 1$	$(- 0 \cdot 161 + 0 \cdot 115)c = \underline{- 0 \cdot 046c}$

Maximum stabilizing or destabilizing effect of elastic wing attachment is $\pm 0 \cdot 091c$.

(g) *Maximum effect of a deformable tailplane attachment with axis coinciding with elevator neutral axis :*

Assume minimum permissible elevator effectiveness factor $F_\eta{}^* = 0 \cdot 2$.

(i) From equation (62):

$$w_\eta = w_{\eta 0} = 0 \cdot 0105 .$$

From equation (68):

$$(w_\alpha)_{\min} = -\,0 \cdot 0208 .$$

(ii) From equations (69)and (70) respectively:

$$F_{\eta 0}{}^* = 0 \cdot 801$$

$$\left(\frac{W c_T}{\lambda}\right)_{\max} = 0 \cdot 1945 .$$

(iii) From equation (63):

$$\Delta w_m = 0 \cdot 00029$$

and hence $w_m = 0 \cdot 00118 - 0 \cdot 00029 = 0 \cdot 00089$.

(iv) $$\Delta\left(\frac{\partial \bar{\alpha}_T}{\partial \bar{C}_L}\bigg|_V\right) = \frac{21 \cdot 4}{0 \cdot 555}\left[0 \cdot 00411 + \frac{0 \cdot 0101}{F_{r.r.}} + 0 \cdot 00089 + 0 \cdot 00440\right]$$

$$= \left\{0 \cdot 362 + \frac{0 \cdot 3895}{F_{r.r.}}\right\} .$$

Therefore

$$\Delta\left(\frac{\partial C_{MT}}{\partial \bar{C}_L}\bigg|_V\right) = -\,1 \cdot 372\left\{0 \cdot 362 + \frac{0 \cdot 3895}{F_{r.r.}}\right\}$$

$$= -\left\{0 \cdot 496 + \frac{0 \cdot 534}{F_{r.r.}}\right\} .$$

Hence, if wing attachment is rigid, the maximum contribution of an elastically mounted tailplane to the rearward shift of manoeuvre point is $0 \cdot 496c$; the limiting values of the rearward contribution with a deformable wing attachment are $-\,0 \cdot 038c$ ($F_{r.r.} = -\,1$) and $1 \cdot 030c$ ($F_{r.r.} = +\,1$).

Combined effect of fuselage and elastically mounted tailplane :

Resultant rearward shifts of manoeuvre point due to fuselage and elastically mounted tailplane are:

Wing attachment	Resultant rearward shift of m.p.
Rigid	$(0 \cdot 496 - 0 \cdot 050)c = \underline{0 \cdot 446c}$
Elastic; $F_{r.r.} = +\,1$	$(1 \cdot 030 - 0 \cdot 215)c = \underline{0 \cdot 815c}$
Elastic; $F_{r.r.} = -\,1$	$(-\,0 \cdot 038 + 0 \cdot 115)c = \underline{0 \cdot 077c}$

The increases in manoeuvre margin due to making the tailplane attachment elastic in the three cases are respectively $0 \cdot 401c$, $0 \cdot 679c$ and $0 \cdot 123c$.

III. *The Complete Aircraft.*—From Appendix III, section 2.2.1, the forward shift of manoeuvre point of wing alone (elastic wash-out effect only) is $0 \cdot 155c$. Hence, the net forward shifts of manoeuvre point for the complete aircraft with the various combinations of wing and tailplane attachments are as follows :

No.	Attachments		Net forward shift of manoeuvre point
	Wing	Tailplane	
1	Rigid	Rigid	$(0 \cdot 155 - 0 \cdot 045)c = 0 \cdot 110c$
2	Elastic $F_{r.r.} = 1$ $F_{r.r.} = -1$	Rigid	$(0 \cdot 155 - 0 \cdot 136)c = 0 \cdot 019c$ $(0 \cdot 155 + 0 \cdot 046)c = 0 \cdot 201c$
3	Rigid	Elastic ($F_\eta{}^* = 0 \cdot 2$)	$(0 \cdot 155 - 0 \cdot 446)c = -0 \cdot 291c$
4	Elastic $F_{r.r.} = 1$ $F_{r.r.} = -1$	Elastic ($F_\eta{}^* = 0 \cdot 2$)	$(0 \cdot 155 - 0 \cdot 815)c = -0 \cdot 660c$ $(0 \cdot 155 - 0 \cdot 077)c = 0 \cdot 078c$

17.3. *Discussion.*—The aircraft considered, with rigidly attached wing and tailplane, will, at its maximum dynamic pressure, suffer a considerable forward shift of manoeuvre point. Deformability of the wing attachment will decrease or increase the forward shift, according as the root region deformability factor $F_{r.r.}$ is positive or negative. By deliberately designing the attachment to give a positive value of $F_{r.r.}$, the forward shift can be practically eliminated. Either on its own, or in conjunction with a suitably designed elastic wing attachment, elastic mounting of the tailplane is a powerful device for eliminating any destabilizing shift of manoeuvre point due to the wing and fuselage.

It should be noted that in practice, flutter considerations may restrict the degree of stabilization which can be achieved by means of elastic attachments, to magnitudes less than those calculated in the above example.

LIST OF SYMBOLS

A, A_T	Aspect ratios of wing and tailplane
$A_1, A_2, \ldots A_n$	Fourier coefficients
B	Bending moment about an axis perpendicular to the elastic axis for the deformable wing
B_r	Bending moment corresponding to B, for the rigid wing
B_F	Vertical bending moment on fuselage
B_1, B_2, B_3, B_4	Construction figures (*see* Table 2)
C_H	Elevator hinge-moment coefficient
$C_L, \bar{C}_L$	Local and overall lift coefficients of wing
C_{LT}	Tailplane lift coefficient
C_M	Pitching-moment coefficient
C_{D0}	Drag coefficient for complete aircraft at zero lift
C_{DP}	Wing profile-drag coefficient

D, H	Unknown coefficients in expression for lift coefficient (equation (9) of Appendix II) corresponding to assumed linear and quadratic modes of elastic wash-out
$D_1, D_2, \ldots D_6$	Construction figures (*see* Table 2)
D_7, D_8	Construction figures (*see* equations 23 and 24)
D_9, D_{10}	Fuselage and tailplane construction figures (*see* equations (80a) and (90))
E	Young's modulus of elasticity
$(EI)_h, (EI)_v$	Bending rigidities about longitudinal and vertical principal axes respectively
$F_b, F_\tau, F_{r.b.}$	Wing stiffness factors (equations (2), (1a) and (3))
$F_{r.r.}$	Wing root-region stiffness factor (section 14.3)
F_F	Fuselage stiffness factor (section 13.1)
F_{F0}	*See* equation (53)
F_{f1}	Fuselage vertical stiffness of Ref. 13(a)
F_τ'	Tailplane torsional stiffness factor
F_I	*See* equation (81)
F_ξ, F_η	Aileron and elevator effectiveness factors (*see* Part I, section 1.1 and Part II, section 13.3)
G	Shear modulus
I	Moment of inertia of cross-section of beam representing wing taken perpendicular to elastic axis
I_F	Moment of inertia of cross-section of beam, representing fuselage for bending considerations
J	Polar moment of inertia of cross-section of equivalent torque tube taken perpendicular to elastic axis
GJ, GJ_T	Section torsional rigidities of wing and tailplane (sections normal to elastic axis)
K	Function of (Q^*B_1) and $(Q^*B_3 \tan \phi)$—*see* equation (35)
K_F, K_α	*See* equations (49) and (51)
L	Lift
L_ϕ	Non-dimensional bending stiffness of Ref. 7.
M	Pitching moment
M_θ	Non-dimensional torsional stiffness of Ref. 7
$\mathcal{M}$	Torque applied about axis of wing attachment
P_α, P_η	Tailplane loads arising respectively from change of tailplane angle of attack and from change of elevator angle (positive downwards)
P_m	Inertia load acting at tail (positive downwards)

P_a, P_b		Accelerating and balancing tail loads (Appendix IV), (positive downwards)
P_{Tp}, P_{Tu}		Proof and ultimate design tail loads (positive downwards)
Q		Dynamic pressure number $\left(= \frac{\text{dynamic pressure}}{\text{wing loading}}\right)$
$Q_{c\eta}$		Critical dynamic pressure number for elevator reversal
R		Radius of curvature of beam
S, S_T		Wing and tailplane areas
T, T_T		Wing and tailplane torques about their elastic axes
T_τ		Tailplane torsional stiffness measured at $0{\cdot}8s_T$ (Ref. 13(a))
V, V'		True air-speed; equivalent air-speed
W		Total aircraft weight
W_c		Weight of middle portion of fuselage (*see* section 13.5)
W_f		Weight of tail-end of fuselage, including complete tail unit
W_T		Weight of tailplane including end plate fins and rudders
W_w		Wing weight
$\mathscr{F}, \mathscr{G}, \mathscr{P}, \mathscr{Q}, \mathscr{W}, \mathscr{X}, \mathscr{Y}, \mathscr{Z}$		Functions of (QB_1) and $(QB_3 \tan \phi)$—*see* equations (22) and (25) of Appendix II
$\mathscr{U}$, $\mathscr{V}$		Functions of B_1, B_2, B_3 and B_4 and ϕ—*see* equation (21) of Appendix II
$\mathscr{C}$, $\mathscr{C}_2$		*See* equations (55) and (62)
b_F		Width of fuselage
c, c_T, c_ξ		Wing, tailplane and aileron chords (taken parallel to plane of symmetry)
e_A		Distance of elastic axis of tailplane attachment behind aerodynamic axis of tailplane
e_F		Chordwise distance of wing elastic axis aft of its axis of aerodynamic centres
e_F'	=	$e_F + \Delta e_F$
Δe_F		Forward displacement of section aerodynamic centre due to elastic camber
e_T		Distance of tailplane elastic axis behind aerodynamic axis
f		Ultimate stress
h, $\bar{h}$		Depth, mean depth, of fuselage structure taking bending
$i_B' \dfrac{W}{g} (l_\alpha^2)$		Pitching moment of inertia of aircraft less tail-end of fuselage (weight W_f)
l_F		Fuselage length
l_N, l_N'		Length of fuselage, ahead of wing-root quarter-chord point, and ahead of wing mean quarter-chord point respectively
l_R		Length of fuselage aft of elastic centre of wing-root section

$l_m(Q)$	Distance of manoeuvre point of aircraft less tail, ahead of aircraft c.g. (a function of Q)
l_α, l_η	Distances from aircraft c.g. to tailplane and elevator neutral points respectively
l_G	Distance of aircraft c.g. aft of section at which fuselage is assumed encastré
l_α', l_η'	$=$ $l_\alpha + l_G$, and $l_\eta + l_G$ respectively
δl	$=$ $l_\eta' - l_\alpha'$
n	Load factor
p	Proof stress
q	Dynamic pressure
$q_{c\xi}$, $q_{c\eta}$	Critical dynamic pressures for aileron reversal and elevator reversal respectively
$\dot{q}$	Angular acceleration (*see* Appendix IV)
r	Chordwise distance of inertia axis aft of elastic axis
s, s_T	Wing and tailplane semi-spans (measured perpendicular to plane of symmetry)
u_{pt}, u_{pc}	Proof tensile and compressive strains
w_α, w_η, w_m	Deformability coefficients defined by equation (47)
$w_{\alpha 0}$, $w_{\eta 0}$, $w_{m 0}$	Values of w_α, etc., corresponding to rigid tailplane attachment
x, y, z	Co-ordinates relative to axes fixed in the aircraft
Δ ; Δ_W	Increment; increment due to wing deformability
α, $\bar{\alpha}$	Angle of attack; mean angle of attack
α_F'	Local angle of attack of fuselage axis in presence of wing
α_δ	Additional angle of attack of tailplane due to angular velocity in pitch
β	$=$ $\sqrt{(1 - (\text{Mach number})^2)} = 1 \div$ (Prandtl-Glauert factor)
γ	Wing camber ratio
ε	Angle of downwash from wing
$\bar{\varepsilon}$	Mean value of ε across tailplane span
ζ	Spanwise parameter $= \dfrac{\lvert y \rvert}{s}$
η, $\bar{\eta}$	Elevator angle; mean elevator angle
η_{TL}	Angular setting of arbitrary tailplane section relative to fuselage datum
η_T	Mean effective tailplane setting relative to fuselage datum
θ	Angle of wash-out of wing
$\dot{\theta}$	Angular velocity in pitch
θ_T	Angle of twist (or negative wash-out) of tailplane

$\varkappa$	=	$\alpha_T - \Delta\eta_T$ = angle of attack which tailplane would have, if forces acting on it did not affect its incidence
λ		Stiffness of hypothetical torsional spring representing deformability of tailplane attachment
μ_1	=	$W/g\rho Sl_\alpha$
ρ		Air density
σ		Weight per unit volume of wing-attachment structural material.
τ		Thickness/chord ratio
ϕ		Angle of sweep, positive for sweepback
χ	=	$(QB_3 \tan\phi)$
ψ		Angular deflection of wing-root section relative to fuselage due to applied torque $\mathscr{M}$

Suffixes

$_A$	Corresponding to actual aspect ratio
$_{A/m}$	Corresponding to an aspect ratio $1/m$ times the actual aspect ratio
$_F$	Fuselage
$_R$	At reference section
$_T$	Tailplane
$_W$	Wing
$_b$	Due to bending
$_c$	Compressive
$_d$	At design diving speed for 10,000 ft.
$_f$	Flexible
$_{m.p.}$	Manoeuvre point
$_0$	Corresponding to $y = 0$
$_p$	Proof
$_r$	Completely rigid
$_{r.b.}$	Due to rib bending
$_t$	Tensile
$_u$	Ultimate
$_\tau$	Due to torsion

Indices

*	At maximum permissible dynamic pressure

Other Symbols

$\left(\frac{\partial}{\partial}\Big\vert_{V,\,etc.}\right)$	Denotes partial differentiation with V, etc., held constant

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Footnote 1956:

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13(*b*)	Vol. 1, Chapter 201 (Amended 1st February, 1954) and Vol. 2, Leaflet 201/1 (Issued 1st May, 1953)
13(*c*)	Vol. 2, Leaflet 500/3 (Issued 1st February, 1954)

APPENDIX I

An Approximate Strip Method for Estimating the Aerodynamic Effects of Elastic Wash-out

1. *Bending Moment in the Plane of Symmetry Due to a Symmetric Wash-out.*—We establish an approximation for the effective local lift slope corresponding to a symmetric wash-out, by obtaining, on the basis of lifting-line theory, an approximate formula for the bending moment in the plane of symmetry.

Consider in the first place an elliptic wing, unswept, of span $2s$ and root chord c_0 (aspect ratio $A = 8s/\pi c_0$).

Write

$$y = -s\cos\theta$$

$$c = c_0 \sin\theta .$$

Assuming the circulation in the form of a Fourier series,

$$\Gamma(\theta) = 4sV \,\Sigma\, A_n \sin n\theta \quad \ldots \quad (1)$$

it is easily shown* that the downwash is given by:

$$w \sin\theta = V \,\Sigma\, nA_n \sin n\theta , \quad \ldots \quad (2)$$

and the local lift coefficient by:

$$C_L = \frac{2\Gamma(\theta)}{cV} = \frac{8s}{c} \,\Sigma\, A_n \sin n\theta . \quad \ldots \quad (3)$$

But C_L is related to the local effective angle of attack. Thus

$$C_L = a_\infty \left(\alpha - \frac{w}{V}\right) \quad \ldots \quad (4)$$

where

$$a_\infty = \left(\frac{\partial C_L}{\partial \alpha}\right)_\infty .$$

Substituting for C_L and w from (3) and (2), we have:

$$\Sigma\, A_n(n\mu + 1) \sin n\theta = \mu\alpha \sin\theta , \quad \ldots \quad (5)$$

where

$$\mu = \frac{a_\infty c_0}{8s} = \frac{a_\infty}{\pi A} .$$

Thus

$$\int_0^\pi \{\Sigma\, A_n(n\mu + 1) \sin n\theta\} \sin n\theta \, d\theta = \int_0^\pi \mu\alpha \sin\theta \, \sin n\theta \, d\theta . \quad \ldots \quad (6)$$

For an arbitrary symmetric angle of attack distribution, the even terms vanish, and the odd terms are given by:

$$A_1 = \frac{2\mu}{\pi(\mu + 1)} \int_0^\pi \alpha \sin^2\theta \, d\theta = \frac{2\mu}{c_0 s\pi(\mu + 1)} \int_{-s}^{s} \alpha c \, dy$$

and

$$A_n = \frac{2\mu}{\pi(n\mu + 1)} \int_0^\pi \alpha \sin\theta \, \sin n\theta \, d\theta .$$

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* *See*, for instance, H. Glauert: '*The Elements of Aerofoil and Airscrew Theory*' (Cambridge University Press).

The bending moment in the plane of symmetry is given by:

$$q\int_0^s C_L cy\,dy = -4qs^3\int_0^{\pi/2}\Sigma A_n \sin n\theta \sin 2\theta\, d\theta$$

$$= 2qs^3\,\Sigma\,(-1)^{(n-1)/2}\frac{4}{n^2-4}A_n\,. \qquad \ldots \quad (7)$$

If the α distribution is a symmetric wash-out giving zero overall lift, then since the total lift is proportional to A_1 we must have $A_1 = 0$ and therefore, in the case of an elliptic wing $\int_0^s \alpha c\,dy = 0$. Equation (7) gives the exact lifting-line theory result for an unswept elliptic wing, and applies to any symmetric wash-out.

If we now assume that A_3 is the only non-zero coefficient then:

$$C_L = \frac{8s}{c}A_3 \sin 3\theta\,, \qquad \ldots \quad (8)$$

and it follows from equation (5) that the wash-out is a parabolic one given by:

$$\alpha = \frac{A_3(3\mu+1)}{\mu}\frac{\sin 3\theta}{\sin\theta} = \left\{\frac{A_3(3\mu+1)}{\mu}\right\}(4\cos^2\theta - 1)\,. \qquad \ldots \quad (9)$$

From equations (8) and (9) we have, since $c = c_0 \sin\theta$ and $A = 8s/\pi c_0$:

$$\left(\frac{\partial C_L}{\partial\alpha}\right) = \frac{\pi A\mu}{3\mu+1} = \frac{a_\infty}{\dfrac{3a_\infty}{\pi A}+1} = a_{A/3} = \text{constant}\,;$$

by lifting-line theory $\qquad \left[a_{A/3} \doteq \left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/3}\right]$.

Thus, in the special case of an elliptic wing with a parabolic wash-out giving zero overall lift, we may use an improved strip method for calculating the bending moment which is given by:

$$B_0 = q\int_0^s C_L cy\,dy = q\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/3}\int_0^s \alpha cy\,dy\,. \qquad \ldots \quad (10)$$

This result is quoted in Ref. 8 by Truckenbrodt who has shown that equation (10) gives a fair approximation to the general result (7) in the case of unswept wings with arbitrary symmetrical wash-outs of the type associated with the problems of this report, for which the major contribution to the zero-lift bending moment, as given by equation (7) with $A_1 = 0$, comes from the term in A_3.

When we come to consider wings of arbitrary chord distribution it must be noted that the strip method and lifting-line theory yield the same condition for zero lift only in the case of an elliptic wing. This implies that in using a strip method for the calculation of the zero lift bending moment, it is necessary, in general, to measure the wash-out from the wing no-lift line appropriate to the strip method. Thus α is replaced in equation (10) by $(\alpha + k)$ where in accordance with the strip method:

$$k = \frac{-\int_0^s \alpha c\,dy}{\int_0^s c\,dy}\,. \qquad \ldots \quad (11)$$

Thus for unswept wings of arbitrary chord distribution, we may assume the zero-lift bending moment to be given by:

$$B_0 \simeq q\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/3}\int_0^s (\alpha + k)cy\,dy\,, \qquad \ldots \quad (12)$$

where k is given by equation (11).

For the special case of constant-chord wings,

$$k = -\frac{1}{s}\int_0^s \alpha \, dy ,$$

and

$$B_0 \simeq q\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/3}\left[\int_0^s \alpha c y \, dy - \frac{c}{s}\int_0^s \alpha \, dy \left\{\int_0^s y \, dy\right\}\right]$$

$$= q\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/3}\int_0^s \alpha\left(y - \frac{s}{2}\right) c \, dy .$$

This equation may also be tolerably accurate for unswept wings with other chord distributions, but for swept wings, some further refinement may be tentatively suggested on the basis of the comparative calculations for a wing of aspect ratio 6 and sweepback 45 deg ($\tan \phi = 1$), discussed in section 11.1 of the report. Those calculations indicated that for such a wing, greater accuracy would be achieved by substituting $(\partial \bar{C}_L/\partial \alpha)_{A/4}$ for $(\partial \bar{C}_L/\partial \alpha)_{A/3}$, and employing the simplest formula of interpolation with respect to $\tan \phi$, it may be suggested that in general $(\partial \bar{C}_L/\partial \alpha)_{A/3}$ should be replaced by $(\partial \bar{C}_L/\partial \alpha)_{A/m}$ with $m = 3 + \tan \phi$

We are thus led to the equation:

$$B_0 = q\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m}\int_0^s \alpha\left(y - \frac{s}{2}\right) c \, dy \quad \ldots \quad (13)$$

as representing the best general approximation in our present state of knowledge. The coefficient m will depend on sweep angle and, to a presumably much smaller extent on aspect ratio and chord distribution. As the best available approximation, subject to modification in the light of future experience, we write:

$$m \simeq 3 + \tan \phi . \quad \ldots \quad (13a)$$

2. *The Pitching Moment on a Swept Wing Due to a Symmetric Wash-out with Zero Overall Lift.*—We may apply the approximate result of the last section to obtain an approximation for the pitching moment due to elastic wash-out on a constant-chord wing with aerodynamic axis swept back at an angle ϕ. If equation (12), with $A/3$ replaced by A/m is assumed to give the no-lift bending moment for such a wing, then the pitching moment for a half-wing, referred to the point in which the aerodynamic axis meets the plane of symmetry, is clearly given by that equation with the moment arm y replaced by $-y \tan \phi$. It then follows from equation (13) that the pitching moment due to an elastic wash-out $\Delta\alpha$, which gives zero overall lift, is given by:

$$\Delta M = -2q\left(\frac{\partial C_L}{\partial \alpha}\right)_{A/m} \tan \phi \int_0^s \Delta\alpha c\left(y - \frac{s}{2}\right) dy \quad \ldots \quad (14)$$

where $m \simeq 3 + \tan \phi$.

3. *Approximate Relationship Between the Local Angle of Attack and the Local Lift Coefficient.*—We can separate the local angle of attack into two parts thus:

$$\alpha = \bar{\alpha} + \Delta\alpha \quad \ldots \quad (15)$$

where $\bar{\alpha}$ is a uniform angle of attack across the span, and $\Delta\alpha$ is a wash-out yielding no overall lift. Now when we have either (*a*) an elliptic wing or (*b*) a constant-chord wing with an assumed rectangular loading, the lift coefficient is constant across the span. It therefore follows, that under these conditions we can write:

$$\bar{\alpha} = \bar{C}_L\left(\frac{\partial \alpha}{\partial \bar{C}_L}\right)_A . \quad \ldots \quad (16)$$

The remaining term $\Delta\alpha$ of equation (15), which is a 'zero-lift wash-out', can be dealt with by the approximate methods of section 1 of this appendix, from which we have derived $(\partial\bar{C}_L/\partial\alpha)_{A/m}$ ($m \simeq 3 + \tan\phi$), as a generally acceptable approximation for the local lift slope. We may thus write:

$$\Delta\alpha \simeq (C_L - \bar{C}_L)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m},$$

so that

$$\alpha \simeq \bar{C}_L\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_A + (C_L - \bar{C}_L)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m}. \qquad (17)$$

The first term is consistent with the assumption made throughout the report, of rectangular loading for the constant-chord wing at uniform angle of attack.

APPENDIX II

The Calculation of Elastic Wash-out for Swept-back Wings with Bending Effect Large in Relation to Torsion Effect

1. *Theory.*—If we consider the deformation of the wing to consist of bending about an axis perpendicular to the elastic axis, and torsion about the elastic axis, we may write:

$$\alpha = \alpha_0 + \cos\phi \int_0^{|y'|} \frac{T}{GJ}\, d|y'| - \sin\phi \int_0^{|y'|} \frac{B}{EI}\, d|y'|,$$

where α_0 is the angle of attack in the plane of symmetry ($y = 0$),

α is the local angle of attack for the section at a perpendicular distance y from the plane of symmetry,

$|y'|$ is distance measured along the elastic axis (so that $|y| = |y'|\cos\phi$),

T is the true torque about the elastic axis (positive if it reduces the wash-out),

B is the bending moment about an axis perpendicular to the elastic axis (positive if it tends to bend the wing upwards),

and GJ, EI denote respectively the torsional and bending rigidities of wing sections normal to the elastic axis.

Then

$$\alpha = \alpha_0 + \int_0^{|y|} \frac{T}{GJ}\, d|y| - \tan\phi \int_0^{|y|} \frac{B}{EI}\, d|y|. \qquad (1)$$

By considering the aerodynamic and inertia forces acting on an elementary chordwise strip at distance y from the plane of symmetry, we have:

$$\frac{dT}{d(s - |y|)} = \left\{C_L c q (e_F + \Delta e_F) + r\frac{W_w}{2s} n\right\} \cos\phi \qquad (2)$$

$$\frac{d^2B}{d(s - |y|)^2} = \frac{1}{\cos\phi}\left\{C_L c q - n\frac{W_w}{2s}\right\} \qquad (3)$$

where e_F is the chordwise distance of the elastic axis behind the line of aerodynamic centres of the undeformed wing profiles,

Δe_F is the forward displacement of the aerodynamic centre due to elastic camber, and r is the chordwise distance of the inertia axis behind the elastic axis.

Hence, writing $e_F' = e_F + \Delta e_F$, we have:

$$\alpha - \alpha_0 = \int_0^{|y|} \int_0^{s-|y|} \frac{\cos\phi}{GJ} \left\{ C_L c q e_F' + \frac{W_w}{2s} rn \right\} d(s-|y|)\, d|y|$$

$$+ \frac{\tan\phi}{(EI_0)\cos\phi} \int_{|y|}^{0} \frac{1}{\left(1 - \frac{|y|}{s}\right)^2} \int_0^{s-|y|} \int_0^{s-|y|} \left\{ C_L c q - n \frac{W_w}{2s} \right\} d(s-|y|)\, d(s-|y|)\, d|y| . \qquad (4)$$

α_0 must be chosen so that

$$\bar{C}_L = \frac{n}{Q} \qquad (5)$$

where $\bar{C}_L$ is the mean value of the lift coefficient. We now assume, in accordance with the approximation established in Appendix I, that α may be expressed in the form:

$$\alpha = \bar{C}_L \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_A + (C_L - \bar{C}_L) \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_{A/m} ,$$

where the suffices A and A/m denote that the respective lift slopes are appropriate to the actual aspect ratio and to an aspect ratio, $1/m$ times the actual. Hence we may write:

$$\alpha = \bar{C}_L \left\{ \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_A - \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_{A/m} \right\} + C_L \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_{A/m}$$

$$= C_L \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_{A/m} - \frac{n}{Q} \left\{ \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_{A/m} - \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_A \right\} . \qquad (6)$$

Also

$$\alpha - \alpha_0 = (C_L - C_{L0}) \left(\frac{\partial\alpha}{\partial \bar{C}_L}\right)_{A/m} . \qquad (7)$$

We now assume the mode of deformation to be such that we may write:

$$\alpha = \alpha_0 + D' \frac{|y|}{s} + H' \left(\frac{y}{s}\right)^2 , \qquad (8)$$

where D' and H' are constants, as yet unknown. Then in virtue of equation (7):

$$C_L = C_{L0} + D \frac{|y|}{s} + H \left(\frac{y}{s}\right)^2 \qquad (9)$$

where

$$D = D' \left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/m} \text{ and } H = H' \left(\frac{\partial \bar{C}_L}{\partial\alpha}\right)_{A/m} .$$

Comparing with the derivation of $\left.\frac{\partial \bar{C}_L}{\partial\alpha}\right|_V$ in section 3 of the main text, we take:*

$$\bar{C}_L = C_{L0} + 0{\cdot}455D + 0{\cdot}285H \qquad (10)$$

$$= \frac{n}{Q}$$

* Equation (10) is correct for a constant-chord wing of aspect ratio 7 with the angle of attack distribution given by (8), although it is not strictly consistent with (9) which is derived using the approximate relationship (7).

from equation (5). Hence

$$C_{L0} = \frac{n}{Q} - 0\cdot455D - 0\cdot285H\,, \quad \ldots \quad (11)$$

and so

$$C_L = \frac{n}{Q} + D\left(\frac{|y|}{s} - 0\cdot455\right) + H\left\{\left(\frac{y}{s}\right)^2 - 0\cdot285\right\}. \quad \ldots \quad (12)$$

If we substitute for C_L from equation (12), then equation (4) gives the change in angle of attack between an arbitrary section and the section in the plane of symmetry due to the lift distribution (12) acting on a wing with the assumed distributions of bending and torsional rigidity. Equation (7) with C_L given by equation (12), gives the corresponding change derived from purely aerodynamic considerations. Physically, the value of $(\alpha - \alpha_0)$ derived from deformability assumptions should, at all points of the span, equal that derived from the lift distribution. We therefore equate the two expressions for $(\alpha - \alpha_0)$ in equations (4) and (7) respectively and, substituting for C_L from (12) we obtain the following relation between the unknowns D and H:

$$\left[D\frac{|y|}{s} + H\left(\frac{y}{s}\right)^2\right]\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m} =$$

$$\frac{\cos\phi}{GJ}\int_0^{|y|}\int_0^{s-|y|}\left[\left\{\frac{n}{Q} + D\left(\frac{|y|}{s} - 0\cdot455\right) + H\left(\left(\frac{y}{s}\right)^2 - 0\cdot285\right)\right\}cqe_F' + \frac{W_w}{2s}rn\right] d(s-|y|)\,d|y|$$

$$+ \frac{\tan\phi}{(EI_0)\cos\phi}\int_{|y|}^0 \frac{1}{\left(1 - \frac{|y|}{s}\right)^2}\int_0^{s-|y|}\int_0^{s-|y|}\left[\frac{n}{Q} + D\left(\frac{|y|}{s} - 0\cdot455\right)\right.$$

$$\left. + H\left\{\left(\frac{y}{s}\right)^2 - 0\cdot285\right\}cq - \frac{W_w}{2s}n\right] d(s-|y|)\,d(s-|y|)\,d|y|\,.$$

Making the substitution

$$\frac{|y|}{s} = \zeta \quad \ldots \quad (13)$$

and differentiating with respect to n, we get:

$$\frac{dD}{dn}\zeta + \frac{dH}{dn}\zeta^2 =$$

$$QB_1\int_0^{\zeta}\int_{\zeta}^1\left\{\frac{1}{Q} + \frac{dD}{dn}(\zeta - 0\cdot455) + \frac{dH}{dn}(\zeta^2 - 0\cdot285)\right\}d\zeta\,d\zeta + B_2\int_0^{\zeta}\int_{\zeta}^1 d\zeta\,d\zeta$$

$$+ \tan\phi\,QB_3\int_{\zeta}^0\frac{1}{(1-\zeta)^2}\int_{\zeta}^1\int_{\zeta}^1\left\{\frac{1}{Q} + \frac{dD}{dn}(\zeta - 0\cdot455) + \frac{dH}{dn}(\zeta^2 - 0\cdot285)\right\}d\zeta\,d\zeta\,d\zeta$$

$$- \tan\phi\,B_4\int_{\zeta}^0\frac{1}{(1-\zeta)^2}\int_{\zeta}^1\int_{\zeta}^1 d\zeta\,d\zeta\,d\zeta\,, \quad \ldots \quad (14)$$

where

$$B_1 = \left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m}\frac{ce_F's^2\cos\phi}{GJ}\frac{W}{S}$$

$$= \frac{1\cdot075}{F_\tau Q^*}\frac{e_F'}{c}\frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2}\left(\frac{\partial\bar{C}_L}{\partial\alpha}\right)_{A/m} \quad \ldots \quad (15)$$

from equation (1a) of the main text,

$$B_2 = \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \frac{s^2 \cos \phi}{GJ} \frac{W_w}{2s} r$$

$$= \frac{1\cdot075\, r}{F_\tau Q^*} \frac{W_w}{c\, W} \frac{(1 - F_\xi^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \qquad (16)$$

$$B_3 = \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \frac{s^3 c}{(EI_0) \cos \phi} \frac{W}{S}$$

$$= \frac{A}{75 n_u \tau F_b \left(1 - \frac{W_w}{W}\right)} \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \qquad (17)$$

from equation (14) of the main text,

$$B_4 = \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \frac{s^3}{(EI)_0 \cos \phi} \frac{W_w}{2s}$$

$$= \frac{A}{75 n_u \tau F_b \left(\frac{W_w}{W} - 1\right)} \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/m} \qquad (18)$$

$$= \frac{W_w}{W} B_3 . \qquad (19)$$

In the expression for B_1, the quantity $e_F' = e_F + \Delta e_F$ depends on Q, since Δe_F is proportional to Q. Apart from this secondary effect, the coefficients B_1, B_2, B_3 and B_4 are independent of the particular flight condition under consideration. They depend on the geometry and the structural features of the wing and may be referred to as ' construction figures '.

Performing the integrations involved in equation (14) we finally arrive at the equation:

$$\frac{dD}{dn}\zeta + \frac{dH}{dn}\zeta^2 =$$

$$QB_1\left[\zeta(1 - 0\cdot5\zeta)\frac{1}{Q} + \zeta(0\cdot045 + 0\cdot2275\zeta - 0\cdot1667\zeta^2)\frac{dD}{dn} + \zeta(0\cdot0483 + 0\cdot1425\zeta - 0\cdot0833\zeta^3)\frac{dH}{dn}\right] + B_2\zeta(1 - 0\cdot5\zeta)$$

$$+ QB_3 \tan \phi \left[-\frac{\zeta}{2}\frac{1}{Q} - \zeta(0\cdot0833\zeta + 0\cdot1058)\frac{dD}{dn} - \zeta(0\cdot02778\zeta^2 + 0\cdot0833\zeta + 0\cdot1075)\frac{dH}{dn}\right]$$

$$+ B_4 \tan \phi \frac{\zeta}{2} . \qquad (20)$$

This equation is always satisfied for $\zeta = 0$, since both sides reduce to zero. From physical conditions, this result is to be expected, because the expression on each side is proportional to the difference in angle of attack between sections lying respectively in the plane of symmetry and at distance ζs from that plane. Clearly each expression must vanish in the plane of symmetry for which $\zeta = 0$. It is not possible to satisfy the equation at all other points of the span because of the restrictive assumption that the wash-out corresponds to a superposition of only two distinct modes. It is in fact possible to satisfy the equation at only two further arbitrarily chosen points, since D and H are then uniquely determined. As our two points we select $\zeta = 1$ and $\zeta = 0{\cdot}5$ and substituting these values in turn, in equation (20) we obtain:

$$\frac{dH}{dn} = \frac{0{\cdot}5\Big[(B_1 + B_2) - \tan\phi\,(B_3 - B_4)\Big] - \dfrac{dD}{dn}\Big[1 - 0{\cdot}1058QB_1 + 0{\cdot}1891QB_3\tan\phi\Big]}{(1 - 0{\cdot}1075QB_1 + 0{\cdot}2186QB_3\tan\phi)},$$

and

$$\frac{dH}{dn} = \frac{\Big[0{\cdot}375(B_1 + B_2) - 0{\cdot}25\tan\phi(B_3 - B_4)\Big] - \dfrac{dD}{dn}\Big[0{\cdot}5 - 0{\cdot}0585QB_1 + 0{\cdot}0738QB_3\tan\phi\Big]}{(0{\cdot}25 - 0{\cdot}0546QB_1 + 0{\cdot}0781QB_3\tan\phi)}.$$

If we write

$$\left.\begin{aligned}\mathscr{U} &\equiv 0{\cdot}5\,[(B_1 + B_2) - \tan\phi(B_3 - B_4)]\\ \mathscr{V} &\equiv 0{\cdot}375\,(B_1 + B_2) - 0{\cdot}25\tan\phi(B_3 - B_4)\end{aligned}\right\} \qquad (21)$$

$$\left.\begin{aligned}\mathscr{W} &\equiv 1 - 0{\cdot}1058QB_1 + 0{\cdot}1891QB_3\tan\phi\\ \mathscr{X} &\equiv 0{\cdot}5 - 0{\cdot}0585QB_1 + 0{\cdot}0738QB_3\tan\phi\\ \mathscr{Y} &\equiv 1 - 0{\cdot}1075QB_1 + 0{\cdot}2186QB_3\tan\phi\\ \mathscr{Z} &\equiv 0{\cdot}25 - 0{\cdot}0546QB_1 + 0{\cdot}0781QB_3\tan\phi\end{aligned}\right\}, \qquad (22)$$

the two expressions for dH/dn become:

$$\frac{dH}{dn} = \frac{\mathscr{U} - \mathscr{W}\dfrac{dD}{dn}}{\mathscr{Y}} = \frac{\mathscr{V} - \mathscr{X}\dfrac{dD}{dn}}{\mathscr{Z}},$$

from which we deduce:

$$\frac{dD}{dn} = \frac{\mathscr{Z}\mathscr{U} - \mathscr{Y}\mathscr{V}}{\mathscr{Z}\mathscr{W} - \mathscr{X}\mathscr{Y}} \qquad (23)$$

$$\frac{dH}{dn} = \frac{\mathscr{W}\mathscr{V} - \mathscr{X}\mathscr{U}}{\mathscr{Z}\mathscr{W} - \mathscr{X}\mathscr{Y}} \qquad (24)$$

or, writing

$$\mathscr{F} = \frac{\mathscr{Y}}{\mathscr{Z}}\,; \quad \mathscr{G} = \frac{\mathscr{X}}{\mathscr{W}}\,; \quad \mathscr{P} = \mathscr{W} - \mathscr{F}\mathscr{X}\,; \quad \mathscr{Q} = -\mathscr{Z} + \mathscr{G}\mathscr{Y} \qquad (25)$$

$$\frac{dD}{dn} = \frac{\mathscr{U} - \mathscr{F}\mathscr{V}}{\mathscr{P}} \qquad (26)$$

$$\frac{dH}{dn} = \frac{-\mathscr{V} + \mathscr{G}\mathscr{U}}{\mathscr{Q}}. \qquad (27)$$

From equations (7), (8) and (13):

$$\alpha - \alpha_0 = (D\zeta + H\zeta^2)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m}$$

where, from equations (6) and (11):

$$\begin{aligned}\alpha_0 &= \frac{n}{Q}\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_A - (0{\cdot}455D + 0{\cdot}285H)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m}\\ &= \alpha_r - (0{\cdot}455D + 0{\cdot}285H)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/m}.\end{aligned}$$

Hence

$$\Delta\alpha = \alpha - \alpha_r = \left\{ D(\zeta - 0\cdot455) + H(\zeta^2 - 0\cdot285) \right\} \left(\frac{\partial\alpha}{\partial\bar{C}_L} \right)_{A/m},$$

or differentiating with respect to n at constant flight speed V we obtain the following relation for elastic wash-out:

$$-\Delta\left(\frac{\partial\alpha}{\partial n}\bigg|_V \right) = -\left\{ \frac{dD}{dn}(\zeta - 0\cdot455) + \frac{dH}{dn}(\zeta^2 - 0\cdot285) \right\} \left(\frac{\partial\alpha}{\partial\bar{C}_L} \right)_{A/m}. \quad \ldots \quad \ldots \quad (28)$$

The change in angle of attack of the fuselage with rigid root region and attachment is given by substituting $|y| = \zeta = 0$ in the last equation. Thus

$$\Delta\left(\frac{\partial\alpha_{y=0}}{\partial n}\bigg|_V \right) = -\left(0\cdot455\frac{dD}{dn} + 0\cdot285\frac{dH}{dn} \right) \left(\frac{\partial\alpha}{\partial\bar{C}_L} \right)_{A/m}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

2. *Graphical Aids to Numerical Calculation.*—As will be seen from equations (28) and (29), the calculation of the quantities $\Delta\left(\frac{\partial\alpha}{\partial n}\bigg|_V \right)$ and $\Delta\left(\frac{\partial\alpha_{y=0}}{\partial n}\bigg|_V \right)$ involves the calculation of dD/dn and dH/dn from equations (26) and (27) which involve in their turn, calculation of the quantities $\mathscr{U}$ and $\mathscr{V}$, $\mathscr{F}$, $\mathscr{G}$, $\mathscr{P}$ and $\mathscr{Q}$. In the expressions for $\mathscr{U}$ and $\mathscr{V}$ (equations (21))

$$(B_1 + B_2) = \frac{1\cdot075}{F_\tau Q^*}\left(\frac{e_F{}'}{c} + \frac{W_w}{W}\frac{r}{c} \right) \frac{(1 - F_\xi{}^*)}{\left(1 - \frac{c_\xi}{c}\right)^2} \left(\frac{\partial\bar{C}_L}{\partial\alpha} \right)_{A/m}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

and

$$(B_3 - B_4) = \left(1 - \frac{W_w}{W} \right) B_3 = \frac{A\left(\frac{\partial\bar{C}_L}{\partial\alpha} \right)_{A/m}}{75n_u\tau F_b}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

Apart from the secondary effect of Q on $(B_1 + B_2)$ through the quantity $e_F{}' = e_F + \Delta e_F$, $(B_1 + B_2)$ and $(B_3 - B_4)$ and hence $\mathscr{U}$ and $\mathscr{V}$ depend solely on the geometrical and structural features of the wing and must be directly calculated.

$\mathscr{F}$, $\mathscr{G}$, $\mathscr{P}$ and $\mathscr{Q}$ are given in (25) as functions of $\mathscr{W}$, $\mathscr{X}$, $\mathscr{Y}$ and $\mathscr{Z}$ which from equations (22) are observed to be linear functions of two variables, *viz.*,

$$(QB_1) = \frac{Q}{Q^*}\frac{1\cdot075\frac{e_F{}'}{c}}{F_\tau}\frac{(1 - F_\xi{}^*)}{\left(1 - \frac{c_\xi}{c}\right)^2}\left(\frac{\partial\bar{C}_L}{\partial\alpha} \right)_{A/m}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

and

$$(QB_3\tan\phi) = \frac{QA\tan\phi\left(\frac{\partial\bar{C}_L}{\partial\alpha} \right)_{A/m}}{75n_u\tau F_b\left(1 - \frac{W_w}{W}\right)}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

and thus directly involve the dynamic pressure number Q in addition to the geometrical and structural features of the wing.

To facilitate calculation, the four functions $\mathscr{F}$, $\mathscr{G}$, $\mathscr{P}$ and $\mathscr{Q}$ have, in Figs. 6 to 9, been plotted against $(QB_3\tan\phi)$ for various values of (QB_1), while for rapid estimation of the quantities $dD/dn = (-\mathscr{U} + \mathscr{F}\mathscr{V})/(-\mathscr{P})$ and $dH/dn = (-\mathscr{V} + \mathscr{G}\mathscr{U})/\mathscr{Q}$, nomograms have been prepared and are presented in Figs. 10 and 11.

3. *The Validity Criterion.*—In section 7.1 of the main text the criterion for the validity of equation (28) is obtained in the form:

$$0\cdot545\,\frac{dD^*}{dn} + 0\cdot715\,\frac{dH^*}{dn} < 0\,.$$

Using equations (23), (24) and (25), this becomes:

$$\frac{1}{\mathscr{X}^*\mathscr{W}^*(1 - \mathscr{F}^*\mathscr{G}^*)}\left\{0\cdot545(\mathscr{X}^*\mathscr{U}^* - \mathscr{Y}^*\mathscr{V}^*) + 0\cdot715(\mathscr{W}^*\mathscr{V}^* - \mathscr{X}^*\mathscr{U}^*)\right\} < 0\,.$$

Now within the practical ranges of values of $Q^*B_1{}^*$ and $Q^*B_3 \tan\phi$ considered for swept-back wings (viz., $0 \leqslant Q^*B_1{}^* \leqslant 2$ and $0 \leqslant Q^*B_3 \tan\phi \leqslant 25$) $\mathscr{X}^*$ and $\mathscr{W}^*$ are seen from inspection of the relevant equations (22) to be both positive, while from Figs. 6 and 7 it can readily be established that $\mathscr{F}^*\mathscr{G}^*$ is greater than unity so that the factor $(1 - \mathscr{F}^*\mathscr{G}^*)$ is negative. It follows that the inequality to be satisfied is:

$$(\mathscr{X}^* - 0\cdot7622\,\mathscr{X}^*)\,\mathscr{U}^* < (\mathscr{W}^* - 0\cdot7622\mathscr{Y}^*)\,\mathscr{V}^*\,,$$

i.e.,

$$\{0\cdot3094 - 0\cdot0169Q^*B_1{}^* + 0\cdot0143Q^*B_3 \tan\phi\}\mathscr{U}^*$$
$$< \{0\cdot2378 - 0\cdot0239Q^*B_1{}^* + 0\cdot0225Q^*B_3 \tan\phi\}\mathscr{V}^*\,.$$

The coefficient of $\mathscr{U}^*$ is always positive within the ranges of $Q^*B_1{}^*$ and $Q^*B_3 \tan\phi$ considered, and hence the inequality may finally be written:

$$\mathscr{U}^* - K\mathscr{V}^* < 0\,, \qquad \ldots \qquad (34)$$

where

$$K = \frac{0\cdot2378 - 0\cdot0239Q^*B_1{}^* + 0\cdot0225Q^*B_3{}^* \tan\phi}{0\cdot3094 - 0\cdot0169Q^*B_1{}^* + 0\cdot0143Q^*B_3 \tan\phi}\,.$$

The function K is plotted against $Q^*B_3 \tan\phi$ for various values of $Q^*B_1{}^*$ in Fig. 12.

APPENDIX III

Numerical Example of the Effect of Elastic Wash-out: Summary of Comparative Calculations by Two Methods

1. *Assumed Data.*—1.1. *General.*—

$W = 12{,}500$ lb

$\dfrac{W_w}{W} = 0\cdot14$

$S = 250$ ft^2

$A = 6$

$\tau = 0\cdot12$

$\phi = 45$ deg

$c_\xi/c = 0\cdot25$

$F_\xi{}^* = 0\cdot20$

$n_u = 12\cdot0$

Maximum normal acceleration

$n = 8\cdot0$

$Q^* = 21\cdot4$ (corresponding to

$V = 647$ m.p.h. at sea-level, Mach number $= 0\cdot85$)

Elastic axis at $0\cdot40c$

Inertia axis at $0\cdot45c$

Therefore $r/c = 0\cdot05$

1.2. *Stiffnesses.*—Bending and torsional rigidities of wing assumed to be as given by equations (14) and (1a) respectively with $F_b = F_\tau = 1 \cdot 0$. Effect of rib bending neglected ($\Delta e_F = 0$).

$$EI = 5 \cdot 13 \times 10^7 (1 - \zeta)^2 \text{ lb ft}^2$$

$$GJ = 7 \cdot 73 \times 10^6 \text{ lb ft}^2 .$$

1.3. *Aerodynamic.*—Low-speed *overall lift slopes* (from Ref. 9, solutions 14 and 25)

$$a = \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A=6} = 3 \cdot 365$$

$$\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/3=2} = 2 \cdot 30 .$$

Since calculations are purely comparative, no compressibility corrections will be applied to these coefficients.

Basic spanwise lift distribution and position of local aerodynamic centre : See Figs. 15 and 16.

(*a*) *For method of Ref.* 1.—Spanwise distribution and local aerodynamic centres as given by Ref. 9.

(*b*) *For method of present report.*—Uniform spanwise distribution. Local aerodynamic centre at $0 \cdot 23c$ (constant). This is the estimated mean value across the span, weighted with respect to distance from centre-line. Corresponding value of e_F/c is $0 \cdot 17$.

2. *Derived Quantities for* $Q = Q^* = 21 \cdot 4$ ($q = 1070$ lb/ft^2).—2.1. *Method of Ref.* 1. (*Notation as given in Ref.* 1.) *Note.—All integrations have been carried out graphically.*—The bending and torsion modes ϕ and χ corresponding to the bending moment B_0 and torque T_0 due to the basic loading are shown in Fig. 17.

Corresponding deflections at reference section (wing tip) are:

$$\left.\begin{aligned} z_{r0} &= 0 \cdot 467n \\ \theta_{r0} &= 0 \cdot 00868n \end{aligned}\right\} \quad \ldots \quad (1)$$

Rates of change of internal elastic energy G with the co-ordinates z_r and θ_r are:

$$\left.\begin{aligned} \frac{\partial G}{\partial z_r} &= z_r \int_0^{s_f} \frac{1}{EI} \left(\frac{B_0}{z_{r0}}\right)^2 dy_f = 3640 z_r \\ \frac{\partial G}{\partial \theta_r} &= \theta_r \int_0^{s_f} \frac{1}{CJ} \left(\frac{T_0}{\theta_{r0}}\right)^2 dy_f = 3 \cdot 72 \times 10^5 \theta_r \end{aligned}\right\} \quad \ldots \quad (2)$$

Wash-out relative to the root is:

$$\psi = -\left(\frac{dz}{dy_f}\right) \sin \Gamma_f + \theta \cos \Gamma_f$$

where $\quad z = \phi z_r , \quad \theta = \chi \theta_r , \quad \Gamma_f = 45 \text{ deg}, \quad y_f = 27 \cdot 4 \zeta ,$

whence

$$\psi = -0 \cdot 04465 z_r \frac{\left(\frac{d\phi}{d\zeta}\right)}{\left(\frac{d\phi}{d\zeta}\right)_{\zeta=1}} + 0 \cdot 707 \theta_r \chi . \quad \ldots \quad (3)$$

The mode $\dfrac{\left(\frac{d\phi}{d\zeta}\right)}{\left(\frac{d\phi}{d\zeta}\right)_{\zeta=1}} = \Phi$ say, is plotted in Fig. 17.

Lift Distributions Due to Unit Modes of Wash-out, Φ and χ.—The lift distribution due to each unit mode has been obtained by superimposition of a ' wash-out without lift ' (W.W.L.) distribution (associated with a certain change α_0 of root angle of attack) and the ' lift without wash-out ' (L.W.W.) distribution corresponding to a uniform angle of attack $-\alpha_0$.

The W.W.L. distributions have been calculated by the method of Ref. 10 and are given by:

Mode Φ

$$\left.\begin{aligned} \frac{C_{LL}}{\alpha_0} &= 0\cdot0483e^{-\zeta_1^2} - 0\cdot0144 \sin\Theta - 1\cdot8336 \sin 3\Theta + 0\cdot2196 \sin 5\Theta \\ \text{where} \quad \Theta &= \cos^{-1}\zeta \text{ and } \zeta_1 = \frac{A}{2}\zeta = 3\zeta\,, \\ \text{with} \quad \alpha_0 &= -0\cdot4897 \end{aligned}\right\} \quad \ldots \quad (4)$$

Mode χ

$$\left.\begin{aligned} \frac{C_{LL}}{\alpha_0} &= 0\cdot0746e^{-\zeta_1^2} - 0\cdot0168 \sin\Theta - 1\cdot5936 \sin 3\Theta + 0\cdot3900 \sin 5\Theta \\ \text{with} \quad \alpha_0 &= -0\cdot5712 \end{aligned}\right\} \quad \ldots \quad (5)$$

The distributions are shown graphically in Fig. 18. The L.W.W. distributions are given by:

$$\frac{C_{LL}}{-\alpha_0} = \left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A=6} \times \frac{C_{LL0}}{\bar{C}_{L0}} = 3\cdot365 \frac{C_{LL0}}{\bar{C}_{L0}}\,. \quad \ldots \quad (6)$$

Using equations (3), (4), (5) and (6) the incremental lift distribution due to the washout ψ has been estimated in terms of z_r and θ_r, and the corresponding increments of bending moment and torque added to B_0 and T_0 to obtain the bending moment B and torque T for the deformed wing. The virtual work W_z and W_θ done by the external forces in unit changes of z_r and θ_r have then been calculated from the expressions:

$$\left.\begin{aligned} W_z &= \int_0^{s_f} \phi \frac{d^2B}{dy_f^2} dy_f \\ W_\theta &= \int_0^{s_f} \chi \left(-\frac{dT}{dy_f}\right) dy_f \end{aligned}\right\}, \quad \ldots \quad (7)$$

and equated to $\partial G/\partial z_r$ and $\partial G/\partial \theta_r$ (equations (2)) respectively. The resulting simultaneous equations in z_r and θ_r for $q = 1070$ lb/ft² and load factor n are:

$$\left.\begin{aligned} 7\cdot398 z_r - 70\cdot9\theta_r &= 1\cdot609n \\ \text{and} \quad 5\cdot51 z_r + 247\cdot1\theta_r &= 3\cdot017n \end{aligned}\right\} \quad \ldots \quad (8)$$

$$\left.\begin{aligned} \text{whence} \quad z_r &= 0\cdot276n \\ \text{and} \quad \theta_r &= 0\cdot00607n \end{aligned}\right\} \quad \ldots \quad (9)$$

Equations (3), (4), (5), (6) and (9) have now been used to obtain the incremental lift distribution per unit load factor $\Delta_1 C_{LL}/n$ at $q = 1070$ lb/ft² when the root angle of attack remains unchanged. The increment in overall lift coefficient has been calculated from the equation:

$$\frac{\Delta_1 \bar{C}_L}{n} = \int_0^1 \frac{\Delta_1 C_{LL}}{n} d\zeta\,. \quad \ldots \quad (10)$$

Estimation of Effective Lift Slope A.—

For the rigid wing:

$$\frac{\bar{C}_{L0}}{n} = \frac{W/S}{q} = \frac{50}{1070} = 0\cdot 04675\,.$$

For the flexible wing at the same root angle of attack:

$$\frac{\bar{C}_L}{n} = \frac{\bar{C}_{L0}}{n} + \frac{\Delta_1 \bar{C}_L}{n}$$

$$= \frac{\bar{C}_{L0}}{n} + \int_0^1 \frac{\Delta_1 C_{LL}}{n}\, d\zeta\,.$$

Therefore

$$\left.\begin{aligned} \frac{\bar{C}_L}{\bar{C}_{L0}} &= \frac{A}{a} = 1 + \frac{\int_0^1 \frac{\Delta_1 C_{LL}}{n}\, d\zeta}{0\cdot 04675}\,, \\ \frac{A}{a} &= 0\cdot 743 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

from detailed calculations so that to maintain same overall lift with deformable wing as with rigid wing, the root angle of attack must be increased in the ratio $1/0\cdot 743 = \underline{1\cdot 346}$.

Elastic Wash-out from Root to Tip for Flexible Wing at Same Overall Lift as Rigid Wing.— From equations (3), (9) and (11),

$$\frac{\Delta\alpha}{n} = 1\cdot 346\,\frac{\psi}{n} = -\,0\cdot 0166\Phi + 0\cdot 00578\chi\,. \quad \ldots \quad \ldots \quad (12)$$

The elastic wash-out calculated from equation (12) is shown in Fig. 19.

Resultant Lift Distribution for Same Total Lift.—

$$\frac{C_{LL}}{n} = 1\cdot 346\left[\frac{C_{LL0}}{n} + \frac{\Delta_1 C_{LL}}{n}\right].$$

Therefore

$$\frac{C_{LL}}{\bar{C}_{L0}} = 1\cdot 346\left[\frac{C_{LL0}}{\bar{C}_{L0}} + \frac{\frac{\Delta_1 C_{LL}}{n}}{0\cdot 04675}\right] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

The resultant lift distribution calculated from equation (13) is shown in Fig. 15.

Second Approximation to Modes of Distortion.—The first part of the calculation has been repeated using the lift distribution given by (13) for the estimation of B_0 and T_0. The resulting modes ϕ_1 and χ_1 are shown in Fig. 17. Since they differ little from ϕ and χ, the remainder of the calculation has not been repeated.

Shift of Manoeuvre Point.—If the effect of rotary damping is neglected, the shift of manoeuvre point due to the change in wing lift distribution is given by:

$$\Delta\left(\frac{x_{m.p.}}{c}\right) = \int_0^1 \Delta\left\{\frac{C_{LL}}{\bar{C}_{L0}}\right\}\frac{x}{c}\, d\zeta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

where

$$\Delta\left\{\frac{C_{LL}}{\bar{C}_{L0}}\right\} = \frac{C_{LL}}{\bar{C}_{L0}} - \frac{C_{LL0}}{\bar{C}_{L0}}$$

is plotted in Fig. 20, and x is the distance of section aerodynamic centre ahead of any convenient datum point, say the leading edge at the root. For the wing considered:

$$-x = \zeta s + e_F$$

or

$$\frac{x}{c} = -\left(3\zeta + \frac{e_F}{c}\right)$$

so that

$$\Delta\left(\frac{x_{m.p.}}{c}\right) = -\int_0^1 \Delta\left\{\frac{C_{LL}}{\bar{C}_{L0}}\right\}\left(3\zeta + \frac{e_F}{c}\right) d\zeta \quad \ldots \quad (15)$$

The integral has been evaluated to obtain the result:

$$\Delta\left(\frac{x_{m.p.}}{c}\right) = \underline{0\cdot 133} .$$

Change in Fuselage Angle of Attack.—

Root angle of attack for rigid wing	$= \bar{C}_{L0}/a$
	$= \dfrac{0\cdot 04675n}{3\cdot 365}$
	$= 0\cdot 0139n$
Increase in root angle of attack for deformable wing	$= 0\cdot 346 \times 0\cdot 0139n$
	$= 0\cdot 0048n$
For $n = 8$,	
Increase in root angle of attack $\Delta\alpha_{y=0}$	
= Increase in fuselage angle of attack	$= 8 \times 57\cdot 3 \times 0\cdot 0048$
	$= \underline{2\cdot 2 \text{ deg.}}$

2.2. *Method of Present Report. Note: The Calculations Summarized Below were Performed for $Q = Q^*$.*

Values of Construction Coefficients $B_1, \ldots B_4$ and the Parameters QB_1 and $QB_3 \tan\phi$.—The values calculated directly from the relevant equations are:

$B_1 = 0\cdot 0280,\quad B_2 = 0\cdot 00115,\quad B_3 = 0\cdot 1487,\quad B_4 = 0\cdot 0208.$

$QB_1 = 0\cdot 599,\quad QB_3 \tan\phi = 3\cdot 18.$

Values of the Functions $\mathscr{F}$, $\mathscr{G}$, $\mathscr{P}$ and $\mathscr{Q}$.—These are derived from Figs. 6 to 9 as $\mathscr{F} = 3\cdot 51$, $\mathscr{G} = 0\cdot 4555$, $\mathscr{P} = -0\cdot 912$, $\mathscr{Q} = 0\cdot 277$.

Values of $\mathscr{U}$ and $\mathscr{V}$.—

$$\mathscr{U} = 0\cdot 5[(B_1 + B_2) - \tan\phi\,(B_3 - B_4)] = -0\cdot 04938$$

$$\mathscr{V} = 0\cdot 375(B_1 + B_2) - 0\cdot 25 \tan\phi\,(B_3 - B_4) = -0\cdot 02105 .$$

Values of dD/dn, dH/dn.—

$$\frac{dD}{dn} = \frac{\mathscr{U} - \mathscr{F}\mathscr{V}}{\mathscr{P}} = -0\cdot 02685$$

$$\frac{dH}{dn} = \frac{-\mathscr{V} + \mathscr{G}\mathscr{U}}{\mathscr{Q}} = -0\cdot 0053 .$$

Elastic Wash-out from Root to Tip for Flexible Wing at Same Overall Lift as Rigid Wing.—

$$\frac{\Delta\alpha}{n} = \left(\frac{dD}{dn}\zeta + \frac{dH}{dn}\zeta^2\right)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/3}$$

$$= -(0 \cdot 01167\zeta + 0 \cdot 002303\zeta^2) \; . \qquad \qquad (16)$$

The elastic wash-out calculated from equation (16) is shown in Fig. 19.

Calculation of Shift of Manoeuvre Point.—

$$\Delta\left(\frac{x_{m.p.}}{c}\right) = \Delta_3\left(\frac{\partial C_{Mw}}{\partial\bar{C}_L}\bigg|_V\right) = -0 \cdot 04165 A \tan\phi \left\{\frac{dD}{dn} + \frac{dH}{dn}\right\} Q$$

$$= \underline{0 \cdot 172} \; .$$

Change in Fuselage Angle of Attack.—

$$\Delta\left(\frac{\partial\alpha_{y=0}}{\partial n}\bigg|_V\right) = -\left(0 \cdot 455\frac{dD}{dn} + 0 \cdot 285\frac{dH}{dn}\right)\left(\frac{\partial\alpha}{\partial\bar{C}_L}\right)_{A/3}$$

$$= 0 \cdot 00597 \; .$$

For $n = 8$, $\Delta\alpha_{y=0} = 8 \times 57 \cdot 3 \times 0 \cdot 00597$

$$= \underline{2 \cdot 74 \text{ deg.}}$$

Estimation of Effective Lift Slope.—

For the rigid wing $\bar{C}_{L0} = 0 \cdot 04675n$

Root angle of attack $= \dfrac{\bar{C}_{L0}}{\left(\dfrac{\partial\bar{C}_L}{\partial\alpha}\right)_A} = \dfrac{0 \cdot 04675n}{3 \cdot 365} = 0 \cdot 0139n$.

For the flexible wing at the same $\bar{C}_L$:

Root angle of attack $= (0 \cdot 0139 + 0 \cdot 00597)n = 0 \cdot 01987n$.

Therefore $\dfrac{\left(\dfrac{\partial\bar{C}_L}{\partial\alpha_F}\right)}{\left(\dfrac{\partial\bar{C}_L}{\partial\alpha_F}\right)_r} = \dfrac{0 \cdot 01390}{0 \cdot 01987} = \underline{0 \cdot 700}$.

Incremental Lift Distribution.—

$$\Delta\left(\frac{C_{LL}}{\bar{C}_{L0}}\right) = Q\frac{dD}{dn}(\zeta - 0 \cdot 455) + Q\frac{dH}{dn}(\zeta^2 - 0 \cdot 285)$$

$$= 0 \cdot 294 - 0 \cdot 575\zeta - 0 \cdot 113\zeta^2 \ldots \qquad \qquad (17)$$

This is shown in Fig. 20 for comparison with the other method; the resultant lift distribution for the deformed wing is shown in Fig. 15.

2.2.1. *Repeat calculation using reduced value for W.W.L. lift slope* (see *section* 11.1 *of main text*).—

$$\left(\frac{\partial \bar{C}_L}{\partial \alpha}\right)_{A/3} = 2 \cdot 30 \text{ is replaced by } 1 \cdot 97 \text{ throughout.}$$

Then:

$$B_1 = 0 \cdot 024, \quad B_2 = 0 \cdot 00099, \quad B_3 = 0 \cdot 1273, \quad B_4 = 0 \cdot 0178.$$

$$QB_1 = 0 \cdot 513 \, ; \quad QB_3 \tan \phi = 2 \cdot 722 \, .$$

$$\mathscr{F} = 3 \cdot 55 \, ; \quad \mathscr{G} = 0 \cdot 461 \, ; \quad \mathscr{P} = -0 \cdot 917 \, ; \quad \mathscr{Q} = 0 \cdot 274$$

$$\mathscr{U} = -0 \cdot 04225 \, ; \quad \mathscr{V} = -0 \cdot 01801$$

$$\frac{dD}{dn} = -0 \cdot 0236 \, ; \quad \frac{dH}{dn} = -0 \cdot 0054$$

Shift of Manoeuvre Point.—

$$\Delta \left(\frac{x_{m.p.}}{c}\right) = \underline{0 \cdot 155}$$

Change in Fuselage Angle of Attack.—

$$\Delta \left(\left.\frac{\partial \alpha_{y=0}}{\partial n}\right|_V\right) = 0 \cdot 00533 \, .$$

For $n = 8$,

$$\Delta \alpha_{y=0} = 8 \times 57 \cdot 3 \times 0 \cdot 00533$$

$$= \underline{2 \cdot 44 \text{ deg.}}$$

Estimation of Effective Lift Slope.—

Root angle of attack of rigid wing $= 0 \cdot 0139n$.

Root angle of attack of flexible wing $= (0 \cdot 0139 + 0 \cdot 00533)n$.

$= 0 \cdot 01923n$.

Therefore

$$\frac{\left(\frac{\partial \bar{C}_L}{\partial \alpha_F}\right)}{\left(\frac{\partial \bar{C}_L}{\partial \alpha_F}\right)_r} = \frac{0 \cdot 01390}{0 \cdot 01923} = \underline{0 \cdot 723}$$

Incremental Lift Distribution.—

$$\Delta \left(\frac{C_{LL}}{\bar{C}_{L0}}\right) = 0 \cdot 262 - 0 \cdot 505\zeta - 0 \cdot 1156\zeta^2 \, . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

This is shown in Fig. 20.

APPENDIX IV

Approximate Estimation of the Ultimate Tail Load P_{Tu} Required in the Calculation of the Deformability Coefficients w_α, etc.

The required tail load P_{Tu} will be the resultant of the aerodynamic tail load corresponding to a particular flight condition, and the relieving inertia load due to the weight W_f of the tail-end of the fuselage (including the complete tail unit). The proposed approximation for P_{Tu} is based on a consideration of three symmetric flight conditions corresponding to points A, B and E, on the basic flight envelope of Ref. 13(*b*) (Fig. 1), under pitching acceleration as specified in paragraph 5 of Ref. 13(*b*).

At a point of the flight envelope where the equivalent air speed is V' (corresponding to dynamic pressure number Q) and the normal acceleration coefficient is n, the specified angular acceleration $\dot{q}$ is:

$$\left.\begin{aligned} \dot{q} &= Kn\frac{V'^*}{V'} \text{ radn/sec}^2 \\ \text{where} \quad K &= \frac{1}{V'^*}\left\{65 + \frac{3\cdot 5 \times 10^5}{W}\right\} \text{ radn/sec}^2 \end{aligned}\right\}, \quad \ldots \quad (1)$$

if V'^* is the maximum permissible equivalent air speed in ft/sec, and

W is the all-up weight of the aeroplane in lb.

The balancing tail load P_b, (measured positively downwards) including inertia relief is given by:

$$-P_b = \frac{nWl_m(Q) + C_{m\,0}QWc}{l_\alpha + l_m(Q)} - nW_f \quad \ldots \quad (2)$$

and the accelerating tail load P_a (measured positively downwards), including inertia relief, by:

$$-P_a = \frac{i_B'Wl_\alpha\dot{q}}{g} = \frac{i_B'KnWl_\alpha}{g}\frac{V'^*}{V'} = -P_a(n, V') \quad \ldots \quad (3)$$

where $C_{m\,0}$ ≡ aircraft pitching moment coefficient at zero lift, assumed for simplicity to be independent of speed,

l_α ≡ tail arm, measured from aircraft c.g.,

$l_m(Q)$ ≡ distance of manoeuvre point of aircraft less tail, ahead of aircraft c.g. (function of Q),

$i_B'\frac{W}{g}l_\alpha^2$ ≡ pitching moment of inertia of aircraft less tail end of fuselage (weight W_f).

The resultant tail load P is this given by:

$$-P = -(P_b + P_a) = \frac{nWl_m(Q) + C_{m\,0}QWc}{l_\alpha + l_m(Q)} - nW_f - P_a(n,V') \,. \quad \ldots \quad (4)$$

It should be noted that in calculating $l_m(Q)$, account must be taken of the shift of manoeuvre point of wing plus fuselage, due to deformability. Furthermore, the minimum value of l_α, corresponding to the aftmost c.g. position will normally give the maximum tail load.

The coefficient $C_{m\,0}$ will generally be negative, but its absolute value may be unknown. We therefore proceed on the arbitrary, but not unreasonable assumption that the value of $C_{m\,0}$ is such as to give equal up and down tail loads respectively in the high speed pull-out case at speed $V_1' = 0\cdot 8V'^*$ and the maximum normal acceleration coefficient n_1, and the high speed inverted pull-out case at speed V_1' and normal acceleration coefficient $-\frac{n_1}{2}$†. Thus, writing

$$-P(n_1, Q_1) = \frac{n_1Wl_m(Q_1) + C_{m\,0}Q_1Wc}{l_\alpha + l_m(Q_1)} - n_1W_f - P_a(n_1, V_1')$$

† (i) These two cases correspond to points B and E of the flight envelope, except that to simplify the analysis somewhat, the speed for the inverted pull-out is assumed equal to that for the normal pull-out, *viz.*, $0\cdot 8V'^*$ instead of $0\cdot 7V'^*$.

(ii) It may be recalled that when investigating the bending deformability of the fuselage (section 13.1), equal up and down values of the ultimate tail load were assumed.

and

$$-P\left(\frac{-n_1}{2}, Q_1\right) = \frac{\frac{-n_1}{2} W l_m(Q_1) + C_{m0} Q_1 W c}{l_\alpha + l_m(Q_1)} + \frac{n_1}{2} W_f - P_a\left(\frac{-n_1}{2}, V_1'\right),$$

we assume

$$P(n_1, Q_1) = -P\left(\frac{-n_1}{2}, Q_1\right),$$

and noting that

$$P_a\left(\frac{-n_1}{2}, V_1'\right) = -\tfrac{1}{2} P_a(n_1, V_1')$$

find

$$C_{m0} = -\frac{n_1}{4Q_1} \frac{l_m(Q_1)}{c} \left[1 - \left\{\frac{W_f}{W} + \frac{P_a(n_1, V_1')}{n_1 W}\right\} \left\{\frac{l_\alpha + l_m(Q_1)}{l_m(Q_1)}\right\}\right] \quad \ldots \quad \ldots \quad (5)$$

and

$$-P(n_1, Q_1) = +P\left(\frac{-n_1}{2}, Q_1\right) = 0{\cdot}75 n_1 W \left\{\frac{l_m(Q_1)}{l_m(Q_1) + l_\alpha} - \frac{W_f}{W} - \frac{P_a(V_1')}{n_1 W}\right\} \quad \ldots \quad (6)$$

where from equation (3)

$$-\frac{P_a(V_1')}{n_1 W} = \frac{i_B' K l_\alpha}{g} \frac{V'^*}{V_1'}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Equation (6) may be assumed to give the maximum unfactored tail load unless, with the value of C_{m0} given by equation (5), the low speed pull-out case corresponding to point A of the flight envelope gives a greater load. At A, $n = n_1$ and if we denote the speed by V_2', the tail load is given by:

$$-P(n_1, Q_2) = \frac{n_1 W l_m(Q_2) + C_{m0} Q_2 W c}{l_\alpha + l_m(Q_2)} - n_1 W_f - \frac{V_1'}{V_2'} P_a(n_1, V_1')$$

$$= n_1 W \left[\frac{l_m(Q_2)}{l_\alpha + l_m(Q_2)} - \frac{W_f}{W} - \frac{V_1'}{V_2'} \frac{P_a(V_1')}{n_1 W}\right]$$

$$- \frac{n_1 W}{4} \frac{Q_2}{Q_1} \frac{l_m(Q_1)}{l_\alpha + l_m(Q_2)} \left[1 - \left\{\frac{W_f}{W} + \frac{P_a(V_1')}{n_1 W}\right\} \left\{\frac{l_\alpha + l_m(Q_1)}{l_m(Q_1)}\right\}\right]$$

$$= n_1 W \left[\frac{l_m(Q_2) - \frac{1}{4}\frac{Q_2}{Q_1} l_m(Q_1)}{l_\alpha + l_m(Q_2)} - \frac{W_f}{W}\left\{1 - \frac{1}{4}\frac{Q_2}{Q_1}\frac{l_\alpha + l_m(Q_1)}{l_\alpha + l_m(Q_2)}\right\}\right.$$

$$\left. - \frac{P_a(V_1')}{n_1 W}\left\{\frac{V_1}{V_2} - \frac{1}{4}\frac{Q_2}{Q_1}\frac{l_\alpha + l_m(Q_1)}{l_\alpha + l_m(Q_2)}\right\}\right]$$

or neglecting a term $\frac{1}{4}\frac{Q_2}{Q_1}\frac{l_m(Q_1) - l_m(Q_2)}{l_\alpha + l_m(Q_2)}$, and putting

$$\frac{l_\alpha + l_m(Q_1)}{l_\alpha + l_m(Q_2)} \simeq 1{\cdot}0,$$

$$-P(n_1, Q_2) \simeq n_1 W \left[\left\{1 - \frac{1}{4}\frac{Q_2}{Q_1}\right\}\left\{\frac{l_m(Q_2)}{l_\alpha + l_m(Q_2)} - \frac{W_f}{W} - \frac{P_a(V_1')}{n_1 W}\right\}\right.$$

$$\left. - \left(\frac{V_1'}{V_2'} - 1\right)\frac{P_a(V_1')}{n_1 W}\right]. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

The magnitude of the ultimate design tail load $|P_{Tu}|$ may now be taken as the greater of the two expressions (6) and (8) with n_1 replaced by n_u. Thus

$$\left.\begin{aligned} \frac{|P_{Tu}|}{n_u W} &= 0\cdot 75\left\{\frac{l_m(Q_1)}{l_\alpha + l_m(Q_1)} - \frac{W_f}{W} - \frac{P_a(V_1')}{n_1 W}\right\} \\ \text{or} \qquad & \left\{1 - \tfrac{1}{4}\frac{Q_2}{Q_1}\right\}\left\{\frac{l_m(Q_2)}{l_\alpha + l_m(Q_2)} - \frac{W_f}{W} - \frac{P_a(V_1')}{n_1 W}\right\} - \left(\frac{V_1'}{V_2'} - 1\right)\frac{P_a(V_1')}{n_1 W} \end{aligned}\right\}, \quad \ldots \quad (9)$$

whichever is the greater,

$\dfrac{P_a(V_1')}{n_1 W}$ being given by equations (7) and (1).

APPENDIX V

The Downwash Field Due to the 'Wash-out without lift' Distribution of a Deformable Wing, and its Effect on Fuselage and Tailplane Contributions to Manoeuvre-Point Shift

1. *Introduction.*—To provide a basis for the rough estimation of the fuselage factor F_{I1} (section 15.1.1) and the tailplane efficiency factor $\left(1 - \dfrac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{C}_L, V}\right)$, (section 15.1.2), the method of Schlichting (Ref. 10) has been applied to calculate the distributions of downwash along the axes of symmetry of wings of the four plan-forms shown in Fig. 23, and also to calculate the spanwise distribution in the plane of each wing at a typical tailplane location.

The wings were of aspect ratios 3 and 6, and angles of sweepback 0 deg and 45 deg, and the spanwise distributions of downwash were worked out at a distance of $1\cdot 02\sqrt{S}$ (S = wing area) behind the mean quarter-chord point of each wing.

In a specific problem, the actual distribution of wash-out would have been calculated using Part I of the report (*see* equation (23)), but for the present purpose, it was considered sufficiently accurate to assume a linear distribution given by

$$\alpha = \alpha_0\left(1 - 2\frac{|y|}{s}\right)$$

in each case.

2. *Résumé of Method of Calculation.*—In this application of the Schlichting method, a single vortex line* at the quarter-chord position was assumed, and with three pivotal points taken on the three-quarter-chord line at $\dfrac{|y|}{s} = 0, 0\cdot 5$ and $0\cdot 866$ respectively, the downwash was calculated as the sum of three Fourier term contributions in the case of the unswept wings, with the addition of a fourth contribution, arising from the 'Middle Function' of Ref. 10, for the swept wings.

In the cases $A = 6$, $\phi = 0$ deg and $A = 6$, $\phi = 45$ deg, the coefficients a_1, a_3, a_5 (and a_0) appertaining to the various contributions were obtained by solving the appropriate sets of equations appearing on page 44 of Ref. 10, the unit coefficients on the right-hand sides of the last two equations in each set being replaced by values of α/α_0 appropriate to the stations $\dfrac{|y|}{s} = 0\cdot 5$ and $0\cdot 866$ respectively (i.e., $\alpha/\alpha_0 = 0, -0\cdot 732$ respectively).

---

* Kinked in the case of the swept wings.

For the remaining wings, $A = 3$, $\phi = 0$ deg, and $A = 3$, $\phi = 45$ deg, Table 14 of Ref. 10 was used to set up the corresponding sets of equations which were then solved as before.

Tables 5 and 8 of Ref. 10 were used in evaluating the Fourier term contributions, while the contribution from the ' Middle Function ' was derived from equations (57) and (58) in the case of the axis of symmetry distributions, and by calculation from equation (37), using Table 2* in the case of the spanwise distributions at the tailplane.

3. *Results of Calculations and their Application.*—The results of the calculations are given in Fig. 24 which shows the variation of the quantity $\left(1 - \frac{\partial \varepsilon(x)}{\partial \alpha_{y=0}}\bigg|_{\bar{c}_L, V}\right)$ along the axis of symmetry of each wing, and in Fig. 25 which gives the spanwise variation of the factor $\left(1 - \frac{\partial \varepsilon(y)}{\partial \alpha_{y=0}}\bigg|_{\bar{c}_L, V}\right)$ at the tailplane location for each case.

The results of Fig. 24 have been applied to estimate the value of the fuselage factor F_{I1} defined by equation (81) of the main text:

$$\left(\int_0^{l_F} b_F^2 \, dx\right) \times F_{I1} = \int_0^{l_F} \left\{1 - \frac{\partial \varepsilon(x)}{\partial \alpha_{y=0}}\bigg|_{\bar{c}_L, V}\right\} b_F^2 \, dx \, .$$

The fuselage has been assumed to have the shape of an ellipsoid of revolution of length $l_F = \sqrt{(6S)}$ and axis ratio 7 : 1, and the fore-and-aft position of the wings relative to the fuselage has been varied. Fig. 23 shows typical arrangements, with the wing mean quarter-chord point at $0 \cdot 4 l_F$ from the fuselage nose. The factor $\left(1 - \frac{\partial \varepsilon(x)}{\partial \alpha_{y=0}}\bigg|_{\bar{c}_L, V}\right)$ has been taken equal to zero along the chord of the wing, where the flow is completely guided (cf. Ref. 15, section 5); elsewhere the distribution was assumed to be given in Fig. 24.

In Fig. 26, the factor F_{I1} has been plotted as a function of l_N/l_F (= fraction of fuselage length ahead of wing-root quarter-chord point) for each wing. Typical values, corresponding to the actual arrangements shown in Fig. 23 are:

	Aspect ratio of wing	
	3	6
Sweepback of wing (deg)	F_{I1}	
0	0·34	0·45
45	0·24	0·45

From the curves of Fig. 26 it is seen that F_{I1} increases with wing aspect ratio and with sweepback and also with the parameter l_N/l_F. It will vary with other parameters such as the shape of the fuselage and its size in relation to the wing, but no detailed investigation of their effect has yet been attempted.

*It was considered sufficiently accurate to assume the value of the Middle Function contribution at the tailplane to be equal to its value at infinity downstream.

The problem of estimating F_{I1} is, of course, closely allied to that of estimating the shift of aerodynamic centre due to a body, and the reader who wishes to pursue this topic further may find Ref. 15 (sweptback wings) and Refs. 16, 17 and 18 (dealing with unswept wing-body combinations) of some assistance.

Reference to the curves of Fig. 24 shows that the value of the downwash behind each wing rapidly approaches its asymptotic value at infinity, so that for all practicable locations of the tailplane in the plane of the wing, values of $\left(1 - \frac{\partial \varepsilon}{\partial \alpha_{y=0}}\bigg|_{\bar{c}_{L},V}\right)$ should differ little from those given in Fig. 25, from which may be deduced, as typical values of the tailplane efficiency factor $\left(1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\bigg|_{\bar{c}_{L},V}\right)$ for a tailplane of span equal to one-third of the wing span:

	Aspect ratio of wing	
	3	6
Sweepback of wing (deg)	$\left\{1 - \frac{\partial \bar{\varepsilon}}{\partial \alpha_{y=0}}\right\}$	
0	0·43	0·56
45	0·51	0·64

These values indicate a tendency to increase both with increasing aspect ratio and with increasing sweepback. The factor would also increase with increasing tailplane span.

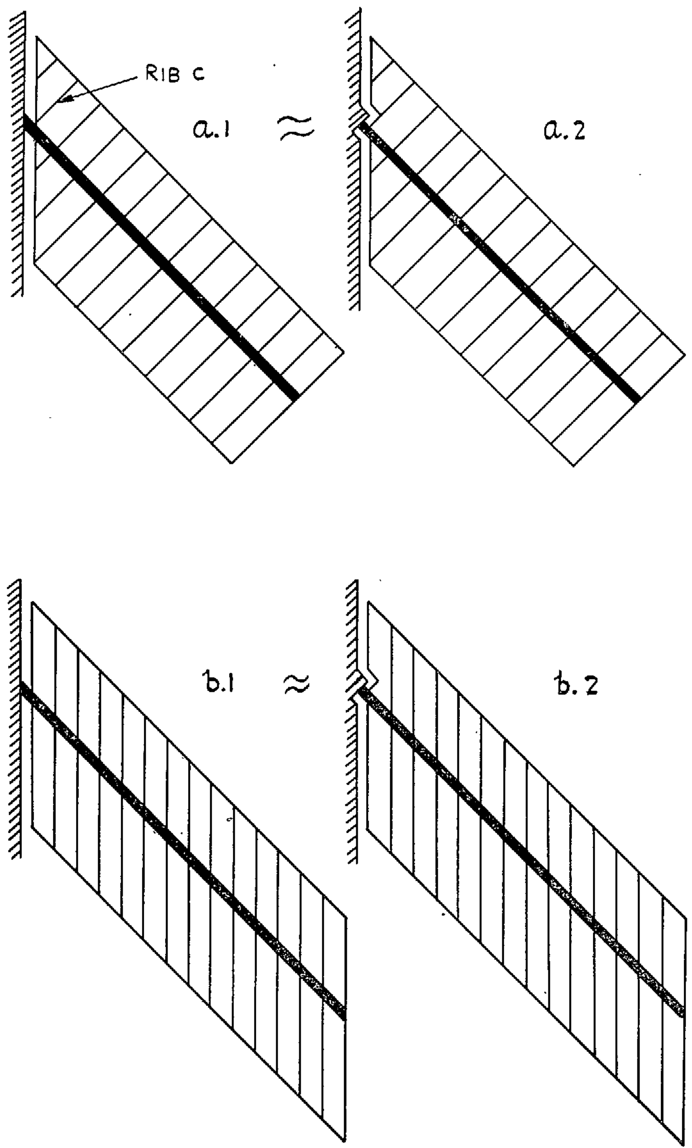


FIG. 1. Structural lay-outs of swept-back wings with a single spar.

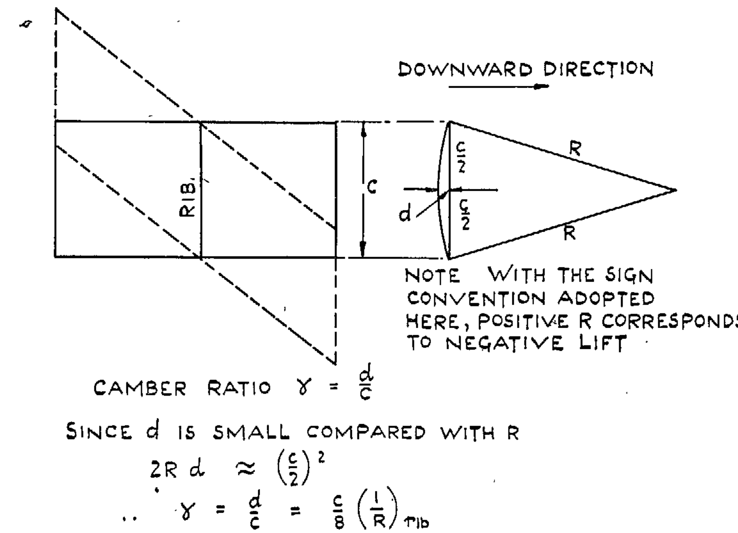


FIG. 2a. Unswept wing and swept wing with ribs in flight direction.

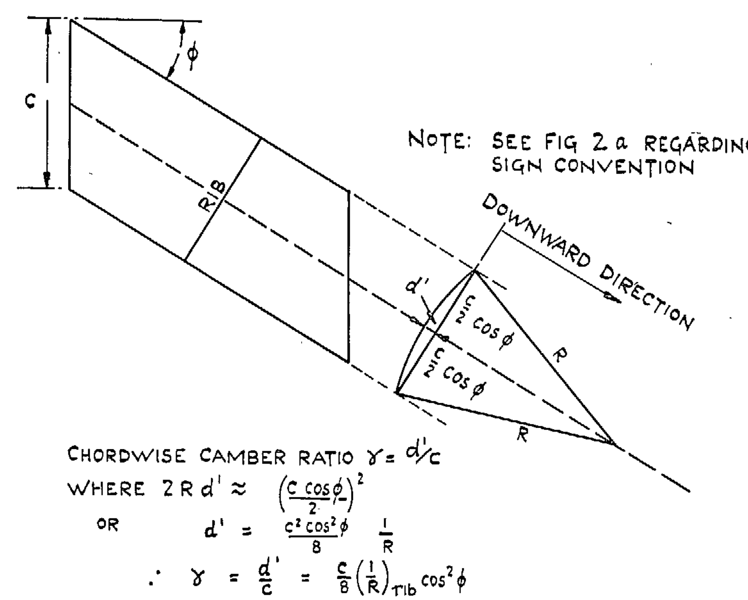
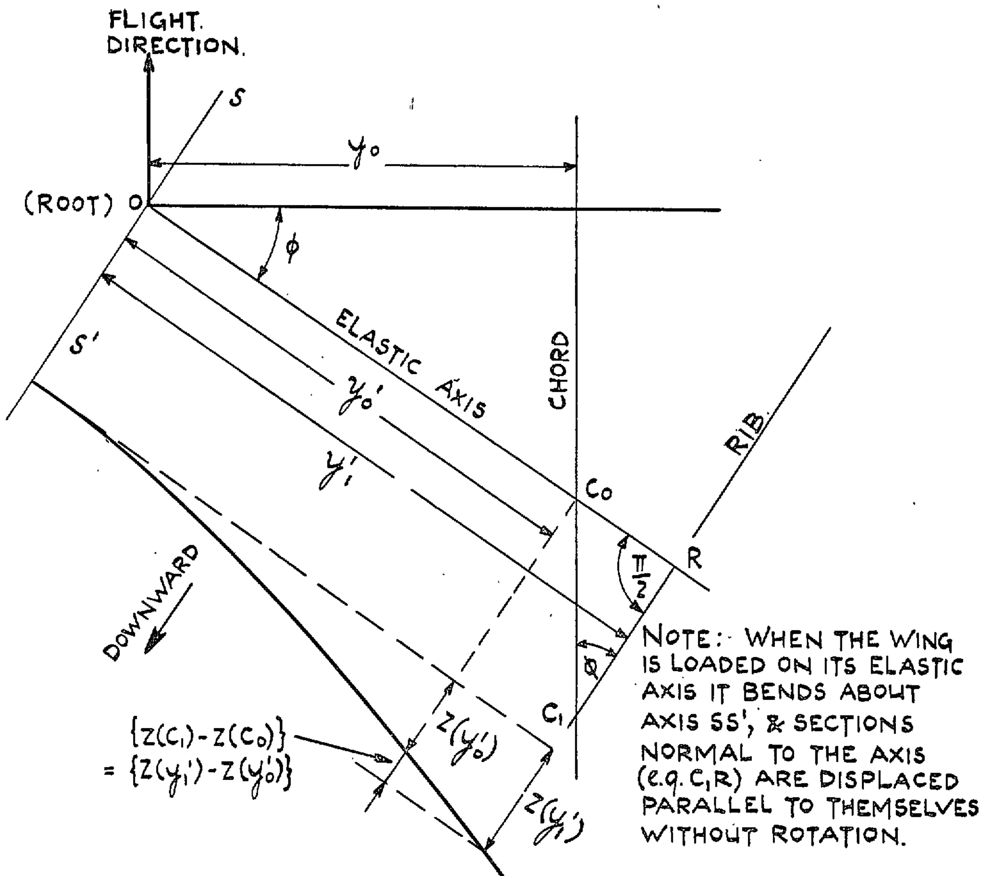


FIG. 2b. Swept wing with ribs perpendicular to spar.

FIGS. 2a and 2b. Elastic camber of chordwise wing sections due to rib bending.



INCREASE IN ANGLE OF ATTACK OF CHORD C_0C_1 IS

$$\Delta\alpha = \lim_{C_1 \to C_0} \left\{ \frac{z(C_1) - z(C_0)}{|C_0 C_1|} \right\} = \lim_{R \to C_0} \left\{ \frac{z(R) - z(C_0)}{|C_0 R|/\sin\phi} \right\}$$

$$= \sin\phi \lim_{y_1' \to y_0'} \left\{ \frac{z(y_1') - z(y_0')}{y_1' - y_0'} \right\} = \frac{dz}{dy'} \sin\phi$$

$$\therefore \quad \frac{\partial\alpha}{\partial y'} = \cos\phi \frac{\partial\alpha}{\partial y} = \frac{d^2 z}{dy'^2} \sin\phi$$

HENCE WASHOUT PER UNIT SPAN DUE TO WING BENDING

$$= -\left[\frac{\partial\alpha}{\partial y}\right]_{\ell} = -\tan\phi \frac{d^2 z}{dy'^2}$$

$$= -\tan\phi \times (\text{CURVATURE OF ELASTIC AXIS})$$

CURVATURE OF CAMBER LINE DUE TO WING BENDING

$$= \lim_{C_1 \to C_0} \left\{ \frac{(\text{SLOPE AT } C_1) - (\text{SLOPE AT } C_0)}{C_1 C_0} \right\}$$

$$= \lim_{y_1' \to y_0'} \frac{\sin\phi \left\{ \left(\frac{dz}{dy'}\right)_{y' = y_1'} - \left(\frac{dz}{dy'}\right)_{y' = y_0'} \right\}}{(y_1' - y_0')/\sin\phi}$$

$$= \sin^2\phi \frac{d^2 z}{dy'^2}$$

$$\sin^2\phi \times (\text{CURVATURE OF ELASTIC AXIS})$$

FIG. 3. Effect of wing bending on wash-out and camber of swept wings with ribs perpendicular to elastic axis.

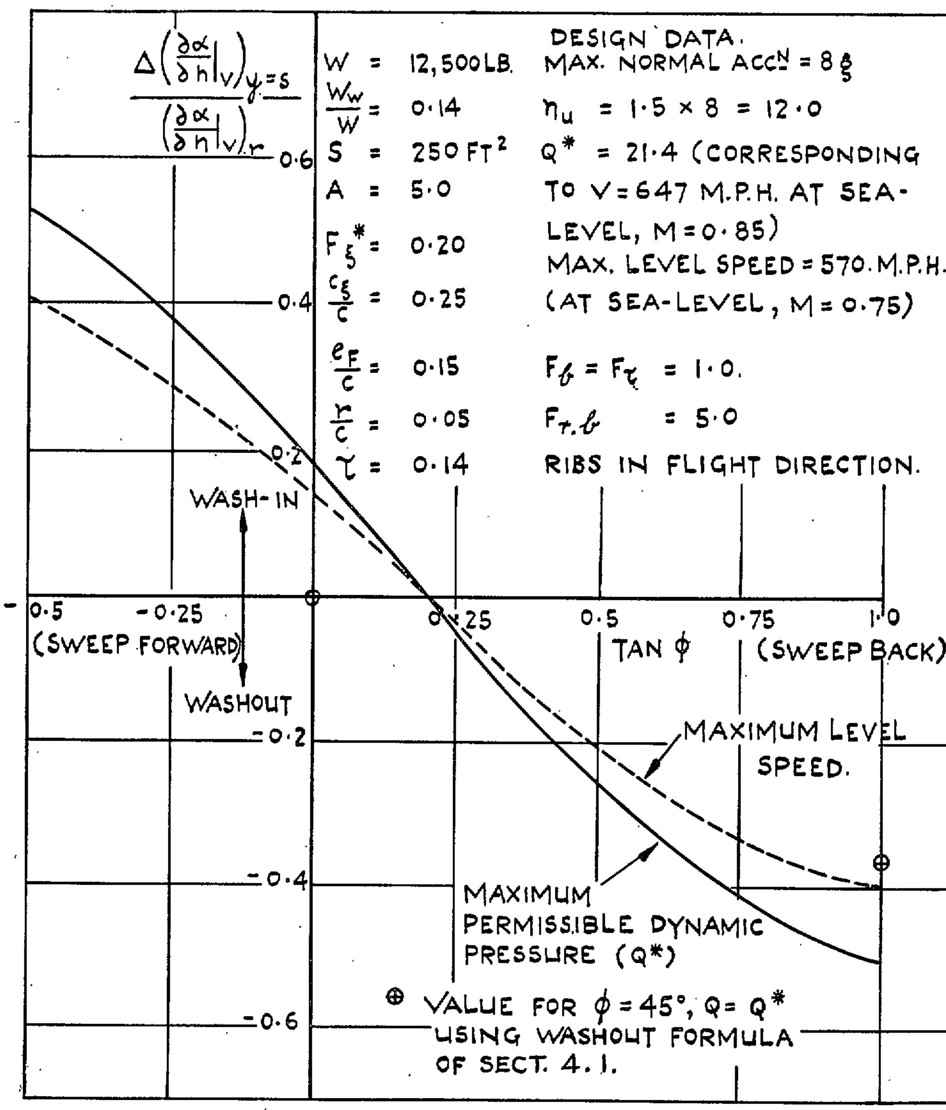


FIG. 4. Relative wash-out at wing tip of typical fighter during a pull-out.

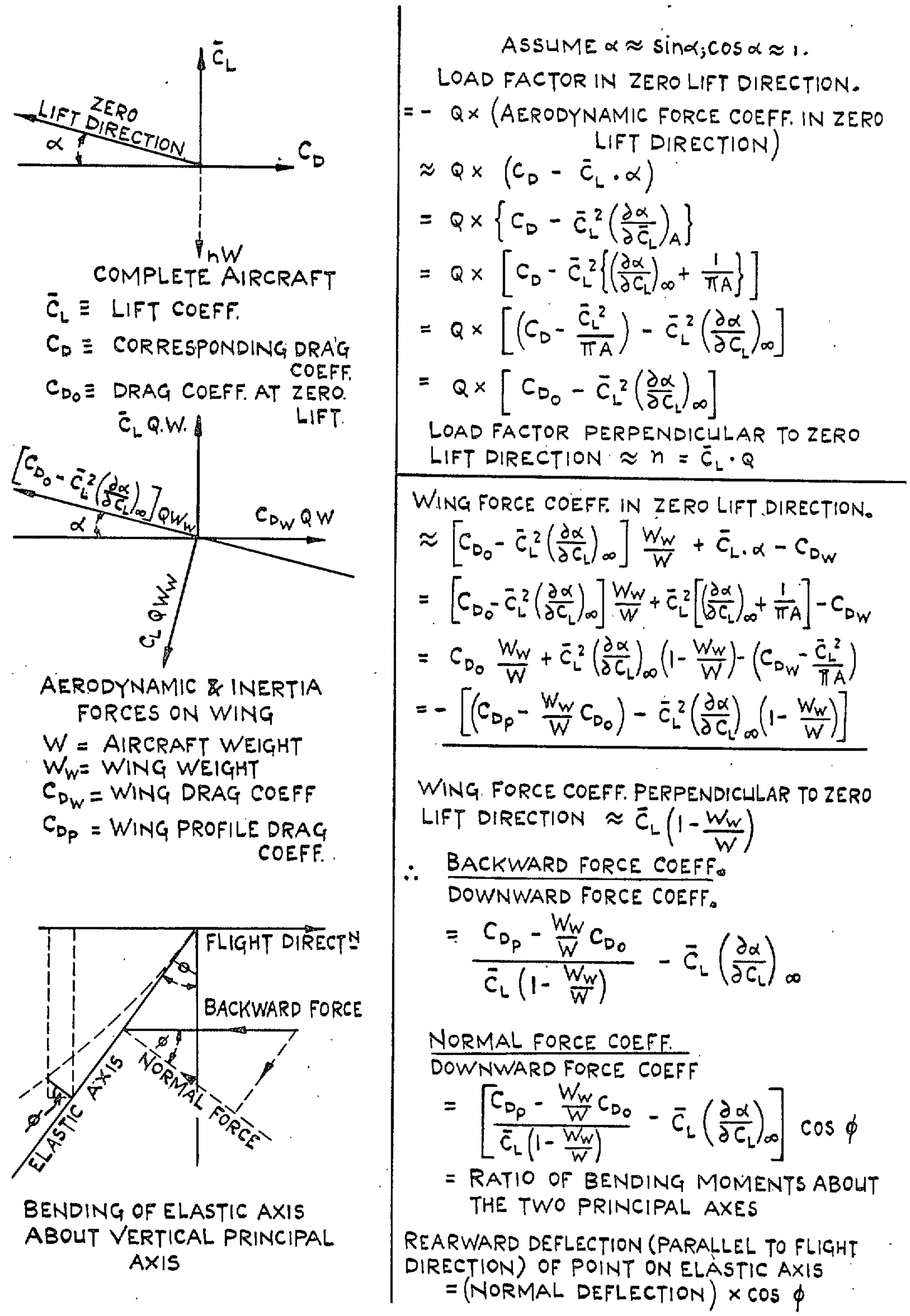


FIG. 5. Relationships required for the calculation of direct effect of wing bending on pitching moment.

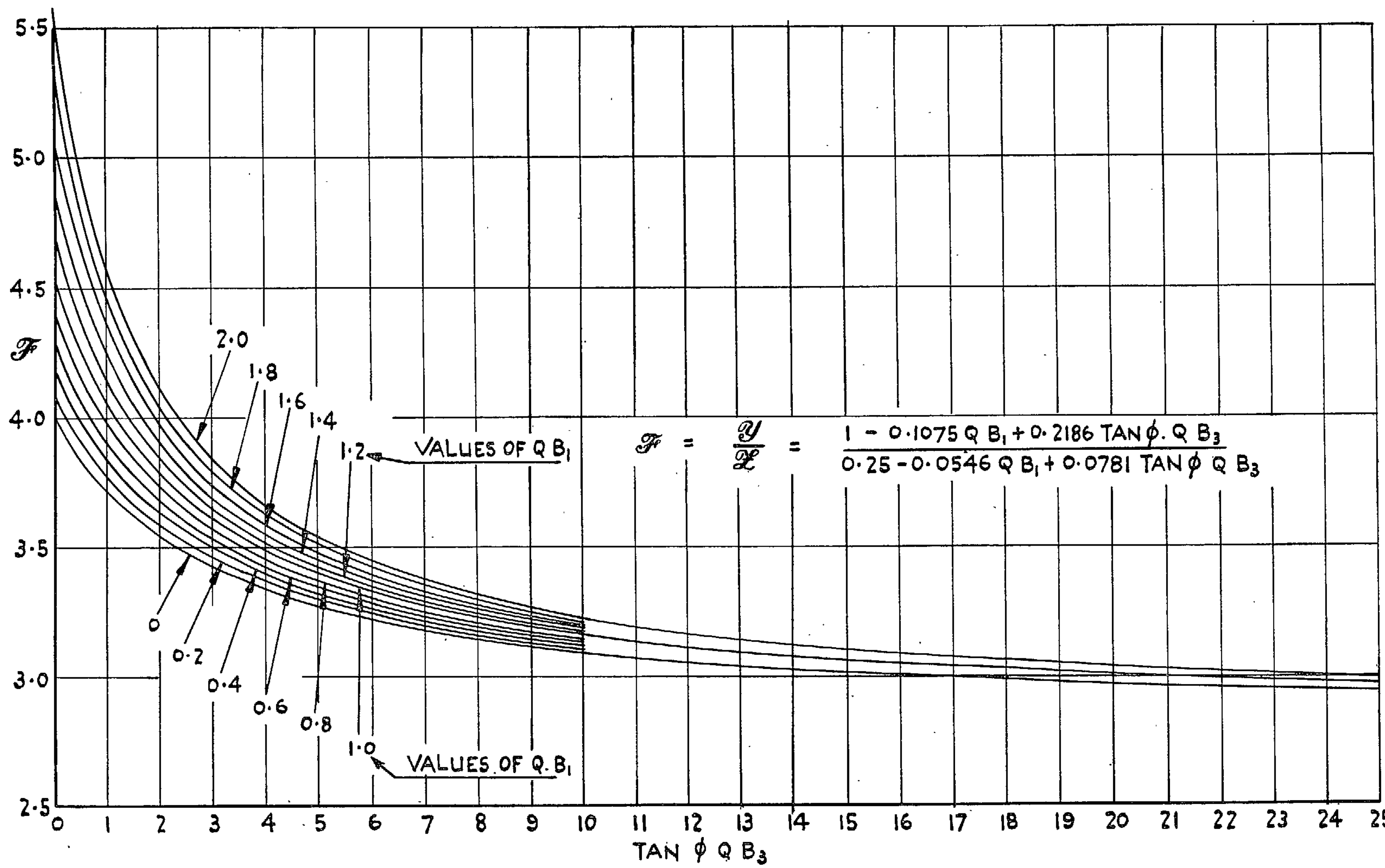


Fig. 6. The function $\mathcal{F}$.

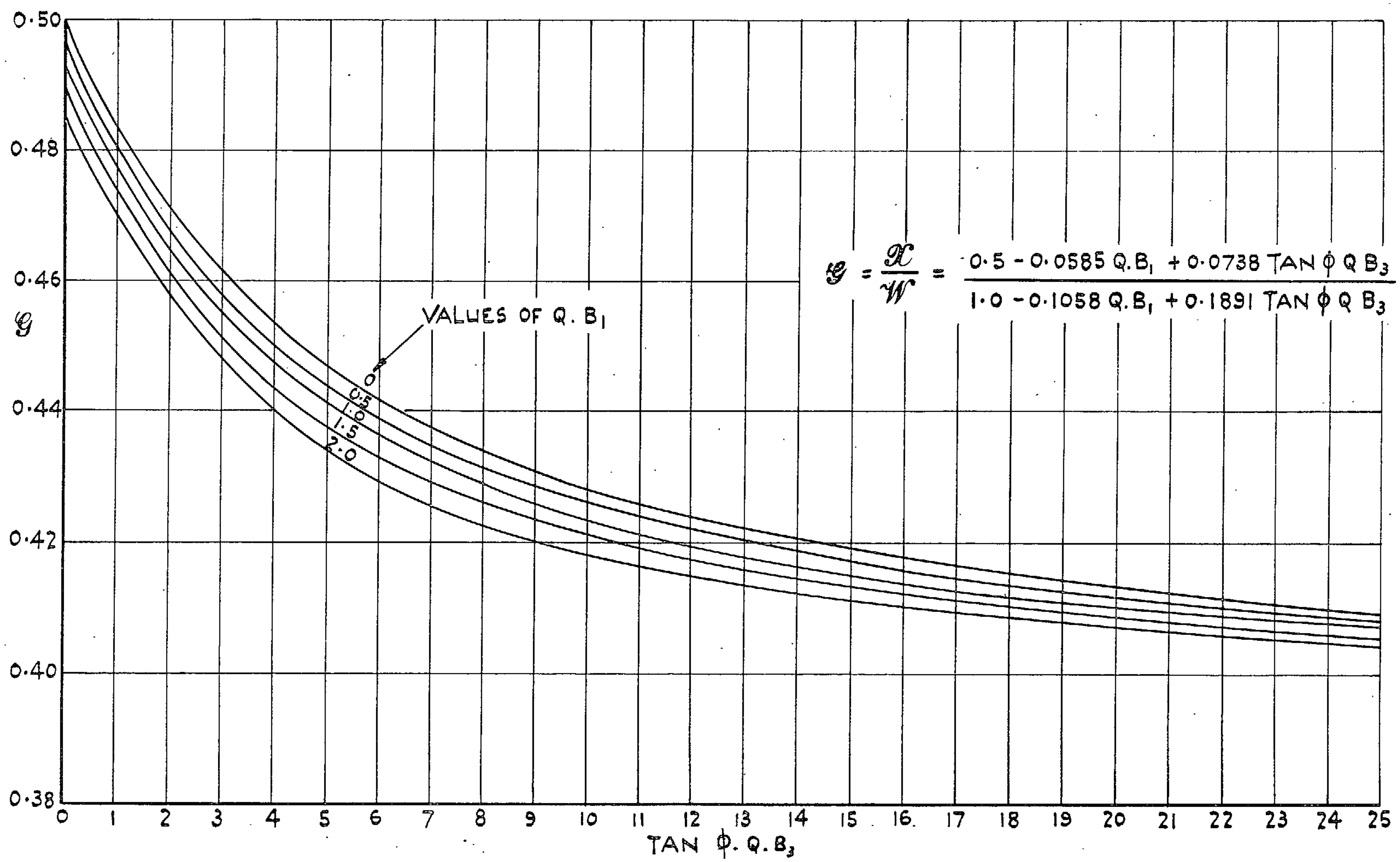


FIG. 7. The function $\mathscr{G}$.

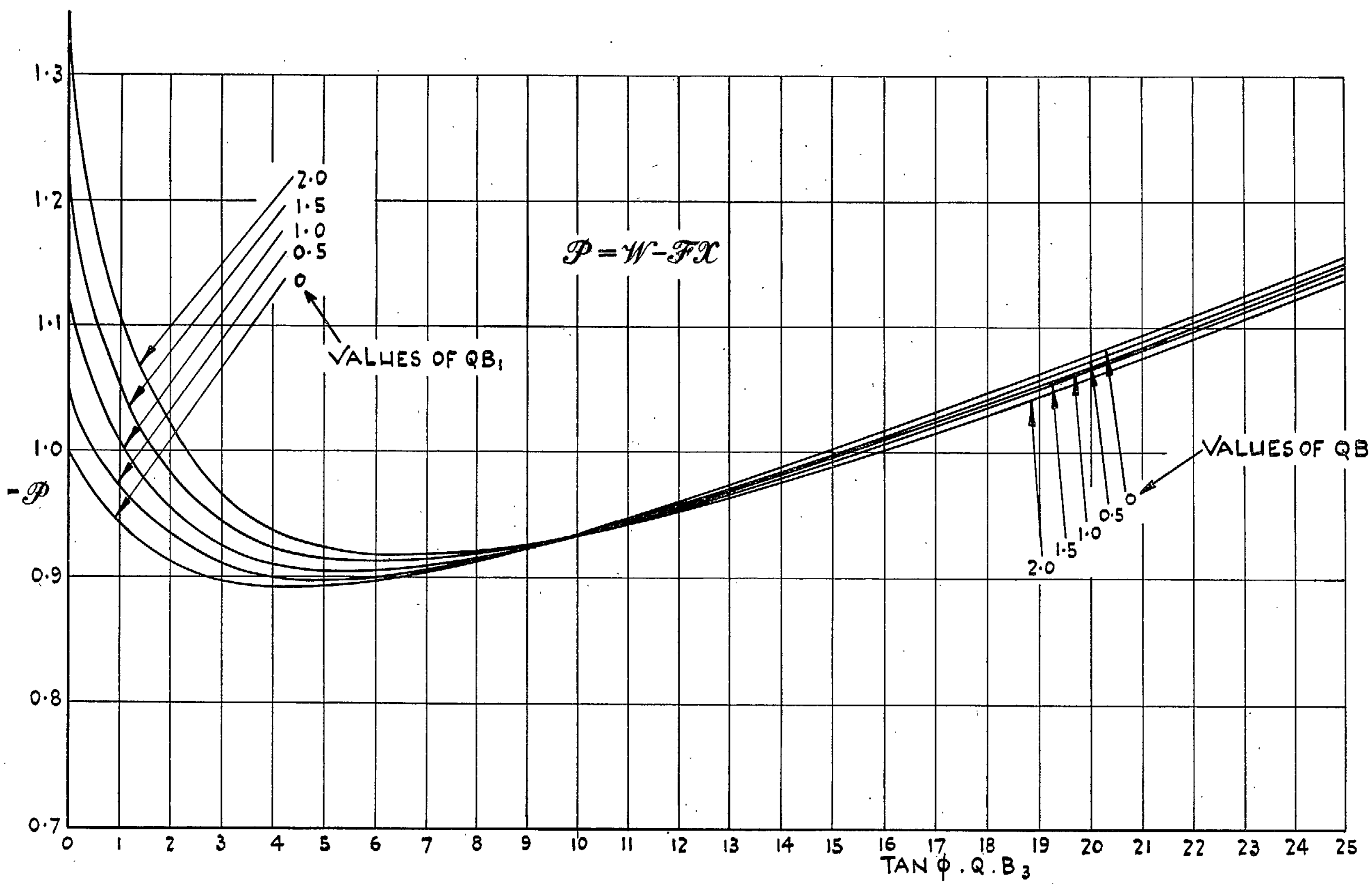


FIG. 8. The function 𝒫.

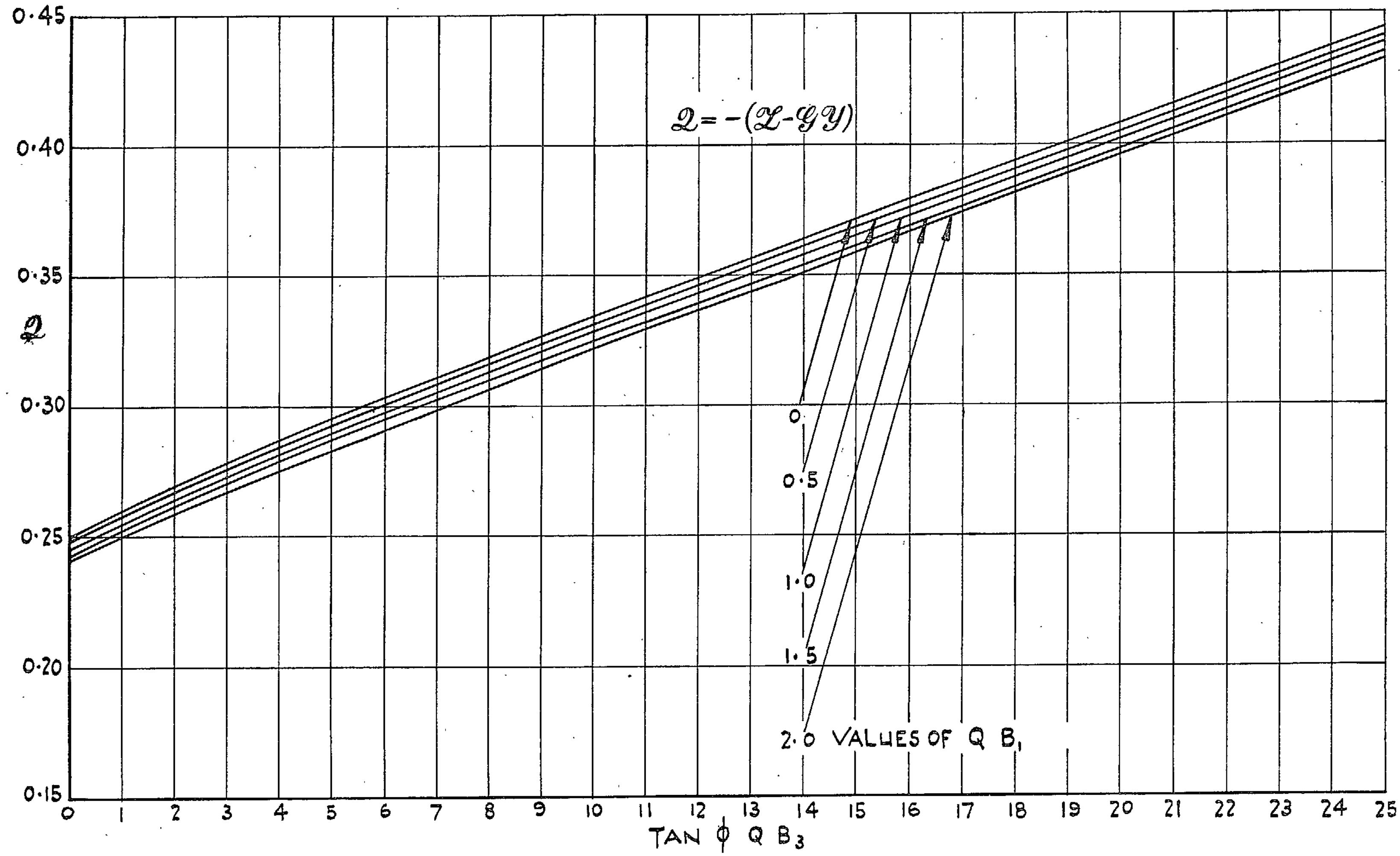
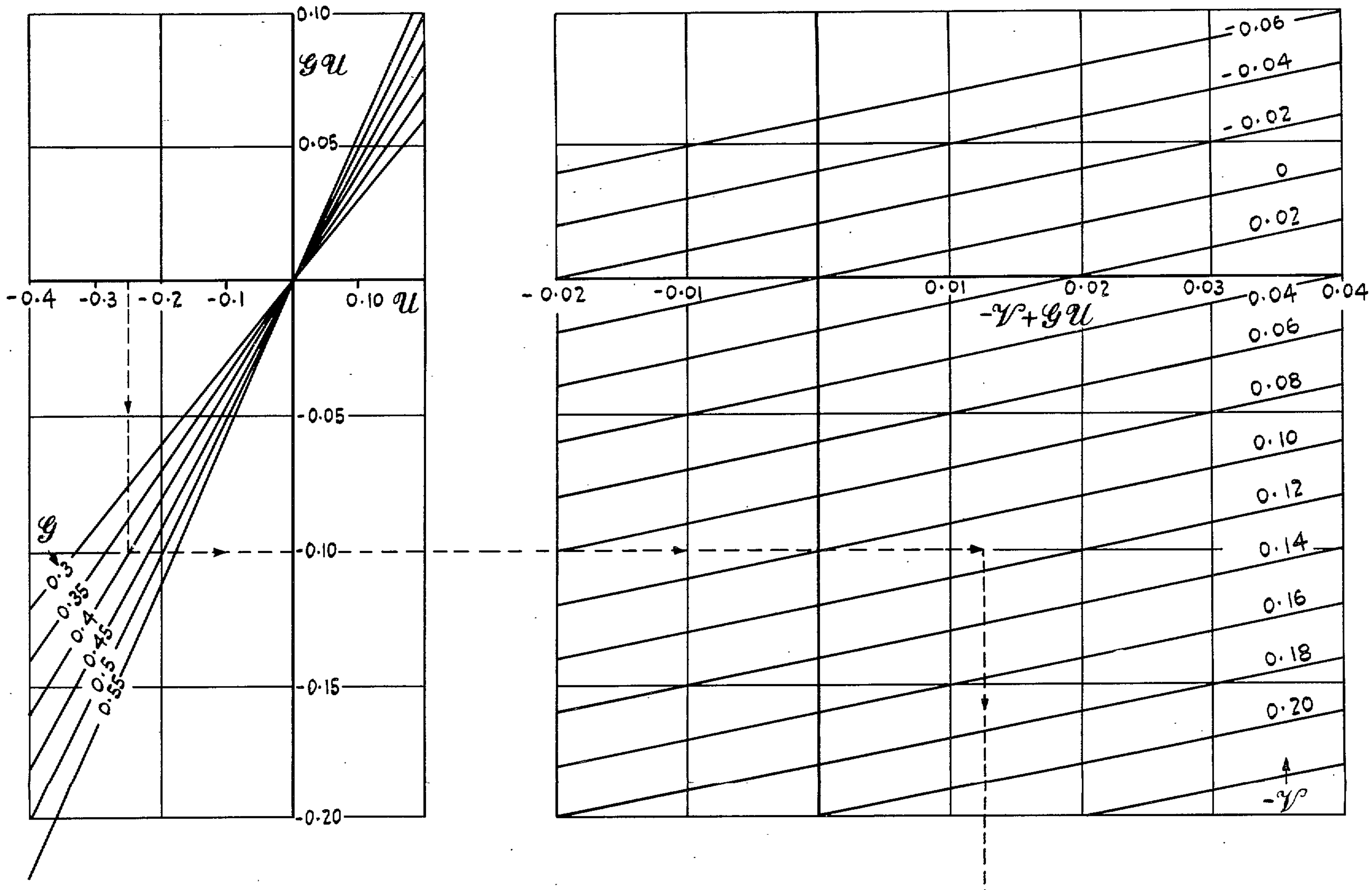
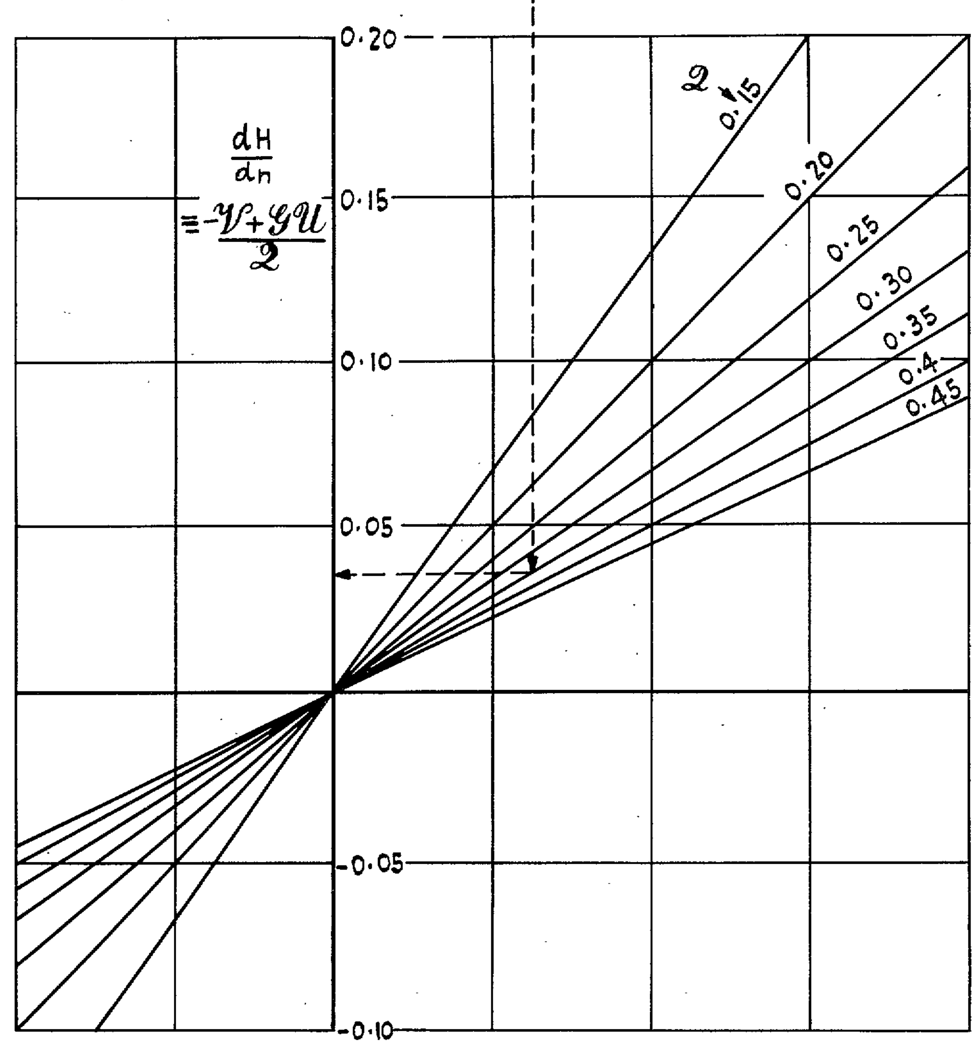
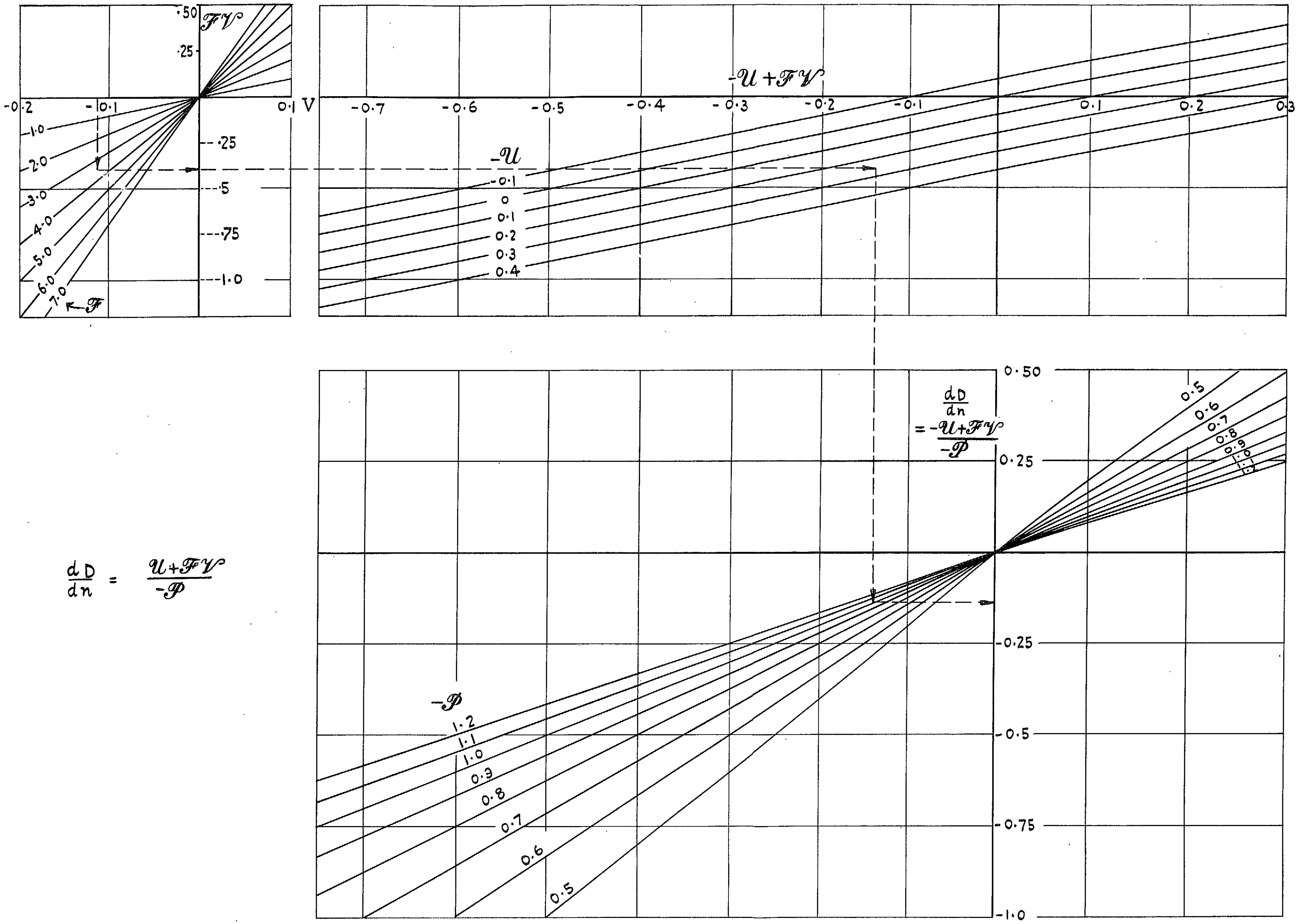


FIG. 9. The function $\mathscr{Q}$.



$$\frac{dH}{dn} = \frac{-\mathcal{V}+\mathcal{GU}}{\mathcal{Q}}$$





·50
FV
·25
-0·2
-0·1
0·1 V
1·0
2·0
3·0
4·0
5·0
6·0
7·0
F
·25
·5
·75
1·0
-U + FV
-0·7
-0·6
-0·5
-0·4
-0·3
-0·2
-0·1
0·1
0·2
0·3
-U
-0·1
0
0·1
0·2
0·3
0·4
0·50
dD/dn = (-U+FV)/(-P)
0·25
-0·25
-0·5
-0·75
-1·0
0·5
0·6
0·7
0·8
0·9
1·0
1·1
1·2
-P
dD/dn = (U+FV)/(-P)

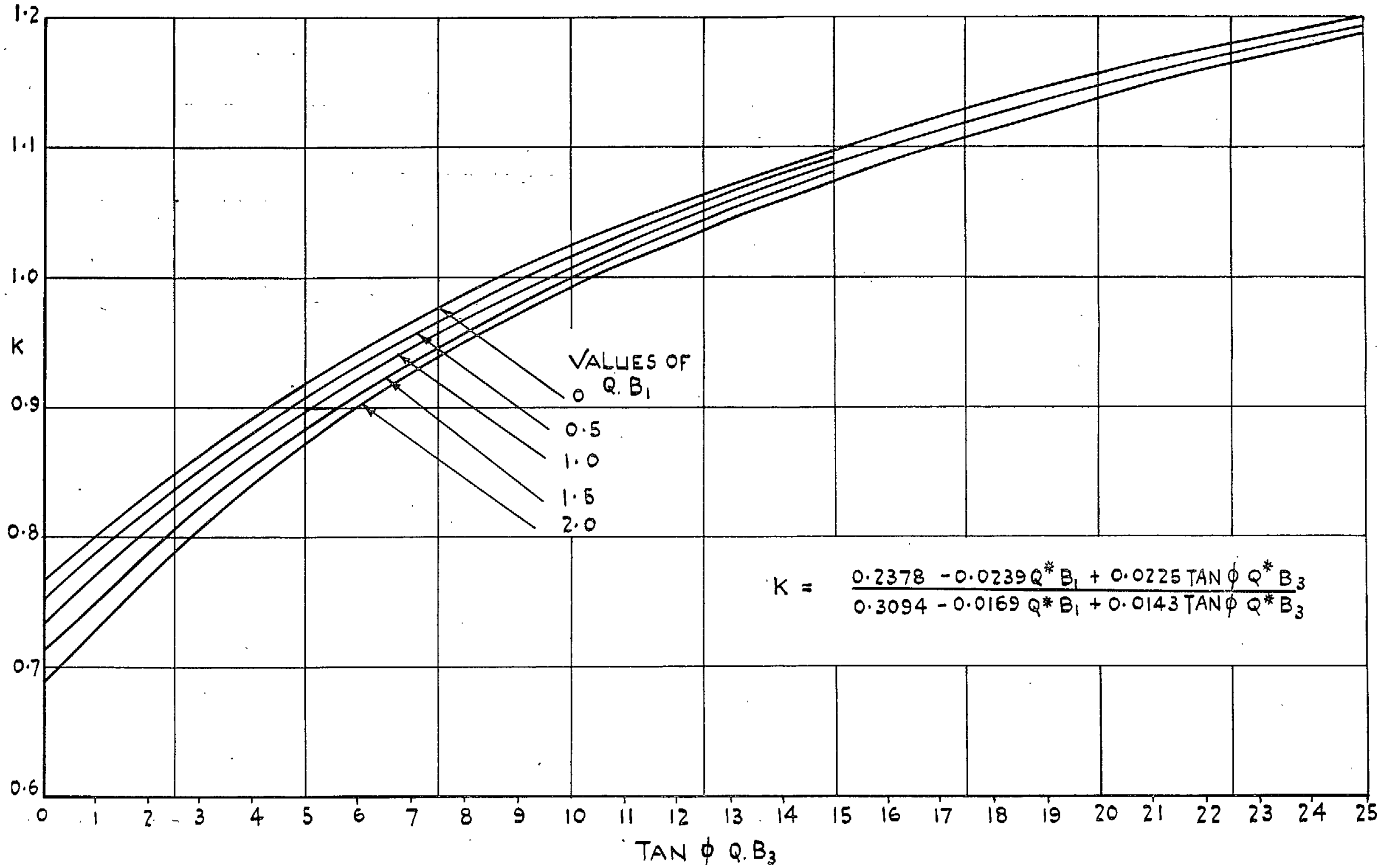


Fig. 12. The function K.

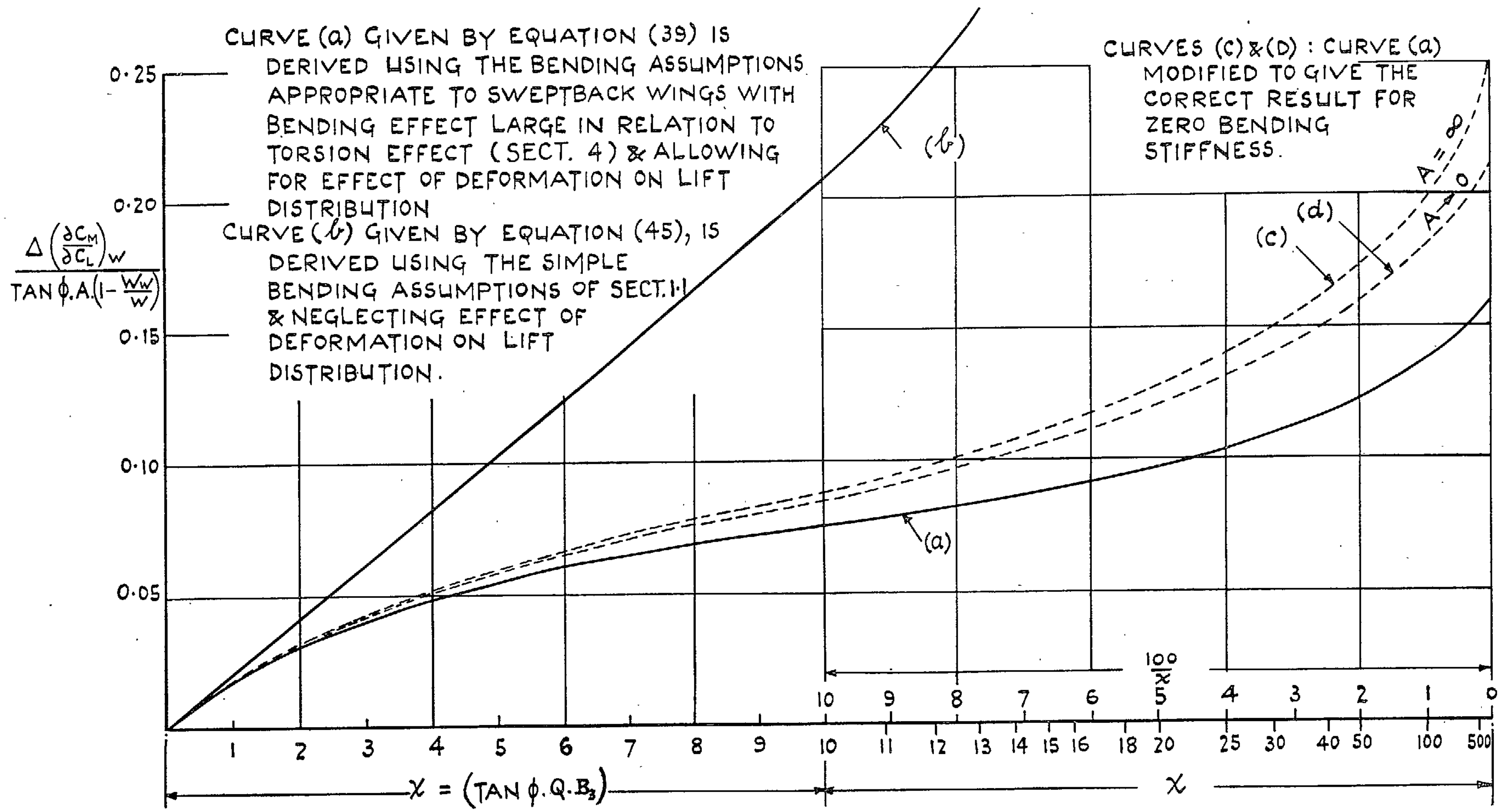


FIG. 13. Swept-back wing of infinite torsional rigidity; shift of manoeuvre point due to bending.

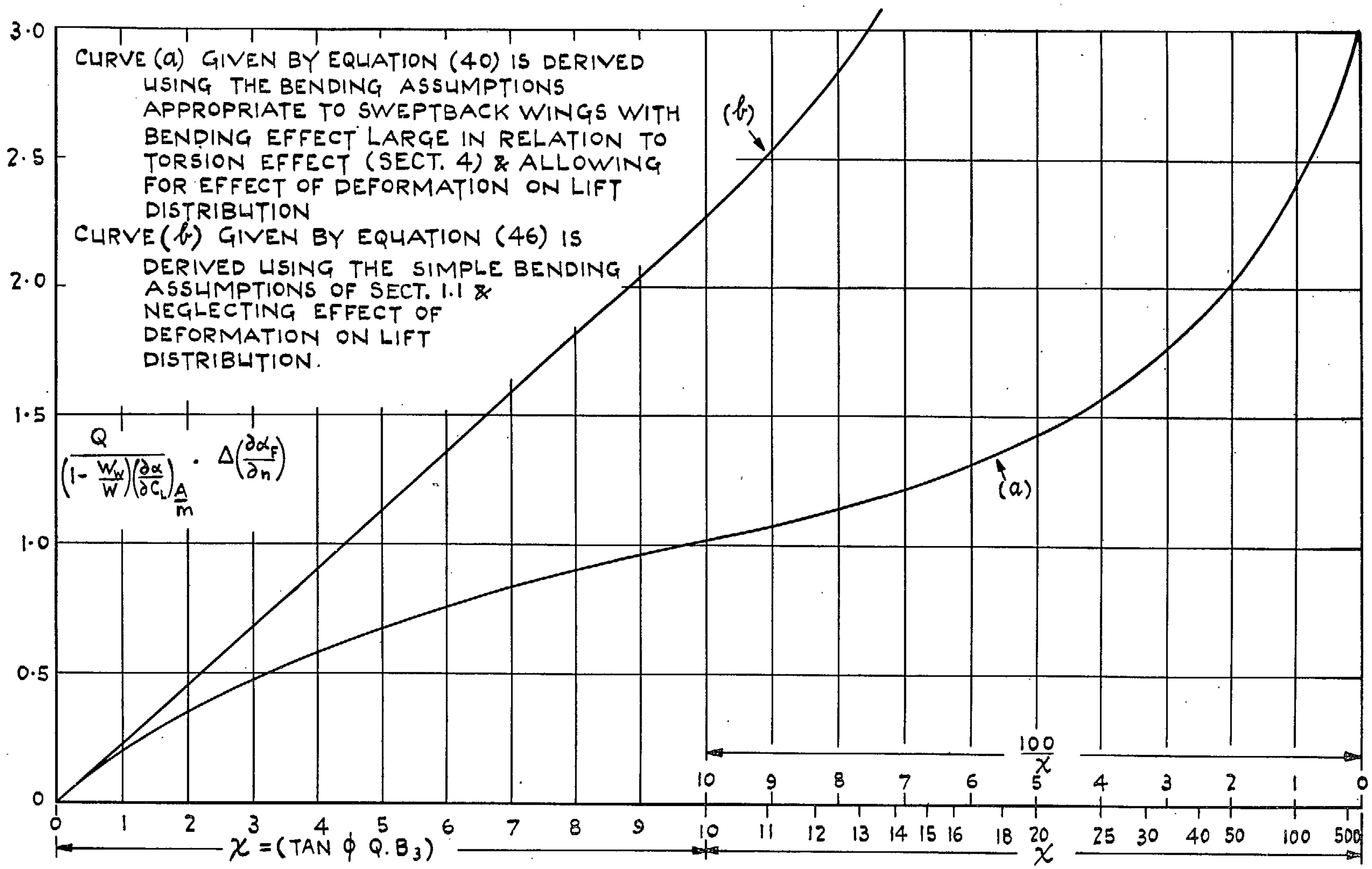


FIG. 14. Swept-back wing of infinite torsional rigidity; change of fuselage angle of attack resulting from bending of wing.

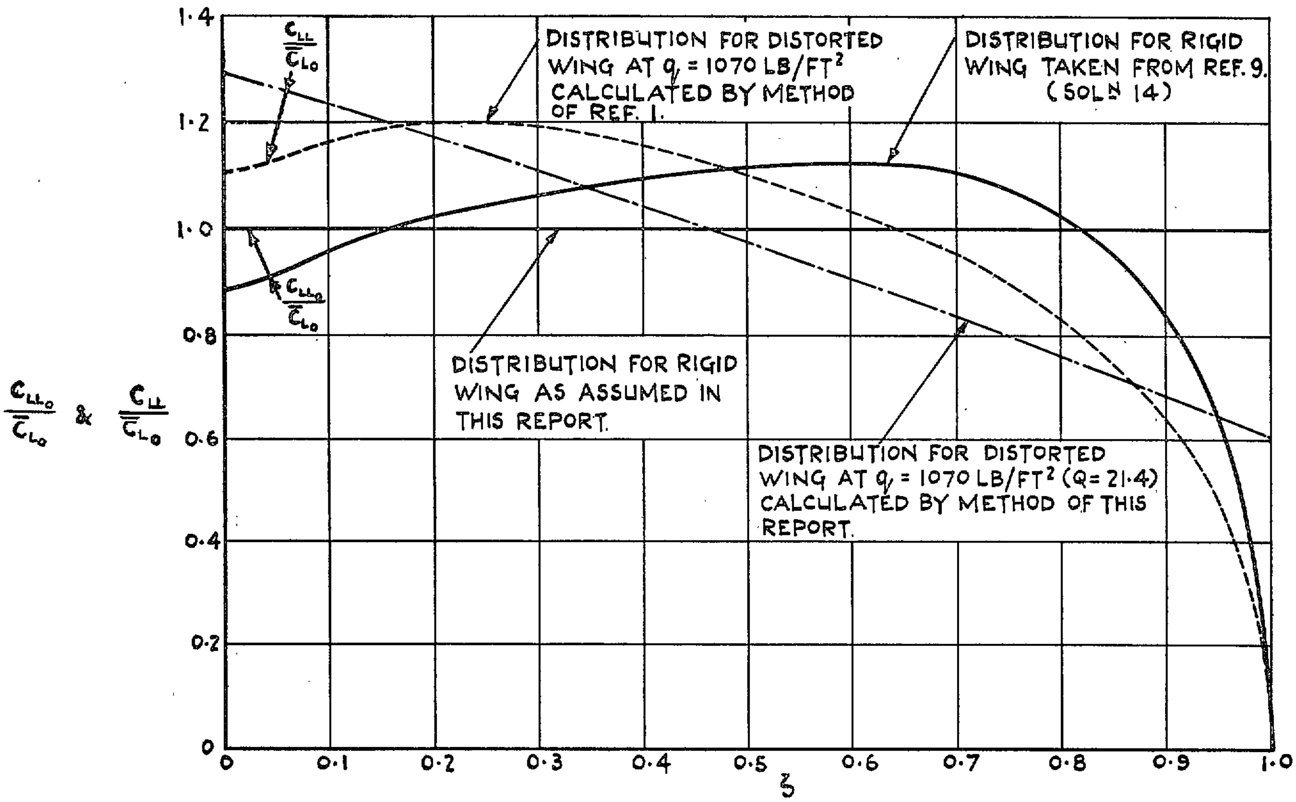


FIG. 15. Constant-chord wing, $A = 6$, $\phi = 45$ deg; spanwise lift distributions.

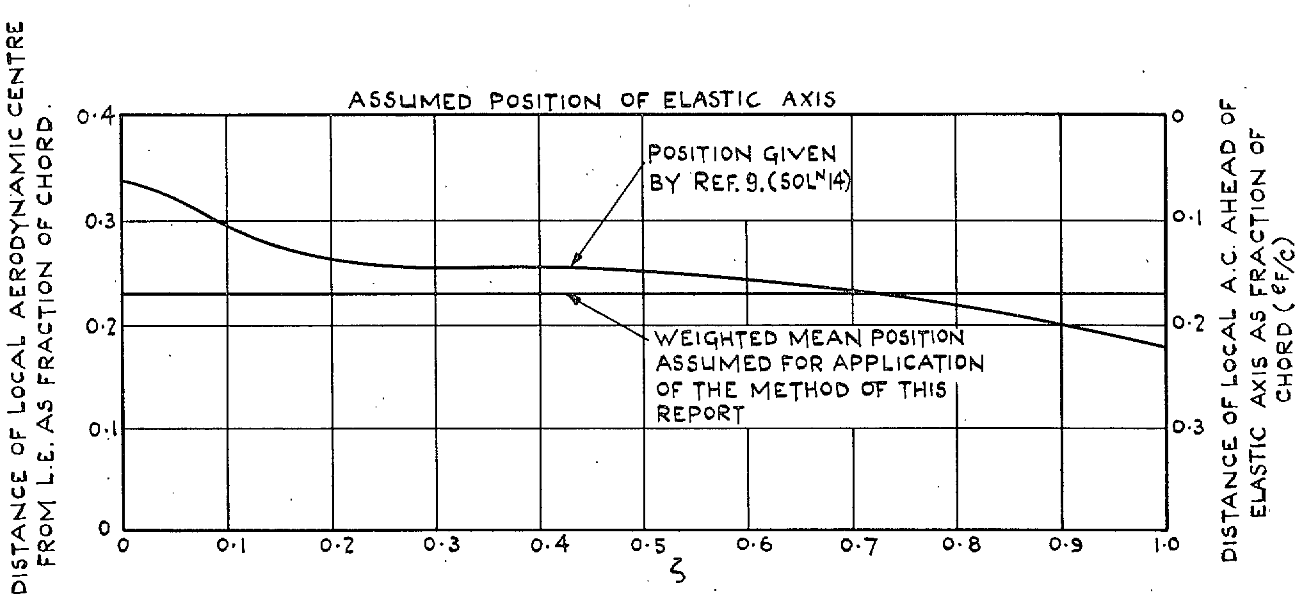


FIG. 16. Constant-chord wing, $A = 6$, $\phi = 45$ deg; position of local aerodynamic centre.

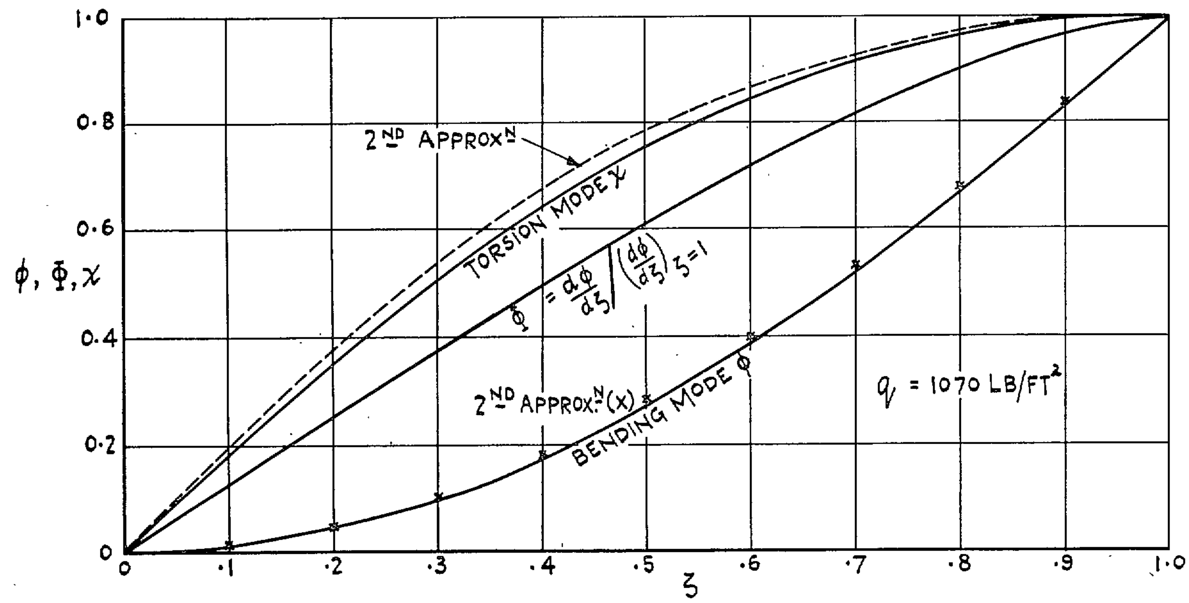


FIG. 17. Modes of distortion calculated by method of Ref. 1.

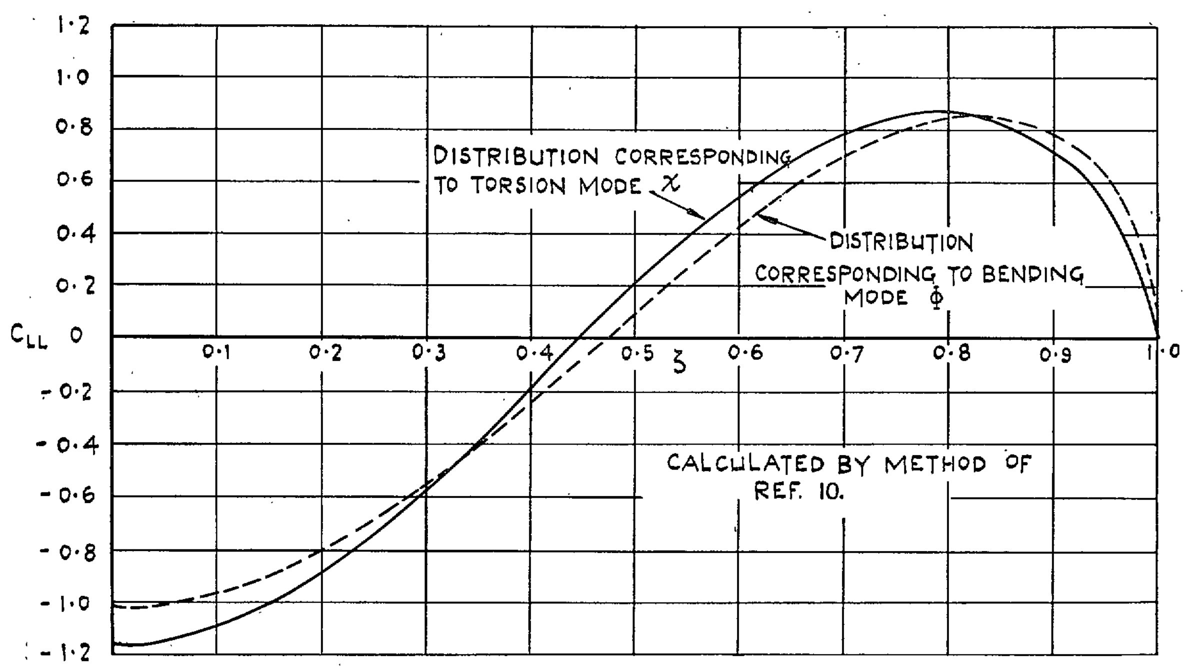


FIG. 18. 'Wash-out without lift' lift distributions corresponding to distortion modes Φ and χ.

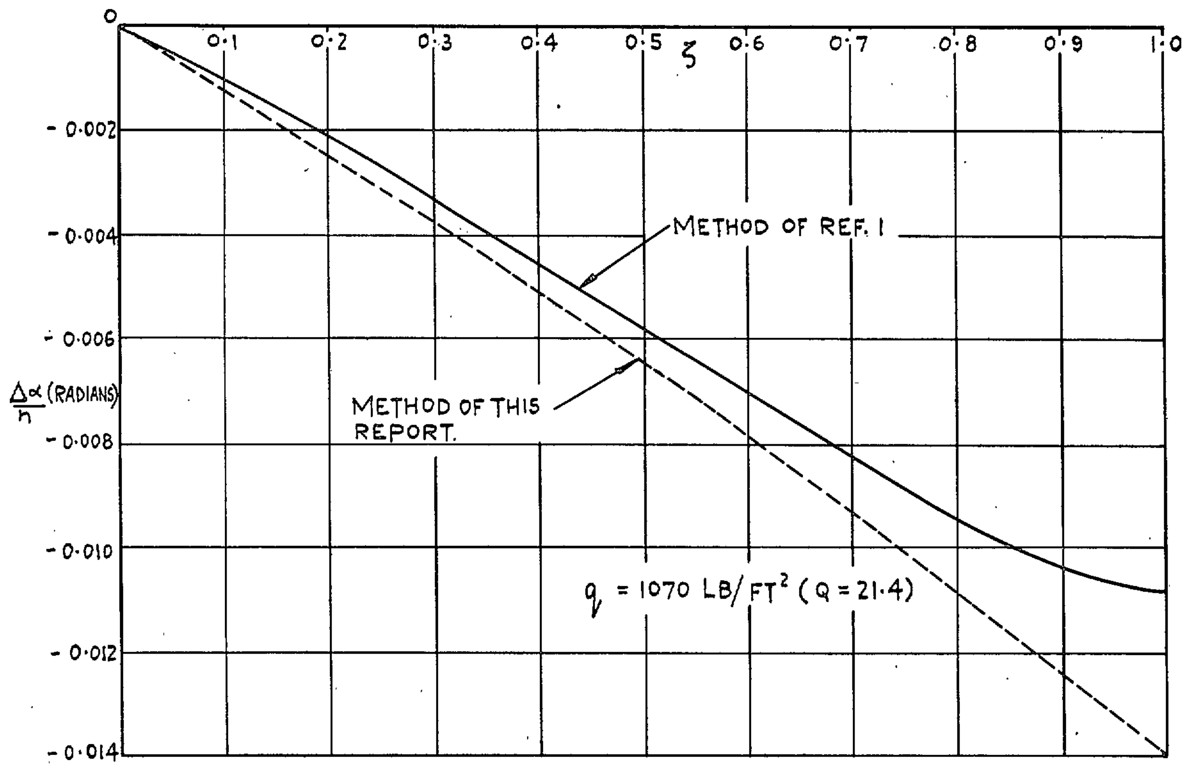


FIG. 19. Elastic wash-out per unit load factor.

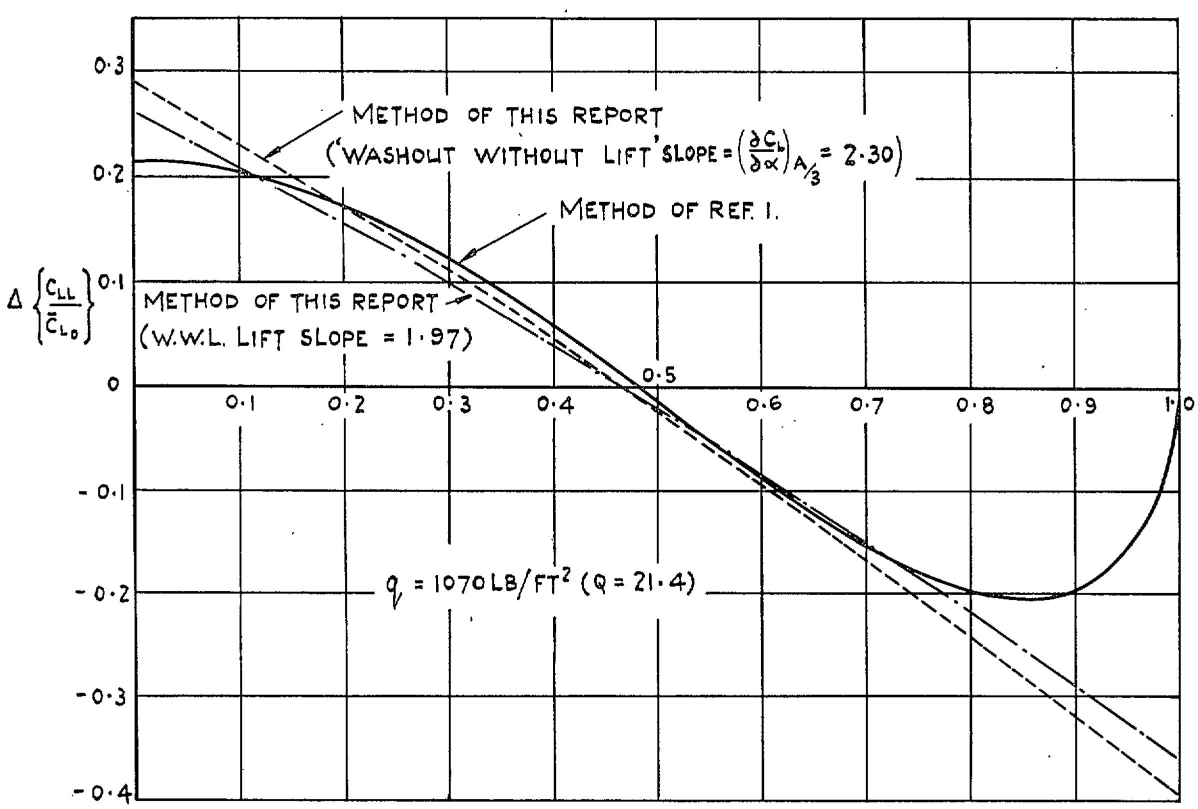


FIG. 20. Incremental lift distribution due to distortion when overall lift coefficient remains constant.

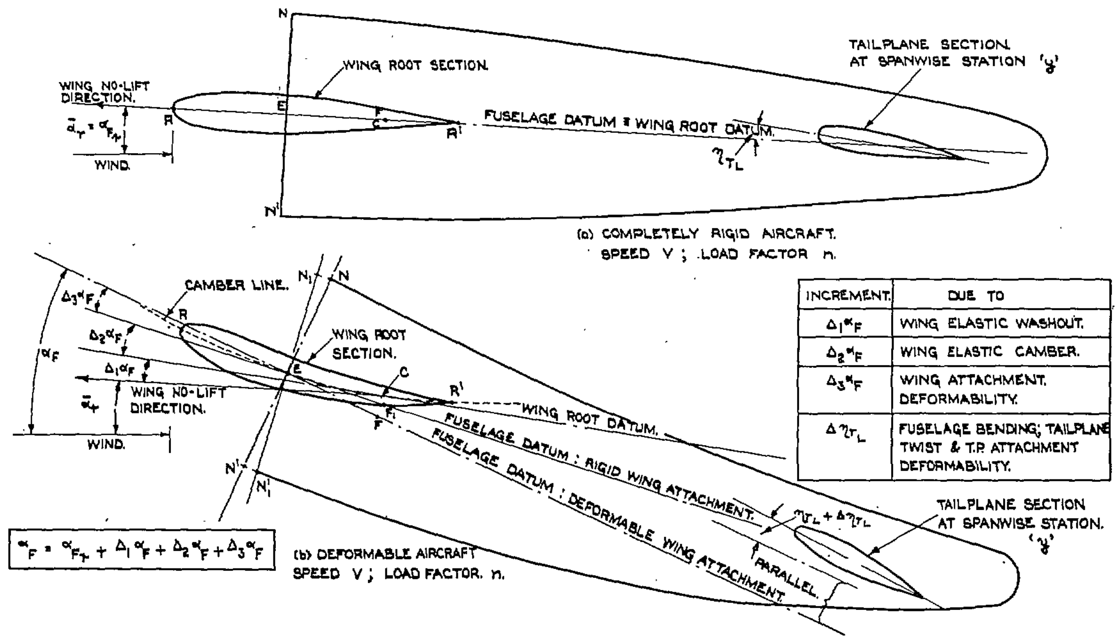


Fig. 21. Effect of deformability on fuselage angle of attack and on tailplane setting.

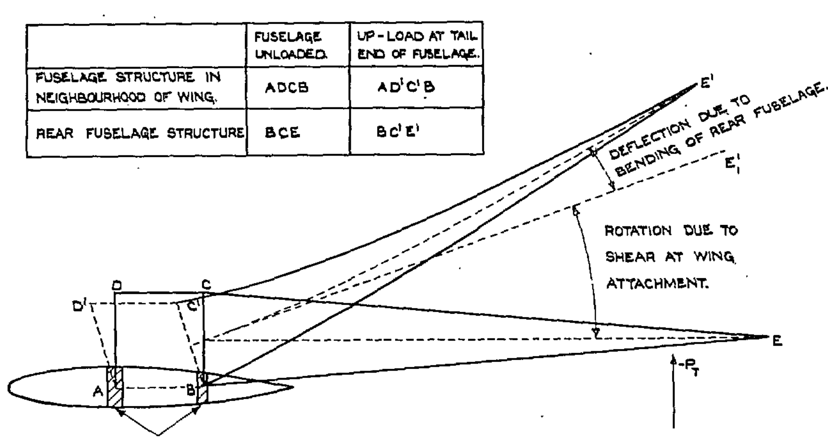


Fig. 22. Diagrammatic representation of fuselage rotation due to shear in neighbourhood of wing attachment.

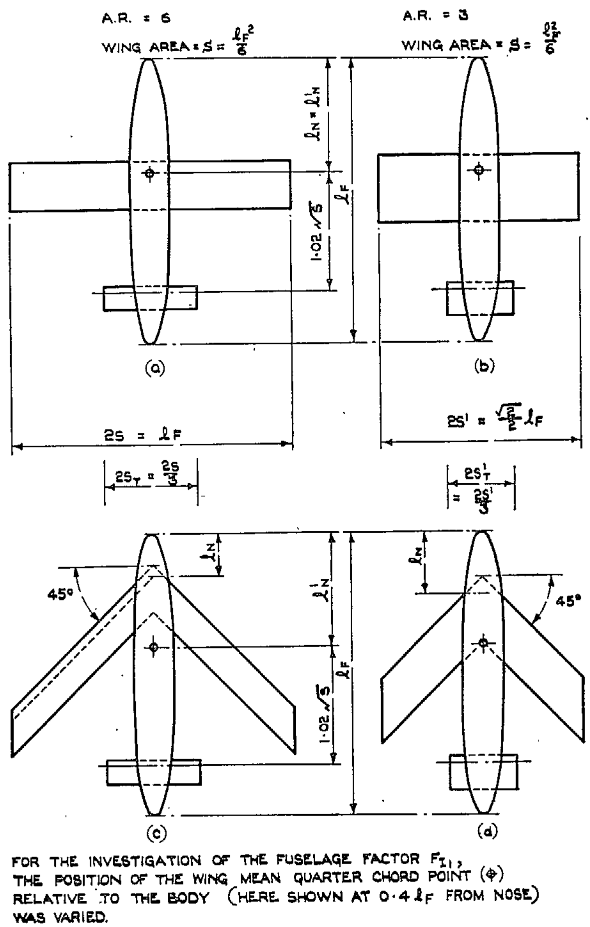


FIG. 23. Planforms considered in Appendix V.

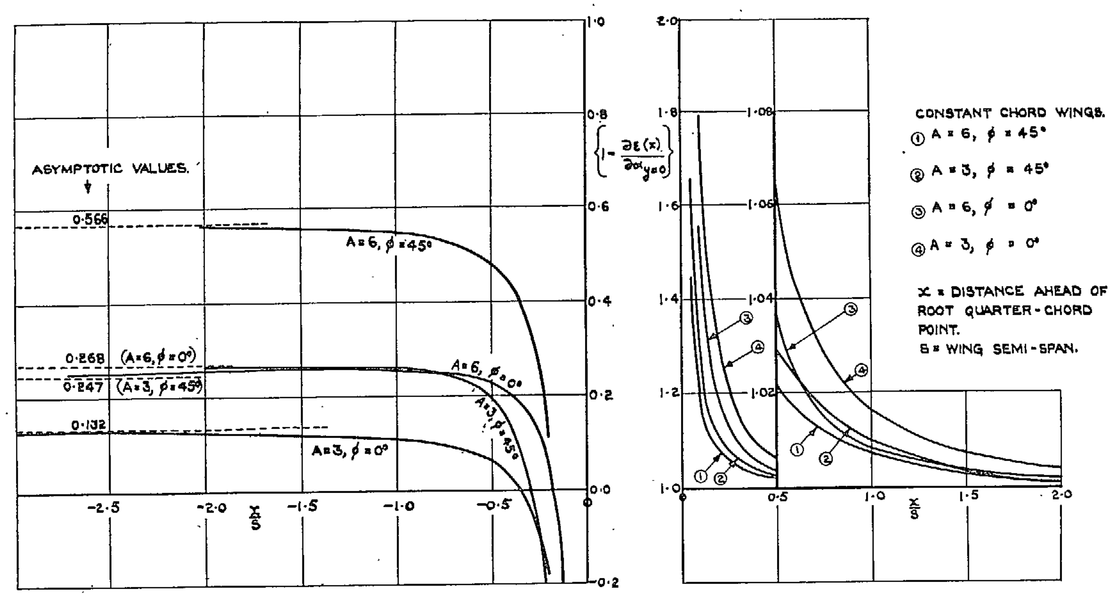


FIG. 24. Curves of upwash and downwash on the axes of symmetry of four constant-chord wings due to linear wash-out: $\alpha = \alpha_0\left(1 - 2\frac{|y|}{s}\right)$.

(5046) Wt. 20/9036 K.7 10/57 Hw.

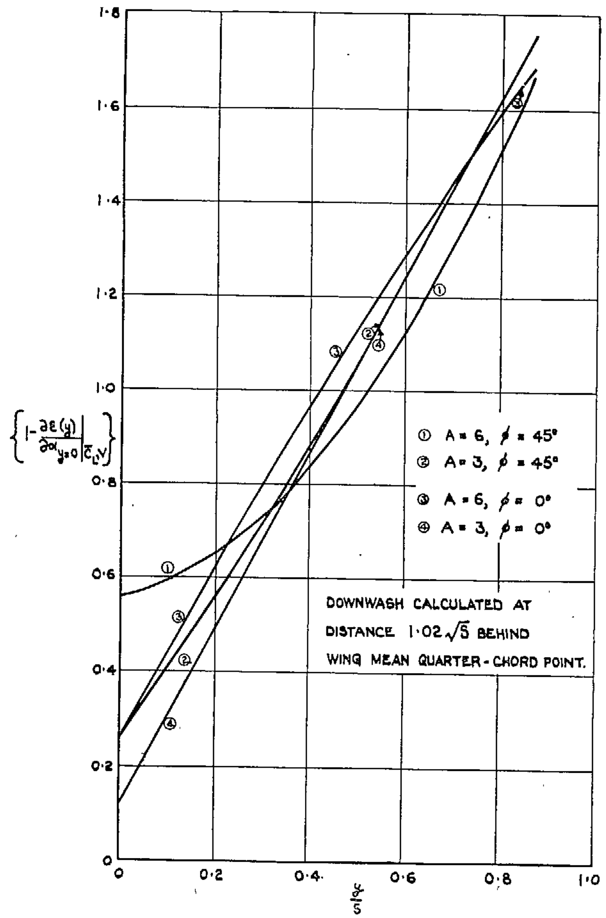


FIG. 25. Spanwise distribution of downwash at tail due to four constant-chord wings with linear wash-out: $\alpha = \alpha_0\left(1 - 2\frac{|y|}{s}\right)$.

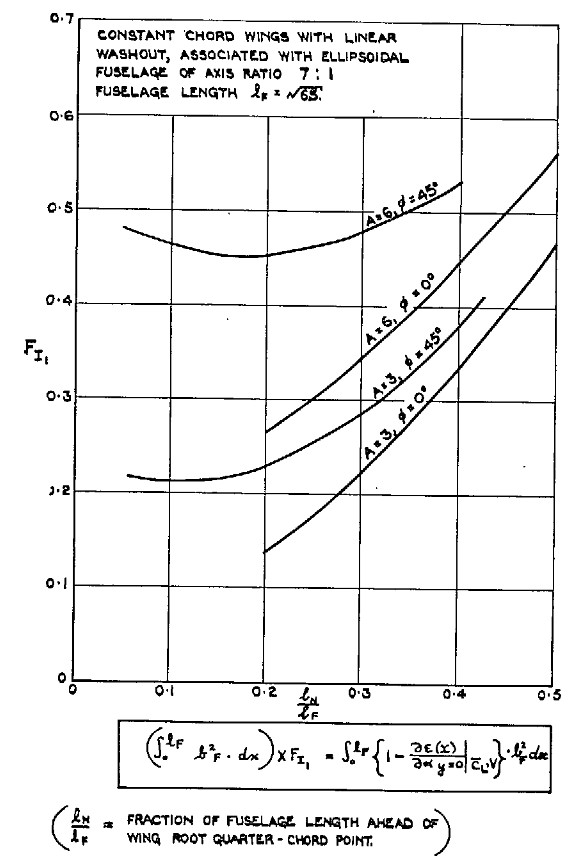


FIG. 26. The fuselage factor F_{I_1}.

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