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Theory of Aerofoil Spoilers

By

L. C. Woods
(New Zealand Scientific Defence Corps,
at present seconded to the Aerodynamics Division of the N.P.L.)

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Summary.—A mathematical theory of aerofoil spoilers† in two-dimensional subsonic flow is presented. Equations are given for load distributions, lift, drag, moments and hinge moments produced by spoiler-flap combinations. The theory is developed for a spoiler in a general position but the trailing-edge spoiler receives special attention. For this important case the theory gives good agreement with experiment, but in the more general case, because of uncertainty about the pressure distribution on the aerofoil to the rear of the spoiler, the agreement is not as good.

NOTATION

(x, y)		The physical plane
z	$=$	$x + iy, i = \sqrt{(-1)}$
n, s		Distances measured normal to and along a streamline respectively
(q, θ)		Velocity vector in polar co-ordinates
ρ, ρ_0		Local and stagnation densities respectively
∞		As a suffix to denote values at infinity
U	$=$	q_∞
M		Local Mach number
β	$=$	$(1 - M^2)^{1/2}$
(ϕ, ψ)		Plane of equipotentials (ϕ = constant) and streamlines (ψ = constant), where

$$d\phi = q\,ds,\ d\psi = \frac{\rho}{\rho_0} q\,dn \qquad (1)$$

ϕ_0, ϕ_1		Values of ϕ at the points of flow separation A and G (Fig. 1)
r		Defined by

$$r = \int_{q=U}^{q} \beta\, d(\log U/q) \qquad (2)$$

† Projections on the aerofoil surface causing flow separation.
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f	Defined by $f = r + i\theta$	(3)
m	,, ,, $m = \beta \dfrac{\rho_0}{\rho}$	(4)
w	,, ,, $w = \phi + im_\infty \psi$	(5
λ	,, ,, $\sin \frac{1}{2}\lambda = \dfrac{\sqrt{\phi_1} - \sqrt{\phi_0}}{\sqrt{\phi_1} + \sqrt{\phi_0}}$	(6)
a	,, ,, $4a = \frac{1}{4}(\sqrt{\phi_1} + \sqrt{\phi_0})^2$	(7)
(η, γ)	Elliptic co-ordinates defined by $w = 4a\,(i \sinh \frac{1}{2}\zeta + \sin \frac{1}{2}\lambda)^2$, $\zeta = \eta + i\gamma$, ... so that the aerofoil surface is $\eta = 0$, and the front stagnation point is at $\eta = 0, \gamma = \lambda$. (*See* Fig. 1b)	(8)
C_p =	$(p - p_\infty)/\frac{1}{2}\rho U^2$, the pressure coefficient	
C_{p0}, C_{pw}, C_{ps}	Pressure coefficients on the aerofoil due to (i) aerofoil alone, (ii) the portion of the wake behind the *trailing edge* alone, and (iii) spoiler-flap combination alone, respectively ; thus the total-pressure coefficient on the aerofoil is $C_p = C_{p0} + C_{pw} + C_{ps}$	(9)
$C_{p\sigma}$	The value of C_{ps} on the aerofoil behind the spoiler	
$\tilde{C}_p$	The change in the pressure coefficient at the trailing edge due to a trailing edge spoiler	
ΔC_p	Jump in C_p across the aerofoil surface, or load coefficient	
C_L, C_D	Lift and drag coefficients	
C_m, C_H	Moment total coefficient about leading edge and hinge-moment coefficient	
α, α'	Absolute incidence (measured from no-lift position) and incidence of the front part of the chord respectively	
c	Chord length	
h	Spoiler height	
ξ, ξ_1	Deflection angles of flap and spoiler respectively	
Ec	The flap chord	
E_1 =	ϕ_1/ϕ_0	
δ^*	Boundary-layer displacement thickness occurring at the spoiler position when $h = 0$, *i.e.*, the spoiler is absent	
ε	Is defined by $\varepsilon = 2\xi_1/\pi(1 + \beta_\infty)$	(10)
$_0$	As a suffix to denote values when the incidence, flap deflection, and spoiler height are all zero.	

1. *Introduction.*—In view of the present interest in the use of spoilers as control devices—either alone[3] or in conjunction with flaps[1,2]—a mathematical account of the effect of these spoilers on ΔC_p, C_L, C_D, C_m and C_H is of some practical value. The calculations of these quantities given in this report are based on an extension of the Helmholtz theory of infinite constant-pressure wakes in incompressible flow to infinite varying-pressure wakes in subsonic compressible flow, recently developed by the author[4]. The Helmholtz constant-pressure wake is physically unrealistic in that with increasing distance downstream the displacement thickness of the wake tends to infinity, whereas of course it should be constant and equal to $cC_D/2$ (Ref. 5).

One of the difficulties with separating flows is the problem of determining how the pressure varies in the wake behind the separation points. The possibility of flow reattachment behind the spoiler (in the interval BG in Fig. 1a,) which is likely with small spoilers, particularly when the flap is deflected in the same direction as the spoiler, complicates matters. Fortunately, in the important case of trailing-edge spoilers, this difficulty is unimportant as the wake pressure makes no contribution to the chordwise loading.

To fix ideas consider the case shown in Fig. 1a. The flow is assumed to separate at points A (the end of the spoiler) and G (usually, but not necessarily, the trailing edge). The front stagnation point is at D, and the shaded surface ABCDEFG can be conveniently termed the 'wetted surface.' The pressures on the streamlines bounding the wake (shown dotted) must be deduced from some plausible assumption. When this has been done the (mixed) boundary conditions are that the shape of the wetted surface is known and the pressures along the separating streamlines are known. For the chordwise pressure distribution behind spoilers we shall make the assumption that the pressure *change* due to the spoiler is constant between the spoiler and the trailing edge, *i.e.*, that $C_{p\sigma}$ is constant. Some experimental evidence supporting this assumption appears in section 5. In the wake downstream of the trailing edge it will be assumed that the pressure is the same at opposite points on the separating streamlines. With this assumption it is obvious that this portion of the wake contributes only a symmetrical term (C_{pw}) to the pressure distribution over the aerofoil, so that as far as ΔC_p, C_L, C_m and C_H are concerned it can be ignored. A likely law of variation of pressure along the streamlines bounding the wake downstream of the trailing edge is discussed in the Appendix, where it is used to determine the value of C_{pw} due to a trailing-edge spoiler. While this theory is in fair agreement with experiment it is relegated to the Appendix because the symmetrical term C_{pw} makes no contribution to lift and moments.

Now it has been shown[12] that a good approximation to the differential equation of compressible subsonic flow is obtained by putting $m = m_\infty$. Then we find that

$$\frac{\partial^2 f}{\partial \phi^2} + \frac{1}{m_\infty^2}\frac{\partial^2 f}{\partial \psi^2} = 0 ,$$

so that from (5), f is approximately an analytic function of w. From this result, it is shown in Ref. 4 that

$$f(\zeta) = i\theta(-\pi) - \frac{1}{\pi}\int_{\gamma^*=-\pi}^{\pi} \log \frac{\sin \frac{1}{4}(\gamma^* + i\zeta)}{\cos \frac{1}{4}(\gamma^* - i\zeta)}\, d\theta(\gamma^*)$$

$$- \frac{\cosh \frac{1}{2}\zeta}{2\pi}\int_0^{-\infty}\left\{\frac{r_+(\eta^*)}{\cosh \frac{1}{2}\eta^* + i \sinh \frac{1}{2}\zeta} + \frac{r_-(\eta^*)}{\cosh \frac{1}{2}\eta^* - i \sinh \frac{1}{2}\zeta}\right\} d\eta^*, \qquad (11)$$

where $\theta(\gamma^*)$ is the flow direction on the wetted surface, and $r_+(\eta^*)$ and $r_-(\eta^*)$ are the values of r on the upper and lower edges of the wake respectively. The ζ-plane, in which this result is calculated, is shown in Fig. 1b.

Suppose θ is measured from an initial direction such that $\theta_\infty = 0$, then since by definition (equation (2)) $r_\infty = 0$, it follows that $f_\infty = 0$. From equations (8) and (11) it is found that this result implies

$$\int_{-\pi}^{\pi} \theta(\gamma^*)\, d\gamma^* = \int_{0}^{-\infty} (r_+ - r_-)\, d\eta^* . \qquad (12)$$

One final general equation required in the subsequent theory is (from equation (1))

$$S = \int_{-\pi}^{\gamma} \frac{d\phi}{q} ,$$

where S is the perimeter distance measured round the perimeter surface from A (*see* Fig. 1a). On this surface ($\eta = 0$) equation (8) becomes

$$\phi = 4a(\sin \tfrac{1}{2}\gamma - \sin \tfrac{1}{2}\lambda)^2 , \qquad (13)$$

so that the equation for S can be written

$$\frac{SU}{4a} = \int_{-\pi}^{\gamma} \frac{U}{q} (\sin \tfrac{1}{2}\lambda - \sin \tfrac{1}{2}\gamma) \cos \tfrac{1}{2}\gamma \, d\gamma . \qquad (14)$$

2. *The Pressure Distribution due to a Spoiler-Flap Combination.*—First consider the contribution from the wake terms in (11) to the *increment* $f - f_0$ due to the spoiler-flap combination (f_0 is the value of f when $\xi = h = \alpha' = 0$). As discussed in the Introduction it will be assumed that the pressure increment is constant on the separated streamline from A to G′ (Fig. 1a), while the pressure on each streamline downstream from the trailing edge can be put equal to its value at infinity without affecting the load distribution. Thus from (2) and Bernoulli's equation we can write

$$r_+ = 0, \quad (0 \leqslant \eta \leqslant -\infty), \quad r_- = \begin{array}{l} -K, (0 \leqslant \eta \leqslant -k) \\ \qquad\qquad\qquad (k > 0) , \\ 0, (-k \leqslant \eta \leqslant -\infty) \end{array} \qquad (15)$$

where K is independent of η, and k is the value of $-\eta$ on $\gamma = -i\pi$ opposite the trailing edge. K is a function of h which must clearly vanish when $h = 0$. From (8) on the separation streamlines $\zeta = \eta_+ + i\pi$, $\zeta = \eta_- - i\pi$,

$$\phi_\pm = 4a\{\cosh \tfrac{1}{2}\eta_\pm \pm (-\sin \tfrac{1}{2}\lambda)\}^2 \qquad (16)$$

with an obvious notation. At the trailing edge $\phi_+ \simeq \phi_-$, $\eta_+ = 0$ and $\eta_- = -k$, so from (16) we find that k is given approximately by

$$\cosh \tfrac{1}{2}k = 1 - 2 \sin \tfrac{1}{2}\lambda . \qquad (17)$$

Greater accuracy is scarcely justified here in view of the crudeness of (15). Substitution of (15) into (11) yields

$$f - f_0 = i\theta(-\pi) - i\theta_0(-\pi) - \frac{1}{\pi}\int_{\gamma^* = -\pi}^{\pi} \log \frac{\sin \frac{1}{4}(\gamma^* + i\zeta)}{\cos \frac{1}{4}(\gamma^* - i\zeta)}\, d(\theta - \theta_0)$$

$$- \frac{2K}{\pi} \tan^{-1}\left\{\tan \frac{\pi + i\zeta}{4} \tanh \frac{k}{4}\right\} . \qquad (18)$$

It will be supposed in the following analysis that the non-separating flow about the aerofoil for $\alpha' = \xi = h = 0$, has already been calculated by one of the usual methods†, so that the relation

$$q = q(s/c),$$

and hence
$$\phi = \phi(s/c) = cU \int_0^{s/c} \frac{q}{U}\, d(s/c) \ , \qquad \ldots \quad (19)$$

where s is the aerofoil perimeter distance measured from the front stagnation point, is known. Now it will be assumed‡ that the relation between ϕ and s remains effectively unchanged by small values of ξ, h, K and α', so that the values of ϕ_0 and ϕ_1 together with the value of ϕ at the hinge of the flap can be calculated from (19). Thus λ and a can be determined from (6) and (7) while λ_2 and λ_3 (the values of γ at the hinge on the upper and lower surfaces respectively ; *see* Fig. 2) follows from (13).

The increments $\theta - \theta_0$ due to the flap and spoiler are shown in Fig. 2. They are as follows : (i) the front stagnation point moves from E ($\gamma = \lambda_0$) to D ($\gamma = \lambda$), thus reversing the flow direction in $\lambda \leqslant \gamma \leqslant \lambda_0$, i.e., increasing θ by π in this interval, (ii) the incidence reduces θ by α' in $-\pi \leqslant \gamma \leqslant \pi$, (iii) the flap deflection reduces θ by ξ in $\lambda_3 \leqslant \gamma \leqslant \pi$, $-\pi \leqslant \gamma \leqslant \lambda_2$, and (iv) the spoiler deflection reduces θ by ξ_1 in $-\pi \leqslant \gamma \leqslant -\pi + \lambda_1$. Thus $\theta - \theta_0$ is a step function with jumps in value as set out in the following Table § :

γ	$-\pi$	$-\pi + \lambda_1$	λ_2	λ	λ_0	λ_3	π
Jump in $\theta - \theta_0$	$-(\alpha' + \xi + \xi_1)$	ξ_1	ξ	π	$-\pi$	$-\xi$	$\alpha' + \xi$

Substituting these discontinuities into equation (18) we find that

$$\begin{aligned} f - f_0 = \; & \log\left\{\frac{\sin \frac{1}{4}(\lambda_0 + i\zeta) \cos \frac{1}{4}(\lambda - i\zeta)}{\cos \frac{1}{4}(\lambda_0 - i\zeta) \sin \frac{1}{4}(\lambda + i\zeta)}\right\} \\ & - \frac{\xi}{\pi} \log\left\{\frac{\sin \frac{1}{4}(\lambda_2 + i\zeta) \cos \frac{1}{4}(\lambda_3 - i\zeta)}{\cos \frac{1}{4}(\lambda_2 - i\zeta) \sin \frac{1}{4}(\lambda_3 + i\zeta)}\right\} \\ & - \frac{\xi_1}{\pi} \log\left\{\frac{\sin \frac{1}{4}(\lambda_1 + i\zeta - \pi)}{\cos \frac{1}{4}(\lambda_1 - i\zeta - \pi)}\right\} - \frac{2K}{\pi} \tan^{-1}\left\{\tan \tfrac{1}{4}(\pi + i\zeta) \tanh \frac{k}{4}\right\} , \quad \ldots \quad (20) \end{aligned}$$

while from (11) the vanishing of this increment at infinity yields

$$2\alpha' + \frac{\xi_1 \lambda_1}{\pi} + \frac{\xi}{\pi}(2\pi + \lambda_2 - \lambda_3) + (\lambda - \lambda_0) - \frac{kK}{\pi} = 0 \ . \qquad \ldots \quad (21)$$

† The method of Ref. 6 would be particularly suitable.

‡ A similar assumption is made in Ref. 7, where it is discussed in detail.

§ When the spoiler coincides with or is in *front* of the hinge λ_2 must be given the value $-\pi$ throughout the following theory. If the spoiler is on the upper surface of the aerofoil it is only necessary to change the signs of C_L, C_m and C_H in the following theory.

On the wetted surface (20) becomes

$$\begin{aligned} r - r_0 = \quad & \log \left| \frac{\sin \frac{1}{4}(\lambda_0 - \gamma) \cos \frac{1}{4}(\lambda + \gamma)}{\cos \frac{1}{4}(\lambda_0 + \gamma) \sin \frac{1}{4}(\lambda - \gamma)} \right| \\ & - \frac{\xi}{\pi} \log \left| \frac{\sin \frac{1}{4}(\lambda_2 - \gamma) \cos \frac{1}{4}(\lambda_3 + \gamma)}{\cos \frac{1}{4}(\lambda_2 + \gamma) \sin \frac{1}{4}(\lambda_3 - \gamma)} \right| \\ & - \frac{\xi_1}{\pi} \log \left| \frac{\sin \frac{1}{4}(\lambda_1 - \gamma - \pi)}{\cos \frac{1}{4}(\lambda_1 + \gamma - \pi)} \right| \\ & - \frac{2K}{\pi} \tan^{-1} \{\tan \tfrac{1}{4}(\pi - \gamma) \tanh \tfrac{1}{4}k\} . \qquad (22) \end{aligned}$$

In section 3, λ_1 will be shown to be a simple function of h, which vanishes when $h = 0$. The numbers α', λ_1 ξ and K will be assumed to be of order t, say, where t is a small number of the first order; terms $O(t^2)$ will be neglected. If $\lambda - \lambda_0 = \delta$ it follows from (21) that δ will also be $O(t)$.

Substituting $\lambda_0 = \lambda - \delta$ in (22) and regarding δ as an independent variable temporarily replacing α', we find that

$$\frac{\partial q}{\partial \delta} = \frac{\partial q}{\partial r} \frac{\partial r}{\partial \delta} = \frac{q}{2\beta} \frac{\cos \frac{1}{2}\gamma}{\sin \frac{1}{2}(\lambda - \delta) - \sin \frac{1}{2}\gamma} ,$$

since from (2) $dq/dr = - q/\beta$. Hence at $\delta = \xi = \lambda_1 = K = 0$;

$$\left(\frac{\partial q}{\partial \delta}\right)_0 = \frac{q_0}{2\beta_0} \frac{\cos \frac{1}{2}\gamma}{\sin \frac{1}{2}\lambda - \sin \frac{1}{2}\gamma} .$$

Similarly $$\left(\frac{\partial q}{\partial \lambda_1}\right)_0 = \frac{\xi_1 q_0}{2\pi\beta_0} \frac{\cos \frac{1}{2}\gamma}{1 + \sin \frac{1}{2}\gamma}$$

$$\left(\frac{\partial q}{\partial \xi}\right)_0 = - \frac{q_0}{\pi\beta_0} \log \left| \frac{\sin \frac{1}{4}(\lambda_2 - \gamma) \cos \frac{1}{4}(\lambda_3 + \gamma)}{\cos \frac{1}{4}(\lambda_2 + \gamma) \sin \frac{1}{4}(\lambda_3 - \gamma)} \right| ,$$

and $$\left(\frac{\partial q}{\partial K}\right)_0 = \frac{2q_0}{\pi\beta_0} \tan^{-1} \{\tan \tfrac{1}{4}(\pi - \gamma) \tanh \tfrac{1}{4}k\} .$$

Thus, using (21) to eliminate δ, we have to first order

$$\begin{aligned} q = q_0 \Bigg[1 - \frac{1}{\beta_0} \left\{ \alpha' + \frac{\xi_1 \lambda_1}{2\pi} + \left(\pi + \frac{\lambda_2 - \lambda_3}{2} \right) \frac{\xi}{\pi} - \frac{kK}{2\pi} \right\} \frac{\cos \frac{1}{2}\gamma}{\sin \frac{1}{2}\lambda - \sin \frac{1}{2}\gamma} \\ - \frac{\xi_1 \lambda_1}{2\beta_0 \pi} \frac{\cos \frac{1}{2}\gamma}{1 + \sin \frac{1}{2}\gamma} + \frac{\xi}{\beta_0 \pi} \log \left| \frac{\sin \frac{1}{4}(\lambda_2 - \gamma) \cos \frac{1}{4}(\lambda_3 + \gamma)}{\cos \frac{1}{4}(\lambda_2 + \gamma) \sin \frac{1}{4}(\lambda_3 - \gamma)} \right| \\ + \frac{2K}{\pi\beta_0} \tan^{-1} \left\{ \tan \tfrac{1}{4}(\pi - \gamma) \tanh \frac{k}{4} \right\} \Bigg] , \end{aligned}$$

an expansion which is plainly not valid in the immediate neighbourhoods of $\gamma = - \pi$, λ, λ_2 and λ_3.

In order to make further algebraic progress it is now necessary to restrict the aerofoil thickness to be $O(t)$, from which it follows that

$$q_0/U = 1 + O(t), \quad \theta_0 = O(t), \quad \beta_0 = \beta_\infty + O(t) \qquad (23)$$

and
$$K = -\tfrac{1}{2}\beta_\infty C_{p\sigma}(1 + O(t)) , \qquad (24)$$

where $C_{p\sigma}$ is the constant value of C_{ps} on the aerofoil behind the spoiler position. From (23), (24) and the expansion for q we deduce that the increment to the pressure coefficient is

$$C_p - C_{p\sigma} = C_{pw} + \frac{1}{\beta_\infty}\left\{2\alpha' + \frac{\xi_1\lambda_1}{\pi} + (2\pi + \lambda_2 - \lambda_3)\frac{\xi}{\pi} + \frac{k\beta_\infty C_{p\sigma}}{2\pi}\right\}\frac{\cos\frac{1}{2}\gamma}{\sin\frac{1}{2}\lambda - \sin\frac{1}{2}\gamma}$$
$$+ \frac{\xi_1\lambda_1}{\beta_\infty\pi}\frac{\cos\frac{1}{2}\gamma}{1+\sin\frac{1}{2}\gamma} - \frac{2\xi}{\beta_\infty\pi}\log\left|\frac{\sin\frac{1}{4}(\lambda_2-\gamma)\cos\frac{1}{4}(\lambda_3+\gamma)}{\cos\frac{1}{4}(\lambda_2+\gamma)\sin\frac{1}{4}(\lambda_3-\gamma)}\right|$$
$$+ \frac{2C_{p\sigma}}{\pi}\tan^{-1}\{\tan\tfrac{1}{4}(\pi-\gamma)\tanh k/4\} . \qquad (25)$$

Equations (23) permit us to write $\phi = Ux + O(t)$ and $\phi_0 = Uc + O(t)$, where x is the distance measured along the chord from the leading edge. Equations (6) and (7) can then be written in the approximate forms

$$\sin\tfrac{1}{2}\lambda = \frac{\sqrt{E_1}-1}{\sqrt{E_1}+1} \qquad (26)$$

and
$$\left(\frac{4a}{Uc}\right) = \tfrac{1}{4}(1+\sqrt{E_1})^2 , \qquad (27)$$

where the spoiler is at $x = E_1 c$. Similarly equation (13) can be written in the approximate form

$$\sin\tfrac{1}{2}\gamma = \pm\frac{2\sqrt{(x/c)}}{1+\sqrt{E_1}} + \sin\tfrac{1}{2}\lambda \qquad (28)$$

where the positive and negative signs apply to the upper $(\gamma > \lambda)$ and lower $(\gamma < \lambda)$ surfaces respectively. From (28) it follows that the values λ_2 and λ_3 defining the hinge position must satisfy

$$\sin\tfrac{1}{2}\lambda_2 + \sin\tfrac{1}{2}\lambda_3 = 2\sin\tfrac{1}{2}\lambda , \qquad (29)$$

since the value of $x, \{(1-E)c\}$ is the same on both surfaces.

Thus the quantities λ, k, λ_2 and λ_3 appearing in (25) can be calculated from (26), (17), (28) and (29), while λ_1 is related to the spoiler height by equation (36) below. The numbers C_{pw} and $C_{p\sigma}$ still remain to be assigned in (25). As C_{pw} is symmetrical across the aerofoil surface, $\Delta C_{pw} = 0$, so that it can be ignored when (25) is employed to calculate load distributions, forces and moments. An equation for C_{pw} in the case of the trailing-edge spoiler is developed in the Appendix. In the absence of any theory on the pressure increment behind spoilers $C_{p\sigma}$ must be assigned from experiment. However, if further experimentation reveals that the simple assumption about this pressure embodied in equation (15) can be improved then, with the aid of (11), there would be no difficulty in modifying the basic result (25) for the pressure increment. An example given in section 5 shows that the assumption (15) is of some value.

In the particular case of a trailing edge spoiler, λ, $k = 0$, and from (29), $-\lambda_2 = \lambda_3 = \lambda_m$ say, so that (25) reduces to

$$C_p - C_{p0} = -\frac{1}{\beta_\infty}\left\{2\alpha' + \frac{\xi_1\lambda_1}{\pi} + 2(\pi - \lambda_m)\frac{\xi}{\pi}\right\}\cot\tfrac{1}{2}\gamma + \frac{\xi_1\lambda_1}{\beta_\infty\pi}\frac{\cos\frac{1}{2}\gamma}{1 + \sin\frac{1}{2}\gamma}$$

$$-\frac{2\xi}{\beta_\infty\pi}\log\left|\frac{\sin\frac{1}{2}(\lambda_m + \gamma)}{\sin\frac{1}{2}(\lambda_m - \gamma)}\right| + \frac{\tilde{C}_p}{1 + b\cos\frac{1}{2}\gamma}\,, \qquad (30)$$

where the wake contribution, calculated in the Appendix has been included. It will be noted that in *this case* the pressure distributions due to incidence and flap deflection are the same as those obtained by the classical theory[8]†. This is not true for other spoiler positions since the spoiler has the additional affect of cancelling out the effectiveness of part of the aerofoil and flap, and thus reducing their efficiency as lifting surfaces.

The load distribution for a trailing-edge spoiler can be calculated quite simply, since from (28), $\sqrt{(x/c)} = \sin(\pm\frac{1}{2}\gamma)$, so that $\Delta C_p = C_p(\gamma) - C_p(-\gamma)$. Thus from (30) it follows that the contribution to the loading due to the spoiler alone is

$$\Delta C_p = -\frac{4\xi_1\lambda_1}{\pi\beta_\infty}\operatorname{cosec}\gamma\,. \qquad (31)$$

3. *The Spoiler Height.*—It is convenient at this stage to establish the relation between λ_1 and h. From (14) and the definition of λ_1 we have

$$\frac{hU}{4a} = \int_{-\pi}^{-\pi+\lambda_1}\frac{U}{q}(\sin\tfrac{1}{2}\lambda - \sin\tfrac{1}{2}\gamma)\cos\tfrac{1}{2}\gamma\,d\gamma\,. \qquad (32)$$

In the interval $-\pi \leqslant \gamma \leqslant -\pi + \lambda_1$ equation (22) can be written approximately

$$\int_{q=U}^{q}\beta\,d(\log U/q) = -\frac{\xi_1}{\pi}\log\left|\frac{\lambda_1 - \gamma - \pi}{\lambda_1 + \gamma + \pi}\right| - K\,, \qquad (33)$$

since $\lambda_1 << \lambda_2$, λ_3, and $\lambda_0 \simeq \lambda$. In the range of integration of (32) q varies from 0 at $\gamma = -\pi + \lambda_1$ to $U + O(t)$‡ at $\gamma = -\pi$, so that an average value of β in the range is approximately $\frac{1}{2}(1 + \beta_\infty)$. To make analytic progress at this point it is necessary to replace β in (33) by this average value. This enables us to write

$$\frac{U}{q} = \frac{U}{q_1}\left(\frac{\lambda_1 + t}{\lambda_1 - t}\right)^\varepsilon, \quad 0 \leqslant t \leqslant \lambda_1$$

where $t = \pi + \gamma$, q_1 is the velocity at the point of flow separation $t = 0$, and ε is defined by equation (10). This result and the fact that λ_1 is small allows (32) to be written

$$\frac{hU}{4a} = \frac{U}{2q_1}\int_0^{\lambda_1}\left(\frac{\lambda_1 + t}{\lambda_1 - t}\right)^\varepsilon(1 + \sin\tfrac{1}{2}\lambda)t\,dt\,;$$

therefore from (26) and (27)

$$h/c = \tfrac{1}{4}\lambda_1^2(E_1 + \sqrt{E_1})\,\frac{U}{q_1}\int_0^1\left(\frac{1 + y}{1 - y}\right)^\varepsilon y\,dy\,.$$

† *See* also the extension of Glauert's theory to thick aerofoils in compressible flow given in Ref. 7.

‡ Admittedly not true for large spoiler heights but this assumption does permit an average value of β in the range to be assigned.

Hence $$\lambda_1 = F\sqrt{\left\{\frac{2h/c}{E_1 + \sqrt{E_1}}\frac{q_1}{U}\right\}}, \qquad (34)$$

where $$F = \left\{\tfrac{1}{2}\int_0^1 \left(\frac{1+y}{1-y}\right)^{\varepsilon} y\, dy\right\}^{-1/2}.$$

The function $F(\varepsilon)$ is set out in the following table, which was established by numerical integration.

ε	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
F	2·000	1·807	1·612	1·423	1·238	1·058	0·883	0·709	0·534	0·347	0

For a spoiler at right-angles to the aerofoil surface in incompressible flow

$$\varepsilon = \tfrac{1}{2} \text{ and } F = \sqrt{\left(\frac{8}{4+\pi}\right)}.$$

Two complications must be considered at this point. Referring to Fig. 3 they are (i) the spoiler causes flow separation to occur at some point E in *front* of B, and (ii) since the boundary-layer displacement thickness will be less at G than at B with the spoiler absent (δ^*), the *effective* spoiler height, $\tilde{h}$, must be less than h. When $h >> \delta^*$ it seems reasonable to write $\tilde{h} = h - \delta^*$, and to replace h by $\tilde{h}$ in (34). When $\tilde{h}$ and δ^* are about the same size, or when $h < \delta^*$, $\tilde{h}$ will be a complicated function of h, δ^* and the Reynolds number. However the following empirical rule, which the author has found to be in moderate agreement with the available experimental data may be of some value. It is that

$$\tilde{h} = \begin{cases} h - \delta^*, & (2\delta^* \leqslant h) \\ h^2/4\delta^*, & (0 \leqslant h \leqslant 2\delta^*), \end{cases} \qquad (35)$$

and equation (34) is replaced by†

$$\lambda_1 = F\sqrt{\left\{\frac{2\tilde{h}/c}{E_1 + \sqrt{E_1}}\frac{q_1}{U}\right\}}. \qquad (36)$$

The value of δ^*/c at the trailing edge is approximately[5]

$$\frac{\delta^*}{c} = \frac{H}{4}\left(\frac{U}{q_t}\right)^{3\cdot 2} C_{D0}, \qquad (37)$$

where q_t is the velocity *at the trailing edge* when the spoiler is absent and H is the ratio of the momentum to the displacement thickness of the boundary layer. It is probably sufficient for our present application to assume that $H = 1\cdot 4$ and δ^* grows linearly along the aerofoil chord. If the value of q_t is not available, then $\delta^*/c = C_{D0}/2$ is probably a fair approximation to (37).

There seems to be no easy way to allow for the flow separation in front of the spoiler, but the success of equations (35) and (36) in predicting experimental results (*see* section (5)) suggests that it is of little importance.

It should be realized that the difficulties mentioned above are significant only in affecting the relation between λ_1 and h; they will have negligible effect on the *law* of load distribution.

† For $h/c < 0\cdot 02$, say, q_1/U can be replaced by unity in (36), with little resulting error.

4.1. *The Forces and Moments Acting on the Aerofoil.*—4.1. *The Lift.*—The lift coefficient is given by

$$C_L = -\frac{1}{c}\oint C_p \cos\theta\, ds$$

$$= -\frac{1}{c}\oint (C_{ps} + C_{p0}) \cos\theta\, ds\ ,$$

from (10) and the symmetry of C_{pw}, and where the contour integral is taken round the aerofoil surface. Since it has been assumed that $C_{ps} = C_{p\sigma}$ in $E_1 c \leqslant x \leqslant c$ on the lower surface, then with the aid of (13) we have

$$C_L - C_{L0} = (1 - E_1) C_{p\sigma} - \left(\frac{4a}{Uc}\right)\int_{-\pi}^{\pi} \frac{U}{q_0} \cos\theta C_{ps} (\sin \tfrac{1}{2}\gamma - \sin \tfrac{1}{2}\lambda) \cos \tfrac{1}{2}\gamma\, d\gamma\ .$$

From (17), (23), (25), (26), (27) and (29) we find that

$$C_L = \frac{2\pi}{\beta_\infty}\left(\frac{4a}{Uc}\right)\left\{\alpha' + \alpha_0 + (1 + \sin \tfrac{1}{2}\lambda)\frac{\xi_1\lambda_1}{\pi} + \frac{\xi}{\pi}\left(\pi + \frac{\lambda_2 - \lambda_3}{2} - \sin\frac{\lambda_2 - \lambda_3}{2}\right)\right\}$$
$$+ \left(\frac{4a}{Uc}\right)(k/2 + \sinh k/2)\, C_{p\sigma}\ , \qquad (38)$$

where $-\alpha_0$ is the no-lift angle, and terms $O(t^2)$ have been ignored.

From this result and equation (27) we have that, in the standard notation

$$\left.\begin{aligned} a_0 &= \frac{\pi}{2\beta_\infty}(1 + \sqrt{E_1})^2 \alpha_0 \\ a_1 &= \frac{\pi}{2\beta_\infty}(1 + \sqrt{E_1})^2 \\ a_2 &= a_1\left(1 - \frac{\lambda_3 - \lambda_2}{2} + \sin\frac{\lambda_3 - \lambda_2}{2}\right) \end{aligned}\right\}\ , \qquad (39)$$

while the contribution to C_L due to the spoiler alone is

$$C_{Ls} = \frac{\xi_1\lambda_1}{\beta_\infty}(\sqrt{E_1} + E_1) + \tfrac{1}{4}(1 + \sqrt{E_1})^2\left(\frac{k}{2} + \sinh\frac{k}{2}\right) C_{p\sigma}\ . \qquad (40)$$

In the case of the trailing-edge spoiler, $\lambda = k = 0$, and $-\lambda_2 = \lambda_3 = \lambda_m$, so that (38) reduces to

$$C_L = \frac{2\pi}{\beta_\infty}\left(\frac{4a}{Uc}\right)\left\{\alpha' + \alpha_0 + \frac{\xi_1\lambda_1}{\pi} + \frac{\xi}{\pi}(\pi - \lambda_m + \sin\lambda_m)\right\}, \qquad (41)$$

of which the incidence and flap-deflection terms are standard results[7].

4.2. *The Drag.*—The increment to the drag coefficient due to the spoiler is

$$C_D - C_{D0} = \frac{1}{c}\oint (C_p - C_{p0}) \sin\theta\, ds\ .$$

Except on the spoiler surface both θ and $C_p - C_{p0}$ are $O(t)$. Hence, ignoring terms $O(t^2)$ we can write

$$C_D - C_{D0} = \frac{\sin\xi_1}{c}\int_{-\pi}^{-\pi+\lambda_1} C_p \frac{ds}{d\phi}\frac{d\phi}{d\gamma}\, d\gamma\ . \qquad (42)$$

From (42) and (13)

$$C_D - C_{D0} = \sin \xi_1 \left(\frac{4a}{Uc}\right) \int_{-\pi}^{-\pi+\lambda_1} \left(\frac{U}{q} - \frac{q}{U}\right) (\sin \tfrac{1}{2}\gamma - \sin \tfrac{1}{2}\lambda) \cos \tfrac{1}{2}\gamma \, d\gamma \, .$$

With the same analysis as used in section 3 to calculate h we can reduce this to

$$C_D - C_{D0} = \pi(\xi\lambda_1)^2 \left(\frac{4a}{Uc}\right) \frac{\sin \xi_1}{\sin \pi\varepsilon} (1 + \sin \tfrac{1}{2}\lambda) \, .$$

i.e., from (26), (27) and (36),

$$C_D - C_{D0} = \pi(\varepsilon F)^2 \frac{\sin \xi_1}{\sin \pi\varepsilon} \frac{\tilde{h}}{c} \, . \qquad \qquad (43)$$

From this result it appears that the drag increment is independent of the spoiler position, but clearly the width of the wake will increase as the spoiler moves forward from the trailing edge. An obvious, but empirical rule would be to replace $\tilde{h}$ in (43) by $(\tilde{h} + h_1)$ where h_1 is defined in Fig. 4. Some experimental justification of this is given in section 5.

4.3. *The Moment about the Leading Edge.*—The moment coefficient about the leading edge is

$$C_m = \frac{1}{c} \oint \left(\frac{x}{c} \cos \theta + \frac{y}{c} \sin \theta\right) C_p \frac{ds}{d\phi} \frac{d\phi}{d\gamma} \, d\gamma \, .$$

From (13) and (23) it is found that the increment to the moment coefficient is given by

$$C_m - C_{m0} = \left(\frac{4a}{Uc}\right)^2 \int_{-\pi}^{\pi} C_{ps} (\sin \tfrac{1}{2}\gamma - \sin \tfrac{1}{2}\lambda)^3 \cos \tfrac{1}{2}\gamma \, d\gamma - \tfrac{1}{2}(1 - E_1{}^2) C_{p\sigma} + O(t^2) \, .$$

Hence from (17), (25) and (29)

$$\begin{aligned} C_m = & - \frac{\pi}{2\beta_\infty} \left(\frac{4a}{Uc}\right)^2 \bigg\{ (1 + 4 \sin^2 \tfrac{1}{2}\lambda)(\alpha' + \alpha_0) \\ & + \frac{2\xi_1\lambda_1}{\pi} (1 + \sin \tfrac{1}{2}\lambda)(2 \sin^2 \tfrac{1}{2}\lambda + 2 \sin \tfrac{1}{2}\lambda + 1) \\ & + \frac{\xi}{2\pi} [(\sin \lambda_2 - \sin \lambda_3)(1 + \sin \tfrac{1}{2}\lambda_2 \sin \tfrac{1}{2}\lambda_3) \\ & + 4 \sin \tfrac{1}{2}(\lambda_3 - \lambda_2) + 2 \cos \tfrac{1}{2}\lambda_2 \sin^3 \tfrac{1}{2}\lambda_3 \\ & - 2 \cos \tfrac{1}{2}\lambda_3 \sin^3 \tfrac{1}{2}\lambda_2 + (2\pi + \gamma_2 - \lambda_3)(1 + 4 \sin^2 \tfrac{1}{2}\gamma)] \bigg\} \\ & - C_{p\sigma} \left(\frac{4a}{Uc}\right)^2 \bigg\{ \tfrac{1}{8}(1 + 4 \sin^2 \tfrac{1}{2}\lambda)(k + 2 \sinh \tfrac{1}{2}k) \\ & + 4 \sin \tfrac{1}{2}\lambda \, (1 - \sin \tfrac{1}{2}\lambda)^2 + \tfrac{1}{2} \sinh \tfrac{1}{2}k \, (1 - \sin \tfrac{1}{2}\lambda) \bigg\} \\ & + 4C_{p\sigma} \left(\frac{4a}{Uc}\right) \sin \tfrac{1}{2}\lambda \, . \end{aligned} \qquad (44)$$

For the trailing-edge spoiler $\lambda = k = 0$, $-\lambda_2 = \lambda_3 = \lambda_m$, and (44) reduces to

$$C_m = -\frac{\pi}{2\beta_\infty}\left(\frac{4a}{Uc}\right)^2\left\{\alpha' + \alpha_0 + \frac{2\xi_1\lambda_1}{\pi} + \frac{\xi}{\pi}[\sin\lambda_m(1 - \cos\lambda_m) + \sin\lambda_m + \pi - \lambda_m]\right\} \quad .. \quad (45)$$

of which the incidence and flap-deflection terms were given in Ref. 7. It is to be noted that, for a trailing-edge spoiler (41), (45) and (27) yield

$$-\left(\frac{C_m}{C_L}\right)_{\alpha'=\alpha_0=\xi=0} = \tfrac{1}{2} ,$$

i.e., the centre of pressure due to the spoiler is at the mid-chord point. It is thus possible to reduce considerably the centre of pressure movement normally experienced in the transonic range by obtaining lift at subsonic speeds mainly from a trailing-edge spoiler, and gradually retracting the spoiler and increasing incidence as supersonic speeds are achieved.

4.4. *Hinge Moments*.—It is easily verified that the equation

$$C_H - C_{H0} = \left(\frac{4a}{Uc}\right)\frac{1}{E^2}\left(\int_{-\pi}^{\lambda_2} + \int_{\lambda_3}^{\pi}\right)C_{ps}\left[\left(\frac{4a}{Uc}\right)(\sin\tfrac{1}{2}\gamma - \sin\tfrac{1}{2}\lambda)^2 - 1 + E\right]$$
$$\times (\sin\tfrac{1}{2}\gamma - \sin\tfrac{1}{2}\lambda)\cos\tfrac{1}{2}\lambda\, d\gamma$$
$$- \tfrac{1}{2}C_{p\sigma}(1 - E_1)(2E - 1 + E_1) ,$$

enables the hinge-moment coefficient to be calculated for any general spoiler position. As the expression for C_H is rather long in the general case, we shall be content to calculate C_H for a trailing-edge spoiler. In this case

$$C_H = C_{H0} + \left(\frac{4a}{Uc}\right)^2\frac{1}{2E^2}\left(\int_{-\pi}^{-\lambda_m} + \int_{\lambda_m}^{\pi}\right)C_{ps}(\sin^2\tfrac{1}{2}\gamma - \sin^2\tfrac{1}{2}\lambda_m)\sin\gamma\, d\gamma ,$$

hence

$$C_H = C_{H0} - \left(\frac{4a}{Uc}\right)^2\frac{1}{\beta_\infty E^2}\Big\{[\sin\lambda_m(1 - \tfrac{1}{2}\cos\lambda_m) + (\pi - \lambda_m)(\cos\lambda_m - \tfrac{1}{2})](\alpha' + \alpha_0)$$
$$+ [\sin\lambda_m + (\pi - \lambda_m)\cos\lambda_m]\frac{\xi_1\lambda_1}{\pi}$$
$$+ [(\pi - \lambda_m)\sin\lambda_m + \tfrac{1}{2}\sin^2\lambda_m - (\tfrac{1}{2} - \cos\lambda_m)(\pi - \lambda_m)^2]\frac{\xi}{\pi}\Big\}, \quad .. \quad (46)$$

where the terms in α' and ξ were given in Ref. 7.

Equations (41) and (46) permit an interesting comparison to be made between the spoiler and flap when used separately as lift-producing devices. From (41) a spoiler will produce the same lift as a flap provided

$$\xi_1\lambda_1 = \xi(\pi - \lambda_m + \sin\lambda_m) ,$$

when from (46) the moments about the flap hinge due to spoiler and flap will be in the ratio

$$C_{Hs}/C_{Hf} = \frac{[\sin\lambda_m + (\pi - \lambda_m)\cos\lambda_m][\pi - \lambda_m + \sin\lambda_m]}{(\pi - \lambda_m)\sin\lambda_m + \tfrac{1}{2}\sin^2\lambda_m - (\tfrac{1}{2} - \cos\lambda_m)(\pi - \lambda_m)^2} , \quad .. \quad .. \quad (47)$$

the suffixes $_s$ and $_f$ denoting *flap* hinge moment due to 'spoiler' and 'flap' respectively. From (28) and the definitions of E and λ_m we find $\cos \lambda_m = 1 - 2E$, so that the following table is easily derived :

E	$0 \cdot 1$	$0 \cdot 2$	$0 \cdot 3$	$0 \cdot 4$
C_{Hs}/C_{Hf}	$3 \cdot 81$	$3 \cdot 63$	$3 \cdot 44$	$3 \cdot 26$

This table illustrates one of the disadvantages of the use of spoilers on aerofoils with flaps, namely that the lift is obtained at the expense of large hinge moments. This disadvantage is mentioned in Refs. 1 and 2.

5. *Comparison with Experiment.*—With the exception of Fig. 8d the experimental results appearing in Figs. 6 to 10 have been selected from a wide range of experimental results on spoilers recently obtained in the Aerodynamics Division of the National Physical Laboratory as part of the programme mentioned in Ref. 1.

Figs. 6a to 6d show theoretical and experimental load distributions for aerofoil RAE 102[9] (a symmetrical aerofoil, 10 per cent thick) fitted with trailing-edge spoilers. The theoretical curves were obtained from equations (31) and (36). Consider for example the curve shown in Fig. 6a for $h = 0 \cdot 019c$. We have the following data:

$$\alpha' = 0, \quad M_\infty = 0 \cdot 4, \quad \xi_1 = \pi/2, \quad h/c = 0 \cdot 019, \quad E_1 = 1, \quad C_{D0} \simeq 1 \cdot 01 \, ,$$

$$\frac{q_t}{U} \simeq 0 \cdot 92 \, ,$$

the last two figures of which were obtained from an experiment on the aerofoil without a spoiler. Hence, from (37), (35), (10) and (36)

$$\delta^*/c = 0 \cdot 005, \quad \tilde{h}/c = 0 \cdot 014, \quad \varepsilon = 0 \cdot 522$$

and (*see* footnote to equation (36))

$$\lambda_1 = F(0 \cdot 522)\sqrt{(0 \cdot 014)} \simeq 0 \cdot 120 \, ,$$

on making use of the table relating F and ε. Thus from (31) $\Delta C_p = -0 \cdot 26 \operatorname{cosec} \gamma$, which yields the theoretical curve shown in the figure. The case shown in Fig. 6d merits some special attention owing to relatively large spoiler height resulting in a large value of C_p behind the spoiler.

For this case

$$\alpha' = 0, \quad M_\infty = 0, \quad \xi_1 = \pi/2, \quad h/c = 0 \cdot 06, \quad E_1 = 1 \, ,$$

and as before

$$\delta^*/c = 0 \cdot 005.$$

Thus from (36), $\lambda_1 = F(0 \cdot 5)\sqrt{(0 \cdot 056 \times q_1/U)}$.

Now the value of C_p behind the spoiler is $-0 \cdot 77$ in this case, *i.e.*, $q_1/U \simeq \sqrt{(1 \cdot 77)} = 1 \cdot 33$, which cannot be ignored without incurring a 15 per cent error in λ_1. We find $\lambda_1 = 0 \cdot 288$, and so from (31) $\Delta C_p = 0 \cdot 58 \operatorname{cosec} \gamma$, which is the curve shown in the figure. It is fair to conclude from the seven examples shown in Fig. 6 that the theory is in good agreement with experiment.

In Figs. 7a and 7b the *increments* to the pressure coefficients due to both the spoiler and the wake, *i.e.*, $C_{ps} + C_{pw}$, are shown for the two examples described in detail above. From (30)

$$C_p - C_{p0} = -\frac{\xi_1 \lambda_1}{\pi \beta_\infty} \frac{\cot \frac{1}{2}\gamma}{1 + \sin \frac{1}{2}\gamma} + \frac{\tilde{C}_p}{1 + b \cos \frac{1}{2}\gamma} \, . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (48)$$

For the case shown in Fig. 7a, from the value of λ_1 calculated above and the experimental values of $\tilde{C}_p$ (the *change* in C_p at the trailing edge due to the presence of the spoiler) we have

$$C_p - C_{p0} = -0\cdot065 \frac{\cot \frac{1}{2}\gamma}{1 + \sin \frac{1}{2}\gamma} - \frac{0\cdot355}{1 + 5\cdot78 \cos \frac{1}{2}\gamma},$$

where the value $b = 5\cdot78$ has been selected to make the equation yield the experimental value, $C_p - C_{p0} = -0\cdot107$, at $\gamma = \pi/2$. An alternative and less empirical procedure is to use equation (55) to derive an approximate value of b. We find in this way that $b = 5\cdot5$, which is surprisingly close to the experimental value. From a similar calculation for the case shown in Fig. 7b, (48) becomes

$$C_p - C_{p0} = -0\cdot145 \frac{\cot \frac{1}{2}\gamma}{1 + \sin \frac{1}{2}\gamma} - \frac{0\cdot92}{1 + 5\cdot15 \cos \frac{1}{2}\gamma},$$

while (55) yields $b = 4\cdot9$.

Figs. 8a to 8c show the variation of C_L, C_D and C_m with h/c for RAE 102 at $M_\infty = 0$, fitted with trailing-edge spoilers. The theoretical curves shown have been computed from equations (41), (43) and (45), while the experimental values were obtained by direct measurement. *Some* of the difference between experiment and theory is apparently due to experimental error, since integration of the experimental loading in Fig. 6d yields the result indicated by a triangle in Fig. 8a, which is 10 per cent larger than the value obtained by direct measurement. Fig. 8d shows some experimental values for the change in the no-lift angle (measured in degrees) due to a trailing-edge spoiler, given in Ref. 3. (The values given in Ref. 3 have been corrected for 'zero gap'.) From (41) the theoretical value of this change is

$$\Delta \alpha_0 = 57\cdot3 \frac{\xi_1 \pi_1}{\pi}$$

i.e., if $M_\infty = 0$, and $\xi_1 = \pi/2$,

$$\Delta \alpha_0 = 30\cdot3 \sqrt{\{(h - \delta^*)c\}} .$$

Now from curves given in Ref. 3, $C_{D0} \simeq 0\cdot016$, therefore $\delta^* \simeq 0\cdot008c$; the theoretical curve in the figure follows from this value of δ^* and the equation for $\Delta \alpha_0$.

The results so far discussed show that the theory of this paper provides a satisfactory explanation of the effect of trailing-edge spoilers. It only remains to establish the relation between h/c and $\tilde{C}_p$—empirically if necessary. From the experimental values of $\tilde{C}_p$ shown in Fig. 8c it appears that the relation is approximately linear up to $h/c = 0\cdot03$. $\tilde{C}_p$ cannot, of course, exceed the value which occurs behind a flat plate normal to the flow (about $\doteq 1\cdot0$) and this explains the flattening out of the curve for large h/c.

When the spoiler is not at the trailing edge one additional empirical constant enters into the calculation of the load distribution and forces, namely $C_{p\sigma}$. The theoretical curve shown in Fig. 9 for a spoiler at $x = 0\cdot65c$ was calculated from (25) on the assumption that $C_{p\sigma} = 0\cdot24$—a value selected to make the average value of ΔC_p in $0\cdot65c \leqslant x \leqslant c$ agree with the experimental average. The agreement between theory and experiment in $0 \leqslant x \leqslant 0\cdot65c$ is certainly some justification of the procedure—although it is probable that the assumption, $C_{ps} =$ constant in $0\cdot65c \leqslant x \leqslant c$, on the lower surface, could be improved on; perhaps the empirical element could be eliminated completely.

The curves given in Fig. 10 clearly show the superiority of the trailing-edge spoiler—it yields the maximum lift for the minimum drag. The lift-coefficient curve was calculated from equation (40) on the assumption that $C_{p\sigma}$ remained equal to $-0\cdot24$ for all values of E_1c, while the drag-coefficient curve was calculated from (43) on the assumption that $\tilde{h}$ is replaced by $\tilde{h} + h_1$ as described in section 4·2†.

† Values of h_1 were obtained from the aerofoil co-ordinates given in Ref. 9.

The figure of $-0 \cdot 24$ was obtained from the experimental results $E_1 = 0 \cdot 65$. These assumptions are of course relatively crude, but nevertheless it appears from Fig. 10 that they have heuristic value.

6. *Further Applications of the Theory.*—Flow separation sometimes occurs when there are no spoilers on the aerofoil surface. For example it may occur at the flap hinge on the upper surface when the flap is deflected downwards, particularly if the hinge is badly designed or the flap chord is quite small. This case is covered by the author's theory by putting $\lambda_2 = -\pi$ and $\lambda_1 = 0$. Equation (38) becomes for example

$$C_L = \frac{\pi}{2\beta_\infty}\{1 + \sqrt{(1-E)}\}^2\left\{\alpha_0' + \alpha_0 + \frac{\xi}{\pi}\left(\frac{\pi}{2} - \frac{\lambda_3}{2} + \cos\frac{\lambda_3}{2}\right)\right\}$$

$$+ \tfrac{1}{4}\{1 + \sqrt{(1-E)}\}^2\left\{\frac{k}{2} + \sinh\frac{k}{2}\right\}C_{p\sigma} ,$$

where from (26) and (29)

$$\sin\frac{\lambda_3}{2} = \frac{3\sqrt{(1-E)} - 1}{\sqrt{(1-E)} + 1} .$$

On the assumption that $C_{p\sigma} = 0$, we have

$$a_2 = \frac{1}{2\beta_\infty}\{1 + \sqrt{(1-E)}\}^2\left\{\frac{\pi - \lambda_3}{2} + \cos\frac{\lambda_3}{2}\right\} ,$$

and hence the ratio of this value, of a_2 to the usual theoretical value, a_{2T} say, (obtained from (39)), with $\lambda_3 = -\lambda_2 = \lambda_m$) can be tabulated thus :

E	0	0·15	0·25	0·35
a_2/a_{2T}	0·500	0·490	0·482	0·472

The importance of the theory of the trailing-edge spoiler is enhanced by the fact that it can be considered as an alternative to the classical theory of flap–tab combinations. The boundary layer requires little inducement to separate from the aerofoil near the trailing edge and provided the value of E for the tab is small enough, say less than $0 \cdot 1$, then the tab would behave more like a spoiler than a flap‡. Some evidence supporting this view is that the average experimental value[10] of a_2/a_{2T} does appear to approach $0 \cdot 5$ as E tends to zero.

The author hopes to give a mathematical account of the effects of spoilers on the unsteady characteristics of aerofoils in a later report.

7. *Acknowledgement.*—The author is pleased to acknowledge that his understanding of the effects of aerofoil spoilers has been enhanced by several discussions on the subject with Mr. H. H. Pearcey of the Aerodynamics Division, N.P.L.

‡ Attention was drawn in Ref. 1 to the possible significance of this at transonic speeds.

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APPENDIX

The Wake Contribution to the Pressure Distribution in the Case of a Trailing-edge Spoiler

When the spoiler is at the trailing edge, $\phi_1 = \phi_0$: thus from (6), (7) and (8), $\lambda = 0, \phi_0 = 4a$, and

$$w = -\,4a \sinh^2 \tfrac{1}{2}\zeta .$$

On each of the separation streamlines bounding the wake, $\zeta = \eta \pm i\pi$ and

$$\phi = 4a \cosh^2 \tfrac{1}{2}\eta . \qquad (50)$$

As stated in the Introduction it will be assumed that the pressure, and hence r, is the same on each side of the wake, *i.e.*, $r_+ = r_-$. Now it is reasonable to assume that some distance downstream of the spoiler the separation streamlines remain a constant distance apart, *i.e.*, the displacement thickness of the wake is constant[5]. Under these conditions the functions r_+ and r_- must be inversely proportional to the value of ϕ, for consider the flow over the step of length h shown in Fig. 5. For this case a simple application of an equation given in Ref. 6 yields

$$r(\phi) = \frac{\xi_1}{\pi} \log\left|\frac{\phi - t}{\phi + t}\right| , \qquad (51)$$

whence for large ϕ, $r_\pm \doteqdot -\dfrac{2t\xi_1}{\pi\phi}$. .. .. (52)

From this result and equation (50), $r_{\pm}\,(\eta^*)$ must be of the form

$$r_{\pm}(\eta^*) = \frac{-K}{1 + b^2 \sinh^2 \frac{1}{2}\eta^*}, \quad \ldots \quad (53)$$

where K and b are constants. Substitution of (53) in equation (11) yields that the contribution of the wake to f is

$$f_w = \frac{-K}{1 + b \cosh \frac{1}{2}\zeta};$$

and in particular the increment to C_p on the aerofoil surface from this wake term is

$$C_{pw} = \frac{-2K}{\beta_\infty(1 + b \cos \frac{1}{2}\gamma)},$$

or from an equation similar to (24)

$$C_{pw} = \frac{\tilde{C}_p}{1 + b \cos \frac{1}{2}\gamma}, \quad \ldots \quad (54)$$

where $\tilde{C}_p$ is the change in the pressure coefficient at the trailing edge due to the spoiler.

It is possible to find an approximate equation for the constant b as follows. From (2) and (51)

$$\frac{U}{q} = \left(\frac{t-\phi}{t+\phi}\right)^{\xi_1/\pi\beta_\infty}, \quad -t \leqslant \phi \leqslant t,$$

hence
$$hU = \int_{-t}^{t} \left(\frac{t-\phi}{t+\phi}\right)^{\xi_1/\pi\beta_\infty} d\phi$$

$$= \frac{2\xi_1 t}{\pi\beta_\infty \sin(\xi_1/\beta_\infty)},$$

i.e. (52) can be written

$$r_{\pm} = -\frac{hU}{\pi}\frac{\beta_\infty \sin(\xi_1/\beta_\infty)}{\phi}.$$

Comparing this result with (50) and (53) we have

$$\frac{4aK}{b^2} = \frac{hU\beta_\infty \sin(\xi_1/\beta_\infty)}{\pi},$$

i.e. from (24) and (27),

$$b = 1\Big/\sqrt{\left\{\frac{2}{\pi}\frac{h}{c}\frac{\sin(\xi_1/\beta_\infty)}{(-\tilde{C}_p)}\right\}}. \quad \ldots \quad (55)$$

Thus b depends essentially on the ratio $(h/c)/\tilde{C}_p$, which has been found experimentally to be approximately constant for $h/c < 0{\cdot}03$. (*See* discussion on this point in section 5.) Thus we should expect b to be approximately constant when $h/c < 0{\cdot}03$.

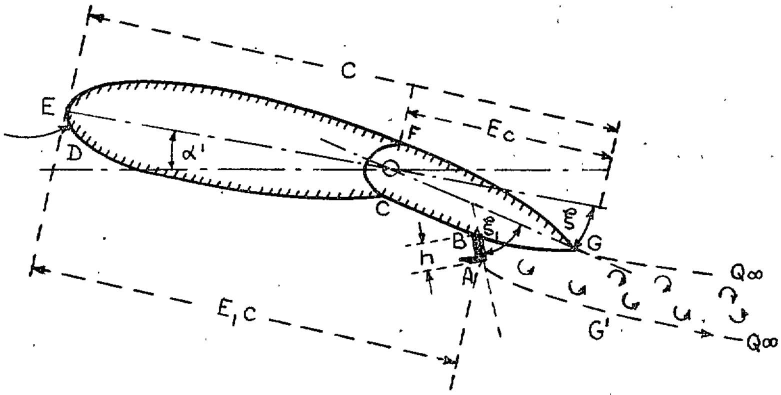

FIG. 1a. Spoiler–flap combination.

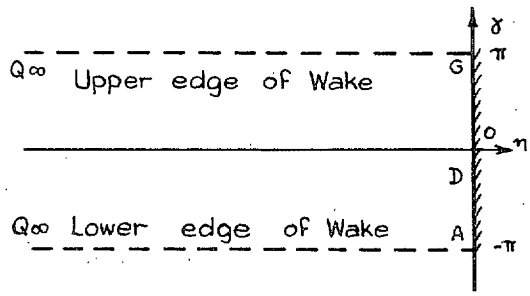

FIG. 1b. The ζ-plane.

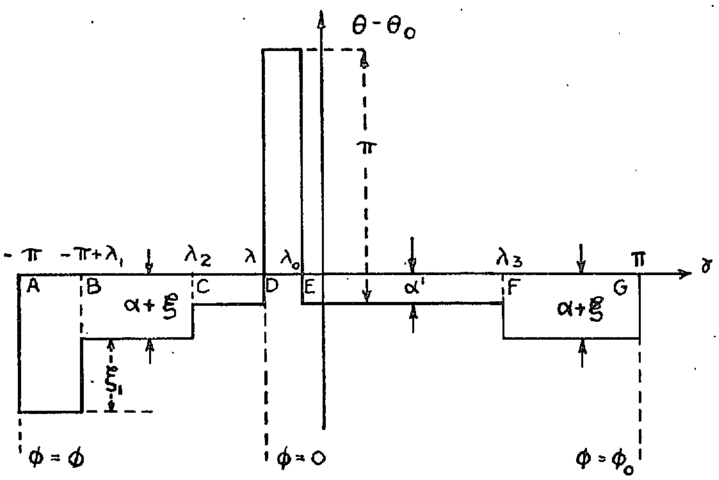

FIG. 2. $\theta - \theta_0$ due to a spoiler–flap combination.

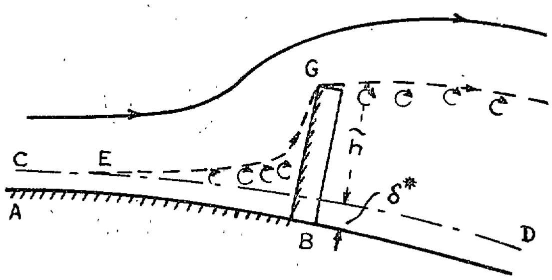

FIG. 3.

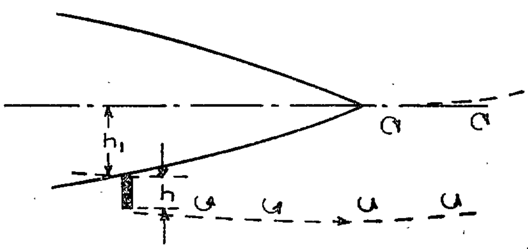

FIG. 4.

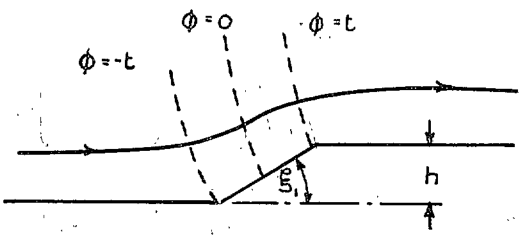

FIG. 5.

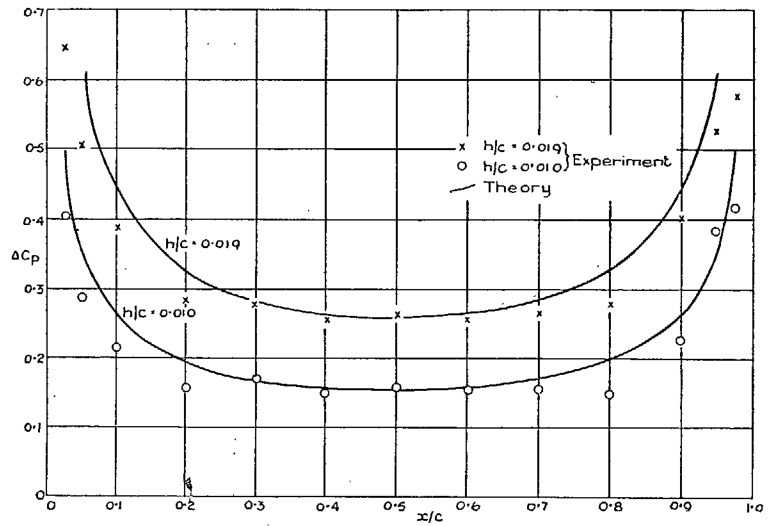

FIG. 6a. $M_\infty = 0{\cdot}4$.

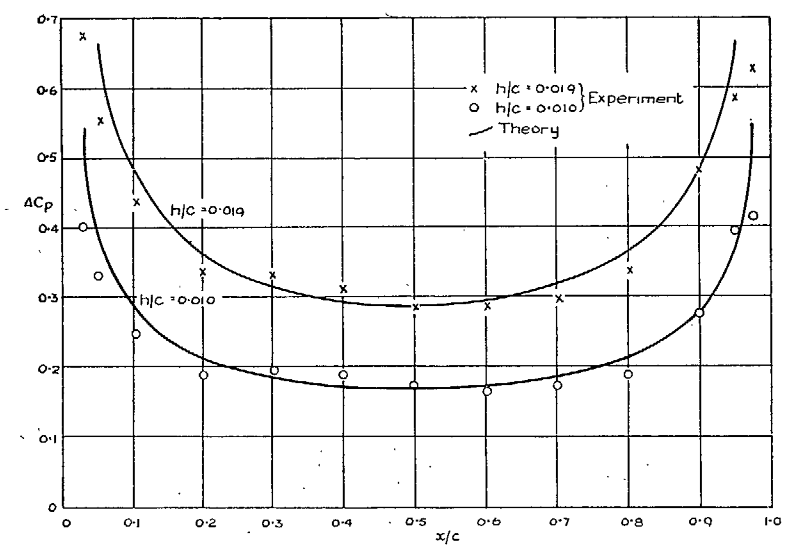

FIG. 6b. $M_\infty = 0{\cdot}60$.

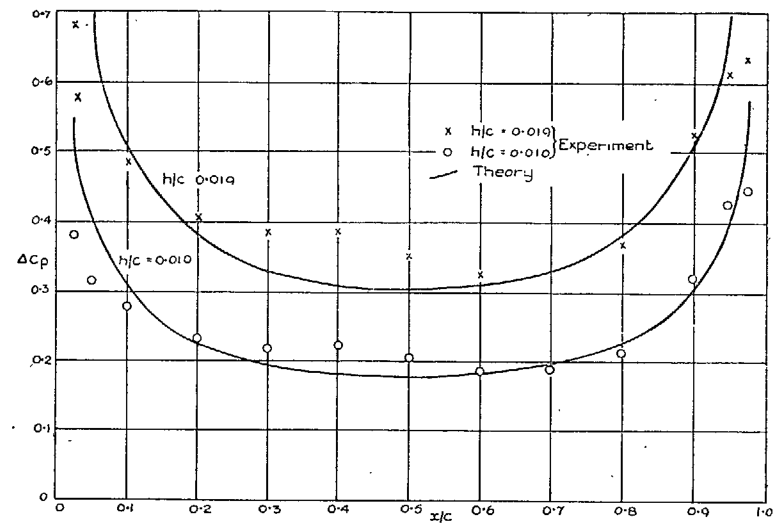

FIG. 6c. $M_\infty = 0{\cdot}7$.

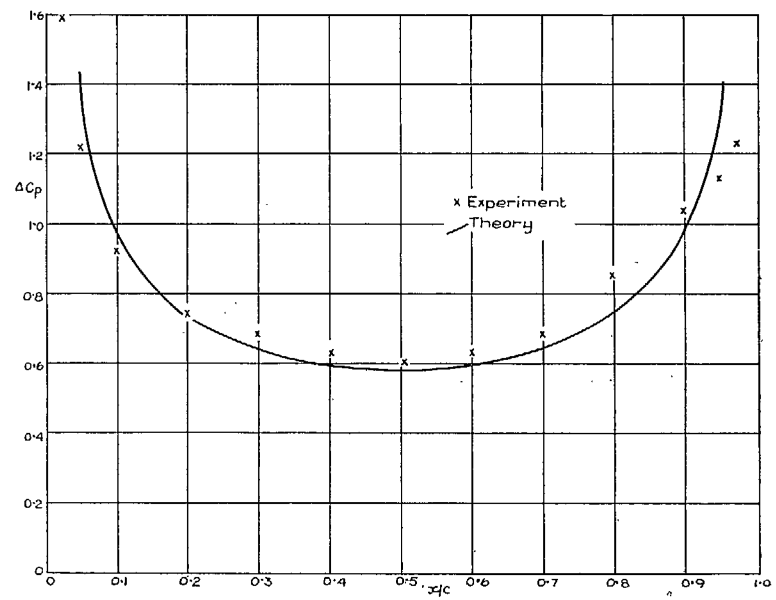

FIG. 6d. $M_\infty = 0$, $h/c = 0{\cdot}08$

FIGS. 6a to 6d. Load distributions due to trailing-edge spoilers.

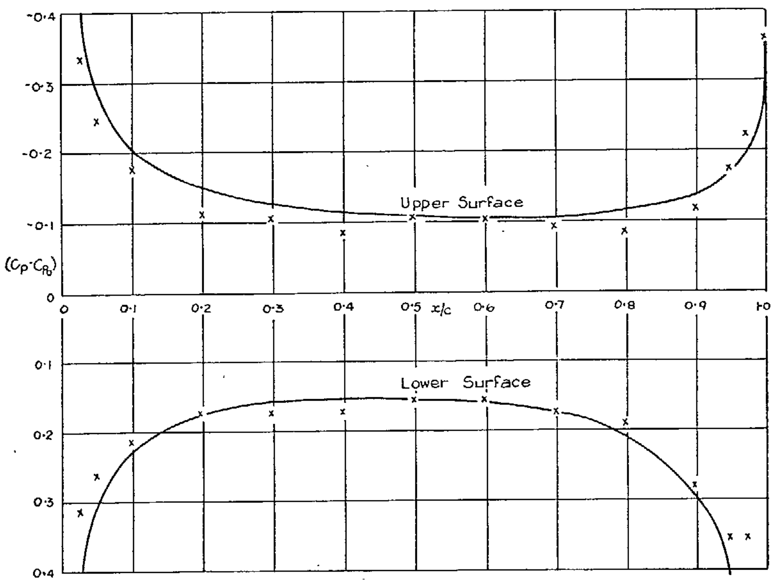

FIG. 7a. $M_\infty = 0 \cdot 40$, $h/c = 0 \cdot 019$.

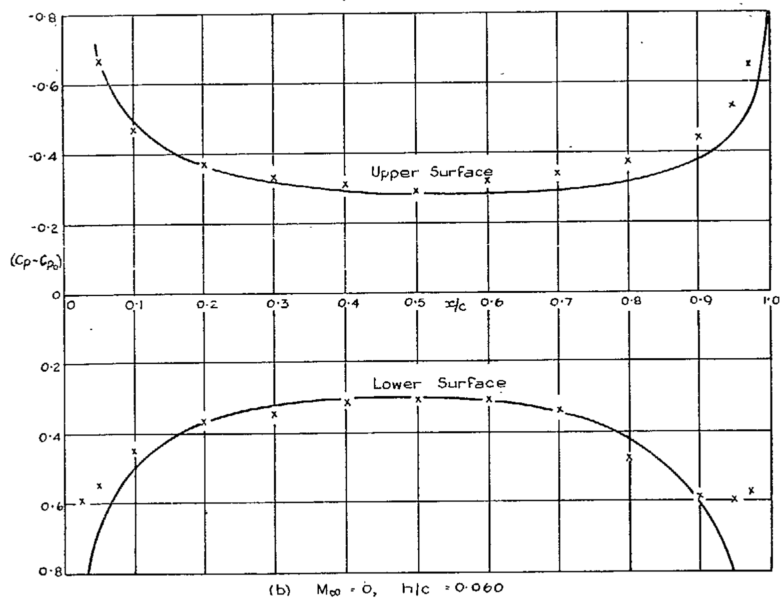

FIG. 7b. $M_\infty = 0$, $h/c = 0 \cdot 060$.

FIGS. 7a and 7b. Pressure distributions due to trailing-edge spoilers. (Spoiler on lower surface.)

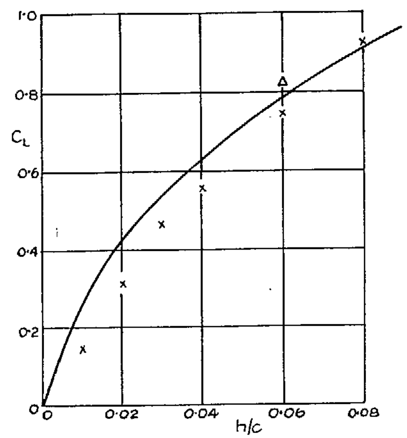

FIG. 8a.

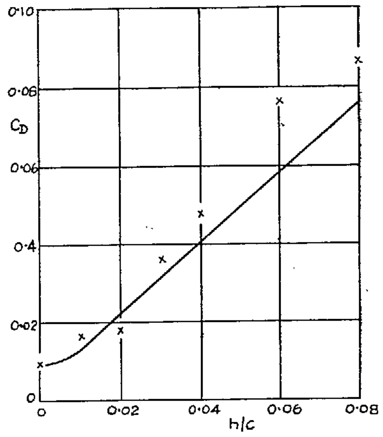

FIG. 8b.

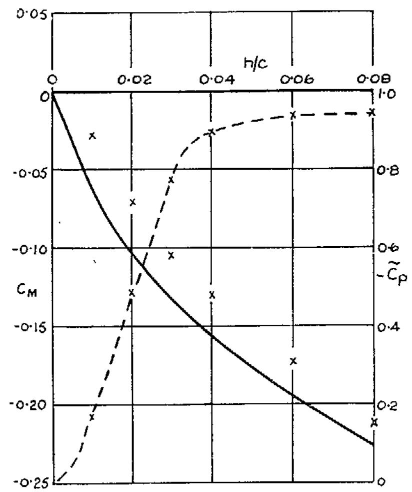

FIG. 8c.

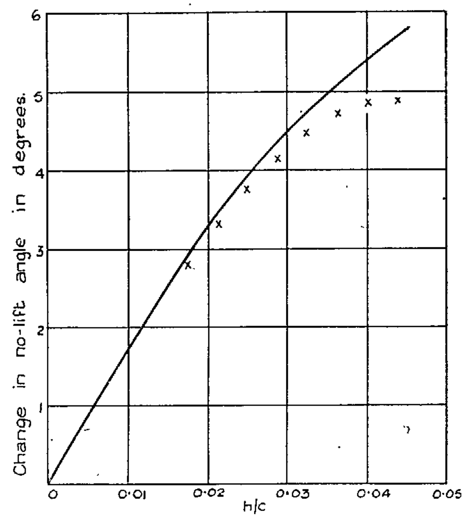

FIG. 8d.

FIGS. 8a to 8d. Forces and moments due to trailing-edge spoilers.

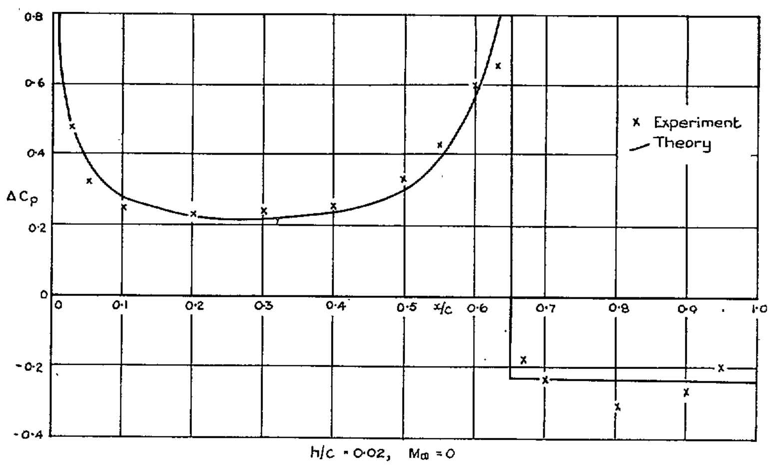

FIG. 9. Load distribution due to a spoiler at $x = 0 \cdot 65c$.

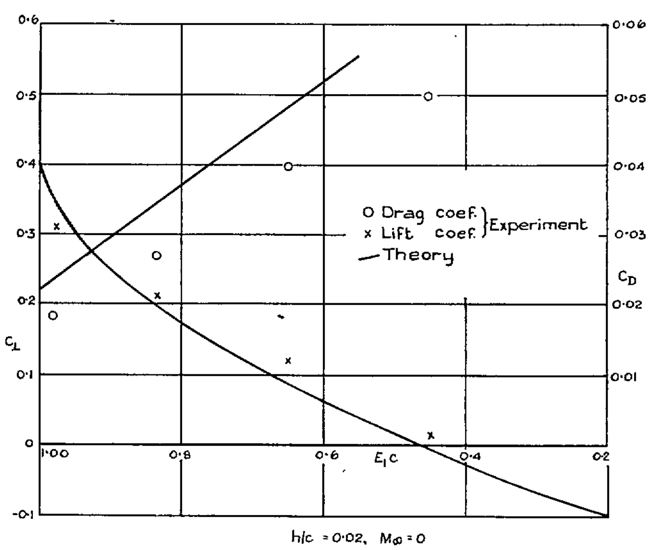

FIG. 10. Variation of C_L and C_D with spoiler position.

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