R. & M. No. 2970
(14,319)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Gust Alleviation Factor

PARTS I, II and III

By

J. K. ZBROŻEK, DIPL. ENG.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1958
PRICE 17*s*. 6*d*. NET

Gust Alleviation Factor

By

J. K. ZBROŻEK, DIPL. ENG.

COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
MINISTRY OF SUPPLY

Reports and Memoranda No. 2970
May, 1953

PART I*

Incompressible Flow

Summary.—The report presents the theoretical calculations of gust alleviation factor made for rigid aircraft and with one degree of freedom only (*i.e.*, vertical motion). It is shown that for average gust lengths and for orthodox (tailed) aircraft the influence of the second degree of freedom (*i.e.*, pitching) on the value of the gust alleviation factor is negligibly small, providing the pitching moment of inertia and damping are not unduly small.

The influence of aspect ratio, the importance of the mass parameter and the gust shape on the values of the gust alleviation factor are shown. The large influence of the gust shape on the value of the alleviation factor makes the gust analysis by simple measurements of maximum aircraft acceleration inadequate, and the full records of the time-history of the aircraft are necessary. An alternative, more direct method of gust measurements is suggested.

The inadequacy of the present gust alleviation curve in Air Publication 970 is pointed out and a suggestion for replacement of this curve by the curves calculated in this report is made.

1. *Introduction.*—The increasing speeds of modern aircraft have made the knowledge of gust loads imposed on the airframe of paramount importance. Up to now most of the experimental work in flight on this subject has been done by measuring the response of a typical aircraft to an encountered gust. The results obtained by this way only give the combined effects of gust structure and aeroplane response, and it is impossible to separate these two factors empirically. In order to obtain information about the gust structure, the response of the aircraft has to be estimated theoretically.

As far back as 1936, Bryant and Jones[10] and Williams and Hanson[11] did some calculations of gust loads for a flexible aeroplane, but their results are applicable to a few particular cases only, and cannot be used for rapid estimation of the response of any aircraft.

Some recent theoretical papers [5,12] try to bring the gust response calculations of flexible aircraft more up to date, but the computational work involved is prohibitive, and cannot be used in gust research, for each individual case would have to be computed separately.

The analysis of the gust loads is considerably simplified if the flexibility of aircraft is neglected. Even with this simplification the calculations of the aeroplane response taking into account two degrees of freedom (*i.e.*, pitching and vertical motion) are very laborious, and no writer has yet succeeded in producing general results which can be used rapidly in estimation of gust loads on any aeroplane. The most useful papers published on this subject are the works of Greidanus and Van de Vooren[1,2], where the wing and tail loads are calculated for a range of aircraft parameters, taking into account two degrees of freedom of the aeroplane. Similar calculations are reported by Donely[3], but his results differ widely from those of Greidanus and Van de Vooren[1,2]. In the opinion of the present writer the results of Donely seem to be in error.

* R.A.E. Report Aero. 2421 dated May, 1951, received 10th September, 1952.

A short description of the factors called into play to control the pitching of a tailed aircraft when it enters a gust may help to show the complexity of the problem. For instance, on the calculations of the gust alleviation factor for an aircraft with two degrees of freedom made in Refs. 1 and 2, the forces and moments acting on the fuselage, wing and tailplane are all taken into account. When the aircraft penetrates into a vertical up-gust, first of all the gust acting on the front portion of the aircraft fuselage produces a small lift and nose-up pitching moment, which is followed by the main forces and moments from the wing, when the wing enters the gust. The pitching moment of the wing can be nose up or down depending on the position of c.g. with respect to the wing. It has to be remembered that, as shown by Glauert, the unsteady aerodynamic forces do not act on the quarter-chord point only; the part of aerodynamic forces due to change of circulation acts at the quarter-chord point and the part due to air inertia at midpoint. The part due to the flow curvature (due to aircraft pitching velocity) acts at the quarter-chord point, but incidence is measured at the three-quarter-chord point.

The further penetration of the aircraft into a gust brings the tailplane into the region of gust velocities and produces a corresponding nose-down pitching moment. At the same time the changes of downwash due to changes in the wing lift forces produce further changes of the tail load and of the pitching moment. It can be seen from the above description that two aircraft with identical static stabilities can have quite different responses in pitch due to a vertical gust, and that the possession of zero static stability does not mean that the aircraft will not pitch when entering a vertical gust.

The above description points to the complexity of gust load calculations retaining two degrees of freedom, and above all to the difficulty of providing some general charts which could be used in rapid estimation of gust loads on a given aircraft.

Further simplification is possible by neglecting the pitching degree of freedom. This line of approach was taken by Küssner and his paper[7] gave results in the practical form of gust alleviation factor calculated for rigid aeroplane with one degree of freedom (vertical motion) only. This work however does not cover the range of parameters required at present and Küssner's mathematical simplifications sometimes lead to numerical errors.

The present paper extends Küssner's work, covering a wider range of mass parameter and gust shapes, and using the exact solution of the equations of motion.

A comparison of the results of the analysis of Ref. 1 and of the present report seems to indicate that the wing loadings can be estimated quite satisfactorily neglecting the pitching degree of freedom. The calculations of Ref. 1 largely disagree with calculations of Ref. 3 on this point; there are many indications, however, that the results of Ref. 1 are the numerically correct ones. Also, the experimental evidence given in Ref. 3 (Fig. 35) substantiates the neglect of pitching motion, provided the correct alleviation factors for vertical motion only as developed in this report are used.

Fig. 1 shows the alleviation factor for a flat-topped gust as a function of mass parameter as obtained in the present report together with points representing the results of Ref. 1 for two lengths of gust gradient. A strict comparison is not possible as the present calculations are for a linear increase in gust strength with distance, and in Ref. 1 a sinusoidal gust shape is assumed, though this is not expected to make a large difference. To check this point some isolated calculation of gust load due to a gust of sinusoidal shape was made, and the agreement between the results of calculation using present method and the result of Ref. 1 was excellent.

An examination of Fig. 1 reveals that, for practical values of the stability parameter dC_M/dC_L and the mass parameter μ_g, the difference between values of the alleviation factor as found in present report and those in Ref. 1 are on average less than 2 per cent; these small discrepancies could be as well attributed to other differences in the assumptions made in both methods.

Calculations of gust loads on sweptback wings have shown that even for comparatively large sweeps the contribution of pitching motion to the value of gust alleviation factor is small, provided the moment of inertia and damping of the aircraft are not unusually small.

It is thought therefore, that the wing loads due to a single gust for a rigid aircraft can be estimated quite satisfactorily by the method using one degree of freedom only.

It is worth stressing that the tail loads cannot be estimated in such a simplified way, especially when one considers that the inertia and aerodynamic loads of the tail can be out of phase, due to certain inertia loads coming from the aerodynamic forces on the wing. For approximate and rapid estimation of tail loads Refs. 1 and 2 should be quite helpful, in spite of fact that they do not cover a wide enough range of mass parameters.

2. *Method of Analysis.*—The calculations were made using the following assumptions:

(*a*) The gust velocity is uniform across the span of the aeroplane at any instant, and is in the vertical direction

(*b*) The aëroplane can rise vertically, but does not pitch

(*c*) The aeroplane before entering gust is in steady level flight

(*d*) The forward speed of aeroplane is not changed during the action of the gust

(*e*) The wing is assumed to be rectangular and unswept.

A wing entering a sharp edge gust, or changing instantaneously its incidence, experiences a change in lift. The change in lift is not instantaneous but gradual and is a function of a distance travelled by the wing. The mode of change of lift is described by the unsteady lift function, which function is the ratio of the momentary to the ultimate steady lift of the wing.

The unsteady lift functions are given in the present paper by the approximate expressions, as suggested by Houbolt[5] and Jones[4]. The unsteady lift function due to penetration into the sharp edge gust is given by the equation:

$$\Psi = 1 - A\,e^{-\alpha s} - B\,e^{-\beta s} \qquad \ldots \quad (1)$$

where s is the distance travelled by the aerofoil measured from the instant at which the leading edge strikes the gust, and expressed in chords. The coefficients A, B, α and β are chosen to achieve the best fit of the function Ψ to the rigorous solution of the unsteady lift function. The values of these coefficients based on Ref. 4 are functions of the aspect ratio only.

The unsteady lift function due to an instantaneous change of wing incidence is assumed to be of the form:

$$\Phi = 1 - D\,e^{-\delta s} \qquad \ldots \quad (2)$$

where again coefficients D and δ are functions of aspect ratio only, and are chosen to give the best approximation to the exact unsteady lift function.

It is worth pointing out that the rigorous solution of the unsteady lift functions can be used in the present method of calculation, but the prohibitive amount of computational work would not be justified by the slight increase of accuracy of the numerical results. The values of the coefficients of the unsteady lift functions are given in the Appendix, Table 1, and the functions themselves are plotted in Figs. 2 and 3. The unsteady lift functions for aspect ratios 3 and 6 are obtained from Refs. 4 and 5, and for infinite aspect ratio approximations are used to conform with equations 1 and 2. The rigorous solution as obtained by von Kármán and Sears[13] for aspect ratio infinity is shown in Figs. 2 and 3 for comparison purposes. The magnitude of error in the values of gust alleviation factor arising from these approximations in the unsteady lift functions, can be judged by the examination of Fig. 4, where the calculated alleviation factors for different aspect ratios are shown. In the worst case, when the value of the mass parameter is small and a sharp-edged gust is assumed, the difference in the values of the alleviation factor for AR $= \infty$ and AR $= 6$ is of the order of 9 per cent. Comparing the shape of the unsteady lift functions in Figs. 2 and 3 for AR $= \infty$ and AR $= 6$, one can deduce that the error due to approximations in unsteady lift functions is of the order of 1 per cent, and this becomes even less for larger values of mass parameter and for gusts different from the sharp-edged gust.

The solution of the equation of motion through a sharp-edged gust gives the vertical force acting on the aeroplane as a function of distance travelled, s. From this, the dimensionless force function Λ can be obtained. This function is defined by:

$$\Lambda = \frac{\text{actual force}}{\frac{1}{2}\rho VSaU}, \quad \ldots \quad (3)$$

where U is the maximum gust velocity. The maximum value of function Λ gives the alleviation factor, K.

The sharp-edged gust force function Λ can be used to calculate the force due to a gust of any shape. The calculations of the present report are limited to triangular and flat-topped gusts with linear increase of gust strength. If the gust shape is given by the expression,

$$u = \frac{U}{H} s; \quad (\text{for } 0 < s < H)$$

and

$$u = U; \quad (\text{for } s > H, \text{ flat-topped gust}) \quad \ldots \quad (4)$$

where H is the gust length expressed in chords and u the momentary gust velocity, the vertical force function, defined as for sharp-edged gust equation 3, is given by

$$\Lambda_s = \frac{1}{H}\int_0^s \Lambda(s - \sigma)\, d\sigma \quad \ldots \quad (5)$$

$$\text{for } 0 < s < H$$

For $s > H$, the vertical force function is obtained by the addition of a new gust starting at the point H. Thus

$$\Lambda_s = \Lambda_s(s) - n \times \Lambda_s(s - H)$$

$$(\text{for } s > H)$$

where $n = 1$ for flat-topped gust and $n = 2$ for triangular gust. The maximum value of the function thus obtained gives the alleviation factor for a flat-topped or a triangular gust, whatever the case may be.

The expressions for the force functions Λ and Λ_s are in the form of series of integrals. The convergence of this series depends only on the value of the mass parameter μ_g, and for large values of μ_g it is quite rapid.

The mass parameter is the measure of the ratio of inertia to aerodynamic forces for a given aeroplane and is given by the formula:

$$\mu_g = \frac{2W}{g\rho Sca}, \quad \ldots \quad (6)$$

where:	W	is weight of aeroplane	lb
	g	acceleration of gravity	ft sec^{-2}
	ρ	air density	slug ft^{-3}
	S	wing area	ft^2
	c	wing chord	ft
	a	lift slope (per radian).	

Substituting values for g and for ρ at sea level the mass parameter can be expressed more simply:

$$\mu_g \simeq 26 \frac{\frac{W}{S}}{\sigma ca} \quad \ldots \quad (7)$$

where $\sigma = \rho/\rho_0$ is the relative density.

A full explanation of the analysis is given in the Appendix.

3. *The Gust Alleviation Factors for Gust of Different Shapes.*—3.1. *Flat-topped Gust.*—It was found the parameters affecting the value of the alleviation factor for flat-topped gust are as follows, in order of their importance:

(i) mass parameter, μ_g

(ii) gust gradient length, H

(iii) aspect ratio of wing.

The gust gradient distance H is the distance measured in chords c along which the gust velocity is increasing linearly from 0 to U; for a sharp-edged gust $H = 0$.

The aspect ratio effect is taken into account by using values of the unsteady lift function corresponding to the chosen aspect ratio. The calculations without the Wagner effect, *i.e.*, with the unsteady lift functions constant and equal to unity, can be treated as the limiting case of zero aspect ratio, for with zero aspect ratio the Wagner effect is absent.

The calculated values of the gust alleviation factor for a flat-topped gust with a linear increase of gust velocity along the distance of H wing chords, are plotted in Figs. 4 and 5.

In Fig. 4 the gust alleviation factors are plotted against the length of gust gradient H for a constant value of mass parameter μ_g, and for three aspect ratios, ∞, 6 and 3 respectively. The curve marked AR $= 0$ also shows the gust alleviation factor when no Wagner effect is taken into account, and for larger values of μ_g (say $\mu_g \geqslant 20$) for all practical purposes can be regarded as the asymptotic curve for all the alleviation factor curves when AR $\neq 0$. It can be seen from Figs. 4a and 4b that for small values of the mass parameter the alleviation factor curves for AR $= 0$ and for AR $= \infty$ cross over and for sufficiently large values of the gust length the numerical values of the gust alleviation factor calculated including the unsteady lift functions are larger than those calculated neglecting unsteady lift functions. This phenomenon depends on the relative magnitudes of the unsteady lift functions $\Psi(s)$ and $\Phi(s)$. If the unsteady lift function due to penetration into gust, $\Psi(s)$, is relatively large in comparison with unsteady lift function due to change of incidence, $\Phi(s)$, then the gain in alleviation due to function $\Psi(s)$ is offset by the decrease in the aerodynamic force due to vertical motion, the magnitude of which is controlled by the function $\Phi(s)$. Thus, for long gusts and light aircraft, where the alleviation due to vertical motion of the aircraft is large, the value of the alleviation factor calculated including Wagner effect can be larger than the value of the alleviation factor due to vertical motion only.

The vertical distance between line $K = 1$ and curve AR $= 0$ in Fig. 4 represents the alleviation due to vertical movement of the aircraft. The amount of alleviation increases with increasing gust gradient distance, because the aircraft has more time to rise before the gust reaches its full strength. For a sharp-edged gust the amount of alleviation, without the Wagner effect, is of course zero and $K = 1$. The increasing value of mass parameter decreases the amount of alleviation due to vertical motion of the aircraft; it is more difficult for the heavy aircraft to rise on the gust.

The vertical distance between curves AR $= 0$ and the appropriate gust alleviation curve is the gain in alleviation due to the combined effect of unsteady lift functions (Wagner effect) and mass parameter.

For the sake of comparison the alleviation factors, as calculated by Küssner[7] for aspect ratio ∞, are replotted from Ref. 3, in Fig. 4. For a sharp-edged gust, $H = 0$, the agreement between Küssner's and the present calculations is good; the small differences of the order of 1 per cent can be attributed to the approximations in the expressions for unsteady lift functions used in the present report. With increasing value of H, however, the influence of Wagner effect on the value of alleviation factor decreases, and the discrepancy between Küssner's and present results should decrease, remembering that both calculations are made under the same assumptions of rigid aircraft and one degree of freedom. An examination of Fig. 4 will show though, that, with increasing H, the discrepancy between Küssner's and the present calculations is in fact increasing. This is, it is thought, due to simplifying assumption made by Küssner. In order to simplify the mathematical analysis, he assumed that peak of gust force coincides with peak of gust.

For small values of μ_g the curves of the gust alleviation factor presented in this paper do not extend throughout the entire range of gust lengths H shown in Fig. 4, because the method of calculating the gust alleviation factor becomes inaccurate for small values of the ratio μ_g/H; and in this region the pitching effects will be appreciable.

Fig. 5 shows the gust alleviation factor plotted in the conventional way against the mass parameter, for the range of gust gradients. Fig. 5a gives the alleviation factor for wings with infinite aspect ratio and covers the range of mass parameters up to 200. Figs. 5b and 5c give the alleviation factor for wings of aspect ratio 6 and 3 respectively, for values of μ_g up to 80. For values of μ_g in excess of 80 the difference in values of alleviation factor for different aspect ratios becomes vanishingly small, and Fig. 5a may be used for any aspect ratio.

3.2. *Triangular Gust.*—Experimental evidence indicates that the flat-topped gust is the exception rather than the rule. In order to find the influence of the gust shape on the alleviation factor, the calculations were extended to triangular and double-triangular gust shapes. The triangular gust is defined as having a maximum gust velocity U, and a total length of $2H$ chords. The diagrams of the gust shape are included in Figs. 6 and 7. The calculations were made for an aspect ratio of infinity only. Fig. 6 shows the alleviation factor plotted against gust gradient for the range of mass parameters. For longer gust gradients, the shapes of the alleviation factor curves for a triangular gust are almost identical with those for a flat-topped gust, and with those obtained by assuming no Wagner effect. For short triangular gusts the value of the alleviation factor decreases rapidly, and in the limit (when the triangular gust degenerates into a pulse of gust) approaches zero value.

Cross-plotting the curves of Fig. 6, Fig. 7 is obtained, showing the alleviation factor as a function of mass parameter for the range of gust lengths. The striking feature of this diagram is the fact that, for larger values of mass parameter, the alleviation for triangular gust is almost independent of mass parameter, but varies with gust length, especially within the practical range of gust lengths, i.e., $H < 10$. This phenomenon has to be borne in mind when analysing the experimental gust loads. In the analysis of the measured gust loads, it has been a general practice up to now, to estimate the gust velocity by using the value of alleviation factor for flat-topped gust, irrespective of the shape of encountered gust. If, for instance, a large and small aircraft are flown through the identical triangular gust, the larger aircraft will apparently register a smaller gust velocity if a flat-topped gust alleviation factor is used. This may lead to an apparent relationship between gust gradient and the length of the wing chord of the aircraft used for gust investigation. This effect is not enough to account completely for the magnitude of such relationship as found experimentally and is quoted in Fig. 12 of Ref. 3, but it is of appreciable order.

3.3. *Double-triangular Gust.*—In the calculations of the aircraft response to a double-triangular gust, it was assumed that the single triangular gust was followed immediately by a similar gust but with reversed direction of gust velocity. The values of peak gust velocities were taken as $+U$ and $-U$ respectively and the total gust length as $4H$ chords, *i.e.*, the distance to the first, positive peak was H chords. The diagrams of the gust shape are included in Figs. 8 and 9. The gust alleviation factor is defined as before:

$$K = (A_s)_{\max} = \frac{\text{maximum actual force}}{\frac{1}{2}\rho V S a U} .$$

Fig. 8 shows the gust alleviation factor for the second peak of gust of the double-triangular gust. (The first peak will be that given by a single triangular gust.) The alleviation factor is plotted against gust gradient length for the range of mass parameters. The alleviation factor curves calculated without the Wagner effect are included and shown by thin lines. Fig. 8 should be examined concurrently with Fig. 6, where the alleviation factor for the first peak of triangular gust is shown. For better comparison of values of alleviation factors for the first and the second gust peaks, Fig. 9 was prepared, where the gust alleviation factors are plotted against mass parameter for the range of gust gradient lengths. The continuous curves show the gust alleviation factor for the second peak as obtained by cross plotting the curves of Fig. 8, and the interrupted curves show the alleviation factor for the first peak replotted from Fig. 7. It can be seen that the gust loads due to second, negative peak could be larger or smaller than those due to first peak, depending on the values of gust length and mass parameter. For all given values of gust length (H = const), the loads at the second peak are larger than the loads at the first peak at the lower end of the μ_g range, but as μ_g is increased (at high H a large increase is necessary) there is a cross over and the second peak load is then smaller than the first peak load. The explanation of this is that the heavy aircraft has had insufficient time to obtain a large vertical velocity due to passage through the first peak of the gust before meeting the second peak, and the loads due to this second peak are diminished by the loads still building up from the first gust peak of opposite sign, which are delayed by the Wagner effect. For the practical range of gust length, however, ($H > 5$) the value of the alleviation factor for the second gust peak is larger than for the first one, even for the practical range of heavy aeroplanes ($\mu_g < 200$). For the very long gust and heavy aircraft the alleviation factor can be larger than unity, but cannot be larger than about 1·04 (*see* Fig. 8, gust alleviation factor with no Wagner effect).

It has to be remembered that, for long gusts, the present analysis is not accurate enough and the presence of the pitching motion of the aircraft will modify the values of the alleviation factor. Nevertheless the present calculations show that the gust shape has a very significant effect upon the value of gust alleviation factor.

4. *Discussion*.—4.1. *General Assumptions*.—The comparison (given in Fig. 1) of the results of the present calculations with those of Ref. 1 shows that the acceleration of the aircraft centre of gravity due to a symmetrical gust can be estimated with a good degree of accuracy by the present method, in which the pitching motion of the aircraft is neglected. A rough calculation made for wider range of parameters than those covered by Ref. 1, indicates that for an aircraft with tailplane, the contribution of pitching to the value of gust alleviation is always small, provided the value of longitudinal moment of inertia, described by coefficient i_B and damping are not too small. Gust alleviation factor curves enable one to estimate the aerodynamic wing loadings, once the gust characteristics are known. In order to estimate the tail loads, a more complete analysis is required. Some approximate estimations can be made using data given in Ref. 1.

The present report considers the aircraft as rigid. This assumption is quite satisfactory for small aeroplanes, say of fighter type, but for large aeroplanes, where the natural frequency of wings is low, the present analysis should be extended and wing flexibility taken into account.

4.2. *Gust Shape*.—The general conclusions of the present calculations for the flat-topped gust are in agreement with those of previous writers*. The numerical values of the gust alleviation factor, calculated by Küssner[7] and quoted by the National Advisory Committee for Aeronautics[3] seem to be in error, and it is felt they should be superseded by the values given in this paper. For a flat-topped gust the mass parameter is the most important factor in the estimation of the alleviation factor, the gust gradient being of secondary importance.

For triangular and double-triangular gusts, the gust length is of primary importance in the estimation of the gust alleviation factor. This is especially evident for larger values of mass parameter (*see* Figs. 7 and 9). This phenomenon, unfortunately, makes the strict investigation of the gust strength or gust structure by a simple measurement of maximum vertical acceleration extremely difficult. In each case the time-history of gust load is required; the analysis of the time-history can then give the full account of gust shape and strength. This process, however, involves prohibitive amount of computational work; it is hoped that in practice a visual examination of the time-history curve will be sufficient to decide what shape of gust and, therefore, what corresponding alleviation factor has to be used in analysis.

* Since completion of the present paper, very similar work has been published in *Journal of the Aeronautical Sciences*, Ref. 18, where the gust alleviation factor for a flat-topped gust and rigid aircraft with one degree of freedom was calculated. The agreement with the results of the present paper is almost complete. Very small numerical differences are due to differences in assumed unsteady lift functions.

The present calculations are limited to gust shapes bounded by straight lines. It is hoped to extend the calculations to the case of a sinusoidal gust, the shape which is most likely to be met in practice.

From the point of view of the airframe stressing the problem of the gust shape is simpler. Once the standard gust shape and strength are established, the appropriate gust alleviation factors can be used. For the time being, assuming that the design gust velocities are known, the flat-topped gust alleviation factors should be retained, as their values are slightly higher than those for the triangular gust. The large values of the gust alleviation factor for the second peak of double-triangular gust (Fig. 9) may be significant from the stressing point of view, but not enough is known of gust structure to assess their importance, and they indicate only what would happen to the aircraft when flying through the series of bumps distributed ' in resonance ' with the aircraft motion.

4.3. *Aspect Ratio.*—The present calculations cover the range of aspect ratios from 0 to ∞. The present, very scanty evidence[3] suggests, however, that for the orthodox aeroplanes the gust alleviation factor for infinite aspect ratio seems to give better agreement with experiment than the alleviation factor for the appropriate aspect ratio; other evidence from the same source suggests that for very low aspect-ratio wings and flying-wing aircraft there is better agreement if the true aspect-ratio is used. There are doubts about the effect of bodies in this problem. The experiments by Kuethe[8], specially designed to check the effect of the aspect ratio on gust loads, have shown that the measured build-up of circulation due to a sharp-edged gust agreed better with the theoretical build-up, if an aspect ratio infinity was assumed instead of the actual aspect ratio corresponding to the wing used in experiment. Unfortunately the results of this experiment are far from conclusive, as the circulation measurements were limited to the wing tip only, and during the same experiment it was shown that the vorticity extended farther inboard from the tip than the area included in the measurement, so that the measurements did not cover the total changes in circulations.

It is felt, therefore, that at the present time the curves for the correct aspect ratio should be used; this involves no difficulty in the design case, if the design gust velocity is known for, as previously suggested, a flat-topped gust should be assumed and all relevant curves are given in this report. In the analysis of gust records, however, when gust shape affects the alleviation factor curve to be used, interpolation between the curves given in this report will be necessary for other than flat-topped gusts.

4.4. *Wing Plan-form.*—The results of the analysis presented in this paper are strictly valid for a rectangular wing. For moderate taper, the present method should be satisfactory if the mean aerodynamic chord is used. For highly tapered wings the local rate of lift build-up will depend on the local chord and the present findings could be in error; this would be especially true in regard to the momentary spanwise lift distribution and the bending moments of the wing.

The effect of the sweepback on the gust loads is not yet properly understood. Experimental investigations in the N.A.C.A. Gust Tunnel[15,16] indicate that the effect of sweep is less than might have been expected. The relief in gust loads due to gradual penetration of swept wing into a gust is counterbalanced by the nose-up pitching motion of the aeroplane.

It was suggested that an approximation to the gust loads on swept wings can be obtained by using strip theory and the lift slope of the equivalent straight wing multiplied by the cosine of the angle of sweep[3,17]. The theoretical calculations using strip theory are presented in Part III of this report.

4.5. *Compressibility.*—It is suggested in Ref. 3 after R. T. Jones that the allowances for air compressibility in the gust alleviation factor for subsonic flight can be made by introducing the ' effective ' aspect ratio obtained by multiplying the geometrical aspect ratio by $\sqrt{(1 - M^2)}$. In addition, the lift slope corresponding to a given Mach number for the particular wing should be used. This suggestion is in disagreement with experiment and theory as shown in Part II of this report.

For supersonic flow, it was further suggested[3], the Wagner effect would vanish altogether. This is not entirely true, as due to sudden change, say of aerofoil incidence, pressure waves are initiated which propagate with the sound velocity a', and the aerofoil has to travel the distance s' before it is clear of these pressure areas created before the sudden change of incidence. It can be expected that the lift will not attain its steady value till the aerofoil covers a distance greater than s'. The value of the distance s' travelled by the aerofoil and necessary to attain its steady lift can be calculated by a simple argument. Let us assume that at the time t' after the sudden change of incidence the aerofoil is just clear of its initial disturbances, *i.e.*, the trailing edge is just clear of the disturbance started by the leading edge t' seconds ago. The distance travelled by this disturbance is $a't'$ and the distance travelled by the aerofoil Vt'; the difference between those two is the chord of the aerofoil. We thus have:

$$s'c = Vt' \text{ and } Vt' - a't' = c$$

and introducing Mach number

$$M = \frac{V}{a'}$$

gives the value of

$$s' = \frac{M}{M-1} .$$

A theoretical analysis of the unsteady lift forces in supersonic flight was made by Heaslet and Lomax [14]; this shows the gradual build-up of lift in supersonic flight, not unlike the Wagner effect in subsonic flight. The unsteady lift forces in supersonic flow act on the aerofoil only during the distance of s' chords. For values of $s > s'$ the theoretical lift of the aerofoil is given by Ackeret's theory of steady supersonic flow.

To show the magnitude of the unsteady supersonic lift forces, Figs. 10 and 11 are reproduced from Ref. 14. In these figures the unsteady lift forces are shown in the form of unsteady lift functions Ψ and Φ as have been used for the subsonic calculations in the present paper. The lift slope used in the supersonic calculations is the two-dimensional lift slope:

$$\frac{4}{\sqrt{(M^2 - 1)}} .$$

The unsteady lift functions shown in Figs. 10 and 11 are calculated for three Mach numbers. The values of Mach numbers were chosen in such a way that the lift slopes are corresponding to lift slopes of wings of aspect ratios ∞, 6 and 3 in incompressible flow.

The curves of function Φ in Fig. 11 are slightly modified as compared with Fig. 6 of Ref. 14, where the horizontal parts of the curves near $s = 0$ are not consistent with the theoretical findings of the paper.

It can be seen that the effect of unsteady lift is diminishing with increasing Mach number, and for very high Mach numbers the gust alleviation factor can be estimated using the curves of Fig. 5d for zero aspect ratio (no Wagner effect).

4.6. *The Comparison with Gust Alleviation Factor as given by A.P.*970.—The comparison of the values of alleviation factor as found in the present report with those as given by the standard alleviation curve in A.P. 970, Chapter 203, indicates that generally A.P. 970 underestimates the gust alleviation factor. The gust alleviation factor as given in A.P. 970 is based on the wing loading only and not on the mass parameter μ_g. This simplification neglects the effects of lift slope, the aeroplane size ($\bar{c}$) and what is most important, the effect of altitude. To show the magnitude of error due to neglect of the altitude parameter Fig. 12 was prepared where the gust alleviation factor is plotted against the altitude of flight. Two aeroplanes were used, geometrically identical ($\bar{c} = 15$ ft, $a = 5$), but one heavily loaded, marked 'aeroplane H', with wing loading of 80 lb/sq ft, and another, 'aeroplane L', lightly loaded with wing loading of 30 lb/sq ft. The gust alleviation factors as calculated in this report are shown for a sharp-edged gust and for a flat-topped gust of 100 ft length, which gives, for this particular aircraft, $H = 6 \cdot 7$. The values obtained from A.P. 970 which are independent of altitude are shown in Fig. 12 by horizontal line.

At sea-level the error in the alleviation factor as given by A.P. 970 is small, but increases with altitude and for altitude of 40,000 ft the error is of the order of 23 per cent. Thus, assuming that the true gust velocities are known, the gust loads at altitude calculated using gust alleviation factor as given in A.P. 970 can be underestimated very considerably. One can argue, however, that since the design gust velocities as used at present were obtained from experimental data using the A.P. 970 gust alleviation curve, and the same gust velocities are used to calculate gust loads using again the gust alleviation factor from A.P. 970, the overall error in load calculations is negligibly small. This reasoning seems to be quite correct as long as we do not have to extrapolate our data, *i.e.*, as long as the aircraft and altitude for which we want to calculate the gust loads are not very different from the aircraft and altitude used to establish the gust strength and shape. If we want to generalize our gust calculations, the true, not 'equivalent', gust strength and shape must be known, and only then can the true gust loads for any altitude and any aeroplane be estimated.

It is expected that in the not too distant future more direct methods of gust measurements will be developed, such as free balloon observation, smoke puffs, etc. The gust velocities measured in such a way would be the true gust velocities, not distorted by the characteristics of the aircraft response, as is the case in the present gust measurements from the analysis of the aircraft accelerations. The comparison of the results of gust measurements by the direct and aircraft response methods is possible only if the true values of gust alleviation factor are used.

It is suggested that for subsonic speeds the present gust alleviation curve in A.P. 970 should be superseded by the curves of the present paper relating to a flat-topped gust. The appropriate aspect ratio should be used and a gust length of 100 ft as used at present should be retained, unless experimental evidence proves otherwise.

5. *Conclusions.*—(i) The estimation of the gust alleviation factor by the method neglecting pitching motion of the aircraft seems to be sufficiently accurate for practical purposes up to moderate values of H and for straight-wing aircraft.

(ii) The numerical results of the present paper indicate that the values of the alleviation factor now generally in use[3,7] can be considerably in error. This may explain, at least partly, the discrepancies between the theoretical and experimental results, which have been found from time to time.

(iii) The value of the gust alleviation factor, as found in this paper, depends on three parameters, the mass parameter, the gust shape measured relative to the aircraft size, and wing aspect ratio.

(iv) The mass parameter takes into account wing loading, altitude of flight and size of the aircraft. The wing loading alone is not sufficient to define the value of the gust alleviation factor, especially when a wide range of aircraft sizes and altitudes of flights has to be considered.

(v) For given aircraft and altitude, the value of gust alleviation factor depends on the relative shape of gust, in other words the acceleration experienced by the aircraft depends not only on the momentary gust strength but also on the past time-history of the aircraft motion after it enters the gust. This is an obvious conclusion, but the present calculations show numerically how large differences in the value of the gust alleviation factor can be expected due to different shapes of the gust. It is worth emphasizing that the theory shows the importance of the relative length of the gust with respect to the aircraft. This phenomenon could be responsible, partly at least, for the curious relationship between the experimentally measured gust length and the size of the aircraft used for this measurement, as found by some experimenters[3].

This relationship between the gust shape and the value of gust alleviation factor makes gust research based on measurements of the aircraft normal acceleration only very difficult. Simple measurements of the maximum accelerations experienced in bumpy air, as by *V-g* or *H-g* records, do not furnish any absolute information either about gust shape or gust strength. However, these simple V-g records do give stressing data suitable for aircraft with the same μ_g and at the same altitude. If we continue to use the aircraft as the instrument for measuring the gust structure, the full time-history of aircraft acceleration has to be measured and analysed. The wing flexibility, which is not taken into account in the present paper, will add to the difficulties of this method. Much better results should be obtained if the gust velocities are measured directly by use of pitchmeters. The corrections due to aircraft vertical and pitching velocities should not be too difficult to apply, provided these corrections are made small enough by mounting the pitchmeter sufficiently far ahead of a suitable aircraft; an aircraft becomes more suitable as wing loading increases, chord decreases, length increases and pitching moment of inertia increases.

(vi) For stressing purposes the gust alleviation curve of A.P. 970 as at present used should be superseded by the curves given in this paper. In the present state of knowledge the flat-topped gust curves for appropriate aspect ratio are suggested. For the time being the standard gust of 50 ft/sec strength and 100 ft length should be retained at sea-level, but corrections should be made to the design gust velocities at other heights, by re-examining the old acceleration records by means of the new gust alleviation factors given herein.

LIST OF SYMBOLS

$\left.\begin{matrix}A\\B\end{matrix}\right\}$	Coefficient describing function Ψ
a	Lift slope (per radian)
a'	Speed of sound (ft sec^{-1})
c	Chord of rectangular wing (ft)
$\bar{c}$	Mean chord (ft)
D	Coefficient describing function Φ
F	Vertical force due to gust (lb)
g	Gravity acceleration (ft sec^{-2})
H	Gust length to the first peak, in chords
K	Gust alleviation factor. $K = \dfrac{\text{maximum actual force}}{\frac{1}{2}\rho VSaU}$
M	$= \dfrac{V}{a'}$, Mach number
s	Distance travelled by the leading edge of wing, measured in chords
s'	Distance within which the unsteady lift functions attain value equal unity, supersonic flow
S	Wing area (ft^2)
t	Time (sec)
u	Gust velocity, function of s (ft sec^{-1})
U	Maximum gust velocity (ft sec^{-1})
V	Forward velocity (ft sec^{-1})
W	Weight of aeroplane (lb)
w	Vertical velocity of aeroplane (ft sec^{-1})
$\left.\begin{matrix}\alpha\\\beta\end{matrix}\right\}$	Coefficients describing the function Ψ
δ	Coefficient describing function Φ

LIST OF SYMBOLS—*continued*

Φ Unsteady lift function due to sudden change of incidence

Ψ Unsteady lift function due to penetration into sharp-edge gust

Λ_1 Gust force function for shapes different from sharp-edged

Λ Gust force function for sharp-edged gust, $\Lambda \equiv \Lambda_1 \equiv \dfrac{\text{actual force}}{\frac{1}{2}\rho VSaU}$

$\mu_g = \dfrac{2W}{g\rho S a \bar{c}}$, Mass parameter

ρ Air density (slug ft^{-3})

σ Distance travelled by the leading edge of wing, measured in chords. σ is variable when s is kept constant

---

REFERENCES

No.	*Author*	*Title, etc.*
1	J. H. Greidanus and A. I. Van de Vooren	Gust load coefficients for wing and tail surfaces of an aeroplane. N.L.L. Report F.28. April, 1948.
2	A. I. Van de Vooren	Loads on wing and tail surfaces of an aeroplane due to sinusoidal gust wave. N.L.L. Report F.33. 1949.
3	P. Donely	Summary of information relating to gust loads on airplanes. N.A.C.A. Tech. Note 1976. November, 1949.
4	R. T. Jones	The unsteady lift of a wing of finite aspect ratio. N.A.C.A. Report 681. 1940.
5	J. C. Houbolt	A recurrence matrix solution for the dynamic response of elastic aircraft. *J.Ae.Sci.* Vol. 17, No. 9. September, 1950.
6	W. V. Lovitt	*Linear Integral Equations.* McGraw-Hill Book Co., Ltd. 1924.
7	H. G. Küssner	Stresses produced in airplane wings by gusts. N.A.C.A. Tech. Memo. 654. 1932.
8	A. M. Kuethe	Circulation measurements about the tip of an airfoil during flight through a gust. N.A.C.A. Tech. Note 685. February, 1939.
9	P. Donely and C.C. Shufflebarger ..	Tests of a gust-alleviating flap in the gust tunnel. N.A.C.A. Tech. Note 745. January, 1940.
10	L. W. Bryant and I. M. W. Jones ..	On the stressing of aeroplane wings due to symmetrical gusts. R. & M. 1690. February, 1936.
11	D. Williams and J. Hanson	Gust Loads on tails and wings. R. & M. 1823. May, 1937.
12	J. R. M. Radok and L. F. Stiles ..	The motion and deformation of aircraft in uniform and non-uniform atmospheric disturbances. Aeronautical Research Report ACA–41. July, 1948.
13	Th. von Kármán and W. R. Sears ..	Airfoil theory for non-uniform motion. *J.Ae.Sci.* Vol. 5, No. 10. August, 1938.
14	Max. A. Heaslet and H. Lomax ..	Two-dimensional unsteady lift problems in supersonic flight. N.A.C.A. Report 945. 1949. A.R.C. 13,481. October, 1950.
15	H. B. Pierce	Tests of a 45° sweptback-wing model in the Langley gust tunnel. N.A.C.A. Tech. Note 1528. February, 1948.
16	H. B. Pierce	Gust-tunnel investigation of a 45° swept forward wing model. N.A.C.A. Tech. Note 1717. October, 1948.
17	T. D. Reisert	Gust-tunnel investigation of a wing model with semichord line swept back 30°. N.A.C.A. Tech. Note 1794. January, 1949.
18	R. L. Bisplinghoff, G. Isakson and T. F. O'Brien	Gust loads on rigid aeroplanes with pitching neglected. *J.Ae.Sci.* Vol. 18, No. 1. January, 1951.

APPENDIX TO PART I

The Details of the Method of Analysis

The gust loads are calculated using the following assumptions:

(*a*) The aeroplane, before entering the gust, is in steady, horizontal flight

(*b*) The forward speed of the aeroplane, V, is not changed during the action of the gust

(*c*) The aeroplane can rise vertically, but does not pitch. The vertical velocity of aeroplane is denoted by w

(*d*) The vertical gust velocity u is uniform across the span of the aeroplane at any instant

(*e*) The wing is assumed to be rectangular and unswept.

1. *Sharp-edge Gust.*—The time and distance are measured from the instant when the leading edge of the wing touches the edge of the gust. The equations of motion are written with respect to variable s and not with respect to time t. The relation between time t and variable distance s is as follows:

$$V \, . \, t = c \, . \, s \qquad \text{(I.1)}$$

and s is the distance travelled by the aerofoil measured in chords.

At any instant after entering the sharp-edged gust of strength U the following forces are acting on the aeroplane:

1.1. The aerodynamic force in the direction of (*i.e.*, normal to the flight path) and due to gust:

$$\tfrac{1}{2}\rho VSaU \times \Psi(s) \qquad \text{(I.2)}$$

where ρ is the air density and S wing area, function $\Psi(s)$ is the unsteady lift function due to penetration into sharp-edged gust. Function $\Psi(s)$ describes the way in which the lift is building up on the wing.

1.2. During the vertical motion of the aircraft the vertical velocity w is variable and is some function of s. At any instant σ measured from the same origin as s, the increment in vertical velocity within distance increment $d\sigma$ is:

$$dw = \frac{dw}{d\sigma} d\sigma$$

which produces corresponding increment in incidence dw/V and the increment of lift dL. This infinitesimal lift increment will not act instantaneously, but will grow gradually according to the function Φ where function Φ is the unsteady lift function due to sudden change of incidence. At the instant s, this infinitesimal lift will be equal to:

$$dL \times \Phi(s - \sigma)$$

the distance travelled being $s - \sigma$. The total lift at instant s, due to all the previous infinitesimal vertical velocities is the integral of all the infinitesimal lifts from instant 0 to s. The integration is performed with respect to σ, s being kept constant. Thus the aerodynamic force due to the vertical velocity of the aircraft w and acting in the direction opposite to that due to the gust is:

$$\tfrac{1}{2}\rho VSa \int_0^s \Phi(s - \sigma) \frac{dw}{d\sigma} d\sigma \, . \qquad \text{(I.3)}$$

1.3. The aerodynamic forces are balanced by the inertia force:

$$\frac{W}{g}\frac{dw}{dt} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (I.4)$$

which can be expressed by the variable s using the equation (I.1);

$$\frac{WV}{gc}\frac{dw}{ds}\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (I.5)$$

The equation of motion due to a sharp-edged gust is obtained by summing all the forces acting on the aircraft and given by the expressions (I.2), (I.3) and (I.5).

$$\tfrac{1}{2}\rho VSaU\,\Psi(s) - \tfrac{1}{2}\rho VSa\int_0^s \Phi(s-\sigma)\frac{dw}{d\sigma}\,d\sigma - \frac{WV}{gc}\frac{dw}{ds} = 0\,. \qquad \ldots \quad \ldots \quad (I.6)$$

After rearranging terms and introducing the mass parameter

$$\mu_g = \frac{2W}{g\rho Sac} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (I.7)$$

the equation (I.6) can be simplified to:

$$\frac{dw}{ds} + \frac{1}{\mu_g}\int_0^s \Phi(s-\sigma)\frac{dw}{d\sigma}\,d\sigma - \frac{1}{\mu_g}U\Psi(s) = 0\,. \qquad \ldots \quad \ldots \quad (I.8)$$

The solution of equation (I.8) gives the vertical velocity, and hence the vertical acceleration of the aircraft as a function of the distance travelled by the wing (measured in chords). To make the analysis entirely dimensionless the vertical force (or vertical acceleration) function can be introduced, defined as follows:

$$\Lambda(s) = \frac{\dfrac{W}{g}}{\tfrac{1}{2}\rho VSaU}\frac{dw}{dt}(s)\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (I.9)$$

The term $(dw/dt)(s)$ is the actual vertical acceleration of the aircraft at point s due to the gust.

Remembering that

$$\frac{dw}{dt}(s) = \frac{V}{\bar{c}}\frac{dw}{ds}(s)$$

and introducing the mass parameter as given by the equation (I.7), the definition of the force function can be expressed in terms of variable s and not in terms of time t:

$$\Lambda(s) = \frac{\mu_g}{U}\frac{dw}{ds}(s)\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (I.10)$$

Substituting the value of dw/ds from the equation (1.10) into the equation of motion (1.8), the equation in terms of the force function is obtained:

$$\Lambda(s) + \frac{1}{\mu_g}\int_0^s \Phi(s-\sigma)\,\Lambda(\sigma)\,d\sigma - \Psi(s) = 0\,. \qquad \ldots \quad \ldots \quad (I.11)$$

The force function $\Lambda(s)$ is the ratio of the actual force acting on the wing to the force that would be acting if the Wagner effect and vertical motion of the aircraft were absent and its maximum value corresponds with gust alleviation factor K.

To minimise the computational work in calculations of the gust loads approximate expressions for unsteady lift functions were used. The form of simplification taken was that suggested by R. T. Jones[4] and Houbolt[5], and for aspect ratios 3 and 6 the actual functions as given in Refs. 4 and 5 were used.

The unsteady lift function due to penetration of the aerofoil into sharp-edged gust is given by:

$$\Psi(s) = 1 - A\mathrm{e}^{-\alpha s} - B\mathrm{e}^{-\beta s} \quad \ldots \quad (\mathrm{I.12})$$

and the unsteady lift function due to sudden change of incidence by:

$$\Phi(s) = 1 - D\mathrm{e}^{-\delta s}. \quad \ldots \quad (\mathrm{I.13})$$

The numerical values of the coefficients as used in this report are given in the following table.

Aspect Ratio	A	α	B	β	D	δ
∞	0·50	0·260	0·50	2·00	0·458	0·265
6	0·48	0·588	0·334	1·93	0·361	0·762
3	0·679	1·116	0·227	6·40	0·283	1·080

The functions Ψ and Φ are shown in Figs. 2 and 3, where for comparison the unsteady lift functions for infinite aspect ratio as calculated by von Kármán and Sears[13] are included. It can be seen that the approximations are generally good, and only the shapes of the function Φ curves, for AR $= \infty$ do not agree; the difference arises from the use of only one exponential term in the expression for function Φ. Some rough estimates show that, for the sharp-edged gust, the error in the alleviation factor due to this approximation should be always less than 1·5 per cent. For gust shapes different from the sharp-edged gust, this error is smaller, for then the alleviation due to vertical movement plays a more important part than the unsteady lift functions. This can be seen by examination of Fig. 4, where, for longer gust gradients, the values of gust alleviation factor are rapidly approaching the values of alleviation factor calculated neglecting Wagner effect.

The solution of the equation (I.8) was obtained by two different methods*. The first method (used, it is believed, for the first time in this application) is the method of successive approximations; the second is the solution of Fredholm's equation by successive substitution.

2. *Solution by the* 1*st Method*.—This is the method of successive approximations. The first approximation is obtained by neglecting the unsteady lift function due to change of incidence, *i.e.*, putting $\Phi = 1$; then the equation (I.11) can be simplified to

$$\Lambda(s) + \frac{1}{\mu_g}\int_0^s \Lambda(\sigma)\, d\sigma - \Psi(s) = 0 \quad \ldots \quad (\mathrm{I.14})$$

or by differentiation further simplified to a standard form:

$$\frac{d\Lambda(s)}{ds} + \frac{1}{\mu_g}\Lambda(s) - \frac{d\Psi(s)}{ds} = 0 \quad \ldots \quad (\mathrm{I.15})$$

and in this form can be solved by any elementary method, once the function $\Psi(s)$ is given. Introducing the form of function $\Psi(s)$ as given by equation (I.12), the solution of the equation (I.15) gives the first approximation of the force function (as defined in I.9) in the form:

$$\Lambda_0 = \left[\mu_g - \left(\frac{A}{\frac{1}{\mu_g} - \alpha} + \frac{B}{\frac{1}{\mu_g} - \beta}\right)\right]\frac{1}{\mu_g}\mathrm{e}^{-\frac{1}{\mu_g}s} + \frac{A}{\frac{1}{\mu_g} - \alpha}\alpha\,\mathrm{e}^{-\alpha s} + \frac{B}{\frac{1}{\mu_g} - \beta}\beta\,\mathrm{e}^{-\beta s}. \quad \ldots \quad (\mathrm{I.16})$$

* A third, much simpler and exact solution is given in Part II.

Using this equation of the vertical forces acting on the aircraft given by the first approximation to the equation (I.11), the difference between the assumed variation of aerodynamic forces due to vertical motion and the next approximate forces derived using the unsteady lift function $\Phi(s)$, can be calculated. These differences in the aerodynamic forces have to be added to those originally calculated, and this correction, expressed as the force function correction, is:

$$(\Delta\Lambda)_1 = \frac{D}{g}\int_0^s e^{-\delta(s-\sigma)}\,\Lambda_0(\sigma)\,d\sigma \quad \ldots \quad (I.17)$$

where D and δ are the coefficients defining the function $\Phi(s)$, equation (I.13).

The second approximation of the expression for the vertical force function is now used to calculate the next correction due to vertical motion, which in turn gives the third approximation of the vertical force function. Proceeding in the way described above the full solution of the equation (I.11) is obtained in the form of series of integrals:

$$\begin{aligned}\Lambda = \Lambda_0 &+ \frac{D}{\mu_g}\int_0^s e^{-\delta(s-\sigma)}\,\Lambda_0(\sigma)\,d\sigma \\ &- \frac{D}{\mu_g^2}\int_0^s \Phi(s-\sigma_1)\int_0^{\sigma_1} e^{-\delta(s-\sigma)}\,\Lambda_0(\sigma)\,d\sigma\,d\sigma_1 \\ &+ \frac{D}{\mu_g^3}\int_0^s \Phi(s-\sigma_2)\int_0^{\sigma_2}\Phi(s-\sigma_1)\int_0^{\sigma_1} e^{-\delta(s-\sigma)}\,\Lambda_0(\sigma)\,d\sigma\,d\sigma_1\,d\sigma_2 - \ldots \quad (I.18)\end{aligned}$$

The convergence of the series (I.18) depends on the value of mass parameter μ_g.

3. *Solution by the 2nd Method.*—The second method for the solution of equation (I.11) is the solution by successive substitutions, and is given in Ref. 6, Chapter II (Fredholm's equation). The solution is given by the series of integrals:

$$\begin{aligned}\Lambda = \Psi(s) &- \frac{1}{\mu_g}\int_0^s \Phi(s-\sigma)\,\Psi(\sigma)\,d\sigma \\ &+ \frac{1}{\mu_g^2}\int_0^s \Phi(s-\sigma_1)\int_0^{\sigma_1}\Phi(s-\sigma)\,\Psi(\sigma)\,d\sigma\,d\sigma_1 \\ &- \frac{1}{\mu_g^3}\int_0^s \Phi(s-\sigma_2)\int_0^{\sigma_2}(s-\sigma_1)\int_0^{\sigma_1}\Phi(s-\sigma)\,\Psi(\sigma)\,d\sigma\,d\sigma_1\,d\sigma_2 + \ldots \quad (I.19)\end{aligned}$$

The solution by the second method is very similar in form to that by the first method, though there is a difference in the physical interpretation. In the first method, the first approximation Λ_0 is the solution when only the Wagner effect on the aerodynamic forces due to a sudden change of incidence is omitted, and hence even the first term in equation (I.18) has a maximum value and gives a fairly good idea about the magnitude of the gust alleviation factor. This especially holds true for large values of the mass parameter.

The first approximation of the second method is the solution for no vertical motion of the aircraft, and the first term in the equation (I.19) is simply the unsteady lift function due to entering the sharp-edged gust.

A comparison of the different degrees of approximation given by the two methods is shown in Fig. 13. The gust force function is calculated for a sharp-edged gust, for a wing aspect ratio of 3 and a mass parameter of 10. The low value of the mass parameter was chosen to show better the differences between different approximations, as the convergence of the series in equations (I.18) and (I.19) is rather slow for $\mu_g = 10$.

The curve calculated by the second method, expanding the series to the term with μ_g^4, can be regarded as the exact solution, as the error is much less than can be shown in Fig. 13. The other two curves were calculated by the first and the second method respectively, but using the expansion up to the term including μ_g^2 only. The difference between these curves and the previous one shows the magnitude of the errors involved in different methods. It can be seen that the accuracy of the first method is not too good near the maximum of the gust force for sharp-edged gust, but it improves as the aerofoil travels through the gust. The second method gives a good approximation near the peak of the gust force function, but diverges rapidly for larger distances travelled.

In the present work the second method was used, and the expression for A was expanded to and including the terms involving μ_g^4.

4. *The Arbitrary Gust.*—Let us assume that the gust shape is given as gust velocity u as a function of the distance s (measured in chords). At any point σ, which is less than s, the increment of gust velocity over a distance $d\sigma$ is given by:

$$du = \frac{du}{d\sigma}\, d\sigma .$$

This infinitesimal increment of gust velocity, du, can be regarded as an infinitely small sharp-edged gust, due to which a gust force dF will build up according to the gust force function A, starting at the point σ. At the instant s this infinitesimal sharp-edged gust force is by definition:

$$dF = \tfrac{1}{2}\rho VSa\, du\, A(s - \sigma) \quad \text{(I.20)}$$

The total gust force at the instant s is the sum of all infinitely small sharp-edged gust forces:

$$F(s) = \tfrac{1}{2}\rho VSa \int_0^s A(s - \sigma) \frac{du}{d\sigma}\, d\sigma \quad \text{(I.21)}$$

The dimensionless gust force function for an arbitrary gust, A_s, is defined as the force $F(s)$ divided by

$$\tfrac{1}{2}\rho VSaU$$

where U is the maximum gust velocity for a given gust shape. Using this definition, the force function for an arbitrary gust shape is given by:

$$A_s = \frac{1}{U} \int_0^s A(s - \sigma) \frac{du}{d\sigma}\, d\sigma \quad \text{(I.22)}$$

and again the force function A_s is the ratio of the actual gust force to the gust force that would be encountered if no vertical motion and no Wagner effects were present.

The maximum value of this function gives the gust alleviation factor K for a given gust shape.

For the linear increase in the gust strength, as assumed in present calculations, the gust velocity is given by

$$u = \frac{U}{H}\, s \quad \text{(I.23)}$$

where U is the gust velocity at the distance H. Differentiating equation (I.23) and putting into (I.22), the force function for linearly increasing gust is obtained:

$$A_s(s) = \frac{1}{H} \int_0^s A(s - \sigma)\, d\sigma \quad \text{(I.24)}$$

For flat-topped or triangular gust, with the maximum velocity U reached at the point H, the force function for $s > H$ is obtained by the addition of a new gust with reversed sign and starting at the point $s = H$. Thus

$$A_H = A_H(s) - n \,.\, A_H(s - H) \qquad \text{(for } s > H\text{)} \qquad \text{(I.25)}$$

where $n = 1$ for flat-topped and $n = 2$ for triangular symmetrical gust.

For double-triangular gust and for $s > 3H$, force function is obtained in a similar way by adding further a new positive gust starting at point $s = 3H$. The force function for double triangular gust of maximum velocity U and for $s > 3H$ is given by:

$$A_H = A_H(s) - 2A_H(s - H) + 2A_H(s - 3H) \,. \qquad \text{(for } s > 3H\text{)} \qquad \text{(1.26)}$$

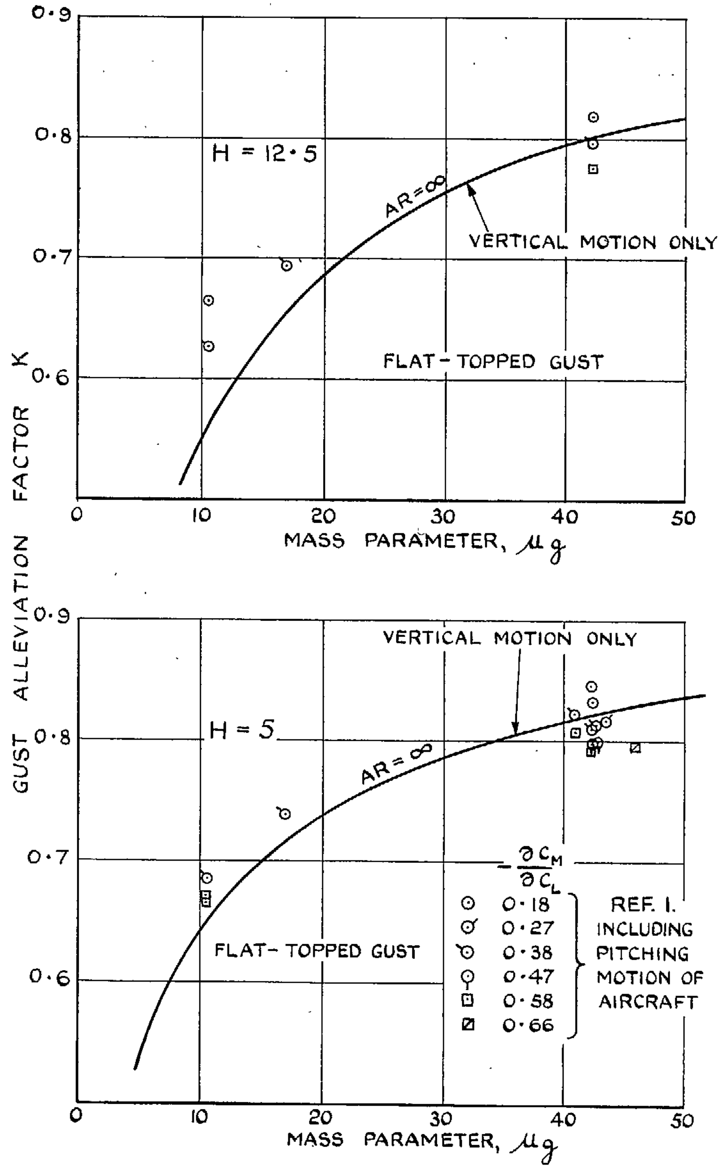


FIG. 1. Comparison of values of gust alleviation factor calculated with and without pitching degree of freedom for two lengths of gust gradient.

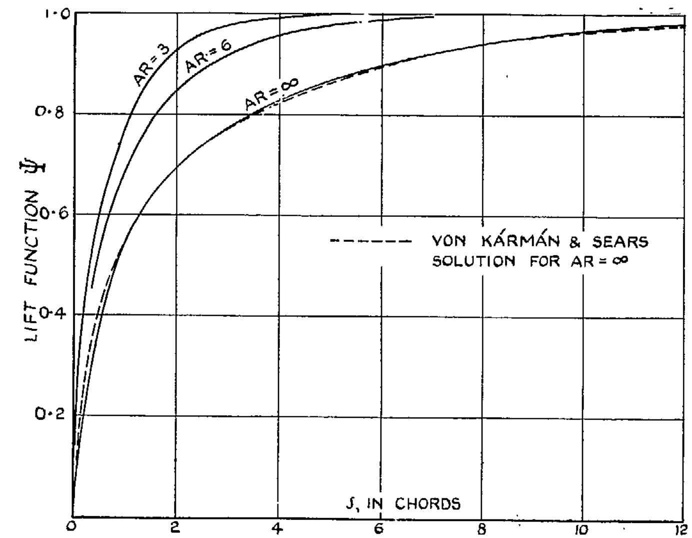


FIG. 2. Unsteady lift functions due to penetration into sharp-edged gust, for different aspect ratios (as used in present report).

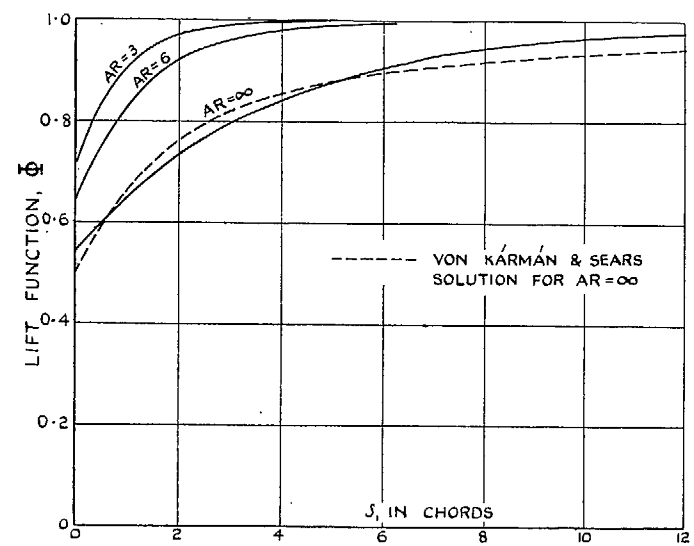


FIG. 3. Unsteady lift functions due to sudden change of incidence, for different aspect ratios (as used in present report).

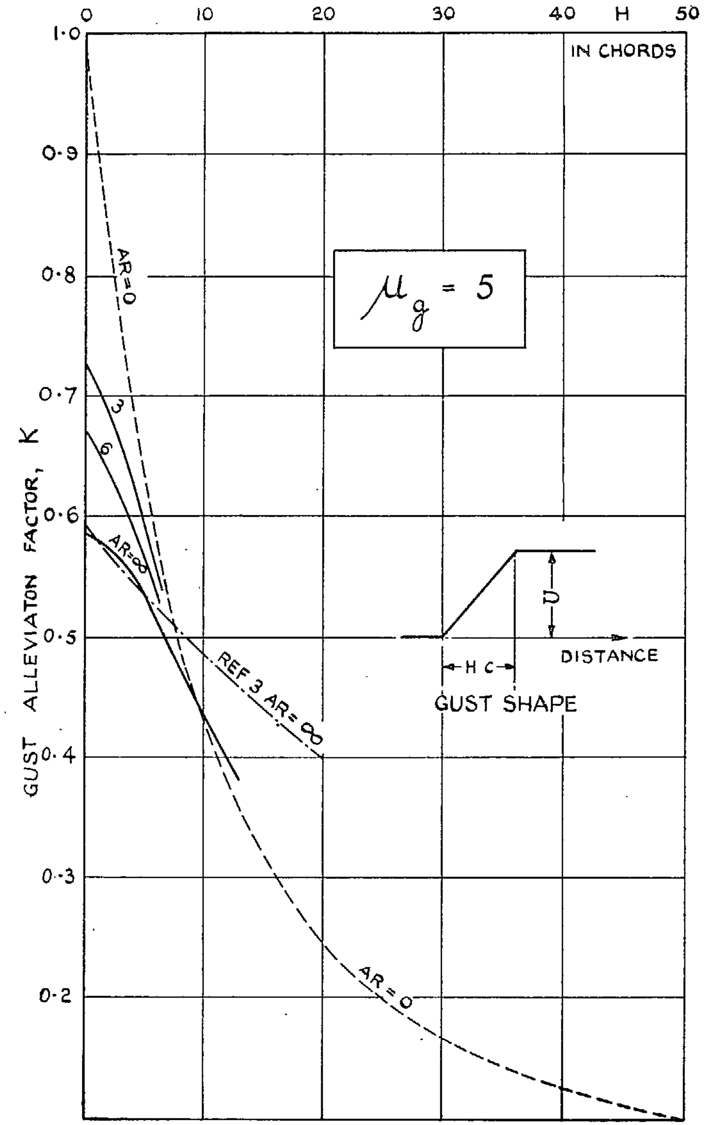


FIG. 4a. Gust alleviation factor *vs.* gust gradient for aspect ratios 0, 3, 6 and ∞. Flat-topped gust.

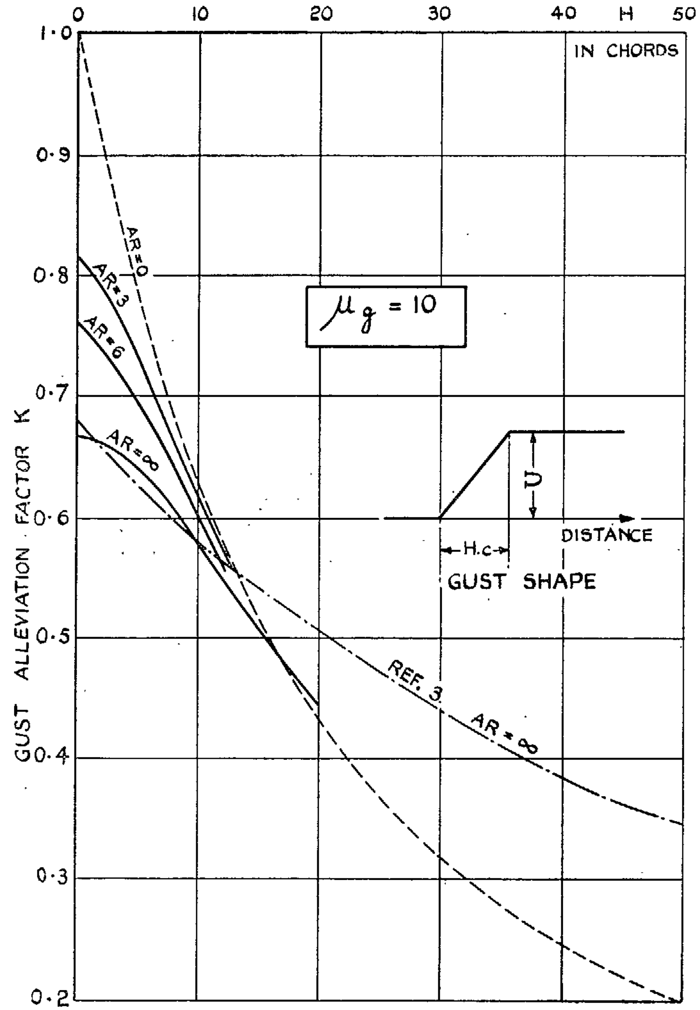


FIG. 4b. Gust alleviation factor *vs.* gust gradient for aspect ratios 0, 3, 6 and ∞. Flat-topped gust.

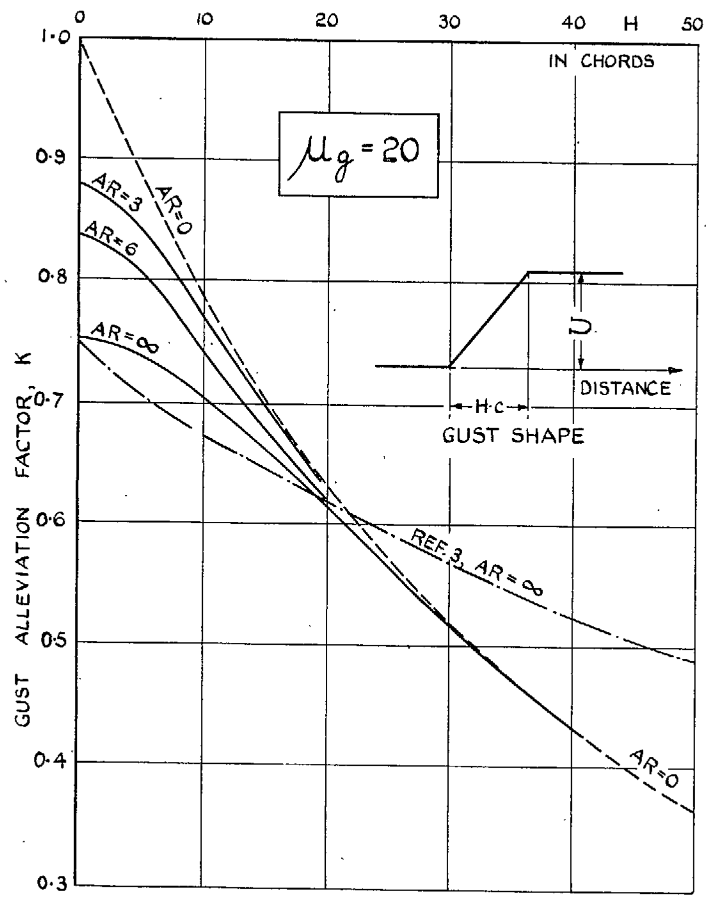


FIG. 4c. Gust alleviation factor *vs.* gust gradient for aspect ratios 0, 3, 6 and ∞. Flat-topped gust.

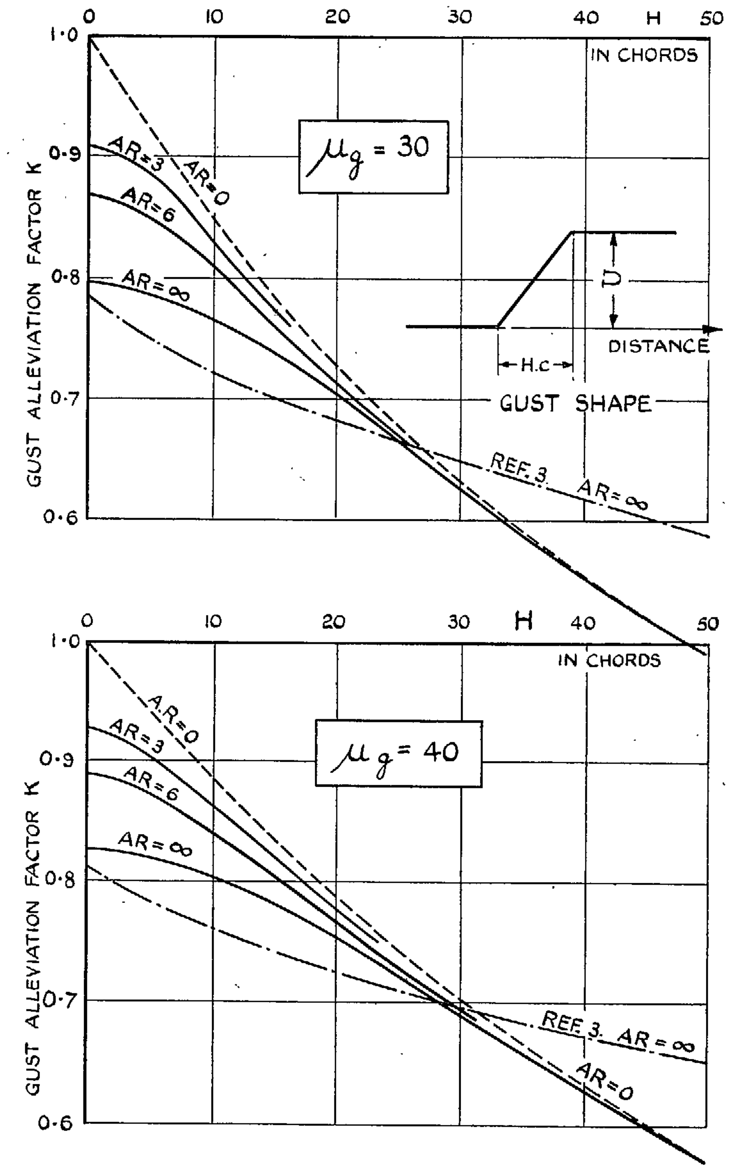


FIG. 4d. Gust alleviation factor *vs.* gust gradient for aspect ratios 0, 3, 6 and ∞. Flat-topped gust.

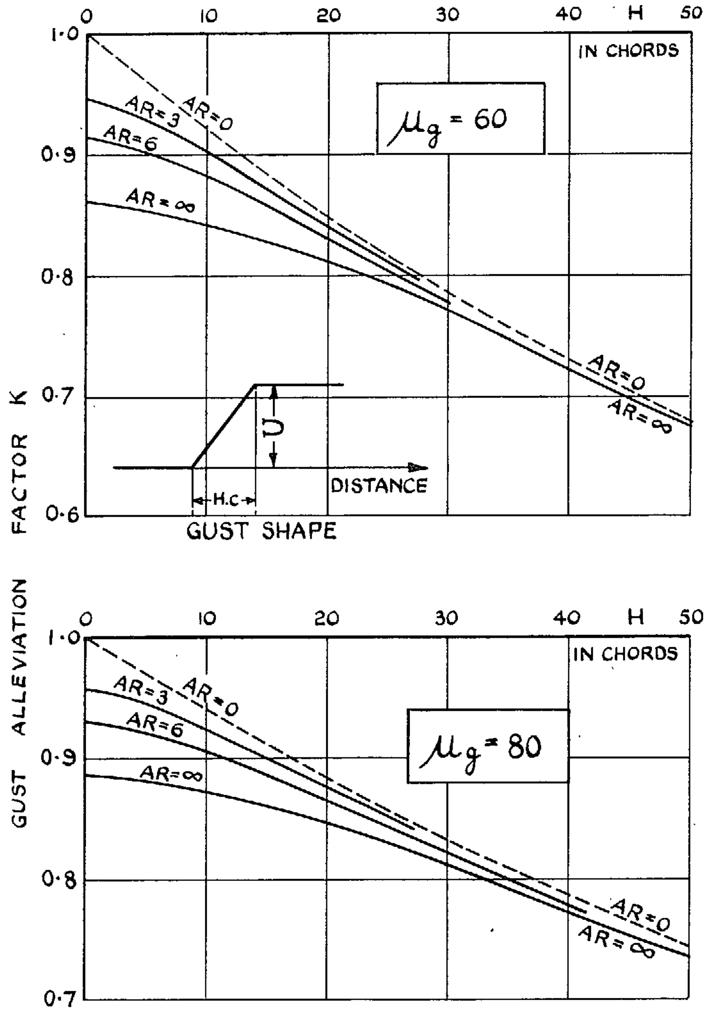


FIG. 4e. Gust alleviation factor *vs.* gust gradient for aspect ratios 0, 3, 6 and ∞. Flat-topped gust.

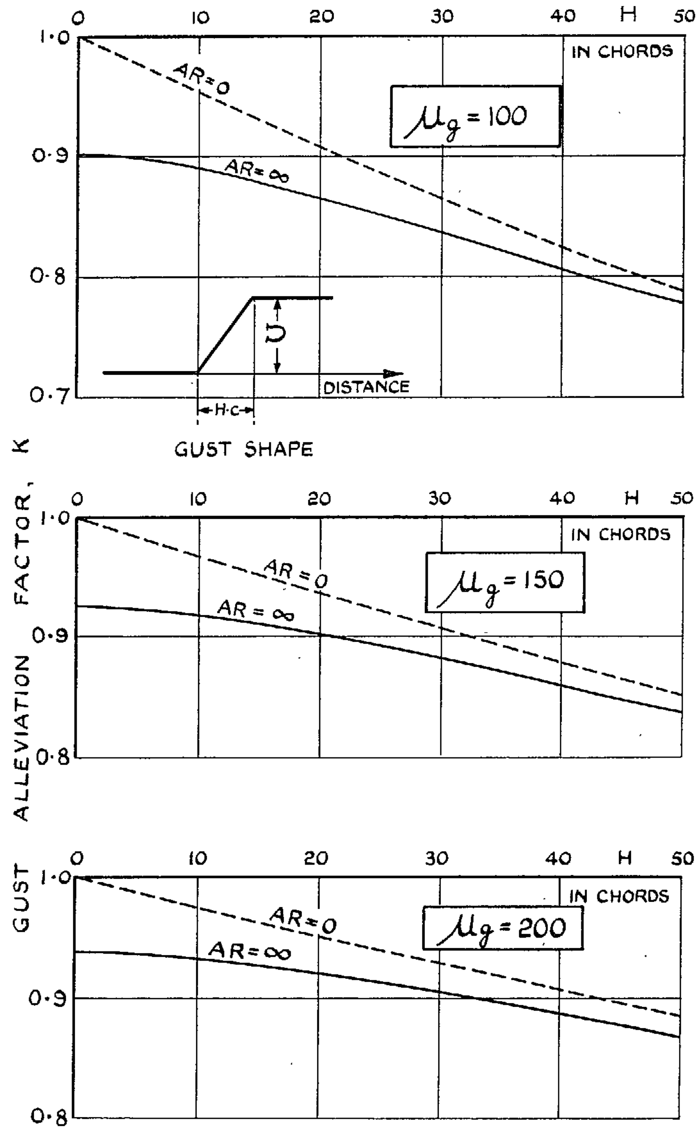


FIG. 4f. Gust alleviation factor *vs.* gust gradient for aspect ratios 0 and ∞. Flat-topped gust.

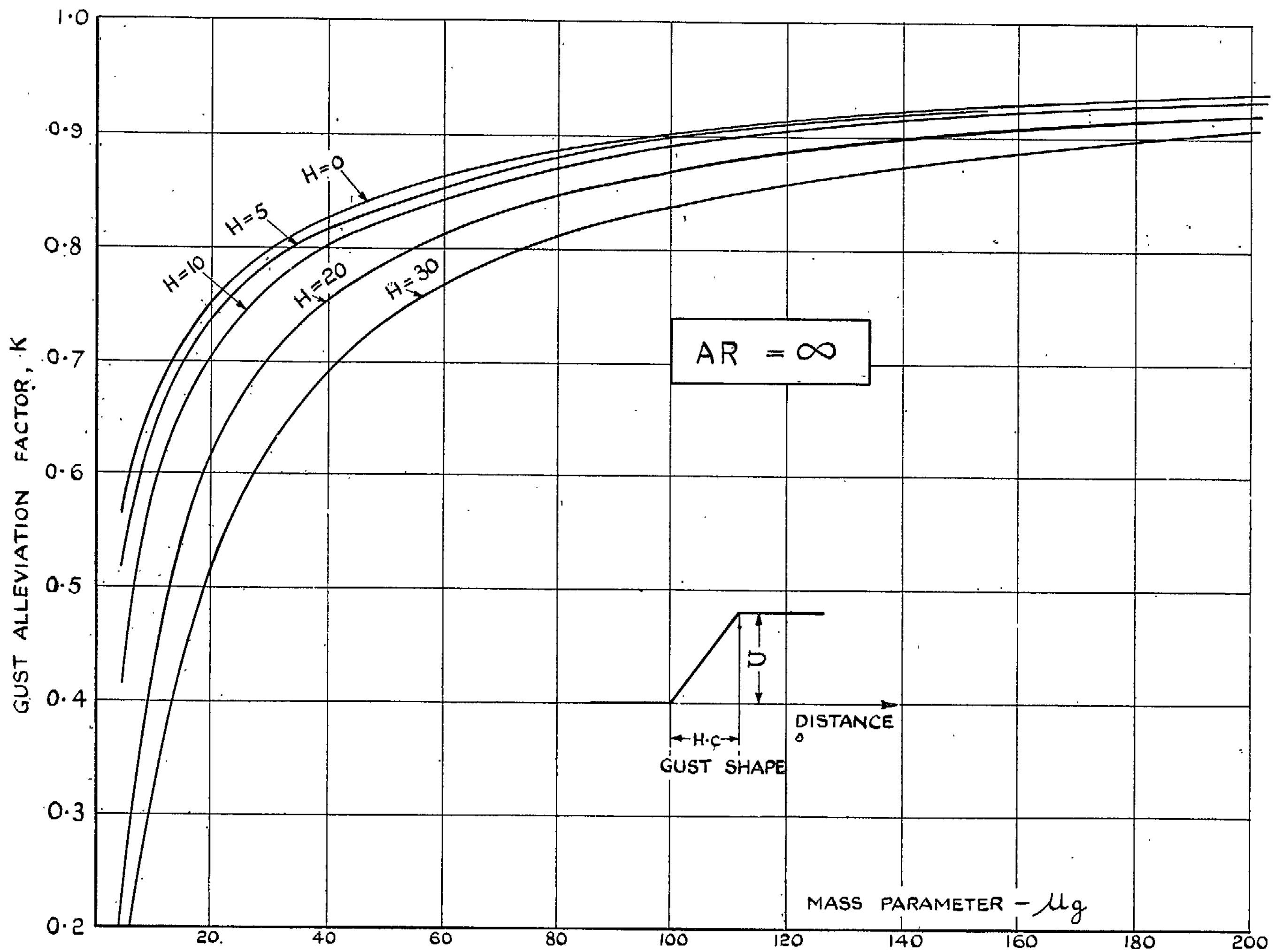


FIG. 5a. Gust alleviation factor *vs.* mass parameter for different gust gradients. Flat-topped gust. Aspect ratio ∞.

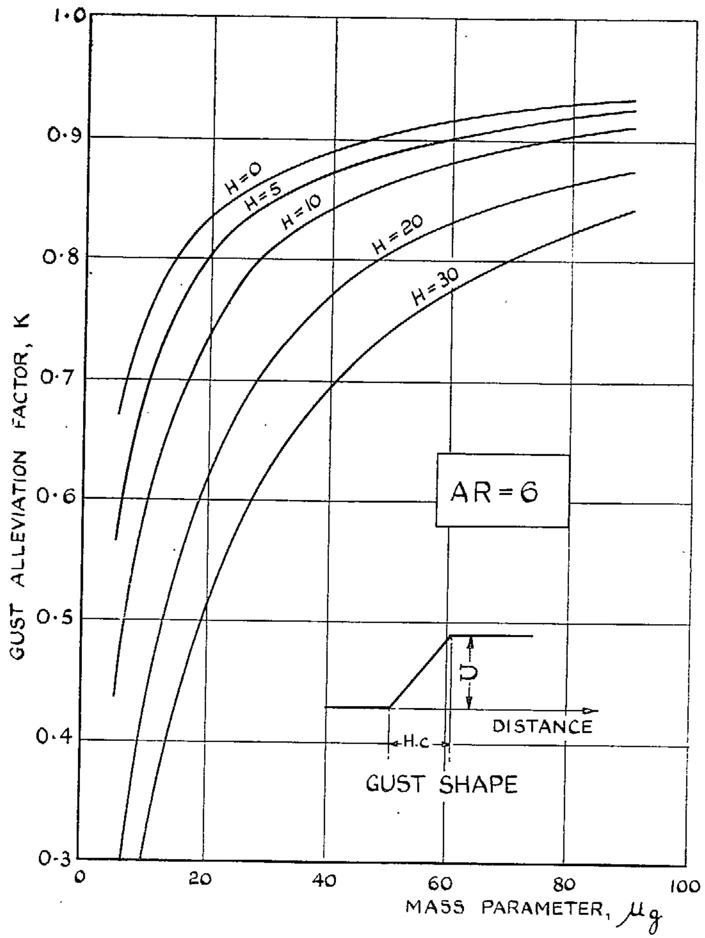


FIG. 5b. Gust alleviation factor *vs.* mass parameter for different gust gradients. Flat-topped gust. Aspect ratio 6.

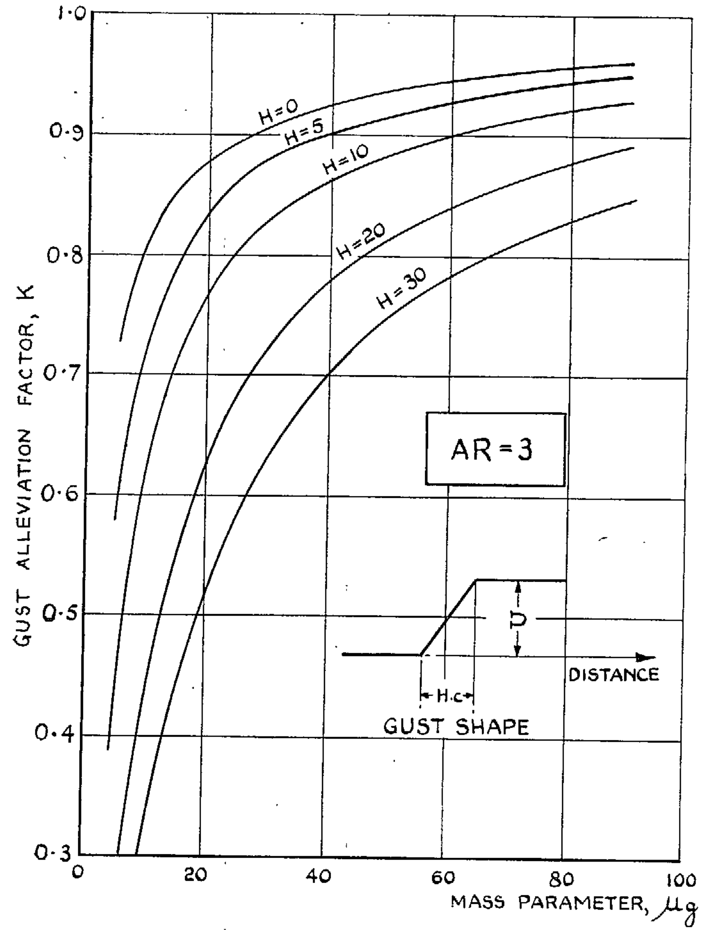


FIG. 5c. Gust alleviation factor *vs.* mass parameter for different gust gradients. Flat-topped gust. Aspect ratio 3.

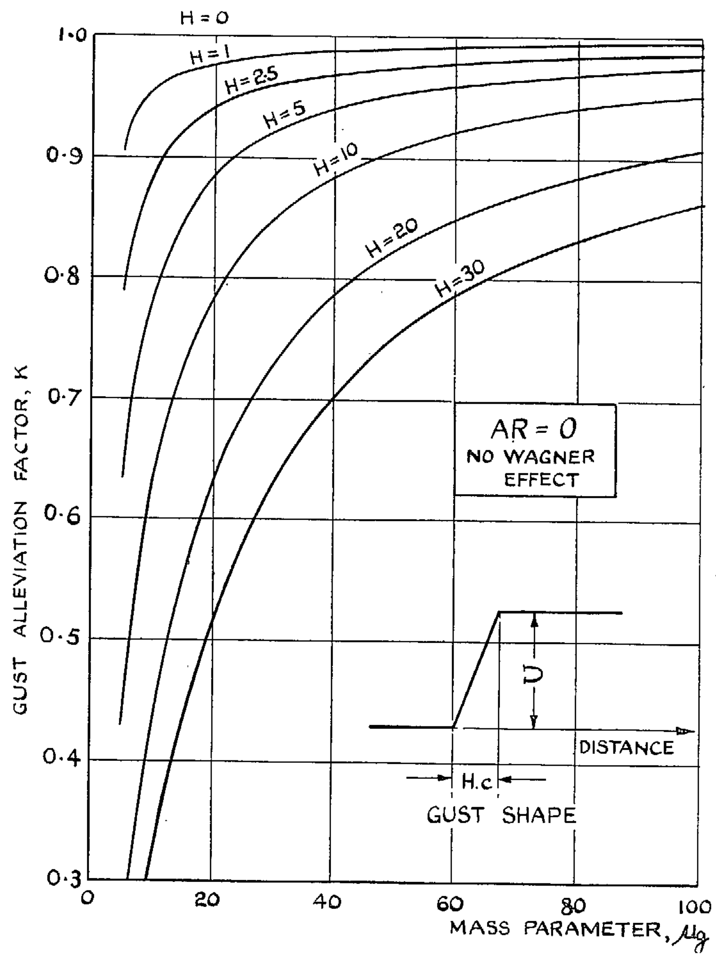


FIG. 5d. Gust alleviation factor *vs.* mass parameter for different gust gradients. Flat-topped gust. Aspect ratio 0. (No Wagner effect.)

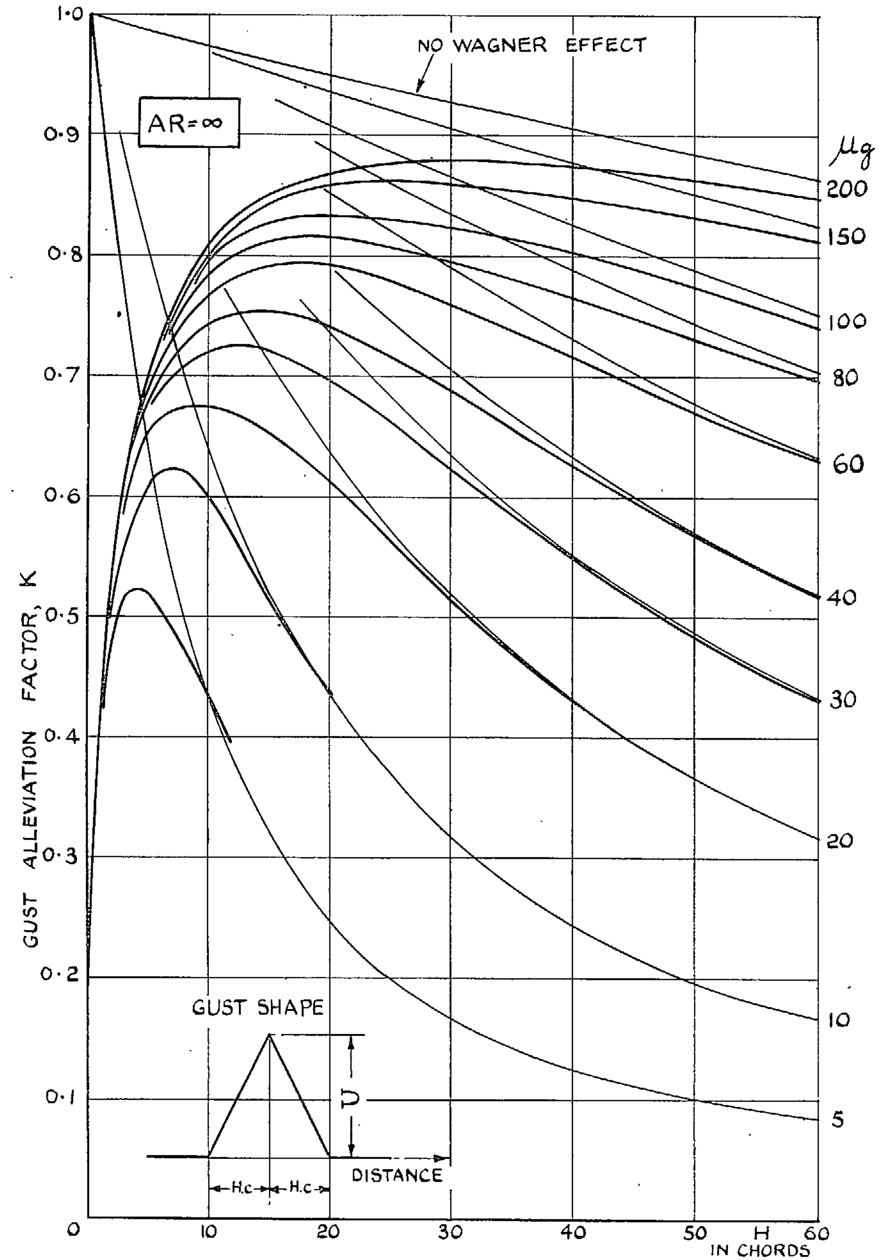


FIG. 6. Gust alleviation factor for triangular gust *vs.* gust gradient. Aspect ratio ∞.

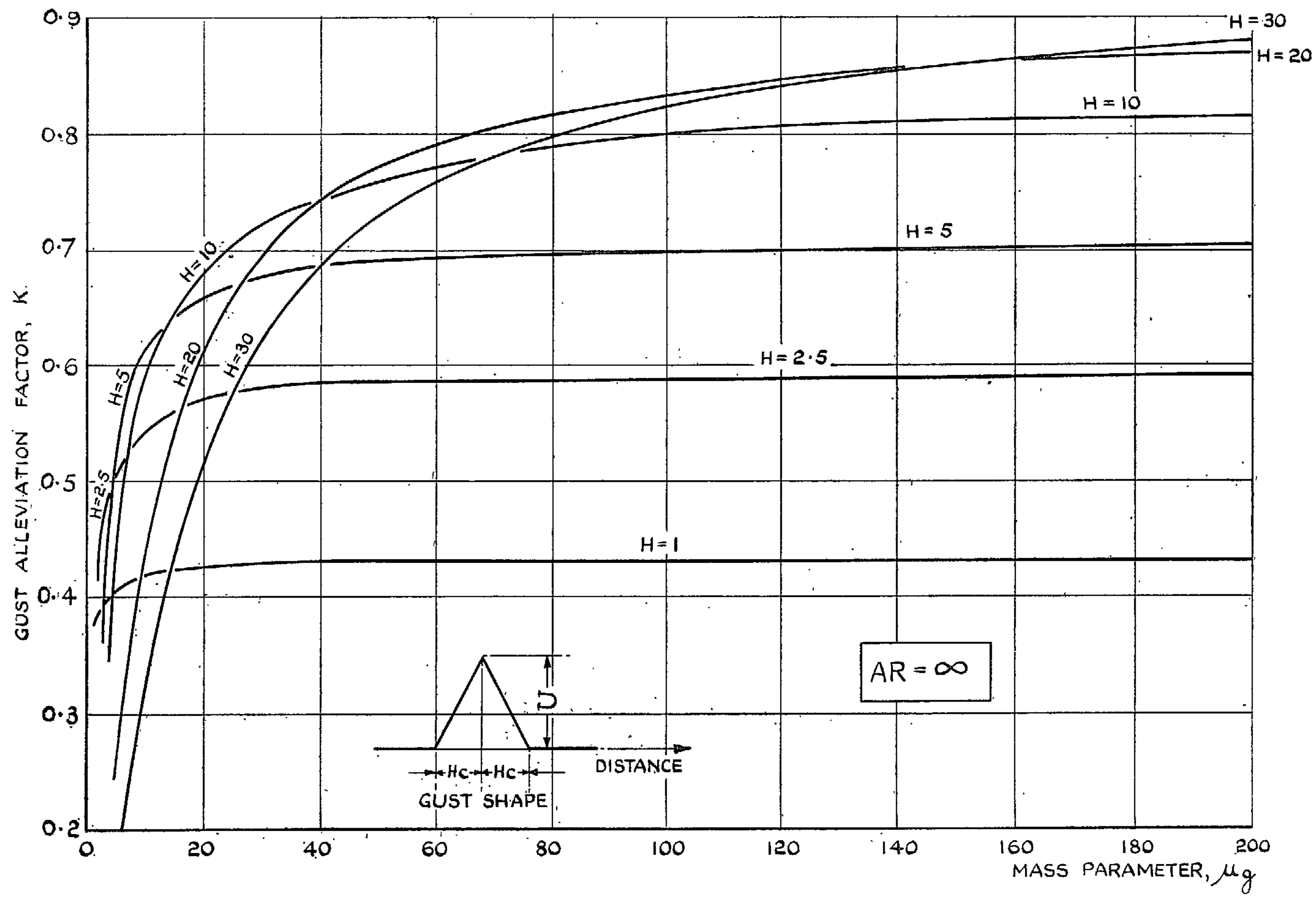


FIG. 7. Gust alleviation factor for triangular gust *vs.* mass parameter for different gust gradients. Aspect ratio ∞.

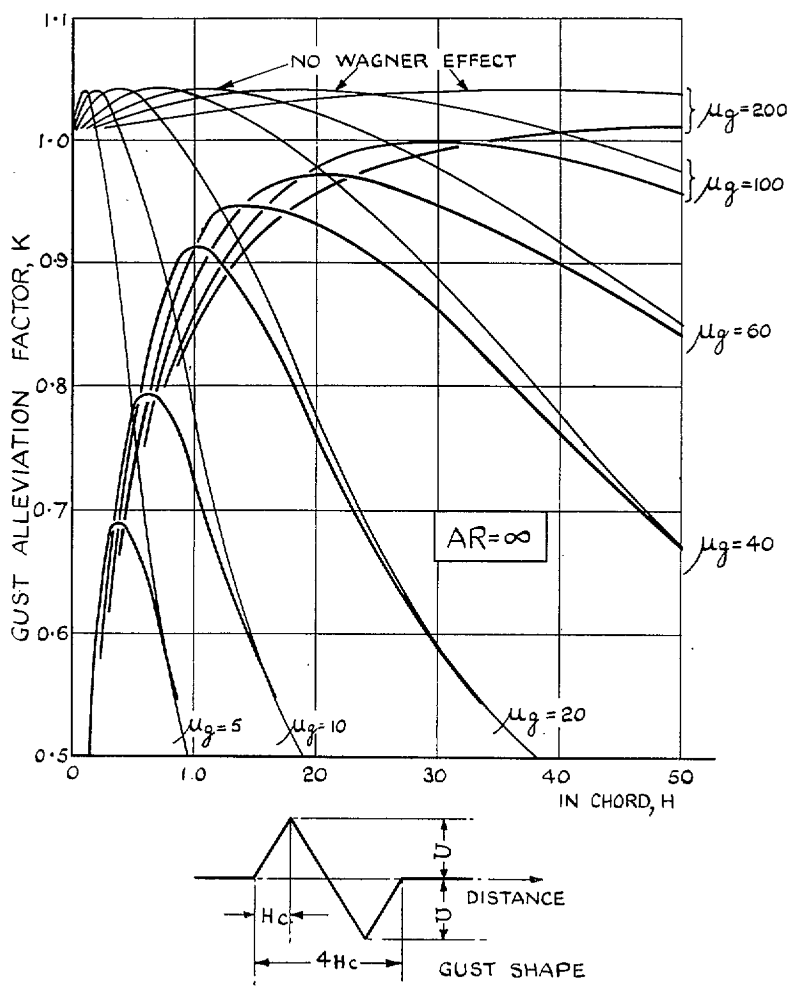


FIG. 8. Gust alleviation factor for the second, negative peak of double-triangular gust plotted against gust length. Aspect ratio ∞.

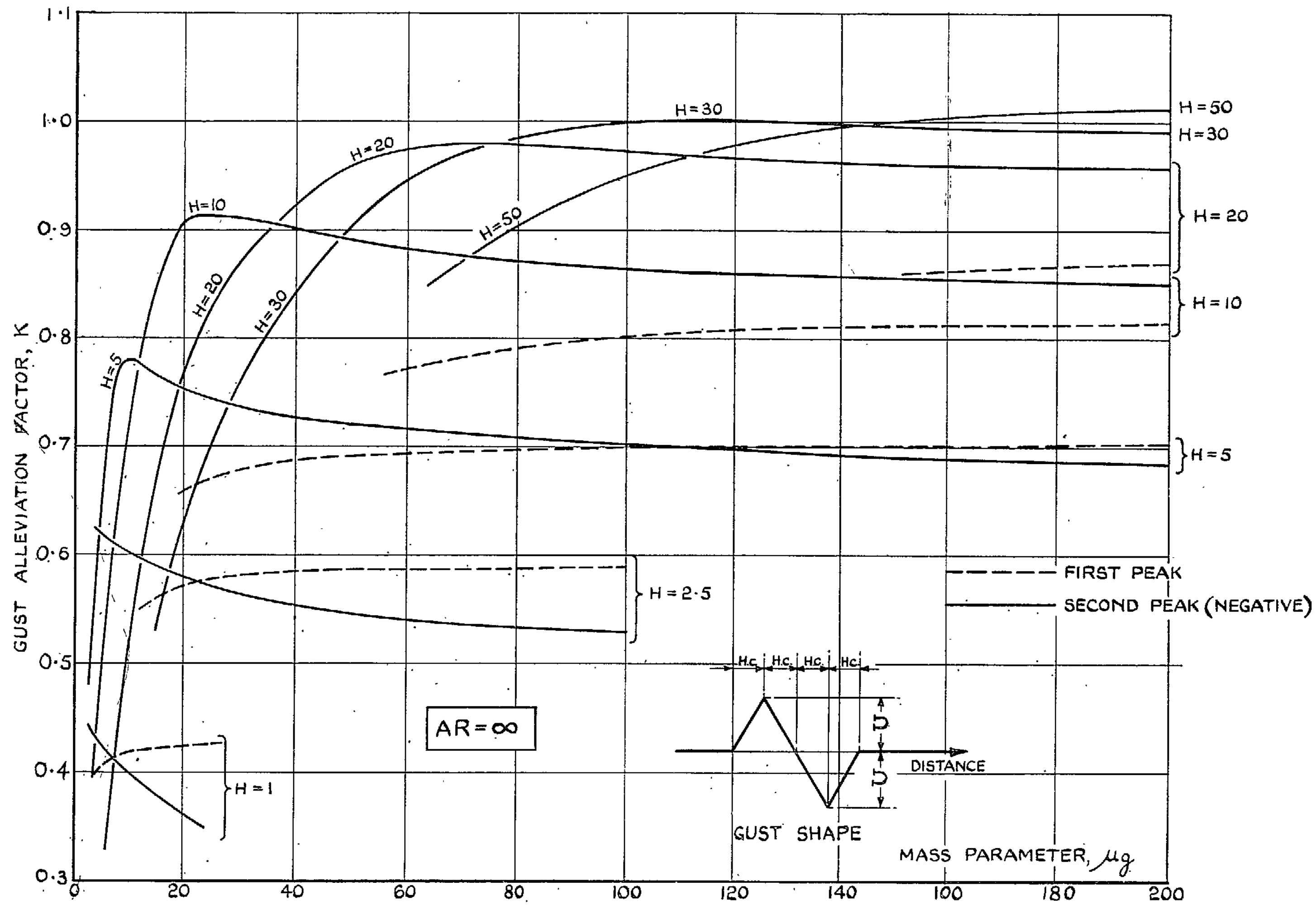


FIG. 9. Gust alleviation factor for double-triangular gust. Comparison of first and second (negative) peaks plotted against mass parameter for the range of gust gradients.

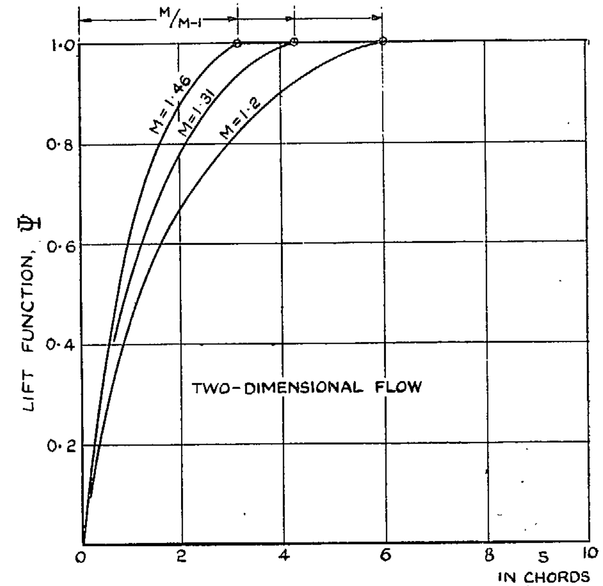


FIG. 10. Unsteady lift functions due to penetration into sharp-edged gust for supersonic speeds.

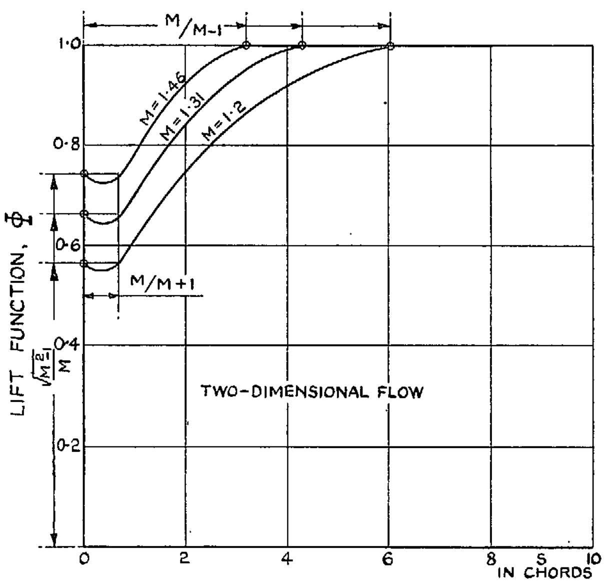


FIG. 11. Unsteady lift functions due to sudden change of incidence for supersonic speeds.

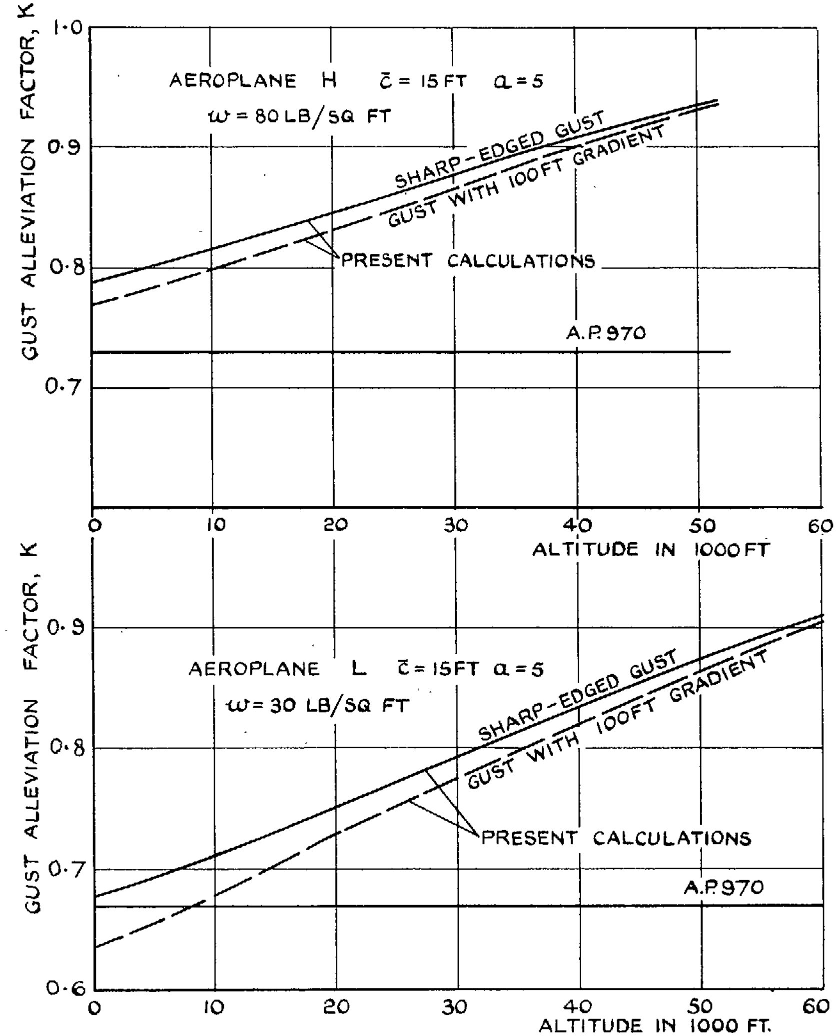


FIG. 12. The effect of altitude on the gust alleviation factor. Comparison with values given by A.P. 970.

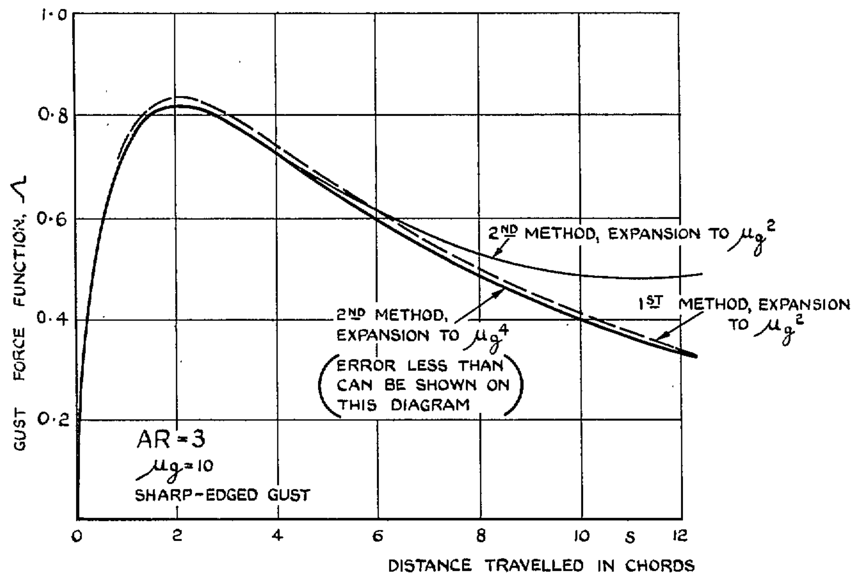


FIG. 13. Comparison of the solutions for gust force function by first and second method.

PART II*

Compressibility Effect

Summary.—Theoretical calculations of the gust alleviation factor for a range of Mach numbers show an appreciable decrease in its value with increasing Mach number. The reduction in the value of the gust factor at $M = 0 \cdot 7$ is about 10 per cent. for sharp-edged gusts and lightly loaded aircraft ($\mu_g = 20$) and decreases to about 5 per cent. for gust length of 10 chords and heavy aircraft ($\mu_g = 100$).

The analysis of existing flight records indicates that the gust loads at high Mach numbers can be estimated satisfactorily if the gust factor and the lift slope are corrected for compressibility.

1. *Introduction.*—It is recommended in A.P. 970, Vol 2, Leaflet 203/1 that the gust load in terms of normal acceleration for high-speed aircraft should be calculated according to the formula

$$\left[1 \pm \tfrac{1}{2}\rho_0 f(M)\, a \frac{KUV}{w}\right] g$$

where ρ_0 is density of air (slugs/ft^3)

- $f(M)$ compressibility correction factor
- a lift-curve slope
- K gust alleviation factor
- U gust velocity (ft/sec E.A.S.)
- V aeroplane velocity (ft/sec E.A.S.)
- w wing loading (lb/sq ft).

The compressibility effects are taken into account by using the compressibility correction factor $f(M)$, which is assumed to be the Glauert factor. No allowances for compressibility effects on the gust alleviation factor K are made, thus increasing the gust loads with increasing Mach number in proportion to the Glauert factor. It has been felt for a long time that this is too great a simplification and that the Mach number must have some effect on the unsteady lift functions and hence on the gust alleviation factor. Theoretical calculations of flutter derivatives show large effects of the Mach number thus providing indirect evidence of Mach number effects on the indicial lift functions. It is possible to calculate the indicial lift and moment functions from known flutter derivatives, if a consistent set of such derivatives is available for a wide enough range of the frequency parameter. This line of approach was used by Mazelsky and Drischler[1] in order to evaluate the indicial lift and moment functions for sinking and pitching motion over a range of Mach numbers. In the same paper the indicial gust function is also calculated for a range of subsonic Mach numbers. The calculations show that the main effect of compressibility is to decrease the rate of lift growth due to a sudden gust.

Experimental investigations of the Mach number effect on gust loads have been made in U.S.A., and to the knowledge of the writer two papers on this subject have been published[2,3]. It might be useful for further discussion to quote in full the summaries of these papers. The investigation made by Goodall[2] is summarised as follows:

> 'The results are presented of a preliminary investigation of effect of Mach number on gust load factor on high-speed airplanes. Tests consisted in measuring load factor, air speed and altitude on a P–47D fighter flying through gusty air at various speeds. Conclusions reached were that gust loads at high speed are apparently affected by other factors than Mach number on slope of lift curve and these factors tend to reduce gust load factor. It was recommended that further tests be conducted at high speeds and with adequate accelerations.'

* R.A.E. Tech. Note Aero. 2254 dated August, 1953, received 7th November, 1953.

The more recent investigation made by Binckley and Funk[3] has the following summary:

> 'Two jet-propelled airplanes were flown at different speeds in rough air to investigate the effects of compressibility on applied loads. Data were obtained over a Mach number range of 0·25 to 0·68 for effective gust velocities up to 15 f.p.s. An analysis of the results indicates that no compressibility correction to the slope of the lift curve was necessary up to a Mach number of 0·68 for gust velocities up to 9 f.p.s. Data obtained for gust velocities greater than about 9 f.p.s. were insufficient for analysis.'

In both cases the experiment points to the conclusion that at high speeds the gust loads are affected by factors other than mere change in lift-slope value, and that these factors tend to decrease the gust loads. Ref. 3 goes even further in that it suggests no compressibility correction to the lift slope for gust load calculations. It is very difficult to accept this suggestion, as this implies discontinuity between the transient and steady state of flight in turbulent air. A more plausible explanation of this phenomenon would be given by a Mach number effect on the unsteady lift functions, as shown in Ref. 1.

In order to obtain a practical measure of the compressibility effects on gust loads, the gust alleviation factor was calculated for compressible flow and the results were compared with the incompressible case.

Analysis of the available experimental results indicates that the present theory should predict the gust loads at high Mach numbers with sufficient accuracy.

2. *Indicial Lift Functions.*—The indicial lift functions for two-dimensional, compressible flow determined theoretically and given in Ref. 1, are approximated by exponential series, expressed as a function of distance travelled in half chords; its asymptotic value being a Glauert factor $(1 - M^2)^{-1/2}$. For the purpose of the gust alleviation factor calculations, these functions are expressed in terms of distance measured in chords and their ultimate value is made equal to unity.

The indicial gust function, being a non-dimensional lift growth function due to penetration into a sharp-edged gust is given in the form:

$$\Psi = 1 + a_1 e^{-\alpha_1 s} + a_2 e^{-\alpha_2 s} + a_3 e^{-\alpha_3 s} ; \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where s is a distance travelled by an airfoil measured in chords. The coefficients a_n and α_n are given below in Table 1 for a range of Mach numbers.

TABLE 1

Indicial Gust Function (Two-dimensional Flow)

Mach number	a_1	a_2	a_3	α_1	α_2	α_3
0	−0·236	−0·513	−0·171	0·116	0·728	4·84
0·5	−0·390	−0·407	−0·203	0·1432	0·748	4·33
0·6	−0·328	−0·430	−0·242	0·1090	0·514	2·922
0·7	−0·402	−0·461	−0·137	0·1084	0·625	2·948

The gust functions are plotted in Fig. 1 and it can be seen at once that the rate of growth of lift due to gust decreases with increasing Mach number.

The indicial incidence function, which is a non-dimensional lift growth function due to sudden change of incidence is given in the form:

$$\Phi = 1 + b_1 e^{-\beta_1 s} + b_2 e^{-\beta_2 s} + b_3 e^{-\beta_3 s} ; \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The Table 2 gives the values of coefficients b_n and β_n for a range of Mach numbers.

TABLE 2

Indicial Incidence Function (Two-dimensional Flow)

Mach number	b_1	b_2	b_3	β_1	β_2	β_3
0	$-0\cdot165$	$-0\cdot335$	—	$0\cdot090$	$0\cdot600$	—
$0\cdot5$	$-0\cdot352$	$-0\cdot216$	$+0\cdot670$	$0\cdot1508$	$0\cdot744$	$3\cdot780$
$0\cdot6$	$-0\cdot362$	$-0\cdot504$	$+0\cdot715$	$0\cdot1292$	$0\cdot962$	$1\cdot916$
$0\cdot7$	$-0\cdot364$	$-0\cdot405$	$+0\cdot419$	$0\cdot1072$	$0\cdot714$	$1\cdot804$

The incidence functions are plotted in Fig. 2. The effect of Mach number on the incidence function is not so simple as that on the gust function, but for a distance traversed of more than one chord, the rate of lift growth is also decreasing with increasing Mach number. The curves of Fig. 2, which are for subsonic flow, are of similar character to curves of the incidence function for supersonic flow calculated by Heaslet and Lomax and shown in Fig. 11 of Part I.

3. *Gust Load Calculations.*—The gust load function, defined as the ratio of the actual gust load to that when no aircraft response and Wagner effect are present, was calculated using the indicial functions for compressible flow. The assumptions made are identical with those in Part I, thus a direct comparison between gust loads for compressible and incompressible flow is possible. The details of calculation are given in the Appendix. The solution of the integral equation obtained by the Operational Method is very convenient for numerical computation and is an improvement on the Fredholm's equation solution used in Part I.

An example of gust load functions for a range of Mach numbers, calculated for a mass parameter $\mu_g = 60$ and for a sharp-edge gust, $H = 0$ is given in Fig. 3. It can be seen that the rate of growth of the gust load function and its maximum value, which is defined as the gust alleviation factor K, are considerably decreased due to the effect of compressibility.

The calculations of the gust alleviation factor were made for a flat-topped gust (Fig. 4) for a range of gust lengths and aircraft mass parameters, using unsteady lift functions for Mach numbers of $0\cdot5$, $0\cdot6$ and $0\cdot7$.

4. *Results.*—The results of calculations are given in Fig. 4 where the gust alleviation factor is plotted against the aircraft mass parameter for four Mach numbers and for a range of gust lengths. The alleviation factor for Mach number $M = 0$ is replotted from Part I. It can be seen that the Mach number effect is to decrease the value of the gust alleviation factor for the whole range of mass parameters and gust lengths. A better picture of the relative decrease of a gust load due to compressibility is given in Fig. 5, where the ratio of gust factor in compressible flow to that in incompressible flow is plotted against Mach number for a range of mass parameters and gust lengths. In the extreme case of the lightly loaded aircraft and a short gust, the additional alleviation due to air compressibility amounts to 10 per cent. This gain progressively decreases with increasing value of the mass parameter and gust length, but is always more than 5 per cent for a practical range of parameters and Mach number $0\cdot7$.

The indicial lift functions are available at present only up to Mach number of $0\cdot7$, whereas for practical application these functions are needed up to at least $M = 0\cdot9$. A rough estimate of the gust loads at higher Mach numbers can probably be obtained by an extrapolation of the curves in Fig. 5, which should not produce large errors up to the critical Mach number of the aircraft considered.

5. *Comparison with Experiment.*—The effects of compressibility on gust loads were measured experimentally and are reported in Ref. 3. The tests consisted of 21 flights over a course of about 55 miles. All flights were made through clear rough air at an altitude of about 2,500 ft. Each

flight consisted of successive runs over the course at speeds of 200, 350 and 450 m.p.h. A single airplane was used in 14 of the flights (six flights with wing-tip tanks installed and eight flights with wing-tip tanks removed); whereas two airplanes were used in the seven remaining flights.

The flights with tip tanks removed represent different elastic characteristic of the wing from the flights with wing-tip tanks installed. The wing fundamental bending frequency with tip tanks full was 2·4 c.p.s., and with tip tanks empty 5·8 c.p.s. The tests have shown that the effect of aero-elasticity may be neglected for the purpose of the present investigation. Changes in stability with speed for the test aeroplane have been shown by flight tests to be negligible over the Mach number range flown.

The results were presented as miles flown to exceed a given gust velocity calculated assuming (*a*) lift slopes uncorrected for compressibility and (*b*) corrected using finite aspect ratio correction and (*c*) Glauert-Prandtl corrections; the alleviation factor was assumed independent of compressibility. The results were not fully conclusive, but indicate that the best results are obtained when no compressibility correction to the lift slope is applied.

The results of Ref. 3 were recalculated by scaling down the gust velocities in the ratio of gust factor times lift slope in compressible flow to gust factor times lift slope in incompressible flow. A constant gust length of 5 chords was assumed and the lift slope was corrected for compressibility from the finite aspect ratio formula:

$$a_c = a_i \frac{\sqrt{(A^2 + 4)} + 2}{\sqrt{\{A^2(1 - M^2) + 4\}} + 2}.$$

The gust alleviation factor was used as calculated in the present paper, *i.e.*, for two-dimensional case.

The distributions of corrected gust velocities are shown in Figs. 6, 7 and 8 together with gust distributions calculated without any compressibility correction and with Glauert compressibility correction to the lift slope only replotted from Ref. 3. It is not known what value of the gust alleviation factor was assumed in the original calculations, but this fact does not invalidate the analysis, which is based on the relative decrease of gust loads due to compressibility. It is worth mentioning that the gust velocities quoted in this paper are so called 'effective gust velocities' as used in U.S.A. and are numerically different from gust velocities as defined in this country.

From inspection of Figs. 6, 7 and 8 it appears that the application of lift slope and gust alleviation factor corrected for compressibility gives better results than those with no compressibility correction to lift slope and gust factor. The application of compressibility correction to the lift slope alone gives quite erroneous results, as was already shown in Refs. 2 and 3. It can be argued that the gust load calculations as suggested in this paper are still pessimistic (Figs. 6 and 8 as an example), but the error involved is small and can as well be attributed to an error in the assumed value of the lift slope at high Mach numbers.

It is also possible that the two-dimensional compressibility corrections are not strictly applicable to wings of finite aspect ratio. The test aircraft had a wing of aspect ratio 6·35 and inspection of Figs. 6 to 8 seems to indicate that, with increasing Mach number, the increase in the value of the product aK for this aircraft is slightly less than predicted by the present theory.

6. *Concluding Remarks.*—The gust load for a given gust is proportional to the product of gust alleviation factor and lift slope, $K \times a$. The present practice of gust load calculations, assuming compressibility effects on the lift slope only[5], leads to an overestimation, which has been confirmed by experiments[2,3]. It has been shown that the compressibility of the air affects the unsteady lift functions[1] and hence the value of the gust alleviation factor. The calculations of the gust alleviation factor presented in this report show an appreciable decrease in its value with increasing Mach number and this decrease is more noticeable for small values of aircraft mass parameter and short gusts. It appears from the analysis of existing flight records that the calculations of gust loads using values of lift slope and alleviation factor corresponding to a given Mach number should give a satisfactory answer. It is worth mentioning that the above argument applies as well

to the reverse process of calculating gust velocities from measured aircraft accelerations. The gust velocity estimates based on acceleration records of a fast aircraft may show lower gust velocities than those obtained from records of a slow aircraft, if no proper compressibility corrections are applied.

The theoretical calculations of the gust alleviation factor are limited to two-dimensional flow and Mach number of $0 \cdot 7$ only.

The experimental results appear to indicate that for wings of finite aspect ratio the gust loads at high Mach numbers are even slightly less than those calculated using two-dimensional, compressible indicial functions. It is quite feasible that for a given aircraft and altitude the product aK is approximately independent of Mach number. In such a case the gust load can be calculated for any Mach number neglecting the compressibility effects on the lift slope and gust alleviation factor.

LIST OF SYMBOLS

A	Aspect ratio
a	Lift slope (per radian)
a_n	Coefficient in the indicial gust function
b_n	Coefficient in the indicial incidence function
c	Wing chord (ft)
c_n	Coefficient in the gust force function
g	Gravity acceleration (ft sec $^{-2}$)
H	Gust length in wing chords
K	Gust alleviation factor
M	Mach number
S	Wing area (ft^2)
s	Distance travelled by leading edge of wing, measured in chords
u	Vertical gust velocity (ft sec $^{-1}$)
U	Maximum value of gust velocity u (ft sec $^{-1}$)
V	Aircraft speed (ft sec $^{-1}$)
W	Weight of aircraft (lb)
α_n	Coefficient in the indicial gust function and force function
β_n	Coefficient in the indicial incidence function
Φ	Indicial incidence function
Ψ	Indicial gust function
Λ	Gust load function for sharp-edged gust
Λ_H	Gust load function for flat-topped gust of length H, $\Lambda \equiv \Lambda_H = \dfrac{\text{actual gust force}}{\frac{1}{2}\rho V S a U}$;
μ_g	$= \dfrac{2W}{g\rho Sac}$, Aircraft mass parameter
ρ	Air density (slug ft $^{-3}$)
σ	Distance travelled by leading edge of wing in chords; σ is variable when s is kept constant.

REFERENCES

No.	Author	Title, etc.
1	B. Mazelsky and J. A. Drischler ..	Numerical determination of indicial lift and moment functions for a two-dimensional sinking and pitching airfoil at Mach numbers 0·5 and 0·6. N.A.C.A. Tech. Note 2739. July, 1952.
2	R. G. Goodall	Gust loads on high-speed airplane. Air Materiel Command, T–3 Engin, Wright Field, Dayton, Ohio. Eng–51–4515–3. August, 1944.
3	E. T. Binckley and J. Funk	A flight investigation of the effect of compressibility on applied gust loads. N.A.C.A. Tech. Note 1937. August, 1949.
4	J. K. Zbrożek	Gust alleviation factor. Incompressible flow. Part I of this Report.
5	.. ..	A.P. 970, Vol. 2, Leaflet 203/1. Gust loads.
6	A. R. Curtis	On a method of calculating the response of an aeroplane to a vertical gust of known form. A.R.C. 9700. June, 1946.
7	P. Donely	Summary of information relating to gust loads on airplane. N.A.C.A. Tech. Note 1976. November, 1949.

APPENDIX TO PART II

Theory

A.1. *Sharp-edged Gust.*—The equation of motion in terms of gust load function as a function of distance travelled, as developed in Part I of this report

$$A(s) + \frac{1}{\mu_g}\int_0^s \Phi(s-\sigma)\, A(\sigma)\, d\sigma - \Psi(s) = 0 \qquad \text{(II.1)}$$

is solved by my more direct and exact method.

Two methods of solving the equation (II.1) were given in Part I, both giving a solution in the form of a series of integrals. The convergence of these series was rather slow for small values of μ_g and a considerable amount of computational work was involved.

In the present paper the solution of equation (II.1) obtained by an Operational Method is exact and the computational work is much simplified. Use is made of the following theorem[6].

Theorem.—If $\bar{\Phi}(p)$ and $\bar{A}(p)$ are the respective Laplace transforms of $\Phi(s)$ and $A(s)$, then $\Phi(p) \,.\, \bar{A}(p)$ is the transform of

$$\int_0^s \Phi(s-\sigma)\, A(\sigma)\, d\sigma$$

and also of

$$\int_0^s \Phi(\sigma)\, A(s-\sigma)\, d\sigma\,.$$

The Laplace transform of equation (II.1) is then:

$$\bar{A}(p) + \frac{1}{\mu_g}\bar{\Phi}(p)\,\bar{A}(p) - \bar{\Psi}(p) = 0 \qquad \text{(II.2)}$$

and the transform of the load function

$$\bar{A}(p) = \frac{\bar{\Psi}(p)}{1 + \dfrac{1}{\mu_g}\bar{\Phi}(p)}\,. \qquad \text{(II.3)}$$

The approximate exponential expression for the gust function is

$$\Psi(s) = 1 + \sum_{n=1}^{n=3} a_n \, e^{-\alpha_n s} \qquad \text{(II.4)}$$

the transform of which is of the form:

$$\bar{\Psi}(p) = \frac{1}{p} + \sum_{n=1}^{n=3} \frac{a_n}{p + \alpha_n} = \frac{Ap^3 + Bp^2 + Cp + D}{p(p + \alpha_1)(p + \alpha_2)(p + \alpha_3)} . \qquad \text{(II.5)}$$

The transform of the incidence function Φ is identical in form and the denominator of the expression (II.3) is given by:

$$1 + \frac{1}{\mu_g} \bar{\Phi}(p) = \frac{p^4 + Ep^3 + Fp^2 + Gp + H}{p(p + \beta_1)(p + \beta_2)(p + \beta_3)} . \qquad \text{(II.6)}$$

Finally the transform of the gust load function is given by:

$$\bar{\Lambda}(p) = \frac{(Ap^3 + Bp^2 + Cp + D)(p + \beta_1)(p + \beta_2)(p + \beta_3)}{(p + \alpha_1)(p + \alpha_2)(p + \alpha_3)(p^4 + Ep^3 + Fp^2 + Gp + H)} . \qquad \text{(II.7)}$$

Finding the roots of the cubic and quartic in equation (II.7) the transform of the load function can be expressed in the form:

$$\bar{\Lambda}(p) = \sum_{n=1}^{n=7} \frac{c_n}{p + \alpha_n} \qquad \text{(II.8)}$$

and transforming it back into s co-ordinates the gust load function is given by

$$\Lambda(s) = \sum_{n=1}^{n=7} c_n \, e^{-\alpha_n s} . \qquad \text{(II.9)}$$

A.2. *Flat-topped Gust.*—The gust load function for any shape of gust is obtained from the following relationship

$$\Lambda_1(s) = \frac{1}{U} \int_0^s \Lambda(s - \sigma) \frac{du}{d\sigma} \, d\sigma \qquad \text{(II.10)}$$

where u is the variable gust velocity with maximum value equal to U.

For a flat-topped gust

$$u = \frac{U}{H} s \text{ when } 0 < s < H \qquad \text{(II.11)}$$

and the load function is:

$$\Lambda_H(s) = \frac{1}{H} \int_0^s \Lambda(s - \sigma) \, d\sigma = \frac{1}{H} \left\{ \sum_{n=1}^{n=7} \frac{c_n}{\alpha_n} - \sum_{n=1}^{n=7} \frac{c_n}{\alpha_n} e^{-\alpha_n s} \right\} \qquad \text{(II.12)}$$

$0 < s < H$.

For $s > H$ the load function is obtained by the addition of a new gust with reversed sign, starting at the point $s = H$. Thus

$$\Lambda_H = \Lambda_H(s) - \Lambda_H(s - H) . \qquad \text{(II.13)}$$

$s > H$.

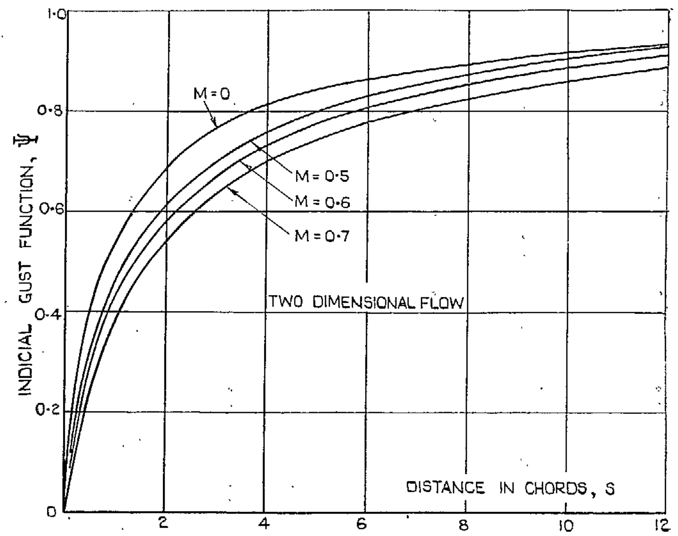


FIG. 1. Unsteady lift functions due to penetration into sharp-edged gust, for different Mach numbers.

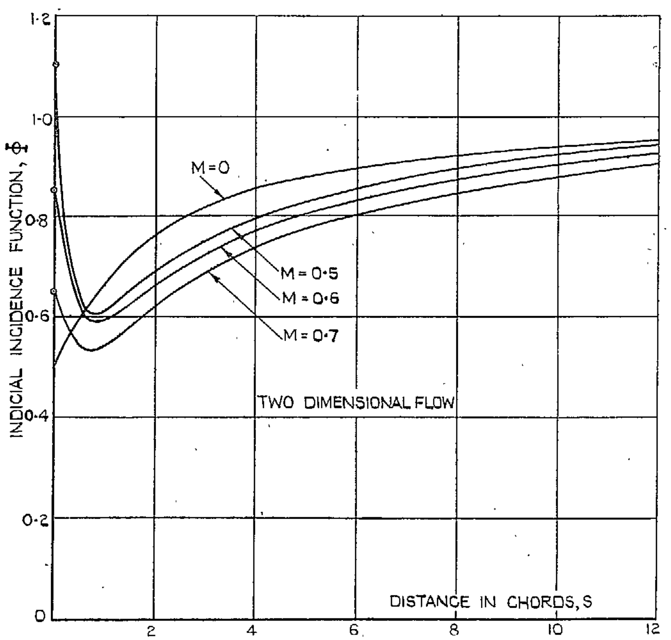


FIG. 2. Unsteady lift function due to sudden change of incidence, for different Mach numbers.

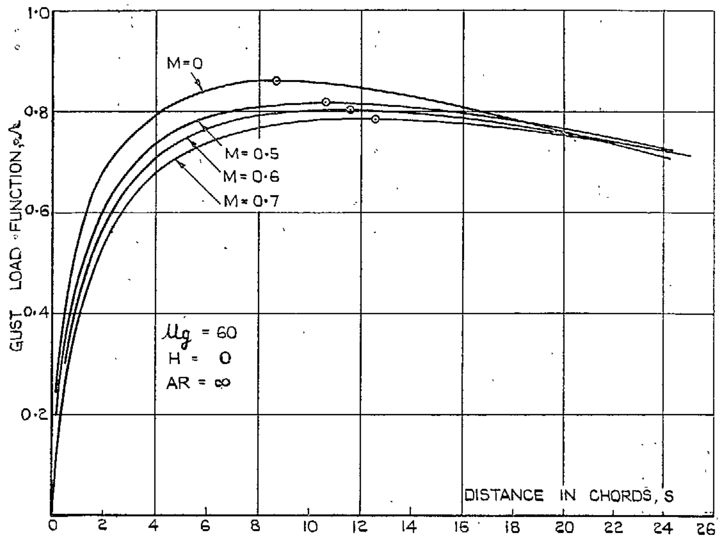


FIG. 3. An example of the Mach number effect on the gust load function due to penetration into sharp-edged gust.

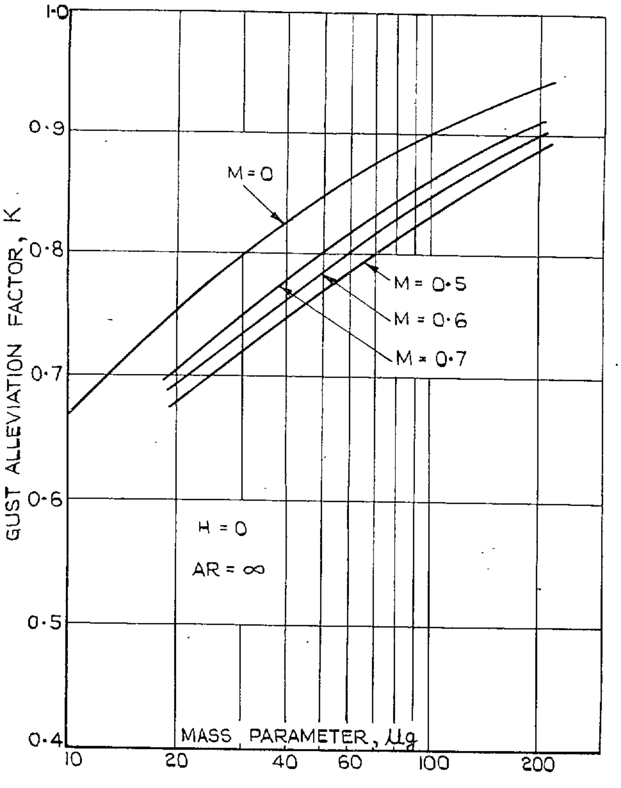


FIG. 4a. Sharp edge gust, H = 0.

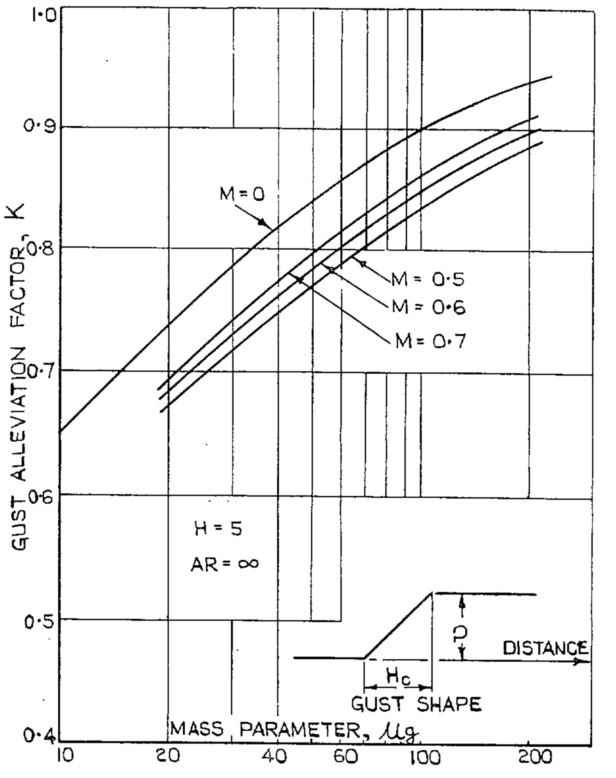


FIG. 4b. Gust length 5 chords, H = 5.

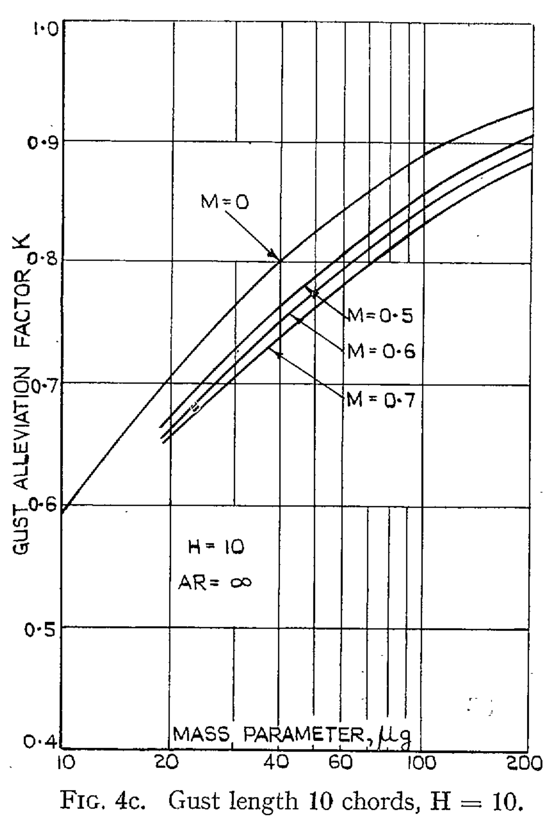


FIG. 4c. Gust length 10 chords, H = 10.

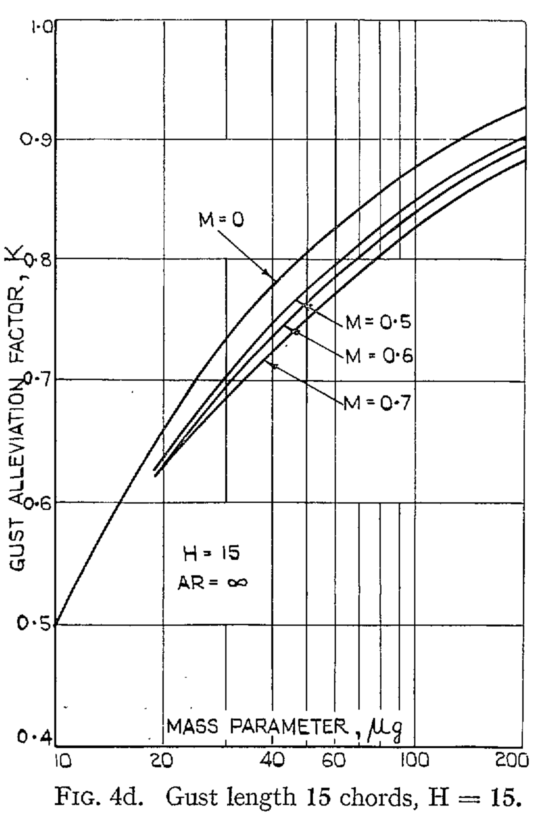


FIG. 4d. Gust length 15 chords, H = 15.

FIGS. 4a to 4d. Gust alleviation factor *vs.* mass parameter for a range of Mach numbers. Flat-topped gust.

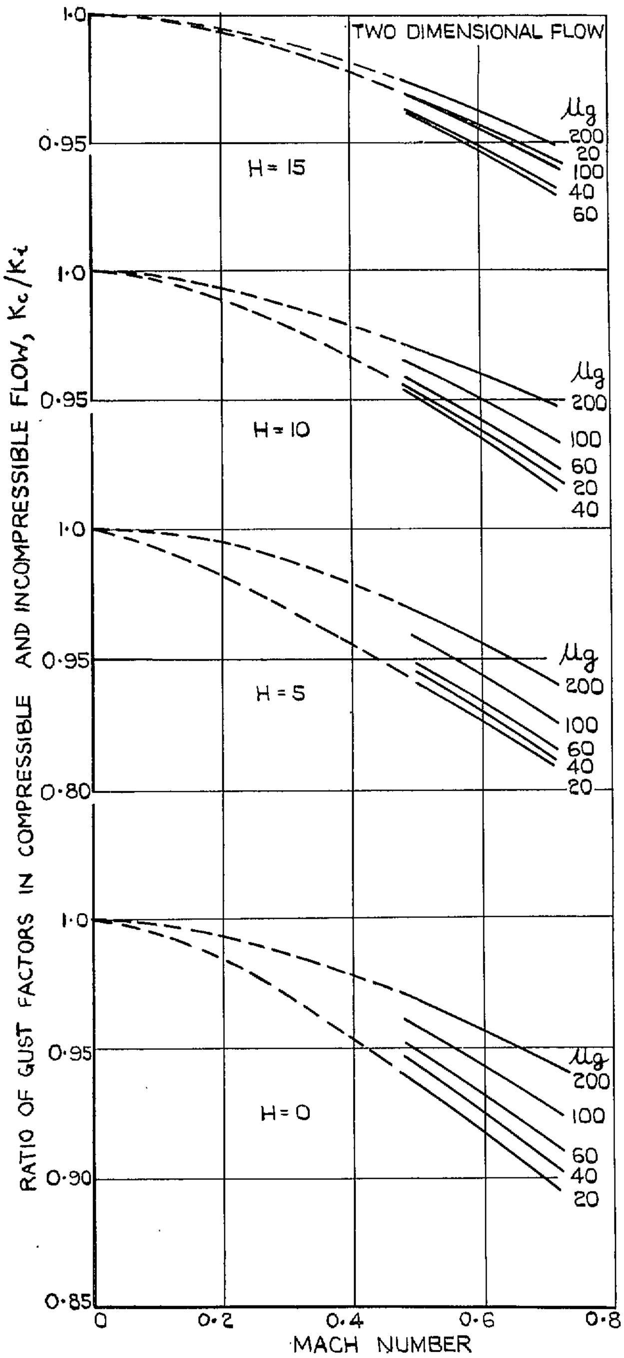
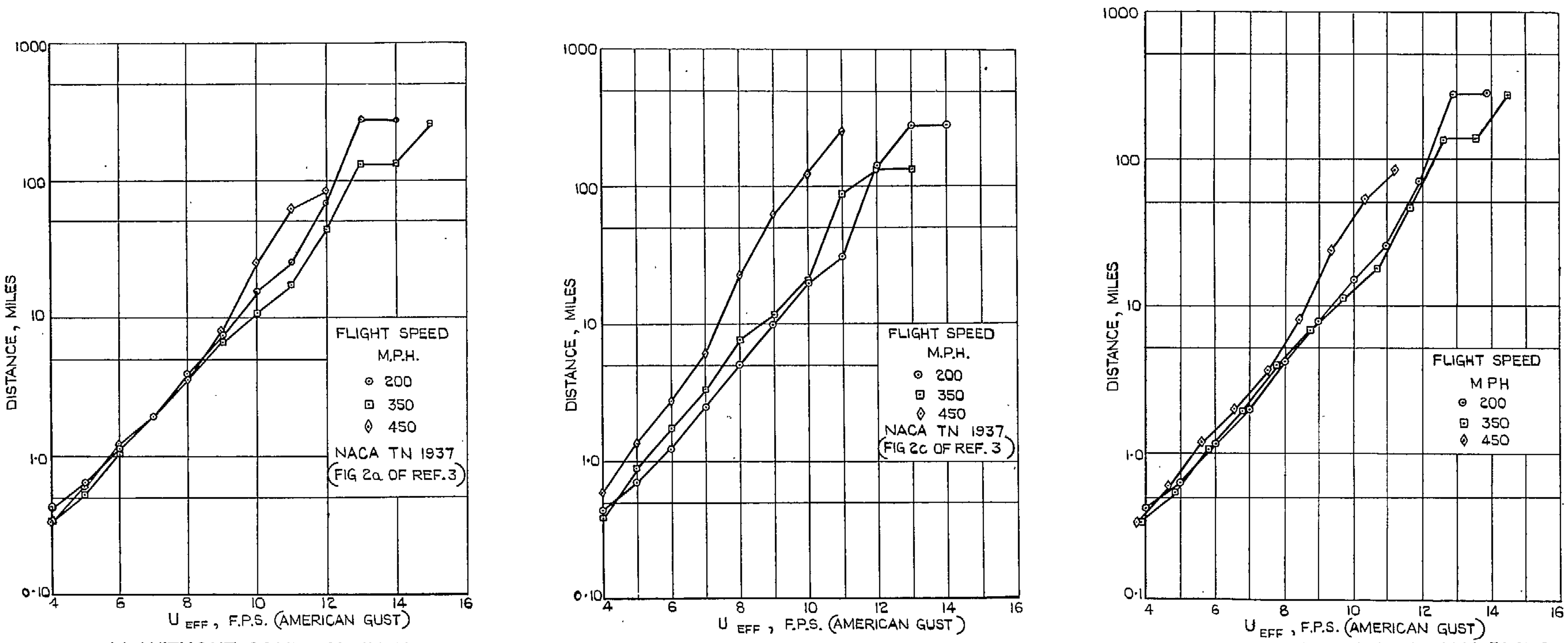


FIG. 5. The relative decrease in gust factor value due to compressibility *vs.* Mach number, for a range of mass parameters and gust lengths.

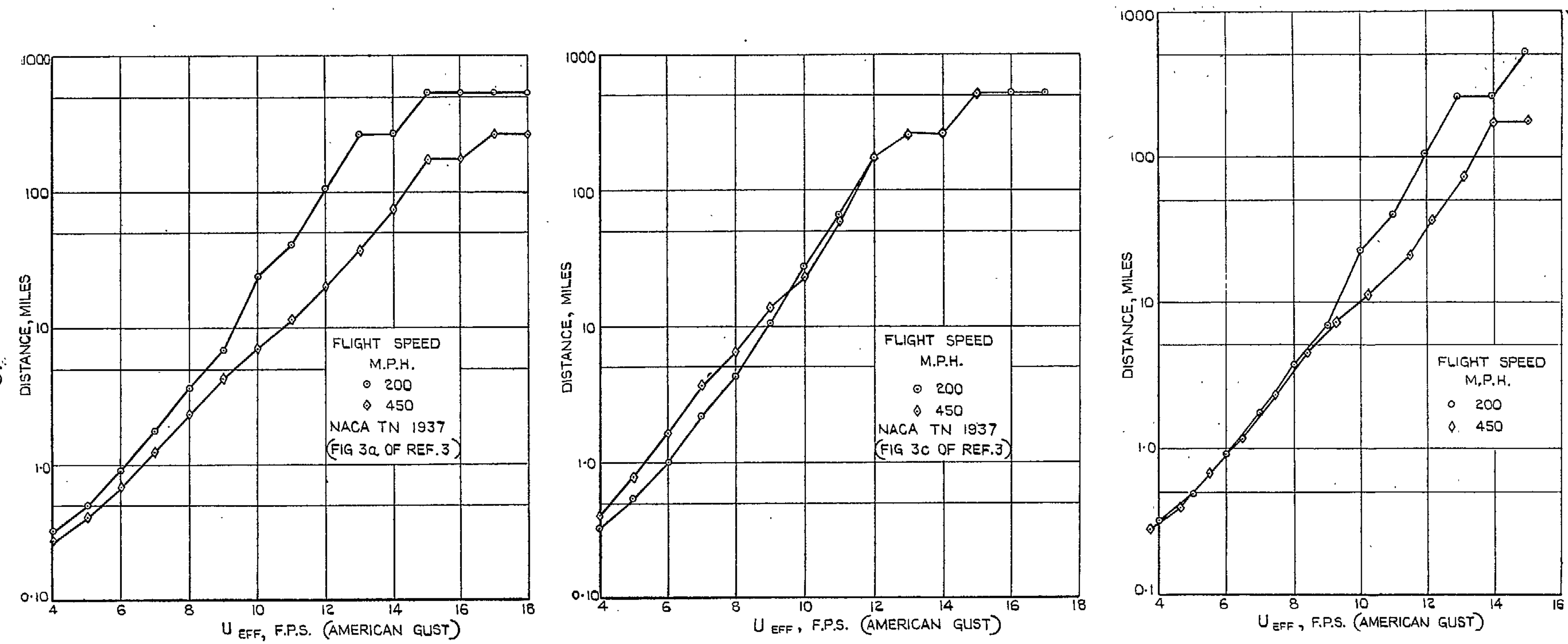


(a) WITHOUT COMPRESSIBILITY CORRECTION.

(b) LIFT SLOPE CORRECTED FOR COMPRESSIBILITY BY GLAUERT FACTOR.

(c) LIFT SLOPE AND ALLEVIATION FACTOR CORRECTED FOR COMPRESSIBILITY.

FIG. 6. Average number of miles to exceed a given gust velocity. Single airplane flights with tip tanks.

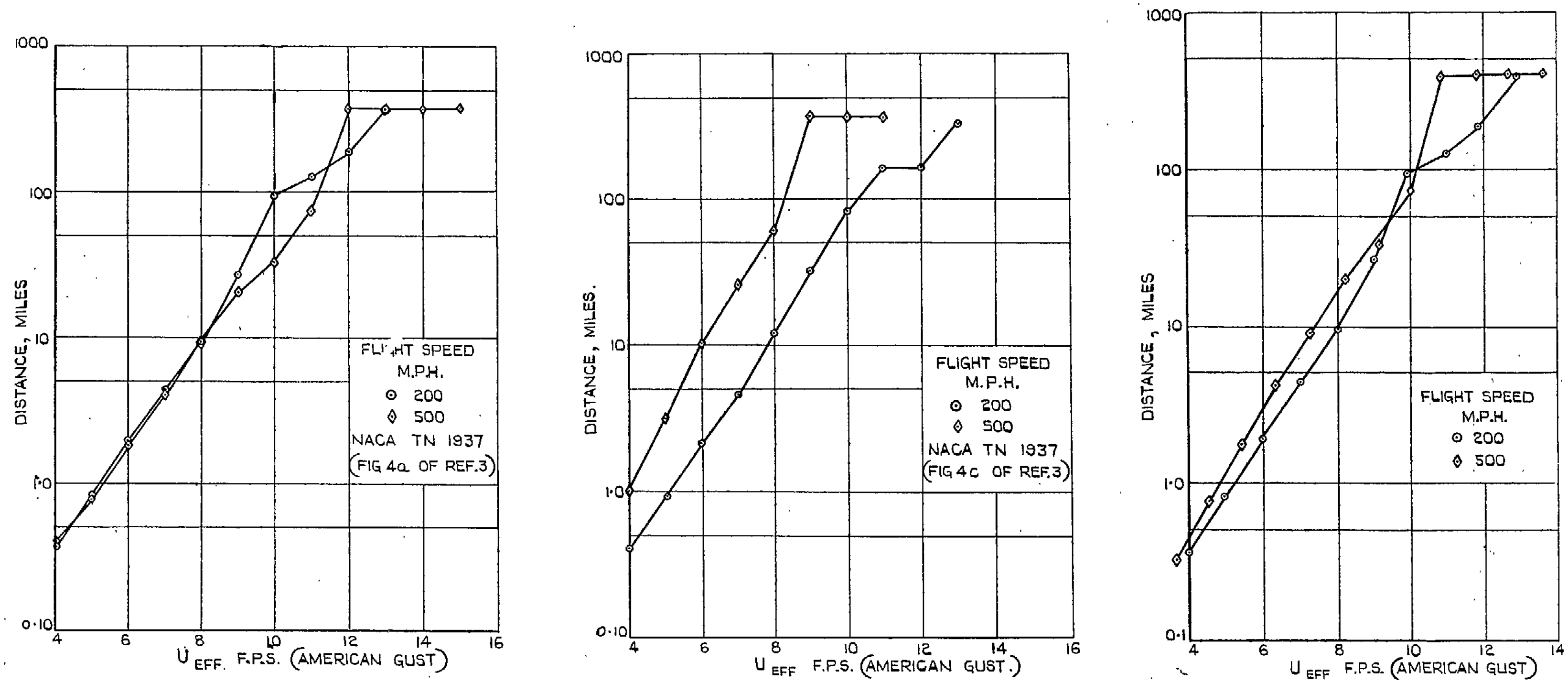


(a) WITHOUT COMPRESSIBILITY CORRECTION.

(b) LIFT SLOPE CORRECTED FOR COMPRESSIBILITY BY GLAUERT FACTOR.

(c) LIFT SLOPE AND ALLEVIATION FACTOR CORRECTED FOR COMPRESSIBILITY.

FIG. 7. Average number of miles to exceed a given gust velocity. Two airplane flights.



(a) WITHOUT COMPRESSIBILITY CORRECTION.

(b) LIFT SLOPE CORRECTED FOR COMPRESSIBILITY BY GLAUERT FACTOR.

(c) LIFT SLOPE AND ALLEVIATION FACTOR CORRECTED FOR COMPRESSION.

FIG. 8. Average number of miles to exceed a given gust velocity. Single airplane flights; wing-tip tanks removed.

PART III*

Gust Loads on Swept Wings
(based on NACA Gust-Tunnel Tests)

Summary.—The gust loads on swept wing aircraft can be split up into two parts, (*a*) gust load neglecting pitching motion and (*b*) correction to gust load due to pitching. Gust loads neglecting pitching can be estimated using the gust alleviation factor of Part I, taking the appropriate aircraft mass parameter and wing aspect ratio and replacing the actual gust length H in wing chords, by an effective gust length $H_{eff} = H + \beta$, where β is the sweep of wing tip expressed in chords. The gust loads computed by this method give satisfactory agreement with gust-tunnel results.

For an aeroplane with high wing loading and small aspect ratio the overall effect of wing sweep on gust loads is small.

---

1. *Introduction.*—The value of the gust alleviation factor for straight-wing aircraft can be satisfactorily estimated using Part I which gives the results of gust factor calculations covering ranges of aircraft mass parameter, wing aspect ratio and gust length. The aircraft pitching due to the gust was neglected. It has been realized for a long time that the gust theory as developed for a straight wing is becoming unsuitable for swept wings, and some form of gust load prediction for swept wings is needed. Unfortunately, no precise predictions are possible because no mathematical theory of unsteady lift on swept wings is as yet available, but it was felt that even a method capable of giving only approximate results would be useful. The method presented in this paper has been checked against N.A.C.A. Gust-Tunnel results and the present method gives results at least as good as the methods proposed by the N.A.C.A., and has the advantage of being more consistent and logical.

2. *Theoretical Considerations.*—For simplification of analysis it is proposed to split the gust effect on the aircraft into two parts, (i) the gust effect neglecting pitching motion, and (ii) the additional load imposed on the aircraft due to the pitching motion.

To estimate the gust load, neglecting the pitching degree of freedom, the following assumptions were made:

(*a*) The gust velocity is uniform across the span of the aircraft at any instant and is in the vertical direction

(*b*) The aeroplane can rise vertically, but does not pitch

(*c*) The aeroplane before entering the gust is in steady level flight

(*d*) The forward speed of the aeroplane is not changed during the action of the gust

(*e*) The wing is of constant chord, equal to the mean aerodynamic chord of the actual wing, $\bar{c}$

(*f*) The aircraft is rigid. The above assumptions are identical with those made in Part I, further assumptions are relevant to the swept wing

(*g*) To allow for gradual penetration of the swept wing into the gust, the strip theory is used

(*h*) The lift growth at each strip is independent of other strips and is a function only of the aspect ratio of the whole wing.

The last assumption has no theoretical justification. It has been used because at present no theory exists which would give even an indication of the effect of angle of sweep on the unsteady lift functions. However, the satisfactory comparison of calculated and measured gust loads (Table 1) seems to indicate that this assumption is not very far wrong.

---

* R.A.E. Tech. Note Aero. 2200 dated November, 1952, received 30th January, 1953.

With the above assumptions, the gust loads can be estimated from the results given in Part I simply by increasing the actual gust length H by the sweep coefficient β, i.e., the effective gust length for the aeroplane with swept wings is

$$H_{\text{eff}} = H + \beta$$

where the dimensionless coefficient of sweep β is defined by

$$\beta = \frac{b \tan \Lambda}{2\bar{c}} \qquad \text{(Fig. 1)}$$

and $\quad b$ is wing span (ft)

$\bar{c}$ mean aerodynamic chord (ft)

Λ angle of sweep measured at quarter-chord line.

β is always positive irrespective of whether the wing is swept forward or back.

3. *Comparison with Experimental Results.*—The gust loads on the aircraft with different degrees of sweep were measured experimentally by the N.A.C.A. in a gust tunnel. The angle of sweep was varied from − 45 deg (forward) to 60 deg back[2, 3, 4, 5], and one series of tests was made on a delta-wing model[6]. The weak point of the above tunnel tests is that all the models were very light, having the gust mass parameter of the order $\mu_g = 10$. This produces large response of the model and hence the corrections due to pitching are fairly large.

In order to correlate the theoretical and experimental values of the aircraft accelerations when flying through gusts, the experimental values of the accelerations were corrected to no pitch conditions using approximate formulae of Ref. 2 (*see* section 4).

The theoretical estimates of gust loads in Refs. 2 to 6 were made assuming strip theory and unsteady lift functions for two-dimensional flow. In order to obtain satisfactory agreement between experiment and theory it was suggested to use: 'the slope of the lift curve of the equivalent straight wing multiplied by the cosine of the angle of sweep rather than the steady flow slope' (Ref. 2, page 9). The above assumption gave good agreement except for delta-wing models, where it was suggested to use the 'slope of the lift curve derived from the simple aspect ratio relation $6A/(A + 2)$' (Ref. 6, page 5).

Calculations of the gust loads were repeated in the present paper, using the gust alleviation factor of Part I and using the appropriate aspect ratio and effective gust length $H_{\text{eff}} = H + \beta$. The gust mass parameters and gust loads were calculated using the value of lift slope obtained from a wind tunnel under steady conditions. In some cases extensive interpolation of curves in Part I was necessary as the aspect ratio of the models varied between 1·46 and 6. The results of the calculations are shown in Table 3 together with the N.A.C.A. theoretical and experimental values. To give a better picture of the magnitudes involved, Fig. 2 was prepared, where the ratio of theoretical to experimental gust load is plotted against angle of sweep. It seems from Fig. 2 that the differences between theoretical and experimental values of gust loads are not related to the angle of sweep and the scatter of points could well be attributed to the experimental errors.

The agreement between the experimental results and the values calculated by the method suggested in the present paper is of the same order as the agreement with the theoretical values predicted by the N.A.C.A. It has to be remembered, however, that the agreement of the N.A.C.A. calculations was obtained by a suitable adjustment of the value of lift slope used in their calculations. The Table 1 below gives a mean value of $(\Delta n)_{\text{theory}}/(\Delta n)_{\text{exper.}}$ and mean error in $(\Delta n)_{\text{theory}}/(\Delta n)_{\text{exper.}}$ as calculated by the two methods.

The initial values of pitching response due to vertical gust, before the tail enters gust, depend upon two factors:

(*a*) *The position of wing with respect to c.g.*—For tailed aircraft the wing aerodynamic centre is usually in front of c.g., and the initial response is nose up, increasing the gust load. For a tailless configuration the wing aerodynamic centre is behind the c.g. and the initial pitching response is nose down.

(*b*) *The amount of wing sweep.*—The gradual penetration of swept wing always produces nose-up pitch, increasing the gust loads.

The penultimate row in Table 2 shows the increment of the gust load due to sweep alone. It was assumed that the 4 per cent correction for an unswept wing, which is due to wing position with respect to c.g., is applicable for the other models. This is not entirely correct because of the variation in c.g. position for different models tested. However, there is some indication that the sweep increases the gust loadings by 6 to 7 per cent. This increment in load is independent of the angle of sweep, but it has to be remembered that the sweep coefficient β is roughly constant for all models. The measured correction to gust load due to pitching and sweep compares very well with the unpublished theoretical calculations by Barrett. From his calculations the value of this correction is about 7 per cent for $\mu_g = 11$, $\beta = 2$, $i_B = 0 \cdot 5$ and $m_q = -0 \cdot 5$. The value of i_B is assumed, as no pitching moments of inertia of models tested are given in the mentioned references.

For the value of $\mu_g = 50$, more representative of modern aeroplane and altitude of flight, and for the same values of β, i_B and m_q the correction to gust load due to pitching due to sweep is of the order of 2 per cent, which indicates, as was already suggested, that for a full-scale aeroplane the pitching effects are of secondary importance.

The results of delta-wing model test shown in the last column of Table 2 point to a very small overall effect of pitching. This is not surprising, as a delta wing has its aerodynamic centre behind c.g., and therefore the wing position and wing sweep effects are of opposite signs.

5. *Conclusions.*—(*a*) The gust load on a swept-wing aircraft, neglecting pitching motion, can be estimated with a good degree of approximation using the gust alleviation factor as given in Part I. The gust mass parameter and wing aspect ratio have to be taken into account and the lift slope used is the lift slope measured in wind tunnel tests under steady conditions. The gradual penetration of swept wings into the gust is taken into account by increasing the actual gust length H by the sweep parameter $\beta = (b \tan \Lambda)/2\bar{c}$. This effect of sweep, which incidentally decreases the gust load, is however small, specially for heavier aircraft.

(*b*) For the models used in gust-tunnel experiments ($\mu_g \simeq 10$) the gust load increment due to pitch is appreciable (about 10 per cent). For full scale aircraft, with mass parameter of the order of $\mu_g \simeq 50$ this increment is substantially smaller (say of the order of 4 per cent). This increment is in the sense of increasing the load.

(*c*) The effect of static stability and wing aerodynamic centre position on the gust loads is not investigated and further experimental and analytical study is desirable.

(*d*) The overall effect of sweep on the gust load, being a sum of gradual penetration effect (negative) and pitching effect (positive) is fairly small for heavily loaded and low aspect ratio aircraft. In these cases it can be said that the main parameters affecting the gust loads on a swept wing aircraft are the same as for straight-wing aircraft, namely mass parameter and aspect ratio; the angle of sweep being of only secondary importance. For light aircraft and for aircraft with high aspect ratio and large angle of sweep, the effect of sweep can modify the gust loads appreciably, but then the problem is complicated by the presence of dynamic overshoot and aeroelastic phenomena and the present analysis, assuming a rigid aeroplane, is not applicable.

TABLE 3

Reference No.	3	3	2	2	4	4	4	2	2	5	6	6
Angle of sweep, Λ (deg) (as defined in N.A.C.A. Reps.)	—45	—45	0	0	30	30	30	45	45	60	Delta 60 $\Lambda_{1/4} =$ 52 deg	Delta 60 $\Lambda_{1/4} =$ 52 deg
β	1·32	1·32	0	0	1·45	1·45	1·45	1·44	1·44	1·41	1·5	1·5
Aspect ratio	3·0	3·0	6·0	6·0	4·44	4·44	4·44	2·99	2·99	1·46	2·33	2·33
W/S (lb/sq ft)	2·0	2·0	1·64	1·64	1·61	1·61	1·61	1·61	1·61	1·56	1·69	1·69
$\bar{c}$ (ft)	1·425	1·425	1·037	1·037	1·16	1·16	1·16	1·478	1·478	2·06	1·73	1·73
a	2·66	2·66	4·41	4·41	3·15	3·15	3·15	2·58	2·58	1·95	2·4	2·4
μ_g	13·7	13·7	9·3	9·3	11·45	11·45	11·45	10·74	10·74	10·1	10·6	10·6
H	0	9	0	9	0	5	9	0	9	0	0	6·5
K (from Ref. 1) gust allev. factor ..	0·834	0·688	0·744	0·612	0·783	0·709	0·638	0·808	0·632	0·846	0·815	0·682
Δn_1 exper. no pitch	1·07	0·89	2·03	1·67	1·72	1·59	1·37	1·34	1·03	1·18* (1·08)	1·27	1·07
Δn_2 theory present calc.	1·157	0·963	2·11	1·73	1·61	1·46	1·312	1·39	1·09	1·11	1·22	1·02
Δn_3 theory, N.A.C.A.	1·09	0·91	1·96	1·65	1·78	1·60	1·56	1·35	1·05	1·00	1·27**	1·10**
$\Delta n_2/\Delta n_1$	1·08	1·08	1·04	1·035	0·935	0·92	0·96	1·035	1·06	0·94 (1·025)	0·96	0·95
$\Delta n_3/\Delta n_1$	1·02	1·02	0·965	0·99	1·035	1·005	1·14	1·01	1·02	0·85 (0·93)	1·00	1·03

* For this particular test seems that the tail contribution to the normal load is of the order of 0·1g. Figure in brackets shows the results corrected for this tail contribution.

** Load on delta wing calculated assuming straight wing and lift slope 3·23.

LIST OF SYMBOLS

A		Aspect ratio
a		Lift slope (per radian)
b		Wing span (ft)
$\bar{c}$		Mean chord (ft)
g		Gravity acceleration (ft sec $^{-2}$)
H		Gust lengths of flat-topped gust, in chords
H_{eff}	$=$	$H + \beta$, Effective gust length, as used for swept wings
K		Gust alleviation factor $K = \dfrac{\text{maximum actual force}}{\frac{1}{2}\rho VSaU}$
n		Vertical acceleration in 'g'
S		Wing area (ft^2)
U		Maximum gust velocity (ft sec $^{-1}$)
V		Forward velocity of aeroplane (ft sec $^{-1}$)
W		Weight of aeroplane (lb)
β	$=$	$(b \tan \Lambda)/2\bar{c}$, Dimensionless coefficient of wing sweep
Λ		Angle of sweep measured at quarter-chord line
μ_g		$2W/g\rho Sa\bar{c}$, Gust mass parameter
ρ		Air density (slug ft $^{-3}$)
θ		Angle of pitch (radians)

REFERENCES

No.	*Author*	*Title, etc.*
1	J. K. Zbrożek	Gust alleviation factor. Incompressible flow. Part I of this report.
2	H. B. Pierce	Test of a 45 deg sweptback wing model in the Langley Gust-Tunnel. N.A.C.A. Tech. Note 1528. February, 1948.
3	H. B. Pierce	Gust-Tunnel investigation of a 45 deg swept forward wing model. N.A.C.A. Tech. Note 1717. October, 1948.
4	T. D. Reisert	Gust-Tunnel investigation of a wing model with semi-chord line swept back 30 deg. N.A.C.A. Tech. Note 1794. January, 1949.
5	H. B. Pierce	Gust-Tunnel investigation of a wing model with semi-chord line swept back 60 deg. N.A.C.A. Tech. Note 2204. October, 1950.
6	H. B. Pierce and Slaton L. Johns ..	Gust-Tunnel investigation of a delta-wing model with the leading edge swept back 60 deg. N.A.C.A. Research Memo. L52B04, TIB/2978. A.R.C. 15,116. April, 1952.

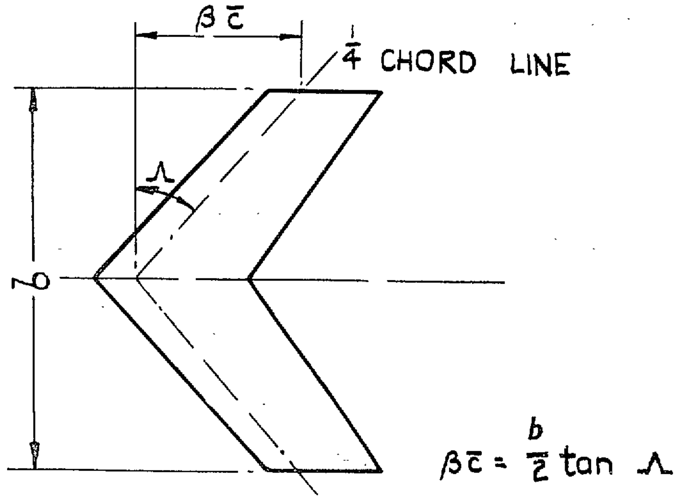


FIG. 1. Definition of sweep coefficient.

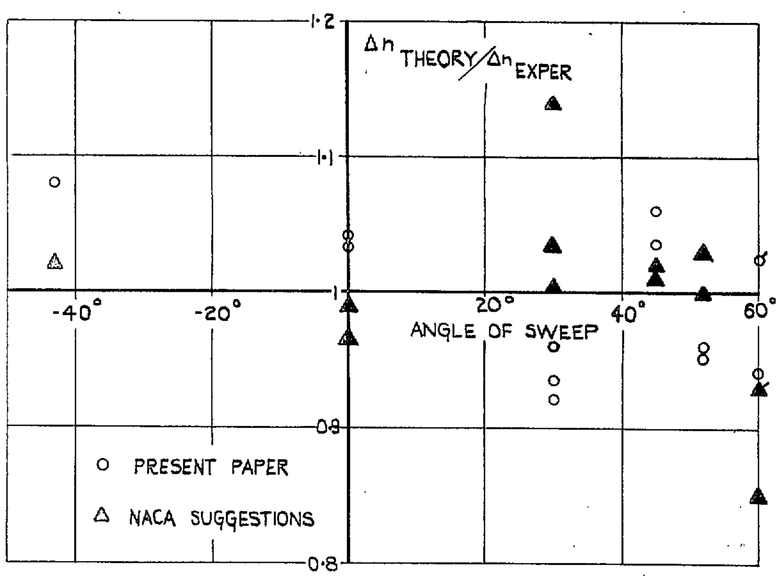


FIG. 2. The ratio of theoretical to experimental gust load for different angles of sweep.

(4074) Wt. 19/8411 K.7 1/58 Hw.

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s*. (52*s*.).
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s*. (65*s*.)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s*. (52*s*.)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 63*s*. (65*s*.)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s*. (77*s*.)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s*. 6*d*. (49*s*. 6*d*.)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80*s*. (82*s*.)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90*s*. (92*s*. 9*d*.)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84*s*. (86*s*. 6*d*.)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84*s*. (86*s*. 6*d*.)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130*s*. (132*s*. 9*d*.)
Vol. II. Aircraft, Airscrews, Controls. 130*s*. (132*s*. 9*d*.)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130*s*. (132*s*. 6*d*.)
Vol. IV. Stability, Structures, Wind Tunnels, Wind Tunnel Technique. 130*s*. (132*s*. 6*d*.)

Annual Reports of the Aeronautical Research Council—

1937 2*s*. (2*s*. 2*d*.) 1938 1*s*. 6*d*. (1*s*. 8*d*.) 1939–48 3*s*. (3*s*. 5*d*.)

Index to all Reports and Memoranda published in the Annual Technical Reports, and separately—

April, 1950 - - - - - R. & M. 2600 2*s*. 6*d*. (2*s*. 10*d*.)

Author Index to all Reports and Memoranda of the Aeronautical Research Council—

1909—January, 1954 R. & M. No. 2570 15*s*. (15*s*. 8*d*.)

Indexes to the Technical Reports of the Aeronautical Research Council—

December 1, 1936—June 30, 1939	R. & M. No. 1850	1*s*. 3*d*. (1*s*. 5*d*.)
July 1, 1939—June 30, 1945	R. & M. No. 1950	1*s*. (1*s*. 2*d*.)
July 1, 1945—June 30, 1946	R. & M. No. 2050	1*s*. (1*s*. 2*d*.)
July 1, 1946—December 31, 1946	R. & M. No. 2150	1*s*. 3*d*. (1*s*. 5*d*.)
January 1, 1947—June 30, 1947	R. & M. No. 2250	1*s*. 3*d*. (1*s*. 5*d*.)

Published Reports and Memoranda of the Aeronautical Research Council—

Between Nos. 2251–2349	R. & M. No. 2350	1*s*. 9*d*. (1*s*. 11*d*.)
Between Nos. 2351–2449	R. & M. No. 2450	2*s*. (2*s*. 2*d*.)
Between Nos. 2451–2549	R. & M. No. 2550	2*s*. 6*d*. (2*s*. 10*d*.)
Between Nos. 2551–2649	R. & M. No. 2650	2*s*. 6*d*. (2*s*. 10*d*.)
Between Nos. 2651–2749	R. & M. No. 2750	2*s*. 6*d*. (2*s*. 10*d*.)

Prices in brackets include postage

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London W.C.2; 423 Oxford Street, London W.1; 13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; 109 St. Mary Street, Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast, *or through any bookseller.*

S.O. Code No. 23–2970

R. & M. No. 29