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An Empirical Analysis of the Planing Lift Characteristics of Rectangular Flat-Plates and Wedges

By

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Summary.—The fluid flow about planing plates and wedges is briefly described and discussed, and on the basis of theoretical and physical considerations of this flow empirical formulae are presented for the lift developed by these planing surfaces. The formulae are mutually compatible and cover the whole range of planing of zero and finite deadrise surfaces including the chine-dry and chine-wet conditions. The lift formulae are extensively compared with experimental data, over a range of trim angles from 2 to 30 deg and deadrise angles from 0 to 40 deg, and very good agreement obtained.

The analysis confirms the existence of an effective critical wetted length for all the planing surfaces studied and shows that the wave-rise about planing wedges is an irrelevant feature of the flow.

The formulae are thought to be of such a nature that they may form the foundation for the lift prediction of more complex planing surfaces than are dealt with here.

Introduction.—This note is one of a series prepared at the Hydrodynamics Department of Short Brothers and Harland, concerned with the impact of seaplanes. The study has been divided into three phases :

(*a*) General seaplane impact theory

(*b*) Theory and analysis of planing as a special case of impact

(*c*) Applied impact and planing theory, including prediction of impact characteristics and experimental comparison.

The present note belongs to the second of these groups. The results of the study so far completed in group (*a*) are presented in Ref. 1.

Whilst this analysis was carried out so that it might be applied in the prediction of impact characteristics, and thus forms part of the general study of impact, it nevertheless stands in its own right as a general study of planing.

The existing empirical studies of planing are unsatisfactory in that they are severally only applicable to certain phases of planing (such as 'dry-chine' or 'wet-chine' planing) and that they consist in the main of little more than fitting arbitrary curves to experimental data. There is therefore need for a general study to cover all phases of planing based on the physics of flow

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about planing surfaces. It is in general desirable that such a study should be based on physical principles especially if it is to be of use in impact theory where its application demands that it be capable of being subject to mathematical manipulation without its validity being impaired.

It is the purpose of this note to present a unified general study of the fully developed planing of rectangular flat-plates and wedges. Because of fundamental differences it does not appear relevant to review existing empirical studies of planing of which Refs. 2 to 4 may be cited as representative.

Wetted Length.—The term ' wetted length ' which appears frequently in this note is not a precise one, as noted in Ref. 4. While it is not intended here to discuss in detail the wetted area of planing surfaces, observations of which are included in Refs. 4 to 6, for the purpose of this study the wetted length may be conveniently and adequately defined as follows.

For a flat-plate the wetted length is defined as the ratio of the wetted area, bounded in front by the stagnation line and at the sides and rear by the chines and step, to the beam.

For a planing wedge the wetted length is defined as the distance along the keel, from the point of intersection of the stagnation line with the keel to the point of the step.

As a matter of convenience the wetted length is referred to in units of beam and is denoted by the symbol λ.

Data.—In order to gain the maximum of generality it was desired to use data from all possible sources. A critical survey showed that reliable experimental planing data are not as plentiful as is sometimes suggested in the literature.

The only reliable flat-plate data are those contained in Ref. 5, where the wetted areas are determined from underwater photographs, and are very extensive. The data reported by Locke and Shoemaker, Refs. 3 and 7, were rejected as unsatisfactory in that the wetted lengths were estimated from visual observations of the water breaking out at the chine.

For information on the planing characteristics of wedges the data in Refs. 4, 6 and 7 were used. In this case Shoemaker's data were admitted largely because they cover a case omitted by Refs. 4 and 6, namely dry-chine planing in which the stagnation line does not cross the chine, and moreover Shoemaker measured the wetted length from the point of intersection of the keel with the water which in the range of trims covered does not differ very much from the stagnation-line-keel intersection. The data in Refs. 4 and 7 were further checked to eliminate non-planing conditions which were taken to be conditions where the buoyancy exceeded 20 per cent of the total lift. The National Advisory Committee for Aeronautics data, Refs. 5 and 6, had already been sorted to eliminate this buoyancy effect. Denoting the lift due to buoyancy of a planing wedge by Δ_s and supposing it to be equal to the vertical component of the resultant of the hydrostatic pressure on that part of the wedge below the still-water line, excepting the region at the back of the step, it can be shown that :

For the chines above the still-water line

$$C\Delta_s = \tfrac{1}{3} \sin^2 \tau \cot \beta \, . \, \lambda^3$$

For the chines below the still-water line

$$C\Delta_s = \tfrac{1}{8} \sin 2\tau \, . \, \lambda^2 - \tfrac{1}{48} \cos^3 \tau \operatorname{cosec} \tau \tan^2 \beta ,$$

the still-water line passing through the chine point when

$$\lambda = \tfrac{1}{2} \cot \tau \tan \beta .$$

These formulae were used to eliminate the non-planing conditions from Refs. 4 and 7 by ensuring that for a given wetted length the load coefficient was at least five times the buoyancy load coefficient as given above.

Analysis.—In framing the following analysis consideration was given to the whole field of planing phenomena, rather than any of its individual aspects and on this basis and from an initial study the following hypothesis was formulated which is basic to the whole analysis. For all planing surfaces there is assumed to exist a critical wetted length, λ_c, such that if the wetted length is greater than critical then the increase in lift with wetted length is directly proportional to the increase in wetted length. Mathematically :

$$\frac{\partial C_{Lb}}{\partial \lambda} = \text{constant for } \lambda \geqslant \lambda_c$$

whereas $$\frac{\partial C_{Lb}}{\partial \lambda} = f(\lambda) \text{ for } \lambda \leqslant \lambda_c.$$

The symbols used are listed later and it may be noted that the lift coefficients based on beam and area are related by :

$$C_{Lb} = \lambda C_L.$$

On this basis it becomes convenient to discuss planing under four heads :

(i) High wetted length planing

(ii) Low wetted length planing of flat-plates

(iii) Low wetted length planing of finite deadrise wedges

(iv) General planing.

These terms are relative and are merely intended to denote that in the régime included in (i) the wetted length is greater than, and in the régimes included in (ii) and (iii) is less than, the critical wetted length, the exact definition of which will appear in the analysis.

High Wetted Length Planing.—A rectangular wedge, which may be of either zero or finite deadrise, is shown planing in this condition in Fig. 1. The wedge is supposed held stationary while the water flows towards it from right to left. The general characteristics of the flow have been fully described in the literature ; the rise of the stagnation line AA_1 with heavy spray shooting out along it, the water peaks A_1A_2 and the trailing wake CD.

The normal force acting on the wedge is supposed to be compounded of two parts, N_1 and N_2, the force N_1 on the portion AB near the free surface being independent of the wetted length when the wetted length is greater than the critical wetted length and therefore :

$$C_{N1b} = \frac{N_1}{\frac{1}{2}\rho V^2 b^2} = N_1(\tau, \beta) \text{ for } \lambda \geqslant \lambda_c. \qquad \ldots \quad (1)$$

Since most of the longitudinal deflection of the flow will take place in the region AB, the force N_2 on BC is considered to be largely due to transverse flow about sections such as BB and if the sections present a drag coefficient C_β, constant along BC, to the transverse flow :

Due to transverse flow

$$C_{N2b} = \frac{\frac{1}{2}C_\beta \rho V^2 \sin^2 \tau . b^2(\lambda - \lambda_c)}{\frac{1}{2}\rho V^2 b^2} = C_\beta(\lambda - \lambda_c) \sin^2 \tau. \qquad \ldots \quad (2)$$

It was supposed that any further effect in this region due to residual longitudinal flow could probably be represented by a sine law, that is :

Due to longitudinal flow

$$C_{N2b} = \frac{\frac{1}{2}(C \sin \tau)\rho V^2 b^2(\lambda - \lambda_c)}{\frac{1}{2}\rho V^2 b^2} = C(\lambda - \lambda_c) \sin \tau. \qquad \ldots \quad (3)$$

Adding equations (1), (2) and (3) :

$$C_{Nb} = C_\beta(\lambda - \lambda_c)\sin^2\tau + C(\lambda - \lambda_c)\sin\tau + N_1(\tau, \beta).$$

Since the lift of the planing surface is the vertical component of the normal force, i.e., $L = N\cos\tau$:

$$C_{Lb} = (C_\beta \sin\tau + C)(\lambda - \lambda_c)\sin\tau\cos\tau + N_1(\tau, \beta) \qquad (4)$$

and differentiating equation (4) with respect to λ :

$$\frac{dC_{Lb}}{d\lambda} = (C_\beta \sin\tau + C)\sin\tau\cos\tau. \qquad (5)$$

Thus a plot of the experimental data in the form C_{Lb} against λ should be linear above the critical wetted length, and if the slopes of these graphs are measured values for C_β and C may be obtained from equation (5). This procedure was carried out using the data in Refs. 4 to 6 whence it was found that :

$$C_\beta = 1\cdot67(1 - \tfrac{\beta}{90}) \qquad (6)$$

$$C = 0\cdot09 \qquad (7)$$

and substituting in equation (4) :

$$C_{Lb} = \{1\cdot67(1 - \tfrac{\beta}{90})\sin\tau + 0\cdot09\}\sin\tau\cos\tau\,.\,(\lambda - \lambda_c) + N_1(\tau, \beta). \qquad (8)$$

Values for the function $N_1(\tau,\beta)$ and λ_c are deduced in the next three sections.

Low Wetted Length Planing of Flat-plates.—For the infinitely wide ($\lambda = 0$) flat-plate planing at an angle of attack α a formula for the two-dimensional lift coefficient may be found from Ref. 8 :

$$C_{L0} = \frac{2\pi\cos\alpha}{\cot\frac{1}{2}\alpha\cos\alpha - \tan\frac{1}{2}\alpha\log(\frac{1}{2} - \frac{1}{2}\cos\alpha) + \pi - \alpha - \sin\alpha}. \qquad (9)$$

Consider a finite beam flat-plate planing as in Fig. 2. The lift on the plate may be supposed to be due to the vertical momentum imparted to the water per second, i.e., $L = \dot{m}w$, where $\dot{m}$ is the time rate of change of the virtual mass of water associated with the plate, which, by analogy with the aerofoil is assumed to be equal to a semi-cylinder of water based on the beam as diameter and of length V, and w is the final downwash velocity of the water. Thus :

$$L = \tfrac{1}{8}\rho\pi b^2 V w.$$

Therefore $$C_L = (\pi/2\lambda)(w/2V). \qquad (10)$$

At the planing surface it can be shown from energy considerations that only half the final downwash velocity is developed so that the downwash angle ε is given by :

$$\tan\varepsilon = w/2V \qquad (11)$$

and substituting for $w/2V$ in equation (10)

$$C_L = (\pi/2\lambda)\tan\varepsilon. \qquad (12)$$

Since there is an infinitely small downwash behind an infinite beam planing surface equation (9) may be taken to be valid for a finite beam planing plate where the lift acts normal to the velocity vector V_R and the angle of attack is measured between the planing surface and this velocity vector. The angle of trim will then be equal to the sum of the attack and downwash angles :

$$\tau = \alpha + \varepsilon. \qquad (13)$$

Thus the lift acting normal to V_R is

$$L_0 = \tfrac{1}{2}C_{L0}\rho V_R^2 \lambda b^2 \qquad (14)$$

and since from Fig. 2, $L = L_0 \cos \varepsilon$ and $V_R = V \sec \varepsilon$ substitution in equation (14) gives :

$$L = \tfrac{1}{2} C_{L0} \rho (V \sec \varepsilon)^2 \lambda b^2 \cos \varepsilon .$$

Therefore $\quad C_L = C_{L0} \sec \varepsilon \qquad \ldots \qquad (15)$

and eliminating C_L between equations (12) and (15)

$$\sin \varepsilon = (2\lambda/\pi) C_{L0} . \qquad \ldots \qquad (16)$$

Equations (9), (13), (15) and (16) may now be solved to yield a plot of C_L against trim with wetted length as parameter as has been done in Fig. 3. Noting that $C_{Lb} = \lambda C_L$ the lift coefficients from Fig. 3 are compared with experimental data for 18-deg trim in Fig. 4a. It will be seen that excellent agreement is obtained up to a wetted length of one beam, beyond this length (aspect ratios less than one) the flow is no longer predominantly longitudinal and equations (10) to (16) would no longer be expected to apply.

This generally excellent agreement up to $\lambda = 1$ between the lift coefficients predicted in Fig. 3 and experimental data was found for all trims for which data were available, that is from 2 deg to 30 deg.

Whilst equations (9) to (16) are well-founded theoretically they do not lend themselves to routine computation ; for this reason rational approximations to these equations were sought. Noting that a plot of $1/C_L$ against λ is very nearly linear as shown in Fig. 4b, and as would be expected from approximations to equations (9) to (16) that a close approximation to equation (9) is :

$$C_{L0} = \frac{2\pi}{\cot \frac{1}{2}\tau + \pi} , \text{ since } \alpha = \tau \text{ at } \lambda = 0$$

it was found that a good expression for the lift coefficient in terms of the wetted length and trim, for wetted lengths up to one beam, was :

$$C_L = \frac{2\pi}{\cot \frac{1}{2}\tau + \pi + (2 \cot \frac{1}{2}\tau - \pi)\lambda} \qquad \ldots \qquad (17)$$

and therefore $$C_{Lb} = \frac{2\pi\lambda}{\cot \frac{1}{2}\tau + \pi + (2 \cot \frac{1}{2}\tau - \pi)\lambda} \text{ for } \lambda \leqslant 1. \qquad \ldots \qquad (18)$$

This approximation is compared with theoretical and experimental values in Fig. 4a and 4b. The chain-dotted line in Fig. 4a is that predicted by equation (8) where the constant in that equation, $N_1(\tau, \beta)$, is adjusted so that the line passes through the point given by $\lambda = 1$ and equation (18).

On the above grounds and from inspection of the experimental data it was concluded that the critical wetted length of a flat-plate was one beam, that is for the flat-plate :

$$\lambda_c = 1.$$

Low Wetted Length Planing of Wedges.—A wedge planing in the above condition, also spoken of as 'dry-chine' planing, is shown in Fig. 5. All observations of the planing wedge refer to the markedly three-dimensional nature of the flow as contrasted with the substantially longitudinal flow about the flat-plate of Fig. 2 planing at low wetted lengths. It is this three-dimensional nature of planing wedge flow which makes analysis so difficult. Nevertheless Pierson and Leshnover in a noteworthy report, Ref. 9, have attempted a synthesis of various two-dimensional studies of wedge planing from which a formula for the lift generated by a planing wedge is derived. Despite the theoretical basis of Pierson's formula it was found after careful study and comparison that the formula exhibited systematic deviations from the experimental data. In default of a better approach the following line of empirical reasoning was resorted to.

Suppose as before that the normal force on the wedge is due to the momentum imparted to the water, i.e., $N = \dot{m}w$, where the time rate of change of the virtual mass, $\dot{m}$, is taken to be a semi-cylinder of water of width c beams and of length $V \cos \tau$ (the velocity component along the keel) and w is the velocity of the water normal to the keel, $V \sin \tau$, then :

$$N = \tfrac{1}{8}\rho\pi(cb)^2 V \cos \tau \,.\, V \sin \tau$$

$$C_{Nb} = \tfrac{1}{4}\pi c^2 \sin \tau \cos \tau. \qquad \ldots \qquad (19)$$

The width c is taken as the width across the still water line at the step, since the width across the stagnation line is a constant percentage of this width, and from Fig. 5 :

$$c = 2\lambda \tan \tau \cot \beta. \qquad \ldots \qquad (20)$$

Substituting for c in equation (19) gives :

$$C_{Nb} = \pi\lambda^2 \sin^3 \tau \cot^2 \beta \sec \tau.$$

This suggested plotting the data in the form

$$C_{Nb}/\lambda^2 \sin^3 \tau \cot^2 \beta \text{ against } \sec \tau \text{ or } f(\tau)$$

and from trial plots it was found that :

$$C_{Nb} = 3 \cdot 6\lambda^2 \sin^3 \tau \cot^2 \beta \,(1 - \sin \tau)$$

therefore $\quad C_{Lb} = 3 \cdot 6\lambda^2 \sin^3 \tau \cot^2 \beta \,(1 - \sin \tau) \cos \tau \qquad \ldots \qquad (21)$

for λ less than λ_c.

Plots were made of the wedge data for all wetted lengths in the form of lift coefficient against wetted length and the parabola given by equation (21) for 'small' wetted lengths, and the straight line given by equation (8) for 'large' wetted lengths, were fitted to the data. The wetted length at which these two curves intersected was read off and found in every case to be close to that at which the still-water line passed through the chine point, that is when $c = 1$. On this basis it was concluded for the wedge, using equation (20), that the critical wetted length is :

$$\lambda_c = \tfrac{1}{2} \cot \tau \tan \beta. \qquad \ldots \qquad (22)$$

General Planing.—The findings of the previous three sections may now be combined and presented together for convenience.

Since for the flat-plate equation (18) applies up to and including a wetted length of one beam, and since at this wetted length equation (8) must predict the same lift, the function $N_1(\tau, \beta)$ of that equation is found by putting $\lambda = 1$ in equation (18), thus :

For the flat-plate $\lambda \leqslant \lambda_2 = 1$

$$C_{Lb} = \frac{2\pi\lambda}{\cot \frac{1}{2}\tau + \pi + (2 \cot \frac{1}{2}\tau - \pi)\lambda}. \qquad \ldots \qquad (23)$$

For the flat-plate $\lambda \geqslant \lambda_c = 1$

$$C_{Lb} = \{1 \cdot 67(1 - \tfrac{\beta}{90}) \sin \tau + 0 \cdot 09\} \sin \tau \cos \tau \,.\, (\lambda - 1) + \frac{2\pi}{3 \cot \frac{1}{2}\tau} \qquad \ldots \qquad (24)$$

where for the flat-plate $\beta = 0$.

Similarly for the wedge equation (21) applies up to and including the critical wetted length given by equation (22), and since at this wetted length equation (8) must predict the same lift, the function $N_1(\tau, \beta)$ of that equation is found by putting $\lambda = \lambda_c$ in equation (21) thus :

For the wedge $\lambda = \lambda_c = \frac{1}{2} \cot \tau \tan \beta$

$$C_{Lb} \leqslant 3 \cdot 6\lambda^2 \cot^2 \beta \sin^3 \tau \,.\, (1 - \sin \tau) \cos \tau. \qquad (25)$$

For the wedge $\lambda = \lambda_c = \frac{1}{2} \cot \tau \tan \beta$

$$C_{Lb} \geqslant \{1 \cdot 67(1 - \tfrac{\beta}{90}) \sin \tau + 0 \cdot 09\} \sin \tau \cos \tau \,.\, (\lambda - \lambda_c)$$
$$+ 0 \cdot 9 \sin \tau \,.\, (1 - \sin \tau) \cos^3 \tau. \qquad (26)$$

In their general form these planing lift equations appear complicated, although they reduce to a simple form for any specific deadrise and trim. In order to facilitate computation, and also to show more clearly the dependence of lift on wetted length, the formulae are re-cast as follows.

The following set of factors are defined :

$$a_1 = \tfrac{1}{2} + 3\pi/2(2 \cot \tfrac{1}{2}\tau - \pi) \qquad (27)$$
$$a_2 = \{1 \cdot 67(1 - \tfrac{\beta}{90}) \sin \tau + 0 \cdot 09\} \sin \tau \cos \tau \qquad (28)$$
$$a_3 = 3\pi/2 \cot \tfrac{1}{2}\tau = 4(a_1 - \tfrac{1}{2})/3(a_1 + 1) \qquad (29)$$
$$a_4 = 3 \cdot 6 \cot^2 \beta \sin^3 \tau \,.\, (1 - \sin \tau) \cos \tau \qquad (30)$$
$$a_5 = 0 \cdot 9 \sin \tau \,.\, (1 - \sin \tau) \cos^3 \tau. \qquad (31)$$

With these factors equations (23) to (26) become :

For the flat-plate

$$C_{Lb} = 4(a_1 - \tfrac{1}{2})\lambda/3(a_1 + \lambda) \quad \text{for } \lambda \leqslant 1 \qquad (23a)$$
$$C_{Lb} = a_2(\lambda - 1) + a_3 \quad \text{for } \lambda \geqslant 1. \qquad (24a)$$

For the wedge

$$C_{Lb} = a_4\lambda^2 \quad \text{for } \lambda \leqslant \lambda_c \qquad (25a)$$
$$C_{Lb} = a_2(\lambda - \lambda_c) + a_5 \quad \text{for } \lambda \geqslant \lambda_c. \qquad (26a)$$

The a factors and λ_c are functions of trim and deadrise angle only and are tabulated in Tables 1 to 4 for angles of trim, 0 deg (2 deg) 30 deg, and deadrise angle, 0 deg (10 deg) 40 deg, to at least four significant figures. This seems to be more accurate than presenting the data in graphical form and individual users should have no trouble in preparing graphs of the a factors either for speed of use or for interpolation.

Results.—The lift coefficients predicted by equations (23) to (26) are compared with the experimental data in Figs. 6 to 10 for deadrise angles from 0 deg to 40 deg and trims from 2 deg to 30 deg in the form of plots of C_{Lb} against λ. Data sources are noted on the figures and in every case the lines drawn through the data are those predicted by equations (23) to (26).

In a study of this nature it is most important that the results of the study be clearly and fairly compared with the experimental data. For this reason some consideration was given to the form in which the results were to be presented, it being apparent from the discontinuous nature of equations (23) to (26) that no one form of presentation was the overall ideal. Since it was not feasible to show more than one form of presentation, because of the large number of data to be handled, that shown in Figs. 6 to 10 was selected as being probably the most general and one from which the forms of equations (23) to (26) could be most readily appreciated and systematic deviations detected. The possibility of deviations being masked by the presentation is discussed in the next section.

The data sources are readily available and with the lift equations in the form (23a) to (26a), together with Tables 1 to 4 of the a factors, there should be no difficulty in preparing plots in any form and to any scale that may seem desirable for comparison purposes.

Discussion.—It is believed that by any standard Figs. 6 to 10 exhibit excellent agreement between the experimental data and the empirical formulae equations (23) to (26), and that this agreement is the more noteworthy in view of the wide variety of configurations and conditions covered. It should be remembered in viewing these comparisons that the data from Refs. 5 and 6 are the most reliable, although it is stated in the former that the accuracy of measurement becomes marginal below a wetted length-beam ratio of $0 \cdot 5$; the data of Ref. 7 were obtained by less precise techniques, and the data from Ref. 4 were gathered at comparatively low speeds.

The possibility of deviations being masked by the form of presentation has been referred to above ; two instances of this may be cited.

The flat-plate formula applicable to wetted lengths less than one beam is shown in two different forms in Figs. 4a and 4b for 18-deg trim, and it is apparent that in Fig. 4b deviations from the empirical formula may be observed near the origin that are not easily detectable in Fig. 4a, an effect referred to as a masking effect. The reason for this breakaway of the experimental points from the theoretical formula, identical with the empirical formula in this region, may be deduced (quite apart from the loss of experimental accuracy for $\lambda < 0 \cdot 5$). The two-dimensional theoretical formula, equation (9), applies to the condition $\lambda = 0$, a condition which is approached theoretically by imagining the beam to be infinite but which was approached in practice, Ref. 5, by letting the wetted length of a finite beam (4 in.) flat-plate approach zero. Thus the breakaway is attributed to the breakdown of the flow about a finite beam planing surface as the wetted length approaches zero (possibly due to effects including the curvature of the stagnation line) and it is argued that for a wider physical beam and any beam likely to be met with in practice the breakdown would be postponed to very much smaller wetted lengths than in Fig. 4b. The presentation shown in Figs. 4a and 6 is therefore held to be justified.

Again in the case of the wedge, *e.g.*, *cf.* Fig. 8b, the lift coefficient varies parabolically with the wetted length when the wetted length is less than critical, and thus a plot of lift coefficient against the square of the wetted length would provide a more searching examination of the correlation between the empirical formulae and the experimental data than that shown. Such plots have been made and examined without any systematic deviations being detected.

In general it may be said that extensive and searching examinations have been made of the empirical-experimental correlation without any unexplainable systematic deviations being detected.

The form assumed for the coefficient C_β in equation (4) deserves mention because a theoretical value for this coefficient has been obtained by Bobyleff, this theoretical value is again fully derived in Ref. 10. In particular the theoretical value of C_β for zero deadrise is $1 \cdot 76$ instead of $1 \cdot 67$ given by equation (6) and the theory predicts a non-linear variation of the coefficient with deadrise whereas a linear variation is found experimentally, equation (6). It is of interest to note than Locke in Ref. 3 also deduced a linear variation for a substantially similar coefficient.

Another point of interest arising from the analysis is the fact that surprisingly the wave-rise, or rise of the stagnation line on a wedge, as in Fig. 5, about which so much has been written is not apparently so fundamental to planing as is generally thought since the wedge behaves as if the wetted area were bounded by the intersection with the still-water line. The phenomenon of water pile-up at the leading edges of flat-plates and wedges, as distinct from wave-rise, will form the subject of a separate report.

Despite Pierson's ingenious theory of dry-chine planing, Ref. 9, there still appears to be need for a further theoretical study of this phase of planing in view of the discrepancies found between that theory and experiment in the course of this analysis. It should be noted however that the experimental data covering this phase of planing are neither as reliable nor as extensive as could be wished.

It will be appreciated that the concept of a critical wetted length introduced in this analysis has proved very useful in rationalising the planing data, and, without reference to the physical validity of this concept, it can be said that it has been equally useful in further planing studies, details of which are to be published.

Concluding Remarks.—A unified general study of the planing of rectangular flat-plates and wedges is presented from which are deduced empirical formulae for computing the lift of such planing surfaces. Good agreement is shown between these formulae and the experimental data over a wide range of trim angles, 2 deg to 30 deg, and deadrise angles, 0 deg to 40 deg. The concept of a critical wetted length, existing for all planing surfaces, is introduced in the analysis and is found to be of the utmost use in rationalising the planing data.

It is observed that the planing wedge behaves as if there were no rise of the stagnation line above the still-water line, and that therefore wave-rise is in some ways an irrelevant feature of planing wedge flow.

Attention is invited to the facts that this analysis derived considerable benefit from theoretical studies such as Ref. 8 and that without the extensive data provided by the N.A.C.A. in Refs. 5 and 6 the analysis could not have been completed. There is, however, a marked lack of data covering the dry-chine planing of wedges.

While the lift formulae for rectangular flat-plates and wedges derived herein are empirical it is considered that because of their physical basis they are unlikely to be entirely superseded and that they should form the foundation for analyses of more complex planing surfaces.

The lift formulae are summarised below. The a factors and λ_c are functions of trim and deadrise, analytical expressions for which are derived in the analysis, and specific values of which are given in tables.

For the flat-plate

$$C_{Lb} = 4(a_1 - \tfrac{1}{2})\lambda/3(a_1 + \lambda) \qquad \text{for } \lambda \leqslant 1$$
$$C_{Lb} = a_2(\lambda - 1) + a_3 \qquad \text{for } \lambda \geqslant 1.$$

For the wedge

$$C_{Lb} = a_4\lambda^2 \qquad \text{for } \lambda \leqslant \lambda_c$$
$$C_{Lb} = a_2(\lambda - \lambda_c) + a_5 \qquad \text{for } \lambda \geqslant \lambda_c.$$

LIST OF SYMBOLS

b	Maximum beam at chine, ft
c	Width of wedge at step across still-water line, beams
C	A constant, equation (7)
C_β	A deadrise function, equation (6)
$C_{\Delta s}$	Hydrostatic beam loading coefficient, $\Delta_s/\rho g b^3$
C_L	Planing lift coefficient, based on area, $L/\frac{1}{2}\rho V^2\lambda b^2$
C_{L0}	Two-dimensional lift coefficient, $L_0/\frac{1}{2}\rho V^2\lambda b^2$
C_{Lb}	Planing lift coefficient, based on beam, $L/\frac{1}{2}\rho V^2 b^2$

LIST OF SYMBOLS—*continued*

C_{Nb}	Normal force coefficient, based on beam, $N/\frac{1}{2}\rho V^2 b^2$
g	Gravity acceleration, $32 \cdot 2$ ft/sec^2
L	Planing lift, normal to undisturbed water surface (lb)
L_0	Two-dimensional lift (lb)
m	Virtual mass, slugs
N	Normal force on planing surface, normal to keel (lb)
V	Horizontal velocity (ft/sec)
w	Downwash velocity (ft/sec)
α	Two-dimensional angle of attach (deg)
β	Deadrise angle (deg)
Δ_s	Lift due to buoyance (lb)
ε	Downwash angle (deg)
λ	Wetted length, *see* text (beams)
λ_c	A critical wetted length (beams)
ρ	Mass density of water (slugs/cu ft)
τ	Trim angle relative to undisturbed water surface (deg)

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TABLE 1

Factors a_1, a_3, and a_5

Trim (deg)	a_1	a_3	a_5
0	0·5000	0	0
2	0·5423	0·03656	0·03026
4	0·5871	0·07314	0·05798
6	0·6346	0·1098	0·08287
8	0·6851	0·1465	0·1047
10	0·7389	0·1832	0·1233
12	0·7966	0·2201	0·1387
14	0·8584	0·2572	0·1508
16	0·9250	0·2943	0·1596
18	0·9968	0·3317	0·1653
20	1·0746	0·3693	0·1681
22	1·1593	0·4071	0·1681
24	1·2519	0·4452	0·1656
26	1·3535	0·4835	0·1609
28	1·4034	0·5012	0·1543
30	1·5902	0·5612	0·1530

TABLE 2

Factor a_2

Trim (deg)	Deadrise 0 deg	Deadrise 10 deg	Deadrise 20 deg	Deadrise 30 deg	Deadrise 40 deg
0	0	0	0	0	0
2	0·005172	0·004946	0·004720	0·004494	0·004269
4	0·01437	0·01347	0·01257	0·01167	0·01077
6	0·02750	0·02549	0·02347	0·02145	0·01944
8	0·04444	0·04088	0·03732	0·03376	0·03020
10	0·06498	0·05947	0·05396	0·04845	0·04294
12	0·08892	0·08107	0·07322	0·06538	0·05753
14	0·1160	0·1054	0·09489	0·08435	0·07381
16	0·1458	0·1323	0·1187	0·1052	0·09161
18	0·1781	0·1613	0·1444	0·1276	0·1107
20	0·2125	0·1921	0·1717	0·1513	0·1309
22	0·2485	0·2244	0·2003	0·1761	0·1520
24	0·2858	0·2578	0·2297	0·2017	0·1737
26	0·3239	0·2919	0·2598	0·2278	0·1957
28	0·3623	0·3262	0·2901	0·2540	0·2179
30	0·4005	0·3604	0·3202	0·2800	0·2398

TABLE 3

Factor a_4

Trim (deg)	Deadrise 10 deg	Deadrise 20 deg	Deadrise 30 deg	Deadrise 40 deg
0	0	0	0	0
2	0·004747	0·001114	0·0004428	0·0002096
4	0·03678	0·008561	0·003402	0·001611
6	0·1178	0·02764	0·01099	0·005201
8	0·2661	0·06245	0·02482	0·01175
10	0·4934	0·1158	0·04602	0·02179
12	0·8602	0·1892	0·07520	0·03560
14	1·2059	0·2830	0·1125	0·05325
16	1·6884	0·3963	0·1575	0·07456
18	2·2454	0·5270	0·2094	0·09915
20	2·8643	0·6722	0·2672	0·1265
22	3·5295	0·8284	0·3292	0·1559
24	4·2227	0·9911	0·3939	0·1865
26	4·9237	1·1556	0·4593	0·2174
28	5·6122	1·3172	0·5235	0·2478
30	6·2672	1·4709	0·5846	0·2767

TABLE 4

Critical Wetted Length c

Trim (deg)	Deadrise 10 deg	Deadrise 20 deg	Deadrise 30 deg	Deadrise 40 deg
0				
2	2·5247	5·2113	8·2665	12·014
4	1·2608	2·6026	4·1283	6·0000
6	0·8388	1·7315	2·7466	3·9918
8	0·6273	1·2949	2·0540	2·9853
10	0·5000	1·0321	1·6372	2·3794
12	0·4148	0·8562	1·3581	1·9738
14	0·3536	0·7299	1·1578	1·6827
16	0·3075	0·6347	1·0067	1·4631
18	0·2713	0·5601	0·8884	1·2912
20	0·2422	0·5000	0·7931	1·1527
22	0·2182	0·4504	0·7145	1·0384
24	0·1980	0·4087	0·6484	0·9423
26	0·1808	0·3731	0·5919	0·8602
28	0·1658	0·3423	0·5429	0·7891
30	0·1527	0·3152	0·5000	0·7267

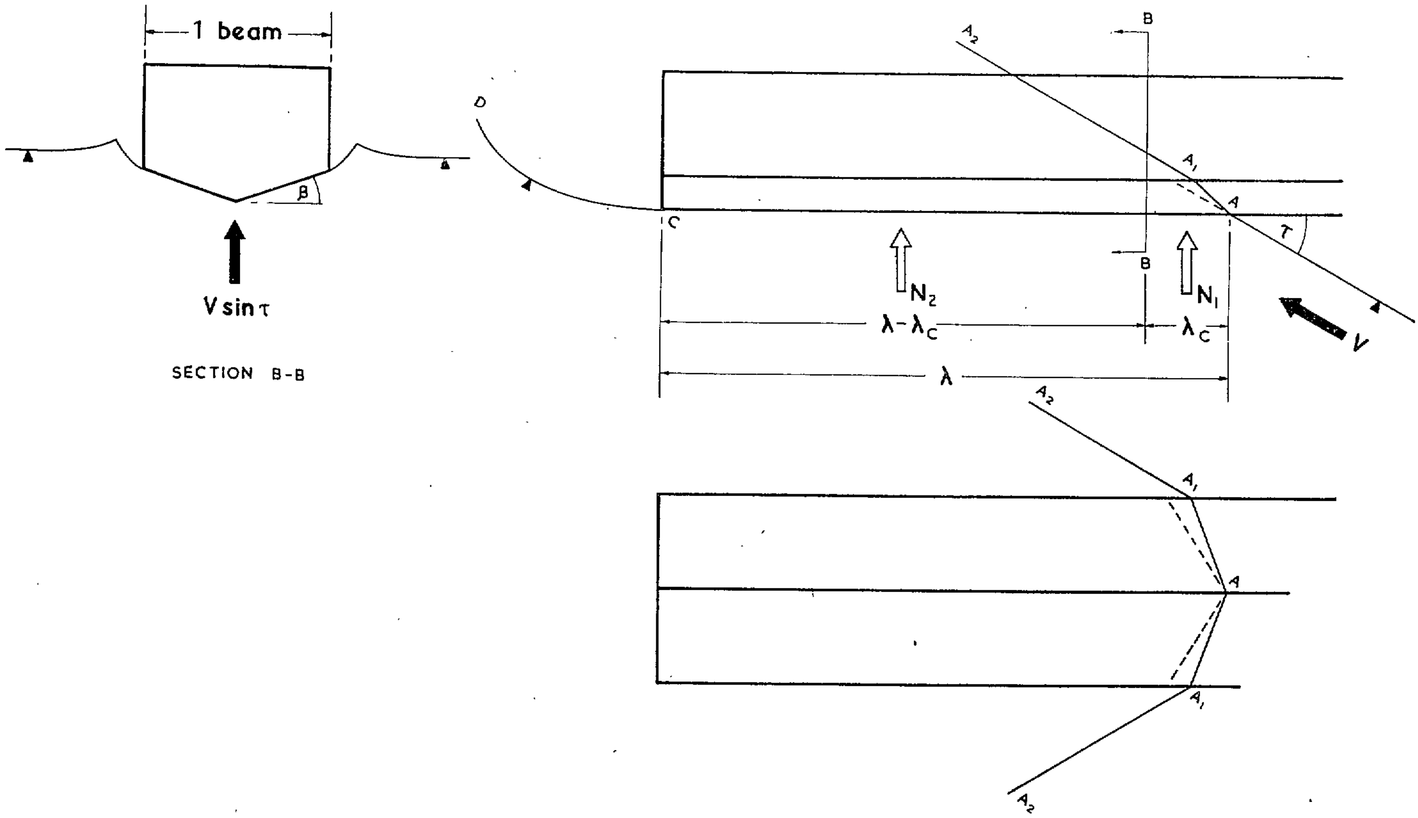

FIG. 1. Wedge planing chines wet.

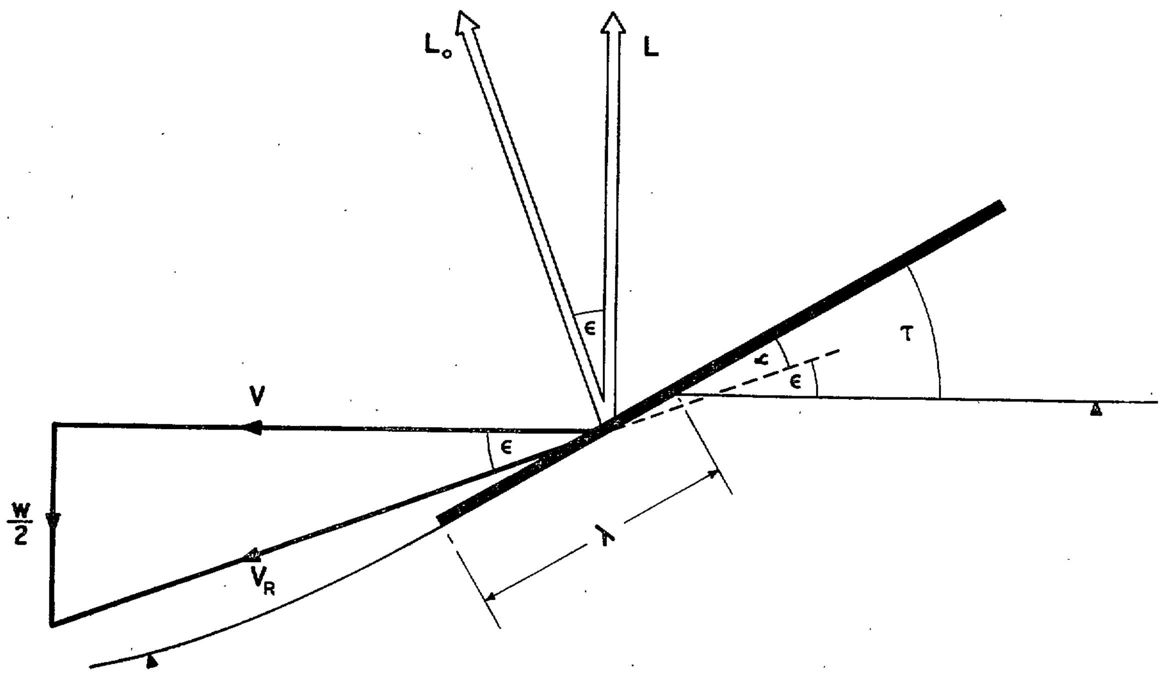

FIG. 2. Flat-plate planing.

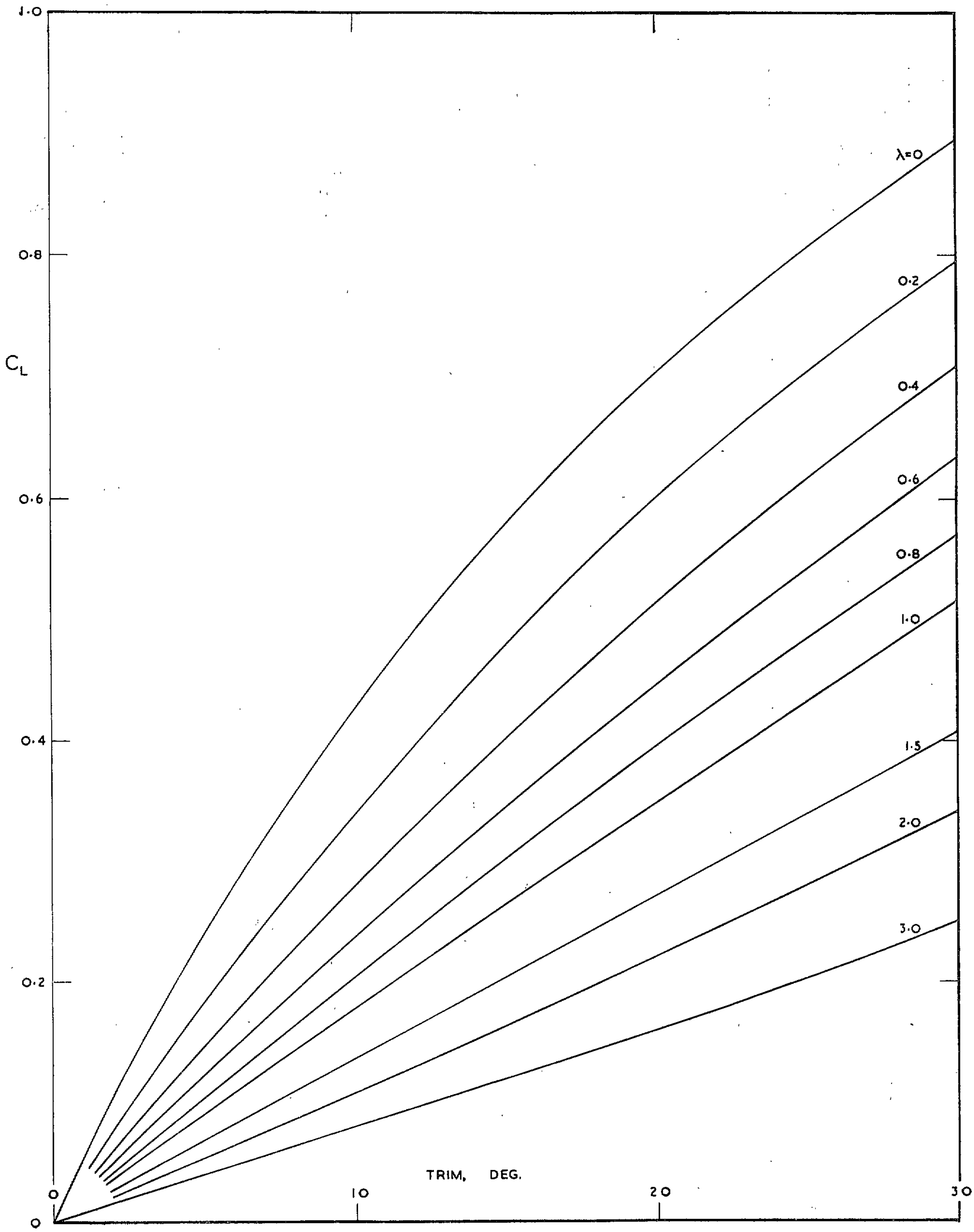

FIG. 3. Theoretical flat-plate lift coefficient.

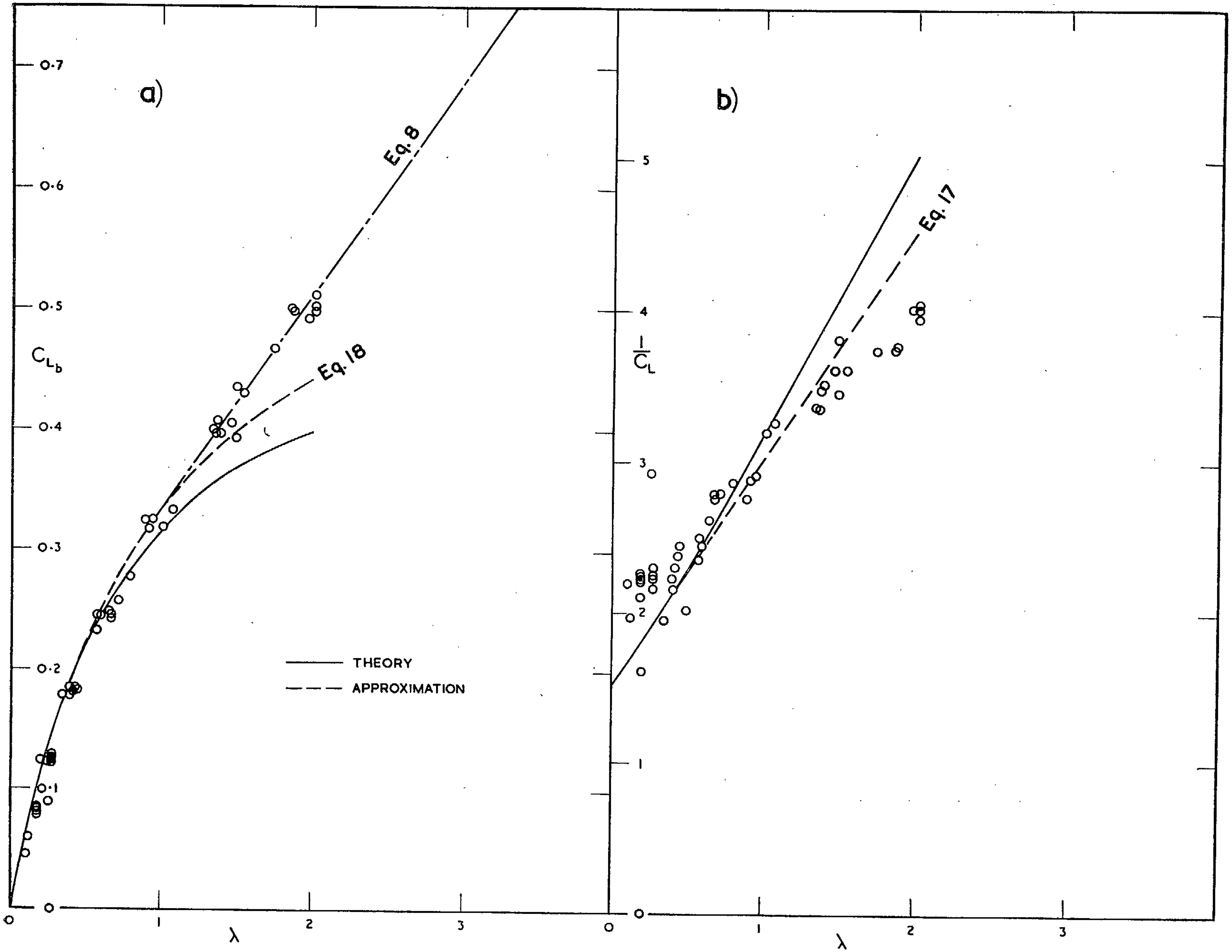

FIG. 4. Theoretical and experimental flat plate lift coefficients. 18-deg trim.

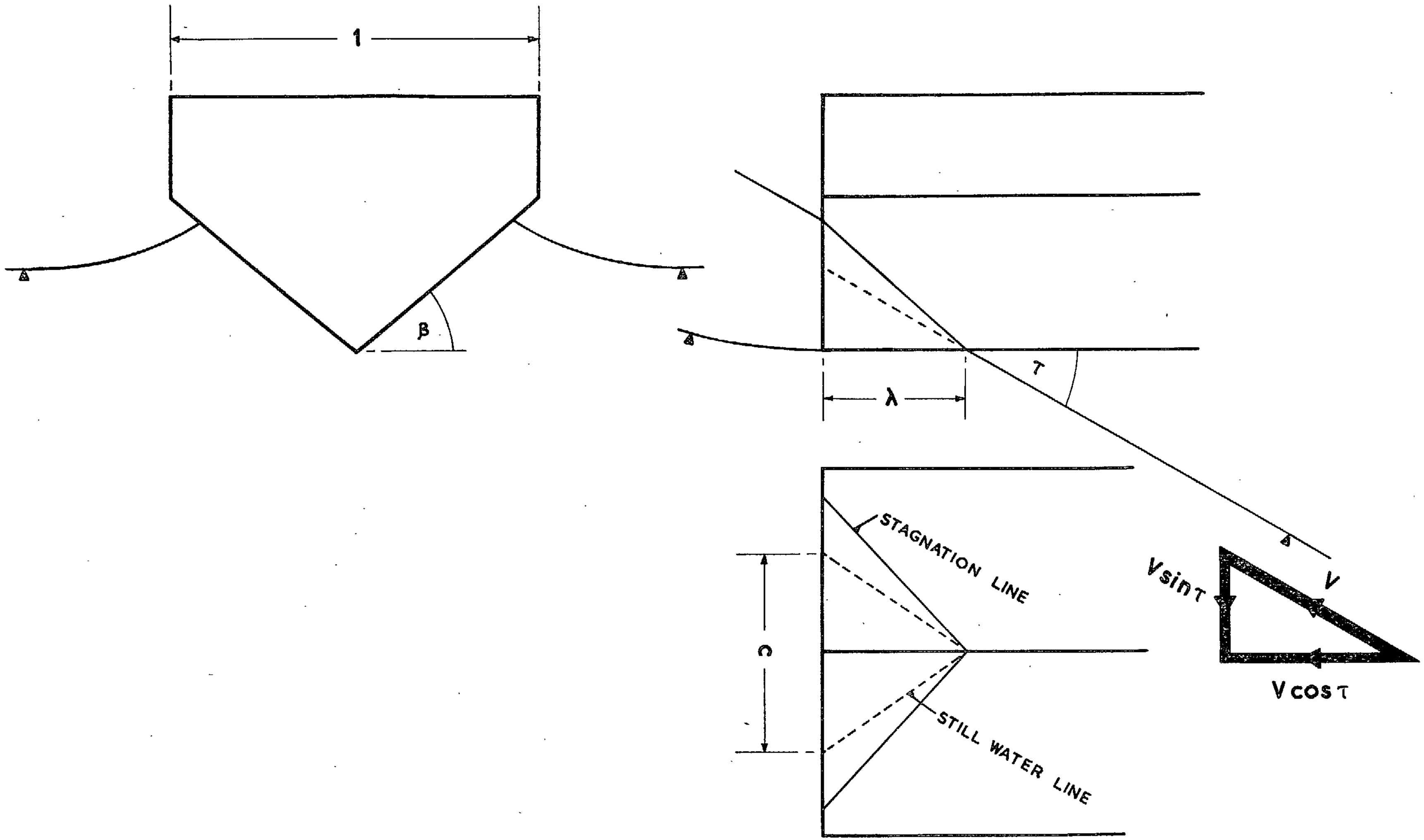

FIG. 5. Wedge planing chines dry.

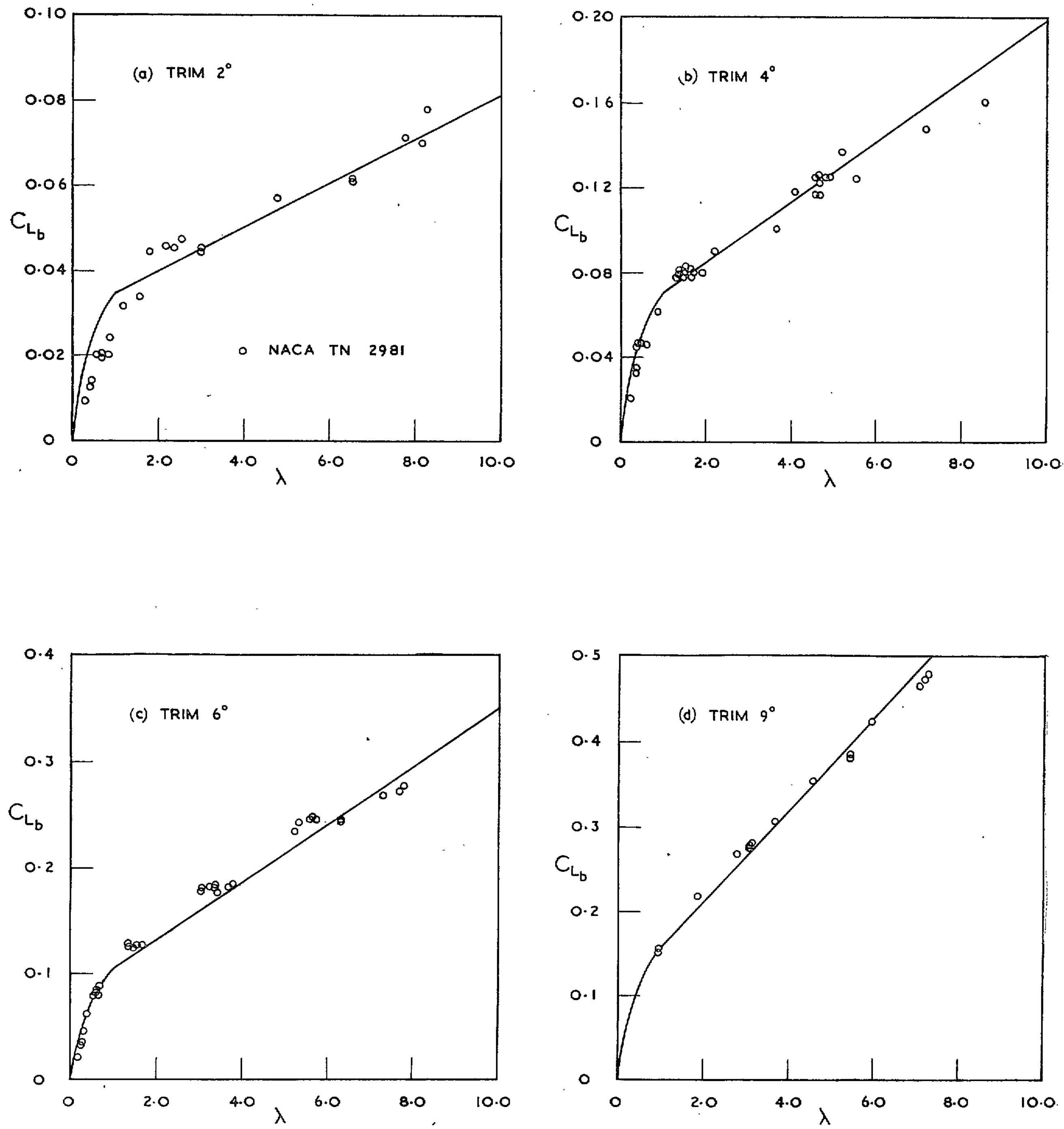

FIG. 6. Lift coefficient against wetted length. Zero deadrise.

B

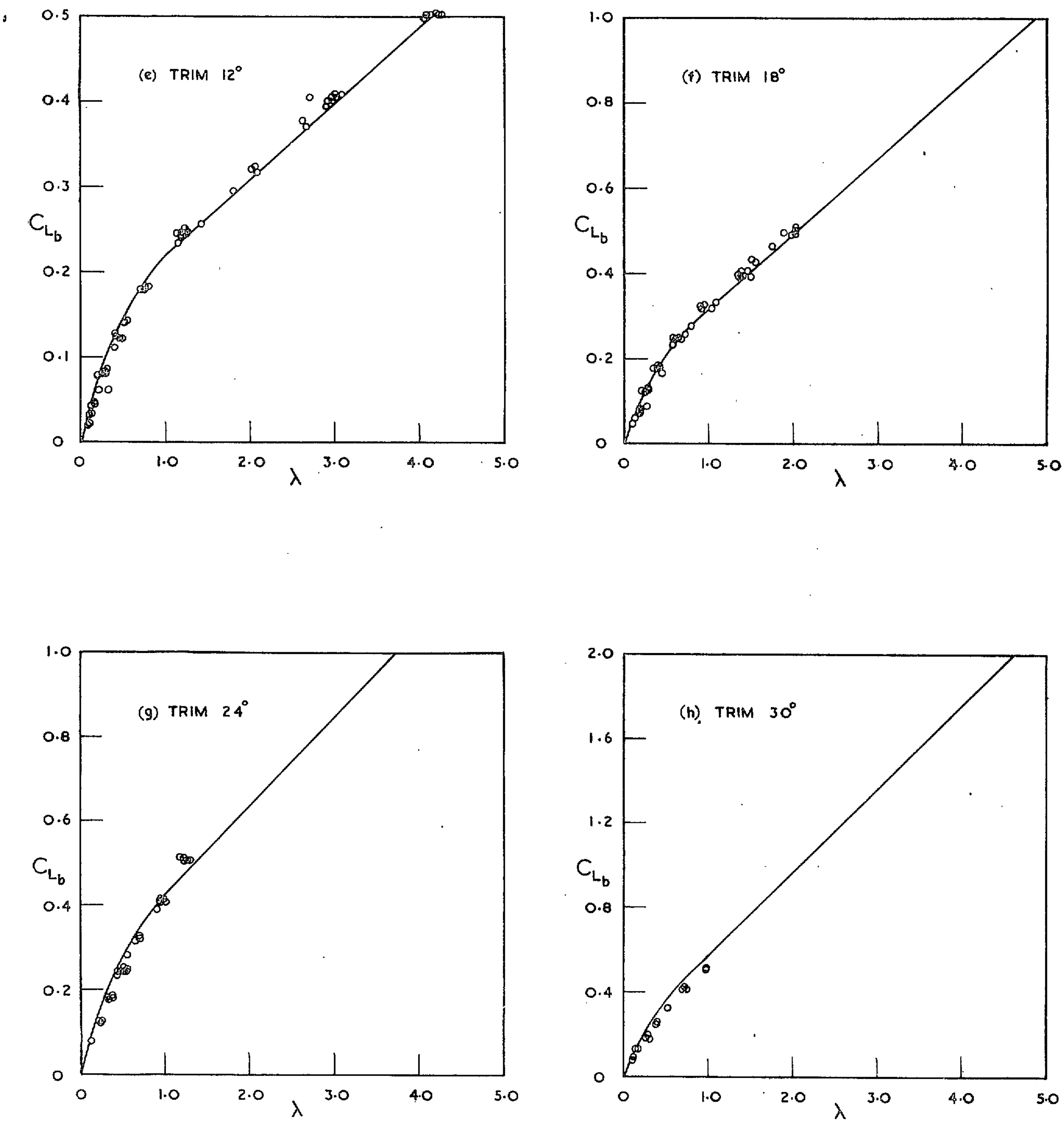

FIG. 6.—*continued.* Lift coefficient against wetted length. Zero deadrise.

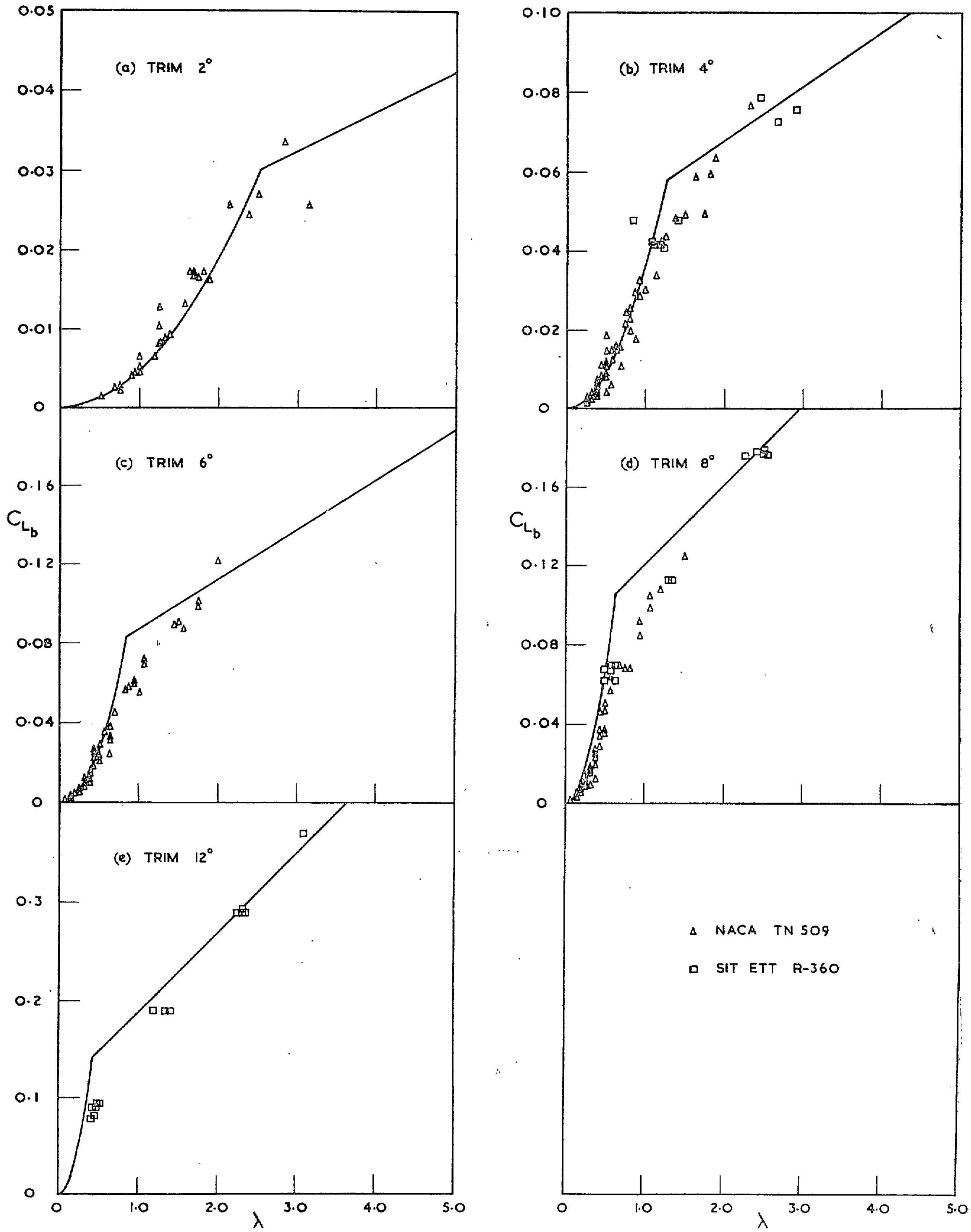

FIG. 7. Lift coefficient against wetted length. 10-deg deadrise.

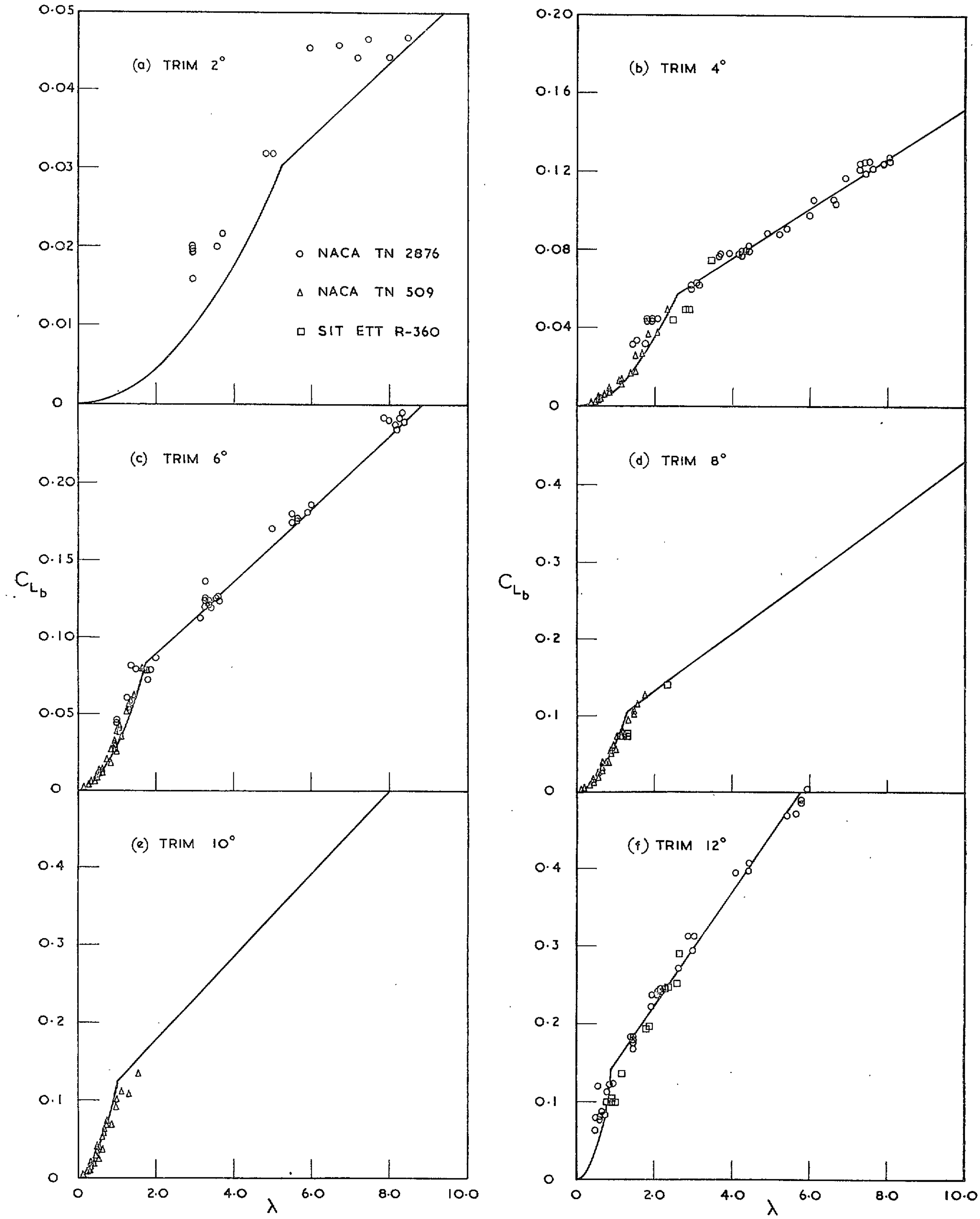

FIG. 8. Lift coefficient against wetted length. 20-deg deadrise.

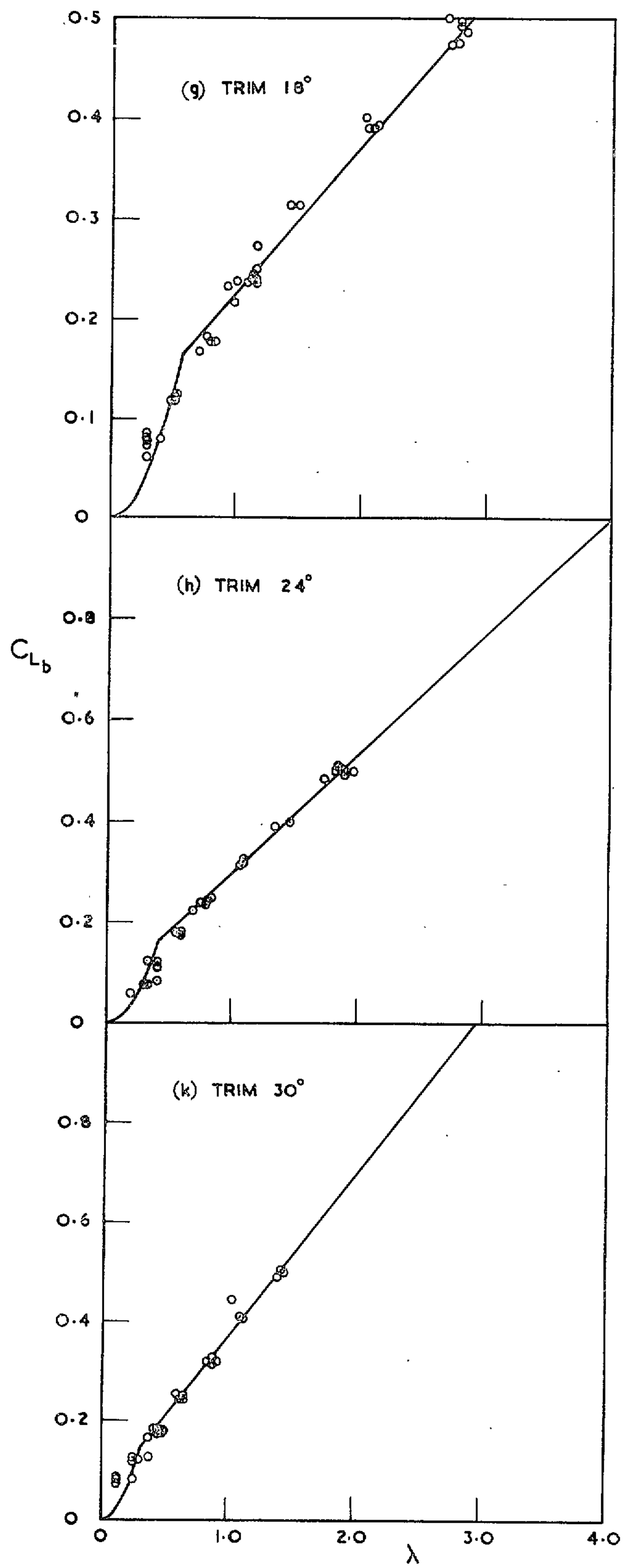

FIG. 8—*continued*. Lift coefficient against wetted length. 20-deg deadrise.

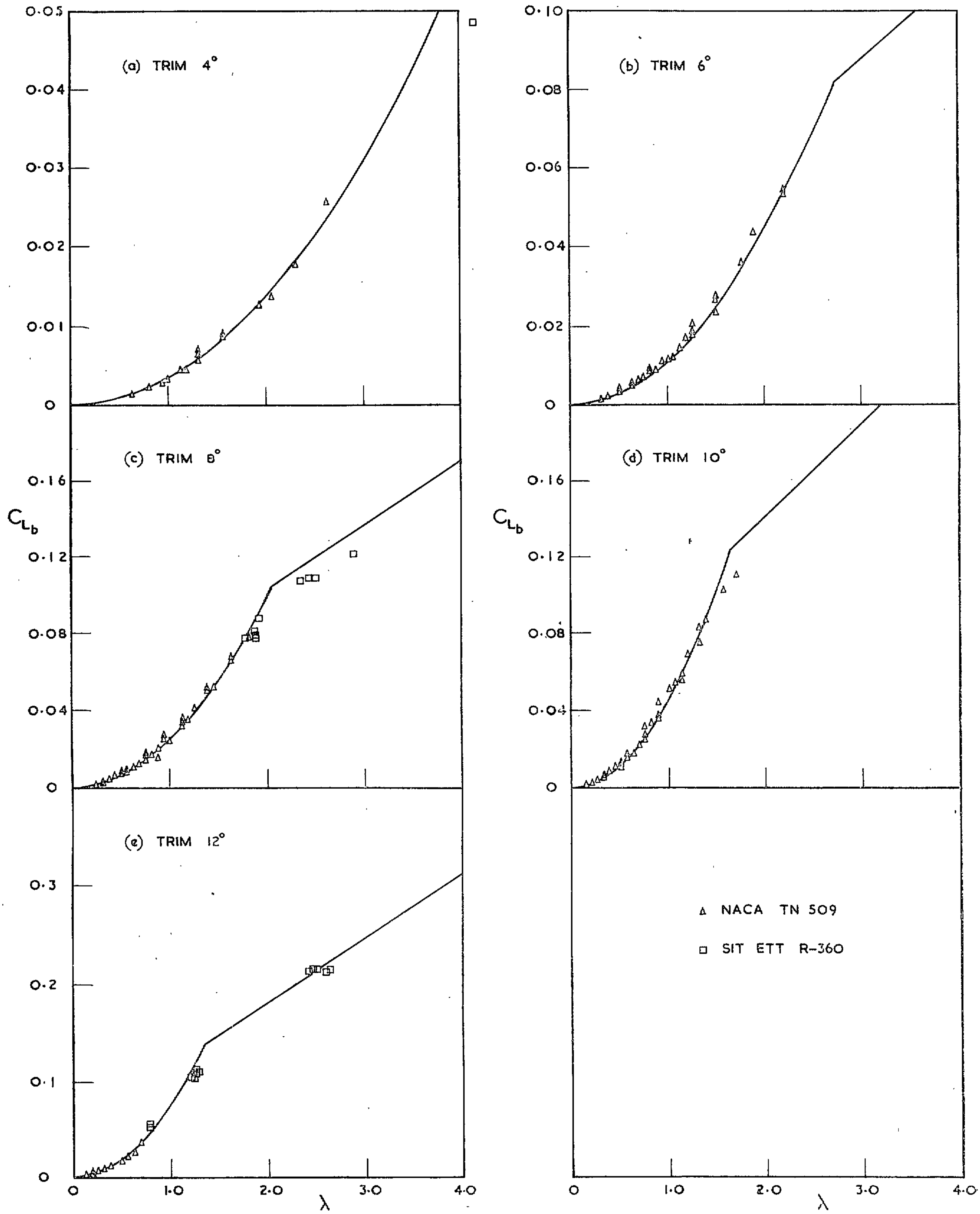

FIG. 9. Lift coefficient against wetted length. 30-deg deadrise.

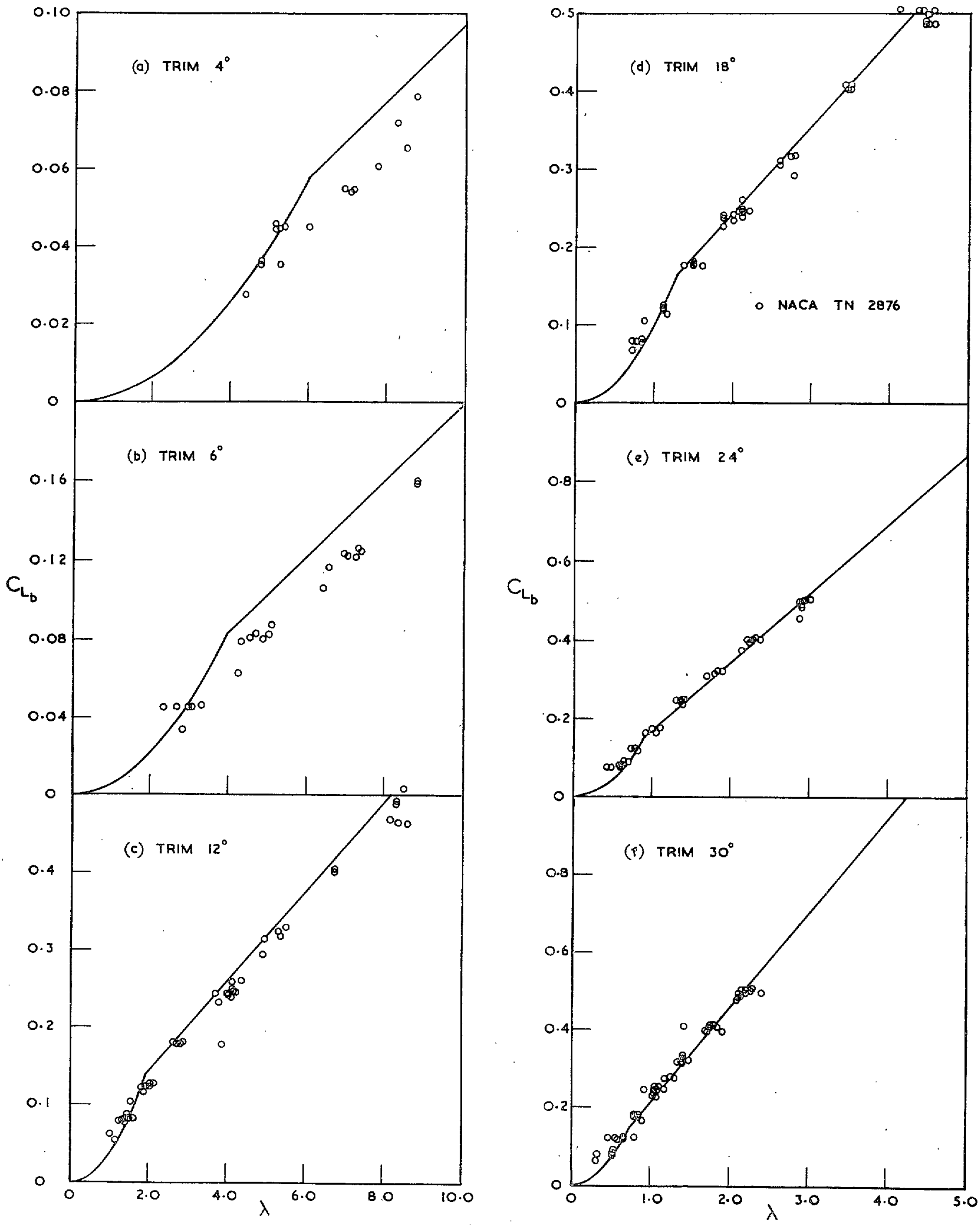

FIG. 10. Lift coefficient against wetted length. 40-deg deadrise.

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